Computer algebra independent integration tests

4-Trig-functions/4.3-Tangent/4.3.1.2-d-sec-^m-a+b-tan-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [700]. This is test number [101].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (700)	% 0. (0)
Mathematica	% 100. (700)	% 0. (0)
Maple	% 82.86 (580)	% 17.14 (120)
Maxima	% 51.71 (362)	% 48.29 (338)
Fricas	% 64.57 (452)	% 35.43 (248)
Sympy	% 14.86 (104)	% 85.14 (596)
Giac	% 35.71 (250)	% 64.29 (450)

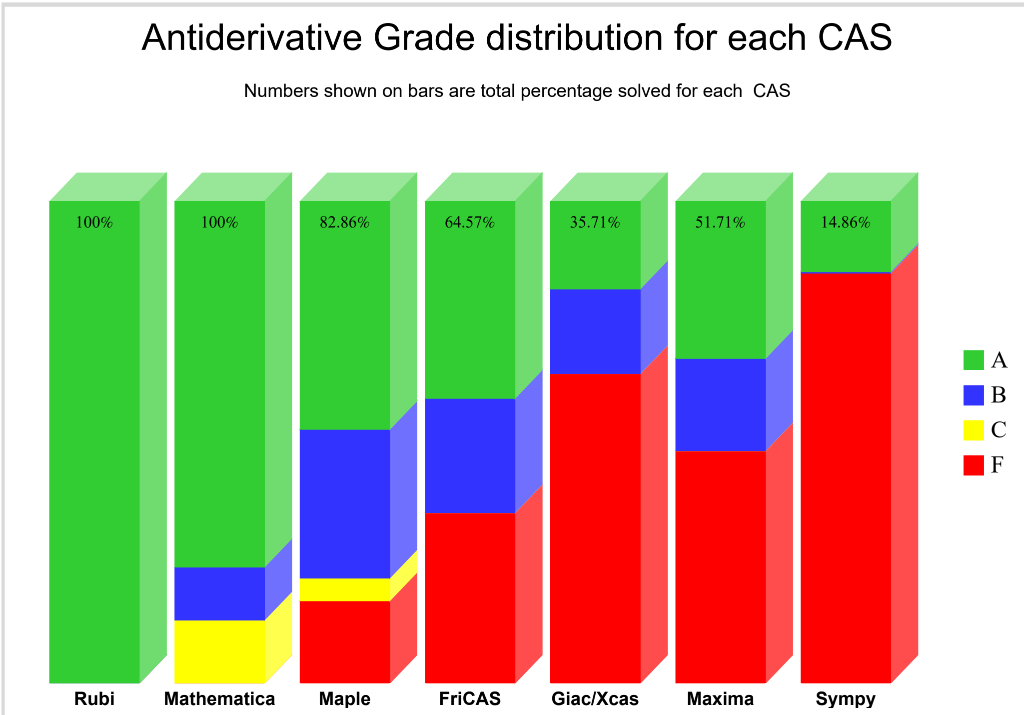
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

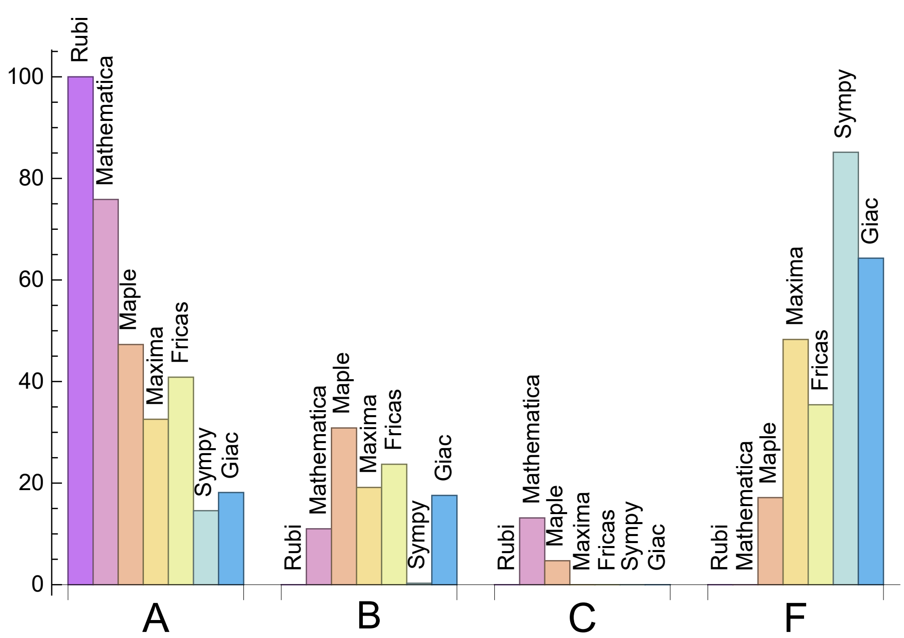
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	75.86	11.	13.14	0.
Maple	47.29	30.86	4.71	17.14
Maxima	32.57	19.14	0.	48.29
Fricas	40.86	23.71	0.	35.43
Sympy	14.57	0.29	0.	85.14
Giac	18.14	17.57	0.	64.29

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	144.93	1.	110.	1.
Mathematica	3.43	498.7	2.07	118.	1.01
Maple	0.54	1815.19	5.2	189.5	1.63
Maxima	2.16	515.18	3.35	165.5	1.7
Fricas	2.04	486.64	4.19	339.	3.74
Sympy	3.63	182.87	1.82	186.	1.54
Giac	1.61	502.64	5.33	202.5	2.33

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {645}

Mathematica {92, 176, 221, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 626, 630, 632, 633, 634, 635, 636, 637, 638, 639, 641, 642, 643, 644, 645, 651, 652, 653, 654, 659, 688, 689, 690, 691, 692, 693, 694, 696, 697, 698, 699, 700}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

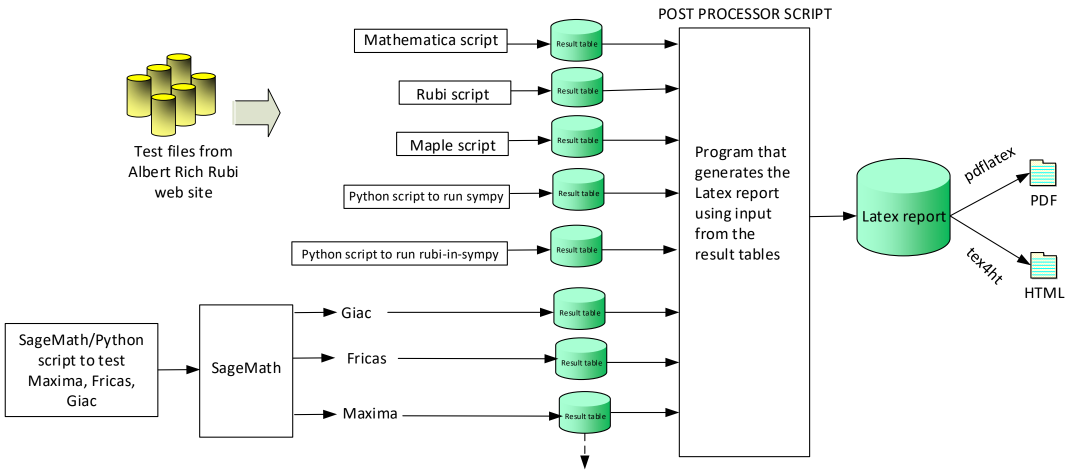
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 32, 33, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 81, 85, 86, 87, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 124, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 151, 153, 155, 156, 157, 158, 161, 162, 163, 164, 165, 169, 170, 171, 173, 174, 175, 178, 179, 180, 181, 182, 183, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228, 230, 232, 234, 236, 238, 240, 242, 244, 246, 248, 250, 252, 254, 256, 258, 260, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 500, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 540, 541, 542, 543, 544, 545, 546, 548, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 566, 567, 568, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 602, 624, 625, 626, 627, 628, 629, 630, 631, 640, 646, 647, 648, 649, 650, 655, 657, 661, 663, 665, 667, 669, 671, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695 }

B grade: { 22, 23, 29, 30, 31, 39, 41, 47, 54, 55, 61, 62, 64, 67, 77, 78, 79, 80, 82, 83, 84, 88, 92, 93, 94, 116, 122, 123, 125, 133, 149, 150, 152, 154, 159, 160, 166, 167, 168, 172, 176, 177, 267, 278, 296, 309, 322, 329, 466, 467, 468, 469, 470, 497, 499, 501, 502, 503, 520, 522, 534, 535, 536, 537, 538, 539, 549, 569, 570, 571, 573, 601, 603, 604, 635, 668, 670 }

C grade: { 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, 243, 245, 247, 249, 251, 253, 255, 257, 259, 261, 547, 560, 561, 572, 574, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 632, 633, 634, 636, 637, 638, 639, 641, 642, 643, 644, 645, 651, 652, 653, 654, 656, 658, 659, 660, 662, 664, 666, 672, 696, 697, 698, 699, 700 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 40, 41, 44, 46, 47, 49, 50, 51, 53, 54, 55, 63, 71, 72, 81, 92, 99, 100, 101, 102, 103, 105, 106, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 148, 149, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 180, 181, 182, 183, 184, 186, 190, 192, 194, 196, 198, 200, 201, 202, 204, 206, 208, 210, 212, 213, 214, 216, 218, 220, 222, 224, 226, 230, 232, 234, 236, 238, 240, 242, 244, 246, 248, 250, 252, 254, 256, 258, 260, 262, 279, 280, 281, 282, 286, 287, 288, 289, 293, 294, 296, 300, 301, 302, 303, 306, 309, 313, 314, 315, 319, 322, 326, 327, 328, 333, 334, 335, 336, 339, 340, 341, 342, 347, 348, 349, 350, 352, 353, 354, 355, 356, 361, 362, 363, 364, 365, 366, 368, 369, 370, 377, 378, 379, 380, 381, 382, 384, 385, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 413, 414, 415, 416, 417, 419, 420, 421, 422, 426, 427, 428, 429, 433, 434, 435, 436, 467, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536, 540, 541, 542, 543, 544, 545, 546, 551, 552, 553, 554, 555, 556, 557, 558, 562, 563, 564, 565, 566, 567, 568, 569, 575, 576, 577, 648, 655, 657, 658, 659, 665, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 685, 686, 687 }

B grade: { 22, 24, 36, 37, 38, 39, 42, 43, 45, 48, 52, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 104, 107, 108, 109, 112, 113, 122, 123, 124, 128, 129, 140, 141, 147, 150, 158, 176, 179, 185, 187, 188, 189, 191, 193, 195, 197, 199, 203, 205, 207, 209, 211, 215, 217, 219, 221, 223, 225, 227, 228, 229, 231, 233, 235, 237, 239, 241, 243, 245, 247, 249, 251, 253, 255, 257, 259, 261, 283, 284, 285, 290, 291, 292, 295, 297, 298, 299, 304, 305, 307, 308, 310, 311, 312, 316, 317, 318, 320, 321, 323, 324, 325, 329, 330, 331, 332, 337, 338, 343, 344, 345, 346, 351, 357, 358, 359, 360, 367, 371, 372, 373, 374, 375, 376, 383, 386, 387, 388, 389, 390, 391, 392, 404, 412, 418, 423, 424, 425, 430, 431, 432, 520, 534, 537, 538, 539, 547, 548, 549, 550, 559, 560, 561, 570, 571, 572, 573, 574, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 656, 660, 661, 662, 663, 664, 666, 684 }

C grade: { 465, 466, 483, 484, 485, 486, 487, 504, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602 }

F grade: { 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 506, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 62, 63, 64, 65, 68, 69, 70, 71, 72, 75, 76, 80, 81, 82, 83, 84, 89, 90, 99, 100, 101, 102, 103, 110, 114, 115, 116, 117, 118, 126, 130, 131, 132, 134, 135, 136, 143, 144, 148, 149, 151, 152, 154, 160, 161, 162, 166, 167, 168, 172, 178, 179, 180, 181, 182, 279, 280, 281, 282, 293, 294, 295, 296, 306, 307, 308, 309, 319, 320, 321, 322, 335, 336, 347, 348, 349, 350, 362, 363, 364, 365, 366, 378, 379, 380, 381, 382, 397, 398, 399, 405, 406, 407, 413, 414, 415, 417, 419, 420, 421, 422, 426, 427, 428, 429, 433, 434, 435, 436, 483, 484, 485, 486, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 554, 555, 556, 557, 558, 566, 567, 568, 569, 673, 674, 675, 681, 682, 683, 685

B grade: { 42, 48, 56, 61, 66, 67, 73, 74, 77, 78, 79, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 107, 108, 109, 122, 123, 124, 125, 133, 140, 141, 142, 150, 153, 158, 159, 169, 170, 171, 176, 177, 288, 290, 291, 292, 300, 301, 303, 304, 315, 316, 317, 327, 328, 329, 330, 333, 334, 340, 341, 342, 343, 345, 346, 354, 355, 356, 357, 358, 360, 361, 369, 370, 371, 372, 373, 374, 376, 377, 385, 386, 387, 388, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 404, 408, 409, 410, 411, 412, 416, 418, 423, 424, 425, 430, 431, 432, 443, 444, 445, 446, 447, 448, 449, 487, 491, 493, 495, 504, 505, 506, 559, 570, 571, 676, 677, 678, 679, 680, 684, 686, 687 }

C grade: { }

F grade: { 104, 105, 106, 111, 112, 113, 119, 120, 121, 127, 128, 129, 137, 138, 139, 145, 146, 147, 155, 156, 157, 163, 164, 165, 173, 174, 175, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 283, 284, 285, 286, 287, 289, 297, 298, 299, 302, 305, 310, 311, 312, 313, 314, 318, 323, 324, 325, 326, 331, 332, 337, 338, 339, 344, 351, 352, 353, 359, 367, 368, 375, 383, 384, 389, 391, 437, 438, 439, 440, 441, 442, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 549, 550, 551, 552, 553, 560, 561, 562, 563, 564, 565, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

2.1.5 FriCAS

A grade: { 6, 7, 8, 9, 10, 15, 16, 17, 18, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 47, 48, 49, 50, 51, 55, 56, 57, 58, 63, 64, 65, 66, 68, 69, 71, 72, 73, 74, 75, 76, 81, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 117, 118, 119, 120, 121, 125, 126, 127, 128, 129, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 279, 285, 286, 287, 288, 289, 292, 299, 300, 301, 302, 303, 313, 314, 315, 318, 326, 327, 328, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 347, 348, 349, 352, 353, 354, 355, 361, 362, 363, 364, 365, 368, 369, 377, 378, 379, 380, 381, 383, 384, 392, 393, 394, 395, 397, 398, 399, 401, 402, 403, 405, 406, 407, 408, 409, 410, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 426, 427, 428, 429, 431, 433, 434, 435, 436, 443, 444, 445, 446, 447, 448, 449, 483, 484, 485, 486, 487, 491, 493, 505, 506, 507, 508, 509, 510, 511, 513, 514, 515, 516, 517, 518, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 553, 558, 559, 564, 565, 647, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 685, 686, 687 }

B grade: { 1, 2, 3, 4, 5, 11, 12, 13, 14, 19, 20, 21, 22, 28, 29, 30, 36, 37, 38, 39, 45, 46, 52, 53, 54, 59, 60, 61, 62, 67, 70, 77, 78, 79, 80, 82, 88, 107, 108, 109, 114, 115, 116, 122, 123, 124, 130, 131, 132, 133, 134, 140, 141, 148, 149, 150, 166, 172, 280, 281, 282, 283, 284, 290, 291, 293, 294, 295, 296, 297, 298, 304, 305, 306, 307, 308, 309, 310, 311, 312, 316, 317, 319, 320, 321, 322, 323, 324, 325, 329, 330, 337, 343, 344, 345, 346, 350, 351, 356, 357, 358, 359, 360, 366, 367, 370, 371, 372, 373, 374, 375, 376, 382, 385, 386, 387, 388, 389, 390, 391, 396, 400, 404, 411, 412, 418, 424, 425, 430, 432, 465, 466, 467, 495, 504, 512, 520, 534, 545, 546, 550, 551, 552, 554, 555, 556, 557, 560, 561, 562, 563, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 646, 648, 684 }

C grade: { }

F grade: { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 437, 438, 439, 440, 441, 442, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 63, 64, 65, 66, 68, 69, 71, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 104, 105, 106, 111, 112, 113, 119, 120, 121, 127, 128, 129, 137, 138, 139, 145, 146, 147, 155, 156, 157, 163, 164, 165, 173, 174, 175, 183, 184, 507, 509, 511, 512 }

B grade: { 67, 88 }

C grade: { }

F grade: { 11, 12, 13, 19, 20, 21, 22, 28, 29, 30, 36, 37, 38, 39, 45, 46, 52, 53, 59, 60, 61, 62, 70, 77, 78, 79, 80, 99, 100, 101, 102, 103, 107, 108, 109, 110, 114, 115, 116, 117, 118, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 140, 141, 142, 143, 144, 148, 149, 150, 151, 152, 153, 154, 158, 159, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 176, 177, 178, 179, 180, 181, 182, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 508, 510, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 19, 20, 21, 23, 24, 26, 30, 31, 36, 37, 38, 41, 45, 46, 52, 53, 59, 60, 64, 70, 99, 100, 101, 102, 105, 106, 107, 108, 110, 111, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 168, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 336, 507, 509, 511, 517, 518, 519, 531, 532, 533, 544, 545, 546, 550, 551, 552, 553, 554, 555, 556, 557, 558, 561, 562, 563, 564, 565, 566, 567, 568, 569, 576, 648 }

B grade: { 11, 12, 13, 14, 15, 16, 17, 18, 22, 25, 27, 28, 29, 32, 33, 34, 35, 39, 40, 42, 43, 44, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 61, 62, 63, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 103, 104, 109, 112, 113, 117, 129, 133, 134, 135, 136, 150, 151, 152, 154, 166, 167, 169, 170, 171, 172, 179, 508, 510, 512, 513, 514, 516, 520, 521, 522, 523, 524, 525, 526, 527, 534, 535, 536, 537, 538, 539, 540, 547, 548, 549, 559, 560, 570, 571, 572, 573, 574, 575, 577 }

C grade: { }

F grade: { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 515, 528, 529, 530, 541, 542, 543, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	79	69	154	629	83	154
normalized size	1	1.	0.84	0.73	1.64	6.69	0.88	1.64
time (sec)	N/A	0.188	0.361	0.093	1.101	1.301	123.932	1.175

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	63	59	124	486	68	124
normalized size	1	1.	0.84	0.79	1.65	6.48	0.91	1.65
time (sec)	N/A	0.041	0.106	0.087	1.038	1.27	53.126	1.153

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	49	95	363	60	95
normalized size	1	1.	0.89	0.79	1.53	5.85	0.97	1.53
time (sec)	N/A	0.037	0.103	0.082	1.097	1.118	18.457	1.139

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	39	65	239	48	65
normalized size	1	1.	0.93	0.85	1.41	5.2	1.04	1.41
time (sec)	N/A	0.034	0.042	0.08	1.062	1.105	8.665	1.129

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	30	30	26	28	123	37	35
normalized size	1	1.11	1.11	0.96	1.04	4.56	1.37	1.3
time (sec)	N/A	0.031	0.013	0.038	1.106	1.103	3.563	1.127

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	23	23	50	24	24
normalized size	1	1.	1.	1.21	1.21	2.63	1.26	1.26
time (sec)	N/A	0.007	0.006	0.001	1.1	1.209	0.493	1.126

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	48	42	51	58	41	31
normalized size	1	1.	1.07	0.93	1.13	1.29	0.91	0.69
time (sec)	N/A	0.031	0.056	0.044	1.67	1.11	0.228	1.121

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	46	53	82	165	138	139
normalized size	1	1.	0.69	0.79	1.22	2.46	2.06	2.07
time (sec)	N/A	0.04	0.042	0.084	1.662	1.137	0.461	1.141

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	56	63	111	252	212	171
normalized size	1	1.	0.63	0.71	1.25	2.83	2.38	1.92
time (sec)	N/A	0.052	0.058	0.084	1.675	1.07	0.783	1.14

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	68	73	139	342	280	204
normalized size	1	1.	0.61	0.66	1.25	3.08	2.52	1.84
time (sec)	N/A	0.067	0.111	0.094	1.662	1.124	1.04	1.161

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	61	95	143	1170	0	247
normalized size	1	1.	0.62	0.97	1.46	11.94	0.	2.52
time (sec)	N/A	0.061	0.293	0.085	1.104	1.09	0.	1.2

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	75	116	836	0	190
normalized size	1	1.	0.92	0.99	1.53	11.	0.	2.5
time (sec)	N/A	0.048	0.152	0.085	1.116	1.251	0.	1.193

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	55	82	520	0	134
normalized size	1	1.	1.	1.02	1.52	9.63	0.	2.48
time (sec)	N/A	0.035	0.013	0.081	1.111	1.119	0.	1.189

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	36	43	220	41	73
normalized size	1	1.	1.	1.33	1.59	8.15	1.52	2.7
time (sec)	N/A	0.016	0.008	0.014	1.066	1.156	5.997	1.18

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	51	24	30	32	26	113
normalized size	1	1.	1.96	0.92	1.15	1.23	1.	4.35
time (sec)	N/A	0.021	0.02	0.036	1.11	1.139	0.232	1.158

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	49	119	107	265
normalized size	1	1.	1.	0.8	1.07	2.59	2.33	5.76
time (sec)	N/A	0.032	0.01	0.078	1.122	1.1	0.559	1.144

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	47	66	208	185	297
normalized size	1	1.	1.	0.76	1.06	3.35	2.98	4.79
time (sec)	N/A	0.035	0.015	0.078	1.142	1.02	1.002	1.202

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	57	78	297	255	329
normalized size	1	1.	1.	0.75	1.03	3.91	3.36	4.33
time (sec)	N/A	0.038	0.042	0.08	1.115	1.071	1.247	1.215

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	99	141	146	590	0	146
normalized size	1	1.	0.91	1.29	1.34	5.41	0.	1.34
time (sec)	N/A	0.066	1.244	0.058	1.105	1.191	0.	1.203

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	90	113	128	464	0	128
normalized size	1	1.	1.1	1.38	1.56	5.66	0.	1.56
time (sec)	N/A	0.056	0.881	0.056	1.095	1.067	0.	1.183

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	77	85	76	329	0	76
normalized size	1	1.	1.4	1.55	1.38	5.98	0.	1.38
time (sec)	N/A	0.043	0.374	0.055	1.124	1.18	0.	1.167

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	68	51	28	212	0	57
normalized size	1	1.	2.52	1.89	1.04	7.85	0.	2.11
time (sec)	N/A	0.038	0.328	0.051	1.114	1.124	0.	1.184

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	100	51	55	151	60	88
normalized size	1	1.	2.63	1.34	1.45	3.97	1.58	2.32
time (sec)	N/A	0.017	0.633	0.005	1.665	1.06	0.938	1.135

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	31	73	43	46	37	23
normalized size	1	1.	1.24	2.92	1.72	1.84	1.48	0.92
time (sec)	N/A	0.037	0.054	0.047	1.647	1.355	0.298	1.192

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	86	100	90	105	88	347
normalized size	1	1.	1.37	1.59	1.43	1.67	1.4	5.51
time (sec)	N/A	0.061	0.44	0.055	1.643	1.146	0.45	1.192

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	116	121	124	217	187	228
normalized size	1	1.	0.99	1.03	1.06	1.85	1.6	1.95
time (sec)	N/A	0.082	0.45	0.056	1.676	1.23	0.583	1.258

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	138	141	155	309	272	462
normalized size	1	1.	0.81	0.82	0.91	1.81	1.59	2.7
time (sec)	N/A	0.107	0.489	0.059	1.656	1.11	1.004	1.256

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	159	169	244	1062	0	323
normalized size	1	1.	1.35	1.43	2.07	9.	0.	2.74
time (sec)	N/A	0.087	1.011	0.053	1.084	1.251	0.	1.232

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	215	123	176	711	0	236
normalized size	1	1.	2.29	1.31	1.87	7.56	0.	2.51
time (sec)	N/A	0.077	1.051	0.052	1.064	1.272	0.	1.213

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	146	79	112	397	0	147
normalized size	1	1.	2.15	1.16	1.65	5.84	0.	2.16
time (sec)	N/A	0.04	0.801	0.021	1.121	1.268	0.	1.199

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	180	53	82	124	68	76
normalized size	1	1.	3.91	1.15	1.78	2.7	1.48	1.65
time (sec)	N/A	0.034	0.214	0.044	1.09	1.209	0.526	1.199

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	54	70	84	76	717
normalized size	1	1.	0.98	1.06	1.37	1.65	1.49	14.06
time (sec)	N/A	0.042	0.178	0.052	1.066	1.196	0.432	1.254

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	72	91	107	169	155	828
normalized size	1	1.	1.04	1.32	1.55	2.45	2.25	12.
time (sec)	N/A	0.049	0.401	0.057	1.076	1.103	0.981	1.285

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	111	111	132	269	240	865
normalized size	1	1.	1.28	1.28	1.52	3.09	2.76	9.94
time (sec)	N/A	0.052	0.415	0.055	1.142	1.019	0.976	1.346

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	133	131	161	365	316	903
normalized size	1	1.	1.27	1.25	1.53	3.48	3.01	8.6
time (sec)	N/A	0.056	1.123	0.09	1.11	1.175	1.354	1.371

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	117	220	146	693	0	146
normalized size	1	1.	1.07	2.02	1.34	6.36	0.	1.34
time (sec)	N/A	0.064	1.791	0.065	1.094	1.231	0.	1.254

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	106	174	146	547	0	146
normalized size	1	1.	1.29	2.12	1.78	6.67	0.	1.78
time (sec)	N/A	0.056	1.206	0.061	1.133	1.04	0.	1.255

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	97	128	111	419	0	111
normalized size	1	1.	1.76	2.33	2.02	7.62	0.	2.02
time (sec)	N/A	0.043	0.888	0.06	1.1	1.122	0.	1.256

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	84	73	28	284	0	76
normalized size	1	1.	3.11	2.7	1.04	10.52	0.	2.81
time (sec)	N/A	0.037	0.489	0.057	1.118	1.191	0.	1.225

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	119	68	103	267	100	158
normalized size	1	1.	1.89	1.08	1.63	4.24	1.59	2.51
time (sec)	N/A	0.032	0.784	0.004	1.667	1.233	1.357	1.116

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	99	87	84	93	53	49
normalized size	1	1.	2.02	1.78	1.71	1.9	1.08	1.
time (sec)	N/A	0.049	0.277	0.051	1.691	1.181	0.709	1.252

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	50	114	77	89	82	182
normalized size	1	1.	1.85	4.22	2.85	3.3	3.04	6.74
time (sec)	N/A	0.038	0.251	0.06	1.701	1.163	0.482	1.291

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	109	156	142	151	133	617
normalized size	1	1.	1.21	1.73	1.58	1.68	1.48	6.86
time (sec)	N/A	0.067	0.539	0.062	1.696	1.212	0.732	1.352

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	131	176	173	271	228	694
normalized size	1	1.	0.91	1.22	1.2	1.88	1.58	4.82
time (sec)	N/A	0.091	0.582	0.062	1.64	1.253	0.825	1.389

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	102	236	209	891	0	258
normalized size	1	1.	0.8	1.86	1.65	7.02	0.	2.03
time (sec)	N/A	0.12	0.652	0.095	1.142	1.237	0.	1.283

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	93	167	147	554	0	171
normalized size	1	1.	0.94	1.69	1.48	5.6	0.	1.73
time (sec)	N/A	0.066	0.497	0.026	1.126	1.227	0.	1.241

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	123	101	111	281	107	316
normalized size	1	1.	2.02	1.66	1.82	4.61	1.75	5.18
time (sec)	N/A	0.05	0.649	0.044	1.226	1.193	0.81	1.313

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	76	101	46	37	1216
normalized size	1	1.	0.97	2.38	3.16	1.44	1.16	38.
time (sec)	N/A	0.037	0.074	0.052	1.035	1.143	0.434	1.424

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	55	126	142	131	117	1254
normalized size	1	1.	0.62	1.43	1.61	1.49	1.33	14.25
time (sec)	N/A	0.071	0.447	0.06	1.208	1.117	0.727	1.506

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	77	146	166	219	192	628
normalized size	1	1.	0.73	1.38	1.57	2.07	1.81	5.92
time (sec)	N/A	0.076	0.573	0.062	1.157	1.078	1.024	1.52

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	116	166	196	319	277	1403
normalized size	1	1.	0.94	1.34	1.58	2.57	2.23	11.31
time (sec)	N/A	0.084	0.652	0.063	1.205	1.183	1.746	1.628

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	171	324	332	1064	0	323
normalized size	1	1.	1.05	1.99	2.04	6.53	0.	1.98
time (sec)	N/A	0.162	1.784	0.064	1.103	1.215	0.	1.353

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	237	231	243	717	0	236
normalized size	1	1.	1.78	1.74	1.83	5.39	0.	1.77
time (sec)	N/A	0.096	1.215	0.028	1.187	1.196	0.	1.338

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	906	141	185	444	153	502
normalized size	1	1.	9.34	1.45	1.91	4.58	1.58	5.18
time (sec)	N/A	0.074	6.432	0.052	1.13	1.203	1.344	1.417

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	246	130	163	176	110	1754
normalized size	1	1.	3.15	1.67	2.09	2.26	1.41	22.49
time (sec)	N/A	0.076	0.486	0.052	1.082	1.191	0.735	1.656

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	139	159	93	82	1235
normalized size	1	1.	0.76	2.11	2.41	1.41	1.24	18.71
time (sec)	N/A	0.074	0.356	0.063	1.145	1.002	0.652	1.632

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	73	203	201	174	158	1791
normalized size	1	1.	0.72	1.99	1.97	1.71	1.55	17.56
time (sec)	N/A	0.089	0.683	0.065	1.103	1.249	1.068	1.811

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	111	233	244	267	230	1902
normalized size	1	1.	0.92	1.94	2.03	2.22	1.92	15.85
time (sec)	N/A	0.101	0.707	0.069	1.061	1.092	1.264	1.887

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	167	377	216	883	0	216
normalized size	1	1.	1.53	3.46	1.98	8.1	0.	1.98
time (sec)	N/A	0.073	3.589	0.083	1.125	1.196	0.	1.438

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	154	295	146	740	0	146
normalized size	1	1.	1.88	3.6	1.78	9.02	0.	1.78
time (sec)	N/A	0.057	2.219	0.079	1.074	1.145	0.	1.49

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	143	213	146	591	0	146
normalized size	1	1.	2.6	3.87	2.65	10.75	0.	2.65
time (sec)	N/A	0.043	1.697	0.078	1.134	1.093	0.	1.537

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	134	115	28	463	0	111
normalized size	1	1.	4.96	4.26	1.04	17.15	0.	4.11
time (sec)	N/A	0.036	1.454	0.076	1.13	1.109	0.	1.583

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	228	101	223	527	185	300
normalized size	1	1.	1.95	0.86	1.91	4.5	1.58	2.56
time (sec)	N/A	0.066	2.563	0.005	1.851	1.061	3.309	1.151

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	649	175	116	352	128	196
normalized size	1	1.	7.82	2.11	1.4	4.24	1.54	2.36
time (sec)	N/A	0.059	6.552	0.107	1.702	1.09	1.387	1.439

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	110	146	119	142	82	608
normalized size	1	1.	1.51	2.	1.63	1.95	1.12	8.33
time (sec)	N/A	0.055	0.625	0.069	1.805	1.159	0.936	1.55

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	50	231	126	93	82	252
normalized size	1	1.	0.91	4.2	2.29	1.69	1.49	4.58
time (sec)	N/A	0.049	0.485	0.084	1.67	1.121	0.605	1.588

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	73	301	139	171	163	360
normalized size	1	1.	2.7	11.15	5.15	6.33	6.04	13.33
time (sec)	N/A	0.038	0.932	0.091	1.641	1.16	0.831	1.618

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	137	331	221	248	211	1157
normalized size	1	1.	0.95	2.3	1.53	1.72	1.47	8.03
time (sec)	N/A	0.086	1.411	0.135	1.731	1.088	1.051	1.808

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	159	361	252	371	303	1234
normalized size	1	1.	0.8	1.82	1.27	1.87	1.53	6.23
time (sec)	N/A	0.114	2.642	0.133	1.677	1.316	1.325	1.869

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	115	329	290	891	0	258
normalized size	1	1.	0.69	1.97	1.74	5.34	0.	1.54
time (sec)	N/A	0.126	1.011	0.072	1.117	1.136	0.	1.469

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	151	214	234	603	196	689
normalized size	1	1.	1.16	1.65	1.8	4.64	1.51	5.3
time (sec)	N/A	0.104	1.547	0.064	1.133	1.189	2.246	1.704

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	130	179	208	332	150	2272
normalized size	1	1.	1.33	1.83	2.12	3.39	1.53	23.18
time (sec)	N/A	0.089	1.637	0.062	1.139	1.282	0.978	2.057

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	170	205	46	37	2253
normalized size	1	1.	0.97	5.31	6.41	1.44	1.16	70.41
time (sec)	N/A	0.036	0.155	0.074	1.194	1.434	0.471	1.956

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	55	257	252	139	122	2291
normalized size	1	1.	0.54	2.54	2.5	1.38	1.21	22.68
time (sec)	N/A	0.114	0.772	0.091	1.16	1.399	0.757	2.107

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	94	287	293	224	194	2329
normalized size	1	1.	0.67	2.04	2.08	1.59	1.38	16.52
time (sec)	N/A	0.122	0.834	0.09	1.191	1.581	1.115	2.181

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	118	317	332	321	267	2439
normalized size	1	1.	0.74	1.99	2.09	2.02	1.68	15.34
time (sec)	N/A	0.126	1.199	0.121	1.136	1.559	1.453	2.307

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	245	611	251	1195	0	251
normalized size	1	1.	2.25	5.61	2.3	10.96	0.	2.3
time (sec)	N/A	0.081	8.613	0.1	1.476	1.442	0.	1.815

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	234	475	234	1041	0	234
normalized size	1	1.	2.85	5.79	2.85	12.7	0.	2.85
time (sec)	N/A	0.071	6.223	0.097	1.119	1.396	0.	1.765

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	223	339	181	890	0	181
normalized size	1	1.	4.05	6.16	3.29	16.18	0.	3.29
time (sec)	N/A	0.045	4.205	0.095	1.343	1.541	0.	1.817

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	212	180	28	741	0	162
normalized size	1	1.	7.85	6.67	1.04	27.44	0.	6.
time (sec)	N/A	0.038	2.864	0.089	1.164	1.291	0.	1.793

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	383	150	163	976	313	510
normalized size	1	1.	1.92	0.75	0.82	4.88	1.56	2.55
time (sec)	N/A	0.139	4.13	0.006	1.628	1.458	13.339	1.212

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	321	406	167	760	252	408
normalized size	1	1.	2.41	3.05	1.26	5.71	1.89	3.07
time (sec)	N/A	0.082	6.051	0.156	1.516	1.543	5.693	1.913

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	566	306	184	536	201	1060
normalized size	1	1.	4.56	2.47	1.48	4.32	1.62	8.55
time (sec)	N/A	0.077	2.231	0.075	1.571	1.537	2.868	2.196

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	414	319	197	323	144	1079
normalized size	1	1.	3.63	2.8	1.73	2.83	1.26	9.46
time (sec)	N/A	0.071	2.094	0.07	1.638	1.494	1.62	2.334

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	451	185	46	37	514
normalized size	1	1.	0.72	10.49	4.3	1.07	0.86	11.95
time (sec)	N/A	0.042	0.295	0.077	2.414	1.425	0.949	2.315

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	55	588	205	140	122	552
normalized size	1	1.	0.69	7.35	2.56	1.75	1.52	6.9
time (sec)	N/A	0.056	1.151	0.092	1.598	1.656	1.325	2.423

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	77	639	219	227	199	590
normalized size	1	1.	1.4	11.62	3.98	4.13	3.62	10.73
time (sec)	N/A	0.048	1.17	0.092	1.525	1.684	1.613	2.546

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	116	689	232	308	280	628
normalized size	1	1.	4.3	25.52	8.59	11.41	10.37	23.26
time (sec)	N/A	0.038	1.635	0.136	1.734	1.911	2.025	2.662

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	166	739	332	405	325	1967
normalized size	1	1.	0.74	3.28	1.48	1.8	1.44	8.74
time (sec)	N/A	0.115	5.351	0.141	1.816	2.601	2.35	3.079

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	188	789	363	528	415	2044
normalized size	1	1.	0.67	2.83	1.3	1.89	1.49	7.33
time (sec)	N/A	0.156	6.716	0.13	1.769	2.825	2.871	3.354

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	205	464	535	1137	326	1247
normalized size	1	1.	0.87	1.97	2.28	4.84	1.39	5.31
time (sec)	N/A	0.204	2.641	0.104	1.189	1.799	12.387	2.3

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	1540	356	475	819	282	3827
normalized size	1	1.	7.51	1.74	2.32	4.	1.38	18.67
time (sec)	N/A	0.193	6.948	0.076	1.193	1.866	4.806	2.998

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	1162	322	440	537	236	3846
normalized size	1	1.	6.72	1.86	2.54	3.1	1.36	22.23
time (sec)	N/A	0.167	6.886	0.074	1.122	1.85	1.99	3.204

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	305	385	417	271	189	3865
normalized size	1	1.	2.01	2.53	2.74	1.78	1.24	25.43
time (sec)	N/A	0.16	2.234	0.082	1.038	1.918	1.518	3.466

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	447	408	95	82	3309
normalized size	1	1.	0.76	6.77	6.18	1.44	1.24	50.14
time (sec)	N/A	0.074	0.565	0.094	1.149	1.906	1.213	3.545

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	73	567	479	190	163	3865
normalized size	1	1.	0.54	4.17	3.52	1.4	1.2	28.42
time (sec)	N/A	0.156	1.274	0.094	1.175	2.091	1.574	3.811

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	111	617	547	292	241	3903
normalized size	1	1.	0.53	2.92	2.59	1.38	1.14	18.5
time (sec)	N/A	0.254	1.273	0.125	1.175	2.244	2.038	4.355

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	133	667	612	382	314	3941
normalized size	1	1.	0.63	3.15	2.89	1.8	1.48	18.59
time (sec)	N/A	0.197	3.103	0.131	1.178	2.368	2.458	4.307

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	71	87	117	458	0	117
normalized size	1	1.	0.66	0.81	1.09	4.28	0.	1.09
time (sec)	N/A	0.07	0.331	0.063	1.056	1.696	0.	1.166

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	60	68	90	333	0	90
normalized size	1	1.	0.75	0.85	1.12	4.16	0.	1.12
time (sec)	N/A	0.062	0.258	0.061	1.134	1.87	0.	1.157

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	47	63	208	0	63
normalized size	1	1.	0.89	0.85	1.15	3.78	0.	1.15
time (sec)	N/A	0.051	0.166	0.06	1.397	1.958	0.	1.198

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	35	26	36	88	0	36
normalized size	1	1.	1.03	0.76	1.06	2.59	0.	1.06
time (sec)	N/A	0.043	0.164	0.054	1.139	1.787	0.	1.159

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	31	23	27	65	0	80
normalized size	1	1.	1.35	1.	1.17	2.83	0.	3.48
time (sec)	N/A	0.041	0.089	0.03	1.164	2.215	0.	1.184

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	45	59	0	86	61	81
normalized size	1	1.	1.36	1.79	0.	2.61	1.85	2.45
time (sec)	N/A	0.012	0.102	0.026	0.	1.918	0.215	1.138

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	78	98	0	159	153	134
normalized size	1	1.	0.95	1.2	0.	1.94	1.87	1.63
time (sec)	N/A	0.068	0.25	0.082	0.	1.913	0.495	1.139

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	109	137	0	242	221	157
normalized size	1	1.	0.81	1.02	0.	1.81	1.65	1.17
time (sec)	N/A	0.087	0.206	0.083	0.	1.944	0.791	1.161

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	60	430	390	810	0	189
normalized size	1	1.	0.71	5.12	4.64	9.64	0.	2.25
time (sec)	N/A	0.077	0.331	0.08	1.279	2.244	0.	1.245

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	50	258	251	505	0	136
normalized size	1	1.	0.83	4.3	4.18	8.42	0.	2.27
time (sec)	N/A	0.054	0.22	0.072	1.036	2.175	0.	1.192

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	34	85	112	217	0	81
normalized size	1	1.	1.1	2.74	3.61	7.	0.	2.61
time (sec)	N/A	0.044	0.129	0.063	1.054	2.105	0.	1.2

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	23	39	35	0	28
normalized size	1	1.	0.89	0.82	1.39	1.25	0.	1.
time (sec)	N/A	0.023	0.027	0.034	0.985	2.162	0.	1.153

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	50	75	0	122	128	90
normalized size	1	1.	1.06	1.6	0.	2.6	2.72	1.91
time (sec)	N/A	0.037	0.117	0.078	0.	1.968	0.487	1.134

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	72	141	0	200	197	161
normalized size	1	1.	1.07	2.1	0.	2.99	2.94	2.4
time (sec)	N/A	0.052	0.152	0.084	0.	2.033	0.856	1.154

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	94	207	0	284	265	231
normalized size	1	1.	1.11	2.44	0.	3.34	3.12	2.72
time (sec)	N/A	0.055	0.173	0.085	0.	2.031	1.328	1.156

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	90	78	104	402	0	104
normalized size	1	1.	1.1	0.95	1.27	4.9	0.	1.27
time (sec)	N/A	0.063	0.418	0.075	1.137	2.11	0.	1.17

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	77	47	63	267	0	63
normalized size	1	1.	1.4	0.85	1.15	4.85	0.	1.15
time (sec)	N/A	0.051	0.328	0.072	1.158	2.128	0.	1.153

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	68	36	47	139	0	47
normalized size	1	1.	2.52	1.33	1.74	5.15	0.	1.74
time (sec)	N/A	0.044	0.209	0.068	1.095	1.939	0.	1.189

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	71	35	43	192	0	138
normalized size	1	1.	1.87	0.92	1.13	5.05	0.	3.63
time (sec)	N/A	0.049	0.366	0.063	0.962	2.369	0.	1.202

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	32	24	28	49	0	41
normalized size	1	1.	1.23	0.92	1.08	1.88	0.	1.58
time (sec)	N/A	0.043	0.04	0.032	0.955	2.264	0.	1.192

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	68	79	0	126	119	97
normalized size	1	1.	1.11	1.3	0.	2.07	1.95	1.59
time (sec)	N/A	0.029	0.165	0.023	0.	2.193	0.34	1.118

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	95	117	0	198	190	139
normalized size	1	1.	0.83	1.03	0.	1.74	1.67	1.22
time (sec)	N/A	0.081	0.258	0.093	0.	2.246	0.68	1.161

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	120	157	0	279	260	171
normalized size	1	1.	0.73	0.95	0.	1.69	1.58	1.04
time (sec)	N/A	0.103	0.252	0.094	0.	2.298	1.171	1.144

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	294	514	568	996	0	277
normalized size	1	1.	2.37	4.15	4.58	8.03	0.	2.23
time (sec)	N/A	0.081	1.797	0.091	1.041	2.496	0.	1.217

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	215	342	398	667	0	207
normalized size	1	1.	2.15	3.42	3.98	6.67	0.	2.07
time (sec)	N/A	0.068	0.708	0.086	1.093	2.376	0.	1.246

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	146	170	225	375	0	131
normalized size	1	1.	1.97	2.3	3.04	5.07	0.	1.77
time (sec)	N/A	0.058	0.365	0.078	1.017	2.427	0.	1.191

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	184	63	158	161	0	80
normalized size	1	1.	3.83	1.31	3.29	3.35	0.	1.67
time (sec)	N/A	0.047	0.178	0.073	1.491	2.305	0.	1.263

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	38	57	61	86	0	63
normalized size	1	1.	0.58	0.88	0.94	1.32	0.	0.97
time (sec)	N/A	0.054	0.074	0.044	0.981	2.213	0.	1.156

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	68	108	0	161	165	126
normalized size	1	1.	0.96	1.52	0.	2.27	2.32	1.77
time (sec)	N/A	0.048	0.272	0.087	0.	2.24	0.723	1.151

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	95	174	0	244	233	196
normalized size	1	1.	1.07	1.96	0.	2.74	2.62	2.2
time (sec)	N/A	0.059	0.191	0.09	0.	2.324	1.227	1.149

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	117	240	0	329	301	266
normalized size	1	1.	1.09	2.24	0.	3.07	2.81	2.49
time (sec)	N/A	0.062	0.348	0.092	0.	2.339	1.726	1.162

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	117	89	117	587	0	117
normalized size	1	1.	1.07	0.82	1.07	5.39	0.	1.07
time (sec)	N/A	0.069	0.683	0.084	1.018	2.61	0.	1.194

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	106	89	117	443	0	117
normalized size	1	1.	1.29	1.09	1.43	5.4	0.	1.43
time (sec)	N/A	0.061	0.469	0.079	0.987	2.347	0.	1.202

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	97	68	90	316	0	90
normalized size	1	1.	1.76	1.24	1.64	5.75	0.	1.64
time (sec)	N/A	0.047	0.436	0.078	1.059	2.354	0.	1.186

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	84	47	63	177	0	63
normalized size	1	1.	3.11	1.74	2.33	6.56	0.	2.33
time (sec)	N/A	0.039	0.379	0.073	0.971	2.303	0.	1.21

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	113	52	61	317	0	176
normalized size	1	1.	1.95	0.9	1.05	5.47	0.	3.03
time (sec)	N/A	0.047	0.375	0.079	0.968	2.339	0.	1.235

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	88	40	89	157	0	138
normalized size	1	1.	1.76	0.8	1.78	3.14	0.	2.76
time (sec)	N/A	0.049	0.215	0.08	0.982	2.267	0.	1.201

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	42	24	28	86	0	77
normalized size	1	1.	1.56	0.89	1.04	3.19	0.	2.85
time (sec)	N/A	0.053	0.07	0.038	0.946	2.225	0.	1.161

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	93	98	0	166	156	108
normalized size	1	1.	1.06	1.11	0.	1.89	1.77	1.23
time (sec)	N/A	0.047	0.195	0.026	0.	2.227	0.592	1.112

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	115	137	0	247	226	161
normalized size	1	1.	0.82	0.97	0.	1.75	1.6	1.14
time (sec)	N/A	0.086	0.224	0.097	0.	2.258	1.042	1.177

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	137	176	0	333	294	184
normalized size	1	1.	0.7	0.9	0.	1.71	1.51	0.94
time (sec)	N/A	0.116	0.374	0.102	0.	2.414	1.338	1.188

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	113	430	460	836	0	224
normalized size	1	1.	0.95	3.61	3.87	7.03	0.	1.88
time (sec)	N/A	0.115	0.376	0.102	1.081	2.599	0.	1.233

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	63	258	290	521	0	154
normalized size	1	1.	0.68	2.77	3.12	5.6	0.	1.66
time (sec)	N/A	0.098	0.38	0.098	1.007	2.491	0.	1.249

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	108	108	444	302	0	151
normalized size	1	1.	1.66	1.66	6.83	4.65	0.	2.32
time (sec)	N/A	0.085	0.273	0.079	1.55	2.36	0.	1.224

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	57	39	49	0	49
normalized size	1	1.	1.	1.78	1.22	1.53	0.	1.53
time (sec)	N/A	0.037	0.047	0.081	0.99	2.223	0.	1.201

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	54	90	93	128	0	99
normalized size	1	1.	0.55	0.92	0.95	1.31	0.	1.01
time (sec)	N/A	0.077	0.118	0.049	0.991	2.029	0.	1.177

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	76	141	0	205	199	161
normalized size	1	1.	0.75	1.4	0.	2.03	1.97	1.59
time (sec)	N/A	0.081	0.177	0.094	0.	2.234	1.059	1.165

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	98	207	0	289	267	231
normalized size	1	1.	0.81	1.71	0.	2.39	2.21	1.91
time (sec)	N/A	0.113	0.225	0.103	0.	2.305	1.595	1.17

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	120	273	0	390	335	301
normalized size	1	1.	0.86	1.96	0.	2.81	2.41	2.17
time (sec)	N/A	0.117	0.547	0.103	0.	2.407	2.182	1.183

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	136	99	131	494	0	131
normalized size	1	1.	1.66	1.21	1.6	6.02	0.	1.6
time (sec)	N/A	0.064	0.56	0.087	0.985	2.804	0.	1.199

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	127	67	90	360	0	90
normalized size	1	1.	2.31	1.22	1.64	6.55	0.	1.64
time (sec)	N/A	0.047	0.347	0.085	0.987	2.501	0.	1.16

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	116	57	77	227	0	74
normalized size	1	1.	4.3	2.11	2.85	8.41	0.	2.74
time (sec)	N/A	0.04	0.368	0.084	0.977	2.413	0.	1.209

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	168	68	72	463	0	211
normalized size	1	1.	1.87	0.76	0.8	5.14	0.	2.34
time (sec)	N/A	0.056	0.67	0.085	0.995	2.485	0.	1.201

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	214	53	130	286	0	198
normalized size	1	1.	3.4	0.84	2.06	4.54	0.	3.14
time (sec)	N/A	0.053	0.572	0.073	0.998	2.427	0.	1.195

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	32	36	90	49	0	59
normalized size	1	1.	1.1	1.24	3.1	1.69	0.	2.03
time (sec)	N/A	0.041	0.051	0.084	1.004	2.372	0.	1.178

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	56	24	28	123	0	115
normalized size	1	1.	2.07	0.89	1.04	4.56	0.	4.26
time (sec)	N/A	0.039	0.143	0.039	0.964	2.39	0.	1.196

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	98	118	0	205	190	124
normalized size	1	1.	0.84	1.02	0.	1.77	1.64	1.07
time (sec)	N/A	0.062	0.183	0.027	0.	2.387	0.818	1.12

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	120	156	0	292	260	166
normalized size	1	1.	0.71	0.92	0.	1.73	1.54	0.98
time (sec)	N/A	0.096	0.253	0.103	0.	2.359	1.232	1.145

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	142	196	0	379	328	198
normalized size	1	1.	0.63	0.88	0.	1.69	1.46	0.88
time (sec)	N/A	0.125	0.495	0.1	0.	2.389	1.525	1.188

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	237	342	398	674	0	207
normalized size	1	1.	1.78	2.57	2.99	5.07	0.	1.56
time (sec)	N/A	0.114	0.961	0.099	1.054	2.512	0.	1.232

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	988	192	630	455	0	155
normalized size	1	1.	9.23	1.79	5.89	4.25	0.	1.45
time (sec)	N/A	0.101	6.124	0.095	1.615	2.41	0.	1.204

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	247	86	190	221	0	99
normalized size	1	1.	3.01	1.05	2.32	2.7	0.	1.21
time (sec)	N/A	0.087	0.241	0.09	1.513	2.513	0.	1.21

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	40	90	72	90	0	99
normalized size	1	1.	0.59	1.32	1.06	1.32	0.	1.46
time (sec)	N/A	0.079	0.08	0.089	0.992	2.321	0.	1.178

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	73	123	123	166	0	134
normalized size	1	1.	0.55	0.93	0.93	1.26	0.	1.02
time (sec)	N/A	0.108	0.185	0.05	1.011	2.243	0.	1.14

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	95	174	0	248	233	196
normalized size	1	1.	0.71	1.3	0.	1.85	1.74	1.46
time (sec)	N/A	0.12	0.191	0.104	0.	2.327	1.637	1.182

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	117	240	0	336	301	266
normalized size	1	1.	0.75	1.54	0.	2.15	1.93	1.71
time (sec)	N/A	0.148	0.353	0.107	0.	2.329	2.022	1.157

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	139	306	0	435	369	336
normalized size	1	1.	0.8	1.76	0.	2.5	2.12	1.93
time (sec)	N/A	0.158	0.791	0.11	0.	2.483	2.605	1.177

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	599	120	313	878	0	340
normalized size	1	1.	4.47	0.9	2.34	6.55	0.	2.54
time (sec)	N/A	0.079	2.712	0.132	1.197	3.335	0.	1.251

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	537	107	288	616	0	304
normalized size	1	1.	4.26	0.85	2.29	4.89	0.	2.41
time (sec)	N/A	0.077	1.386	0.12	1.243	3.008	0.	1.217

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	397	92	255	370	0	270
normalized size	1	1.	3.42	0.79	2.2	3.19	0.	2.33
time (sec)	N/A	0.068	0.886	0.094	1.253	2.853	0.	1.219

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	63	217	49	0	95
normalized size	1	1.	0.74	1.47	5.05	1.14	0.	2.21
time (sec)	N/A	0.043	0.059	0.115	1.148	2.523	0.	1.195

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	56	49	192	132	0	185
normalized size	1	1.	0.69	0.6	2.37	1.63	0.	2.28
time (sec)	N/A	0.056	0.154	0.102	1.15	2.466	0.	1.185

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	78	36	165	205	0	220
normalized size	1	1.	1.42	0.65	3.	3.73	0.	4.
time (sec)	N/A	0.048	0.21	0.12	1.159	2.327	0.	1.175

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	100	24	28	278	0	255
normalized size	1	1.	3.7	0.89	1.04	10.3	0.	9.44
time (sec)	N/A	0.039	0.229	0.054	1.192	2.352	0.	1.243

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	148	196	0	396	326	178
normalized size	1	1.	0.65	0.86	0.	1.73	1.42	0.78
time (sec)	N/A	0.153	0.484	0.029	0.	2.404	2.112	1.107

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	170	234	0	479	396	220
normalized size	1	1.	0.61	0.84	0.	1.72	1.42	0.79
time (sec)	N/A	0.144	0.966	0.119	0.	2.445	2.735	1.187

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	192	274	0	571	464	254
normalized size	1	1.	0.58	0.82	0.	1.71	1.39	0.76
time (sec)	N/A	0.181	1.5	0.122	0.	2.552	2.011	1.17

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	1704	409	1075	826	0	266
normalized size	1	1.	8.31	2.	5.24	4.03	0.	1.3
time (sec)	N/A	0.217	6.213	0.146	2.092	2.956	0.	1.248

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	1244	282	730	554	0	225
normalized size	1	1.	6.8	1.54	3.99	3.03	0.	1.23
time (sec)	N/A	0.2	6.184	0.12	1.91	3.025	0.	1.257

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	304	176	250	306	0	169
normalized size	1	1.	1.95	1.13	1.6	1.96	0.	1.08
time (sec)	N/A	0.216	1.034	0.11	1.853	2.652	0.	1.231

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	40	156	72	92	0	169
normalized size	1	1.	0.59	2.29	1.06	1.35	0.	2.49
time (sec)	N/A	0.08	0.101	0.108	1.216	2.445	0.	1.229

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	73	189	131	177	0	204
normalized size	1	1.	0.53	1.37	0.95	1.28	0.	1.48
time (sec)	N/A	0.175	0.209	0.109	1.237	2.43	0.	1.252

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	95	222	190	267	0	239
normalized size	1	1.	0.45	1.04	0.89	1.25	0.	1.12
time (sec)	N/A	0.276	0.25	0.125	1.243	2.32	0.	1.196

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	117	255	242	355	0	274
normalized size	1	1.	0.43	0.95	0.9	1.32	0.	1.02
time (sec)	N/A	0.26	0.429	0.059	1.253	2.534	0.	1.163

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	139	306	0	452	369	336
normalized size	1	1.	0.51	1.13	0.	1.67	1.36	1.24
time (sec)	N/A	0.312	0.901	0.118	0.	2.666	3.499	1.177

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	161	372	0	543	437	406
normalized size	1	1.	0.53	1.24	0.	1.8	1.45	1.35
time (sec)	N/A	0.384	1.319	0.125	0.	2.549	4.395	1.2

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	156	365	0	0	0	0
normalized size	1	1.	1.27	2.97	0.	0.	0.	0.
time (sec)	N/A	0.091	2.139	0.365	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	57	192	0	0	0	0
normalized size	1	1.	0.61	2.04	0.	0.	0.	0.
time (sec)	N/A	0.06	0.504	0.232	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	102	351	0	0	0	0
normalized size	1	1.	1.13	3.9	0.	0.	0.	0.
time (sec)	N/A	0.061	0.801	0.21	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	44	164	0	0	0	0
normalized size	1	1.	0.73	2.73	0.	0.	0.	0.
time (sec)	N/A	0.043	0.236	0.238	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	73	910	0	0	0	0
normalized size	1	1.	1.22	15.17	0.	0.	0.	0.
time (sec)	N/A	0.047	0.344	0.243	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	62	170	0	0	0	0
normalized size	1	1.	0.65	1.77	0.	0.	0.	0.
time (sec)	N/A	0.068	0.395	0.199	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	99	339	0	0	0	0
normalized size	1	1.	1.03	3.53	0.	0.	0.	0.
time (sec)	N/A	0.065	0.755	0.191	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	121	187	0	0	0	0
normalized size	1	1.	0.97	1.5	0.	0.	0.	0.
time (sec)	N/A	0.082	0.674	0.196	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	267	374	0	0	0	0
normalized size	1	1.	1.93	2.71	0.	0.	0.	0.
time (sec)	N/A	0.117	2.502	0.266	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	67	201	0	0	0	0
normalized size	1	1.	0.63	1.9	0.	0.	0.	0.
time (sec)	N/A	0.089	0.614	0.237	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	132	1099	0	0	0	0
normalized size	1	1.	1.23	10.27	0.	0.	0.	0.
time (sec)	N/A	0.081	0.979	0.266	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	114	173	0	0	0	0
normalized size	1	1.	1.34	2.04	0.	0.	0.	0.
time (sec)	N/A	0.071	0.437	0.224	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	114	343	0	0	0	0
normalized size	1	1.	1.34	4.04	0.	0.	0.	0.
time (sec)	N/A	0.072	0.983	0.203	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	133	189	0	0	0	0
normalized size	1	1.	1.15	1.63	0.	0.	0.	0.
time (sec)	N/A	0.087	0.877	0.203	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	133	353	0	0	0	0
normalized size	1	1.	1.15	3.04	0.	0.	0.	0.
time (sec)	N/A	0.087	1.704	0.254	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	155	205	0	0	0	0
normalized size	1	1.	1.05	1.39	0.	0.	0.	0.
time (sec)	N/A	0.106	1.242	0.283	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	442	402	0	0	0	0
normalized size	1	1.	2.19	1.99	0.	0.	0.	0.
time (sec)	N/A	0.201	7.557	0.371	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	89	229	0	0	0	0
normalized size	1	1.	0.51	1.31	0.	0.	0.	0.
time (sec)	N/A	0.188	1.697	0.292	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	129	392	0	0	0	0
normalized size	1	1.	0.74	2.24	0.	0.	0.	0.
time (sec)	N/A	0.185	2.46	0.281	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	79	213	0	0	0	0
normalized size	1	1.	0.57	1.53	0.	0.	0.	0.
time (sec)	N/A	0.139	1.249	0.276	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	146	101	2552	0	0	0	0
normalized size	1	1.18	0.81	20.58	0.	0.	0.	0.
time (sec)	N/A	0.134	1.561	0.317	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	123	175	0	0	0	0
normalized size	1	1.	1.11	1.58	0.	0.	0.	0.
time (sec)	N/A	0.102	0.655	0.274	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	108	1086	0	0	0	0
normalized size	1	1.	0.97	9.78	0.	0.	0.	0.
time (sec)	N/A	0.101	1.257	0.245	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	133	199	0	0	0	0
normalized size	1	1.	1.07	1.6	0.	0.	0.	0.
time (sec)	N/A	0.127	0.933	0.231	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	118	370	0	0	0	0
normalized size	1	1.	0.95	2.98	0.	0.	0.	0.
time (sec)	N/A	0.127	2.409	0.283	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	148	216	0	0	0	0
normalized size	1	1.	0.95	1.39	0.	0.	0.	0.
time (sec)	N/A	0.155	1.25	0.287	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	155	380	0	0	0	0
normalized size	1	1.	1.	2.45	0.	0.	0.	0.
time (sec)	N/A	0.148	6.361	0.37	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	170	232	0	0	0	0
normalized size	1	1.	0.91	1.25	0.	0.	0.	0.
time (sec)	N/A	0.173	1.851	0.378	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	429	401	0	0	0	0
normalized size	1	1.	2.	1.87	0.	0.	0.	0.
time (sec)	N/A	0.257	7.481	0.325	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	101	230	0	0	0	0
normalized size	1	1.	0.55	1.26	0.	0.	0.	0.
time (sec)	N/A	0.202	1.465	0.318	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	123	1618	0	0	0	0
normalized size	1	1.	0.69	9.09	0.	0.	0.	0.
time (sec)	N/A	0.186	3.981	0.337	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	130	198	0	0	0	0
normalized size	1	1.	0.89	1.36	0.	0.	0.	0.
time (sec)	N/A	0.149	1.165	0.332	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	110	3762	0	0	0	0
normalized size	1	1.	0.71	24.12	0.	0.	0.	0.
time (sec)	N/A	0.139	2.515	0.336	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	133	200	0	0	0	0
normalized size	1	1.	1.06	1.6	0.	0.	0.	0.
time (sec)	N/A	0.134	1.031	0.245	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	108	370	0	0	0	0
normalized size	1	1.	0.86	2.96	0.	0.	0.	0.
time (sec)	N/A	0.131	3.667	0.285	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	148	215	0	0	0	0
normalized size	1	1.	0.95	1.38	0.	0.	0.	0.
time (sec)	N/A	0.147	1.389	0.306	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	450	380	0	0	0	0
normalized size	1	1.	2.88	2.44	0.	0.	0.	0.
time (sec)	N/A	0.147	6.929	0.357	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	155	232	0	0	0	0
normalized size	1	1.	0.83	1.24	0.	0.	0.	0.
time (sec)	N/A	0.167	2.048	0.364	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	128	375	0	0	0	0
normalized size	1	1.	0.94	2.76	0.	0.	0.	0.
time (sec)	N/A	0.108	1.433	0.274	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	62	202	0	0	0	0
normalized size	1	1.	0.59	1.92	0.	0.	0.	0.
time (sec)	N/A	0.089	0.585	0.239	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	102	361	0	0	0	0
normalized size	1	1.	1.01	3.57	0.	0.	0.	0.
time (sec)	N/A	0.088	0.722	0.233	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	49	174	0	0	0	0
normalized size	1	1.	0.7	2.49	0.	0.	0.	0.
time (sec)	N/A	0.074	0.302	0.237	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	74	916	0	0	0	0
normalized size	1	1.	1.06	13.09	0.	0.	0.	0.
time (sec)	N/A	0.073	0.383	0.248	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	83	192	0	0	0	0
normalized size	1	1.	1.04	2.4	0.	0.	0.	0.
time (sec)	N/A	0.067	0.305	0.319	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	109	358	0	0	0	0
normalized size	1	1.	1.36	4.47	0.	0.	0.	0.
time (sec)	N/A	0.07	0.799	0.444	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	125	218	0	0	0	0
normalized size	1	1.	1.1	1.91	0.	0.	0.	0.
time (sec)	N/A	0.093	0.488	0.336	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	134	376	0	0	0	0
normalized size	1	1.	1.18	3.3	0.	0.	0.	0.
time (sec)	N/A	0.092	0.982	0.408	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	142	236	0	0	0	0
normalized size	1	1.	0.98	1.63	0.	0.	0.	0.
time (sec)	N/A	0.109	0.721	0.397	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	302	384	0	0	0	0
normalized size	1	1.	1.65	2.1	0.	0.	0.	0.
time (sec)	N/A	0.127	2.284	0.312	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	85	219	0	0	0	0
normalized size	1	1.	0.56	1.44	0.	0.	0.	0.
time (sec)	N/A	0.109	0.487	0.279	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	374	0	0	0	0
normalized size	1	1.	0.81	2.46	0.	0.	0.	0.
time (sec)	N/A	0.107	0.87	0.284	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	67	201	0	0	0	0
normalized size	1	1.	0.56	1.69	0.	0.	0.	0.
time (sec)	N/A	0.089	0.302	0.248	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	80	1093	0	0	0	0
normalized size	1	1.	0.7	9.5	0.	0.	0.	0.
time (sec)	N/A	0.089	0.474	0.239	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	101	173	0	0	0	0
normalized size	1	1.	1.12	1.92	0.	0.	0.	0.
time (sec)	N/A	0.073	0.365	0.24	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	102	341	0	0	0	0
normalized size	1	1.	1.13	3.79	0.	0.	0.	0.
time (sec)	N/A	0.075	0.496	0.228	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	112	208	0	0	0	0
normalized size	1	1.	0.97	1.79	0.	0.	0.	0.
time (sec)	N/A	0.083	0.384	0.325	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	123	366	0	0	0	0
normalized size	1	1.	1.06	3.16	0.	0.	0.	0.
time (sec)	N/A	0.084	1.267	0.386	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	134	234	0	0	0	0
normalized size	1	1.	0.89	1.56	0.	0.	0.	0.
time (sec)	N/A	0.11	0.484	0.339	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	149	386	0	0	0	0
normalized size	1	1.	0.99	2.57	0.	0.	0.	0.
time (sec)	N/A	0.109	2.022	0.723	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	151	252	0	0	0	0
normalized size	1	1.	0.83	1.39	0.	0.	0.	0.
time (sec)	N/A	0.131	0.874	0.492	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	128	392	0	0	0	0
normalized size	1	1.	0.72	2.2	0.	0.	0.	0.
time (sec)	N/A	0.17	1.541	0.323	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	74	213	0	0	0	0
normalized size	1	1.	0.52	1.51	0.	0.	0.	0.
time (sec)	N/A	0.15	0.577	0.285	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	93	1562	0	0	0	0
normalized size	1	1.	0.66	11.08	0.	0.	0.	0.
time (sec)	N/A	0.149	0.919	0.298	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	125	175	0	0	0	0
normalized size	1	1.	1.08	1.51	0.	0.	0.	0.
time (sec)	N/A	0.134	0.383	0.272	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	117	1086	0	0	0	0
normalized size	1	1.	1.01	9.36	0.	0.	0.	0.
time (sec)	N/A	0.132	0.576	0.243	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	104	199	0	0	0	0
normalized size	1	1.	0.79	1.51	0.	0.	0.	0.
time (sec)	N/A	0.127	0.574	0.257	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	140	368	0	0	0	0
normalized size	1	1.	1.06	2.79	0.	0.	0.	0.
time (sec)	N/A	0.129	0.743	0.299	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	129	236	0	0	0	0
normalized size	1	1.	0.85	1.55	0.	0.	0.	0.
time (sec)	N/A	0.14	0.425	0.39	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	145	395	0	0	0	0
normalized size	1	1.	0.95	2.6	0.	0.	0.	0.
time (sec)	N/A	0.145	1.322	0.394	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	151	261	0	0	0	0
normalized size	1	1.	0.81	1.4	0.	0.	0.	0.
time (sec)	N/A	0.182	0.582	0.411	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	124	1628	0	0	0	0
normalized size	1	1.	0.65	8.48	0.	0.	0.	0.
time (sec)	N/A	0.175	1.337	0.339	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	134	198	0	0	0	0
normalized size	1	1.	0.85	1.26	0.	0.	0.	0.
time (sec)	N/A	0.155	0.5	0.3	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	106	3582	0	0	0	0
normalized size	1	1.	0.65	21.98	0.	0.	0.	0.
time (sec)	N/A	0.154	0.564	0.335	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	137	200	0	0	0	0
normalized size	1	1.	1.04	1.52	0.	0.	0.	0.
time (sec)	N/A	0.137	0.494	0.251	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	149	370	0	0	0	0
normalized size	1	1.	1.13	2.8	0.	0.	0.	0.
time (sec)	N/A	0.138	0.74	0.269	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	144	216	0	0	0	0
normalized size	1	1.	0.88	1.33	0.	0.	0.	0.
time (sec)	N/A	0.144	0.556	0.329	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	142	378	0	0	0	0
normalized size	1	1.	0.87	2.32	0.	0.	0.	0.
time (sec)	N/A	0.146	1.428	0.381	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	137	252	0	0	0	0
normalized size	1	1.	0.72	1.32	0.	0.	0.	0.
time (sec)	N/A	0.192	0.487	0.356	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	104	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	1.244	0.109	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	92	0	0	0	0	0
normalized size	1	1.	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	0.555	0.102	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	98	0	0	0	0	0
normalized size	1	1.	1.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.441	0.143	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	106	0	0	0	0	0
normalized size	1	1.	1.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	0.443	0.151	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	267	0	0	0	0	0
normalized size	1	1.	3.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.177	2.718	0.132	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	128	0	0	0	0	0
normalized size	1	1.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.966	0.132	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	132	0	0	0	0	0
normalized size	1	1.	1.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	1.119	0.129	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	105	0	0	0	0	0
normalized size	1	1.	1.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	0.636	0.159	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	84	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.175	0.461	0.136	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	103	0	0	0	0	0
normalized size	1	1.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	0.429	0.154	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	112	0	0	0	0	0
normalized size	1	1.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.908	0.128	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	119	0	0	0	0	0
normalized size	1	1.	1.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	0.862	0.133	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	128	0	0	0	0	0
normalized size	1	1.	1.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.704	0.174	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	121	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.17	0.562	0.174	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	141	0	0	0	0	0
normalized size	1	1.	1.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	1.358	0.15	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	143	0	0	0	0	0
normalized size	1	1.	2.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.19	0.81	0.151	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	95	141	103	522	0	0
normalized size	1	1.	0.81	1.21	0.88	4.46	0.	0.
time (sec)	N/A	0.075	0.819	2.026	0.98	2.376	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	77	114	78	392	0	0
normalized size	1	1.	0.88	1.3	0.89	4.45	0.	0.
time (sec)	N/A	0.069	0.408	0.536	0.999	2.31	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	58	87	54	270	0	0
normalized size	1	1.	0.98	1.47	0.92	4.58	0.	0.
time (sec)	N/A	0.062	0.267	0.34	0.948	2.328	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	34	24	28	132	0	0
normalized size	1	1.	1.17	0.83	0.97	4.55	0.	0.
time (sec)	N/A	0.056	0.123	0.043	0.971	2.27	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	105	397	0	772	0	0
normalized size	1	1.	0.88	3.31	0.	6.43	0.	0.
time (sec)	N/A	0.098	0.439	0.39	0.	2.698	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	133	741	0	868	0	0
normalized size	1	1.	0.69	3.84	0.	4.5	0.	0.
time (sec)	N/A	0.112	0.37	0.355	0.	2.845	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	159	1085	0	973	0	0
normalized size	1	1.	0.6	4.08	0.	3.66	0.	0.
time (sec)	N/A	0.142	0.609	0.427	0.	2.937	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	95	141	0	467	0	0
normalized size	1	1.	0.65	0.96	0.	3.18	0.	0.
time (sec)	N/A	0.25	0.586	0.841	0.	2.31	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	77	114	0	342	0	0
normalized size	1	1.	0.7	1.04	0.	3.11	0.	0.
time (sec)	N/A	0.178	0.37	0.385	0.	2.236	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	87	304	227	0	0
normalized size	1	1.	0.86	1.19	4.16	3.11	0.	0.
time (sec)	N/A	0.107	0.218	0.317	116.376	2.34	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	50	0	66	0	0
normalized size	1	1.	1.26	1.61	0.	2.13	0.	0.
time (sec)	N/A	0.028	0.144	0.242	0.	2.34	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	87	217	1045	690	0	0
normalized size	1	1.	1.05	2.61	12.59	8.31	0.	0.
time (sec)	N/A	0.094	0.517	0.322	1.974	2.353	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	126	569	1261	813	0	0
normalized size	1	1.	0.82	3.69	8.19	5.28	0.	0.
time (sec)	N/A	0.223	0.463	0.387	2.233	2.386	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	152	913	2990	906	0	0
normalized size	1	1.	0.68	4.09	13.41	4.06	0.	0.
time (sec)	N/A	0.391	0.556	0.352	2.807	2.441	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	111	152	103	578	0	0
normalized size	1	1.	0.95	1.3	0.88	4.94	0.	0.
time (sec)	N/A	0.089	1.126	5.216	1.041	2.594	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	93	125	78	446	0	0
normalized size	1	1.	1.06	1.42	0.89	5.07	0.	0.
time (sec)	N/A	0.077	0.491	0.335	1.001	2.326	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	98	54	311	0	0
normalized size	1	1.	1.37	1.66	0.92	5.27	0.	0.
time (sec)	N/A	0.069	0.474	0.289	1.029	2.351	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	69	24	28	170	0	0
normalized size	1	1.	2.38	0.83	0.97	5.86	0.	0.
time (sec)	N/A	0.061	0.265	0.03	1.107	2.348	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	97	396	0	671	0	0
normalized size	1	1.	1.04	4.26	0.	7.22	0.	0.
time (sec)	N/A	0.083	0.58	0.332	0.	2.366	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	143	742	0	856	0	0
normalized size	1	1.	0.86	4.47	0.	5.16	0.	0.
time (sec)	N/A	0.106	0.73	0.313	0.	2.669	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	169	1086	0	959	0	0
normalized size	1	1.	0.71	4.54	0.	4.01	0.	0.
time (sec)	N/A	0.134	0.982	0.365	0.	2.768	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	109	125	1345	437	0	0
normalized size	1	1.	0.74	0.85	9.15	2.97	0.	0.
time (sec)	N/A	0.241	0.955	0.321	69.607	2.337	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	91	98	788	312	0	0
normalized size	1	1.	0.83	0.89	7.16	2.84	0.	0.
time (sec)	N/A	0.177	0.415	0.286	3.53	2.211	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	57	71	0	194	0	0
normalized size	1	1.	0.83	1.03	0.	2.81	0.	0.
time (sec)	N/A	0.064	0.249	0.237	0.	2.308	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	42	271	105	0	0
normalized size	1	1.	1.	1.35	8.74	3.39	0.	0.
time (sec)	N/A	0.049	0.136	0.259	1.876	2.209	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	101	570	1192	764	0	0
normalized size	1	1.	0.83	4.67	9.77	6.26	0.	0.
time (sec)	N/A	0.149	0.882	0.36	2.216	2.372	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	160	914	0	892	0	0
normalized size	1	1.	0.83	4.76	0.	4.65	0.	0.
time (sec)	N/A	0.27	1.377	0.313	0.	2.397	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	113	171	103	635	0	0
normalized size	1	1.	0.97	1.46	0.88	5.43	0.	0.
time (sec)	N/A	0.086	1.416	15.969	1.111	2.524	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	97	144	78	497	0	0
normalized size	1	1.	1.1	1.64	0.89	5.65	0.	0.
time (sec)	N/A	0.077	0.745	0.581	1.096	2.612	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	85	117	54	360	0	0
normalized size	1	1.	1.44	1.98	0.92	6.1	0.	0.
time (sec)	N/A	0.071	0.532	0.327	1.077	2.612	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	73	24	28	209	0	0
normalized size	1	1.	2.52	0.83	0.97	7.21	0.	0.
time (sec)	N/A	0.063	0.314	0.032	1.127	2.615	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	116	398	0	693	0	0
normalized size	1	1.	1.3	4.47	0.	7.79	0.	0.
time (sec)	N/A	0.083	0.597	0.332	0.	2.494	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	116	744	0	761	0	0
normalized size	1	1.	0.85	5.43	0.	5.55	0.	0.
time (sec)	N/A	0.095	0.704	0.336	0.	2.479	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	142	1088	0	936	0	0
normalized size	1	1.	0.68	5.18	0.	4.46	0.	0.
time (sec)	N/A	0.12	0.825	0.402	0.	3.177	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	103	117	0	402	0	0
normalized size	1	1.	0.7	0.8	0.	2.73	0.	0.
time (sec)	N/A	0.24	0.68	0.293	0.	2.486	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	93	90	0	282	0	0
normalized size	1	1.	0.89	0.87	0.	2.71	0.	0.
time (sec)	N/A	0.101	0.314	0.247	0.	2.034	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	46	53	447	116	0	0
normalized size	1	1.	0.71	0.82	6.88	1.78	0.	0.
time (sec)	N/A	0.097	0.253	0.279	1.94	1.97	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	69	63	443	157	0	0
normalized size	1	1.	1.97	1.8	12.66	4.49	0.	0.
time (sec)	N/A	0.06	0.323	0.293	2.145	2.049	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	118	916	1451	840	0	0
normalized size	1	1.	0.74	5.76	9.13	5.28	0.	0.
time (sec)	N/A	0.218	1.011	0.324	2.299	2.225	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	155	1260	0	971	0	0
normalized size	1	1.	0.67	5.45	0.	4.2	0.	0.
time (sec)	N/A	0.339	1.047	0.484	0.	2.046	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	113	181	103	682	0	0
normalized size	1	1.	0.97	1.55	0.88	5.83	0.	0.
time (sec)	N/A	0.084	1.699	47.629	1.109	2.328	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	97	154	78	537	0	0
normalized size	1	1.	1.1	1.75	0.89	6.1	0.	0.
time (sec)	N/A	0.077	0.858	0.628	1.12	2.06	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	85	127	54	405	0	0
normalized size	1	1.	1.44	2.15	0.92	6.86	0.	0.
time (sec)	N/A	0.068	0.591	0.365	1.093	1.98	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	73	24	28	244	0	0
normalized size	1	1.	2.52	0.83	0.97	8.41	0.	0.
time (sec)	N/A	0.062	0.339	0.03	1.185	2.147	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	137	412	0	709	0	0
normalized size	1	1.	1.18	3.55	0.	6.11	0.	0.
time (sec)	N/A	0.088	1.243	0.374	0.	2.152	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	152	742	0	737	0	0
normalized size	1	1.	1.11	5.42	0.	5.38	0.	0.
time (sec)	N/A	0.094	1.347	0.344	0.	2.275	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	129	1088	0	813	0	0
normalized size	1	1.	0.71	6.01	0.	4.49	0.	0.
time (sec)	N/A	0.108	1.004	0.402	0.	2.245	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	109	100	0	363	0	0
normalized size	1	1.	0.78	0.72	0.	2.61	0.	0.
time (sec)	N/A	0.14	0.642	0.253	0.	2.042	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	59	73	564	244	0	0
normalized size	1	1.	0.57	0.7	5.42	2.35	0.	0.
time (sec)	N/A	0.153	0.357	0.283	2.059	1.978	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	86	71	680	157	0	0
normalized size	1	1.	1.21	1.	9.58	2.21	0.	0.
time (sec)	N/A	0.122	0.347	0.278	2.179	1.942	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	73	73	613	198	0	0
normalized size	1	1.	2.09	2.09	17.51	5.66	0.	0.
time (sec)	N/A	0.059	0.515	0.349	2.143	2.228	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	131	1260	1688	891	0	0
normalized size	1	1.	0.67	6.43	8.61	4.55	0.	0.
time (sec)	N/A	0.287	1.782	0.422	2.465	2.265	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	188	1604	0	1034	0	0
normalized size	1	1.	0.7	5.99	0.	3.86	0.	0.
time (sec)	N/A	0.411	3.205	0.615	0.	2.251	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	194	1948	0	1149	0	0
normalized size	1	1.	0.57	5.7	0.	3.36	0.	0.
time (sec)	N/A	0.565	6.327	0.816	0.	2.269	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	95	127	401	498	0	0
normalized size	1	1.	0.81	1.09	3.43	4.26	0.	0.
time (sec)	N/A	0.08	0.467	0.583	1.124	2.094	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	77	100	228	362	0	0
normalized size	1	1.	0.88	1.14	2.59	4.11	0.	0.
time (sec)	N/A	0.072	0.28	0.375	1.134	2.218	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	65	73	107	242	0	0
normalized size	1	1.	1.1	1.24	1.81	4.1	0.	0.
time (sec)	N/A	0.064	0.175	0.319	1.096	1.913	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	32	24	28	95	0	53
normalized size	1	1.	1.19	0.89	1.04	3.52	0.	1.96
time (sec)	N/A	0.057	0.144	0.043	1.058	1.923	0.	1.677

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	126	341	0	837	0	0
normalized size	1	1.	0.86	2.34	0.	5.73	0.	0.
time (sec)	N/A	0.098	0.477	0.334	0.	2.469	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	152	368	0	922	0	0
normalized size	1	1.	0.69	1.68	0.	4.21	0.	0.
time (sec)	N/A	0.124	0.627	0.361	0.	2.577	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	178	395	0	1031	0	0
normalized size	1	1.	0.61	1.35	0.	3.53	0.	0.
time (sec)	N/A	0.153	0.959	0.506	0.	3.04	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	95	154	821	529	0	0
normalized size	1	1.	0.65	1.05	5.59	3.6	0.	0.
time (sec)	N/A	0.258	0.501	1.273	2.055	2.109	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	77	127	640	398	0	0
normalized size	1	1.	0.7	1.15	5.82	3.62	0.	0.
time (sec)	N/A	0.187	0.379	0.388	1.901	2.026	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	65	100	459	274	0	0
normalized size	1	1.	0.89	1.37	6.29	3.75	0.	0.
time (sec)	N/A	0.118	0.214	0.322	1.643	2.108	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	40	73	278	153	0	0
normalized size	1	1.	1.14	2.09	7.94	4.37	0.	0.
time (sec)	N/A	0.054	0.12	0.298	1.535	1.999	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	70	137	0	502	0	0
normalized size	1	1.	1.35	2.63	0.	9.65	0.	0.
time (sec)	N/A	0.041	0.335	0.255	0.	2.2	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	96	319	1130	778	0	0
normalized size	1	1.	0.79	2.61	9.26	6.38	0.	0.
time (sec)	N/A	0.134	0.455	0.325	2.178	2.146	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	117	346	2618	865	0	0
normalized size	1	1.	0.61	1.79	13.56	4.48	0.	0.
time (sec)	N/A	0.266	0.528	0.329	2.634	2.034	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	110	117	103	468	0	0
normalized size	1	1.	0.94	1.	0.88	4.	0.	0.
time (sec)	N/A	0.087	0.669	0.318	1.102	2.065	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	92	90	78	332	0	0
normalized size	1	1.	1.05	1.02	0.89	3.77	0.	0.
time (sec)	N/A	0.079	0.285	0.279	1.085	2.235	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	80	61	51	180	0	0
normalized size	1	1.	1.4	1.07	0.89	3.16	0.	0.
time (sec)	N/A	0.072	0.191	0.262	1.053	2.072	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	28	130	0	0
normalized size	1	1.	1.	0.89	1.04	4.81	0.	0.
time (sec)	N/A	0.065	0.143	0.032	1.093	2.13	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	142	368	0	906	0	0
normalized size	1	1.	0.81	2.1	0.	5.18	0.	0.
time (sec)	N/A	0.118	0.78	0.276	0.	2.493	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	168	395	0	999	0	0
normalized size	1	1.	0.68	1.59	0.	4.03	0.	0.
time (sec)	N/A	0.144	1.115	0.398	0.	2.738	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	203	422	0	1106	0	0
normalized size	1	1.	0.63	1.31	0.	3.45	0.	0.
time (sec)	N/A	0.18	1.769	0.648	0.	3.264	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	108	171	1031	598	0	0
normalized size	1	1.	0.73	1.16	7.01	4.07	0.	0.
time (sec)	N/A	0.262	0.77	3.044	2.405	2.099	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	92	144	845	463	0	0
normalized size	1	1.	0.84	1.31	7.68	4.21	0.	0.
time (sec)	N/A	0.191	0.405	0.342	2.369	2.31	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	80	117	659	325	0	0
normalized size	1	1.	1.1	1.6	9.03	4.45	0.	0.
time (sec)	N/A	0.129	0.275	0.295	1.723	2.084	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	59	90	473	198	0	0
normalized size	1	1.	1.69	2.57	13.51	5.66	0.	0.
time (sec)	N/A	0.062	0.192	0.267	1.506	2.151	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	101	158	1099	690	0	0
normalized size	1	1.	1.17	1.84	12.78	8.02	0.	0.
time (sec)	N/A	0.102	0.722	0.238	2.025	2.107	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	95	318	0	760	0	0
normalized size	1	1.	1.09	3.66	0.	8.74	0.	0.
time (sec)	N/A	0.075	0.605	0.258	0.	2.138	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	120	346	2457	845	0	0
normalized size	1	1.	0.76	2.2	15.65	5.38	0.	0.
time (sec)	N/A	0.192	0.979	0.246	2.162	2.083	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	145	373	3553	934	0	0
normalized size	1	1.	0.62	1.6	15.25	4.01	0.	0.
time (sec)	N/A	0.337	1.52	0.301	2.444	2.235	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	116	127	127	566	0	0
normalized size	1	1.	0.79	0.87	0.87	3.88	0.	0.
time (sec)	N/A	0.093	0.693	0.349	0.979	2.279	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	108	100	103	414	0	0
normalized size	1	1.	0.92	0.85	0.88	3.54	0.	0.
time (sec)	N/A	0.086	0.494	0.3	1.189	2.218	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	94	73	78	262	0	0
normalized size	1	1.	1.09	0.85	0.91	3.05	0.	0.
time (sec)	N/A	0.079	0.263	0.273	1.069	2.117	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	65	59	135	0	0
normalized size	1	1.	0.65	1.18	1.07	2.45	0.	0.
time (sec)	N/A	0.07	0.254	0.266	1.113	2.145	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	39	24	28	176	0	0
normalized size	1	1.	1.34	0.83	0.97	6.07	0.	0.
time (sec)	N/A	0.063	0.214	0.03	1.086	2.16	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	163	395	0	949	0	0
normalized size	1	1.	0.8	1.94	0.	4.65	0.	0.
time (sec)	N/A	0.127	1.004	0.302	0.	2.536	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	189	422	0	1064	0	0
normalized size	1	1.	0.68	1.52	0.	3.84	0.	0.
time (sec)	N/A	0.157	1.464	0.452	0.	2.852	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	112	181	1218	649	0	0
normalized size	1	1.	0.76	1.23	8.29	4.41	0.	0.
time (sec)	N/A	0.265	0.953	15.287	2.873	2.247	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	94	154	1031	509	0	0
normalized size	1	1.	0.85	1.4	9.37	4.63	0.	0.
time (sec)	N/A	0.193	0.592	0.819	2.5	2.218	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	80	127	845	373	0	0
normalized size	1	1.	1.1	1.74	11.58	5.11	0.	0.
time (sec)	N/A	0.126	0.434	0.416	2.176	2.07	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	57	100	659	240	0	0
normalized size	1	1.	1.63	2.86	18.83	6.86	0.	0.
time (sec)	N/A	0.063	0.256	0.284	1.715	2.077	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	82	281	1445	872	0	0
normalized size	1	1.	0.67	2.28	11.75	7.09	0.	0.
time (sec)	N/A	0.17	0.728	0.285	2.177	2.161	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	149	443	1115	761	0	0
normalized size	1	1.	1.73	5.15	12.97	8.85	0.	0.
time (sec)	N/A	0.144	0.976	0.282	1.997	1.997	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	121	346	0	807	0	0
normalized size	1	1.	0.99	2.84	0.	6.61	0.	0.
time (sec)	N/A	0.115	0.752	0.283	0.	2.118	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	143	373	3101	892	0	0
normalized size	1	1.	0.74	1.94	16.15	4.65	0.	0.
time (sec)	N/A	0.259	1.167	0.276	2.224	2.113	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	165	400	5107	988	0	0
normalized size	1	1.	0.61	1.48	18.91	3.66	0.	0.
time (sec)	N/A	0.422	1.191	0.343	2.623	2.125	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	114	117	127	513	0	0
normalized size	1	1.	0.78	0.8	0.87	3.51	0.	0.
time (sec)	N/A	0.093	0.853	0.317	0.994	2.153	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	110	90	103	343	0	0
normalized size	1	1.	0.97	0.8	0.91	3.04	0.	0.
time (sec)	N/A	0.086	0.44	0.282	0.981	2.158	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	61	88	84	243	0	0
normalized size	1	1.	0.73	1.05	1.	2.89	0.	0.
time (sec)	N/A	0.08	0.296	0.28	0.978	2.162	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	80	88	43	177	0	0
normalized size	1	1.	1.4	1.54	0.75	3.11	0.	0.
time (sec)	N/A	0.074	0.205	0.271	0.977	2.05	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	39	24	28	212	0	0
normalized size	1	1.	1.34	0.83	0.97	7.31	0.	0.
time (sec)	N/A	0.064	0.253	0.034	0.974	2.153	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	176	422	0	1010	0	0
normalized size	1	1.	0.76	1.81	0.	4.33	0.	0.
time (sec)	N/A	0.142	1.517	0.365	0.	2.749	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	202	449	0	1106	0	0
normalized size	1	1.	0.66	1.47	0.	3.61	0.	0.
time (sec)	N/A	0.177	2.148	0.706	0.	2.854	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	92	171	1218	555	0	0
normalized size	1	1.	0.84	1.55	11.07	5.05	0.	0.
time (sec)	N/A	0.194	0.66	0.629	2.556	2.349	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	82	144	1031	420	0	0
normalized size	1	1.	1.12	1.97	14.12	5.75	0.	0.
time (sec)	N/A	0.127	0.418	0.37	2.174	2.084	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	59	115	845	281	0	0
normalized size	1	1.	1.69	3.29	24.14	8.03	0.	0.
time (sec)	N/A	0.062	0.291	0.303	1.929	2.1	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	130	399	1574	1052	0	0
normalized size	1	1.	0.81	2.49	9.84	6.58	0.	0.
time (sec)	N/A	0.251	1.118	0.302	2.35	2.253	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	126	318	0	756	0	0
normalized size	1	1.	1.04	2.63	0.	6.25	0.	0.
time (sec)	N/A	0.217	0.767	0.279	0.	2.117	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	120	345	1318	801	0	0
normalized size	1	1.	0.96	2.76	10.54	6.41	0.	0.
time (sec)	N/A	0.188	0.882	0.313	2.057	2.085	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	119	373	0	851	0	0
normalized size	1	1.	0.76	2.38	0.	5.42	0.	0.
time (sec)	N/A	0.165	1.363	0.236	0.	1.991	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	141	400	3754	940	0	0
normalized size	1	1.	0.62	1.76	16.54	4.14	0.	0.
time (sec)	N/A	0.375	1.944	0.303	2.251	2.121	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	175	427	7834	1054	0	0
normalized size	1	1.	0.57	1.39	25.52	3.43	0.	0.
time (sec)	N/A	0.52	2.158	0.452	2.808	2.163	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	524	373	309	2531	1427	0	0
normalized size	1	1.	0.71	0.59	4.83	2.72	0.	0.
time (sec)	N/A	0.445	1.798	0.391	2.304	2.212	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	289	225	1890	1172	0	0
normalized size	1	1.	0.89	0.7	5.85	3.63	0.	0.
time (sec)	N/A	0.191	0.931	0.369	2.209	2.245	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	56	103	176	0	0
normalized size	1	1.	1.	1.56	2.86	4.89	0.	0.
time (sec)	N/A	0.061	0.044	0.322	1.616	1.864	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	48	75	73	220	0	0
normalized size	1	1.	0.59	0.93	0.9	2.72	0.	0.
time (sec)	N/A	0.132	0.168	0.333	1.884	1.937	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	63	85	176	262	0	0
normalized size	1	1.	0.52	0.7	1.44	2.15	0.	0.
time (sec)	N/A	0.197	0.186	0.35	1.942	2.093	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	80	102	240	301	0	0
normalized size	1	1.	0.49	0.62	1.46	1.84	0.	0.
time (sec)	N/A	0.283	0.219	0.378	1.95	2.096	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	358	414	4070	1989	0	0
normalized size	1	1.	0.79	0.91	8.98	4.39	0.	0.
time (sec)	N/A	0.506	3.335	0.325	2.989	2.273	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	571	571	395	363	3213	1808	0	0
normalized size	1	1.	0.69	0.64	5.63	3.17	0.	0.
time (sec)	N/A	0.543	21.077	0.337	2.554	2.308	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	348	304	2539	1376	0	0
normalized size	1	1.	0.96	0.84	6.98	3.78	0.	0.
time (sec)	N/A	0.315	1.745	0.336	2.35	2.136	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	520	520	354	286	1974	1543	0	0
normalized size	1	1.	0.68	0.55	3.8	2.97	0.	0.
time (sec)	N/A	0.434	0.669	0.293	2.152	2.186	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	76	103	186	0	0
normalized size	1	1.	1.	2.	2.71	4.89	0.	0.
time (sec)	N/A	0.091	0.062	0.288	1.662	2.028	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	84	86	80	227	0	0
normalized size	1	1.	1.04	1.06	0.99	2.8	0.	0.
time (sec)	N/A	0.158	0.365	0.296	1.889	2.112	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	98	103	113	273	0	0
normalized size	1	1.	0.78	0.82	0.9	2.18	0.	0.
time (sec)	N/A	0.23	0.427	0.303	1.925	1.989	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	113	113	216	317	0	0
normalized size	1	1.	0.68	0.68	1.29	1.9	0.	0.
time (sec)	N/A	0.286	0.552	0.372	1.949	2.167	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	612	612	417	424	4074	2105	0	0
normalized size	1	1.	0.68	0.69	6.66	3.44	0.	0.
time (sec)	N/A	0.69	20.564	0.294	3.391	2.299	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	397	371	3291	1701	0	0
normalized size	1	1.	0.97	0.9	8.01	4.14	0.	0.
time (sec)	N/A	0.453	2.569	0.359	2.755	2.362	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	563	367	347	2720	1615	0	0
normalized size	1	1.	0.65	0.62	4.83	2.87	0.	0.
time (sec)	N/A	0.575	1.932	0.345	2.48	2.358	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	362	355	323	2014	1539	0	0
normalized size	1	1.	0.98	0.89	5.56	4.25	0.	0.
time (sec)	N/A	0.33	2.415	0.331	2.434	2.27	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	88	103	213	0	0
normalized size	1	1.	1.	2.32	2.71	5.61	0.	0.
time (sec)	N/A	0.075	0.081	0.294	1.801	2.469	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	92	105	127	240	0	0
normalized size	1	1.	1.14	1.3	1.57	2.96	0.	0.
time (sec)	N/A	0.154	0.398	0.305	2.074	2.328	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	104	115	130	284	0	0
normalized size	1	1.	0.83	0.92	1.04	2.27	0.	0.
time (sec)	N/A	0.222	0.459	0.329	2.193	2.301	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	121	132	167	288	0	0
normalized size	1	1.	0.72	0.78	0.99	1.7	0.	0.
time (sec)	N/A	0.312	0.646	0.477	1.917	2.342	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	366	316	3069	1420	0	0
normalized size	1	1.	0.99	0.86	8.32	3.85	0.	0.
time (sec)	N/A	0.308	1.851	0.35	2.266	2.291	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	483	483	302	237	980	1239	0	0
normalized size	1	1.	0.63	0.49	2.03	2.57	0.	0.
time (sec)	N/A	0.353	1.103	0.333	2.049	2.162	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	74	103	176	0	0
normalized size	1	1.	1.	2.06	2.86	4.89	0.	0.
time (sec)	N/A	0.068	0.05	0.308	1.603	1.965	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	48	85	108	220	0	0
normalized size	1	1.	0.6	1.06	1.35	2.75	0.	0.
time (sec)	N/A	0.138	0.101	0.319	1.876	2.028	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	68	105	176	265	0	0
normalized size	1	1.	0.56	0.87	1.45	2.19	0.	0.
time (sec)	N/A	0.22	0.25	0.319	1.933	2.064	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	79	115	240	304	0	0
normalized size	1	1.	0.48	0.7	1.45	1.84	0.	0.
time (sec)	N/A	0.291	0.339	0.332	1.97	2.082	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	87	132	305	354	0	0
normalized size	1	1.	0.42	0.64	1.48	1.72	0.	0.
time (sec)	N/A	0.387	0.367	0.352	2.023	2.195	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	529	405	1022	2456	1602	0	0
normalized size	1	1.	0.77	1.93	4.64	3.03	0.	0.
time (sec)	N/A	0.567	15.992	0.311	2.225	2.289	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	350	957	1050	1663	0	0
normalized size	1	1.	0.96	2.62	2.88	4.56	0.	0.
time (sec)	N/A	0.316	2.683	0.337	2.112	2.295	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	87	103	186	0	0
normalized size	1	1.	1.	2.29	2.71	4.89	0.	0.
time (sec)	N/A	0.076	0.069	0.275	1.636	2.045	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	101	108	219	0	0
normalized size	1	1.	0.79	1.26	1.35	2.74	0.	0.
time (sec)	N/A	0.145	0.12	0.308	1.918	2.103	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	83	106	176	265	0	0
normalized size	1	1.	0.69	0.88	1.45	2.19	0.	0.
time (sec)	N/A	0.209	0.288	0.308	1.937	2.049	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	100	132	240	309	0	0
normalized size	1	1.	0.61	0.8	1.45	1.87	0.	0.
time (sec)	N/A	0.306	0.438	0.304	1.973	1.981	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	100	142	305	358	0	0
normalized size	1	1.	0.48	0.68	1.46	1.71	0.	0.
time (sec)	N/A	0.393	0.471	0.312	2.037	2.05	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	369	1439	3312	1704	0	0
normalized size	1	1.	0.9	3.5	8.06	4.15	0.	0.
time (sec)	N/A	0.434	5.437	0.346	2.253	2.514	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	527	527	422	1324	1979	1663	0	0
normalized size	1	1.	0.8	2.51	3.76	3.16	0.	0.
time (sec)	N/A	0.454	11.63	0.303	2.269	2.406	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	105	103	192	0	0
normalized size	1	1.	1.	2.76	2.71	5.05	0.	0.
time (sec)	N/A	0.078	0.22	0.281	1.619	2.09	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	112	116	232	0	0
normalized size	1	1.	0.79	1.4	1.45	2.9	0.	0.
time (sec)	N/A	0.166	0.199	0.291	1.902	2.016	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	85	128	176	262	0	0
normalized size	1	1.	0.7	1.06	1.45	2.17	0.	0.
time (sec)	N/A	0.22	0.306	0.313	1.89	1.878	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	102	140	240	271	0	0
normalized size	1	1.	0.63	0.86	1.48	1.67	0.	0.
time (sec)	N/A	0.301	0.4	0.32	1.942	2.126	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	107	159	305	360	0	0
normalized size	1	1.	0.52	0.77	1.48	1.75	0.	0.
time (sec)	N/A	0.399	0.548	0.315	2.052	2.166	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	118	0	0	0	0	0
normalized size	1	1.	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	0.663	0.331	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	116	0	0	0	0	0
normalized size	1	1.	1.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	0.55	0.33	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	116	0	0	0	0	0
normalized size	1	1.	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.19	0.429	0.448	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	95	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	0.52	0.408	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	95	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	0.739	0.359	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	112	0	0	0	0	0
normalized size	1	1.	1.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.21	0.655	0.322	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	437	437	240	0	5268	1608	0	0
normalized size	1	1.	0.55	0.	12.05	3.68	0.	0.
time (sec)	N/A	0.398	1.795	0.2	2.828	2.334	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	220	0	2572	1496	0	0
normalized size	1	1.	0.58	0.	6.8	3.96	0.	0.
time (sec)	N/A	0.343	1.233	0.157	2.325	2.076	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	340	161	0	2367	1064	0	0
normalized size	1	1.	0.47	0.	6.96	3.13	0.	0.
time (sec)	N/A	0.171	0.636	0.144	2.257	2.146	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	47	0	144	155	0	0
normalized size	1	1.	1.27	0.	3.89	4.19	0.	0.
time (sec)	N/A	0.077	0.324	0.129	1.99	2.086	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	429	167	0	0
normalized size	1	1.	0.86	0.	5.3	2.06	0.	0.
time (sec)	N/A	0.154	0.514	0.14	2.005	2.075	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	100	0	543	305	0	0
normalized size	1	1.	0.82	0.	4.45	2.5	0.	0.
time (sec)	N/A	0.235	0.596	0.14	2.209	2.062	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	116	0	1319	389	0	0
normalized size	1	1.	0.71	0.	8.09	2.39	0.	0.
time (sec)	N/A	0.321	1.11	0.141	2.263	2.197	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	151	0	0	0	0	0
normalized size	1	1.	1.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	3.204	0.474	0.	0.	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	154	0	0	0	0	0
normalized size	1	1.	1.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	1.356	0.497	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	141	0	0	0	0	0
normalized size	1	1.	1.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	0.956	0.364	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	130	0	0	0	0	0
normalized size	1	1.	1.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.609	0.477	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	150	0	0	0	0	0
normalized size	1	1.	1.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.608	0.79	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	154	0	0	0	0	0
normalized size	1	1.	1.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.159	18.194	1.02	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	151	0	0	0	0	0
normalized size	1	1.	1.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	11.833	1.264	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	178	0	0	0	0	0
normalized size	1	1.	1.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	2.409	0.335	0.	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	163	0	0	0	0	0
normalized size	1	1.	1.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.198	1.652	0.316	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	163	0	0	0	0	0
normalized size	1	1.	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	1.03	0.324	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	162	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	0.658	0.403	0.	0.	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	162	0	0	0	0	0
normalized size	1	1.	1.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	1.127	0.357	0.	0.	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	178	0	0	0	0	0
normalized size	1	1.	1.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	1.583	0.337	0.	0.	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	178	0	0	0	0	0
normalized size	1	1.	1.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.21	3.149	0.398	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	159	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	8.787	0.73	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	171	3316	0	686	0	0
normalized size	1	1.	1.76	34.19	0.	7.07	0.	0.
time (sec)	N/A	0.07	14.105	0.577	0.	2.207	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	143	1668	0	374	0	0
normalized size	1	1.	2.2	25.66	0.	5.75	0.	0.
time (sec)	N/A	0.056	13.435	0.236	0.	2.176	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	111	31	0	165	0	0
normalized size	1	1.	3.47	0.97	0.	5.16	0.	0.
time (sec)	N/A	0.046	12.984	0.019	0.	2.007	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	141	0	0	0	0	0
normalized size	1	1.	2.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	13.187	1.096	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	143	0	0	0	0	0
normalized size	1	1.	2.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	4.134	0.743	0.	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	143	0	0	0	0	0
normalized size	1	1.	2.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	5.486	0.801	0.	0.	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	149	0	0	0	0	0
normalized size	1	1.	1.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	13.909	0.345	0.	0.	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	149	0	0	0	0	0
normalized size	1	1.	1.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.182	13.218	0.607	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	134	0	0	0	0	0
normalized size	1	1.	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	8.206	0.444	0.	0.	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	136	0	0	0	0	0
normalized size	1	1.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	12.841	0.858	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	149	0	0	0	0	0
normalized size	1	1.	1.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	13.461	1.84	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	149	0	0	0	0	0
normalized size	1	1.	1.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	6.018	0.734	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	156	0	0	0	0	0
normalized size	1	1.	1.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	8.869	0.198	0.	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	156	0	0	0	0	0
normalized size	1	1.	1.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	8.711	0.181	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	156	0	0	0	0	0
normalized size	1	1.	1.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.175	8.349	0.194	0.	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	129	0	0	0	0	0
normalized size	1	1.	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.178	9.535	0.199	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	129	0	0	0	0	0
normalized size	1	1.	1.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	10.195	0.19	0.	0.	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	147	0	0	0	0	0
normalized size	1	1.	1.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.21	10.413	0.181	0.	0.	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	165	5866	583	844	0	0
normalized size	1	1.	0.61	21.81	2.17	3.14	0.	0.
time (sec)	N/A	0.406	0.622	1.361	2.056	2.202	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	119	4990	464	653	0	0
normalized size	1	1.	0.58	24.34	2.26	3.19	0.	0.
time (sec)	N/A	0.305	0.615	1.178	1.904	2.134	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	82	3376	235	406	0	0
normalized size	1	1.	0.55	22.81	1.59	2.74	0.	0.
time (sec)	N/A	0.198	0.22	0.823	2.09	2.394	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	58	2488	153	271	0	0
normalized size	1	1.	0.62	26.47	1.63	2.88	0.	0.
time (sec)	N/A	0.116	0.189	0.87	1.99	2.414	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	874	116	154	0	0
normalized size	1	1.	1.	23.62	3.14	4.16	0.	0.
time (sec)	N/A	0.048	0.037	0.387	1.783	2.197	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	87	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.218	4.44	0.971	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	112	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	12.551	0.967	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	116	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.235	11.064	0.886	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	122	0	1434	419	0	0
normalized size	1	1.	0.78	0.	9.19	2.69	0.	0.
time (sec)	N/A	0.226	2.127	0.758	12.563	2.079	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	166	0	0	0	0	0
normalized size	1	1.	1.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	13.225	0.781	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	91	0	802	317	0	0
normalized size	1	1.	0.93	0.	8.18	3.23	0.	0.
time (sec)	N/A	0.132	1.18	0.746	3.264	1.967	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	166	0	0	0	0	0
normalized size	1	1.	1.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	11.8	0.782	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	59	0	293	240	0	0
normalized size	1	1.	1.28	0.	6.37	5.22	0.	0.
time (sec)	N/A	0.056	0.603	0.782	2.266	1.886	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	154	0	0	0	0	0
normalized size	1	1.	1.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	8.101	0.741	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	146	0	0	0	0	0
normalized size	1	1.	2.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	1.654	0.487	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	157	0	0	0	0	0
normalized size	1	1.	1.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	12.826	1.25	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	151	0	0	0	0	0
normalized size	1	1.	2.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	13.051	1.239	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	166	0	0	0	0	0
normalized size	1	1.	1.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	13.367	1.461	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	165	0	0	0	0	0
normalized size	1	1.	2.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	118.315	1.473	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	165	0	0	0	0	0
normalized size	1	1.	2.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	13.964	1.344	0.	0.	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	150	0	0	0	0	0
normalized size	1	1.	2.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	1.055	0.486	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	1294	185	236	0	0
normalized size	1	1.	1.	32.35	4.62	5.9	0.	0.
time (sec)	N/A	0.057	0.411	0.596	2.472	1.905	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	61	0	410	304	0	0
normalized size	1	1.	0.66	0.	4.46	3.3	0.	0.
time (sec)	N/A	0.128	1.075	0.809	3.06	1.905	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	129	0	833	401	0	0
normalized size	1	1.	0.87	0.	5.63	2.71	0.	0.
time (sec)	N/A	0.211	1.841	0.862	10.37	1.935	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	48	95	147	56	95
normalized size	1	1.	0.88	0.8	1.58	2.45	0.93	1.58
time (sec)	N/A	0.037	0.161	0.042	1.476	1.832	5.741	1.265

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	68	74	116	242	0	190
normalized size	1	1.	0.92	1.	1.57	3.27	0.	2.57
time (sec)	N/A	0.046	0.199	0.039	1.524	1.916	0.	1.302

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	65	116	44	65
normalized size	1	1.	0.93	0.86	1.48	2.64	1.	1.48
time (sec)	N/A	0.032	0.082	0.037	1.583	1.883	3.364	1.212

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	54	82	203	0	134
normalized size	1	1.	1.	1.04	1.58	3.9	0.	2.58
time (sec)	N/A	0.035	0.016	0.039	1.522	1.964	0.	1.316

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	27	81	34	34
normalized size	1	1.	1.	0.89	0.96	2.89	1.21	1.21
time (sec)	N/A	0.028	0.013	0.035	1.504	1.761	2.194	1.287

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	34	42	144	37	73
normalized size	1	1.	1.	1.42	1.75	6.	1.54	3.04
time (sec)	N/A	0.015	0.009	0.011	1.304	1.815	3.832	1.274

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	46	23	31	51	0	174
normalized size	1	1.	1.92	0.96	1.29	2.12	0.	7.25
time (sec)	N/A	0.02	0.019	0.033	1.184	1.847	0.	1.245

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	41	51	86	0	197
normalized size	1	1.	1.07	0.95	1.19	2.	0.	4.58
time (sec)	N/A	0.028	0.054	0.042	2.348	1.841	0.	1.357

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	47	90	0	0
normalized size	1	1.	1.	0.82	1.07	2.05	0.	0.
time (sec)	N/A	0.032	0.012	0.046	1.755	1.765	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	82	127	0	575
normalized size	1	1.	0.95	0.8	1.26	1.95	0.	8.85
time (sec)	N/A	0.041	0.092	0.04	2.318	1.812	0.	1.528

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	133	138	180	282	0	211
normalized size	1	1.	1.12	1.16	1.51	2.37	0.	1.77
time (sec)	N/A	0.107	0.483	0.054	1.599	2.142	0.	1.338

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	104	110	140	231	0	159
normalized size	1	1.	1.07	1.13	1.44	2.38	0.	1.64
time (sec)	N/A	0.083	0.639	0.053	1.352	1.954	0.	1.392

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	54	82	96	184	0	108
normalized size	1	1.	0.72	1.09	1.28	2.45	0.	1.44
time (sec)	N/A	0.066	0.187	0.052	1.525	1.888	0.	1.32

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	46	48	27	131	0	55
normalized size	1	1.	2.09	2.18	1.23	5.95	0.	2.5
time (sec)	N/A	0.037	0.042	0.048	1.363	1.715	0.	1.364

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	52	70	74	120	0	331
normalized size	1	1.	1.06	1.43	1.51	2.45	0.	6.76
time (sec)	N/A	0.053	0.125	0.046	2.248	1.779	0.	1.472

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	216	97	115	170	0	3086
normalized size	1	1.	2.45	1.1	1.31	1.93	0.	35.07
time (sec)	N/A	0.078	2.915	0.054	2.446	1.81	0.	5.973

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	131	235	297	397	0	590
normalized size	1	1.	0.8	1.44	1.82	2.44	0.	3.62
time (sec)	N/A	0.132	0.786	0.053	1.298	2.046	0.	1.637

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	104	189	243	343	0	463
normalized size	1	1.	0.79	1.44	1.85	2.62	0.	3.53
time (sec)	N/A	0.115	0.505	0.052	1.415	1.935	0.	1.554

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	120	143	174	289	0	336
normalized size	1	1.	1.21	1.44	1.76	2.92	0.	3.39
time (sec)	N/A	0.099	0.059	0.054	1.422	2.043	0.	1.493

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	67	98	111	234	0	165
normalized size	1	1.	1.03	1.51	1.71	3.6	0.	2.54
time (sec)	N/A	0.052	0.039	0.021	1.367	1.79	0.	1.49

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	84	63	81	155	0	1777
normalized size	1	1.	1.79	1.34	1.72	3.3	0.	37.81
time (sec)	N/A	0.033	0.139	0.046	1.385	1.802	0.	1.943

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	64	52	70	120	0	0
normalized size	1	1.	0.71	0.58	0.78	1.33	0.	0.
time (sec)	N/A	0.095	0.456	0.056	1.431	1.861	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	116	88	104	169	0	0
normalized size	1	1.	1.02	0.77	0.91	1.48	0.	0.
time (sec)	N/A	0.102	0.21	0.055	1.365	1.905	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	154	108	132	221	0	0
normalized size	1	1.	1.12	0.78	0.96	1.6	0.	0.
time (sec)	N/A	0.107	0.426	0.057	1.522	1.934	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	177	219	238	344	0	297
normalized size	1	1.	0.91	1.13	1.23	1.77	0.	1.53
time (sec)	N/A	0.147	1.96	0.072	1.188	2.065	0.	1.863

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	115	173	192	304	0	224
normalized size	1	1.	0.83	1.25	1.39	2.2	0.	1.62
time (sec)	N/A	0.125	0.571	0.069	1.163	2.12	0.	1.769

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	54	127	132	246	0	151
normalized size	1	1.	0.72	1.69	1.76	3.28	0.	2.01
time (sec)	N/A	0.071	0.347	0.063	1.154	1.886	0.	1.756

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	57	72	27	181	0	77
normalized size	1	1.	2.59	3.27	1.23	8.23	0.	3.5
time (sec)	N/A	0.036	0.155	0.061	1.188	1.839	0.	1.827

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	401	123	109	181	0	811
normalized size	1	1.	4.66	1.43	1.27	2.1	0.	9.43
time (sec)	N/A	0.093	0.746	0.059	1.737	1.982	0.	1.912

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	257	114	149	228	0	3370
normalized size	1	1.	3.06	1.36	1.77	2.71	0.	40.12
time (sec)	N/A	0.069	3.519	0.071	1.778	1.86	0.	22.987

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	177	637	328	281	410	0	628
normalized size	1	1.11	4.01	2.06	1.77	2.58	0.	3.95
time (sec)	N/A	0.145	2.143	0.074	1.174	2.089	0.	1.892

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	144	464	256	212	362	0	450
normalized size	1	1.14	3.68	2.03	1.68	2.87	0.	3.57
time (sec)	N/A	0.13	1.273	0.068	1.068	1.945	0.	1.863

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	109	293	187	150	304	0	231
normalized size	1	1.2	3.22	2.05	1.65	3.34	0.	2.54
time (sec)	N/A	0.085	1.555	0.027	1.122	1.942	0.	1.837

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	102	131	126	113	273	0	6400
normalized size	1	1.21	1.56	1.5	1.35	3.25	0.	76.19
time (sec)	N/A	0.08	1.065	0.049	1.142	2.044	0.	6.002

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	81	75	104	173	0	0
normalized size	1	1.	1.16	1.07	1.49	2.47	0.	0.
time (sec)	N/A	0.069	0.356	0.055	1.122	1.753	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	150	125	144	230	0	0
normalized size	1	1.	1.43	1.19	1.37	2.19	0.	0.
time (sec)	N/A	0.095	0.691	0.065	1.14	1.776	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	204	145	170	278	0	0
normalized size	1	1.	1.44	1.02	1.2	1.96	0.	0.
time (sec)	N/A	0.152	1.054	0.069	1.158	1.985	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	99	162	146	439	0	162
normalized size	1	1.	0.85	1.4	1.26	3.78	0.	1.4
time (sec)	N/A	0.102	1.214	0.08	1.147	2.131	0.	1.738

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	52	72	72	294	0	73
normalized size	1	1.	0.88	1.22	1.22	4.98	0.	1.24
time (sec)	N/A	0.065	0.126	0.064	1.11	1.913	0.	1.506

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	144	0	26
normalized size	1	1.	1.	1.06	1.33	8.	0.	1.44
time (sec)	N/A	0.042	0.014	0.029	1.136	1.956	0.	1.829

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	143	236	190	278	0	246
normalized size	1	1.	1.54	2.54	2.04	2.99	0.	2.65
time (sec)	N/A	0.136	0.225	0.098	1.641	1.935	0.	1.609

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	218	524	366	471	0	435
normalized size	1	1.	1.43	3.45	2.41	3.1	0.	2.86
time (sec)	N/A	0.197	0.395	0.086	1.802	2.054	0.	1.46

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	152	321	488	0	633	0	375
normalized size	1	1.09	2.29	3.49	0.	4.52	0.	2.68
time (sec)	N/A	0.198	1.909	0.088	0.	2.958	0.	1.608

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	109	174	0	471	0	184
normalized size	1	1.	1.38	2.2	0.	5.96	0.	2.33
time (sec)	N/A	0.094	0.141	0.069	0.	2.11	0.	1.655

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	43	0	312	0	100
normalized size	1	1.	0.98	0.93	0.	6.78	0.	2.17
time (sec)	N/A	0.031	0.045	0.031	0.	1.921	0.	1.587

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	79	90	0	436	0	159
normalized size	1	1.	0.88	1.	0.	4.84	0.	1.77
time (sec)	N/A	0.101	0.306	0.08	0.	2.053	0.	1.529

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	137	221	0	598	0	386
normalized size	1	1.	0.83	1.34	0.	3.62	0.	2.34
time (sec)	N/A	0.194	1.349	0.082	0.	1.942	0.	1.541

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	229	305	251	890	0	342
normalized size	1	1.	1.29	1.71	1.41	5.	0.	1.92
time (sec)	N/A	0.152	2.419	0.106	1.135	2.556	0.	1.315

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	122	174	155	656	0	201
normalized size	1	1.	1.05	1.5	1.34	5.66	0.	1.73
time (sec)	N/A	0.097	2.77	0.098	1.134	2.296	0.	1.441

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	51	78	81	440	0	96
normalized size	1	1.	0.84	1.28	1.33	7.21	0.	1.57
time (sec)	N/A	0.066	0.061	0.083	1.068	2.002	0.	1.35

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	32	21	27	132	0	27
normalized size	1	1.	1.6	1.05	1.35	6.6	0.	1.35
time (sec)	N/A	0.039	0.044	0.043	1.129	1.768	0.	1.338

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	304	292	381	614	0	338
normalized size	1	1.	2.	1.92	2.51	4.04	0.	2.22
time (sec)	N/A	0.163	3.751	0.102	1.668	2.122	0.	1.43

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	416	661	678	944	0	626
normalized size	1	1.	1.77	2.81	2.89	4.02	0.	2.66
time (sec)	N/A	0.266	3.165	0.103	1.734	2.546	0.	1.386

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	1152	989	0	1118	0	716
normalized size	1	1.	4.9	4.21	0.	4.76	0.	3.05
time (sec)	N/A	0.268	6.179	0.129	0.	3.881	0.	1.648

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	709	440	0	860	0	378
normalized size	1	1.	4.03	2.5	0.	4.89	0.	2.15
time (sec)	N/A	0.168	6.122	0.111	0.	2.625	0.	1.644

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	132	120	174	0	699	0	224
normalized size	1	1.45	1.32	1.91	0.	7.68	0.	2.46
time (sec)	N/A	0.107	0.748	0.102	0.	2.319	0.	1.657

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	105	78	118	0	502	0	186
normalized size	1	1.28	0.95	1.44	0.	6.12	0.	2.27
time (sec)	N/A	0.073	0.356	0.048	0.	1.81	0.	1.433

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	153	172	0	697	0	386
normalized size	1	1.	0.97	1.1	0.	4.44	0.	2.46
time (sec)	N/A	0.127	0.529	0.098	0.	1.941	0.	1.498

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	249	320	0	946	0	591
normalized size	1	1.	1.03	1.33	0.	3.93	0.	2.45
time (sec)	N/A	0.258	1.169	0.116	0.	2.215	0.	1.613

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	272	321	270	1076	0	328
normalized size	1	1.	1.47	1.74	1.46	5.82	0.	1.77
time (sec)	N/A	0.156	1.331	0.128	1.035	2.538	0.	1.432

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	140	184	173	814	0	189
normalized size	1	1.	1.16	1.52	1.43	6.73	0.	1.56
time (sec)	N/A	0.102	3.109	0.124	1.177	2.134	0.	1.456

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	57	84	105	647	0	84
normalized size	1	1.	0.83	1.22	1.52	9.38	0.	1.22
time (sec)	N/A	0.073	0.177	0.116	1.148	2.264	0.	1.41

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	58	21	27	313	0	27
normalized size	1	1.	2.64	0.95	1.23	14.23	0.	1.23
time (sec)	N/A	0.043	0.174	0.051	1.193	2.164	0.	1.488

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	458	453	618	1102	0	593
normalized size	1	1.	2.27	2.24	3.06	5.46	0.	2.94
time (sec)	N/A	0.233	6.272	0.122	1.826	2.885	0.	1.267

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	596	824	996	1497	0	792
normalized size	1	1.	2.02	2.79	3.38	5.07	0.	2.68
time (sec)	N/A	0.349	6.248	0.13	1.816	2.871	0.	1.389

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	688	1125	0	1299	0	689
normalized size	1	1.	2.88	4.71	0.	5.44	0.	2.88
time (sec)	N/A	0.243	2.21	0.148	0.	3.121	0.	1.636

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	189	396	611	0	1175	0	424
normalized size	1	1.28	2.68	4.13	0.	7.94	0.	2.86
time (sec)	N/A	0.16	2.349	0.127	0.	2.531	0.	1.718

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	118	132	191	0	679	0	298
normalized size	1	1.24	1.39	2.01	0.	7.15	0.	3.14
time (sec)	N/A	0.092	0.293	0.118	0.	2.018	0.	1.911

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	110	280	0	795	0	396
normalized size	1	1.	0.71	1.81	0.	5.13	0.	2.55
time (sec)	N/A	0.114	0.894	0.069	0.	2.034	0.	2.249

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	183	283	0	1073	0	539
normalized size	1	1.	0.83	1.28	0.	4.86	0.	2.44
time (sec)	N/A	0.214	1.842	0.129	0.	2.304	0.	2.279

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	371	457	0	1391	0	864
normalized size	1	1.	1.2	1.47	0.	4.49	0.	2.79
time (sec)	N/A	0.385	1.833	0.155	0.	2.797	0.	2.2

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	69	371	0	0	0	0
normalized size	1	1.	0.57	3.07	0.	0.	0.	0.
time (sec)	N/A	0.093	0.619	0.327	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	58	195	0	0	0	0
normalized size	1	1.	0.63	2.12	0.	0.	0.	0.
time (sec)	N/A	0.069	0.408	0.214	0.	0.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	58	355	0	0	0	0
normalized size	1	1.	0.66	4.03	0.	0.	0.	0.
time (sec)	N/A	0.069	0.288	0.211	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	42	168	0	0	0	0
normalized size	1	1.	0.72	2.9	0.	0.	0.	0.
time (sec)	N/A	0.047	0.193	0.25	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	916	0	0	0	0
normalized size	1	1.	0.93	15.79	0.	0.	0.	0.
time (sec)	N/A	0.049	0.274	0.258	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	69	172	0	0	0	0
normalized size	1	1.	0.73	1.83	0.	0.	0.	0.
time (sec)	N/A	0.069	0.195	0.197	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	74	345	0	0	0	0
normalized size	1	1.	0.79	3.67	0.	0.	0.	0.
time (sec)	N/A	0.07	0.605	0.204	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	94	190	0	0	0	0
normalized size	1	1.	0.76	1.54	0.	0.	0.	0.
time (sec)	N/A	0.091	0.348	0.199	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	127	382	0	0	0	0
normalized size	1	1.	0.89	2.67	0.	0.	0.	0.
time (sec)	N/A	0.16	0.772	0.313	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	126	712	0	0	0	0
normalized size	1	1.	0.88	4.98	0.	0.	0.	0.
time (sec)	N/A	0.163	0.637	0.296	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	87	339	0	0	0	0
normalized size	1	1.	0.84	3.29	0.	0.	0.	0.
time (sec)	N/A	0.123	0.617	0.282	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	64	2564	0	0	0	0
normalized size	1	1.	0.67	26.99	0.	0.	0.	0.
time (sec)	N/A	0.126	0.886	0.309	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	101	320	0	0	0	0
normalized size	1	1.	0.73	2.3	0.	0.	0.	0.
time (sec)	N/A	0.162	0.519	0.277	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	92	670	0	0	0	0
normalized size	1	1.	0.63	4.62	0.	0.	0.	0.
time (sec)	N/A	0.166	0.922	0.275	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	127	359	0	0	0	0
normalized size	1	1.	0.69	1.95	0.	0.	0.	0.
time (sec)	N/A	0.192	2.222	0.281	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	126	697	0	0	0	0
normalized size	1	1.	0.68	3.79	0.	0.	0.	0.
time (sec)	N/A	0.192	2.843	0.352	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	157	414	0	0	0	0
normalized size	1	1.	0.79	2.09	0.	0.	0.	0.
time (sec)	N/A	0.154	1.691	0.382	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	155	759	0	0	0	0
normalized size	1	1.	0.88	4.31	0.	0.	0.	0.
time (sec)	N/A	0.146	1.784	0.34	0.	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	132	373	0	0	0	0
normalized size	1	1.	1.02	2.89	0.	0.	0.	0.
time (sec)	N/A	0.112	2.031	0.327	0.	0.	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	130	5006	0	0	0	0
normalized size	1	1.	0.73	28.12	0.	0.	0.	0.
time (sec)	N/A	0.141	1.891	0.346	0.	0.	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	117	342	0	0	0	0
normalized size	1	1.	0.8	2.34	0.	0.	0.	0.
time (sec)	N/A	0.124	1.308	0.303	0.	0.	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	150	1920	0	0	0	0
normalized size	1	1.	0.74	9.41	0.	0.	0.	0.
time (sec)	N/A	0.157	1.461	0.314	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	150	391	0	0	0	0
normalized size	1	1.	0.88	2.3	0.	0.	0.	0.
time (sec)	N/A	0.143	2.441	0.304	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	372	745	0	0	0	0
normalized size	1	1.	2.11	4.23	0.	0.	0.	0.
time (sec)	N/A	0.139	6.378	0.384	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	296	430	0	0	0	0
normalized size	1	1.	1.36	1.97	0.	0.	0.	0.
time (sec)	N/A	0.167	6.438	0.387	0.	0.	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	456	456	11149	26371	0	0	0	0
normalized size	1	1.	24.45	57.83	0.	0.	0.	0.
time (sec)	N/A	0.408	29.331	0.654	0.	0.	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	396	396	6631	10704	0	0	0	0
normalized size	1	1.	16.74	27.03	0.	0.	0.	0.
time (sec)	N/A	0.384	24.628	0.497	0.	0.	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	334	334	6301	3737	0	0	0	0
normalized size	1	1.	18.87	11.19	0.	0.	0.	0.
time (sec)	N/A	0.294	21.676	0.326	0.	0.	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	4648	3130	0	0	0	0
normalized size	1	1.	14.35	9.66	0.	0.	0.	0.
time (sec)	N/A	0.301	24.067	0.357	0.	0.	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	451	451	4693	8821	0	0	0	0
normalized size	1	1.	10.41	19.56	0.	0.	0.	0.
time (sec)	N/A	0.423	30.349	0.433	0.	0.	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	422	422	9313	6252	0	0	0	0
normalized size	1	1.	22.07	14.82	0.	0.	0.	0.
time (sec)	N/A	0.432	26.574	0.313	0.	0.	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	568	568	17838	14547	0	0	0	0
normalized size	1	1.	31.4	25.61	0.	0.	0.	0.
time (sec)	N/A	0.609	32.674	0.536	0.	0.	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	480	480	1129	44463	0	0	0	0
normalized size	1	1.	2.35	92.63	0.	0.	0.	0.
time (sec)	N/A	0.382	22.145	4.405	0.	0.	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	440	440	3091	5329	0	0	0	0
normalized size	1	1.	7.02	12.11	0.	0.	0.	0.
time (sec)	N/A	0.395	21.802	0.607	0.	0.	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	477	477	6560	25422	0	0	0	0
normalized size	1	1.	13.75	53.3	0.	0.	0.	0.
time (sec)	N/A	0.385	29.786	1.04	0.	0.	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	430	430	8876	14318	0	0	0	0
normalized size	1	1.	20.64	33.3	0.	0.	0.	0.
time (sec)	N/A	0.38	26.126	0.877	0.	0.	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	555	555	17812	38627	0	0	0	0
normalized size	1	1.	32.09	69.6	0.	0.	0.	0.
time (sec)	N/A	0.537	32.126	2.362	0.	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	520	520	11962	15455	0	0	0	0
normalized size	1	1.	23.	29.72	0.	0.	0.	0.
time (sec)	N/A	0.566	27.634	0.838	0.	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	700	700	18542	44337	0	0	0	0
normalized size	1	1.	26.49	63.34	0.	0.	0.	0.
time (sec)	N/A	0.714	31.922	1.676	0.	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	583	583	14257	101372	0	0	0	0
normalized size	1	1.	24.45	173.88	0.	0.	0.	0.
time (sec)	N/A	0.517	29.018	5.917	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	532	532	4504	45973	0	0	0	0
normalized size	1	1.	8.47	86.42	0.	0.	0.	0.
time (sec)	N/A	0.498	25.755	3.185	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	566	566	14396	80250	0	0	0	0
normalized size	1	1.	25.43	141.78	0.	0.	0.	0.
time (sec)	N/A	0.539	29.113	4.376	0.	0.	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	515	515	4471	82060	0	0	0	0
normalized size	1	1.	8.68	159.34	0.	0.	0.	0.
time (sec)	N/A	0.522	25.245	4.334	0.	0.	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	664	664	14684	100394	0	0	0	0
normalized size	1	1.	22.11	151.2	0.	0.	0.	0.
time (sec)	N/A	0.761	29.233	11.363	0.	0.	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	620	620	4725	82289	0	0	0	0
normalized size	1	1.	7.62	132.72	0.	0.	0.	0.
time (sec)	N/A	0.759	24.883	2.92	0.	0.	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	814	814	15513	114407	0	0	0	0
normalized size	1	1.	19.06	140.55	0.	0.	0.	0.
time (sec)	N/A	0.915	28.701	4.79	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	126	0	0	0	0	0
normalized size	1	1.	1.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.507	0.1	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	58	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.174	0.124	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	119	0	0	0	0	0
normalized size	1	1.	1.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.929	0.138	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	94	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.434	0.094	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	108	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	1.737	0.147	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	83	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	0.461	0.141	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	209	0	0	0	0	0
normalized size	1	1.	1.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	3.918	0.144	0.	0.	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.488	0.142	0.	0.	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	552	552	276	0	0	0	0	0
normalized size	1	1.	0.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.849	5.833	0.174	0.	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	552	552	280	0	0	0	0	0
normalized size	1	1.	0.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.769	4.347	0.175	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	579	579	285	0	0	0	0	0
normalized size	1	1.	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.85	21.344	0.167	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	581	581	6862	0	0	0	0	0
normalized size	1	1.	11.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.819	31.737	0.169	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	687	687	3398	0	0	0	0	0
normalized size	1	1.	4.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.944	39.189	0.278	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	687	687	4485	0	0	0	0	0
normalized size	1	1.	6.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.884	25.496	0.274	0.	0.	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	715	715	9626	0	0	0	0	0
normalized size	1	1.	13.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.994	51.389	0.278	0.	0.	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	717	717	5235	0	0	0	0	0
normalized size	1	1.	7.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.969	27.466	0.272	0.	0.	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	167	175	0	0	0	0	0
normalized size	1	0.97	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	57.256	0.543	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	11095	0	0	0	0	0
normalized size	1	1.	75.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	26.543	0.325	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	3302	0	0	0	0	0
normalized size	1	1.	35.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	16.828	0.501	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	141	141	1158	0	0	0	0	0
normalized size	1	1.	8.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	15.053	0.228	0.	0.	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	356	0	0	0	0	0
normalized size	1	1.	1.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.198	3.249	0.267	0.	0.	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	181	187	699	0	0	0	0	0
normalized size	1	1.03	3.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	6.203	0.284	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	161	0	0	899	0	0
normalized size	1	1.	1.	0.	0.	5.58	0.	0.
time (sec)	N/A	0.123	2.962	0.245	0.	3.442	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	71	0	0	392	0	0
normalized size	1	1.	0.81	0.	0.	4.45	0.	0.
time (sec)	N/A	0.076	0.24	0.197	0.	2.777	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	0	155	0	35
normalized size	1	1.	1.	1.04	0.	5.96	0.	1.35
time (sec)	N/A	0.044	0.178	0.017	0.	2.559	0.	2.963

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	225	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.457	1.24	0.377	0.	0.	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	434	434	360	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.687	4.34	0.328	0.	0.	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	306	0	0	0	0	0
normalized size	1	1.	1.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.164	3.915	0.183	0.	0.	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	340	0	0	0	0	0
normalized size	1	1.	2.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	3.759	0.346	0.	0.	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	341	0	0	0	0	0
normalized size	1	1.	2.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	4.786	0.156	0.	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	341	0	0	0	0	0
normalized size	1	1.	2.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	6.176	0.613	0.	0.	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	133	241	0	0	0	0
normalized size	1	1.	1.07	1.94	0.	0.	0.	0.
time (sec)	N/A	0.135	0.689	2.749	0.	0.	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	387	205	0	0	0	0
normalized size	1	1.	4.3	2.28	0.	0.	0.	0.
time (sec)	N/A	0.111	12.476	2.346	0.	0.	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	100	168	0	0	0	0
normalized size	1	1.	1.11	1.87	0.	0.	0.	0.
time (sec)	N/A	0.109	0.343	2.457	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	244	108	0	0	0	0
normalized size	1	1.	4.07	1.8	0.	0.	0.	0.
time (sec)	N/A	0.078	0.691	1.352	0.	0.	0.	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	143	94	0	0	0	0
normalized size	1	1.	2.38	1.57	0.	0.	0.	0.
time (sec)	N/A	0.079	1.114	2.592	0.	0.	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	92	369	214	0	0	0	0
normalized size	1	1.03	4.15	2.4	0.	0.	0.	0.
time (sec)	N/A	0.105	3.953	3.768	0.	0.	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	57	283	0	0	0	0
normalized size	1	1.	0.59	2.95	0.	0.	0.	0.
time (sec)	N/A	0.104	0.495	4.727	0.	0.	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	596	396	0	0	0	0
normalized size	1	1.	4.58	3.05	0.	0.	0.	0.
time (sec)	N/A	0.127	6.471	8.012	0.	0.	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	156	387	0	0	0	0
normalized size	1	1.	0.82	2.04	0.	0.	0.	0.
time (sec)	N/A	0.221	1.236	3.086	0.	0.	0.	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	471	351	0	0	0	0
normalized size	1	1.	3.06	2.28	0.	0.	0.	0.
time (sec)	N/A	0.191	3.124	3.112	0.	0.	0.	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	131	315	0	0	0	0
normalized size	1	1.	0.85	2.05	0.	0.	0.	0.
time (sec)	N/A	0.198	0.661	2.975	0.	0.	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	126	420	277	0	0	0	0
normalized size	1	1.05	3.5	2.31	0.	0.	0.	0.
time (sec)	N/A	0.167	1.75	2.736	0.	0.	0.	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	126	158	240	0	0	0	0
normalized size	1	1.05	1.32	2.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.64	2.564	0.	0.	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	244	207	0	0	0	0
normalized size	1	1.	2.65	2.25	0.	0.	0.	0.
time (sec)	N/A	0.154	0.675	2.392	0.	0.	0.	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	116	170	0	0	0	0
normalized size	1	1.	1.26	1.85	0.	0.	0.	0.
time (sec)	N/A	0.15	0.369	2.291	0.	0.	0.	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	255	135	0	0	0	0
normalized size	1	1.	2.09	1.11	0.	0.	0.	0.
time (sec)	N/A	0.171	0.985	1.97	0.	0.	0.	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	67	208	0	0	0	0
normalized size	1	1.	0.53	1.65	0.	0.	0.	0.
time (sec)	N/A	0.172	0.358	3.309	0.	0.	0.	0.

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	406	321	0	0	0	0
normalized size	1	1.	2.48	1.96	0.	0.	0.	0.
time (sec)	N/A	0.191	5.812	5.024	0.	0.	0.	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	80	97	273	300	0	0
normalized size	1	1.	0.45	0.54	1.53	1.68	0.	0.
time (sec)	N/A	0.383	0.536	0.388	2.924	2.073	0.	0.

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	63	80	200	255	0	0
normalized size	1	1.	0.48	0.61	1.52	1.93	0.	0.
time (sec)	N/A	0.288	0.337	0.373	3.116	2.065	0.	0.

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	86	56	70	80	204	0	0
normalized size	1	1.01	0.66	0.82	0.94	2.4	0.	0.
time (sec)	N/A	0.212	0.203	0.338	3.022	2.061	0.	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	45	103	157	0	0
normalized size	1	1.	1.	1.25	2.86	4.36	0.	0.
time (sec)	N/A	0.129	0.171	0.322	2.362	2.103	0.	0.

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	125	226	1890	917	0	0
normalized size	1	1.	0.37	0.67	5.64	2.74	0.	0.
time (sec)	N/A	0.214	0.849	0.371	3.137	2.263	0.	0.

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	620	274	308	2476	1303	0	0
normalized size	1	1.18	0.52	0.59	4.73	2.49	0.	0.
time (sec)	N/A	0.59	3.893	0.412	3.34	2.248	0.	0.

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	512	512	227	366	3058	1628	0	0
normalized size	1	1.	0.44	0.71	5.97	3.18	0.	0.
time (sec)	N/A	0.564	2.387	0.432	3.603	2.577	0.	0.

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	719	719	305	417	3606	1883	0	0
normalized size	1	1.	0.42	0.58	5.02	2.62	0.	0.
time (sec)	N/A	0.86	2.981	0.379	4.327	2.729	0.	0.

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	80	110	273	305	0	0
normalized size	1	1.	0.46	0.63	1.56	1.74	0.	0.
time (sec)	N/A	0.378	0.53	0.336	3.248	2.135	0.	0.

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	63	100	184	252	0	0
normalized size	1	1.	0.5	0.79	1.46	2.	0.	0.
time (sec)	N/A	0.313	0.336	0.35	3.569	2.063	0.	0.

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	48	74	108	204	0	0
normalized size	1	1.	0.6	0.92	1.35	2.55	0.	0.
time (sec)	N/A	0.209	0.19	0.364	3.144	2.098	0.	0.

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	69	103	165	0	0
normalized size	1	1.	1.	1.92	2.86	4.58	0.	0.
time (sec)	N/A	0.139	0.173	0.45	2.307	2.049	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	495	209	232	964	973	0	0
normalized size	1	1.	0.42	0.47	1.95	1.97	0.	0.
time (sec)	N/A	0.332	9.516	0.362	3.32	2.298	0.	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	470	470	250	313	2909	1331	0	0
normalized size	1	1.	0.53	0.67	6.19	2.83	0.	0.
time (sec)	N/A	0.441	1.691	0.356	3.326	2.259	0.	0.

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	682	682	245	371	3056	1669	0	0
normalized size	1	1.	0.36	0.54	4.48	2.45	0.	0.
time (sec)	N/A	0.778	1.577	0.392	3.931	2.431	0.	0.

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	192	0	0	0	0	0
normalized size	1	1.	1.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.231	13.21	0.725	0.	0.	0.	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	125	0	0	0	0	0
normalized size	1	1.	1.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.219	1.586	0.419	0.	0.	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	131	0	0	0	0	0
normalized size	1	1.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.175	7.578	0.506	0.	0.	0.	0.

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	147	0	0	0	0	0
normalized size	1	1.	1.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.238	59.159	0.868	0.	0.	0.	0.

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	154	0	0	0	0	0
normalized size	1	1.	1.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.237	69.348	1.24	0.	0.	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	106	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.286	0.807	0.388	0.	0.	0.	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	143	0	0	0	0	0
normalized size	1	1.	1.38	0.	0.	0.	0.	0.
time (sec)	N/A	0.29	14.894	0.363	0.	0.	0.	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	175	215	0	0	0	0	0
normalized size	1	0.96	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.285	59.059	0.644	0.	0.	0.	0.

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	330	0	0	0	0	0
normalized size	1	1.	2.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.238	3.735	0.371	0.	0.	0.	0.

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	91	203	0	0	0	0	0
normalized size	1	1.01	2.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	1.027	0.558	0.	0.	0.	0.

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	1132	0	0	0	0	0
normalized size	1	1.	8.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.211	14.102	0.216	0.	0.	0.	0.

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	361	0	0	0	0	0
normalized size	1	1.	1.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.277	3.128	0.306	0.	0.	0.	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	698	0	0	0	0	0
normalized size	1	1.	3.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	22.484	0.239	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [616] had the largest ratio of [0.68]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	22	0.091
2	A	3	2	1.	22	0.091
3	A	3	2	1.	22	0.091
4	A	3	2	1.	22	0.091
5	A	3	3	1.11	22	0.136
6	A	2	1	1.	13	0.077
7	A	3	3	1.	22	0.136
8	A	4	3	1.	22	0.136
9	A	5	3	1.	22	0.136
10	A	6	3	1.	22	0.136
11	A	5	3	1.	22	0.136
12	A	4	3	1.	22	0.136
13	A	3	3	1.	22	0.136
14	A	2	2	1.	20	0.1
15	A	2	2	1.	20	0.1
16	A	3	2	1.	22	0.091
17	A	3	2	1.	22	0.091
18	A	3	2	1.	22	0.091
19	A	3	2	1.	24	0.083
20	A	3	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	3	2	1.	24	0.083
22	A	2	2	1.	24	0.083
23	A	2	2	1.	15	0.133
24	A	2	2	1.	24	0.083
25	A	4	3	1.	24	0.125
26	A	4	3	1.	24	0.125
27	A	4	3	1.	24	0.125
28	A	5	4	1.	24	0.167
29	A	4	4	1.	24	0.167
30	A	3	3	1.	22	0.136
31	A	2	2	1.	22	0.091
32	A	2	2	1.	24	0.083
33	A	3	2	1.	24	0.083
34	A	3	2	1.	24	0.083
35	A	3	2	1.	24	0.083
36	A	3	2	1.	24	0.083
37	A	3	2	1.	24	0.083
38	A	3	2	1.	24	0.083
39	A	2	2	1.	24	0.083
40	A	3	3	1.	15	0.2
41	A	3	2	1.	24	0.083
42	A	2	2	1.	24	0.083
43	A	4	3	1.	24	0.125
44	A	4	3	1.	24	0.125
45	A	5	4	1.	24	0.167
46	A	4	3	1.	22	0.136
47	A	3	3	1.	22	0.136
48	A	1	1	1.	24	0.042
49	A	4	3	1.	24	0.125
50	A	4	3	1.	24	0.125
51	A	4	3	1.	24	0.125
52	A	6	4	1.	24	0.167
53	A	5	3	1.	22	0.136
54	A	4	4	1.	22	0.182
55	A	3	2	1.	24	0.083
56	A	2	2	1.	24	0.083
57	A	4	2	1.	24	0.083
58	A	4	2	1.	24	0.083
59	A	3	2	1.	24	0.083
60	A	3	2	1.	24	0.083
61	A	3	2	1.	24	0.083
62	A	2	2	1.	24	0.083
63	A	5	3	1.	15	0.2
64	A	3	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
65	A	3	2	1.	24	0.083
66	A	3	2	1.	24	0.083
67	A	2	2	1.	24	0.083
68	A	4	3	1.	24	0.125
69	A	4	3	1.	24	0.125
70	A	6	3	1.	22	0.136
71	A	5	4	1.	22	0.182
72	A	4	3	1.	24	0.125
73	A	1	1	1.	24	0.042
74	A	3	2	1.	24	0.083
75	A	5	3	1.	24	0.125
76	A	5	3	1.	24	0.125
77	A	3	2	1.	24	0.083
78	A	3	2	1.	24	0.083
79	A	3	2	1.	24	0.083
80	A	2	2	1.	24	0.083
81	A	8	3	1.	15	0.2
82	A	3	2	1.	24	0.083
83	A	3	2	1.	24	0.083
84	A	3	2	1.	24	0.083
85	A	2	2	1.	24	0.083
86	A	3	2	1.	24	0.083
87	A	3	2	1.	24	0.083
88	A	2	2	1.	24	0.083
89	A	4	3	1.	24	0.125
90	A	4	3	1.	24	0.125
91	A	8	4	1.	22	0.182
92	A	7	4	1.	24	0.167
93	A	6	4	1.	24	0.167
94	A	5	2	1.	24	0.083
95	A	2	2	1.	24	0.083
96	A	4	2	1.	24	0.083
97	A	6	2	1.	24	0.083
98	A	6	2	1.	24	0.083
99	A	3	2	1.	24	0.083
100	A	3	2	1.	24	0.083
101	A	3	2	1.	24	0.083
102	A	2	1	1.	24	0.042
103	A	2	2	1.	24	0.083
104	A	2	2	1.	15	0.133
105	A	4	3	1.	24	0.125
106	A	4	3	1.	24	0.125
107	A	4	3	1.	24	0.125
108	A	3	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
109	A	2	2	1.	24	0.083
110	A	1	1	1.	22	0.045
111	A	2	2	1.	22	0.091
112	A	3	2	1.	24	0.083
113	A	3	2	1.	24	0.083
114	A	3	2	1.	24	0.083
115	A	3	2	1.	24	0.083
116	A	2	2	1.	24	0.083
117	A	3	2	1.	24	0.083
118	A	2	2	1.	24	0.083
119	A	3	2	1.	15	0.133
120	A	4	3	1.	24	0.125
121	A	4	3	1.	24	0.125
122	A	5	3	1.	24	0.125
123	A	4	3	1.	24	0.125
124	A	3	3	1.	24	0.125
125	A	2	2	1.	24	0.083
126	A	2	2	1.	22	0.091
127	A	3	2	1.	22	0.091
128	A	3	2	1.	24	0.083
129	A	3	2	1.	24	0.083
130	A	3	2	1.	24	0.083
131	A	3	2	1.	24	0.083
132	A	3	2	1.	24	0.083
133	A	2	2	1.	24	0.083
134	A	3	2	1.	24	0.083
135	A	3	2	1.	24	0.083
136	A	2	2	1.	24	0.083
137	A	4	2	1.	15	0.133
138	A	4	3	1.	24	0.125
139	A	4	3	1.	24	0.125
140	A	5	4	1.	24	0.167
141	A	4	4	1.	24	0.167
142	A	3	3	1.	24	0.125
143	A	1	1	1.	24	0.042
144	A	3	2	1.	22	0.091
145	A	4	3	1.	22	0.136
146	A	4	3	1.	24	0.125
147	A	4	3	1.	24	0.125
148	A	3	2	1.	24	0.083
149	A	3	2	1.	24	0.083
150	A	2	2	1.	24	0.083
151	A	3	2	1.	24	0.083
152	A	3	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	2	2	1.	24	0.083
154	A	2	2	1.	24	0.083
155	A	5	2	1.	15	0.133
156	A	4	3	1.	24	0.125
157	A	4	3	1.	24	0.125
158	A	5	3	1.	24	0.125
159	A	4	3	1.	24	0.125
160	A	3	2	1.	24	0.083
161	A	2	2	1.	24	0.083
162	A	4	2	1.	22	0.091
163	A	5	3	1.	22	0.136
164	A	5	3	1.	24	0.125
165	A	5	3	1.	24	0.125
166	A	3	2	1.	24	0.083
167	A	3	2	1.	24	0.083
168	A	3	2	1.	24	0.083
169	A	2	2	1.	24	0.083
170	A	3	2	1.	24	0.083
171	A	3	2	1.	24	0.083
172	A	2	2	1.	24	0.083
173	A	9	2	1.	15	0.133
174	A	4	3	1.	24	0.125
175	A	4	3	1.	24	0.125
176	A	7	3	1.	24	0.125
177	A	6	3	1.	24	0.125
178	A	5	2	1.	24	0.083
179	A	2	2	1.	24	0.083
180	A	4	2	1.	24	0.083
181	A	6	2	1.	24	0.083
182	A	8	2	1.	22	0.091
183	A	9	3	1.	22	0.136
184	A	9	3	1.	24	0.125
185	A	5	4	1.	26	0.154
186	A	4	4	1.	26	0.154
187	A	4	4	1.	26	0.154
188	A	3	3	1.	26	0.115
189	A	3	3	1.	26	0.115
190	A	4	4	1.	26	0.154
191	A	4	4	1.	26	0.154
192	A	5	4	1.	26	0.154
193	A	5	5	1.	28	0.179
194	A	4	4	1.	28	0.143
195	A	4	4	1.	28	0.143
196	A	3	3	1.	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	3	3	1.	28	0.107
198	A	4	4	1.	28	0.143
199	A	4	4	1.	28	0.143
200	A	5	4	1.	28	0.143
201	A	7	5	1.	28	0.179
202	A	6	5	1.	28	0.179
203	A	6	5	1.	28	0.179
204	A	5	4	1.	28	0.143
205	A	5	5	1.18	28	0.179
206	A	4	4	1.	28	0.143
207	A	4	4	1.	28	0.143
208	A	4	4	1.	28	0.143
209	A	4	4	1.	28	0.143
210	A	5	5	1.	28	0.179
211	A	5	5	1.	28	0.179
212	A	6	5	1.	28	0.179
213	A	7	5	1.	28	0.179
214	A	6	4	1.	28	0.143
215	A	6	6	1.	28	0.214
216	A	5	5	1.	28	0.179
217	A	5	4	1.	28	0.143
218	A	4	3	1.	28	0.107
219	A	4	3	1.	28	0.107
220	A	5	4	1.	28	0.143
221	A	5	4	1.	28	0.143
222	A	6	4	1.	28	0.143
223	A	5	4	1.	28	0.143
224	A	4	4	1.	28	0.143
225	A	4	4	1.	28	0.143
226	A	3	3	1.	28	0.107
227	A	3	3	1.	28	0.107
228	A	3	3	1.	28	0.107
229	A	3	3	1.	28	0.107
230	A	4	4	1.	28	0.143
231	A	4	4	1.	28	0.143
232	A	5	4	1.	28	0.143
233	A	6	4	1.	28	0.143
234	A	5	4	1.	28	0.143
235	A	5	4	1.	28	0.143
236	A	4	4	1.	28	0.143
237	A	4	4	1.	28	0.143
238	A	3	3	1.	28	0.107
239	A	3	3	1.	28	0.107
240	A	4	4	1.	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
241	A	4	4	1.	28	0.143
242	A	5	4	1.	28	0.143
243	A	5	4	1.	28	0.143
244	A	6	4	1.	28	0.143
245	A	6	5	1.	28	0.179
246	A	5	5	1.	28	0.179
247	A	5	5	1.	28	0.179
248	A	4	4	1.	28	0.143
249	A	4	4	1.	28	0.143
250	A	4	4	1.	28	0.143
251	A	4	4	1.	28	0.143
252	A	5	5	1.	28	0.179
253	A	5	5	1.	28	0.179
254	A	6	5	1.	28	0.179
255	A	6	4	1.	28	0.143
256	A	5	4	1.	28	0.143
257	A	5	4	1.	28	0.143
258	A	4	3	1.	28	0.107
259	A	4	3	1.	28	0.107
260	A	5	4	1.	28	0.143
261	A	5	4	1.	28	0.143
262	A	6	5	1.	28	0.179
263	A	4	4	1.	26	0.154
264	A	4	4	1.	26	0.154
265	A	4	4	1.	26	0.154
266	A	4	4	1.	26	0.154
267	A	4	4	1.	28	0.143
268	A	4	4	1.	28	0.143
269	A	4	4	1.	28	0.143
270	A	4	4	1.	28	0.143
271	A	4	4	1.	28	0.143
272	A	4	4	1.	28	0.143
273	A	4	4	1.	28	0.143
274	A	4	4	1.	28	0.143
275	A	4	4	1.	28	0.143
276	A	4	4	1.	28	0.143
277	A	4	4	1.	28	0.143
278	A	4	4	1.	28	0.143
279	A	3	2	1.	26	0.077
280	A	3	2	1.	26	0.077
281	A	3	2	1.	26	0.077
282	A	2	2	1.	26	0.077
283	A	5	4	1.	26	0.154
284	A	7	4	1.	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
285	A	9	4	1.	26	0.154
286	A	4	2	1.	26	0.077
287	A	3	2	1.	26	0.077
288	A	2	2	1.	26	0.077
289	A	1	1	1.	24	0.042
290	A	3	3	1.	24	0.125
291	A	5	5	1.	26	0.192
292	A	7	5	1.	26	0.192
293	A	3	2	1.	26	0.077
294	A	3	2	1.	26	0.077
295	A	3	2	1.	26	0.077
296	A	2	2	1.	26	0.077
297	A	4	4	1.	26	0.154
298	A	6	4	1.	26	0.154
299	A	8	4	1.	26	0.154
300	A	4	2	1.	26	0.077
301	A	3	2	1.	26	0.077
302	A	2	2	1.	24	0.083
303	A	1	1	1.	24	0.042
304	A	4	3	1.	26	0.115
305	A	6	5	1.	26	0.192
306	A	3	2	1.	26	0.077
307	A	3	2	1.	26	0.077
308	A	3	2	1.	26	0.077
309	A	2	2	1.	26	0.077
310	A	4	4	1.	26	0.154
311	A	5	4	1.	26	0.154
312	A	7	4	1.	26	0.154
313	A	4	2	1.	26	0.077
314	A	3	2	1.	24	0.083
315	A	2	2	1.	24	0.083
316	A	1	1	1.	26	0.038
317	A	5	3	1.	26	0.115
318	A	7	5	1.	26	0.192
319	A	3	2	1.	26	0.077
320	A	3	2	1.	26	0.077
321	A	3	2	1.	26	0.077
322	A	2	2	1.	26	0.077
323	A	5	5	1.	26	0.192
324	A	5	5	1.	26	0.192
325	A	6	4	1.	26	0.154
326	A	4	2	1.	24	0.083
327	A	3	2	1.	24	0.083
328	A	2	2	1.	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	1	1	1.	26	0.038
330	A	6	3	1.	26	0.115
331	A	8	5	1.	26	0.192
332	A	10	5	1.	26	0.192
333	A	3	2	1.	26	0.077
334	A	3	2	1.	26	0.077
335	A	3	2	1.	26	0.077
336	A	2	2	1.	26	0.077
337	A	6	4	1.	26	0.154
338	A	8	4	1.	26	0.154
339	A	10	4	1.	26	0.154
340	A	4	2	1.	26	0.077
341	A	3	2	1.	26	0.077
342	A	2	2	1.	26	0.077
343	A	1	1	1.	26	0.038
344	A	2	2	1.	24	0.083
345	A	4	4	1.	24	0.167
346	A	6	5	1.	26	0.192
347	A	3	2	1.	26	0.077
348	A	3	2	1.	26	0.077
349	A	3	2	1.	26	0.077
350	A	2	2	1.	26	0.077
351	A	7	4	1.	26	0.154
352	A	9	4	1.	26	0.154
353	A	11	4	1.	26	0.154
354	A	4	2	1.	26	0.077
355	A	3	2	1.	26	0.077
356	A	2	2	1.	26	0.077
357	A	1	1	1.	26	0.038
358	A	3	3	1.	26	0.115
359	A	3	3	1.	24	0.125
360	A	5	4	1.	24	0.167
361	A	7	5	1.	26	0.192
362	A	3	2	1.	26	0.077
363	A	3	2	1.	26	0.077
364	A	3	2	1.	26	0.077
365	A	3	2	1.	26	0.077
366	A	2	2	1.	26	0.077
367	A	8	4	1.	26	0.154
368	A	10	4	1.	26	0.154
369	A	4	2	1.	26	0.077
370	A	3	2	1.	26	0.077
371	A	2	2	1.	26	0.077
372	A	1	1	1.	26	0.038

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
373	A	4	3	1.	26	0.115
374	A	4	4	1.	26	0.154
375	A	4	3	1.	24	0.125
376	A	6	4	1.	24	0.167
377	A	8	5	1.	26	0.192
378	A	3	2	1.	26	0.077
379	A	3	2	1.	26	0.077
380	A	3	2	1.	26	0.077
381	A	3	2	1.	26	0.077
382	A	2	2	1.	26	0.077
383	A	9	4	1.	26	0.154
384	A	11	4	1.	26	0.154
385	A	3	2	1.	26	0.077
386	A	2	2	1.	26	0.077
387	A	1	1	1.	26	0.038
388	A	5	3	1.	26	0.115
389	A	5	4	1.	26	0.154
390	A	5	4	1.	26	0.154
391	A	5	3	1.	24	0.125
392	A	7	4	1.	24	0.167
393	A	9	5	1.	26	0.192
394	A	12	9	1.	30	0.3
395	A	10	7	1.	30	0.233
396	A	1	1	1.	30	0.033
397	A	2	2	1.	30	0.067
398	A	3	3	1.	30	0.1
399	A	4	3	1.	30	0.1
400	A	13	9	1.	30	0.3
401	A	13	9	1.	30	0.3
402	A	11	8	1.	30	0.267
403	A	12	9	1.	30	0.3
404	A	1	1	1.	30	0.033
405	A	2	2	1.	30	0.067
406	A	3	2	1.	30	0.067
407	A	4	3	1.	30	0.1
408	A	14	9	1.	30	0.3
409	A	12	8	1.	30	0.267
410	A	13	10	1.	30	0.333
411	A	11	8	1.	30	0.267
412	A	1	1	1.	30	0.033
413	A	2	2	1.	30	0.067
414	A	3	2	1.	30	0.067
415	A	4	2	1.	30	0.067
416	A	11	8	1.	30	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
417	A	11	8	1.	30	0.267
418	A	1	1	1.	30	0.033
419	A	2	2	1.	30	0.067
420	A	3	3	1.	30	0.1
421	A	4	3	1.	30	0.1
422	A	5	3	1.	30	0.1
423	A	13	10	1.	30	0.333
424	A	11	8	1.	30	0.267
425	A	1	1	1.	30	0.033
426	A	2	2	1.	30	0.067
427	A	3	2	1.	30	0.067
428	A	4	3	1.	30	0.1
429	A	5	3	1.	30	0.1
430	A	12	9	1.	30	0.3
431	A	12	9	1.	30	0.3
432	A	1	1	1.	30	0.033
433	A	2	2	1.	30	0.067
434	A	3	2	1.	30	0.067
435	A	4	2	1.	30	0.067
436	A	5	3	1.	30	0.1
437	A	4	4	1.	30	0.133
438	A	4	4	1.	30	0.133
439	A	4	4	1.	30	0.133
440	A	4	4	1.	30	0.133
441	A	4	4	1.	30	0.133
442	A	4	4	1.	30	0.133
443	A	9	8	1.	30	0.267
444	A	8	8	1.	30	0.267
445	A	6	6	1.	30	0.2
446	A	1	1	1.	30	0.033
447	A	2	2	1.	30	0.067
448	A	3	2	1.	30	0.067
449	A	4	2	1.	30	0.067
450	A	4	4	1.	26	0.154
451	A	4	4	1.	26	0.154
452	A	4	4	1.	26	0.154
453	A	4	4	1.	24	0.167
454	A	4	4	1.	26	0.154
455	A	4	4	1.	26	0.154
456	A	4	4	1.	26	0.154
457	A	4	4	1.	28	0.143
458	A	4	4	1.	28	0.143
459	A	4	4	1.	28	0.143
460	A	4	4	1.	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
461	A	4	4	1.	28	0.143
462	A	4	4	1.	28	0.143
463	A	4	4	1.	28	0.143
464	A	4	4	1.	26	0.154
465	A	3	2	1.	24	0.083
466	A	3	2	1.	24	0.083
467	A	2	2	1.	24	0.083
468	A	2	2	1.	24	0.083
469	A	2	2	1.	24	0.083
470	A	2	2	1.	24	0.083
471	A	4	4	1.	24	0.167
472	A	4	4	1.	24	0.167
473	A	4	4	1.	22	0.182
474	A	4	4	1.	22	0.182
475	A	4	4	1.	24	0.167
476	A	4	4	1.	24	0.167
477	A	4	4	1.	28	0.143
478	A	4	4	1.	28	0.143
479	A	4	4	1.	28	0.143
480	A	4	4	1.	28	0.143
481	A	4	4	1.	28	0.143
482	A	4	4	1.	28	0.143
483	A	5	2	1.	30	0.067
484	A	4	2	1.	30	0.067
485	A	3	2	1.	30	0.067
486	A	2	2	1.	30	0.067
487	A	1	1	1.	28	0.036
488	A	4	4	1.	30	0.133
489	A	4	4	1.	30	0.133
490	A	4	4	1.	30	0.133
491	A	3	2	1.	30	0.067
492	A	5	5	1.	30	0.167
493	A	2	2	1.	30	0.067
494	A	5	5	1.	30	0.167
495	A	1	1	1.	30	0.033
496	A	5	5	1.	30	0.167
497	A	3	3	1.	28	0.107
498	A	5	5	1.	30	0.167
499	A	4	4	1.	30	0.133
500	A	5	5	1.	30	0.167
501	A	4	4	1.	32	0.125
502	A	4	4	1.	32	0.125
503	A	3	3	1.	30	0.1
504	A	1	1	1.	32	0.031

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
505	A	2	2	1.	32	0.062
506	A	3	2	1.	32	0.062
507	A	3	2	1.	19	0.105
508	A	4	3	1.	19	0.158
509	A	3	2	1.	19	0.105
510	A	3	3	1.	19	0.158
511	A	3	3	1.	19	0.158
512	A	2	2	1.	17	0.118
513	A	2	2	1.	17	0.118
514	A	3	3	1.	19	0.158
515	A	3	2	1.	19	0.105
516	A	4	3	1.	19	0.158
517	A	4	3	1.	21	0.143
518	A	4	3	1.	21	0.143
519	A	3	2	1.	21	0.095
520	A	2	2	1.	21	0.095
521	A	3	3	1.	21	0.143
522	A	4	4	1.	21	0.19
523	A	6	4	1.	21	0.19
524	A	5	4	1.	21	0.19
525	A	4	4	1.	21	0.19
526	A	3	3	1.	19	0.158
527	A	1	1	1.	19	0.053
528	A	4	3	1.	21	0.143
529	A	4	3	1.	21	0.143
530	A	4	3	1.	21	0.143
531	A	4	3	1.	21	0.143
532	A	3	2	1.	21	0.095
533	A	3	2	1.	21	0.095
534	A	2	2	1.	21	0.095
535	A	6	6	1.	21	0.286
536	A	4	4	1.	21	0.19
537	A	6	5	1.11	21	0.238
538	A	5	5	1.14	21	0.238
539	A	4	4	1.2	19	0.21
540	A	4	4	1.21	19	0.21
541	A	3	3	1.	21	0.143
542	A	4	4	1.	21	0.19
543	A	5	5	1.	21	0.238
544	A	3	2	1.	21	0.095
545	A	3	2	1.	21	0.095
546	A	2	2	1.	21	0.095
547	A	7	6	1.	21	0.286
548	A	8	7	1.	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
549	A	9	6	1.09	21	0.286
550	A	5	5	1.	21	0.238
551	A	2	2	1.	19	0.105
552	A	5	5	1.	19	0.263
553	A	9	6	1.	21	0.286
554	A	3	2	1.	21	0.095
555	A	3	2	1.	21	0.095
556	A	3	2	1.	21	0.095
557	A	2	2	1.	21	0.095
558	A	7	6	1.	21	0.286
559	A	8	7	1.	21	0.333
560	A	8	7	1.	21	0.333
561	A	7	7	1.	21	0.333
562	A	6	6	1.45	21	0.286
563	A	4	4	1.28	19	0.21
564	A	5	5	1.	19	0.263
565	A	6	6	1.	21	0.286
566	A	3	2	1.	21	0.095
567	A	3	2	1.	21	0.095
568	A	3	2	1.	21	0.095
569	A	2	2	1.	21	0.095
570	A	7	6	1.	21	0.286
571	A	8	7	1.	21	0.333
572	A	8	8	1.	21	0.381
573	A	7	7	1.28	21	0.333
574	A	4	4	1.24	21	0.19
575	A	5	5	1.	19	0.263
576	A	6	6	1.	19	0.316
577	A	7	7	1.	21	0.333
578	A	5	4	1.	23	0.174
579	A	4	4	1.	23	0.174
580	A	4	4	1.	23	0.174
581	A	3	3	1.	23	0.13
582	A	3	3	1.	23	0.13
583	A	4	4	1.	23	0.174
584	A	4	4	1.	23	0.174
585	A	5	4	1.	23	0.174
586	A	5	5	1.	25	0.2
587	A	5	5	1.	25	0.2
588	A	4	4	1.	25	0.16
589	A	4	4	1.	25	0.16
590	A	5	5	1.	25	0.2
591	A	5	5	1.	25	0.2
592	A	6	5	1.	25	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
593	A	6	5	1.	25	0.2
594	A	5	5	1.	25	0.2
595	A	5	5	1.	25	0.2
596	A	4	4	1.	25	0.16
597	A	5	5	1.	25	0.2
598	A	4	4	1.	25	0.16
599	A	5	5	1.	25	0.2
600	A	4	4	1.	25	0.16
601	A	4	4	1.	25	0.16
602	A	5	5	1.	25	0.2
603	A	17	15	1.	25	0.6
604	A	17	15	1.	25	0.6
605	A	13	11	1.	25	0.44
606	A	14	12	1.	25	0.48
607	A	17	15	1.	25	0.6
608	A	17	15	1.	25	0.6
609	A	18	16	1.	25	0.64
610	A	17	15	1.	25	0.6
611	A	17	15	1.	25	0.6
612	A	17	15	1.	25	0.6
613	A	17	15	1.	25	0.6
614	A	18	16	1.	25	0.64
615	A	18	16	1.	25	0.64
616	A	19	17	1.	25	0.68
617	A	18	16	1.	25	0.64
618	A	18	16	1.	25	0.64
619	A	18	16	1.	25	0.64
620	A	18	16	1.	25	0.64
621	A	19	16	1.	25	0.64
622	A	19	16	1.	25	0.64
623	A	20	17	1.	25	0.68
624	A	3	3	1.	23	0.13
625	A	3	3	1.	23	0.13
626	A	3	3	1.	23	0.13
627	A	3	3	1.	23	0.13
628	A	4	4	1.	25	0.16
629	A	4	4	1.	25	0.16
630	A	4	4	1.	25	0.16
631	A	4	4	1.	25	0.16
632	A	16	11	1.	25	0.44
633	A	16	11	1.	25	0.44
634	A	17	12	1.	25	0.48
635	A	17	12	1.	25	0.48
636	A	18	13	1.	25	0.52

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
637	A	18	13	1.	25	0.52
638	A	19	13	1.	25	0.52
639	A	19	13	1.	25	0.52
640	A	4	4	0.97	23	0.174
641	A	4	4	1.	23	0.174
642	A	3	3	1.	21	0.143
643	A	6	5	1.	23	0.217
644	A	7	6	1.	23	0.261
645	A	3	3	1.03	23	0.13
646	A	3	2	1.	21	0.095
647	A	3	2	1.	21	0.095
648	A	2	2	1.	21	0.095
649	A	6	4	1.	21	0.19
650	A	7	5	1.	21	0.238
651	A	3	3	1.	21	0.143
652	A	3	3	1.	19	0.158
653	A	3	3	1.	19	0.158
654	A	3	3	1.	21	0.143
655	A	6	5	1.	26	0.192
656	A	5	5	1.	26	0.192
657	A	5	5	1.	26	0.192
658	A	4	4	1.	26	0.154
659	A	4	4	1.	26	0.154
660	A	5	5	1.03	26	0.192
661	A	5	5	1.	26	0.192
662	A	6	5	1.	26	0.192
663	A	7	5	1.	28	0.179
664	A	6	5	1.	28	0.179
665	A	6	5	1.	28	0.179
666	A	5	5	1.05	28	0.179
667	A	5	5	1.05	28	0.179
668	A	4	4	1.	28	0.143
669	A	4	4	1.	28	0.143
670	A	5	5	1.	28	0.179
671	A	5	5	1.	28	0.179
672	A	6	5	1.	28	0.179
673	A	5	4	1.	30	0.133
674	A	4	4	1.	30	0.133
675	A	3	3	1.01	30	0.1
676	A	2	2	1.	30	0.067
677	A	10	7	1.	30	0.233
678	A	13	10	1.18	30	0.333
679	A	13	10	1.	30	0.333
680	A	15	11	1.	30	0.367

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
681	A	5	4	1.	30	0.133
682	A	4	4	1.	30	0.133
683	A	3	3	1.	30	0.1
684	A	2	2	1.	30	0.067
685	A	11	8	1.	30	0.267
686	A	12	9	1.	30	0.3
687	A	14	11	1.	30	0.367
688	A	5	5	1.	26	0.192
689	A	5	5	1.	26	0.192
690	A	5	5	1.	24	0.208
691	A	5	5	1.	26	0.192
692	A	5	5	1.	26	0.192
693	A	5	5	1.	28	0.179
694	A	5	5	1.	28	0.179
695	A	5	5	0.96	23	0.217
696	A	5	5	1.	23	0.217
697	A	4	4	1.01	21	0.19
698	A	7	6	1.	23	0.261
699	A	8	7	1.	23	0.304
700	A	4	4	1.	23	0.174

Chapter 3

Listing of integrals

3.1 $\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=94

$$\frac{a\tan^9(c+dx)}{9d}+\frac{4a\tan^7(c+dx)}{7d}+\frac{6a\tan^5(c+dx)}{5d}+\frac{4a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^{10}(c+dx)}{10d}$$

```
[Out] ((I/10)*a*Sec[c + d*x]^10)/d + (a*Tan[c + d*x])/d + (4*a*Tan[c + d*x]^3)/(3
*d) + (6*a*Tan[c + d*x]^5)/(5*d) + (4*a*Tan[c + d*x]^7)/(7*d) + (a*Tan[c +
d*x]^9)/(9*d)
```

Rubi [A] time = 0.188289, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3486, 3767}

$$\frac{a\tan^9(c+dx)}{9d}+\frac{4a\tan^7(c+dx)}{7d}+\frac{6a\tan^5(c+dx)}{5d}+\frac{4a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^{10}(c+dx)}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/10)*a*Sec[c + d*x]^10)/d + (a*Tan[c + d*x])/d + (4*a*Tan[c + d*x]^3)/(3
*d) + (6*a*Tan[c + d*x]^5)/(5*d) + (4*a*Tan[c + d*x]^7)/(7*d) + (a*Tan[c +
d*x]^9)/(9*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^{10}(c+dx)}{10d} + a\int \sec^{10}(c+dx)\,dx$$
$$= \frac{ia\sec^{10}(c+dx)}{10d} - \frac{a\,\mathrm{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)dx, x, -\tan(c+dx)\right)}{d}$$
$$= \frac{ia\sec^{10}(c+dx)}{10d} + \frac{a\tan(c+dx)}{d} + \frac{4a\tan^3(c+dx)}{3d} + \frac{6a\tan^5(c+dx)}{5d} + \frac{4a\tan}{}$$

Mathematica [A] time = 0.361365, size = 79, normalized size = 0.84

$$\frac{a\left(\frac{1}{9}\tan^9(c+dx)+\frac{4}{7}\tan^7(c+dx)+\frac{6}{5}\tan^5(c+dx)+\frac{4}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{ia\sec^{10}(c+dx)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/10)*a*Sec[c + d*x]^10)/d + (a*(Tan[c + d*x] + (4*Tan[c + d*x]^3)/3 + (6
*Tan[c + d*x]^5)/5 + (4*Tan[c + d*x]^7)/7 + Tan[c + d*x]^9/9))/d
```

Maple [A] time = 0.093, size = 69, normalized size = 0.7

$$\frac{1}{d}\left(\frac{\frac{i}{10}a}{(\cos(dx+c))^{10}} - a\left(-\frac{128}{315} - \frac{(\sec(dx+c))^8}{9} - \frac{8(\sec(dx+c))^6}{63} - \frac{16(\sec(dx+c))^4}{105} - \frac{64(\sec(dx+c))^2}{315}\right)\tan(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/10*I*a/cos(d*x+c)^10-a*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-
16/105*sec(d*x+c)^4-64/315*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.10113, size = 154, normalized size = 1.64

$$\frac{63i\,a\tan(dx+c)^{10}+70\,a\tan(dx+c)^9+315i\,a\tan(dx+c)^8+360\,a\tan(dx+c)^7+630i\,a\tan(dx+c)^6+756\,a\tan(d}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/630*(63*I*a*tan(d*x + c)^10 + 70*a*tan(d*x + c)^9 + 315*I*a*tan(d*x + c)^
8 + 360*a*tan(d*x + c)^7 + 630*I*a*tan(d*x + c)^6 + 756*a*tan(d*x + c)^5 +
630*I*a*tan(d*x + c)^4 + 840*a*tan(d*x + c)^3 + 315*I*a*tan(d*x + c)^2 + 63
0*a*tan(d*x + c))/d
```

Fricas [B] time = 1.301, size = 629, normalized size = 6.69

$$\frac{64512i\,ae^{(10i\,dx+10i\,c)}+53760i\,ae^{(8i\,dx+8i\,c)}+30720i\,ae^{(6i\,dx+6i\,c)}+11520i\,ae^{(4i\,dx+4i\,c)}+}{315\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de^{(10i\,dx+10i\,c)}+210\,de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/315*(64512*I*a*e^(10*I*d*x + 10*I*c) + 53760*I*a*e^(8*I*d*x + 8*I*c) + 30
720*I*a*e^(6*I*d*x + 6*I*c) + 11520*I*a*e^(4*I*d*x + 4*I*c) + 2560*I*a*e^(2
*I*d*x + 2*I*c) + 256*I*a)/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18
*I*c) + 45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^
(12*I*d*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*
c) + 120*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x
 + 2*I*c) + d)
```

Sympy [A] time = 123.932, size = 83, normalized size = 0.88

$$\begin{cases} \frac{a\left(\frac{\tan^9(c+dx)}{9}+\frac{4\tan^7(c+dx)}{7}+\frac{6\tan^5(c+dx)}{5}+\frac{4\tan^3(c+dx)}{3}+\tan(c+dx)\right)+\frac{ia\sec^{10}(c+dx)}{10}}{d} & \text{for } d\neq 0 \\ x\left(ia\tan(c)+a\right)\sec^{10}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**9/9 + 4*tan(c + d*x)**7/7 + 6*tan(c + d*x)**5/
5 + 4*tan(c + d*x)**3/3 + tan(c + d*x)) + I*a*sec(c + d*x)**10/10)/d, Ne(d,
 0)), (x*(I*a*tan(c) + a)*sec(c)**10, True))
```

Giac [A] time = 1.17452, size = 154, normalized size = 1.64

$$-\frac{-63i\,a\tan(dx+c)^{10}-70\,a\tan(dx+c)^{9}-315i\,a\tan(dx+c)^{8}-360\,a\tan(dx+c)^{7}-630i\,a\tan(dx+c)^{6}-756\,a\,\mathrm{t}}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/630*(-63*I*a*tan(d*x + c)^10 - 70*a*tan(d*x + c)^9 - 315*I*a*tan(d*x + c
)^8 - 360*a*tan(d*x + c)^7 - 630*I*a*tan(d*x + c)^6 - 756*a*tan(d*x + c)^5
- 630*I*a*tan(d*x + c)^4 - 840*a*tan(d*x + c)^3 - 315*I*a*tan(d*x + c)^2 -
630*a*tan(d*x + c))/d
```

3.2 $\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=75

$$\frac{a\tan^7(c+dx)}{7d}+\frac{3a\tan^5(c+dx)}{5d}+\frac{a\tan^3(c+dx)}{d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^8(c+dx)}{8d}$$

```
[Out] ((I/8)*a*Sec[c + d*x]^8)/d + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/d + (3
*a*Tan[c + d*x]^5)/(5*d) + (a*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0407898, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3486, 3767}

$$\frac{a\tan^7(c+dx)}{7d}+\frac{3a\tan^5(c+dx)}{5d}+\frac{a\tan^3(c+dx)}{d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/8)*a*Sec[c + d*x]^8)/d + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/d + (3
*a*Tan[c + d*x]^5)/(5*d) + (a*Tan[c + d*x]^7)/(7*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec^8(c+dx)}{8d}+a\int \sec^8(c+dx)\,dx\\ &= \frac{ia\sec^8(c+dx)}{8d}-\frac{a\,\mathrm{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{ia\sec^8(c+dx)}{8d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{d}+\frac{3a\tan^5(c+dx)}{5d}+\frac{a\tan^7(c}{7d}\end{aligned}$$

Mathematica [A] time = 0.10612, size = 63, normalized size = 0.84

$$\frac{a\left(\frac{1}{7}\tan^7(c+dx)+\frac{3}{5}\tan^5(c+dx)+\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{ia\sec^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/8)*a*Sec[c + d*x]^8)/d + (a*(Tan[c + d*x] + Tan[c + d*x]^3 + (3*Tan[c +
 d*x]^5)/5 + Tan[c + d*x]^7/7))/d
```

Maple [A] time = 0.087, size = 59, normalized size = 0.8

$$\frac{1}{d}\left(\frac{\frac{i}{8}a}{(\cos(dx+c))^{8}}-a\left(-\frac{16}{35}-\frac{(\sec(dx+c))^{6}}{7}-\frac{6\,(\sec(dx+c))^{4}}{35}-\frac{8\,(\sec(dx+c))^{2}}{35}\right)\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/8*I*a/cos(d*x+c)^8-a*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35
*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.03796, size = 124, normalized size = 1.65

$$\frac{35i\,a\tan(dx+c)^{8}+40\,a\tan(dx+c)^{7}+140i\,a\tan(dx+c)^{6}+168\,a\tan(dx+c)^{5}+210i\,a\tan(dx+c)^{4}+280\,a\tan}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/280*(35*I*a*tan(d*x + c)^8 + 40*a*tan(d*x + c)^7 + 140*I*a*tan(d*x + c)^6
 + 168*a*tan(d*x + c)^5 + 210*I*a*tan(d*x + c)^4 + 280*a*tan(d*x + c)^3 + 1
40*I*a*tan(d*x + c)^2 + 280*a*tan(d*x + c))/d
```

Fricas [B] time = 1.27001, size = 486, normalized size = 6.48

$$\frac{2240i\,ae^{(8i\,dx+8i\,c)}+1792i\,ae^{(6i\,dx+6i\,c)}+896i\,ae^{(4i\,dx+4i\,c)}+256i\,ae^{(2i\,dx+2i\,c)}+32i\,a}{35\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i\,c)}+28\,de^{(4i\,dx+4i}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/35*(2240*I*a*e^(8*I*d*x + 8*I*c) + 1792*I*a*e^(6*I*d*x + 6*I*c) + 896*I*a
*e^(4*I*d*x + 4*I*c) + 256*I*a*e^(2*I*d*x + 2*I*c) + 32*I*a)/(d*e^(16*I*d*x
 + 16*I*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*
e^(10*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c)
 + 28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 53.1259, size = 68, normalized size = 0.91

$$\begin{cases}\frac{a\left(\frac{\tan^{7}(c+dx)}{7}+\frac{3\tan^{5}(c+dx)}{5}+\tan^{3}(c+dx)+\tan(c+dx)\right)+\frac{ia\sec^{8}(c+dx)}{8}}{d} & \text{for}\ d\neq 0\\ x\,(ia\tan(c)+a)\sec^{8}(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**7/7 + 3*tan(c + d*x)**5/5 + tan(c + d*x)**3 +
tan(c + d*x)) + I*a*sec(c + d*x)**8/8)/d, Ne(d, 0)), (x*(I*a*tan(c) + a)*se
c(c)**8, True))
```

Giac [A] time = 1.1527, size = 124, normalized size = 1.65

$$-\frac{-35i\,a\tan\left(dx+c\right)^{8}-40\,a\tan\left(dx+c\right)^{7}-140i\,a\tan\left(dx+c\right)^{6}-168\,a\tan\left(dx+c\right)^{5}-210i\,a\tan\left(dx+c\right)^{4}-280\,a\tan\left(\right.}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/280*(-35*I*a*tan(d*x + c)^8 - 40*a*tan(d*x + c)^7 - 140*I*a*tan(d*x + c)
^6 - 168*a*tan(d*x + c)^5 - 210*I*a*tan(d*x + c)^4 - 280*a*tan(d*x + c)^3 -
 140*I*a*tan(d*x + c)^2 - 280*a*tan(d*x + c))/d
```

3.3 $\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=62

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^6(c+dx)}{6d}$$

```
[Out] ((I/6)*a*Sec[c + d*x]^6)/d + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d
) + (a*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0373904, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3486, 3767}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/6)*a*Sec[c + d*x]^6)/d + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d
) + (a*Tan[c + d*x]^5)/(5*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec^6(c+dx)}{6d}+a\int \sec^6(c+dx)\,dx\\ &= \frac{ia\sec^6(c+dx)}{6d}-\frac{a\,\text{Subst}\left(\int\left(1+2x^2+x^4\right)dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{ia\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.10323, size = 55, normalized size = 0.89

$$\frac{a\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{ia\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/6)*a*Sec[c + d*x]^6)/d + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[
c + d*x]^5/5))/d
```

Maple [A] time = 0.082, size = 49, normalized size = 0.8

$$\frac{1}{d}\left(\frac{\frac{i}{6}a}{(\cos(dx+c))^6}-a\left(-\frac{8}{15}-\frac{(\sec(dx+c))^4}{5}-\frac{4(\sec(dx+c))^2}{15}\right)\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/6*I*a/cos(d*x+c)^6-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(
d*x+c))
```

Maxima [A] time = 1.09728, size = 95, normalized size = 1.53

$$\frac{5ia\tan(dx+c)^6+6a\tan(dx+c)^5+15ia\tan(dx+c)^4+20a\tan(dx+c)^3+15ia\tan(dx+c)^2+30a\tan(dx+c)}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(5*I*a*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*I*a*tan(d*x + c)^4 + 2
0*a*tan(d*x + c)^3 + 15*I*a*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Fricas [B] time = 1.11756, size = 363, normalized size = 5.85

$$\frac{320ia e^{(6idx+6ic)}+240ia e^{(4idx+4ic)}+96ia e^{(2idx+2ic)}+16ia}{15\left(de^{(12idx+12ic)}+6de^{(10idx+10ic)}+15de^{(8idx+8ic)}+20de^{(6idx+6ic)}+15de^{(4idx+4ic)}+6de^{(2idx+2ic)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(320*I*a*e^(6*I*d*x + 6*I*c) + 240*I*a*e^(4*I*d*x + 4*I*c) + 96*I*a*e^
(2*I*d*x + 2*I*c) + 16*I*a)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10
*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d
*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 18.4573, size = 60, normalized size = 0.97

$$\begin{cases}\frac{a\left(\frac{\tan^5(c+dx)}{5}+\frac{2\tan^3(c+dx)}{3}+\tan(c+dx)\right)+\frac{ia\sec^6(c+dx)}{6}}{d} & \text{for } d\neq 0\\ x(ia\tan(c)+a)\sec^6(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**5/5 + 2*tan(c + d*x)**3/3 + tan(c + d*x)) + I*
a*sec(c + d*x)**6/6)/d, Ne(d, 0)), (x*(I*a*tan(c) + a)*sec(c)**6, True))
```

Giac [A] time = 1.13942, size = 95, normalized size = 1.53

$$-\frac{-5i\,a\tan\left(dx+c\right)^{6}-6\,a\tan\left(dx+c\right)^{5}-15i\,a\tan\left(dx+c\right)^{4}-20\,a\tan\left(dx+c\right)^{3}-15i\,a\tan\left(dx+c\right)^{2}-30\,a\tan\left(dx+\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(-5*I*a*tan(d*x + c)^6 - 6*a*tan(d*x + c)^5 - 15*I*a*tan(d*x + c)^4 -
 20*a*tan(d*x + c)^3 - 15*I*a*tan(d*x + c)^2 - 30*a*tan(d*x + c))/d
```

3.4 $\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=46

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^4(c+dx)}{4d}$$

```
[Out] ((I/4)*a*Sec[c + d*x]^4)/d + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0342313, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3486, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/4)*a*Sec[c + d*x]^4)/d + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec^4(c+dx)}{4d}+a\int \sec^4(c+dx)\,dx\\
&= \frac{ia\sec^4(c+dx)}{4d}-\frac{a\,\text{Subst}\left(\int \left(1+x^2\right)dx, x, -\tan(c+dx)\right)}{d}\\
&= \frac{ia\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0422621, size = 43, normalized size = 0.93

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{ia\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/4)*a*Sec[c + d*x]^4)/d + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.08, size = 39, normalized size = 0.9

$$\frac{1}{d}\left(\frac{\frac{i}{4}a}{\left(\cos\left(dx+c\right)\right)^{4}}-a\left(-\frac{2}{3}-\frac{\left(\sec\left(dx+c\right)\right)^{2}}{3}\right)\tan\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/4*I*a/cos(d*x+c)^4-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.06162, size = 65, normalized size = 1.41

$$\frac{3i\,a\tan\left(dx+c\right)^{4}+4\,a\tan\left(dx+c\right)^{3}+6i\,a\tan\left(dx+c\right)^{2}+12\,a\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*I*a*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*I*a*tan(d*x + c)^2 + 12
*a*tan(d*x + c))/d
```

Fricas [B] time = 1.10474, size = 239, normalized size = 5.2

$$\frac{24i\,ae^{(4i\,dx+4i\,c)}+16i\,ae^{(2i\,dx+2i\,c)}+4i\,a}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(24*I*a*e^(4*I*d*x + 4*I*c) + 16*I*a*e^(2*I*d*x + 2*I*c) + 4*I*a)/(d*e^
(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d
*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 8.66531, size = 48, normalized size = 1.04

$$\begin{cases}\frac{a\left(\frac{\tan^{3}(c+dx)}{3}+\tan(c+dx)\right)+\frac{ia\sec^{4}(c+dx)}{4}}{d} & \text{for } d\neq 0\\ x\left(ia\tan\left(c\right)+a\right)\sec^{4}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**3/3 + tan(c + d*x)) + I*a*sec(c + d*x)**4/4)/d
, Ne(d, 0)), (x*(I*a*tan(c) + a)*sec(c)**4, True))
```

Giac [A] time = 1.12926, size = 65, normalized size = 1.41

$$-\frac{-3i\,a\tan\left(dx+c\right)^{4}-4\,a\tan\left(dx+c\right)^{3}-6i\,a\tan\left(dx+c\right)^{2}-12\,a\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(-3*I*a*tan(d*x + c)^4 - 4*a*tan(d*x + c)^3 - 6*I*a*tan(d*x + c)^2 -
12*a*tan(d*x + c))/d
```

3.5 $\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=27

$$-\frac{i(a+ia\tan(c+dx))^2}{2ad}$$

```
[Out] ((-I/2)*(a + I*a*Tan[c + d*x])^2)/(a*d)
```

Rubi [A] time = 0.0306043, antiderivative size = 30, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3486, 3767, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{ia\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/2)*a*Sec[c + d*x]^2)/d + (a*Tan[c + d*x])/d
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec^2(c+dx)}{2d}+a\int \sec^2(c+dx)\,dx\\ &= \frac{ia\sec^2(c+dx)}{2d}-\frac{a\,\mathrm{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{ia\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0132175, size = 30, normalized size = 1.11

$$\frac{a\tan(c+dx)}{d}+\frac{ia\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/2)*a*Sec[c + d*x]^2)/d + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.038, size = 26, normalized size = 1.

$$\frac{1}{d}\left(\frac{\frac{i}{2}a}{\left(\cos\left(dx+c\right)\right)^{2}}+a\tan\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/2*I*a/cos(d*x+c)^2+a*tan(d*x+c))
```

Maxima [A] time = 1.1059, size = 28, normalized size = 1.04

$$-\frac{i\left(i\,a\tan\left(dx+c\right)+a\right)^{2}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*I*(I*a*tan(d*x + c) + a)^2/(a*d)
```

Fricas [B] time = 1.1028, size = 123, normalized size = 4.56

$$\frac{4i\,ae^{(2i\,dx+2i\,c)}+2i\,a}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (4*I*a*e^(2*I*d*x + 2*I*c) + 2*I*a)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x
 + 2*I*c) + d)
```

Sympy [A] time = 3.56283, size = 37, normalized size = 1.37

$$\begin{cases}\frac{\frac{ia\tan^{2}(c+dx)}{2}+a\tan(c+dx)}{d} & \text{for}\ d\neq 0\\ x\left(ia\tan\left(c\right)+a\right)\sec^{2}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((I*a*tan(c + d*x)**2/2 + a*tan(c + d*x))/d, Ne(d, 0)), (x*(I*a*t
an(c) + a)*sec(c)**2, True))
```

Giac [A] time = 1.12703, size = 35, normalized size = 1.3

$$-\frac{-i\,a\tan\left(dx+c\right)^{2}-2\,a\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(-I*a*tan(d*x + c)^2 - 2*a*tan(d*x + c))/d
```

3.6 $\int (a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=19

$$ax-\frac{ia\log(\cos(c+dx))}{d}$$

```
[Out] a*x - (I*a*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.0074034, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3475}

$$ax-\frac{ia\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + I*a*Tan[c + d*x],x]
```

```
[Out] a*x - (I*a*Log[Cos[c + d*x]])/d
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(c+dx))\,dx &= ax+(ia)\int \tan(c+dx)\,dx\\ &= ax-\frac{ia\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0058757, size = 19, normalized size = 1.

$$ax-\frac{ia\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + I*a*Tan[c + d*x],x]
```

```
[Out] a*x - (I*a*Log[Cos[c + d*x]])/d
```

Maple [A] time = 0.001, size = 23, normalized size = 1.2

$$ax+\frac{\frac{i}{2}a\ln\left(1+(\tan(dx+c))^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+I*a*tan(d*x+c),x)
```

```
[Out] a*x+1/2*I*a/d*ln(1+tan(d*x+c)^2)
```

Maxima [A] time = 1.09955, size = 23, normalized size = 1.21

$$ax + \frac{i\,a\log\left(\sec\left(dx + c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + I*a*log(sec(d*x + c))/d
```

Fricas [A] time = 1.20903, size = 50, normalized size = 2.63

$$-\frac{i\,a\log\left(e^{(2i\,dx+2i\,c)} + 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -I*a*log(e^(2*I*d*x + 2*I*c) + 1)/d
```

Sympy [A] time = 0.49256, size = 24, normalized size = 1.26

$$-\frac{ia\log\left(e^{2idx} + e^{-2ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x)
```

```
[Out] -I*a*log(exp(2*I*d*x) + exp(-2*I*c))/d
```

Giac [A] time = 1.12551, size = 24, normalized size = 1.26

$$ax - \frac{i\,a\log\left(|\cos\left(dx + c\right)|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="giac")
```

```
[Out] a*x - I*a*log(abs(cos(d*x + c)))/d
```

3.7 $\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=45

$$-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] (a*x)/2 - ((I/2)*a*Cos[c + d*x]^2)/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0310949, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3486, 2635, 8}

$$-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*x)/2 - ((I/2)*a*Cos[c + d*x]^2)/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos^2(c+dx)}{2d}+a\int \cos^2(c+dx)\,dx\\ &= -\frac{ia\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\ &= \frac{ax}{2}-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0560812, size = 48, normalized size = 1.07

$$\frac{a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}-\frac{ia\cos^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) - ((I/2)*a*Cos[c + d*x]^2)/d + (a*Sin[2*(c + d*x)])/(4*
d)
```

Maple [A] time = 0.044, size = 42, normalized size = 0.9

$$\frac{1}{d}\left(-\frac{i}{2}a(\cos(dx+c))^2+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/2*I*a*cos(d*x+c)^2+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.66952, size = 51, normalized size = 1.13

$$\frac{(dx+c)a+\frac{a\tan(dx+c)-ia}{\tan(dx+c)^2+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)*a + (a*tan(d*x + c) - I*a)/(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.10973, size = 58, normalized size = 1.29

$$\frac{2\,adx-i\,ae^{(2i\,dx+2i\,c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*d*x - I*a*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A] time = 0.227572, size = 41, normalized size = 0.91

$$\frac{ax}{2}+\begin{cases}-\frac{iae^{2ic}e^{2idx}}{4d} & \text{for } 4d\neq 0\\ \frac{axe^{2ic}}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*x/2 + Piecewise((-I*a*exp(2*I*c)*exp(2*I*d*x)/(4*d), Ne(4*d, 0)), (a*x*ex
p(2*I*c)/2, True))
```

Giac [A] time = 1.12101, size = 31, normalized size = 0.69

$$\frac{2\,adx - i\,ae^{(2i\,dx+2i\,c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*d*x - I*a*e^(2*I*d*x + 2*I*c))/d
```

3.8 $\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=67

$$-\frac{ia\cos^4(c+dx)}{4d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

```
[Out] (3*a*x)/8 - ((I/4)*a*Cos[c + d*x]^4)/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8
*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.03995, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3486, 2635, 8}

$$-\frac{ia\cos^4(c+dx)}{4d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - ((I/4)*a*Cos[c + d*x]^4)/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8
*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos^4(c+dx)}{4d}+a\int \cos^4(c+dx)\,dx\\
&= -\frac{ia\cos^4(c+dx)}{4d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3a)\int \cos^2(c+dx)\,dx\\
&= -\frac{ia\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\\
&= \frac{3ax}{8}-\frac{ia\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+}{4d}
\end{aligned}$$

Mathematica [A] time = 0.0422158, size = 46, normalized size = 0.69

$$\frac{a\left(8\sin(2(c+dx))+\sin(4(c+dx))-8i\cos^4(c+dx)+12c+12dx\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*(12*c + 12*d*x - (8*I)*Cos[c + d*x]^4 + 8*Sin[2*(c + d*x)] + Sin[4*(c +
d*x)]))/(32*d)
```

Maple [A] time = 0.084, size = 53, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{i}{4}a\left(\cos\left(dx+c\right)\right)^4+a\left(\frac{\sin\left(dx+c\right)}{4}\left(\left(\cos\left(dx+c\right)\right)^3+\frac{3\cos\left(dx+c\right)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/4*I*a*cos(d*x+c)^4+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+
3/8*d*x+3/8*c))
```

Maxima [A] time = 1.6623, size = 82, normalized size = 1.22

$$\frac{3\left(dx+c\right)a+\frac{3\,a\tan\left(dx+c\right)^3+5\,a\tan\left(dx+c\right)-2i\,a}{\tan\left(dx+c\right)^4+2\tan\left(dx+c\right)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(d*x + c)*a + (3*a*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 2*I*a)/(tan(d
*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.13684, size = 165, normalized size = 2.46

$$\frac{\left(12\,adxe^{(2i\,dx+2i\,c)}-i\,ae^{(6i\,dx+6i\,c)}-6i\,ae^{(4i\,dx+4i\,c)}+2i\,a\right)e^{(-2i\,dx-2i\,c)}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(12*a*d*x*e^(2*I*d*x + 2*I*c) - I*a*e^(6*I*d*x + 6*I*c) - 6*I*a*e^(4*I
*d*x + 4*I*c) + 2*I*a)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A] time = 0.4609, size = 138, normalized size = 2.06

$$\frac{3ax}{8} + \begin{cases} \frac{\left(-256iad^2e^{6ic}e^{4idx} - 1536iad^2e^{4ic}e^{2idx} + 512iad^2e^{-2idx}\right)e^{-2ic}}{8192d^3} & \text{for } 8192d^3e^{2ic} \neq 0 \\ x\left(-\frac{3a}{8} + \frac{\left(ae^{6ic} + 3ae^{4ic} + 3ae^{2ic} + a\right)e^{-2ic}}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 3*a*x/8 + Piecewise(((-256*I*a*d**2*exp(6*I*c)*exp(4*I*d*x) - 1536*I*a*d**2
*exp(4*I*c)*exp(2*I*d*x) + 512*I*a*d**2*exp(-2*I*d*x))*exp(-2*I*c)/(8192*d*
*3), Ne(8192*d**3*exp(2*I*c), 0)), (x*(-3*a/8 + (a*exp(6*I*c) + 3*a*exp(4*I
*c) + 3*a*exp(2*I*c) + a)*exp(-2*I*c)/8), True))
```

Giac [A] time = 1.14142, size = 139, normalized size = 2.07

$$\frac{\left(12\,adxe^{(2i\,dx+2i\,c)} + i\,ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - i\,ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right) - i\,ae^{(6i\,dx+6i\,c)} - 6i\,ae^{(4i\,dx+4i\,c)}\right.}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(12*a*d*x*e^(2*I*d*x + 2*I*c) + I*a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) - I*a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) - I*
a*e^(6*I*d*x + 6*I*c) - 6*I*a*e^(4*I*d*x + 4*I*c) + 2*I*a)*e^(-2*I*d*x - 2*
I*c)/d
```

3.9 $\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=89

$$-\frac{ia\cos^6(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}$$

```
[Out] (5*a*x)/16 - ((I/6)*a*Cos[c + d*x]^6)/d + (5*a*Cos[c + d*x]*Sin[c + d*x])/(
16*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c
+ d*x])/(6*d)
```

Rubi [A] time = 0.0522472, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3486, 2635, 8}

$$-\frac{ia\cos^6(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*a*x)/16 - ((I/6)*a*Cos[c + d*x]^6)/d + (5*a*Cos[c + d*x]*Sin[c + d*x])/(
16*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c
+ d*x])/(6*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos^6(c+dx)}{6d}+a\int \cos^6(c+dx)\,dx\\ &= -\frac{ia\cos^6(c+dx)}{6d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}(5a)\int \cos^4(c+dx)\,dx\\ &= -\frac{ia\cos^6(c+dx)}{6d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{8}\ldots\\ &= -\frac{ia\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\ldots\\ &= \frac{5ax}{16}-\frac{ia\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}\ldots\end{aligned}$$

Mathematica [A] time = 0.05835, size = 56, normalized size = 0.63

$$\frac{a\left(45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))-32i\cos^6(c+dx)+60c+60dx\right)}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*(60*c + 60*d*x - (32*I)*Cos[c + d*x]^6 + 45*Sin[2*(c + d*x)] + 9*Sin[4*(
c + d*x)] + Sin[6*(c + d*x)]))/(192*d)
```

Maple [A] time = 0.084, size = 63, normalized size = 0.7

$$\frac{1}{d}\left(-\frac{i}{6}a\left(\cos\left(dx+c\right)\right)^{6}+a\left(\frac{\sin\left(dx+c\right)}{6}\left(\left(\cos\left(dx+c\right)\right)^{5}+\frac{5\left(\cos\left(dx+c\right)\right)^{3}}{4}+\frac{15\cos\left(dx+c\right)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/6*I*a*cos(d*x+c)^6+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d
*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.67519, size = 111, normalized size = 1.25

$$\frac{15\,(dx+c)a+\frac{15\,a\tan(dx+c)^5+40\,a\tan(dx+c)^3+33\,a\tan(dx+c)-8i\,a}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(15*(d*x + c)*a + (15*a*tan(d*x + c)^5 + 40*a*tan(d*x + c)^3 + 33*a*ta
n(d*x + c) - 8*I*a)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 +
 1))/d
```

Fricas [A] time = 1.06969, size = 252, normalized size = 2.83

$$\frac{\left(120\,adxe^{(4i\,dx+4i\,c)}-2i\,ae^{(10i\,dx+10i\,c)}-15i\,ae^{(8i\,dx+8i\,c)}-60i\,ae^{(6i\,dx+6i\,c)}+30i\,ae^{(2i\,dx+2i\,c)}+3i\,a\right)e^{(-4i\,dx-4i\,c)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/384*(120*a*d*x*e^(4*I*d*x + 4*I*c) - 2*I*a*e^(10*I*d*x + 10*I*c) - 15*I*a
*e^(8*I*d*x + 8*I*c) - 60*I*a*e^(6*I*d*x + 6*I*c) + 30*I*a*e^(2*I*d*x + 2*I
*c) + 3*I*a)*e^(-4*I*d*x - 4*I*c)/d
```

Sympy [A] time = 0.782871, size = 212, normalized size = 2.38

$$\frac{5ax}{16}+\begin{cases}\frac{\left(-33554432iad^4e^{12ic}e^{6idx}-251658240iad^4e^{10ic}e^{4idx}-1006632960iad^4e^{8ic}e^{2idx}+503316480iad^4e^{4ic}e^{-2idx}+50331648iad^4e^{2ic}e^{-4idx}\right)e^{-6ic}}{6442450944d^5} & \text{for } 64424 \\ x\left(-\frac{5a}{16}+\frac{\left(ae^{10ic}+5ae^{8ic}+10ae^{6ic}+10ae^{4ic}+5ae^{2ic}+a\right)e^{-4ic}}{32}\right) & \text{otherwis}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 5*a*x/16 + Piecewise(((-33554432*I*a*d**4*exp(12*I*c)*exp(6*I*d*x) - 251658
240*I*a*d**4*exp(10*I*c)*exp(4*I*d*x) - 1006632960*I*a*d**4*exp(8*I*c)*exp(
2*I*d*x) + 503316480*I*a*d**4*exp(4*I*c)*exp(-2*I*d*x) + 50331648*I*a*d**4*
exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(6442450944*d**5), Ne(6442450944*d**5
*exp(6*I*c), 0)), (x*(-5*a/16 + (a*exp(10*I*c) + 5*a*exp(8*I*c) + 10*a*exp(
6*I*c) + 10*a*exp(4*I*c) + 5*a*exp(2*I*c) + a)*exp(-4*I*c)/32), True))
```

Giac [A] time = 1.13989, size = 171, normalized size = 1.92

$$\frac{\left(120\,adxe^{(4i\,dx+2i\,c)}+12i\,ae^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,ae^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)-2i\,ae^{(10i\,dx+8i\,c)}-15i\,ae^{(8i\,dx}\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(120*a*d*x*e^(4*I*d*x + 2*I*c) + 12*I*a*e^(4*I*d*x + 2*I*c)*log(e^(2*
I*d*x + 2*I*c) + 1) - 12*I*a*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*
c)) - 2*I*a*e^(10*I*d*x + 8*I*c) - 15*I*a*e^(8*I*d*x + 6*I*c) - 60*I*a*e^(6
*I*d*x + 4*I*c) + 30*I*a*e^(2*I*d*x) + 3*I*a*e^(-2*I*c))*e^(-4*I*d*x - 2*I*
c)/d
```

3.10 $\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=111

$$-\frac{ia\cos^8(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a\sin$$

```
[Out] (35*a*x)/128 - ((I/8)*a*Cos[c + d*x]^8)/d + (35*a*Cos[c + d*x]*Sin[c + d*x]
)/(128*d) + (35*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (7*a*Cos[c + d*x]^
5*Sin[c + d*x])/(48*d) + (a*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rubi [A] time = 0.067046, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3486, 2635, 8}

$$-\frac{ia\cos^8(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a\sin$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```


```
[Out] (35*a*x)/128 - ((I/8)*a*Cos[c + d*x]^8)/d + (35*a*Cos[c + d*x]*Sin[c + d*x]
)/(128*d) + (35*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (7*a*Cos[c + d*x]^
5*Sin[c + d*x])/(48*d) + (a*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```


Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^8(c+dx)}{8d} + a\int \cos^8(c+dx)\,dx$$
$$= -\frac{ia\cos^8(c+dx)}{8d} + \frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}(7a)\int \cos^6(c+dx)\,dx$$
$$= -\frac{ia\cos^8(c+dx)}{8d} + \frac{7a\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{4}$$
$$= -\frac{ia\cos^8(c+dx)}{8d} + \frac{35a\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{7a\cos^5(c+dx)\sin(c+dx)}{48d} +$$
$$= -\frac{ia\cos^8(c+dx)}{8d} + \frac{35a\cos(c+dx)\sin(c+dx)}{128d} + \frac{35a\cos^3(c+dx)\sin(c+dx)}{192d}$$
$$= \frac{35ax}{128} - \frac{ia\cos^8(c+dx)}{8d} + \frac{35a\cos(c+dx)\sin(c+dx)}{128d} + \frac{35a\cos^3(c+dx)\sin(c}{192d}$$

Mathematica [A] time = 0.111135, size = 68, normalized size = 0.61

$$\frac{a\left(672\sin(2(c+dx)) + 168\sin(4(c+dx)) + 32\sin(6(c+dx)) + 3\sin(8(c+dx)) - 384i\cos^8(c+dx) + 840c + 840dx\right)}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*(840*c + 840*d*x - (384*I)*Cos[c + d*x]^8 + 672*Sin[2*(c + d*x)] + 168*S
in[4*(c + d*x)] + 32*Sin[6*(c + d*x)] + 3*Sin[8*(c + d*x)]))/(3072*d)
```

Maple [A] time = 0.094, size = 73, normalized size = 0.7

$$\frac{1}{d}\left(-\frac{i}{8}a(\cos(dx+c))^8 + a\left(\frac{\sin(dx+c)}{8}\left((\cos(dx+c))^7 + \frac{7(\cos(dx+c))^5}{6} + \frac{35(\cos(dx+c))^3}{24} + \frac{35\cos(dx+c)}{16}\right) + \frac{3}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/8*I*a*cos(d*x+c)^8+a*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(
d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [A] time = 1.6621, size = 139, normalized size = 1.25

$$\frac{105(dx+c)a + \frac{105a\tan(dx+c)^7 + 385a\tan(dx+c)^5 + 511a\tan(dx+c)^3 + 279a\tan(dx+c) - 48ia}{\tan(dx+c)^8 + 4\tan(dx+c)^6 + 6\tan(dx+c)^4 + 4\tan(dx+c)^2 + 1}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/384*(105*(d*x + c)*a + (105*a*tan(d*x + c)^7 + 385*a*tan(d*x + c)^5 + 511
*a*tan(d*x + c)^3 + 279*a*tan(d*x + c) - 48*I*a)/(tan(d*x + c)^8 + 4*tan(d*
x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.12426, size = 342, normalized size = 3.08

$$\frac{\left(840\,adxe^{(6i\,dx+6i\,c)} - 3i\,ae^{(14i\,dx+14i\,c)} - 28i\,ae^{(12i\,dx+12i\,c)} - 126i\,ae^{(10i\,dx+10i\,c)} - 420i\,ae^{(8i\,dx+8i\,c)} + 252i\,ae^{(4i\,dx+4i\,c)} + 42i\,\ldots}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3072*(840*a*d*x*e^(6*I*d*x + 6*I*c) - 3*I*a*e^(14*I*d*x + 14*I*c) - 28*I*
a*e^(12*I*d*x + 12*I*c) - 126*I*a*e^(10*I*d*x + 10*I*c) - 420*I*a*e^(8*I*d*
x + 8*I*c) + 252*I*a*e^(4*I*d*x + 4*I*c) + 42*I*a*e^(2*I*d*x + 2*I*c) + 4*I
*a)*e^(-6*I*d*x - 6*I*c)/d
```

Sympy [A] time = 1.04046, size = 280, normalized size = 2.52

$$\frac{35ax}{128} + \begin{cases} \frac{\left(-10133099161583616iad^6e^{20ic}e^{8idx} - 94575592174780416iad^6e^{18ic}e^{6idx} - 425590164786511872iad^6e^{16ic}e^{4idx} - 1418633882621706240iad^6e^{14ic}e^{2i\ldots}{10376293541461622784d\ldots} \\ x\left(-\frac{35a}{128} + \frac{\left(ae^{14ic}+7ae^{12ic}+21ae^{10ic}+35ae^{8ic}+35ae^{6ic}+21ae^{4ic}+7ae^{2ic}+a\right)e^{-6ic}}{128}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 35*a*x/128 + Piecewise(((-10133099161583616*I*a*d**6*exp(20*I*c)*exp(8*I*d*
x) - 94575592174780416*I*a*d**6*exp(18*I*c)*exp(6*I*d*x) - 4255901647865118
72*I*a*d**6*exp(16*I*c)*exp(4*I*d*x) - 1418633882621706240*I*a*d**6*exp(14*
I*c)*exp(2*I*d*x) + 851180329573023744*I*a*d**6*exp(10*I*c)*exp(-2*I*d*x) +
 141863388262170624*I*a*d**6*exp(8*I*c)*exp(-4*I*d*x) + 13510798882111488*I
*a*d**6*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(10376293541461622784*d**7),
 Ne(10376293541461622784*d**7*exp(12*I*c), 0)), (x*(-35*a/128 + (a*exp(14*I
*c) + 7*a*exp(12*I*c) + 21*a*exp(10*I*c) + 35*a*exp(8*I*c) + 35*a*exp(6*I*c
) + 21*a*exp(4*I*c) + 7*a*exp(2*I*c) + a)*exp(-6*I*c)/128), True))
```

Giac [A] time = 1.16079, size = 204, normalized size = 1.84

$$\frac{\left(840\,adxe^{(6i\,dx+2i\,c)} + 84i\,ae^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - 84i\,ae^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right) - 3i\,ae^{(14i\,dx+10i\,c)} - 28i\,a\ldots}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(840*a*d*x*e^(6*I*d*x + 2*I*c) + 84*I*a*e^(6*I*d*x + 2*I*c)*log(e^(2
*I*d*x + 2*I*c) + 1) - 84*I*a*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I
*c)) - 3*I*a*e^(14*I*d*x + 10*I*c) - 28*I*a*e^(12*I*d*x + 8*I*c) - 126*I*a*
e^(10*I*d*x + 6*I*c) - 420*I*a*e^(8*I*d*x + 4*I*c) + 42*I*a*e^(2*I*d*x - 2*
I*c) + 252*I*a*e^(4*I*d*x) + 4*I*a*e^(-4*I*c))*e^(-6*I*d*x - 2*I*c)/d
```

3.11 $\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=98

$$\frac{ia\sec^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a\tan(c+dx)\mathrm{se}}{16d}$$

```
[Out] (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((I/7)*a*Sec[c + d*x]^7)/d + (5*a*Sec[
c + d*x]*Tan[c + d*x])/(16*d) + (5*a*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) +
(a*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.0606778, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3486, 3768, 3770}

$$\frac{ia\sec^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a\tan(c+dx)\mathrm{se}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((I/7)*a*Sec[c + d*x]^7)/d + (5*a*Sec[
c + d*x]*Tan[c + d*x])/(16*d) + (5*a*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) +
(a*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec^7(c+dx)}{7d} + a\int \sec^7(c+dx)\,dx \\
&= \frac{ia\sec^7(c+dx)}{7d} + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{1}{6}(5a)\int \sec^5(c+dx)\,dx \\
&= \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d} + \\
&= \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} + \\
&= \frac{5a\tanh^{-1}(\sin(c+dx))}{16d} + \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a}{}
\end{aligned}
$$

Mathematica [A] time = 0.293266, size = 61, normalized size = 0.62

$$\frac{a\left(3360\tanh^{-1}(\sin(c+dx)) + (1981\sin(2(c+dx)) + 700\sin(4(c+dx)) + 105\sin(6(c+dx)) + 1536i)\sec^7(c+dx)\right)}{10752d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*(3360*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]^7*(1536*I + 1981*Sin[2*(c + d
*x)] + 700*Sin[4*(c + d*x)] + 105*Sin[6*(c + d*x)])))/(10752*d)
```

Maple [A] time = 0.085, size = 95, normalized size = 1.

$$\frac{\frac{i}{7}a}{d(\cos(dx+c))^7} + \frac{a(\sec(dx+c))^5\tan(dx+c)}{6d} + \frac{5a(\sec(dx+c))^3\tan(dx+c)}{24d} + \frac{5a\sec(dx+c)\tan(dx+c)}{16d} + \frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/7*I/d*a/cos(d*x+c)^7+1/6*a*sec(d*x+c)^5*tan(d*x+c)/d+5/24*a*sec(d*x+c)^3*
tan(d*x+c)/d+5/16*a*sec(d*x+c)*tan(d*x+c)/d+5/16/d*a*ln(sec(d*x+c)+tan(d*x+
c))
```

Maxima [A] time = 1.10413, size = 143, normalized size = 1.46

$$-\frac{7a\left(\frac{2(15\sin(dx+c)^5-40\sin(dx+c)^3+33\sin(dx+c))}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1} - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1)\right) - \frac{96ia}{\cos(dx+c)^7}}{672d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/672*(7*a*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x +
 c) + 1) + 15*log(sin(d*x + c) - 1)) - 96*I*a/cos(d*x + c)^7)/d
```

Fricas [B] time = 1.08979, size = 1170, normalized size = 11.94

$$\frac{-210i\,ae^{(13i\,dx+13i\,c)} - 1400i\,ae^{(11i\,dx+11i\,c)} - 3962i\,ae^{(9i\,dx+9i\,c)} + 6144i\,ae^{(7i\,dx+7i\,c)} + 3962i\,ae^{(5i\,dx+5i\,c)} + 1400i\,ae^{(3i\,dx+3i\,c)} + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/336*(-210*I*a*e^(13*I*d*x + 13*I*c) - 1400*I*a*e^(11*I*d*x + 11*I*c) - 39
62*I*a*e^(9*I*d*x + 9*I*c) + 6144*I*a*e^(7*I*d*x + 7*I*c) + 3962*I*a*e^(5*I
*d*x + 5*I*c) + 1400*I*a*e^(3*I*d*x + 3*I*c) + 210*I*a*e^(I*d*x + I*c) + 10
5*(a*e^(14*I*d*x + 14*I*c) + 7*a*e^(12*I*d*x + 12*I*c) + 21*a*e^(10*I*d*x +
 10*I*c) + 35*a*e^(8*I*d*x + 8*I*c) + 35*a*e^(6*I*d*x + 6*I*c) + 21*a*e^(4*
I*d*x + 4*I*c) + 7*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) + I) - 10
5*(a*e^(14*I*d*x + 14*I*c) + 7*a*e^(12*I*d*x + 12*I*c) + 21*a*e^(10*I*d*x +
 10*I*c) + 35*a*e^(8*I*d*x + 8*I*c) + 35*a*e^(6*I*d*x + 6*I*c) + 21*a*e^(4*
I*d*x + 4*I*c) + 7*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) - I))/(d*
e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I
*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x
 + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int i\tan\left(c+dx\right)\sec^{7}\left(c+dx\right)dx + \int \sec^{7}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*(Integral(I*tan(c + d*x)*sec(c + d*x)**7, x) + Integral(sec(c + d*x)**7,
x))
```

Giac [B] time = 1.20048, size = 247, normalized size = 2.52

$$\frac{105\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 105\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(231\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} - 336i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12} - 196\,a\tan\left(\frac{1}{2}\cdots\right.\right.}{\cdots}}{336\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/336*(105*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) + 2*(231*a*tan(1/2*d*x + 1/2*c)^13 - 336*I*a*tan(1/2*d*x +
1/2*c)^12 - 196*a*tan(1/2*d*x + 1/2*c)^11 + 595*a*tan(1/2*d*x + 1/2*c)^9 -
1680*I*a*tan(1/2*d*x + 1/2*c)^8 - 595*a*tan(1/2*d*x + 1/2*c)^5 - 1008*I*a*t
an(1/2*d*x + 1/2*c)^4 + 196*a*tan(1/2*d*x + 1/2*c)^3 - 231*a*tan(1/2*d*x +
1/2*c) - 48*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.12 $\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=76

$$\frac{ia\sec^5(c+dx)}{5d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + ((I/5)*a*Sec[c + d*x]^5)/d + (3*a*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.0483808, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3486, 3768, 3770}

$$\frac{ia\sec^5(c+dx)}{5d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + ((I/5)*a*Sec[c + d*x]^5)/d + (3*a*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec^5(c+dx)}{5d}+a\int \sec^5(c+dx)\,dx\\ &= \frac{ia\sec^5(c+dx)}{5d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}(3a)\int \sec^3(c+dx)\,dx\\ &= \frac{ia\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\\ &= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{ia\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\,\mathrm{s}}{}\end{aligned}$$

Mathematica [A] time = 0.151544, size = 70, normalized size = 0.92

$$\frac{ia\sec^5(c+dx)}{5d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/5)*a*Sec[c + d*x]^5)/d + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(
ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A] time = 0.085, size = 75, normalized size = 1.

$$\frac{\frac{i}{5}a}{d\left(\cos(dx+c)\right)^5}+\frac{a\left(\sec(dx+c)\right)^3\tan(dx+c)}{4d}+\frac{3a\sec(dx+c)\tan(dx+c)}{8d}+\frac{3a\ln\left(\sec(dx+c)+\tan(dx+c)\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/5*I/d*a/cos(d*x+c)^5+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan
(d*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.11564, size = 116, normalized size = 1.53

$$-\frac{5a\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin(dx+c)+1\right)+3\log\left(\sin(dx+c)-1\right)\right)-\frac{16ia}{\cos(dx+c)^5}}{80d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*I*
a/cos(d*x + c)^5)/d
```

Fricas [B] time = 1.2512, size = 836, normalized size = 11.

$$\frac{-30i\,ae^{(9i\,dx+9i\,c)}-140i\,ae^{(7i\,dx+7i\,c)}+256i\,ae^{(5i\,dx+5i\,c)}+140i\,ae^{(3i\,dx+3i\,c)}+30i\,ae^{(i\,dx+i\,c)}+15\left(ae^{(10i\,dx+10i\,c)}+5\,ae^{(8i\,dx+8i\,c)}\right.}{40\left(de^{(10i\,dx+10i\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(-30*I*a*e^(9*I*d*x + 9*I*c) - 140*I*a*e^(7*I*d*x + 7*I*c) + 256*I*a*e
^(5*I*d*x + 5*I*c) + 140*I*a*e^(3*I*d*x + 3*I*c) + 30*I*a*e^(I*d*x + I*c) +
 15*(a*e^(10*I*d*x + 10*I*c) + 5*a*e^(8*I*d*x + 8*I*c) + 10*a*e^(6*I*d*x +
6*I*c) + 10*a*e^(4*I*d*x + 4*I*c) + 5*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d
```

```
*x + I*c) + I) - 15*(a*e^(10*I*d*x + 10*I*c) + 5*a*e^(8*I*d*x + 8*I*c) + 10
*a*e^(6*I*d*x + 6*I*c) + 10*a*e^(4*I*d*x + 4*I*c) + 5*a*e^(2*I*d*x + 2*I*c)
 + a)*log(e^(I*d*x + I*c) - I))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x +
 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*
d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int i\tan\left(c+dx\right)\sec^{5}\left(c+dx\right)dx+\int\sec^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*(Integral(I*tan(c + d*x)*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**5,
x))
```

Giac [B] time = 1.19293, size = 190, normalized size = 2.5

$$\frac{15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-40i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\right.}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*log(abs(tan(1/2*d*x +
1/2*c) - 1)) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*I*a*tan(1/2*d*x + 1/2*c)
^8 - 10*a*tan(1/2*d*x + 1/2*c)^7 - 80*I*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan
(1/2*d*x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*I*a)/(tan(1/2*d*x + 1/2
*c)^2 - 1)^5)/d
```

3.13 $\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=54

$$\frac{ia\sec^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((I/3)*a*Sec[c + d*x]^3)/d + (a*Sec[c + d
*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0352547, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3486, 3768, 3770}

$$\frac{ia\sec^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((I/3)*a*Sec[c + d*x]^3)/d + (a*Sec[c + d
*x]*Tan[c + d*x])/(2*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{ia\sec^3(c+dx)}{3d}+a\int \sec^3(c+dx)\,dx\\ &= \frac{ia\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a\int \sec(c+dx)\,dx\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{ia\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0129146, size = 54, normalized size = 1.

$$\frac{ia\sec^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((I/3)*a*Sec[c + d*x]^3)/d + (a*Sec[c + d
*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.081, size = 55, normalized size = 1.

$$\frac{\frac{i}{3}a}{d\left(\cos\left(dx+c\right)\right)^{3}}+\frac{a\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/3*I/d*a/cos(d*x+c)^3+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+
tan(d*x+c))
```

Maxima [A] time = 1.11126, size = 82, normalized size = 1.52

$$-\frac{3\,a\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{4i\,a}{\cos\left(dx+c\right)^{3}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*I*a/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.11928, size = 520, normalized size = 9.63

$$\frac{-6i\,ae^{\left(5i\,dx+5i\,c\right)}+16i\,ae^{\left(3i\,dx+3i\,c\right)}+6i\,ae^{\left(i\,dx+i\,c\right)}+3\left(ae^{\left(6i\,dx+6i\,c\right)}+3\,ae^{\left(4i\,dx+4i\,c\right)}+3\,ae^{\left(2i\,dx+2i\,c\right)}+a\right)\log\left(e^{\left(i\,dx+i\,c\right)}+i\right)-3}{6\left(de^{\left(6i\,dx+6i\,c\right)}+3\,de^{\left(4i\,dx+4i\,c\right)}+3\,de^{\left(2i\,dx+2i\,c\right)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(-6*I*a*e^(5*I*d*x + 5*I*c) + 16*I*a*e^(3*I*d*x + 3*I*c) + 6*I*a*e^(I*d
*x + I*c) + 3*(a*e^(6*I*d*x + 6*I*c) + 3*a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2*I
*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) + I) - 3*(a*e^(6*I*d*x + 6*I*c) + 3*
a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) -
I))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I
*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int i\tan(c+dx)\sec^3(c+dx)\,dx+\int\sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*(Integral(I*tan(c + d*x)*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3,
x))
```

Giac [B] time = 1.18881, size = 134, normalized size = 2.48

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2i\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*I*a*tan(1/2*d*x + 1/2*c)^4 -
3*a*tan(1/2*d*x + 1/2*c) - 2*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.14 $\int \sec(c + dx)(a + ia \tan(c + dx))\, dx$

Optimal. Leaf size=27

$$\frac{a \tanh^{-1}(\sin(c + dx))}{d} + \frac{ia \sec(c + dx)}{d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d
```

Rubi [A] time = 0.0158266, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3486, 3770}

$$\frac{a \tanh^{-1}(\sin(c + dx))}{d} + \frac{ia \sec(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c + dx)(a + ia \tan(c + dx))\, dx &= \frac{ia \sec(c + dx)}{d} + a \int \sec(c + dx)\, dx\\ &= \frac{a \tanh^{-1}(\sin(c + dx))}{d} + \frac{ia \sec(c + dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0080077, size = 27, normalized size = 1.

$$\frac{a \tanh^{-1}(\sin(c + dx))}{d} + \frac{ia \sec(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d
```

Maple [A] time = 0.014, size = 36, normalized size = 1.3

$$\frac{ia}{d\cos\left(dx+c\right)}+\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I/d*a/cos(d*x+c)+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.06567, size = 43, normalized size = 1.59

$$\frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+\frac{ia}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + I*a/cos(d*x + c))/d
```

Fricas [B] time = 1.15591, size = 220, normalized size = 8.15

$$\frac{2i\,ae^{(i\,dx+i\,c)}+\left(ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-\left(ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*I*a*e^(I*d*x + I*c) + (a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) +
I) - (a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) - I))/(d*e^(2*I*d*x +
2*I*c) + d)
```

Sympy [A] time = 5.99679, size = 41, normalized size = 1.52

$$\begin{cases}\frac{a\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)+ia\sec\left(c+dx\right)}{d} & \text{for}\ d\neq 0\\ x\left(ia\tan\left(c\right)+a\right)\sec\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + I*a*sec(c + d*x))/d, Ne(d,
 0)), (x*(I*a*tan(c) + a)*sec(c), True))
```

Giac [B] time = 1.18006, size = 73, normalized size = 2.7

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*I*a/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.15 $\int \cos(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

```
[Out] ((-I)*a*Cos[c + d*x])/d + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.020528, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3486, 2637}

$$\frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*a*Cos[c + d*x])/d + (a*Sin[c + d*x])/d
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos(c+dx)}{d}+a\int \cos(c+dx)\,dx\\ &= -\frac{ia\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0198565, size = 51, normalized size = 1.96

$$\frac{ia\sin(c)\sin(dx)}{d}-\frac{ia\cos(c)\cos(dx)}{d}+\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*a*Cos[c]*Cos[d*x])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
+ (I*a*Sin[c]*Sin[d*x])/d
```

Maple [A] time = 0.036, size = 24, normalized size = 0.9

$$\frac{-ia\cos(dx+c)+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(-I*a*cos(d*x+c)+a*sin(d*x+c))
```

Maxima [A] time = 1.10954, size = 30, normalized size = 1.15

$$\frac{-i\,a\cos(dx+c)+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (-I*a*cos(d*x + c) + a*sin(d*x + c))/d
```

Fricas [A] time = 1.13935, size = 32, normalized size = 1.23

$$-\frac{i\,ae^{(i\,dx+i\,c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -I*a*e^(I*d*x + I*c)/d
```

Sympy [A] time = 0.231601, size = 26, normalized size = 1.

$$\begin{cases} -\frac{iae^{ic}e^{idx}}{d} & \text{for}\ d \neq 0 \\ axe^{ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((-I*a*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (a*x*exp(I*c), True))
```

Giac [B] time = 1.15785, size = 113, normalized size = 4.35

$$-\frac{4i\,ae^{(i\,dx+i\,c)}+a\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+a\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-a\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-a\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*I*a*e^(I*d*x + I*c) + a*log(I*e^(I*d*x + I*c) + 1) + a*log(I*e^(I*d
*x + I*c) - 1) - a*log(-I*e^(I*d*x + I*c) + 1) - a*log(-I*e^(I*d*x + I*c) -
 1))/d
```

3.16 $\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=46

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^3(c+dx)}{3d}$$

```
[Out] ((-I/3)*a*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0323892, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3486, 2633}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I/3)*a*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos^3(c+dx)}{3d}+a\int \cos^3(c+dx)\,dx\\
&= -\frac{ia\cos^3(c+dx)}{3d}-\frac{a\,\text{Subst}\left(\int \left(1-x^2\right)dx,x,-\sin(c+dx)\right)}{d}\\
&= -\frac{ia\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0095743, size = 46, normalized size = 1.

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I/3)*a*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.078, size = 37, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{i}{3}a\left(\cos\left(dx+c\right)\right)^{3}+\frac{a\left(2+\left(\cos\left(dx+c\right)\right)^{2}\right)\sin\left(dx+c\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/3*I*a*cos(d*x+c)^3+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.12166, size = 49, normalized size = 1.07

$$-\frac{i\,a\cos\left(dx+c\right)^{3}+\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(I*a*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

Fricas [A] time = 1.10006, size = 119, normalized size = 2.59

$$\frac{\left(-i\,ae^{(4i\,dx+4i\,c)}-6i\,ae^{(2i\,dx+2i\,c)}+3i\,a\right)e^{(-i\,dx-i\,c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^(2*I*d*x + 2*I*c) + 3*I*a)*e^(-I*d
*x - I*c)/d
```

Sympy [A] time = 0.558693, size = 107, normalized size = 2.33

$$\begin{cases}\frac{\left(-8iad^{2}e^{4ic}e^{3idx}-48iad^{2}e^{2ic}e^{idx}+24iad^{2}e^{-idx}\right)e^{-ic}}{96d^{3}} & \text{for}\ 96d^{3}e^{ic}\neq 0\\ \frac{x\left(ae^{4ic}+2ae^{2ic}+a\right)e^{-ic}}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-8*I*a*d**2*exp(4*I*c)*exp(3*I*d*x) - 48*I*a*d**2*exp(2*I*c)*ex
p(I*d*x) + 24*I*a*d**2*exp(-I*d*x))*exp(-I*c)/(96*d**3), Ne(96*d**3*exp(I*c
```

```
), 0)), (x*(a*exp(4*I*c) + 2*a*exp(2*I*c) + a)*exp(-I*c)/4, True))
```

Giac [B] time = 1.14382, size = 265, normalized size = 5.76

$$-\frac{\left(9\,ae^{(i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+6\,ae^{(i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-9\,ae^{(i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-6\,ae^{(i\,dx+i\,c)}\log\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/48*(9*a*e^(I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 6*a*e^(I*d*x + I*c)
*log(I*e^(I*d*x + I*c) - 1) - 9*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x + I*c) +
1) - 6*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3*a*e^(I*d*x + I*c)*
log(I*e^(I*d*x) + e^(-I*c)) + 3*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I*
c)) + 4*I*a*e^(4*I*d*x + 4*I*c) + 24*I*a*e^(2*I*d*x + 2*I*c) - 12*I*a)*e^(-
I*d*x - I*c)/d
```

3.17 $\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=62

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^5(c+dx)}{5d}$$

```
[Out] ((-I/5)*a*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*
d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0347422, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3486, 2633}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I/5)*a*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*
d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos^5(c+dx)}{5d}+a\int \cos^5(c+dx)\,dx\\ &= -\frac{ia\cos^5(c+dx)}{5d}-\frac{a\,\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)dx,x,-\sin(c+dx)\right)}{d}\\ &= -\frac{ia\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0150096, size = 62, normalized size = 1.

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I/5)*a*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*
d) + (a*Sin[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.078, size = 47, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{i}{5}a\left(\cos\left(dx+c\right)\right)^{5}+\frac{a\sin\left(dx+c\right)}{5}\left(\frac{8}{3}+\left(\cos\left(dx+c\right)\right)^{4}+\frac{4\left(\cos\left(dx+c\right)\right)^{2}}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/5*I*a*cos(d*x+c)^5+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*
x+c))
```

Maxima [A] time = 1.14187, size = 66, normalized size = 1.06

$$-\frac{3i\,a\cos\left(dx+c\right)^{5}-\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)a}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/15*(3*I*a*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*si
n(d*x + c))*a)/d
```

Fricas [A] time = 1.02029, size = 208, normalized size = 3.35

$$\frac{\left(-3i\,ae^{(8i\,dx+8i\,c)}-20i\,ae^{(6i\,dx+6i\,c)}-90i\,ae^{(4i\,dx+4i\,c)}+60i\,ae^{(2i\,dx+2i\,c)}+5i\,a\right)e^{(-3i\,dx-3i\,c)}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(-3*I*a*e^(8*I*d*x + 8*I*c) - 20*I*a*e^(6*I*d*x + 6*I*c) - 90*I*a*e^(
4*I*d*x + 4*I*c) + 60*I*a*e^(2*I*d*x + 2*I*c) + 5*I*a)*e^(-3*I*d*x - 3*I*c)
/d
```

Sympy [A] time = 1.00191, size = 185, normalized size = 2.98

$$\begin{cases}\frac{\left(-18432iad^{4}e^{9ic}e^{5idx}-122880iad^{4}e^{7ic}e^{3idx}-552960iad^{4}e^{5ic}e^{idx}+368640iad^{4}e^{3ic}e^{-idx}+30720iad^{4}e^{ic}e^{-3idx}\right)e^{-4ic}}{1474560d^{5}} & \text{for}\ 1474560d^{5}e^{4ic}\neq 0\\ \frac{x\left(ae^{8ic}+4ae^{6ic}+6ae^{4ic}+4ae^{2ic}+a\right)e^{-3ic}}{16} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-18432*I*a*d**4*exp(9*I*c)*exp(5*I*d*x) - 122880*I*a*d**4*exp(7
*I*c)*exp(3*I*d*x) - 552960*I*a*d**4*exp(5*I*c)*exp(I*d*x) + 368640*I*a*d**
4*exp(3*I*c)*exp(-I*d*x) + 30720*I*a*d**4*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*
c)/(1474560*d**5), Ne(1474560*d**5*exp(4*I*c), 0)), (x*(a*exp(8*I*c) + 4*a*
exp(6*I*c) + 6*a*exp(4*I*c) + 4*a*exp(2*I*c) + a)*exp(-3*I*c)/16, True))
```

Giac [B] time = 1.20179, size = 297, normalized size = 4.79

$$-\frac{\left(135\, ae^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+90\, ae^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-135\, ae^{(3i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-90\, ae^{(3i\,dx+i\,c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/960*(135*a*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 90*a*e^(3*I*d*
x + I*c)*log(I*e^(I*d*x + I*c) - 1) - 135*a*e^(3*I*d*x + I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 90*a*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 45*a*
e^(3*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 45*a*e^(3*I*d*x + I*c)*log(
-I*e^(I*d*x) + e^(-I*c)) + 12*I*a*e^(8*I*d*x + 6*I*c) + 80*I*a*e^(6*I*d*x +
 4*I*c) + 360*I*a*e^(4*I*d*x + 2*I*c) - 240*I*a*e^(2*I*d*x) - 20*I*a*e^(-2*
I*c))*e^(-3*I*d*x - I*c)/d
```

3.18 $\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=76

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^7(c+dx)}{7d}$$

```
[Out] ((-I/7)*a*Cos[c + d*x]^7)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (
3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0381817, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3486, 2633}

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I/7)*a*Cos[c + d*x]^7)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (
3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx &= -\frac{ia\cos^7(c+dx)}{7d}+a\int \cos^7(c+dx)\,dx\\ &= -\frac{ia\cos^7(c+dx)}{7d}-\frac{a\,\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{d}\\ &= -\frac{ia\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^7}{}\end{aligned}$$

Mathematica [A] time = 0.0422304, size = 76, normalized size = 1.

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I/7)*a*Cos[c + d*x]^7)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (
3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Maple [A] time = 0.08, size = 57, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{i}{7}a\left(\cos\left(dx+c\right)\right)^{7}+\frac{a\sin\left(dx+c\right)}{7}\left(\frac{16}{5}+\left(\cos\left(dx+c\right)\right)^{6}+\frac{6\left(\cos\left(dx+c\right)\right)^{4}}{5}+\frac{8\left(\cos\left(dx+c\right)\right)^{2}}{5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/7*I*a*cos(d*x+c)^7+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*co
s(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.11542, size = 78, normalized size = 1.03

$$-\frac{5i\,a\cos\left(dx+c\right)^{7}+\left(5\,\sin\left(dx+c\right)^{7}-21\,\sin\left(dx+c\right)^{5}+35\,\sin\left(dx+c\right)^{3}-35\,\sin\left(dx+c\right)\right)a}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/35*(5*I*a*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*si
n(d*x + c)^3 - 35*sin(d*x + c))*a)/d
```

Fricas [A] time = 1.07054, size = 297, normalized size = 3.91

$$\frac{\left(-5i\,ae^{(12i\,dx+12i\,c)}-42i\,ae^{(10i\,dx+10i\,c)}-175i\,ae^{(8i\,dx+8i\,c)}-700i\,ae^{(6i\,dx+6i\,c)}+525i\,ae^{(4i\,dx+4i\,c)}+70i\,ae^{(2i\,dx+2i\,c)}+7i\,a\right)e^{(-5i\,d}}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2240*(-5*I*a*e^(12*I*d*x + 12*I*c) - 42*I*a*e^(10*I*d*x + 10*I*c) - 175*I
*a*e^(8*I*d*x + 8*I*c) - 700*I*a*e^(6*I*d*x + 6*I*c) + 525*I*a*e^(4*I*d*x +
 4*I*c) + 70*I*a*e^(2*I*d*x + 2*I*c) + 7*I*a)*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [A] time = 1.24666, size = 255, normalized size = 3.36

$$\begin{cases}\frac{\left(-107374182400iad^{6}e^{16ic}e^{7idx}-901943132160iad^{6}e^{14ic}e^{5idx}-3758096384000iad^{6}e^{12ic}e^{3idx}-15032385536000iad^{6}e^{10ic}e^{idx}+11274289152000iad^{6}e^{8ic}e^{-idx}+15032\right.}{48103633715200d^{7}} \\ \frac{x\left(ae^{12ic}+6ae^{10ic}+15ae^{8ic}+20ae^{6ic}+15ae^{4ic}+6ae^{2ic}+a\right)e^{-5ic}}{64}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-107374182400*I*a*d**6*exp(16*I*c)*exp(7*I*d*x) - 901943132160*
I*a*d**6*exp(14*I*c)*exp(5*I*d*x) - 3758096384000*I*a*d**6*exp(12*I*c)*exp(
3*I*d*x) - 15032385536000*I*a*d**6*exp(10*I*c)*exp(I*d*x) + 11274289152000*
I*a*d**6*exp(8*I*c)*exp(-I*d*x) + 1503238553600*I*a*d**6*exp(6*I*c)*exp(-3*
I*d*x) + 150323855360*I*a*d**6*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(48103
633715200*d**7), Ne(48103633715200*d**7*exp(9*I*c), 0)), (x*(a*exp(12*I*c)
+ 6*a*exp(10*I*c) + 15*a*exp(8*I*c) + 20*a*exp(6*I*c) + 15*a*exp(4*I*c) + 6
*a*exp(2*I*c) + a)*exp(-5*I*c)/64, True))
```

Giac [B] time = 1.21458, size = 329, normalized size = 4.33

$$-\frac{\left(1015\,ae^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+700\,ae^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-1015\,ae^{(5i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-700\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8960*(1015*a*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 700*a*e^(5*I
*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) - 1015*a*e^(5*I*d*x + I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 700*a*e^(5*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) -
315*a*e^(5*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 315*a*e^(5*I*d*x + I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) + 20*I*a*e^(12*I*d*x + 8*I*c) + 168*I*a*e^(
10*I*d*x + 6*I*c) + 700*I*a*e^(8*I*d*x + 4*I*c) + 2800*I*a*e^(6*I*d*x + 2*I
*c) - 280*I*a*e^(2*I*d*x - 2*I*c) - 2100*I*a*e^(4*I*d*x) - 28*I*a*e^(-4*I*c
))*e^(-5*I*d*x - I*c)/d
```

3.19 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=109

$$\frac{i(a+ia\tan(c+dx))^9}{9a^7d}-\frac{3i(a+ia\tan(c+dx))^8}{4a^6d}+\frac{12i(a+ia\tan(c+dx))^7}{7a^5d}-\frac{4i(a+ia\tan(c+dx))^6}{3a^4d}$$

```
[Out] (((-4*I)/3)*(a + I*a*Tan[c + d*x])^6)/(a^4*d) + (((12*I)/7)*(a + I*a*Tan[c
+ d*x])^7)/(a^5*d) - (((3*I)/4)*(a + I*a*Tan[c + d*x])^8)/(a^6*d) + ((I/9)*
(a + I*a*Tan[c + d*x])^9)/(a^7*d)
```

Rubi [A] time = 0.0662851, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a+ia\tan(c+dx))^9}{9a^7d}-\frac{3i(a+ia\tan(c+dx))^8}{4a^6d}+\frac{12i(a+ia\tan(c+dx))^7}{7a^5d}-\frac{4i(a+ia\tan(c+dx))^6}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((-4*I)/3)*(a + I*a*Tan[c + d*x])^6)/(a^4*d) + (((12*I)/7)*(a + I*a*Tan[c
+ d*x])^7)/(a^5*d) - (((3*I)/4)*(a + I*a*Tan[c + d*x])^8)/(a^6*d) + ((I/9)*
(a + I*a*Tan[c + d*x])^9)/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^3(a+x)^5\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(8a^3(a+x)^5-12a^2(a+x)^6+6a(a+x)^7-(a+x)^8\right)dx,x,ia\tan(c\right.}{a^7d}\\ &= -\frac{4i(a+ia\tan(c+dx))^6}{3a^4d}+\frac{12i(a+ia\tan(c+dx))^7}{7a^5d}-\frac{3i(a+ia\tan(c+dx))^8}{4a^6d}+\end{aligned}$$

Mathematica [A] time = 1.24373, size = 99, normalized size = 0.91

$$\frac{a^2\sec(c)\sec^9(c+dx)(-63\sin(2c+dx)+84\sin(2c+3dx)+36\sin(4c+5dx)+9\sin(6c+7dx)+\sin(8c+9dx)+63i\,\text{cc}}{504d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c]*Sec[c + d*x]^9*((63*I)*Cos[d*x] + (63*I)*Cos[2*c + d*x] + 63*Si
n[d*x] - 63*Sin[2*c + d*x] + 84*Sin[2*c + 3*d*x] + 36*Sin[4*c + 5*d*x] + 9*
Sin[6*c + 7*d*x] + Sin[8*c + 9*d*x]))/(504*d)
```

Maple [A] time = 0.058, size = 141, normalized size = 1.3

$$\frac{1}{d}\left(-a^2\left(\frac{(\sin(dx+c))^3}{9(\cos(dx+c))^9}+\frac{2(\sin(dx+c))^3}{21(\cos(dx+c))^7}+\frac{8(\sin(dx+c))^3}{105(\cos(dx+c))^5}+\frac{16(\sin(dx+c))^3}{315(\cos(dx+c))^3}\right)+\frac{\frac{i}{4}a^2}{(\cos(dx+c))^8}-a^2\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/1
05*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+1/4*I*a^2/co
s(d*x+c)^8-a^2*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2
)*tan(d*x+c))
```

Maxima [A] time = 1.10516, size = 146, normalized size = 1.34

$$-\frac{140\,a^2\tan(dx+c)^9-315i\,a^2\tan(dx+c)^8+360\,a^2\tan(dx+c)^7-1260i\,a^2\tan(dx+c)^6-1890i\,a^2\tan(dx+c)^4-}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/1260*(140*a^2*tan(d*x + c)^9 - 315*I*a^2*tan(d*x + c)^8 + 360*a^2*tan(d*
x + c)^7 - 1260*I*a^2*tan(d*x + c)^6 - 1890*I*a^2*tan(d*x + c)^4 - 840*a^2*
tan(d*x + c)^3 - 1260*I*a^2*tan(d*x + c)^2 - 1260*a^2*tan(d*x + c))/d
```

Fricas [B] time = 1.19107, size = 590, normalized size = 5.41

$$\frac{8064i\,a^2e^{(10i\,dx+10i\,c)}+8064i\,a^2e^{(8i\,dx+8i\,c)}+5376i\,a^2e^{(6i\,dx+6i\,c)}+2304i\,a^2e^{(4i\,dx+4i\,c)}+576i\,a^2e}{63\left(de^{(18i\,dx+18i\,c)}+9\,de^{(16i\,dx+16i\,c)}+36\,de^{(14i\,dx+14i\,c)}+84\,de^{(12i\,dx+12i\,c)}+126\,de^{(10i\,dx+10i\,c)}+126\,de^{(8i\,dx+8i\,c)}+84\,de^{(6i}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/63*(8064*I*a^2*e^(10*I*d*x + 10*I*c) + 8064*I*a^2*e^(8*I*d*x + 8*I*c) + 5
376*I*a^2*e^(6*I*d*x + 6*I*c) + 2304*I*a^2*e^(4*I*d*x + 4*I*c) + 576*I*a^2*
e^(2*I*d*x + 2*I*c) + 64*I*a^2)/(d*e^(18*I*d*x + 18*I*c) + 9*d*e^(16*I*d*x
+ 16*I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d*x + 12*I*c) + 126*d
*e^(10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 84*d*e^(6*I*d*x + 6*I*
c) + 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int -\tan^2(c+dx)\sec^8(c+dx)\,dx+\int 2i\tan(c+dx)\sec^8(c+dx)\,dx+\int \sec^8(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(-tan(c + d*x)**2*sec(c + d*x)**8, x) + Integral(2*I*tan(c +
d*x)*sec(c + d*x)**8, x) + Integral(sec(c + d*x)**8, x))
```

Giac [A] time = 1.20306, size = 146, normalized size = 1.34

$$-\frac{28\,a^2\tan(dx+c)^9-63i\,a^2\tan(dx+c)^8+72\,a^2\tan(dx+c)^7-252i\,a^2\tan(dx+c)^6-378i\,a^2\tan(dx+c)^4-168\,a^2\,\mathrm{t}}{252\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/252*(28*a^2*tan(d*x + c)^9 - 63*I*a^2*tan(d*x + c)^8 + 72*a^2*tan(d*x +
c)^7 - 252*I*a^2*tan(d*x + c)^6 - 378*I*a^2*tan(d*x + c)^4 - 168*a^2*tan(d*
x + c)^3 - 252*I*a^2*tan(d*x + c)^2 - 252*a^2*tan(d*x + c))/d
```

3.20 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=82

$$-\frac{i(a+ia\tan(c+dx))^7}{7a^5d}+\frac{2i(a+ia\tan(c+dx))^6}{3a^4d}-\frac{4i(a+ia\tan(c+dx))^5}{5a^3d}$$

```
[Out] (((-4*I)/5)*(a + I*a*Tan[c + d*x])^5)/(a^3*d) + (((2*I)/3)*(a + I*a*Tan[c +
 d*x])^6)/(a^4*d) - ((I/7)*(a + I*a*Tan[c + d*x])^7)/(a^5*d)
```

Rubi [A] time = 0.056461, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{i(a+ia\tan(c+dx))^7}{7a^5d}+\frac{2i(a+ia\tan(c+dx))^6}{3a^4d}-\frac{4i(a+ia\tan(c+dx))^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((-4*I)/5)*(a + I*a*Tan[c + d*x])^5)/(a^3*d) + (((2*I)/3)*(a + I*a*Tan[c +
 d*x])^6)/(a^4*d) - ((I/7)*(a + I*a*Tan[c + d*x])^7)/(a^5*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^2(a+x)^4\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a+x)^4-4a(a+x)^5+(a+x)^6\right)dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{4i(a+ia\tan(c+dx))^5}{5a^3d}+\frac{2i(a+ia\tan(c+dx))^6}{3a^4d}-\frac{i(a+ia\tan(c+dx))^7}{7a^5d}\end{aligned}$$

Mathematica [A] time = 0.880934, size = 90, normalized size = 1.1

$$\frac{a^2\sec(c)\sec^7(c+dx)(-35\sin(2c+dx)+42\sin(2c+3dx)+14\sin(4c+5dx)+2\sin(6c+7dx)+35i\cos(2c+dx)+}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c]*Sec[c + d*x]^7*((35*I)*Cos[d*x] + (35*I)*Cos[2*c + d*x] + 35*Si
n[d*x] - 35*Sin[2*c + d*x] + 42*Sin[2*c + 3*d*x] + 14*Sin[4*c + 5*d*x] + 2*
Sin[6*c + 7*d*x]))/(210*d)
```

Maple [A] time = 0.056, size = 113, normalized size = 1.4

$$\frac{1}{d}\left(-a^{2}\left(\frac{(\sin(dx+c))^{3}}{7(\cos(dx+c))^{7}}+\frac{4(\sin(dx+c))^{3}}{35(\cos(dx+c))^{5}}+\frac{8(\sin(dx+c))^{3}}{105(\cos(dx+c))^{3}}\right)+\frac{\frac{i}{3}a^{2}}{(\cos(dx+c))^{6}}-a^{2}\left(-\frac{8}{15}-\frac{(\sec(dx+c))^{4}}{5}-\frac{4}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/1
05*sin(d*x+c)^3/cos(d*x+c)^3)+1/3*I*a^2/cos(d*x+c)^6-a^2*(-8/15-1/5*sec(d*x
+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.09504, size = 128, normalized size = 1.56

$$-\frac{15\,a^{2}\tan(dx+c)^{7}-35i\,a^{2}\tan(dx+c)^{6}+21\,a^{2}\tan(dx+c)^{5}-105i\,a^{2}\tan(dx+c)^{4}-35\,a^{2}\tan(dx+c)^{3}-105i\,a^{2}\tan}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(15*a^2*tan(d*x + c)^7 - 35*I*a^2*tan(d*x + c)^6 + 21*a^2*tan(d*x +
c)^5 - 105*I*a^2*tan(d*x + c)^4 - 35*a^2*tan(d*x + c)^3 - 105*I*a^2*tan(d*x
 + c)^2 - 105*a^2*tan(d*x + c))/d
```

Fricas [B] time = 1.06656, size = 464, normalized size = 5.66

$$\frac{4480i\,a^{2}e^{(8i\,dx+8i\,c)}+4480i\,a^{2}e^{(6i\,dx+6i\,c)}+2688i\,a^{2}e^{(4i\,dx+4i\,c)}+896i\,a^{2}e^{(2i\,dx+2i\,c)}+128i\,a^{2}}{105\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4i\,c)}+7\,de^{(2i\,dx+2i\,c)}+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(4480*I*a^2*e^(8*I*d*x + 8*I*c) + 4480*I*a^2*e^(6*I*d*x + 6*I*c) + 26
88*I*a^2*e^(4*I*d*x + 4*I*c) + 896*I*a^2*e^(2*I*d*x + 2*I*c) + 128*I*a^2)/(
d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10
*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d
*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int -\tan^2(c+dx)\sec^6(c+dx)\,dx + \int 2i\tan(c+dx)\sec^6(c+dx)\,dx + \int \sec^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(-tan(c + d*x)**2*sec(c + d*x)**6, x) + Integral(2*I*tan(c +
d*x)*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**6, x))
```

Giac [A] time = 1.18313, size = 128, normalized size = 1.56

$$-\frac{15\,a^2\tan(dx+c)^7 - 35i\,a^2\tan(dx+c)^6 + 21\,a^2\tan(dx+c)^5 - 105i\,a^2\tan(dx+c)^4 - 35\,a^2\tan(dx+c)^3 - 105i\,a^2}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/105*(15*a^2*tan(d*x + c)^7 - 35*I*a^2*tan(d*x + c)^6 + 21*a^2*tan(d*x +
c)^5 - 105*I*a^2*tan(d*x + c)^4 - 35*a^2*tan(d*x + c)^3 - 105*I*a^2*tan(d*x
 + c)^2 - 105*a^2*tan(d*x + c))/d
```

3.21 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{i(a+ia\tan(c+dx))^5}{5a^3d}-\frac{i(a+ia\tan(c+dx))^4}{2a^2d}$$

```
[Out] ((-I/2)*(a + I*a*Tan[c + d*x])^4)/(a^2*d) + ((I/5)*(a + I*a*Tan[c + d*x])^5
)/(a^3*d)
```

Rubi [A] time = 0.0431053, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a+ia\tan(c+dx))^5}{5a^3d}-\frac{i(a+ia\tan(c+dx))^4}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I/2)*(a + I*a*Tan[c + d*x])^4)/(a^2*d) + ((I/5)*(a + I*a*Tan[c + d*x])^5
)/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)(a+x)^3\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(2a(a+x)^3-(a+x)^4\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i(a+ia\tan(c+dx))^4}{2a^2d}+\frac{i(a+ia\tan(c+dx))^5}{5a^3d}\end{aligned}$$

Mathematica [A] time = 0.373512, size = 77, normalized size = 1.4

$$\frac{a^2\sec(c)\sec^5(c+dx)(-5\sin(2c+dx)+5\sin(2c+3dx)+\sin(4c+5dx)+5i\cos(2c+dx)+5\sin(dx)+5i\cos(dx))}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c]*Sec[c + d*x]^5*((5*I)*Cos[d*x] + (5*I)*Cos[2*c + d*x] + 5*Sin[d
*x] - 5*Sin[2*c + d*x] + 5*Sin[2*c + 3*d*x] + Sin[4*c + 5*d*x]))/(20*d)
```

Maple [A] time = 0.055, size = 85, normalized size = 1.6

$$\frac{1}{d}\left(-a^2\left(\frac{(\sin(dx+c))^3}{5(\cos(dx+c))^5}+\frac{2(\sin(dx+c))^3}{15(\cos(dx+c))^3}\right)+\frac{\frac{i}{2}a^2}{(\cos(dx+c))^4}-a^2\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+1/
2*I*a^2/cos(d*x+c)^4-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.12444, size = 76, normalized size = 1.38

$$-\frac{6\,a^2\tan(dx+c)^5-15i\,a^2\tan(dx+c)^4-30i\,a^2\tan(dx+c)^2-30\,a^2\tan(dx+c)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(6*a^2*tan(d*x + c)^5 - 15*I*a^2*tan(d*x + c)^4 - 30*I*a^2*tan(d*x +
c)^2 - 30*a^2*tan(d*x + c))/d
```

Fricas [B] time = 1.18037, size = 329, normalized size = 5.98

$$\frac{80i\,a^2e^{(6i\,dx+6i\,c)}+80i\,a^2e^{(4i\,dx+4i\,c)}+40i\,a^2e^{(2i\,dx+2i\,c)}+8i\,a^2}{5\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5*(80*I*a^2*e^(6*I*d*x + 6*I*c) + 80*I*a^2*e^(4*I*d*x + 4*I*c) + 40*I*a^2
*e^(2*I*d*x + 2*I*c) + 8*I*a^2)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x +
 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*
d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int -\tan^2(c+dx)\sec^4(c+dx)\,dx+\int 2i\tan(c+dx)\sec^4(c+dx)\,dx+\int \sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(-tan(c + d*x)**2*sec(c + d*x)**4, x) + Integral(2*I*tan(c +
d*x)*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

Giac [A] time = 1.1673, size = 76, normalized size = 1.38

$$-\frac{2\,a^2\tan\left(dx+c\right)^5-5i\,a^2\tan\left(dx+c\right)^4-10i\,a^2\tan\left(dx+c\right)^2-10\,a^2\tan\left(dx+c\right)}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*a^2*tan(d*x + c)^5 - 5*I*a^2*tan(d*x + c)^4 - 10*I*a^2*tan(d*x + c
)^2 - 10*a^2*tan(d*x + c))/d
```

3.22 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=27

$$-\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

```
[Out] ((-I/3)*(a + I*a*Tan[c + d*x])^3)/(a*d)
```

Rubi [A] time = 0.0375375, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$-\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I/3)*(a + I*a*Tan[c + d*x])^3)/(a*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{i\,\text{Subst}\left(\int (a+x)^2\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{i(a+ia\tan(c+dx))^3}{3ad}\end{aligned}$$

Mathematica [B] time = 0.327802, size = 68, normalized size = 2.52

$$\frac{a^2\sec(c)\sec^3(c+dx)(-3\sin(2c+dx)+2\sin(2c+3dx)+3i\cos(2c+dx)+3\sin(dx)+3i\cos(dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c]*Sec[c + d*x]^3*((3*I)*Cos[d*x] + (3*I)*Cos[2*c + d*x] + 3*Sin[d
*x] - 3*Sin[2*c + d*x] + 2*Sin[2*c + 3*d*x]))/(6*d)
```

Maple [B] time = 0.051, size = 51, normalized size = 1.9

$$\frac{1}{d}\left(-\frac{a^2(\sin(dx+c))^3}{3(\cos(dx+c))^3}+\frac{ia^2}{(\cos(dx+c))^2}+a^2\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/3*a^2*sin(d*x+c)^3/cos(d*x+c)^3+I*a^2/cos(d*x+c)^2+a^2*tan(d*x+c))
```

Maxima [A] time = 1.11372, size = 28, normalized size = 1.04

$$-\frac{i(ia\tan(dx+c)+a)^3}{3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*I*(I*a*tan(d*x + c) + a)^3/(a*d)
```

Fricas [B] time = 1.12419, size = 212, normalized size = 7.85

$$\frac{24i\,a^2e^{(4i\,dx+4i\,c)}+24i\,a^2e^{(2i\,dx+2i\,c)}+8i\,a^2}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(24*I*a^2*e^(4*I*d*x + 4*I*c) + 24*I*a^2*e^(2*I*d*x + 2*I*c) + 8*I*a^2)
/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int-\tan^2(c+dx)\sec^2(c+dx)\,dx+\int 2i\tan(c+dx)\sec^2(c+dx)\,dx+\int\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(-tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(2*I*tan(c +
d*x)*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

Giac [B] time = 1.18351, size = 57, normalized size = 2.11

$$-\frac{a^2\tan\left(dx+c\right)^3-3i\,a^2\tan\left(dx+c\right)^2-3\,a^2\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(a^2*tan(d*x + c)^3 - 3*I*a^2*tan(d*x + c)^2 - 3*a^2*tan(d*x + c))/d
```

3.23 $\int (a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=38

$$-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x$$

```
[Out] 2*a^2*x - ((2*I)*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.0169765, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3477, 3475}

$$-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] 2*a^2*x - ((2*I)*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c + d*x])/d
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(c+dx))^2\,dx &= 2a^2x-\frac{a^2\tan(c+dx)}{d}+\left(2ia^2\right)\int \tan(c+dx)\,dx\\ &= 2a^2x-\frac{2ia^2\log(\cos(c+dx))}{d}-\frac{a^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.632605, size = 100, normalized size = 2.63

$$-\frac{a^2\sec(c)\sec(c+dx)\left(-4dx\cos(2c+dx)+\cos(dx)\left(-4dx+i\log\left(\cos^2(c+dx)\right)\right)+i\cos(2c+dx)\log\left(\cos^2(c+dx)\right)+4\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -(a^2*Sec[c]*Sec[c + d*x]*(4*ArcTan[Tan[3*c + d*x]]*Cos[c]*Cos[c + d*x] - 4
*d*x*Cos[2*c + d*x] + Cos[d*x]*(-4*d*x + I*Log[Cos[c + d*x]^2]) + I*Cos[2*c
 + d*x]*Log[Cos[c + d*x]^2] + 2*Sin[d*x]))/(2*d)
```

Maple [A] time = 0.005, size = 51, normalized size = 1.3

$$\frac{ia^2\ln\left(1+(\tan(dx+c))^2\right)}{d}+2\frac{a^2\arctan(\tan(dx+c))}{d}-\frac{a^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2,x)
```

```
[Out] I/d*a^2*ln(1+tan(d*x+c)^2)+2/d*a^2*arctan(tan(d*x+c))-a^2*tan(d*x+c)/d
```

Maxima [A] time = 1.66464, size = 55, normalized size = 1.45

$$a^2x+\frac{(dx+c-\tan(dx+c))a^2}{d}+\frac{2i\,a^2\log(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + (d*x + c - tan(d*x + c))*a^2/d + 2*I*a^2*log(sec(d*x + c))/d
```

Fricas [A] time = 1.06008, size = 151, normalized size = 3.97

$$\frac{-2i\,a^2+\left(-2i\,a^2e^{(2i\,dx+2i\,c)}-2i\,a^2\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (-2*I*a^2 + (-2*I*a^2*e^(2*I*d*x + 2*I*c) - 2*I*a^2)*log(e^(2*I*d*x + 2*I*c
) + 1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 0.937538, size = 60, normalized size = 1.58

$$-\frac{2ia^2\log\left(e^{2idx}+e^{-2ic}\right)}{d}-\frac{2ia^2e^{-2ic}}{d\left(e^{2idx}+e^{-2ic}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -2*I*a**2*log(exp(2*I*d*x) + exp(-2*I*c))/d - 2*I*a**2*exp(-2*I*c)/(d*(exp(
2*I*d*x) + exp(-2*I*c)))
```

Giac [A] time = 1.13516, size = 88, normalized size = 2.32

$$\frac{-2i\,a^2e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-2i\,a^2\log\left(e^{(2i\,dx+2i\,c)}+1\right)-2i\,a^2}{de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (-2*I*a^2*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2*I*a^2*log(e^
(2*I*d*x + 2*I*c) + 1) - 2*I*a^2)/(d*e^(2*I*d*x + 2*I*c) + d)
```

3.24 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=25

$$-\frac{ia^3}{d(a-ia\tan(c+dx))}$$

```
[Out] ((-I)*a^3)/(d*(a - I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0372104, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$-\frac{ia^3}{d(a-ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*a^3)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{\left(ia^3\right)\text{Subst}\left(\int \frac{1}{(a-x)^2}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{ia^3}{d(a-ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0535852, size = 31, normalized size = 1.24

$$-\frac{ia^2(\cos(c+dx)+i\sin(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I/2)*a^2*(Cos[c + d*x] + I*Sin[c + d*x])^2)/d
```

Maple [B] time = 0.047, size = 73, normalized size = 2.9

$$\frac{1}{d}\left(-a^{2}\left(-\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ia^{2}\left(\cos\left(dx+c\right)\right)^{2}+a^{2}\left(\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-I*a^2*cos(d*x+c)^2+a^2
*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.64713, size = 43, normalized size = 1.72

$$\frac{a^{2}\tan\left(dx+c\right)-i\,a^{2}}{\left(\tan\left(dx+c\right)^{2}+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (a^2*tan(d*x + c) - I*a^2)/((tan(d*x + c)^2 + 1)*d)
```

Fricas [A] time = 1.35485, size = 46, normalized size = 1.84

$$-\frac{i\,a^{2}e^{(2i\,dx+2i\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

Sympy [A] time = 0.298465, size = 37, normalized size = 1.48

$$\begin{cases}-\frac{ia^{2}e^{2ic}e^{2idx}}{2d} & \text{for } 2d\neq 0\\ a^{2}xe^{2ic} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((-I*a**2*exp(2*I*c)*exp(2*I*d*x)/(2*d), Ne(2*d, 0)), (a**2*x*exp(
2*I*c), True))
```

Giac [A] time = 1.19159, size = 23, normalized size = 0.92

$$-\frac{i\,a^2e^{(2i\,dx+2i\,c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

3.25 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=63

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2}-\frac{ia^3}{4d(a-ia\tan(c+dx))}+\frac{a^2x}{4}$$

```
[Out] (a^2*x)/4 - ((I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2) - ((I/4)*a^3)/(d*(a -
I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0607932, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2}-\frac{ia^3}{4d(a-ia\tan(c+dx))}+\frac{a^2x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*x)/4 - ((I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2) - ((I/4)*a^3)/(d*(a -
I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^3}+\frac{1}{4a^2(a-x)^2}+\frac{1}{4a^2(a^2-x^2)}\right)dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2}-\frac{ia^3}{4d(a-ia\tan(c+dx))}-\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,d\right.}{4d}$$

$$= \frac{a^2x}{4}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2}-\frac{ia^3}{4d(a-ia\tan(c+dx))}$$

Mathematica [A] time = 0.440161, size = 86, normalized size = 1.37

$$\frac{a^2((1-4idx)\sin(2(c+dx))+(4dx-i)\cos(2(c+dx))-4i)(\cos(2(c+2dx))+i\sin(2(c+2dx)))}{16d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(-4*I + (-I + 4*d*x)*Cos[2*(c + d*x)] + (1 - (4*I)*d*x)*Sin[2*(c + d*x
)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(16*d*(Cos[d*x] + I*Sin[d*
x])^2)
```

Maple [A] time = 0.055, size = 100, normalized size = 1.6

$$\frac{1}{d}\left(-a^2\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{i}{2}a^2(\cos(dx+c))^4+a^2\left(\frac{\sin(dx+c)}{4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1
/8*c)-1/2*I*a^2*cos(d*x+c)^4+a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x
+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.64277, size = 90, normalized size = 1.43

$$\frac{(dx+c)a^2+\frac{a^2\tan(dx+c)^3+3\,a^2\tan(dx+c)-2i\,a^2}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((d*x + c)*a^2 + (a^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c) - 2*I*a^2)/(t
an(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.14616, size = 105, normalized size = 1.67

$$\frac{4\,a^2dx - i\,a^2e^{(4i\,dx+4i\,c)} - 4i\,a^2e^{(2i\,dx+2i\,c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a^2*d*x - I*a^2*e^(4*I*d*x + 4*I*c) - 4*I*a^2*e^(2*I*d*x + 2*I*c))/
d
```

Sympy [A] time = 0.449745, size = 88, normalized size = 1.4

$$\frac{a^2x}{4} + \begin{cases} \frac{-4ia^2de^{4ic}e^{4idx}-16ia^2de^{2ic}e^{2idx}}{64d^2} & \text{for } 64d^2 \neq 0 \\ x\left(\frac{a^2e^{4ic}}{4} + \frac{a^2e^{2ic}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*x/4 + Piecewise(((-4*I*a**2*d*exp(4*I*c)*exp(4*I*d*x) - 16*I*a**2*d*ex
p(2*I*c)*exp(2*I*d*x))/(64*d**2), Ne(64*d**2, 0)), (x*(a**2*exp(4*I*c)/4 +
a**2*exp(2*I*c)/2), True))
```

Giac [B] time = 1.19227, size = 347, normalized size = 5.51

$$\frac{8\,a^2dxe^{(4i\,dx+2i\,c)} + 16\,a^2dxe^{(2i\,dx)} + 8\,a^2dxe^{(-2i\,c)} - i\,a^2e^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - 2i\,a^2e^{(2i\,dx)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - i\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/32*(8*a^2*d*x*e^(4*I*d*x + 2*I*c) + 16*a^2*d*x*e^(2*I*d*x) + 8*a^2*d*x*e^
(-2*I*c) - I*a^2*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2*I*a^2
*e^(2*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - I*a^2*e^(-2*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) + I*a^2*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) +
2*I*a^2*e^(2*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) + I*a^2*e^(-2*I*c)*log(e^
(2*I*d*x) + e^(-2*I*c)) - 2*I*a^2*e^(8*I*d*x + 6*I*c) - 12*I*a^2*e^(6*I*d*x
 + 4*I*c) - 18*I*a^2*e^(4*I*d*x + 2*I*c) - 8*I*a^2*e^(2*I*d*x))/(d*e^(4*I*d
*x + 2*I*c) + 2*d*e^(2*I*d*x) + d*e^(-2*I*c))
```

3.26 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=117

$$-\frac{ia^5}{12d(a-ia\tan(c+dx))^3}-\frac{ia^4}{8d(a-ia\tan(c+dx))^2}-\frac{3ia^3}{16d(a-ia\tan(c+dx))}+\frac{ia^3}{16d(a+ia\tan(c+dx))}+\frac{a^2x}{4}$$

```
[Out] (a^2*x)/4 - ((I/12)*a^5)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/8)*a^4)/(d*(a -
 I*a*Tan[c + d*x])^2) - (((3*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])) + ((I/1
6)*a^3)/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0815212, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{ia^5}{12d(a-ia\tan(c+dx))^3}-\frac{ia^4}{8d(a-ia\tan(c+dx))^2}-\frac{3ia^3}{16d(a-ia\tan(c+dx))}+\frac{ia^3}{16d(a+ia\tan(c+dx))}+\frac{a^2x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*x)/4 - ((I/12)*a^5)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/8)*a^4)/(d*(a -
 I*a*Tan[c + d*x])^2) - (((3*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])) + ((I/1
6)*a^3)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{\left(ia^7\right)\mathrm{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{\left(ia^7\right)\mathrm{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^4} + \frac{1}{4a^3(a-x)^3} + \frac{3}{16a^4(a-x)^2} + \frac{1}{16a^4(a+x)^2} + \frac{1}{4a^4\left(a^2-x^2\right)}\right) dx, x}{d}$$

$$= -\frac{ia^5}{12d(a-ia\tan(c+dx))^3} - \frac{ia^4}{8d(a-ia\tan(c+dx))^2} - \frac{3ia^3}{16d(a-ia\tan(c+dx))}$$

$$= \frac{a^2x}{4} - \frac{ia^5}{12d(a-ia\tan(c+dx))^3} - \frac{ia^4}{8d(a-ia\tan(c+dx))^2} - \frac{3ia^3}{16d(a-ia\tan(c+}$$

Mathematica [A] time = 0.449529, size = 116, normalized size = 0.99

$$\frac{a^2(-12idx\sin(2(c+dx)) + 3\sin(2(c+dx)) + 2\sin(4(c+dx)) + 3(4dx-i)\cos(2(c+dx)) + i\cos(4(c+dx)) - 9i)(\cos(2(c}{48d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(-9*I + 3*(-I + 4*d*x)*Cos[2*(c + d*x)] + I*Cos[4*(c + d*x)] + 3*Sin[2
*(c + d*x)] - (12*I)*d*x*Sin[2*(c + d*x)] + 2*Sin[4*(c + d*x)])*(Cos[2*(c +
 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(48*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.056, size = 121, normalized size = 1.

$$\frac{1}{d}\left(-a^2\left(-\frac{(\cos(dx+c))^5\sin(dx+c)}{6} + \frac{\sin(dx+c)}{24}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right) + \frac{dx}{16} + \frac{c}{16}\right) - \frac{i}{3}a^2(\cos(dx+c))^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-1/6*cos(d*x+c)^5*sin(d*x+c)+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*
sin(d*x+c)+1/16*d*x+1/16*c)-1/3*I*a^2*cos(d*x+c)^6+a^2*(1/6*(cos(d*x+c)^5+5
/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.6762, size = 124, normalized size = 1.06

$$\frac{3(dx+c)a^2 + \frac{3a^2\tan(dx+c)^5 + 8a^2\tan(dx+c)^3 + 9a^2\tan(dx+c) - 4ia^2}{\tan(dx+c)^6 + 3\tan(dx+c)^4 + 3\tan(dx+c)^2 + 1}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(d*x + c)*a^2 + (3*a^2*tan(d*x + c)^5 + 8*a^2*tan(d*x + c)^3 + 9*a^
2*tan(d*x + c) - 4*I*a^2)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x +
c)^2 + 1))/d
```

Fricas [A] time = 1.22996, size = 217, normalized size = 1.85

$$\frac{\left(24\,a^{2}dxe^{(2i\,dx+2i\,c)} - i\,a^{2}e^{(8i\,dx+8i\,c)} - 6i\,a^{2}e^{(6i\,dx+6i\,c)} - 18i\,a^{2}e^{(4i\,dx+4i\,c)} + 3i\,a^{2}\right)e^{(-2i\,dx-2i\,c)}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/96*(24*a^2*d*x*e^(2*I*d*x + 2*I*c) - I*a^2*e^(8*I*d*x + 8*I*c) - 6*I*a^2*
e^(6*I*d*x + 6*I*c) - 18*I*a^2*e^(4*I*d*x + 4*I*c) + 3*I*a^2)*e^(-2*I*d*x -
 2*I*c)/d
```

Sympy [A] time = 0.582573, size = 187, normalized size = 1.6

$$\frac{a^{2}x}{4} + \begin{cases} \frac{\left(-8192ia^{2}d^{3}e^{8ic}e^{6idx}-49152ia^{2}d^{3}e^{6ic}e^{4idx}-147456ia^{2}d^{3}e^{4ic}e^{2idx}+24576ia^{2}d^{3}e^{-2idx}\right)e^{-2ic}}{786432d^{4}} & \text{for}\ 786432d^{4}e^{2ic} \neq 0 \\ x\left(-\frac{a^{2}}{4} + \frac{\left(a^{2}e^{8ic}+4a^{2}e^{6ic}+6a^{2}e^{4ic}+4a^{2}e^{2ic}+a^{2}\right)e^{-2ic}}{16}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*x/4 + Piecewise(((-8192*I*a**2*d**3*exp(8*I*c)*exp(6*I*d*x) - 49152*I*
a**2*d**3*exp(6*I*c)*exp(4*I*d*x) - 147456*I*a**2*d**3*exp(4*I*c)*exp(2*I*d
*x) + 24576*I*a**2*d**3*exp(-2*I*d*x))*exp(-2*I*c)/(786432*d**4), Ne(786432
*d**4*exp(2*I*c), 0)), (x*(-a**2/4 + (a**2*exp(8*I*c) + 4*a**2*exp(6*I*c) +
 6*a**2*exp(4*I*c) + 4*a**2*exp(2*I*c) + a**2)*exp(-2*I*c)/16), True))
```

Giac [A] time = 1.25757, size = 228, normalized size = 1.95

$$\frac{96\,a^{2}dxe^{(6i\,dx+4i\,c)} + 192\,a^{2}dxe^{(4i\,dx+2i\,c)} + 96\,a^{2}dxe^{(2i\,dx)} - 4i\,a^{2}e^{(12i\,dx+10i\,c)} - 32i\,a^{2}e^{(10i\,dx+8i\,c)} - 124i\,a^{2}e^{(8i\,dx+6i\,c)} - 168i}{384\left(de^{(6i\,dx+4i\,c)} + 2\,de^{(4i\,dx+2i\,c)} + de^{(2i\,dx)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/384*(96*a^2*d*x*e^(6*I*d*x + 4*I*c) + 192*a^2*d*x*e^(4*I*d*x + 2*I*c) + 9
6*a^2*d*x*e^(2*I*d*x) - 4*I*a^2*e^(12*I*d*x + 10*I*c) - 32*I*a^2*e^(10*I*d*
x + 8*I*c) - 124*I*a^2*e^(8*I*d*x + 6*I*c) - 168*I*a^2*e^(6*I*d*x + 4*I*c)
- 60*I*a^2*e^(4*I*d*x + 2*I*c) + 24*I*a^2*e^(2*I*d*x) + 12*I*a^2*e^(-2*I*c)
)/(d*e^(6*I*d*x + 4*I*c) + 2*d*e^(4*I*d*x + 2*I*c) + d*e^(2*I*d*x))
```

3.27 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=171

$$-\frac{ia^6}{32d(a-ia\tan(c+dx))^4}-\frac{ia^5}{16d(a-ia\tan(c+dx))^3}-\frac{3ia^4}{32d(a-ia\tan(c+dx))^2}+\frac{ia^4}{64d(a+ia\tan(c+dx))^2}-\frac{}{32d(a-i}$$

```
[Out] (15*a^2*x)/64 - ((I/32)*a^6)/(d*(a - I*a*Tan[c + d*x])^4) - ((I/16)*a^5)/(d
*(a - I*a*Tan[c + d*x])^3) - (((3*I)/32)*a^4)/(d*(a - I*a*Tan[c + d*x])^2)
- (((5*I)/32)*a^3)/(d*(a - I*a*Tan[c + d*x])) + ((I/64)*a^4)/(d*(a + I*a*Ta
n[c + d*x])^2) + (((5*I)/64)*a^3)/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.106948, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{ia^6}{32d(a-ia\tan(c+dx))^4}-\frac{ia^5}{16d(a-ia\tan(c+dx))^3}-\frac{3ia^4}{32d(a-ia\tan(c+dx))^2}+\frac{ia^4}{64d(a+ia\tan(c+dx))^2}-\frac{}{32d(a-i}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```



```
[Out] (15*a^2*x)/64 - ((I/32)*a^6)/(d*(a - I*a*Tan[c + d*x])^4) - ((I/16)*a^5)/(d
*(a - I*a*Tan[c + d*x])^3) - (((3*I)/32)*a^4)/(d*(a - I*a*Tan[c + d*x])^2)
- (((5*I)/32)*a^3)/(d*(a - I*a*Tan[c + d*x])) + ((I/64)*a^4)/(d*(a + I*a*Ta
n[c + d*x])^2) + (((5*I)/64)*a^3)/(d*(a + I*a*Tan[c + d*x]))
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{(ia^9)\,\text{Subst}\left(\int \frac{1}{(a-x)^5(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^9)\,\text{Subst}\left(\int\left(\frac{1}{8a^3(a-x)^5} + \frac{3}{16a^4(a-x)^4} + \frac{3}{16a^5(a-x)^3} + \frac{5}{32a^6(a-x)^2} + \frac{1}{32a^5(a+x)^3} + \right.\right.}{d}$$

$$= -\frac{ia^6}{32d(a-ia\tan(c+dx))^4} - \frac{ia^5}{16d(a-ia\tan(c+dx))^3} - \frac{3ia^4}{32d(a-ia\tan(c+}$$

$$= \frac{15a^2x}{64} - \frac{ia^6}{32d(a-ia\tan(c+dx))^4} - \frac{ia^5}{16d(a-ia\tan(c+dx))^3} - \frac{3i}{32d(a-ia\,\text{t}}$$

Mathematica [A] time = 0.488739, size = 138, normalized size = 0.81

$$\frac{a^2(-120idx\sin(2(c+dx)) + 30\sin(2(c+dx)) + 32\sin(4(c+dx)) + 3\sin(6(c+dx)) + 30(4dx-i)\cos(2(c+dx)) + 16}{512d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(-80*I + 30*(-I + 4*d*x)*Cos[2*(c + d*x)] + (16*I)*Cos[4*(c + d*x)] +
I*Cos[6*(c + d*x)] + 30*Sin[2*(c + d*x)] - (120*I)*d*x*Sin[2*(c + d*x)] + 3
2*Sin[4*(c + d*x)] + 3*Sin[6*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c +
 2*d*x)]))/(512*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.059, size = 141, normalized size = 0.8

$$\frac{1}{d}\left(-a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^7}{8} + \frac{\sin(dx+c)}{48}\left((\cos(dx+c))^5 + \frac{5(\cos(dx+c))^3}{4} + \frac{15\cos(dx+c)}{8}\right) + \frac{5dx}{128} + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3
+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-1/4*I*a^2*cos(d*x+c)^8+a^2*
(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*si
n(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [A] time = 1.65595, size = 155, normalized size = 0.91

$$\frac{15\,(dx+c)a^2 + \frac{15\,a^2\tan(dx+c)^7 + 55\,a^2\tan(dx+c)^5 + 73\,a^2\tan(dx+c)^3 + 49\,a^2\tan(dx+c) - 16i\,a^2}{\tan(dx+c)^8 + 4\tan(dx+c)^6 + 6\tan(dx+c)^4 + 4\tan(dx+c)^2 + 1}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/64*(15*(d*x + c)*a^2 + (15*a^2*tan(d*x + c)^7 + 55*a^2*tan(d*x + c)^5 + 7
3*a^2*tan(d*x + c)^3 + 49*a^2*tan(d*x + c) - 16*I*a^2)/(tan(d*x + c)^8 + 4*
```

```
tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.11037, size = 309, normalized size = 1.81

$$\frac{\left(120\,a^2dxe^{(4i\,dx+4i\,c)} - i\,a^2e^{(12i\,dx+12i\,c)} - 8i\,a^2e^{(10i\,dx+10i\,c)} - 30i\,a^2e^{(8i\,dx+8i\,c)} - 80i\,a^2e^{(6i\,dx+6i\,c)} + 24i\,a^2e^{(2i\,dx+2i\,c)} + 2i\,a^2\right)e^{(-4}}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/512*(120*a^2*d*x*e^(4*I*d*x + 4*I*c) - I*a^2*e^(12*I*d*x + 12*I*c) - 8*I*
a^2*e^(10*I*d*x + 10*I*c) - 30*I*a^2*e^(8*I*d*x + 8*I*c) - 80*I*a^2*e^(6*I*
d*x + 6*I*c) + 24*I*a^2*e^(2*I*d*x + 2*I*c) + 2*I*a^2)*e^(-4*I*d*x - 4*I*c)
/d
```

Sympy [A] time = 1.00391, size = 272, normalized size = 1.59

$$\frac{15a^2x}{64} + \begin{cases} \frac{\left(-8589934592ia^2d^5e^{14ic}e^{8idx} - 68719476736ia^2d^5e^{12ic}e^{6idx} - 257698037760ia^2d^5e^{10ic}e^{4idx} - 687194767360ia^2d^5e^{8ic}e^{2idx} + 206158430208ia^2d^5e^{4ic}e^{-2id}\right.}{4398046511104d^6} \\ x\left(-\frac{15a^2}{64} + \frac{\left(a^2e^{12ic} + 6a^2e^{10ic} + 15a^2e^{8ic} + 20a^2e^{6ic} + 15a^2e^{4ic} + 6a^2e^{2ic} + a^2\right)e^{-4ic}}{64}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] 15*a**2*x/64 + Piecewise(((-8589934592*I*a**2*d**5*exp(14*I*c)*exp(8*I*d*x)
 - 68719476736*I*a**2*d**5*exp(12*I*c)*exp(6*I*d*x) - 257698037760*I*a**2*d
**5*exp(10*I*c)*exp(4*I*d*x) - 687194767360*I*a**2*d**5*exp(8*I*c)*exp(2*I*
d*x) + 206158430208*I*a**2*d**5*exp(4*I*c)*exp(-2*I*d*x) + 17179869184*I*a*
*2*d**5*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(4398046511104*d**6), Ne(4398
046511104*d**6*exp(6*I*c), 0)), (x*(-15*a**2/64 + (a**2*exp(12*I*c) + 6*a**
2*exp(10*I*c) + 15*a**2*exp(8*I*c) + 20*a**2*exp(6*I*c) + 15*a**2*exp(4*I*c
) + 6*a**2*exp(2*I*c) + a**2)*exp(-4*I*c)/64), True))
```

Giac [B] time = 1.25628, size = 462, normalized size = 2.7

$$120\,a^2dxe^{(8i\,dx+4i\,c)} + 240\,a^2dxe^{(6i\,dx+2i\,c)} + 120\,a^2dxe^{(4i\,dx)} + 8i\,a^2e^{(8i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) + 16i\,a^2e^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*(120*a^2*d*x*e^(8*I*d*x + 4*I*c) + 240*a^2*d*x*e^(6*I*d*x + 2*I*c) +
120*a^2*d*x*e^(4*I*d*x) + 8*I*a^2*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*
c) + 1) + 16*I*a^2*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 8*I*a
^2*e^(4*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 8*I*a^2*e^(8*I*d*x + 4*I*c)*l
og(e^(2*I*d*x) + e^(-2*I*c)) - 16*I*a^2*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x)
 + e^(-2*I*c)) - 8*I*a^2*e^(4*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) - I*a^2*
e^(16*I*d*x + 12*I*c) - 10*I*a^2*e^(14*I*d*x + 10*I*c) - 47*I*a^2*e^(12*I*d
```

```
*x + 8*I*c) - 148*I*a^2*e^(10*I*d*x + 6*I*c) - 190*I*a^2*e^(8*I*d*x + 4*I*c
) - 56*I*a^2*e^(6*I*d*x + 2*I*c) + 28*I*a^2*e^(2*I*d*x - 2*I*c) + 50*I*a^2*
e^(4*I*d*x) + 2*I*a^2*e^(-4*I*c))/(d*e^(8*I*d*x + 4*I*c) + 2*d*e^(6*I*d*x +
 2*I*c) + d*e^(4*I*d*x))
```

3.28 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=118

$$\frac{7ia^2\sec^5(c+dx)}{30d}+\frac{7a^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}+\frac{7a^2\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{7a}{}$$

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(16*d) + (((7*I)/30)*a^2*Sec[c + d*x]^5)/d +
(7*a^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (7*a^2*Sec[c + d*x]^3*Tan[c + d*
x])/(24*d) + ((I/6)*Sec[c + d*x]^5*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rubi [A] time = 0.0871915, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3498, 3486, 3768, 3770}

$$\frac{7ia^2\sec^5(c+dx)}{30d}+\frac{7a^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}+\frac{7a^2\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{7a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(16*d) + (((7*I)/30)*a^2*Sec[c + d*x]^5)/d +
(7*a^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (7*a^2*Sec[c + d*x]^3*Tan[c + d*
x])/(24*d) + ((I/6)*Sec[c + d*x]^5*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d} + \frac{1}{6}(7a)\int \sec^5(c+dx)(a+ia\tan(c+$$

$$= \frac{7ia^2\sec^5(c+dx)}{30d} + \frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d} + \frac{1}{6}\left(7a^2\right)\int \sec^5($$

$$= \frac{7ia^2\sec^5(c+dx)}{30d} + \frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{i\sec^5(c+dx)\left(a^2+ia^2\mathrm{t}\right.}{6d}$$

$$= \frac{7ia^2\sec^5(c+dx)}{30d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{16d} + \frac{7a^2\sec^3(c+dx)\tan(c+}{24d}$$

$$= \frac{7a^2\tanh^{-1}(\sin(c+dx))}{16d} + \frac{7ia^2\sec^5(c+dx)}{30d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{16d}$$

Mathematica [A] time = 1.01147, size = 159, normalized size = 1.35

$$\frac{a^2(\cos(2c)-i\sin(2c))(\tan(c+dx)-i)^2\sec^4(c+dx)\left(150\sin(c+dx)-35(17\sin(3(c+dx))+3\sin(5(c+dx)))-153\right.}{3840d(\cos(d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c + d*x]^4*(Cos[2*c] - I*Sin[2*c])*((-1536*I)*Cos[c + d*x] + 1680*
Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]]) + 150*Sin[c + d*x] - 35*(17*Sin[3*(c + d*x)] + 3*
Sin[5*(c + d*x)]))*(-I + Tan[c + d*x])^2)/(3840*d*(Cos[d*x] + I*Sin[d*x])^2
)
```

Maple [A] time = 0.053, size = 169, normalized size = 1.4

$$-\frac{a^2(\sin(dx+c))^3}{6d(\cos(dx+c))^6} - \frac{a^2(\sin(dx+c))^3}{8d(\cos(dx+c))^4} - \frac{a^2(\sin(dx+c))^3}{16d(\cos(dx+c))^2} - \frac{a^2\sin(dx+c)}{16d} + \frac{7a^2\ln(\sec(dx+c)+\tan(dx+c)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -1/6/d*a^2*sin(d*x+c)^3/cos(d*x+c)^6-1/8/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4-1/
16/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2-1/16*a^2*sin(d*x+c)/d+7/16/d*a^2*ln(sec(
d*x+c)+tan(d*x+c))+2/5*I/d*a^2/cos(d*x+c)^5+1/4*a^2*sec(d*x+c)^3*tan(d*x+c)
/d+3/8*a^2*sec(d*x+c)*tan(d*x+c)/d
```

Maxima [A] time = 1.08428, size = 244, normalized size = 2.07

$$-\frac{5a^2\left(\frac{2\left(3\sin(dx+c)^5-8\sin(dx+c)^3-3\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right) + 30a^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin\right.}{\sin(dx+c)^4-2\sin(d}\right.}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/480*(5*a^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) + 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x
 + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) +
3*log(sin(d*x + c) - 1)) - 192*I*a^2/cos(d*x + c)^5)/d
```

Fricas [B] time = 1.25052, size = 1062, normalized size = 9.

$$\frac{-210i\,a^2e^{(11i\,dx+11i\,c)}-1190i\,a^2e^{(9i\,dx+9i\,c)}+3372i\,a^2e^{(7i\,dx+7i\,c)}+2772i\,a^2e^{(5i\,dx+5i\,c)}+1190i\,a^2e^{(3i\,dx+3i\,c)}+210i\,a^2e^{(i\,dx+i\,c)}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(-210*I*a^2*e^(11*I*d*x + 11*I*c) - 1190*I*a^2*e^(9*I*d*x + 9*I*c) +
3372*I*a^2*e^(7*I*d*x + 7*I*c) + 2772*I*a^2*e^(5*I*d*x + 5*I*c) + 1190*I*a^
2*e^(3*I*d*x + 3*I*c) + 210*I*a^2*e^(I*d*x + I*c) + 105*(a^2*e^(12*I*d*x +
12*I*c) + 6*a^2*e^(10*I*d*x + 10*I*c) + 15*a^2*e^(8*I*d*x + 8*I*c) + 20*a^2
*e^(6*I*d*x + 6*I*c) + 15*a^2*e^(4*I*d*x + 4*I*c) + 6*a^2*e^(2*I*d*x + 2*I*
c) + a^2)*log(e^(I*d*x + I*c) + I) - 105*(a^2*e^(12*I*d*x + 12*I*c) + 6*a^2
*e^(10*I*d*x + 10*I*c) + 15*a^2*e^(8*I*d*x + 8*I*c) + 20*a^2*e^(6*I*d*x + 6
*I*c) + 15*a^2*e^(4*I*d*x + 4*I*c) + 6*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e
^(I*d*x + I*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) +
 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*
I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int -\tan^2(c+dx)\sec^5(c+dx)\,dx+\int 2i\tan(c+dx)\sec^5(c+dx)\,dx+\int \sec^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(-tan(c + d*x)**2*sec(c + d*x)**5, x) + Integral(2*I*tan(c +
d*x)*sec(c + d*x)**5, x) + Integral(sec(c + d*x)**5, x))
```

Giac [B] time = 1.23203, size = 323, normalized size = 2.74

$$105\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(135\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-445\,a^2\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(105*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^2*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) + 2*(135*a^2*tan(1/2*d*x + 1/2*c)^11 - 480*I*a^2*tan(1/
```

```
2*d*x + 1/2*c)^10 - 445*a^2*tan(1/2*d*x + 1/2*c)^9 + 480*I*a^2*tan(1/2*d*x
+ 1/2*c)^8 - 330*a^2*tan(1/2*d*x + 1/2*c)^7 - 960*I*a^2*tan(1/2*d*x + 1/2*c
)^6 - 330*a^2*tan(1/2*d*x + 1/2*c)^5 + 960*I*a^2*tan(1/2*d*x + 1/2*c)^4 - 4
45*a^2*tan(1/2*d*x + 1/2*c)^3 - 96*I*a^2*tan(1/2*d*x + 1/2*c)^2 + 135*a^2*t
an(1/2*d*x + 1/2*c) + 96*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.29 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=94

$$\frac{5ia^2\sec^3(c+dx)}{12d}+\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{5a^2\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (((5*I)/12)*a^2*Sec[c + d*x]^3)/d + (
5*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((I/4)*Sec[c + d*x]^3*(a^2 + I*a^2
*Tan[c + d*x]))/d
```

Rubi [A] time = 0.0766581, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3498, 3486, 3768, 3770}

$$\frac{5ia^2\sec^3(c+dx)}{12d}+\frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{5a^2\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (((5*I)/12)*a^2*Sec[c + d*x]^3)/d + (
5*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((I/4)*Sec[c + d*x]^3*(a^2 + I*a^2
*Tan[c + d*x]))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d} + \frac{1}{4}(5a)\int \sec^3(c+dx)(a+ia\tan(c+$$

$$= \frac{5ia^2\sec^3(c+dx)}{12d} + \frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d} + \frac{1}{4}\left(5a^2\right)\int \sec^3($$

$$= \frac{5ia^2\sec^3(c+dx)}{12d} + \frac{5a^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{i\sec^3(c+dx)\left(a^2+ia^2\,\text{ta}\right.}{4d}$$

$$= \frac{5a^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{5ia^2\sec^3(c+dx)}{12d} + \frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}$$

Mathematica [B] time = 1.051, size = 215, normalized size = 2.29

$$\frac{a^2\sec^4(c+dx)\left(-18\sin(c+dx)+30\sin(3(c+dx))+128i\cos(c+dx)-45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sec[c + d*x]^4*((128*I)*Cos[c + d*x] - 45*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - 60*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 15*Cos[4*(c + d*x)]*(Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]) + 45*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 18*Sin[c + d*x] + 30*Si
n[3*(c + d*x)]))/(192*d)
```

Maple [A] time = 0.052, size = 123, normalized size = 1.3

$$-\frac{a^2(\sin(dx+c))^3}{4d(\cos(dx+c))^4} - \frac{a^2(\sin(dx+c))^3}{8d(\cos(dx+c))^2} - \frac{a^2\sin(dx+c)}{8d} + \frac{5a^2\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{\frac{2i}{3}a^2}{d(\cos(dx+c))^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2*sin(d*x+c)^3/cos(d*x+c)^4-1/8/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2-1/
8*a^2*sin(d*x+c)/d+5/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/3*I/d*a^2/cos(d*x+
c)^3+1/2*a^2*sec(d*x+c)*tan(d*x+c)/d
```

Maxima [A] time = 1.06383, size = 176, normalized size = 1.87

$$-\frac{3a^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 12a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/48*(3*a^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*(2*s
```

```
in(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
 - 1)) - 32*I*a^2/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.2719, size = 711, normalized size = 7.56

$$\frac{-30i\,a^2e^{(7i\,dx+7i\,c)}+146i\,a^2e^{(5i\,dx+5i\,c)}+110i\,a^2e^{(3i\,dx+3i\,c)}+30i\,a^2e^{(i\,dx+i\,c)}+15\left(a^2e^{(8i\,dx+8i\,c)}+4\,a^2e^{(6i\,dx+6i\,c)}+6\,a^2e^{(4i\,dx+4i\,c}\right.}{24\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-30*I*a^2*e^(7*I*d*x + 7*I*c) + 146*I*a^2*e^(5*I*d*x + 5*I*c) + 110*I
*a^2*e^(3*I*d*x + 3*I*c) + 30*I*a^2*e^(I*d*x + I*c) + 15*(a^2*e^(8*I*d*x +
8*I*c) + 4*a^2*e^(6*I*d*x + 6*I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) + 4*a^2*e^(2
*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I) - 15*(a^2*e^(8*I*d*x + 8*I*
c) + 4*a^2*e^(6*I*d*x + 6*I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) + 4*a^2*e^(2*I*d
*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e
^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int -\tan^2(c+dx)\sec^3(c+dx)\,dx+\int 2i\tan(c+dx)\sec^3(c+dx)\,dx+\int \sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(-tan(c + d*x)**2*sec(c + d*x)**3, x) + Integral(2*I*tan(c +
d*x)*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x))
```

Giac [B] time = 1.2133, size = 236, normalized size = 2.51

$$\frac{15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-33\,a^2\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*I*a^2*tan(1/2*d*x +
 1/2*c)^6 - 33*a^2*tan(1/2*d*x + 1/2*c)^5 + 48*I*a^2*tan(1/2*d*x + 1/2*c)^4
 - 33*a^2*tan(1/2*d*x + 1/2*c)^3 - 16*I*a^2*tan(1/2*d*x + 1/2*c)^2 + 9*a^2*
tan(1/2*d*x + 1/2*c) + 16*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.30 $\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=68

$$\frac{3ia^2\sec(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (((3*I)/2)*a^2*Sec[c + d*x])/d + ((I/
2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rubi [A] time = 0.0399934, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3498, 3486, 3770}

$$\frac{3ia^2\sec(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (((3*I)/2)*a^2*Sec[c + d*x])/d + ((I/
2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{1}{2}(3a)\int \sec(c+dx)(a+ia\tan(c+dx$$

$$=\frac{3ia^2\sec(c+dx)}{2d}+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{1}{2}\left(3a^2\right)\int \sec(c+$$

$$=\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ia^2\sec(c+dx)}{2d}+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+\right.}{2d}$$

Mathematica [B] time = 0.801456, size = 146, normalized size = 2.15

$$-\frac{a^2\sec^2(c+dx)\left(2\sin(c+dx)-8i\cos(c+dx)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\left(\log\left(\cos\left(\right.\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -(a^2*Sec[c + d*x]^2*((-8*I)*Cos[c + d*x] + 3*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 3*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + 2*Sin[c + d*x]))/(4*d)
```

Maple [A] time = 0.021, size = 79, normalized size = 1.2

$$-\frac{a^2(\sin(dx+c))^3}{2d(\cos(dx+c))^2}-\frac{a^2\sin(dx+c)}{2d}+\frac{3a^2\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2ia^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2-1/2*a^2*sin(d*x+c)/d+3/2/d*a^2*ln(sec(
d*x+c)+tan(d*x+c))+2*I/d*a^2/cos(d*x+c)
```

Maxima [A] time = 1.12143, size = 112, normalized size = 1.65

$$\frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)+4a^2\log(\sec(dx+c)+\tan(dx+c))+\frac{8ia^2}{\cos(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 4*a^2*log(sec(d*x + c) + tan(d*x + c)) + 8*I*a^2/cos(
d*x + c))/d
```

Fricas [B] time = 1.26849, size = 397, normalized size = 5.84

$$\frac{10i\,a^2e^{(3i\,dx+3i\,c)}+6i\,a^2e^{(i\,dx+i\,c)}+3\left(a^2e^{(4i\,dx+4i\,c)}+2\,a^2e^{(2i\,dx+2i\,c)}+a^2\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(a^2e^{(4i\,dx+4i\,c)}+2\,a^2e^{(2i\,dx+2i\,c)}\right.}{2\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(10*I*a^2*e^(3*I*d*x + 3*I*c) + 6*I*a^2*e^(I*d*x + I*c) + 3*(a^2*e^(4*I
*d*x + 4*I*c) + 2*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I) -
```

```
3*(a^2*e^(4*I*d*x + 4*I*c) + 2*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x
+ I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int -\tan^2(c+dx)\sec(c+dx)\,dx+\int 2i\tan(c+dx)\sec(c+dx)\,dx+\int \sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(-tan(c + d*x)**2*sec(c + d*x), x) + Integral(2*I*tan(c + d*x
)*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

Giac [A] time = 1.19868, size = 147, normalized size = 2.16

$$\frac{3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4ia^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(a^2*tan(1/2*d*x + 1/2*c)^3 + 4*I*a^2*tan(1/2*d*x + 1/2*c
)^2 + a^2*tan(1/2*d*x + 1/2*c) - 4*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.31 $\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

```
[Out] -((a^2*ArcTanh[Sin[c + d*x]])/d) - ((2*I)*Cos[c + d*x]*(a^2 + I*a^2*Tan[c +
 d*x]))/d
```

Rubi [A] time = 0.0344902, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3496, 3770}

$$-\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTanh[Sin[c + d*x]])/d) - ((2*I)*Cos[c + d*x]*(a^2 + I*a^2*Tan[c +
 d*x]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}-a^2\int \sec(c+dx)\,dx\\ &= -\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\end{aligned}$$

Mathematica [B] time = 0.214237, size = 180, normalized size = 3.91

$$\frac{a^2\left(\cos\left(\frac{1}{2}(c+5dx)\right)+i\sin\left(\frac{1}{2}(c+5dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{d(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(Cos[(c + d*x)/2]*(-2*I + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 - I*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[(c + d*x)
/2])*(Cos[(c + 5*d*x)/2] + I*Sin[(c + 5*d*x)/2]))/(d*(Cos[d*x] + I*Sin[d*x]
)^2)
```

Maple [A] time = 0.044, size = 53, normalized size = 1.2

$$\frac{-2\,ia^2\cos(dx+c)}{d}-\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\,\frac{a^2\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -2*I/d*a^2*cos(d*x+c)-1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*sin(d*x+c)/d
```

Maxima [A] time = 1.08992, size = 82, normalized size = 1.78

$$-\frac{a^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))+4i\,a^2\cos(dx+c)-2\,a^2\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))
+ 4*I*a^2*cos(d*x + c) - 2*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.20895, size = 124, normalized size = 2.7

$$\frac{-2i\,a^2e^{(i\,dx+i\,c)}-a^2\log\left(e^{(i\,dx+i\,c)}+i\right)+a^2\log\left(e^{(i\,dx+i\,c)}-i\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (-2*I*a^2*e^(I*d*x + I*c) - a^2*log(e^(I*d*x + I*c) + I) + a^2*log(e^(I*d*x
 + I*c) - I))/d
```

Sympy [A] time = 0.525563, size = 68, normalized size = 1.48

$$\frac{a^2\left(\log\left(e^{idx}-ie^{-ic}\right)-\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}+\begin{cases}-\frac{2ia^2e^{ic}e^{idx}}{d} & \text{for } d\neq 0\\ 2a^2xe^{ic} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*(log(exp(I*d*x) - I*exp(-I*c)) - log(exp(I*d*x) + I*exp(-I*c)))/d + Pi
ecewise((-2*I*a**2*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (2*a**2*x*exp(I*c), Tr
ue))
```

Giac [A] time = 1.19859, size = 76, normalized size = 1.65

$$\frac{-2i\,a^2e^{(i\,dx+i\,c)} - a^2\log\left(i\,e^{(i\,dx+i\,c)} - 1\right) + a^2\log\left(-i\,e^{(i\,dx+i\,c)} - 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (-2*I*a^2*e^(I*d*x + I*c) - a^2*log(I*e^(I*d*x + I*c) - 1) + a^2*log(-I*e^(
I*d*x + I*c) - 1))/d
```

3.32 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=51

$$\frac{a^2\sin(c+dx)}{3d}-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

```
[Out] (a^2*Sin[c + d*x])/(3*d) - (((2*I)/3)*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d
*x]))/d
```

Rubi [A] time = 0.0419488, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3496, 2637}

$$\frac{a^2\sin(c+dx)}{3d}-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/(3*d) - (((2*I)/3)*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d
*x]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}+\frac{1}{3}a^2\int \cos(c+dx)\,dx\\ &= \frac{a^2\sin(c+dx)}{3d}-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}\end{aligned}$$

Mathematica [A] time = 0.178426, size = 50, normalized size = 0.98

$$\frac{a^2(2\cos(c+dx)-i\sin(c+dx))(\sin(2(c+dx))-i\cos(2(c+dx)))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(2*Cos[c + d*x] - I*Sin[c + d*x])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(3*d)
```

Maple [A] time = 0.052, size = 54, normalized size = 1.1

$$\frac{1}{d}\left(-\frac{a^2\left(\sin\left(dx+c\right)\right)^3}{3}-\frac{2\,i}{3}a^2\left(\cos\left(dx+c\right)\right)^3+\frac{a^2\left(2+\left(\cos\left(dx+c\right)\right)^2\right)\sin\left(dx+c\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/3*a^2*sin(d*x+c)^3-2/3*I*a^2*cos(d*x+c)^3+1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.06649, size = 70, normalized size = 1.37

$$-\frac{2i\,a^2\cos\left(dx+c\right)^3+a^2\sin\left(dx+c\right)^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*I*a^2*cos(d*x + c)^3 + a^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a^2)/d
```

Fricas [A] time = 1.19644, size = 84, normalized size = 1.65

$$\frac{-i\,a^2e^{(3i\,dx+3i\,c)}-3i\,a^2e^{(i\,dx+i\,c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(-I*a^2*e^(3*I*d*x + 3*I*c) - 3*I*a^2*e^(I*d*x + I*c))/d
```

Sympy [A] time = 0.43183, size = 76, normalized size = 1.49

$$\begin{cases}\frac{-2ia^2de^{3ic}e^{3idx}-6ia^2de^{ic}e^{idx}}{12d^2} & \text{for}\ 12d^2\neq 0\\ x\left(\frac{a^2e^{3ic}}{2}+\frac{a^2e^{ic}}{2}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-2*I*a**2*d*exp(3*I*c)*exp(3*I*d*x) - 6*I*a**2*d*exp(I*c)*exp(I
*d*x))/(12*d**2), Ne(12*d**2, 0)), (x*(a**2*exp(3*I*c)/2 + a**2*exp(I*c)/2)
, True))
```

Giac [B] time = 1.25377, size = 717, normalized size = 14.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*(24*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 48*a^2*e^(2*
I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 24*a^2*e^(-2*I*c)*log(I*e^(I*d*x + I*c)
 + 1) + 27*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 54*a^2*e^(2
*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 27*a^2*e^(-2*I*c)*log(I*e^(I*d*x + I*c
) - 1) - 24*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 48*a^2*e^
(2*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 24*a^2*e^(-2*I*c)*log(-I*e^(I*d*x +
 I*c) + 1) - 27*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 54*a^
2*e^(2*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 27*a^2*e^(-2*I*c)*log(-I*e^(I*d
*x + I*c) - 1) + 3*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 6*
a^2*e^(2*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^2*e^(-2*I*c)*log(I*e^(I*d
*x) + e^(-I*c)) - 3*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) -
6*a^2*e^(2*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) - 3*a^2*e^(-2*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) + 16*I*a^2*e^(7*I*d*x + 5*I*c) + 80*I*a^2*e^(5*I*d*x +
3*I*c) + 112*I*a^2*e^(3*I*d*x + I*c) + 48*I*a^2*e^(I*d*x - I*c))/(d*e^(4*I*
d*x + 2*I*c) + 2*d*e^(2*I*d*x) + d*e^(-2*I*c))
```

3.33 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=69

$$-\frac{a^2\sin^3(c+dx)}{5d}+\frac{3a^2\sin(c+dx)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

```
[Out] (3*a^2*Sin[c + d*x])/(5*d) - (a^2*Sin[c + d*x]^3)/(5*d) - (((2*I)/5)*Cos[c
+ d*x]^5*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rubi [A] time = 0.0491498, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3496, 2633}

$$-\frac{a^2\sin^3(c+dx)}{5d}+\frac{3a^2\sin(c+dx)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*a^2*Sin[c + d*x])/(5*d) - (a^2*Sin[c + d*x]^3)/(5*d) - (((2*I)/5)*Cos[c
+ d*x]^5*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}+\frac{1}{5}\left(3a^2\right)\int \cos^3(c+dx)\,dx\\ &= -\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}-\frac{\left(3a^2\right)\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(\right.}{5d}\\ &= \frac{3a^2\sin(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}\end{aligned}$$

Mathematica [A] time = 0.401438, size = 72, normalized size = 1.04

$$\frac{a^2(\sin(2(c+dx))-i\cos(2(c+dx)))(-5i\sin(c+dx)+3i\sin(3(c+dx))+10\cos(c+dx)-2\cos(3(c+dx)))}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(10*Cos[c + d*x] - 2*Cos[3*
(c + d*x)] - (5*I)*Sin[c + d*x] + (3*I)*Sin[3*(c + d*x)]))/(20*d)
```

Maple [A] time = 0.057, size = 91, normalized size = 1.3

$$\frac{1}{d}\left(-a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{15}\right)-\frac{2i}{5}a^2(\cos(dx+c))^5+\frac{a^2\sin(dx+c)}{5}\left(\frac{8}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-2
/5*I*a^2*cos(d*x+c)^5+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c
))
```

Maxima [A] time = 1.07611, size = 107, normalized size = 1.55

$$-\frac{6i\,a^2\cos(dx+c)^5-\left(3\sin(dx+c)^5-5\sin(dx+c)^3\right)a^2-\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^2}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(6*I*a^2*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^2 -
 (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2)/d
```

Fricas [A] time = 1.10349, size = 169, normalized size = 2.45

$$\frac{\left(-i\,a^2e^{(6i\,dx+6i\,c)}-5i\,a^2e^{(4i\,dx+4i\,c)}-15i\,a^2e^{(2i\,dx+2i\,c)}+5i\,a^2\right)e^{(-i\,dx-i\,c)}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/40*(-I*a^2*e^(6*I*d*x + 6*I*c) - 5*I*a^2*e^(4*I*d*x + 4*I*c) - 15*I*a^2*e
^(2*I*d*x + 2*I*c) + 5*I*a^2)*e^(-I*d*x - I*c)/d
```

Sympy [A] time = 0.981247, size = 155, normalized size = 2.25

$$\begin{cases} \frac{\left(-512ia^2d^3e^{6ic}e^{5idx}-2560ia^2d^3e^{4ic}e^{3idx}-7680ia^2d^3e^{2ic}e^{idx}+2560ia^2d^3e^{-idx}\right)e^{-ic}}{20480d^4} & \text{for } 20480d^4e^{ic}\neq 0 \\ \frac{x\left(a^2e^{6ic}+3a^2e^{4ic}+3a^2e^{2ic}+a^2\right)e^{-ic}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-512*I*a**2*d**3*exp(6*I*c)*exp(5*I*d*x) - 2560*I*a**2*d**3*exp
(4*I*c)*exp(3*I*d*x) - 7680*I*a**2*d**3*exp(2*I*c)*exp(I*d*x) + 2560*I*a**2
*d**3*exp(-I*d*x))*exp(-I*c)/(20480*d**4), Ne(20480*d**4*exp(I*c), 0)), (x*
(a**2*exp(6*I*c) + 3*a**2*exp(4*I*c) + 3*a**2*exp(2*I*c) + a**2)*exp(-I*c)/
8, True))
```

Giac [B] time = 1.28452, size = 828, normalized size = 12.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/160*(45*a^2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 90*a^2*e^(3
*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 45*a^2*e^(I*d*x - I*c)*log(I*e^(
I*d*x + I*c) + 1) + 40*a^2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 80*a^2*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 40*a^2*e^(I*d*x - I*
c)*log(I*e^(I*d*x + I*c) - 1) - 45*a^2*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 90*a^2*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 45*a^2
*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 40*a^2*e^(5*I*d*x + 3*I*c)*l
og(-I*e^(I*d*x + I*c) - 1) - 80*a^2*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c
) - 1) - 40*a^2*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5*a^2*e^(5*I*
d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 10*a^2*e^(3*I*d*x + I*c)*log(I*e
^(I*d*x) + e^(-I*c)) - 5*a^2*e^(I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) +
5*a^2*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 10*a^2*e^(3*I*d*x
+ I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 5*a^2*e^(I*d*x - I*c)*log(-I*e^(I*d*x
) + e^(-I*c)) + 4*I*a^2*e^(10*I*d*x + 8*I*c) + 28*I*a^2*e^(8*I*d*x + 6*I*c)
 + 104*I*a^2*e^(6*I*d*x + 4*I*c) + 120*I*a^2*e^(4*I*d*x + 2*I*c) + 20*I*a^2
*e^(2*I*d*x) - 20*I*a^2*e^(-2*I*c))/(d*e^(5*I*d*x + 3*I*c) + 2*d*e^(3*I*d*x
 + I*c) + d*e^(I*d*x - I*c))
```

3.34 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=87

$$\frac{a^2\sin^5(c+dx)}{7d}-\frac{10a^2\sin^3(c+dx)}{21d}+\frac{5a^2\sin(c+dx)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

```
[Out] (5*a^2*Sin[c + d*x])/(7*d) - (10*a^2*Sin[c + d*x]^3)/(21*d) + (a^2*Sin[c +
d*x]^5)/(7*d) - (((2*I)/7)*Cos[c + d*x]^7*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rubi [A] time = 0.0523546, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3496, 2633}

$$\frac{a^2\sin^5(c+dx)}{7d}-\frac{10a^2\sin^3(c+dx)}{21d}+\frac{5a^2\sin(c+dx)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*a^2*Sin[c + d*x])/(7*d) - (10*a^2*Sin[c + d*x]^3)/(21*d) + (a^2*Sin[c +
d*x]^5)/(7*d) - (((2*I)/7)*Cos[c + d*x]^7*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}+\frac{1}{7}\left(5a^2\right)\int \cos^5(c+dx)\,dx\\ &= -\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}-\frac{\left(5a^2\right)\operatorname{Subst}\left(\int\left(1-2x^2+x^4\right)dx\right.}{7d}\\ &= \frac{5a^2\sin(c+dx)}{7d}-\frac{10a^2\sin^3(c+dx)}{21d}+\frac{a^2\sin^5(c+dx)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2\right.}{7}\end{aligned}$$

Mathematica [A] time = 0.414968, size = 111, normalized size = 1.28

$$\frac{a^2(-70\sin(c+dx)+63\sin(3(c+dx))+5\sin(5(c+dx))-140i\cos(c+dx)+42i\cos(3(c+dx))+2i\cos(5(c+dx)))(}{336d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*((-140*I)*Cos[c + d*x] + (42*I)*Cos[3*(c + d*x)] + (2*I)*Cos[5*(c + d*
x)] - 70*Sin[c + d*x] + 63*Sin[3*(c + d*x)] + 5*Sin[5*(c + d*x)])*(Cos[2*(c
 + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(336*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.055, size = 111, normalized size = 1.3

$$\frac{1}{d}\left(-a^2\left(-\frac{(\cos(dx+c))^6\sin(dx+c)}{7}+\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)\right)-\frac{2\,i}{7}a^2(\cos(dx+c))^7+\frac{a}{\cdot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-1/7*cos(d*x+c)^6*sin(d*x+c)+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+
c)^2)*sin(d*x+c))-2/7*I*a^2*cos(d*x+c)^7+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*cos
(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.14223, size = 132, normalized size = 1.52

$$-\frac{30i\,a^2\cos(dx+c)^7+\left(15\sin(dx+c)^7-42\sin(dx+c)^5+35\sin(dx+c)^3\right)a^2+3\left(5\sin(dx+c)^7-21\sin(dx+c)^5+\cdots\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(30*I*a^2*cos(d*x + c)^7 + (15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 +
35*sin(d*x + c)^3)*a^2 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d
*x + c)^3 - 35*sin(d*x + c))*a^2)/d
```

Fricas [A] time = 1.01903, size = 269, normalized size = 3.09

$$\frac{\left(-3i\,a^2e^{(10i\,dx+10i\,c)}-21i\,a^2e^{(8i\,dx+8i\,c)}-70i\,a^2e^{(6i\,dx+6i\,c)}-210i\,a^2e^{(4i\,dx+4i\,c)}+105i\,a^2e^{(2i\,dx+2i\,c)}+7i\,a^2\right)e^{(-3i\,dx-3i\,c)}}{672\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/672*(-3*I*a^2*e^(10*I*d*x + 10*I*c) - 21*I*a^2*e^(8*I*d*x + 8*I*c) - 70*I
*a^2*e^(6*I*d*x + 6*I*c) - 210*I*a^2*e^(4*I*d*x + 4*I*c) + 105*I*a^2*e^(2*I
*d*x + 2*I*c) + 7*I*a^2)*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [A] time = 0.976075, size = 240, normalized size = 2.76

$$\begin{cases} \frac{\left(-75497472ia^2d^5e^{11ic}e^{7idx}-528482304ia^2d^5e^{9ic}e^{5idx}-1761607680ia^2d^5e^{7ic}e^{3idx}-5284823040ia^2d^5e^{5ic}e^{idx}+2642411520ia^2d^5e^{3ic}e^{-idx}+176160768ia^2d^5e^{ic}e^{-3}\right.}{16911433728d^6} \\ \frac{x\left(a^2e^{10ic}+5a^2e^{8ic}+10a^2e^{6ic}+10a^2e^{4ic}+5a^2e^{2ic}+a^2\right)e^{-3ic}}{32} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-75497472*I*a**2*d**5*exp(11*I*c)*exp(7*I*d*x) - 528482304*I*a*
*2*d**5*exp(9*I*c)*exp(5*I*d*x) - 1761607680*I*a**2*d**5*exp(7*I*c)*exp(3*I
*d*x) - 5284823040*I*a**2*d**5*exp(5*I*c)*exp(I*d*x) + 2642411520*I*a**2*d*
*5*exp(3*I*c)*exp(-I*d*x) + 176160768*I*a**2*d**5*exp(I*c)*exp(-3*I*d*x))*e
xp(-4*I*c)/(16911433728*d**6), Ne(16911433728*d**6*exp(4*I*c), 0)), (x*(a**
2*exp(10*I*c) + 5*a**2*exp(8*I*c) + 10*a**2*exp(6*I*c) + 10*a**2*exp(4*I*c)
 + 5*a**2*exp(2*I*c) + a**2)*exp(-3*I*c)/32, True))
```

Giac [B] time = 1.34624, size = 865, normalized size = 9.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10752*(2583*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5166*a^
2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 2583*a^2*e^(3*I*d*x - I*c)
*log(I*e^(I*d*x + I*c) + 1) + 2121*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x +
 I*c) - 1) + 4242*a^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 2121*a
^2*e^(3*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 2583*a^2*e^(7*I*d*x + 3*I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 5166*a^2*e^(5*I*d*x + I*c)*log(-I*e^(I*d*
x + I*c) + 1) - 2583*a^2*e^(3*I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 21
21*a^2*e^(7*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 4242*a^2*e^(5*I*d*
x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2121*a^2*e^(3*I*d*x - I*c)*log(-I*e^
(I*d*x + I*c) - 1) - 462*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)
) - 924*a^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 462*a^2*e^(3*I*
d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 462*a^2*e^(7*I*d*x + 3*I*c)*log(-I
*e^(I*d*x) + e^(-I*c)) + 924*a^2*e^(5*I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) + 462*a^2*e^(3*I*d*x - I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 48*I*a^2*e^
(14*I*d*x + 10*I*c) + 432*I*a^2*e^(12*I*d*x + 8*I*c) + 1840*I*a^2*e^(10*I*d
*x + 6*I*c) + 5936*I*a^2*e^(8*I*d*x + 4*I*c) + 6160*I*a^2*e^(6*I*d*x + 2*I*
c) - 1904*I*a^2*e^(2*I*d*x - 2*I*c) - 112*I*a^2*e^(4*I*d*x) - 112*I*a^2*e^(
-4*I*c))/(d*e^(7*I*d*x + 3*I*c) + 2*d*e^(5*I*d*x + I*c) + d*e^(3*I*d*x - I*
c))
```

3.35 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=105

$$-\frac{a^2\sin^7(c+dx)}{9d}+\frac{7a^2\sin^5(c+dx)}{15d}-\frac{7a^2\sin^3(c+dx)}{9d}+\frac{7a^2\sin(c+dx)}{9d}-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

```
[Out] (7*a^2*Sin[c + d*x])/(9*d) - (7*a^2*Sin[c + d*x]^3)/(9*d) + (7*a^2*Sin[c +
d*x]^5)/(15*d) - (a^2*Sin[c + d*x]^7)/(9*d) - (((2*I)/9)*Cos[c + d*x]^9*(a^
2 + I*a^2*Tan[c + d*x]))/d
```

Rubi [A] time = 0.0559433, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3496, 2633}

$$-\frac{a^2\sin^7(c+dx)}{9d}+\frac{7a^2\sin^5(c+dx)}{15d}-\frac{7a^2\sin^3(c+dx)}{9d}+\frac{7a^2\sin(c+dx)}{9d}-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*a^2*Sin[c + d*x])/(9*d) - (7*a^2*Sin[c + d*x]^3)/(9*d) + (7*a^2*Sin[c +
d*x]^5)/(15*d) - (a^2*Sin[c + d*x]^7)/(9*d) - (((2*I)/9)*Cos[c + d*x]^9*(a^
2 + I*a^2*Tan[c + d*x]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}+\frac{1}{9}\left(7a^2\right)\int \cos^7(c+dx)\,dx\\ &= -\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}-\frac{\left(7a^2\right)\operatorname{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\right.}{9d}\\ &= \frac{7a^2\sin(c+dx)}{9d}-\frac{7a^2\sin^3(c+dx)}{9d}+\frac{7a^2\sin^5(c+dx)}{15d}-\frac{a^2\sin^7(c+dx)}{9d}-\frac{2i\,\mathrm{c}}{}\end{aligned}$$

Mathematica [A] time = 1.12329, size = 133, normalized size = 1.27

$$\frac{a^2(-525\sin(c+dx)+567\sin(3(c+dx))+75\sin(5(c+dx))+7\sin(7(c+dx))-1050i\cos(c+dx)+378i\cos(3(c+\cdots}{2880d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*((-1050*I)*Cos[c + d*x] + (378*I)*Cos[3*(c + d*x)] + (30*I)*Cos[5*(c +
 d*x)] + (2*I)*Cos[7*(c + d*x)] - 525*Sin[c + d*x] + 567*Sin[3*(c + d*x)] +
 75*Sin[5*(c + d*x)] + 7*Sin[7*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c
 + 2*d*x)]))/(2880*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.09, size = 131, normalized size = 1.3

$$\frac{1}{d}\left(-a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^8}{9}+\frac{\sin(dx+c)}{63}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6(\cos(dx+c))^4}{5}+\frac{8(\cos(dx+c))^2}{5}\right)\right)-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x
+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-2/9*I*a^2*cos(d*x+c)^9+1/9*a^2*(128/35+
cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*
x+c))
```

Maxima [A] time = 1.11045, size = 161, normalized size = 1.53

$$-\frac{70i\,a^2\cos(dx+c)^9-\left(35\sin(dx+c)^9-135\sin(dx+c)^7+189\sin(dx+c)^5-105\sin(dx+c)^3\right)a^2-\left(35\sin(dx\cdots\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/315*(70*I*a^2*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 +
 189*sin(d*x + c)^5 - 105*sin(d*x + c)^3)*a^2 - (35*sin(d*x + c)^9 - 180*si
n(d*x + c)^7 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*
a^2)/d
```

Fricas [A] time = 1.17465, size = 365, normalized size = 3.48

$$\frac{\left(-5i\,a^2e^{(14i\,dx+14i\,c)}-45i\,a^2e^{(12i\,dx+12i\,c)}-189i\,a^2e^{(10i\,dx+10i\,c)}-525i\,a^2e^{(8i\,dx+8i\,c)}-1575i\,a^2e^{(6i\,dx+6i\,c)}+945i\,a^2e^{(4i\,dx+4i\,c)}\cdots\right.}{5760\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5760*(-5*I*a^2*e^(14*I*d*x + 14*I*c) - 45*I*a^2*e^(12*I*d*x + 12*I*c) - 1
89*I*a^2*e^(10*I*d*x + 10*I*c) - 525*I*a^2*e^(8*I*d*x + 8*I*c) - 1575*I*a^2
*e^(6*I*d*x + 6*I*c) + 945*I*a^2*e^(4*I*d*x + 4*I*c) + 105*I*a^2*e^(2*I*d*x
 + 2*I*c) + 9*I*a^2)*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [A] time = 1.35423, size = 316, normalized size = 3.01

$$\begin{cases} \frac{\left(-126663739519795200ia^2d^7e^{18ic}e^{9idx}-113997365567815680 0ia^2d^7e^{16ic}e^{7idx}-4787889353848258560ia^2d^7e^{14ic}e^{5idx}-13299692649578496000ia^2d^7e^{12ic}e^{3idx}-3\cdots\right.}{145916627926\cdots} \\ \frac{x\left(a^2e^{14ic}+7a^2e^{12ic}+21a^2e^{10ic}+35a^2e^{8ic}+35a^2e^{6ic}+21a^2e^{4ic}+7a^2e^{2ic}+a^2\right)e^{-5ic}}{128} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-126663739519795200*I*a**2*d**7*exp(18*I*c)*exp(9*I*d*x) - 1139
973655678156800*I*a**2*d**7*exp(16*I*c)*exp(7*I*d*x) - 4787889353848258560*
I*a**2*d**7*exp(14*I*c)*exp(5*I*d*x) - 13299692649578496000*I*a**2*d**7*exp
(12*I*c)*exp(3*I*d*x) - 39899077948735488000*I*a**2*d**7*exp(10*I*c)*exp(I*
d*x) + 23939446769241292800*I*a**2*d**7*exp(8*I*c)*exp(-I*d*x) + 2659938529
915699200*I*a**2*d**7*exp(6*I*c)*exp(-3*I*d*x) + 227994731135631360*I*a**2*
d**7*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(145916627926804070400*d**8), Ne
(145916627926804070400*d**8*exp(9*I*c), 0)), (x*(a**2*exp(14*I*c) + 7*a**2*
exp(12*I*c) + 21*a**2*exp(10*I*c) + 35*a**2*exp(8*I*c) + 35*a**2*exp(6*I*c)
 + 21*a**2*exp(4*I*c) + 7*a**2*exp(2*I*c) + a**2)*exp(-5*I*c)/128, True))
```

Giac [B] time = 1.37149, size = 903, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/92160*(18585*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 37170*
a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 18585*a^2*e^(5*I*d*x - I
*c)*log(I*e^(I*d*x + I*c) + 1) + 14625*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 29250*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 1
4625*a^2*e^(5*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 18585*a^2*e^(9*I*d*
x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 37170*a^2*e^(7*I*d*x + I*c)*log(-I
*e^(I*d*x + I*c) + 1) - 18585*a^2*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 14625*a^2*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 29250*a^
2*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 14625*a^2*e^(5*I*d*x - I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 3960*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*
x) + e^(-I*c)) - 7920*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 3
960*a^2*e^(5*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 3960*a^2*e^(9*I*d*x
 + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 7920*a^2*e^(7*I*d*x + I*c)*log(-I*
e^(I*d*x) + e^(-I*c)) + 3960*a^2*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) + 80*I*a^2*e^(18*I*d*x + 12*I*c) + 880*I*a^2*e^(16*I*d*x + 10*I*c) + 4
544*I*a^2*e^(14*I*d*x + 8*I*c) + 15168*I*a^2*e^(12*I*d*x + 6*I*c) + 45024*I
*a^2*e^(10*I*d*x + 4*I*c) + 43680*I*a^2*e^(8*I*d*x + 2*I*c) - 18624*I*a^2*e
^(4*I*d*x - 2*I*c) - 1968*I*a^2*e^(2*I*d*x - 4*I*c) - 6720*I*a^2*e^(6*I*d*x
) - 144*I*a^2*e^(-6*I*c))/(d*e^(9*I*d*x + 3*I*c) + 2*d*e^(7*I*d*x + I*c) +
d*e^(5*I*d*x - I*c))
```

3.36 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=109

$$\frac{i(a+ia\tan(c+dx))^{10}}{10a^7d}-\frac{2i(a+ia\tan(c+dx))^9}{3a^6d}+\frac{3i(a+ia\tan(c+dx))^8}{2a^5d}-\frac{8i(a+ia\tan(c+dx))^7}{7a^4d}$$

```
[Out] (((-8*I)/7)*(a + I*a*Tan[c + d*x])^7)/(a^4*d) + (((3*I)/2)*(a + I*a*Tan[c +
 d*x])^8)/(a^5*d) - (((2*I)/3)*(a + I*a*Tan[c + d*x])^9)/(a^6*d) + ((I/10)*
(a + I*a*Tan[c + d*x])^10)/(a^7*d)
```

Rubi [A] time = 0.0637078, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a+ia\tan(c+dx))^{10}}{10a^7d}-\frac{2i(a+ia\tan(c+dx))^9}{3a^6d}+\frac{3i(a+ia\tan(c+dx))^8}{2a^5d}-\frac{8i(a+ia\tan(c+dx))^7}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-8*I)/7)*(a + I*a*Tan[c + d*x])^7)/(a^4*d) + (((3*I)/2)*(a + I*a*Tan[c +
 d*x])^8)/(a^5*d) - (((2*I)/3)*(a + I*a*Tan[c + d*x])^9)/(a^6*d) + ((I/10)*
(a + I*a*Tan[c + d*x])^10)/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^3(a+x)^6\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(8a^3(a+x)^6-12a^2(a+x)^7+6a(a+x)^8-(a+x)^9\right)dx,x,ia\,\text{ta}\right.}{a^7d}\\ &= -\frac{8i(a+ia\tan(c+dx))^7}{7a^4d}+\frac{3i(a+ia\tan(c+dx))^8}{2a^5d}-\frac{2i(a+ia\tan(c+dx))^9}{3a^6d}\end{aligned}$$

Mathematica [A] time = 1.79137, size = 117, normalized size = 1.07

$$\frac{a^3\sec(c)\sec^{10}(c+dx)(105\sin(c+2dx)-105\sin(3c+2dx)+120\sin(3c+4dx)+45\sin(5c+6dx)+10\sin(7c+8dx}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]^10*((126*I)*Cos[c] + (105*I)*Cos[c + 2*d*x] + (105
*I)*Cos[3*c + 2*d*x] - 126*Sin[c] + 105*Sin[c + 2*d*x] - 105*Sin[3*c + 2*d*
x] + 120*Sin[3*c + 4*d*x] + 45*Sin[5*c + 6*d*x] + 10*Sin[7*c + 8*d*x] + Sin
[9*c + 10*d*x]))/(840*d)
```

Maple [B] time = 0.065, size = 220, normalized size = 2.

$$\frac{1}{d}\left(-ia^3\left(\frac{(\sin(dx+c))^4}{10(\cos(dx+c))^{10}}+\frac{3(\sin(dx+c))^4}{40(\cos(dx+c))^8}+\frac{(\sin(dx+c))^4}{20(\cos(dx+c))^6}+\frac{(\sin(dx+c))^4}{40(\cos(dx+c))^4}\right)-3a^3\left(1/9\frac{(\sin(dx+c))^3}{(\cos(dx+c))^9}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-I*a^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8
+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4)-3*a^3*(1/9*
sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3
/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+3/8*I*a^3/cos(d*x+c)^8-a^3*
(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.09396, size = 146, normalized size = 1.34

$$\frac{-84i\,a^3\tan(dx+c)^{10}-280\,a^3\tan(dx+c)^9-960\,a^3\tan(dx+c)^7+840i\,a^3\tan(dx+c)^6-1008\,a^3\tan(dx+c)^5+168\cdots}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(-84*I*a^3*tan(d*x + c)^10 - 280*a^3*tan(d*x + c)^9 - 960*a^3*tan(d*x
 + c)^7 + 840*I*a^3*tan(d*x + c)^6 - 1008*a^3*tan(d*x + c)^5 + 1680*I*a^3*t
an(d*x + c)^4 + 1260*I*a^3*tan(d*x + c)^2 + 840*a^3*tan(d*x + c))/d
```

Fricas [B] time = 1.2306, size = 693, normalized size = 6.36

$$\frac{26880i\,a^3e^{(12i\,dx+12i\,c)}+32256i\,a^3e^{(10i\,dx+10i\,c)}+26880i\,a^3e^{(8i\,dx+8i\,c)}+15360i\,a^3e^{(6i\,dx+6i\,c)}+5760i\,\cdots}{105\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de^{(10i\,dx+10i\,c)}+210\,d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(26880*I*a^3*e^(12*I*d*x + 12*I*c) + 32256*I*a^3*e^(10*I*d*x + 10*I*c
) + 26880*I*a^3*e^(8*I*d*x + 8*I*c) + 15360*I*a^3*e^(6*I*d*x + 6*I*c) + 576
0*I*a^3*e^(4*I*d*x + 4*I*c) + 1280*I*a^3*e^(2*I*d*x + 2*I*c) + 128*I*a^3)/(
d*e^(20*I*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) + 45*d*e^(16*I*d*x + 1
6*I*c) + 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d*x + 12*I*c) + 252*d*
```

```
e^(10*I*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 120*d*e^(6*I*d*x + 6*I*
c) + 45*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25443, size = 146, normalized size = 1.34

$$-\frac{21i\,a^3\tan\left(dx+c\right)^{10}+70\,a^3\tan\left(dx+c\right)^9+240\,a^3\tan\left(dx+c\right)^7-210i\,a^3\tan\left(dx+c\right)^6+252\,a^3\tan\left(dx+c\right)^5-420}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/210*(21*I*a^3*tan(d*x + c)^10 + 70*a^3*tan(d*x + c)^9 + 240*a^3*tan(d*x
+ c)^7 - 210*I*a^3*tan(d*x + c)^6 + 252*a^3*tan(d*x + c)^5 - 420*I*a^3*tan(
d*x + c)^4 - 315*I*a^3*tan(d*x + c)^2 - 210*a^3*tan(d*x + c))/d
```

3.37 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=82

$$-\frac{i(a+ia\tan(c+dx))^8}{8a^5d}+\frac{4i(a+ia\tan(c+dx))^7}{7a^4d}-\frac{2i(a+ia\tan(c+dx))^6}{3a^3d}$$

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^6)/(a^3*d) + (((4*I)/7)*(a + I*a*Tan[c +
 d*x])^7)/(a^4*d) - ((I/8)*(a + I*a*Tan[c + d*x])^8)/(a^5*d)
```

Rubi [A] time = 0.0563161, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{i(a+ia\tan(c+dx))^8}{8a^5d}+\frac{4i(a+ia\tan(c+dx))^7}{7a^4d}-\frac{2i(a+ia\tan(c+dx))^6}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^6)/(a^3*d) + (((4*I)/7)*(a + I*a*Tan[c +
 d*x])^7)/(a^4*d) - ((I/8)*(a + I*a*Tan[c + d*x])^8)/(a^5*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^2(a+x)^5\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a+x)^5-4a(a+x)^6+(a+x)^7\right)dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{2i(a+ia\tan(c+dx))^6}{3a^3d}+\frac{4i(a+ia\tan(c+dx))^7}{7a^4d}-\frac{i(a+ia\tan(c+dx))^8}{8a^5d}\end{aligned}$$

Mathematica [A] time = 1.20586, size = 106, normalized size = 1.29

$$\frac{a^3\sec(c)\sec^8(c+dx)(28\sin(c+2dx)-28\sin(3c+2dx)+28\sin(3c+4dx)+8\sin(5c+6dx)+\sin(7c+8dx)+28i\cos}{168d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]^8*((35*I)*Cos[c] + (28*I)*Cos[c + 2*d*x] + (28*I)*
Cos[3*c + 2*d*x] - 35*Sin[c] + 28*Sin[c + 2*d*x] - 28*Sin[3*c + 2*d*x] + 28
*Sin[3*c + 4*d*x] + 8*Sin[5*c + 6*d*x] + Sin[7*c + 8*d*x]))/(168*d)
```

Maple [B] time = 0.061, size = 174, normalized size = 2.1

$$\frac{1}{d}\left(-ia^{3}\left(\frac{(\sin(dx+c))^{4}}{8\,(\cos(dx+c))^{8}}+\frac{(\sin(dx+c))^{4}}{12\,(\cos(dx+c))^{6}}+\frac{(\sin(dx+c))^{4}}{24\,(\cos(dx+c))^{4}}\right)-3\,a^{3}\left(1/7\,\frac{(\sin(dx+c))^{3}}{(\cos(dx+c))^{7}}+\frac{4\,(\sin(dx+c))^{3}}{35\,(\cos(dx+c))^{5}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-I*a^3*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1
/24*sin(d*x+c)^4/cos(d*x+c)^4)-3*a^3*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*si
n(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+1/2*I*a^3/cos(d*x+
c)^6-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.13342, size = 146, normalized size = 1.78

$$\frac{-105i\,a^{3}\tan(dx+c)^{8}-360\,a^{3}\tan(dx+c)^{7}+140i\,a^{3}\tan(dx+c)^{6}-840\,a^{3}\tan(dx+c)^{5}+1050i\,a^{3}\tan(dx+c)^{4}-}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(-105*I*a^3*tan(d*x + c)^8 - 360*a^3*tan(d*x + c)^7 + 140*I*a^3*tan(d
*x + c)^6 - 840*a^3*tan(d*x + c)^5 + 1050*I*a^3*tan(d*x + c)^4 - 280*a^3*ta
n(d*x + c)^3 + 1260*I*a^3*tan(d*x + c)^2 + 840*a^3*tan(d*x + c))/d
```

Fricas [B] time = 1.03985, size = 547, normalized size = 6.67

$$\frac{1792i\,a^{3}e^{(10i\,dx+10i\,c)}+2240i\,a^{3}e^{(8i\,dx+8i\,c)}+1792i\,a^{3}e^{(6i\,dx+6i\,c)}+896i\,a^{3}e^{(4i\,dx+4i\,c)}+256i\,a^{3}e^{(2i\,dx+2i\,c)}+}{21\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i\,c)}+28\,de^{(4i\,dx+4i}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/21*(1792*I*a^3*e^(10*I*d*x + 10*I*c) + 2240*I*a^3*e^(8*I*d*x + 8*I*c) + 1
792*I*a^3*e^(6*I*d*x + 6*I*c) + 896*I*a^3*e^(4*I*d*x + 4*I*c) + 256*I*a^3*e
^(2*I*d*x + 2*I*c) + 32*I*a^3)/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x +
 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e
^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) +
8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int -3\tan^2(c+dx)\sec^6(c+dx)\,dx+\int 3i\tan(c+dx)\sec^6(c+dx)\,dx+\int -i\tan^3(c+dx)\sec^6(c+dx)\,dx+\int \mathrm{se}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(-3*tan(c + d*x)**2*sec(c + d*x)**6, x) + Integral(3*I*tan(c
+ d*x)*sec(c + d*x)**6, x) + Integral(-I*tan(c + d*x)**3*sec(c + d*x)**6, x
) + Integral(sec(c + d*x)**6, x))
```

Giac [A] time = 1.2548, size = 146, normalized size = 1.78

$$-\frac{21i\,a^3\tan(dx+c)^8+72\,a^3\tan(dx+c)^7-28i\,a^3\tan(dx+c)^6+168\,a^3\tan(dx+c)^5-210i\,a^3\tan(dx+c)^4+56\,a^3\,\mathrm{ta}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/168*(21*I*a^3*tan(d*x + c)^8 + 72*a^3*tan(d*x + c)^7 - 28*I*a^3*tan(d*x
+ c)^6 + 168*a^3*tan(d*x + c)^5 - 210*I*a^3*tan(d*x + c)^4 + 56*a^3*tan(d*x
 + c)^3 - 252*I*a^3*tan(d*x + c)^2 - 168*a^3*tan(d*x + c))/d
```

3.38 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=55

$$\frac{i(a+ia\tan(c+dx))^6}{6a^3d}-\frac{2i(a+ia\tan(c+dx))^5}{5a^2d}$$

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^5)/(a^2*d) + ((I/6)*(a + I*a*Tan[c + d*x
])^6)/(a^3*d)
```

Rubi [A] time = 0.0426855, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a+ia\tan(c+dx))^6}{6a^3d}-\frac{2i(a+ia\tan(c+dx))^5}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^5)/(a^2*d) + ((I/6)*(a + I*a*Tan[c + d*x
])^6)/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)(a+x)^4\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\
&= -\frac{i\,\text{Subst}\left(\int \left(2a(a+x)^4-(a+x)^5\right)dx, x, ia\tan(c+dx)\right)}{a^3d}\\
&= -\frac{2i(a+ia\tan(c+dx))^5}{5a^2d}+\frac{i(a+ia\tan(c+dx))^6}{6a^3d}
\end{aligned}$$

Mathematica [A] time = 0.888484, size = 97, normalized size = 1.76

$$\frac{a^3\sec(c)\sec^6(c+dx)(15\sin(c+2dx)-15\sin(3c+2dx)+12\sin(3c+4dx)+2\sin(5c+6dx)+15i\cos(c+2dx)+15}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]^6*((20*I)*Cos[c] + (15*I)*Cos[c + 2*d*x] + (15*I)*
Cos[3*c + 2*d*x] - 20*Sin[c] + 15*Sin[c + 2*d*x] - 15*Sin[3*c + 2*d*x] + 12
*Sin[3*c + 4*d*x] + 2*Sin[5*c + 6*d*x]))/(60*d)
```

Maple [B] time = 0.06, size = 128, normalized size = 2.3

$$\frac{1}{d}\left(-ia^3\left(\frac{(\sin(dx+c))^4}{6\,(\cos(dx+c))^6}+\frac{(\sin(dx+c))^4}{12\,(\cos(dx+c))^4}\right)-3\,a^3\left(1/5\,\frac{(\sin(dx+c))^3}{(\cos(dx+c))^5}+2/15\,\frac{(\sin(dx+c))^3}{(\cos(dx+c))^3}\right)+\frac{\frac{3i}{4}a^3}{(\cos(dx+c))^4}-a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-I*a^3*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)-
3*a^3*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+3/4*I*
a^3/cos(d*x+c)^4-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.09965, size = 111, normalized size = 2.02

$$\frac{-10i\,a^3\tan(dx+c)^6-36\,a^3\tan(dx+c)^5+30i\,a^3\tan(dx+c)^4-40\,a^3\tan(dx+c)^3+90i\,a^3\tan(dx+c)^2+60\,a^3\tan(}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(-10*I*a^3*tan(d*x + c)^6 - 36*a^3*tan(d*x + c)^5 + 30*I*a^3*tan(d*x +
 c)^4 - 40*a^3*tan(d*x + c)^3 + 90*I*a^3*tan(d*x + c)^2 + 60*a^3*tan(d*x +
c))/d
```

Fricas [B] time = 1.12241, size = 419, normalized size = 7.62

$$\frac{480i\,a^3e^{(8i\,dx+8i\,c)}+640i\,a^3e^{(6i\,dx+6i\,c)}+480i\,a^3e^{(4i\,dx+4i\,c)}+192i\,a^3e^{(2i\,dx+2i\,c)}+32i\,a^3}{15\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(480*I*a^3*e^(8*I*d*x + 8*I*c) + 640*I*a^3*e^(6*I*d*x + 6*I*c) + 480*I
*a^3*e^(4*I*d*x + 4*I*c) + 192*I*a^3*e^(2*I*d*x + 2*I*c) + 32*I*a^3)/(d*e^(
12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) +
 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I
*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int -3\tan^2(c+dx)\sec^4(c+dx)\,dx + \int 3i\tan(c+dx)\sec^4(c+dx)\,dx + \int -i\tan^3(c+dx)\sec^4(c+dx)\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(-3*tan(c + d*x)**2*sec(c + d*x)**4, x) + Integral(3*I*tan(c
+ d*x)*sec(c + d*x)**4, x) + Integral(-I*tan(c + d*x)**3*sec(c + d*x)**4, x
) + Integral(sec(c + d*x)**4, x))
```

Giac [A] time = 1.25639, size = 111, normalized size = 2.02

$$-\frac{5i\,a^3\tan(dx+c)^6 + 18\,a^3\tan(dx+c)^5 - 15i\,a^3\tan(dx+c)^4 + 20\,a^3\tan(dx+c)^3 - 45i\,a^3\tan(dx+c)^2 - 30\,a^3\tan}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/30*(5*I*a^3*tan(d*x + c)^6 + 18*a^3*tan(d*x + c)^5 - 15*I*a^3*tan(d*x +
c)^4 + 20*a^3*tan(d*x + c)^3 - 45*I*a^3*tan(d*x + c)^2 - 30*a^3*tan(d*x + c
))/d
```

3.39 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=27

$$-\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

```
[Out] ((-I/4)*(a + I*a*Tan[c + d*x])^4)/(a*d)
```

Rubi [A] time = 0.0365777, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$-\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-I/4)*(a + I*a*Tan[c + d*x])^4)/(a*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{i\,\text{Subst}\left(\int (a+x)^3\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{i(a+ia\tan(c+dx))^4}{4ad}\end{aligned}$$

Mathematica [B] time = 0.488852, size = 84, normalized size = 3.11

$$\frac{a^3\sec(c)\sec^4(c+dx)(2\sin(c+2dx)-2\sin(3c+2dx)+\sin(3c+4dx)+2i\cos(c+2dx)+2i\cos(3c+2dx)-3\sin(c)+}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]^4*((3*I)*Cos[c] + (2*I)*Cos[c + 2*d*x] + (2*I)*Cos
[3*c + 2*d*x] - 3*Sin[c] + 2*Sin[c + 2*d*x] - 2*Sin[3*c + 2*d*x] + Sin[3*c
+ 4*d*x]))/(4*d)
```

Maple [B] time = 0.057, size = 73, normalized size = 2.7

$$\frac{1}{d}\left(\frac{-\frac{i}{4}a^3(\sin(dx+c))^4}{(\cos(dx+c))^4}-\frac{a^3(\sin(dx+c))^3}{(\cos(dx+c))^3}+\frac{\frac{3i}{2}a^3}{(\cos(dx+c))^2}+a^3\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/4*I*a^3*sin(d*x+c)^4/cos(d*x+c)^4-a^3*sin(d*x+c)^3/cos(d*x+c)^3+3/2
*I*a^3/cos(d*x+c)^2+a^3*tan(d*x+c))
```

Maxima [A] time = 1.11796, size = 28, normalized size = 1.04

$$-\frac{i(ia\tan(dx+c)+a)^4}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*I*(I*a*tan(d*x + c) + a)^4/(a*d)
```

Fricas [B] time = 1.19093, size = 284, normalized size = 10.52

$$\frac{16i\,a^3e^{(6i\,dx+6i\,c)}+24i\,a^3e^{(4i\,dx+4i\,c)}+16i\,a^3e^{(2i\,dx+2i\,c)}+4i\,a^3}{de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (16*I*a^3*e^(6*I*d*x + 6*I*c) + 24*I*a^3*e^(4*I*d*x + 4*I*c) + 16*I*a^3*e^(
2*I*d*x + 2*I*c) + 4*I*a^3)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c
) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int -3\tan^2(c+dx)\sec^2(c+dx)\,dx+\int 3i\tan(c+dx)\sec^2(c+dx)\,dx+\int -i\tan^3(c+dx)\sec^2(c+dx)\,dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(-3*tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(3*I*tan(c
+ d*x)*sec(c + d*x)**2, x) + Integral(-I*tan(c + d*x)**3*sec(c + d*x)**2, x
) + Integral(sec(c + d*x)**2, x))
```

Giac [B] time = 1.22505, size = 76, normalized size = 2.81

$$-\frac{i\,a^3\tan\left(dx+c\right)^4+4\,a^3\tan\left(dx+c\right)^3-6i\,a^3\tan\left(dx+c\right)^2-4\,a^3\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(I*a^3*tan(d*x + c)^4 + 4*a^3*tan(d*x + c)^3 - 6*I*a^3*tan(d*x + c)^2
- 4*a^3*tan(d*x + c))/d
```

3.40 $\int (a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=63

$$-\frac{2a^3\tan(c+dx)}{d}-\frac{4ia^3\log(\cos(c+dx))}{d}+4a^3x+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

```
[Out] 4*a^3*x - ((4*I)*a^3*Log[Cos[c + d*x]])/d - (2*a^3*Tan[c + d*x])/d + ((I/2)
*a*(a + I*a*Tan[c + d*x])^2)/d
```

Rubi [A] time = 0.0317299, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3478, 3477, 3475}

$$-\frac{2a^3\tan(c+dx)}{d}-\frac{4ia^3\log(\cos(c+dx))}{d}+4a^3x+\frac{ia(a+ia\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] 4*a^3*x - ((4*I)*a^3*Log[Cos[c + d*x]])/d - (2*a^3*Tan[c + d*x])/d + ((I/2)
*a*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+ia\tan(c+dx))^3\,dx &= \frac{ia(a+ia\tan(c+dx))^2}{2d}+(2a)\int (a+ia\tan(c+dx))^2\,dx\\ &= 4a^3x-\frac{2a^3\tan(c+dx)}{d}+\frac{ia(a+ia\tan(c+dx))^2}{2d}+\left(4ia^3\right)\int \tan(c+dx)\,dx\\ &= 4a^3x-\frac{4ia^3\log(\cos(c+dx))}{d}-\frac{2a^3\tan(c+dx)}{d}+\frac{ia(a+ia\tan(c+dx))^2}{2d}\end{aligned}$$

Mathematica [A] time = 0.784299, size = 119, normalized size = 1.89

$$\frac{a^3\sec(c)\sec^2(c+dx)\left(-3\sin(c+2dx)+2dx\cos(3c+2dx)-i\cos(3c+2dx)\log\left(\cos^2(c+dx)\right)+\cos(c+2dx)\left(2dx-\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c]*Sec[c + d*x]^2*(2*d*x*Cos[3*c + 2*d*x] + Cos[c + 2*d*x]*(2*d*x
- I*Log[Cos[c + d*x]^2]) + Cos[c]*(-I + 4*d*x - (2*I)*Log[Cos[c + d*x]^2])
- I*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]^2] + 3*Sin[c] - 3*Sin[c + 2*d*x]))/(2
*d)
```

Maple [A] time = 0.004, size = 68, normalized size = 1.1

$$-3\,\frac{a^{3}\tan\left(dx+c\right)}{d}-\frac{\frac{i}{2}a^{3}\left(\tan\left(dx+c\right)\right)^{2}}{d}+\frac{2\,ia^{3}\ln\left(1+\left(\tan\left(dx+c\right)\right)^{2}\right)}{d}+4\,\frac{a^{3}\arctan\left(\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -3*a^3*tan(d*x+c)/d-1/2*I/d*a^3*tan(d*x+c)^2+2*I/d*a^3*ln(1+tan(d*x+c)^2)+4
/d*a^3*arctan(tan(d*x+c))
```

Maxima [A] time = 1.66684, size = 103, normalized size = 1.63

$$a^{3}x+\frac{3\left(dx+c-\tan\left(dx+c\right)\right)a^{3}}{d}+\frac{i\,a^{3}\left(\frac{1}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)^{2}-1\right)\right)}{2\,d}+\frac{3i\,a^{3}\log\left(\sec\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3*(d*x + c - tan(d*x + c))*a^3/d + 1/2*I*a^3*(1/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c)^2 - 1))/d + 3*I*a^3*log(sec(d*x + c))/d
```

Fricas [A] time = 1.23346, size = 267, normalized size = 4.24

$$\frac{-8i\,a^{3}e^{\left(2i\,dx+2i\,c\right)}-6i\,a^{3}+\left(-4i\,a^{3}e^{\left(4i\,dx+4i\,c\right)}-8i\,a^{3}e^{\left(2i\,dx+2i\,c\right)}-4i\,a^{3}\right)\log\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)}{de^{\left(4i\,dx+4i\,c\right)}+2\,de^{\left(2i\,dx+2i\,c\right)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (-8*I*a^3*e^(2*I*d*x + 2*I*c) - 6*I*a^3 + (-4*I*a^3*e^(4*I*d*x + 4*I*c) - 8
*I*a^3*e^(2*I*d*x + 2*I*c) - 4*I*a^3)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4
*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 1.35666, size = 100, normalized size = 1.59

$$-\frac{4ia^{3}\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-\frac{8ia^{3}e^{-2ic}e^{2idx}}{d}-\frac{6ia^{3}e^{-4ic}}{d}}{e^{4idx}+2e^{-2ic}e^{2idx}+e^{-4ic}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -4*I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-8*I*a**3*exp(-2*I*c)*exp(2*
I*d*x)/d - 6*I*a**3*exp(-4*I*c)/d)/(exp(4*I*d*x) + 2*exp(-2*I*c)*exp(2*I*d*
x) + exp(-4*I*c))
```

Giac [B] time = 1.11637, size = 158, normalized size = 2.51

$$\frac{-4i\,a^3e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-8i\,a^3e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-8i\,a^3e^{(2i\,dx+2i\,c)}-4i\,a^3\log\left(e^{(2i\,dx+2i\,c)}+1\right)-}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (-4*I*a^3*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 8*I*a^3*e^(2*I
*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 8*I*a^3*e^(2*I*d*x + 2*I*c) -
4*I*a^3*log(e^(2*I*d*x + 2*I*c) + 1) - 6*I*a^3)/(d*e^(4*I*d*x + 4*I*c) + 2*
d*e^(2*I*d*x + 2*I*c) + d)
```

3.41 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=49

$$-\frac{2ia^4}{d(a-ia\tan(c+dx))}+\frac{ia^3\log(\cos(c+dx))}{d}-a^3x$$

```
[Out] -(a^3*x) + (I*a^3*Log[Cos[c + d*x]])/d - ((2*I)*a^4)/(d*(a - I*a*Tan[c + d*
x]))
```

Rubi [A] time = 0.0486997, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{2ia^4}{d(a-ia\tan(c+dx))}+\frac{ia^3\log(\cos(c+dx))}{d}-a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (I*a^3*Log[Cos[c + d*x]])/d - ((2*I)*a^4)/(d*(a - I*a*Tan[c + d*
x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{\left(ia^3\right)\operatorname{Subst}\left(\int \frac{a+x}{(a-x)^2}\,dx,x,ia\tan(c+dx)\right)}{d}\\
&= -\frac{\left(ia^3\right)\operatorname{Subst}\left(\int \left(\frac{2a}{(a-x)^2}+\frac{1}{-a+x}\right)dx,x,ia\tan(c+dx)\right)}{d}\\
&= -a^3x+\frac{ia^3\log(\cos(c+dx))}{d}-\frac{2ia^4}{d(a-ia\tan(c+dx))}
\end{aligned}$$

Mathematica [B] time = 0.277116, size = 99, normalized size = 2.02

$$-\frac{a^3(\cos(c+4dx)+i\sin(c+4dx))\left(\cos(c+dx)\left(-i\log\left(\cos^2(c+dx)\right)+2dx+2i\right)+\sin(c+dx)\left(-\log\left(\cos^2(c+dx)\right)-2\right.\right.}{2d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -(a^3*(Cos[c + d*x]*(2*I + 2*d*x - I*Log[Cos[c + d*x]^2]) + (-2 - (2*I)*d*x
 - Log[Cos[c + d*x]^2])*Sin[c + d*x])*(Cos[c + 4*d*x] + I*Sin[c + 4*d*x]))/
(2*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] time = 0.051, size = 87, normalized size = 1.8

$$\frac{\frac{i}{2}a^3(\sin(dx+c))^2}{d}+\frac{ia^3\ln(\cos(dx+c))}{d}+2\frac{a^3\sin(dx+c)\cos(dx+c)}{d}-a^3x-\frac{a^3c}{d}-\frac{\frac{3i}{2}a^3(\cos(dx+c))^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/2*I/d*a^3*sin(d*x+c)^2+I*a^3*ln(cos(d*x+c))/d+2/d*a^3*sin(d*x+c)*cos(d*x+
c)-a^3*x-1/d*a^3*c-3/2*I/d*a^3*cos(d*x+c)^2
```

Maxima [A] time = 1.69114, size = 84, normalized size = 1.71

$$-\frac{2(dx+c)a^3+i\,a^3\log\left(\tan(dx+c)^2+1\right)-\frac{4\left(a^3\tan(dx+c)-i\,a^3\right)}{\tan(dx+c)^2+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c)*a^3 + I*a^3*log(tan(d*x + c)^2 + 1) - 4*(a^3*tan(d*x + c)
 - I*a^3)/(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.18139, size = 93, normalized size = 1.9

$$\frac{-i\,a^3e^{(2i\,dx+2i\,c)}+i\,a^3\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (-I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3*log(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [A] time = 0.709498, size = 53, normalized size = 1.08

$$2a^3\left(\begin{cases}-\frac{ie^{2idx}}{2d} & \text{for } d\neq 0\\ x & \text{otherwise}\end{cases}\right)e^{2ic}+\frac{ia^3\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] 2*a**3*Piecewise((-I*exp(2*I*d*x)/(2*d), Ne(d, 0)), (x, True))*exp(2*I*c) +
 I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d
```

Giac [A] time = 1.25177, size = 49, normalized size = 1.

$$\frac{-i\,a^3e^{(2i\,dx+2i\,c)} + i\,a^3\log\left(e^{(2i\,dx+2i\,c)} + 1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (-I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3*log(e^(2*I*d*x + 2*I*c) + 1))/d
```

3.42 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=27

$$-\frac{ia^5}{2d(a-ia\tan(c+dx))^2}$$

```
[Out] ((-I/2)*a^5)/(d*(a - I*a*Tan[c + d*x])^2)
```

Rubi [A] time = 0.0378988, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$-\frac{ia^5}{2d(a-ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-I/2)*a^5)/(d*(a - I*a*Tan[c + d*x])^2)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{\left(ia^5\right)\operatorname{Subst}\left(\int \frac{1}{(a-x)^3}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{ia^5}{2d(a-ia\tan(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.251446, size = 50, normalized size = 1.85

$$\frac{a^3(3\cos(c+dx)-i\sin(c+dx))(\sin(3(c+dx))-i\cos(3(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(3*Cos[c + d*x] - I*Sin[c + d*x])*((-I)*Cos[3*(c + d*x)] + Sin[3*(c +
d*x)]))/(8*d)
```

Maple [B] time = 0.06, size = 114, normalized size = 4.2

$$\frac{1}{d}\left(-\frac{i}{4}a^3(\sin(dx+c))^4-3a^3\left(-1/4(\cos(dx+c))^3\sin(dx+c)+1/8\cos(dx+c)\sin(dx+c)+1/8dx+c/8\right)-\frac{3i}{4}a^3(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/4*I*a^3*sin(d*x+c)^4-3*a^3*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*
x+c)*sin(d*x+c)+1/8*d*x+1/8*c)-3/4*I*a^3*cos(d*x+c)^4+a^3*(1/4*(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [B] time = 1.70067, size = 77, normalized size = 2.85

$$\frac{4i\,a^3\tan(dx+c)^2+8\,a^3\tan(dx+c)-4i\,a^3}{8\left(\tan(dx+c)^4+2\,\tan(dx+c)^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*I*a^3*tan(d*x + c)^2 + 8*a^3*tan(d*x + c) - 4*I*a^3)/((tan(d*x + c)^
4 + 2*tan(d*x + c)^2 + 1)*d)
```

Fricas [A] time = 1.16282, size = 89, normalized size = 3.3

$$\frac{-i\,a^3e^{(4i\,dx+4i\,c)}-2i\,a^3e^{(2i\,dx+2i\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(-I*a^3*e^(4*I*d*x + 4*I*c) - 2*I*a^3*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A] time = 0.481706, size = 82, normalized size = 3.04

$$\begin{cases} \frac{-4ia^3de^{4ic}e^{4idx}-8ia^3de^{2ic}e^{2idx}}{32d^2} & \text{for } 32d^2 \neq 0 \\ x\left(\frac{a^3e^{4ic}}{2}+\frac{a^3e^{2ic}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-4*I*a**3*d*exp(4*I*c)*exp(4*I*d*x) - 8*I*a**3*d*exp(2*I*c)*exp
(2*I*d*x))/(32*d**2), Ne(32*d**2, 0)), (x*(a**3*exp(4*I*c)/2 + a**3*exp(2*I
*c)/2), True))
```

Giac [B] time = 1.29083, size = 182, normalized size = 6.74

$$\frac{-8i\,a^3e^{(12i\,dx+8i\,c)} - 48i\,a^3e^{(10i\,dx+6i\,c)} - 112i\,a^3e^{(8i\,dx+4i\,c)} - 128i\,a^3e^{(6i\,dx+2i\,c)} - 16i\,a^3e^{(2i\,dx-2i\,c)} - 72i\,a^3e^{(4i\,dx)}}{64\left(de^{(8i\,dx+4i\,c)} + 4\,de^{(6i\,dx+2i\,c)} + 4\,de^{(2i\,dx-2i\,c)} + 6\,de^{(4i\,dx)} + de^{(-4i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(-8*I*a^3*e^(12*I*d*x + 8*I*c) - 48*I*a^3*e^(10*I*d*x + 6*I*c) - 112*I
*a^3*e^(8*I*d*x + 4*I*c) - 128*I*a^3*e^(6*I*d*x + 2*I*c) - 16*I*a^3*e^(2*I*
d*x - 2*I*c) - 72*I*a^3*e^(4*I*d*x))/(d*e^(8*I*d*x + 4*I*c) + 4*d*e^(6*I*d*
x + 2*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x) + d*e^(-4*I*c))
```

3.43 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=90

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3}-\frac{ia^5}{8d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}+\frac{a^3x}{8}$$

```
[Out] (a^3*x)/8 - ((I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/8)*a^5)/(d*(a -
I*a*Tan[c + d*x])^2) - ((I/8)*a^4)/(d*(a - I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0667252, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3}-\frac{ia^5}{8d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}+\frac{a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*x)/8 - ((I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/8)*a^5)/(d*(a -
I*a*Tan[c + d*x])^2) - ((I/8)*a^4)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{(ia^7)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^4(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^7)\,\mathrm{Subst}\left(\int \left(\frac{1}{2a(a-x)^4} + \frac{1}{4a^2(a-x)^3} + \frac{1}{8a^3(a-x)^2} + \frac{1}{8a^3(a^2-x^2)}\right) dx, x, ia\tan(c\right.}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3} - \frac{ia^5}{8d(a-ia\tan(c+dx))^2} - \frac{ia^4}{8d(a-ia\tan(c+dx)}$$

$$= \frac{a^3x}{8} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3} - \frac{ia^5}{8d(a-ia\tan(c+dx))^2} - \frac{ia^4}{8d(a-ia\tan(c+}$$

Mathematica [A] time = 0.538751, size = 109, normalized size = 1.21

$$\frac{a^3(-9\sin(c+dx) - 12idx\sin(3(c+dx)) + 2\sin(3(c+dx)) - 27i\cos(c+dx) + 2(6dx-i)\cos(3(c+dx)))(\cos(3(c+2d}{96d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*((-27*I)*Cos[c + d*x] + 2*(-I + 6*d*x)*Cos[3*(c + d*x)] - 9*Sin[c + d*
x] + 2*Sin[3*(c + d*x)] - (12*I)*d*x*Sin[3*(c + d*x)])*(Cos[3*(c + 2*d*x)]
+ I*Sin[3*(c + 2*d*x)]))/(96*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [B] time = 0.062, size = 156, normalized size = 1.7

$$\frac{1}{d}\left(-ia^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^4}{6} - \frac{(\cos(dx+c))^4}{12}\right) - 3a^3\left(-1/6\,(\cos(dx+c))^5\sin(dx+c) + 1/24\,\left((\cos(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-I*a^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)-3*a^3*(-1/6*
cos(d*x+c)^5*sin(d*x+c)+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*
d*x+1/16*c)-1/2*I*a^3*cos(d*x+c)^6+a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.69633, size = 142, normalized size = 1.58

$$\frac{6\,(dx+c)a^3 + \frac{6\,a^3\tan(dx+c)^5 + 16\,a^3\tan(dx+c)^3 + 12i\,a^3\tan(dx+c)^2 + 42\,a^3\tan(dx+c) - 20i\,a^3}{\tan(dx+c)^6 + 3\,\tan(dx+c)^4 + 3\,\tan(dx+c)^2 + 1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(6*(d*x + c)*a^3 + (6*a^3*tan(d*x + c)^5 + 16*a^3*tan(d*x + c)^3 + 12*
I*a^3*tan(d*x + c)^2 + 42*a^3*tan(d*x + c) - 20*I*a^3)/(tan(d*x + c)^6 + 3*
```

```
tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.21181, size = 151, normalized size = 1.68

$$\frac{12\,a^3dx - 2i\,a^3e^{(6i\,dx+6i\,c)} - 9i\,a^3e^{(4i\,dx+4i\,c)} - 18i\,a^3e^{(2i\,dx+2i\,c)}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(12*a^3*d*x - 2*I*a^3*e^(6*I*d*x + 6*I*c) - 9*I*a^3*e^(4*I*d*x + 4*I*c
) - 18*I*a^3*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A] time = 0.731826, size = 133, normalized size = 1.48

$$\frac{a^3x}{8} + \begin{cases} \frac{-512ia^3d^2e^{6ic}e^{6idx} - 2304ia^3d^2e^{4ic}e^{4idx} - 4608ia^3d^2e^{2ic}e^{2idx}}{24576d^3} & \text{for } 24576d^3 \neq 0 \\ x\left(\frac{a^3e^{6ic}}{8} + \frac{3a^3e^{4ic}}{8} + \frac{3a^3e^{2ic}}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] a**3*x/8 + Piecewise(((-512*I*a**3*d**2*exp(6*I*c)*exp(6*I*d*x) - 2304*I*a*
*3*d**2*exp(4*I*c)*exp(4*I*d*x) - 4608*I*a**3*d**2*exp(2*I*c)*exp(2*I*d*x))
/(24576*d**3), Ne(24576*d**3, 0)), (x*(a**3*exp(6*I*c)/8 + 3*a**3*exp(4*I*c
)/8 + 3*a**3*exp(2*I*c)/8), True))
```

Giac [B] time = 1.35198, size = 617, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/384*(48*a^3*d*x*e^(8*I*d*x + 4*I*c) + 192*a^3*d*x*e^(6*I*d*x + 2*I*c) + 1
92*a^3*d*x*e^(2*I*d*x - 2*I*c) + 288*a^3*d*x*e^(4*I*d*x) + 48*a^3*d*x*e^(-4
*I*c) - 12*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 48*I*a^
3*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 48*I*a^3*e^(2*I*d*x -
2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 72*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*x +
 2*I*c) + 1) - 12*I*a^3*e^(-4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 12*I*a^3*
e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 48*I*a^3*e^(6*I*d*x + 2
*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 48*I*a^3*e^(2*I*d*x - 2*I*c)*log(e^(2
*I*d*x) + e^(-2*I*c)) + 72*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c))
+ 12*I*a^3*e^(-4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) - 8*I*a^3*e^(14*I*d*x +
 10*I*c) - 68*I*a^3*e^(12*I*d*x + 8*I*c) - 264*I*a^3*e^(10*I*d*x + 6*I*c) -
 536*I*a^3*e^(8*I*d*x + 4*I*c) - 584*I*a^3*e^(6*I*d*x + 2*I*c) - 72*I*a^3*e
^(2*I*d*x - 2*I*c) - 324*I*a^3*e^(4*I*d*x))/(d*e^(8*I*d*x + 4*I*c) + 4*d*e^
(6*I*d*x + 2*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x) + d*e^(-4*I*c
))
```

3.44 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=144

$$-\frac{ia^7}{16d(a-ia\tan(c+dx))^4}-\frac{ia^6}{12d(a-ia\tan(c+dx))^3}-\frac{3ia^5}{32d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}+\frac{}{32d(a+}$$

```
[Out] (5*a^3*x)/32 - ((I/16)*a^7)/(d*(a - I*a*Tan[c + d*x])^4) - ((I/12)*a^6)/(d*
(a - I*a*Tan[c + d*x])^3) - (((3*I)/32)*a^5)/(d*(a - I*a*Tan[c + d*x])^2) -
 ((I/8)*a^4)/(d*(a - I*a*Tan[c + d*x])) + ((I/32)*a^4)/(d*(a + I*a*Tan[c +
d*x]))
```

Rubi [A] time = 0.091226, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{ia^7}{16d(a-ia\tan(c+dx))^4}-\frac{ia^6}{12d(a-ia\tan(c+dx))^3}-\frac{3ia^5}{32d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}+\frac{}{32d(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/32 - ((I/16)*a^7)/(d*(a - I*a*Tan[c + d*x])^4) - ((I/12)*a^6)/(d*
(a - I*a*Tan[c + d*x])^3) - (((3*I)/32)*a^5)/(d*(a - I*a*Tan[c + d*x])^2) -
 ((I/8)*a^4)/(d*(a - I*a*Tan[c + d*x])) + ((I/32)*a^4)/(d*(a + I*a*Tan[c +
d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{(ia^9)\,\text{Subst}\left(\int \frac{1}{(a-x)^5(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^9)\,\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^5}+\frac{1}{4a^3(a-x)^4}+\frac{3}{16a^4(a-x)^3}+\frac{1}{8a^5(a-x)^2}+\frac{1}{32a^5(a+x)^2}+\frac{}{32a^5(}\right.\right.}{d}$$

$$= -\frac{ia^7}{16d(a-ia\tan(c+dx))^4}-\frac{ia^6}{12d(a-ia\tan(c+dx))^3}-\frac{3ia^5}{32d(a-ia\tan(c+dx)}$$

$$= \frac{5a^3x}{32}-\frac{ia^7}{16d(a-ia\tan(c+dx))^4}-\frac{ia^6}{12d(a-ia\tan(c+dx))^3}-\frac{3ia^5}{32d(a-ia\tan(c}$$

Mathematica [A] time = 0.581832, size = 131, normalized size = 0.91

$$\frac{a^3(-60\sin(c+dx)-120idx\sin(3(c+dx))+20\sin(3(c+dx))+15\sin(5(c+dx))-180i\cos(c+dx)+20(6dx-i)\cos(3}{768d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*((-180*I)*Cos[c + d*x] + 20*(-I + 6*d*x)*Cos[3*(c + d*x)] + (9*I)*Cos[
5*(c + d*x)] - 60*Sin[c + d*x] + 20*Sin[3*(c + d*x)] - (120*I)*d*x*Sin[3*(c
 + d*x)] + 15*Sin[5*(c + d*x)])*(Cos[3*(c + 2*d*x)] + I*Sin[3*(c + 2*d*x)])
)/(768*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] time = 0.062, size = 176, normalized size = 1.2

$$\frac{1}{d}\left(-ia^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{8}-\frac{(\cos(dx+c))^6}{24}\right)-3a^3\left(-1/8\sin(dx+c)(\cos(dx+c))^7+1/48\left((\cos(dx+c)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-I*a^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)-3*a^3*(-1/8*
sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c)
)*sin(d*x+c)+5/128*d*x+5/128*c)-3/8*I*a^3*cos(d*x+c)^8+a^3*(1/8*(cos(d*x+c)
^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*
d*x+35/128*c))
```

Maxima [A] time = 1.63956, size = 173, normalized size = 1.2

$$\frac{60\,(dx+c)a^3+\frac{60\,a^3\tan(dx+c)^7+220\,a^3\tan(dx+c)^5+292\,a^3\tan(dx+c)^3+64i\,a^3\tan(dx+c)^2+324\,a^3\tan(dx+c)-128i\,a^3}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/384*(60*(d*x + c)*a^3 + (60*a^3*tan(d*x + c)^7 + 220*a^3*tan(d*x + c)^5 +
 292*a^3*tan(d*x + c)^3 + 64*I*a^3*tan(d*x + c)^2 + 324*a^3*tan(d*x + c) -
128*I*a^3)/(tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*
x + c)^2 + 1))/d
```

Fricas [A] time = 1.25267, size = 271, normalized size = 1.88

$$\frac{\left(120\,a^{3}dxe^{(2i\,dx+2i\,c)} - 3i\,a^{3}e^{(10i\,dx+10i\,c)} - 20i\,a^{3}e^{(8i\,dx+8i\,c)} - 60i\,a^{3}e^{(6i\,dx+6i\,c)} - 120i\,a^{3}e^{(4i\,dx+4i\,c)} + 12i\,a^{3}\right)e^{(-2i\,dx-2i\,c)}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/768*(120*a^3*d*x*e^(2*I*d*x + 2*I*c) - 3*I*a^3*e^(10*I*d*x + 10*I*c) - 20
*I*a^3*e^(8*I*d*x + 8*I*c) - 60*I*a^3*e^(6*I*d*x + 6*I*c) - 120*I*a^3*e^(4*
I*d*x + 4*I*c) + 12*I*a^3)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A] time = 0.825254, size = 228, normalized size = 1.58

$$\frac{5a^3x}{32} + \begin{cases} \frac{\left(-25165824ia^3d^4e^{10ic}e^{8idx} - 167772160ia^3d^4e^{8ic}e^{6idx} - 503316480ia^3d^4e^{6ic}e^{4idx} - 1006632960ia^3d^4e^{4ic}e^{2idx} + 100663296ia^3d^4e^{-2idx}\right)e^{-2ic}}{6442450944d^5} & \text{for 6} \\ x\left(-\frac{5a^3}{32} + \frac{\left(a^3e^{10ic}+5a^3e^{8ic}+10a^3e^{6ic}+10a^3e^{4ic}+5a^3e^{2ic}+a^3\right)e^{-2ic}}{32}\right) & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] 5*a**3*x/32 + Piecewise(((-25165824*I*a**3*d**4*exp(10*I*c)*exp(8*I*d*x) -
167772160*I*a**3*d**4*exp(8*I*c)*exp(6*I*d*x) - 503316480*I*a**3*d**4*exp(6
*I*c)*exp(4*I*d*x) - 1006632960*I*a**3*d**4*exp(4*I*c)*exp(2*I*d*x) + 10066
3296*I*a**3*d**4*exp(-2*I*d*x))*exp(-2*I*c)/(6442450944*d**5), Ne(644245094
4*d**5*exp(2*I*c), 0)), (x*(-5*a**3/32 + (a**3*exp(10*I*c) + 5*a**3*exp(8*I
*c) + 10*a**3*exp(6*I*c) + 10*a**3*exp(4*I*c) + 5*a**3*exp(2*I*c) + a**3)*e
xp(-2*I*c)/32), True))
```

Giac [B] time = 1.3893, size = 694, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3072*(480*a^3*d*x*e^(10*I*d*x + 6*I*c) + 1920*a^3*d*x*e^(8*I*d*x + 4*I*c)
 + 2880*a^3*d*x*e^(6*I*d*x + 2*I*c) + 480*a^3*d*x*e^(2*I*d*x - 2*I*c) + 192
0*a^3*d*x*e^(4*I*d*x) - 66*I*a^3*e^(10*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*
c) + 1) - 264*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 396*
I*a^3*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 66*I*a^3*e^(2*I*d*
x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 264*I*a^3*e^(4*I*d*x)*log(e^(2*I*
d*x + 2*I*c) + 1) + 66*I*a^3*e^(10*I*d*x + 6*I*c)*log(e^(2*I*d*x) + e^(-2*I
```

```
*c)) + 264*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 396*I*
a^3*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 66*I*a^3*e^(2*I*d*x
 - 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 264*I*a^3*e^(4*I*d*x)*log(e^(2*I*
d*x) + e^(-2*I*c)) - 12*I*a^3*e^(18*I*d*x + 14*I*c) - 128*I*a^3*e^(16*I*d*x
 + 12*I*c) - 632*I*a^3*e^(14*I*d*x + 10*I*c) - 1968*I*a^3*e^(12*I*d*x + 8*I
*c) - 3692*I*a^3*e^(10*I*d*x + 6*I*c) - 3872*I*a^3*e^(8*I*d*x + 4*I*c) - 19
68*I*a^3*e^(6*I*d*x + 2*I*c) + 192*I*a^3*e^(2*I*d*x - 2*I*c) - 192*I*a^3*e^
(4*I*d*x) + 48*I*a^3*e^(-4*I*c))/(d*e^(10*I*d*x + 6*I*c) + 4*d*e^(8*I*d*x +
 4*I*c) + 6*d*e^(6*I*d*x + 2*I*c) + d*e^(2*I*d*x - 2*I*c) + 4*d*e^(4*I*d*x)
)
```

3.45 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=127

$$\frac{7ia^3\sec^3(c+dx)}{12d}+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{7i\sec^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d}+\frac{7a^3\tan(c+dx)\sec(c+dx)}{8d}+$$

```
[Out] (7*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (((7*I)/12)*a^3*Sec[c + d*x]^3)/d + (
7*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((I/5)*a*Sec[c + d*x]^3*(a + I*a*T
an[c + d*x])^2)/d + (((7*I)/20)*Sec[c + d*x]^3*(a^3 + I*a^3*Tan[c + d*x]))/
d
```

Rubi [A] time = 0.119956, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3498, 3486, 3768, 3770}

$$\frac{7ia^3\sec^3(c+dx)}{12d}+\frac{7a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{7i\sec^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d}+\frac{7a^3\tan(c+dx)\sec(c+dx)}{8d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (7*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (((7*I)/12)*a^3*Sec[c + d*x]^3)/d + (
7*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((I/5)*a*Sec[c + d*x]^3*(a + I*a*T
an[c + d*x])^2)/d + (((7*I)/20)*Sec[c + d*x]^3*(a^3 + I*a^3*Tan[c + d*x]))/
d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d} + \frac{1}{5}(7a)\int \sec^3(c+dx)(a+ia\tan(c+dx)$$
$$= \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d} + \frac{7i\sec^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d} +$$
$$= \frac{7ia^3\sec^3(c+dx)}{12d} + \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d} + \frac{7i\sec^3(c+dx)\left(a^3+\right.}{20d}$$
$$= \frac{7ia^3\sec^3(c+dx)}{12d} + \frac{7a^3\sec(c+dx)\tan(c+dx)}{8d} + \frac{ia\sec^3(c+dx)(a+ia\tan(c}{5d}$$
$$= \frac{7a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{7ia^3\sec^3(c+dx)}{12d} + \frac{7a^3\sec(c+dx)\tan(c+dx)}{8d} + \frac{i}{}$$

Mathematica [A] time = 0.652026, size = 102, normalized size = 0.8

$$\frac{a^3(\cos(3dx)+i\sin(3dx))\left(1680\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)+\sec^5(c+dx)(-150\sin(2(c+dx))+105\sin(4(c+dx}{960d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(Cos[3*d*x] + I*Sin[3*d*x])*(1680*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]
] + Sec[c + d*x]^5*(448*I + (640*I)*Cos[2*(c + d*x)] - 150*Sin[2*(c + d*x)]
 + 105*Sin[4*(c + d*x)])))/(960*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [B] time = 0.095, size = 236, normalized size = 1.9

$$\frac{-\frac{i}{5}a^3(\sin(dx+c))^4}{d(\cos(dx+c))^5} - \frac{\frac{i}{15}a^3(\sin(dx+c))^4}{d(\cos(dx+c))^3} + \frac{\frac{i}{15}a^3(\sin(dx+c))^4}{d\cos(dx+c)} + \frac{\frac{i}{15}a^3(\sin(dx+c))^2\cos(dx+c)}{d} + \frac{\frac{2i}{15}a^3\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -1/5*I/d*a^3*sin(d*x+c)^4/cos(d*x+c)^5-1/15*I/d*a^3*sin(d*x+c)^4/cos(d*x+c)
^3+1/15*I/d*a^3*sin(d*x+c)^4/cos(d*x+c)+1/15*I/d*a^3*sin(d*x+c)^2*cos(d*x+c
)+2/15*I/d*a^3*cos(d*x+c)-3/4/d*a^3*sin(d*x+c)^3/cos(d*x+c)^4-3/8/d*a^3*sin
(d*x+c)^3/cos(d*x+c)^2-3/8*a^3*sin(d*x+c)/d+7/8/d*a^3*ln(sec(d*x+c)+tan(d*x
+c))+I/d*a^3/cos(d*x+c)^3+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Maxima [A] time = 1.14215, size = 209, normalized size = 1.65

$$-\frac{45a^3\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 60a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1)\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(45*a^3*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^3*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) - 240*I*a^3/cos(d*x + c)^3 - 16*I*(5*cos(d*x + c)^2 - 3)*a^3/cos(d
*x + c)^5)/d
```

Fricas [B] time = 1.23709, size = 891, normalized size = 7.02

$$\frac{-210i\,a^3e^{(9i\,dx+9i\,c)} + 1580i\,a^3e^{(7i\,dx+7i\,c)} + 1792i\,a^3e^{(5i\,dx+5i\,c)} + 980i\,a^3e^{(3i\,dx+3i\,c)} + 210i\,a^3e^{(i\,dx+i\,c)} + 105\left(a^3e^{(10i\,dx+10i\,c)}\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(-210*I*a^3*e^(9*I*d*x + 9*I*c) + 1580*I*a^3*e^(7*I*d*x + 7*I*c) + 17
92*I*a^3*e^(5*I*d*x + 5*I*c) + 980*I*a^3*e^(3*I*d*x + 3*I*c) + 210*I*a^3*e^
(I*d*x + I*c) + 105*(a^3*e^(10*I*d*x + 10*I*c) + 5*a^3*e^(8*I*d*x + 8*I*c)
+ 10*a^3*e^(6*I*d*x + 6*I*c) + 10*a^3*e^(4*I*d*x + 4*I*c) + 5*a^3*e^(2*I*d*
x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) - 105*(a^3*e^(10*I*d*x + 10*I*c)
 + 5*a^3*e^(8*I*d*x + 8*I*c) + 10*a^3*e^(6*I*d*x + 6*I*c) + 10*a^3*e^(4*I*d
*x + 4*I*c) + 5*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) - I))/(d
*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c)
 + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int -3\tan^2(c+dx)\sec^3(c+dx)\,dx + \int 3i\tan(c+dx)\sec^3(c+dx)\,dx + \int -i\tan^3(c+dx)\sec^3(c+dx)\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(-3*tan(c + d*x)**2*sec(c + d*x)**3, x) + Integral(3*I*tan(c
+ d*x)*sec(c + d*x)**3, x) + Integral(-I*tan(c + d*x)**3*sec(c + d*x)**3, x
) + Integral(sec(c + d*x)**3, x))
```

Giac [A] time = 1.28311, size = 258, normalized size = 2.03

$$\frac{105\,a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 105\,a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + \frac{2\left(15\,a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 - 360i\,a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8 - 390\,a^3\right.}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(105*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^9 - 360*I*a^3*tan(1/2*
d*x + 1/2*c)^8 - 390*a^3*tan(1/2*d*x + 1/2*c)^7 + 960*I*a^3*tan(1/2*d*x + 1
```

```
/2*c)^6 - 400*I*a^3*tan(1/2*d*x + 1/2*c)^4 + 390*a^3*tan(1/2*d*x + 1/2*c)^3
 + 320*I*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*a^3*tan(1/2*d*x + 1/2*c) - 136*I*a
^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.46 $\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=99

$$\frac{5ia^3\sec(c+dx)}{2d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5i\sec(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx)}{3d}$$

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (((5*I)/2)*a^3*Sec[c + d*x])/d + ((I/
3)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d + (((5*I)/6)*Sec[c + d*x]*(a^
3 + I*a^3*Tan[c + d*x]))/d
```

Rubi [A] time = 0.0660193, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3498, 3486, 3770}

$$\frac{5ia^3\sec(c+dx)}{2d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5i\sec(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```


```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (((5*I)/2)*a^3*Sec[c + d*x])/d + ((I/
3)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d + (((5*I)/6)*Sec[c + d*x]*(a^
3 + I*a^3*Tan[c + d*x]))/d
```


Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} + \frac{1}{3}(5a)\int \sec(c+dx)(a+ia\tan(c+dx))^2\,d$$
$$= \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} + \frac{5i\sec(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d} + \frac{1}{2}($$
$$= \frac{5ia^3\sec(c+dx)}{2d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} + \frac{5i\sec(c+dx)\left(a^3+ia^3\right.}{6d}$$
$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{5ia^3\sec(c+dx)}{2d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))}{3d}$$

Mathematica [A] time = 0.496575, size = 93, normalized size = 0.94

$$\frac{a^3(\cos(3dx)+i\sin(3dx))\left(60\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)+i\sec^3(c+dx)(9i\sin(2(c+dx))+24\cos(2(c+dx))+2\right.}{12d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(Cos[3*d*x] + I*Sin[3*d*x])*(60*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]
+ I*Sec[c + d*x]^3*(20 + 24*Cos[2*(c + d*x)] + (9*I)*Sin[2*(c + d*x)])))/(1
2*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] time = 0.026, size = 167, normalized size = 1.7

$$\frac{-\frac{i}{3}a^3(\sin(dx+c))^4}{d(\cos(dx+c))^3} + \frac{\frac{i}{3}a^3(\sin(dx+c))^4}{d\cos(dx+c)} + \frac{\frac{i}{3}a^3\cos(dx+c)(\sin(dx+c))^2}{d} + \frac{\frac{2i}{3}a^3\cos(dx+c)}{d} - \frac{3a^3(\sin(dx+c))^3}{2d(\cos(dx+c))^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -1/3*I/d*a^3*sin(d*x+c)^4/cos(d*x+c)^3+1/3*I/d*a^3*sin(d*x+c)^4/cos(d*x+c)+
1/3*I/d*a^3*cos(d*x+c)*sin(d*x+c)^2+2/3*I/d*a^3*cos(d*x+c)-3/2/d*a^3*sin(d*
x+c)^3/cos(d*x+c)^2-3/2*a^3*sin(d*x+c)/d+5/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c)
)+3*I/d*a^3/cos(d*x+c)
```

Maxima [A] time = 1.12551, size = 147, normalized size = 1.48

$$\frac{9a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)+12a^3\log(\sec(dx+c)+\tan(dx+c))+\frac{36ia^3}{\cos(dx+c)}+\frac{4i(}{}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(9*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) + 12*a^3*log(sec(d*x + c) + tan(d*x + c)) + 36*I*a^3
/cos(d*x + c) + 4*I*(3*cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.22749, size = 554, normalized size = 5.6

$$\frac{66i\,a^3e^{(5i\,dx+5i\,c)} + 80i\,a^3e^{(3i\,dx+3i\,c)} + 30i\,a^3e^{(i\,dx+i\,c)} + 15\left(a^3e^{(6i\,dx+6i\,c)} + 3\,a^3e^{(4i\,dx+4i\,c)} + 3\,a^3e^{(2i\,dx+2i\,c)} + a^3\right)\log\left(e^{(i\,dx+i}\right.}{6\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4i\,dx+4i\,c)} + 3\,de^{(2i\,dx+2i\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(66*I*a^3*e^(5*I*d*x + 5*I*c) + 80*I*a^3*e^(3*I*d*x + 3*I*c) + 30*I*a^3
*e^(I*d*x + I*c) + 15*(a^3*e^(6*I*d*x + 6*I*c) + 3*a^3*e^(4*I*d*x + 4*I*c)
+ 3*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) - 15*(a^3*e^(6*
I*d*x + 6*I*c) + 3*a^3*e^(4*I*d*x + 4*I*c) + 3*a^3*e^(2*I*d*x + 2*I*c) + a^
3)*log(e^(I*d*x + I*c) - I))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*
c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int -3\tan^2{(c+dx)}\sec{(c+dx)}\,dx + \int 3i\tan{(c+dx)}\sec{(c+dx)}\,dx + \int -i\tan^3{(c+dx)}\sec{(c+dx)}\,dx + \int \mathrm{se}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(-3*tan(c + d*x)**2*sec(c + d*x), x) + Integral(3*I*tan(c + d
*x)*sec(c + d*x), x) + Integral(-I*tan(c + d*x)**3*sec(c + d*x), x) + Integ
ral(sec(c + d*x), x))
```

Giac [A] time = 1.24123, size = 171, normalized size = 1.73

$$\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-48i\,a^3\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 + 18*I*a^3*tan(1/2*d*x +
1/2*c)^4 - 48*I*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) + 2
2*I*a^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.47 $\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=61

$$-\frac{3ia^3\sec(c+dx)}{d}-\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

```
[Out] (-3*a^3*ArcTanh[Sin[c + d*x]])/d - ((3*I)*a^3*Sec[c + d*x])/d - ((2*I)*a*Co
s[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d
```

Rubi [A] time = 0.0501246, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3496, 3486, 3770}

$$-\frac{3ia^3\sec(c+dx)}{d}-\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Sin[c + d*x]])/d - ((3*I)*a^3*Sec[c + d*x])/d - ((2*I)*a*Co
s[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx &= -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}-\left(3a^2\right)\int \sec(c+dx)(a+ia\tan(c+dx))\\ &= -\frac{3ia^3\sec(c+dx)}{d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}-\left(3a^3\right)\int \sec(c+dx)\\ &= -\frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{3ia^3\sec(c+dx)}{d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+d}{d}\end{aligned}$$

Mathematica [B] time = 0.649382, size = 123, normalized size = 2.02

$$\frac{a^3\cos^2(c+dx)(\tan(c+dx)-i)^3\left((-\cos(2c-dx)+i\sin(2c-dx))(5\cos(c+dx)-i\sin(c+dx))+6(\sin(3c)+i\cos(3\right.}{d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]^2*(6*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]*(
I*Cos[3*c] + Sin[3*c]) + (-Cos[2*c - d*x] + I*Sin[2*c - d*x])*(5*Cos[c + d*
x] - I*Sin[c + d*x]))*(-I + Tan[c + d*x])^3)/(d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] time = 0.044, size = 101, normalized size = 1.7

$$\frac{-ia^3\left(\sin(dx+c)\right)^4}{d\cos(dx+c)}-\frac{ia^3\left(\sin(dx+c)\right)^2\cos(dx+c)}{d}-\frac{5\,ia^3\cos(dx+c)}{d}-3\,\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+4\,\frac{a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -I/d*a^3*sin(d*x+c)^4/cos(d*x+c)-I/d*a^3*sin(d*x+c)^2*cos(d*x+c)-5*I/d*a^3*
cos(d*x+c)-3/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+4*a^3*sin(d*x+c)/d
```

Maxima [A] time = 1.22594, size = 111, normalized size = 1.82

$$-\frac{2i\,a^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+3\,a^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))+6i\,a^3\cos(dx+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*a^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a^3*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) + 6*I*a^3*cos(d*x + c) - 2*a^3*
sin(d*x + c))/d
```

Fricas [A] time = 1.19292, size = 281, normalized size = 4.61

$$\frac{-4i\,a^3e^{(3i\,dx+3i\,c)}-6i\,a^3e^{(i\,dx+i\,c)}-3\left(a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(i\,dx+i\,c)}+i\right)+3\left(a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (-4*I*a^3*e^(3*I*d*x + 3*I*c) - 6*I*a^3*e^(I*d*x + I*c) - 3*(a^3*e^(2*I*d*x
 + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) + 3*(a^3*e^(2*I*d*x + 2*I*c) + a^
3)*log(e^(I*d*x + I*c) - I))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 0.810117, size = 107, normalized size = 1.75

$$\frac{3a^3\left(\log\left(e^{idx}-ie^{-ic}\right)-\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}-\frac{2ia^3e^{-ic}e^{idx}}{d\left(e^{2idx}+e^{-2ic}\right)}+\begin{cases}-\frac{4ia^3e^{ic}e^{idx}}{d} & \text{for } d\neq 0\\ 4a^3xe^{ic} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] 3*a**3*(log(exp(I*d*x) - I*exp(-I*c)) - log(exp(I*d*x) + I*exp(-I*c)))/d -
2*I*a**3*exp(-I*c)*exp(I*d*x)/(d*(exp(2*I*d*x) + exp(-2*I*c))) + Piecewise(
(-4*I*a**3*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (4*a**3*x*exp(I*c), True))
```

Giac [B] time = 1.31265, size = 316, normalized size = 5.18

$$\frac{63\,a^3e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)-33\,a^3e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-63\,a^3e^{(2i\,dx+2i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)+33\,a^3e^{(2i\,dx+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/32*(63*a^3*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 33*a^3*e^(2*I
*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) - 63*a^3*e^(2*I*d*x + 2*I*c)*log(-
I*e^(I*d*x + I*c) + 1) + 33*a^3*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c)
- 1) - 128*I*a^3*e^(3*I*d*x + 3*I*c) - 192*I*a^3*e^(I*d*x + I*c) + 63*a^3*l
og(I*e^(I*d*x + I*c) + 1) - 33*a^3*log(I*e^(I*d*x + I*c) - 1) - 63*a^3*log(
-I*e^(I*d*x + I*c) + 1) + 33*a^3*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(2*I*d*x
 + 2*I*c) + d)
```

3.48 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=32

$$-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

```
[Out] ((-I/3)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d
```

Rubi [A] time = 0.0366968, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3488}

$$-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]

[Out] ((-I/3)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

Mathematica [A] time = 0.0739692, size = 31, normalized size = 0.97

$$-\frac{ia^3(\cos(c+dx)+i\sin(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]

[Out] ((-I/3)*a^3*(Cos[c + d*x] + I*Sin[c + d*x])^3)/d
```

Maple [B] time = 0.052, size = 76, normalized size = 2.4

$$\frac{1}{d}\left(\frac{i}{3}a^3\left(2+(\sin(dx+c))^2\right)\cos(dx+c)-a^3(\sin(dx+c))^3-ia^3(\cos(dx+c))^3+\frac{a^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*I*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)-a^3*sin(d*x+c)^3-I*a^3*cos(d*x+c
)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 1.0345, size = 101, normalized size = 3.16

$$-\frac{3i a^3 \cos(dx+c)^3 + 3a^3 \sin(dx+c)^3 + i\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)a^3 + \left(\sin(dx+c)^3 - 3\sin(dx+c)\right)a^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3*(3*I*a^3*cos(d*x + c)^3 + 3*a^3*sin(d*x + c)^3 + I*(cos(d*x + c)^3 - 3
*cos(d*x + c))*a^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a^3)/d
```

Fricas [A] time = 1.14286, size = 46, normalized size = 1.44

$$-\frac{i a^3 e^{(3i dx+3i c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*I*a^3*e^(3*I*d*x + 3*I*c)/d
```

Sympy [A] time = 0.433956, size = 37, normalized size = 1.16

$$\begin{cases} -\frac{ia^3e^{3ic}e^{3idx}}{3d} & \text{for } 3d \neq 0 \\ a^3xe^{3ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((-I*a**3*exp(3*I*c)*exp(3*I*d*x)/(3*d), Ne(3*d, 0)), (a**3*x*exp(
3*I*c), True))
```

Giac [B] time = 1.42433, size = 1216, normalized size = 38.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/384*(108*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 432*a^3*e^
(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 432*a^3*e^(2*I*d*x - 2*I*c)*
log(I*e^(I*d*x + I*c) + 1) + 648*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) + 1)
 + 108*a^3*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 111*a^3*e^(8*I*d*x + 4*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 444*a^3*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x
 + I*c) - 1) + 444*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 666
*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 111*a^3*e^(-4*I*c)*log(I*e^(I
*d*x + I*c) - 1) - 108*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 432*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 432*a^3*e^(2*I*
d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 648*a^3*e^(4*I*d*x)*log(-I*e^(I*
d*x + I*c) + 1) - 108*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 111*a^3*
e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 444*a^3*e^(6*I*d*x + 2*I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 444*a^3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*
x + I*c) - 1) - 666*a^3*e^(4*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 111*a^3*e
^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 3*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(
I*d*x) + e^(-I*c)) + 12*a^3*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c))
 + 12*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 18*a^3*e^(4*I*d
*x)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^3*e^(-4*I*c)*log(I*e^(I*d*x) + e^(-I*
c)) - 3*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 12*a^3*e^(6*
I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 12*a^3*e^(2*I*d*x - 2*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) - 18*a^3*e^(4*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)
) - 3*a^3*e^(-4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 128*I*a^3*e^(11*I*d*x +
 7*I*c) + 512*I*a^3*e^(9*I*d*x + 5*I*c) + 768*I*a^3*e^(7*I*d*x + 3*I*c) + 5
12*I*a^3*e^(5*I*d*x + I*c) + 128*I*a^3*e^(3*I*d*x - I*c))/(d*e^(8*I*d*x + 4
*I*c) + 4*d*e^(6*I*d*x + 2*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x)
 + d*e^(-4*I*c))
```

3.49 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=88

$$-\frac{a^3\sin^3(c+dx)}{15d}+\frac{a^3\sin(c+dx)}{5d}-\frac{ia^3\cos^3(c+dx)}{15d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

```
[Out] ((-I/15)*a^3*Cos[c + d*x]^3)/d + (a^3*Sin[c + d*x])/(5*d) - (a^3*Sin[c + d*
x]^3)/(15*d) - (((2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2)/d
```

Rubi [A] time = 0.0710485, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3496, 3486, 2633}

$$-\frac{a^3\sin^3(c+dx)}{15d}+\frac{a^3\sin(c+dx)}{5d}-\frac{ia^3\cos^3(c+dx)}{15d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-I/15)*a^3*Cos[c + d*x]^3)/d + (a^3*Sin[c + d*x])/(5*d) - (a^3*Sin[c + d*
x]^3)/(15*d) - (((2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d} + \frac{1}{5}a^2\int \cos^3(c+dx)(a+ia\tan(c+$$
$$= -\frac{ia^3\cos^3(c+dx)}{15d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d} + \frac{1}{5}a^3\int \cos^3(c+$$
$$= -\frac{ia^3\cos^3(c+dx)}{15d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d} - \frac{a^3\,\text{Subst}\left(\int (1\right.}{}$$
$$= -\frac{ia^3\cos^3(c+dx)}{15d} + \frac{a^3\sin(c+dx)}{5d} - \frac{a^3\sin^3(c+dx)}{15d} - \frac{2ia\cos^5(c+dx)(a+}{5d}$$

Mathematica [A] time = 0.447172, size = 55, normalized size = 0.62

$$\frac{a^3(-6i\sin(2(c+dx))+9\cos(2(c+dx))+5)(\sin(3(c+dx))-i\cos(3(c+dx)))}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(5 + 9*Cos[2*(c + d*x)] - (6*I)*Sin[2*(c + d*x)])*((-I)*Cos[3*(c + d*x
)] + Sin[3*(c + d*x)]))/(30*d)
```

Maple [A] time = 0.06, size = 126, normalized size = 1.4

$$\frac{1}{d}\left(-ia^3\left(-\frac{(\cos(dx+c))^3(\sin(dx+c))^2}{5} - \frac{2(\cos(dx+c))^3}{15}\right) - 3a^3\left(-1/5\sin(dx+c)(\cos(dx+c))^4 + 1/15\left(2+(\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-I*a^3*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(d*x+c)^3)-3*a^3*(-1/5*
sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/5*I*a^3*cos(d*x
+c)^5+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.20822, size = 142, normalized size = 1.61

$$-\frac{9i\,a^3\cos(dx+c)^5 + i\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a^3 - 3\left(3\sin(dx+c)^5 - 5\sin(dx+c)^3\right)a^3 - \left(3\sin(dx+c)\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(9*I*a^3*cos(d*x + c)^5 + I*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^3
 - 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^3 - (3*sin(d*x + c)^5 - 10*sin
(d*x + c)^3 + 15*sin(d*x + c))*a^3)/d
```

Fricas [A] time = 1.1175, size = 131, normalized size = 1.49

$$\frac{-3i\,a^3e^{(5i\,dx+5i\,c)}-10i\,a^3e^{(3i\,dx+3i\,c)}-15i\,a^3e^{(i\,dx+i\,c)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(-3*I*a^3*e^(5*I*d*x + 5*I*c) - 10*I*a^3*e^(3*I*d*x + 3*I*c) - 15*I*a^
3*e^(I*d*x + I*c))/d
```

Sympy [A] time = 0.727356, size = 117, normalized size = 1.33

$$\begin{cases} \frac{-24ia^3d^2e^{5ic}e^{5idx}-80ia^3d^2e^{3ic}e^{3idx}-120ia^3d^2e^{ic}e^{idx}}{480d^3} & \text{for } 480d^3 \neq 0 \\ x\left(\frac{a^3e^{5ic}}{4}+\frac{a^3e^{3ic}}{2}+\frac{a^3e^{ic}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-24*I*a**3*d**2*exp(5*I*c)*exp(5*I*d*x) - 80*I*a**3*d**2*exp(3*
I*c)*exp(3*I*d*x) - 120*I*a**3*d**2*exp(I*c)*exp(I*d*x))/(480*d**3), Ne(480
*d**3, 0)), (x*(a**3*exp(5*I*c)/4 + a**3*exp(3*I*c)/2 + a**3*exp(I*c)/4), T
rue))
```

Giac [B] time = 1.50648, size = 1254, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/7680*(1785*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7140*a^3*
e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7140*a^3*e^(2*I*d*x - 2*I*
c)*log(I*e^(I*d*x + I*c) + 1) + 10710*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c)
 + 1) + 1785*a^3*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1530*a^3*e^(8*I*d*
x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 6120*a^3*e^(6*I*d*x + 2*I*c)*log(I*
e^(I*d*x + I*c) - 1) + 6120*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 9180*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 1530*a^3*e^(-4*I*c)
*log(I*e^(I*d*x + I*c) - 1) - 1785*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 7140*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 71
40*a^3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 10710*a^3*e^(4*I*d
*x)*log(-I*e^(I*d*x + I*c) + 1) - 1785*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c
) + 1) - 1530*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6120*a^
3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6120*a^3*e^(2*I*d*x - 2
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9180*a^3*e^(4*I*d*x)*log(-I*e^(I*d*x +
I*c) - 1) - 1530*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 255*a^3*e^(8*
I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1020*a^3*e^(6*I*d*x + 2*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) - 1020*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x) +
 e^(-I*c)) - 1530*a^3*e^(4*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 255*a^3*e^(
-4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 255*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^
```

```
(I*d*x) + e^(-I*c)) + 1020*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) + 1020*a^3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1530*a^3
*e^(4*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 255*a^3*e^(-4*I*c)*log(-I*e^(I*
d*x) + e^(-I*c)) - 384*I*a^3*e^(13*I*d*x + 9*I*c) - 2816*I*a^3*e^(11*I*d*x
+ 7*I*c) - 9344*I*a^3*e^(9*I*d*x + 5*I*c) - 16896*I*a^3*e^(7*I*d*x + 3*I*c)
 - 17024*I*a^3*e^(5*I*d*x + I*c) - 8960*I*a^3*e^(3*I*d*x - I*c) - 1920*I*a^
3*e^(I*d*x - 3*I*c))/(d*e^(8*I*d*x + 4*I*c) + 4*d*e^(6*I*d*x + 2*I*c) + 4*d
*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x) + d*e^(-4*I*c))
```

3.50 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=106

$$\frac{3a^3\sin^5(c+dx)}{35d}-\frac{2a^3\sin^3(c+dx)}{7d}+\frac{3a^3\sin(c+dx)}{7d}-\frac{3ia^3\cos^5(c+dx)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

```
[Out] (((-3*I)/35)*a^3*Cos[c + d*x]^5)/d + (3*a^3*Sin[c + d*x])/(7*d) - (2*a^3*Si
n[c + d*x]^3)/(7*d) + (3*a^3*Sin[c + d*x]^5)/(35*d) - (((2*I)/7)*a*Cos[c +
d*x]^7*(a + I*a*Tan[c + d*x])^2)/d
```

Rubi [A] time = 0.0757118, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3496, 3486, 2633}

$$\frac{3a^3\sin^5(c+dx)}{35d}-\frac{2a^3\sin^3(c+dx)}{7d}+\frac{3a^3\sin(c+dx)}{7d}-\frac{3ia^3\cos^5(c+dx)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-3*I)/35)*a^3*Cos[c + d*x]^5)/d + (3*a^3*Sin[c + d*x])/(7*d) - (2*a^3*Si
n[c + d*x]^3)/(7*d) + (3*a^3*Sin[c + d*x]^5)/(35*d) - (((2*I)/7)*a*Cos[c +
d*x]^7*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d} + \frac{1}{7}\left(3a^2\right)\int \cos^5(c+dx)(a+ia\tan$$
$$= -\frac{3ia^3\cos^5(c+dx)}{35d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d} + \frac{1}{7}\left(3a^3\right)\int \cos$$
$$= -\frac{3ia^3\cos^5(c+dx)}{35d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d} - \frac{\left(3a^3\right)\text{Subst}\,(}{}$$
$$= -\frac{3ia^3\cos^5(c+dx)}{35d} + \frac{3a^3\sin(c+dx)}{7d} - \frac{2a^3\sin^3(c+dx)}{7d} + \frac{3a^3\sin^5(c+dx)}{35d}$$

Mathematica [A] time = 0.573259, size = 77, normalized size = 0.73

$$\frac{a^3(\sin(3(c+dx)) - i\cos(3(c+dx)))(-56i\sin(2(c+dx)) + 20i\sin(4(c+dx)) + 84\cos(2(c+dx)) - 15\cos(4(c+dx))}{280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(35 + 84*Cos[2*(c + d*x)] -
 15*Cos[4*(c + d*x)] - (56*I)*Sin[2*(c + d*x)] + (20*I)*Sin[4*(c + d*x)]))/
(280*d)
```

Maple [A] time = 0.062, size = 146, normalized size = 1.4

$$\frac{1}{d}\left(-ia^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^5}{7} - \frac{2(\cos(dx+c))^5}{35}\right) - 3a^3\left(-1/7(\cos(dx+c))^6\sin(dx+c) + 1/35\left(8/3 + (c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-I*a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)-3*a^3*(-1/7*
cos(d*x+c)^6*sin(d*x+c)+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
)-3/7*I*a^3*cos(d*x+c)^7+1/7*a^3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*co
s(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.15661, size = 166, normalized size = 1.57

$$-\frac{15i\,a^3\cos(dx+c)^7 + i\left(5\cos(dx+c)^7 - 7\cos(dx+c)^5\right)a^3 + \left(15\sin(dx+c)^7 - 42\sin(dx+c)^5 + 35\sin(dx+c)^3\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35*(15*I*a^3*cos(d*x + c)^7 + I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^
3 + (15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^3 + (5*si
n(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3
)/d
```

Fricas [A] time = 1.07833, size = 219, normalized size = 2.07

$$\frac{\left(-5i\,a^3e^{(8i\,dx+8i\,c)}-28i\,a^3e^{(6i\,dx+6i\,c)}-70i\,a^3e^{(4i\,dx+4i\,c)}-140i\,a^3e^{(2i\,dx+2i\,c)}+35i\,a^3\right)e^{(-i\,dx-i\,c)}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(-5*I*a^3*e^(8*I*d*x + 8*I*c) - 28*I*a^3*e^(6*I*d*x + 6*I*c) - 70*I*a
^3*e^(4*I*d*x + 4*I*c) - 140*I*a^3*e^(2*I*d*x + 2*I*c) + 35*I*a^3)*e^(-I*d*
x - I*c)/d
```

Sympy [A] time = 1.02439, size = 192, normalized size = 1.81

$$\begin{cases}\frac{\left(-10240ia^3d^4e^{8ic}e^{7idx}-57344ia^3d^4e^{6ic}e^{5idx}-143360ia^3d^4e^{4ic}e^{3idx}-286720ia^3d^4e^{2ic}e^{idx}+71680ia^3d^4e^{-idx}\right)e^{-ic}}{1146880d^5} & \text{for } 1146880d^5e^{ic}\neq 0\\ \frac{x\left(a^3e^{8ic}+4a^3e^{6ic}+6a^3e^{4ic}+4a^3e^{2ic}+a^3\right)e^{-ic}}{16} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-10240*I*a**3*d**4*exp(8*I*c)*exp(7*I*d*x) - 57344*I*a**3*d**4*
exp(6*I*c)*exp(5*I*d*x) - 143360*I*a**3*d**4*exp(4*I*c)*exp(3*I*d*x) - 2867
20*I*a**3*d**4*exp(2*I*c)*exp(I*d*x) + 71680*I*a**3*d**4*exp(-I*d*x))*exp(-
I*c)/(1146880*d**5), Ne(1146880*d**5*exp(I*c), 0)), (x*(a**3*exp(8*I*c) + 4
*a**3*exp(6*I*c) + 6*a**3*exp(4*I*c) + 4*a**3*exp(2*I*c) + a**3)*exp(-I*c)/
16, True))
```

Giac [B] time = 1.51986, size = 628, normalized size = 5.92

$$19635\,a^3e^{(5i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+39270\,a^3e^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+19635\,a^3e^{(i\,dx-i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+196$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/71680*(19635*a^3*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 39270*a
^3*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 19635*a^3*e^(I*d*x - I*c)
*log(I*e^(I*d*x + I*c) + 1) + 19635*a^3*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x
+ I*c) - 1) + 39270*a^3*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 1963
5*a^3*e^(I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 19635*a^3*e^(5*I*d*x + 3
*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 39270*a^3*e^(3*I*d*x + I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 19635*a^3*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) -
19635*a^3*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 39270*a^3*e^(3*
I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 19635*a^3*e^(I*d*x - I*c)*log(-I
*e^(I*d*x + I*c) - 1) - 640*I*a^3*e^(12*I*d*x + 10*I*c) - 4864*I*a^3*e^(10*
I*d*x + 8*I*c) - 16768*I*a^3*e^(8*I*d*x + 6*I*c) - 39424*I*a^3*e^(6*I*d*x +
 4*I*c) - 40320*I*a^3*e^(4*I*d*x + 2*I*c) - 8960*I*a^3*e^(2*I*d*x) + 4480*I
*a^3*e^(-2*I*c))/(d*e^(5*I*d*x + 3*I*c) + 2*d*e^(3*I*d*x + I*c) + d*e^(I*d*
x - I*c))
```

3.51 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=124

$$-\frac{5a^3\sin^7(c+dx)}{63d}+\frac{a^3\sin^5(c+dx)}{3d}-\frac{5a^3\sin^3(c+dx)}{9d}+\frac{5a^3\sin(c+dx)}{9d}-\frac{5ia^3\cos^7(c+dx)}{63d}-\frac{2ia\cos^9(c+dx)(a+}{9d}$$

```
[Out] (((-5*I)/63)*a^3*Cos[c + d*x]^7)/d + (5*a^3*Sin[c + d*x])/(9*d) - (5*a^3*Si
n[c + d*x]^3)/(9*d) + (a^3*Sin[c + d*x]^5)/(3*d) - (5*a^3*Sin[c + d*x]^7)/(
63*d) - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2)/d
```

Rubi [A] time = 0.083663, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3496, 3486, 2633}

$$-\frac{5a^3\sin^7(c+dx)}{63d}+\frac{a^3\sin^5(c+dx)}{3d}-\frac{5a^3\sin^3(c+dx)}{9d}+\frac{5a^3\sin(c+dx)}{9d}-\frac{5ia^3\cos^7(c+dx)}{63d}-\frac{2ia\cos^9(c+dx)(a+}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-5*I)/63)*a^3*Cos[c + d*x]^7)/d + (5*a^3*Sin[c + d*x])/(9*d) - (5*a^3*Si
n[c + d*x]^3)/(9*d) + (a^3*Sin[c + d*x]^5)/(3*d) - (5*a^3*Sin[c + d*x]^7)/(
63*d) - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d} + \frac{1}{9}\left(5a^2\right)\int \cos^7(c+dx)(a+ia\tan(c$$
$$= -\frac{5ia^3\cos^7(c+dx)}{63d} - \frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d} + \frac{1}{9}\left(5a^3\right)\int \cos^7(c$$
$$= -\frac{5ia^3\cos^7(c+dx)}{63d} - \frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d} - \frac{\left(5a^3\right)\text{Subst}\left(\int (\right.}{}$$
$$= -\frac{5ia^3\cos^7(c+dx)}{63d} + \frac{5a^3\sin(c+dx)}{9d} - \frac{5a^3\sin^3(c+dx)}{9d} + \frac{a^3\sin^5(c+dx)}{3d} - 5$$

Mathematica [A] time = 0.651795, size = 116, normalized size = 0.94

$$\frac{a^3(-378i\sin(2(c+dx))+216i\sin(4(c+dx))+14i\sin(6(c+dx))+567\cos(2(c+dx))-162\cos(4(c+dx))-7\cos(6(c+}{2016d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(210 + 567*Cos[2*(c + d*x)] - 162*Cos[4*(c + d*x)] - 7*Cos[6*(c + d*x)
] - (378*I)*Sin[2*(c + d*x)] + (216*I)*Sin[4*(c + d*x)] + (14*I)*Sin[6*(c +
 d*x)])*((-I)*Cos[3*(c + 2*d*x)] + Sin[3*(c + 2*d*x)]))/(2016*d*(Cos[d*x] +
 I*Sin[d*x])^3)
```

Maple [A] time = 0.063, size = 166, normalized size = 1.3

$$\frac{1}{d}\left(-ia^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^7}{9} - \frac{2(\cos(dx+c))^7}{63}\right) - 3a^3\left(-1/9\sin(dx+c)(\cos(dx+c))^8 + \frac{\sin(dx+c)}{63}\left(\frac{16}{5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-I*a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)-3*a^3*(-1/9*
sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*
x+c)^2)*sin(d*x+c))-1/3*I*a^3*cos(d*x+c)^9+1/9*a^3*(128/35+cos(d*x+c)^8+8/7
*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.20519, size = 196, normalized size = 1.58

$$-\frac{105ia^3\cos(dx+c)^9 + 5i\left(7\cos(dx+c)^9 - 9\cos(dx+c)^7\right)a^3 - 3\left(35\sin(dx+c)^9 - 135\sin(dx+c)^7 + 189\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/315*(105*I*a^3*cos(d*x + c)^9 + 5*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7
)*a^3 - 3*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 10
5*sin(d*x + c)^3)*a^3 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d
*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^3)/d
```

Fricas [A] time = 1.18284, size = 319, normalized size = 2.57

$$\frac{\left(-7i\,a^3e^{(12i\,dx+12i\,c)}-54i\,a^3e^{(10i\,dx+10i\,c)}-189i\,a^3e^{(8i\,dx+8i\,c)}-420i\,a^3e^{(6i\,dx+6i\,c)}-945i\,a^3e^{(4i\,dx+4i\,c)}+378i\,a^3e^{(2i\,dx+2i\,c)}+\cdots\right.}{4032\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4032*(-7*I*a^3*e^(12*I*d*x + 12*I*c) - 54*I*a^3*e^(10*I*d*x + 10*I*c) - 1
89*I*a^3*e^(8*I*d*x + 8*I*c) - 420*I*a^3*e^(6*I*d*x + 6*I*c) - 945*I*a^3*e^
(4*I*d*x + 4*I*c) + 378*I*a^3*e^(2*I*d*x + 2*I*c) + 21*I*a^3)*e^(-3*I*d*x -
 3*I*c)/d
```

Sympy [A] time = 1.74558, size = 277, normalized size = 2.23

$$\begin{cases} \frac{\left(-270582939648ia^3d^6e^{13ic}e^{9idx}-2087354105856ia^3d^6e^{11ic}e^{7idx}-7305739370496ia^3d^6e^{9ic}e^{5idx}-16234976378880ia^3d^6e^{7ic}e^{3idx}-36528696852480ia^3d^6e^{5ic}e^{idx}\cdots\right.}{155855773237248d^7} \\ \frac{x\left(a^3e^{12ic}+6a^3e^{10ic}+15a^3e^{8ic}+20a^3e^{6ic}+15a^3e^{4ic}+6a^3e^{2ic}+a^3\right)e^{-3ic}}{64} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-270582939648*I*a**3*d**6*exp(13*I*c)*exp(9*I*d*x) - 2087354105
856*I*a**3*d**6*exp(11*I*c)*exp(7*I*d*x) - 7305739370496*I*a**3*d**6*exp(9*
I*c)*exp(5*I*d*x) - 16234976378880*I*a**3*d**6*exp(7*I*c)*exp(3*I*d*x) - 36
528696852480*I*a**3*d**6*exp(5*I*c)*exp(I*d*x) + 14611478740992*I*a**3*d**6
*exp(3*I*c)*exp(-I*d*x) + 811748818944*I*a**3*d**6*exp(I*c)*exp(-3*I*d*x))*
exp(-4*I*c)/(155855773237248*d**7), Ne(155855773237248*d**7*exp(4*I*c), 0))
, (x*(a**3*exp(12*I*c) + 6*a**3*exp(10*I*c) + 15*a**3*exp(8*I*c) + 20*a**3*
exp(6*I*c) + 15*a**3*exp(4*I*c) + 6*a**3*exp(2*I*c) + a**3)*exp(-3*I*c)/64,
 True))
```

Giac [B] time = 1.62819, size = 1403, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/516096*(119511*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4780
44*a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 717066*a^3*e^(7*I*d
*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 478044*a^3*e^(5*I*d*x - I*c)*log(I*e
^(I*d*x + I*c) + 1) + 119511*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c)
+ 1) + 128898*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 515592*
a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 773388*a^3*e^(7*I*d*x
+ I*c)*log(I*e^(I*d*x + I*c) - 1) + 515592*a^3*e^(5*I*d*x - I*c)*log(I*e^(I
*d*x + I*c) - 1) + 128898*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1
) - 119511*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 478044*a^
3*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 717066*a^3*e^(7*I*d*x +
```

```
I*c)*log(-I*e^(I*d*x + I*c) + 1) - 478044*a^3*e^(5*I*d*x - I*c)*log(-I*e^(
I*d*x + I*c) + 1) - 119511*a^3*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x + I*c) +
 1) - 128898*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 515592*
a^3*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 773388*a^3*e^(7*I*d*x
 + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 515592*a^3*e^(5*I*d*x - I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 128898*a^3*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x + I*c)
 - 1) + 9387*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 37548*a
^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 56322*a^3*e^(7*I*d*x +
 I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 37548*a^3*e^(5*I*d*x - I*c)*log(I*e^(I*
d*x) + e^(-I*c)) + 9387*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x) + e^(-I*c))
 - 9387*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 37548*a^3*e
^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 56322*a^3*e^(7*I*d*x + I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) - 37548*a^3*e^(5*I*d*x - I*c)*log(-I*e^(I*d
*x) + e^(-I*c)) - 9387*a^3*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c))
 - 896*I*a^3*e^(20*I*d*x + 14*I*c) - 10496*I*a^3*e^(18*I*d*x + 12*I*c) - 57
216*I*a^3*e^(16*I*d*x + 10*I*c) - 195584*I*a^3*e^(14*I*d*x + 8*I*c) - 50969
6*I*a^3*e^(12*I*d*x + 6*I*c) - 861696*I*a^3*e^(10*I*d*x + 4*I*c) - 768768*I
*a^3*e^(8*I*d*x + 2*I*c) + 88704*I*a^3*e^(4*I*d*x - 2*I*c) + 59136*I*a^3*e^
(2*I*d*x - 4*I*c) - 236544*I*a^3*e^(6*I*d*x) + 2688*I*a^3*e^(-6*I*c))/(d*e^
(11*I*d*x + 5*I*c) + 4*d*e^(9*I*d*x + 3*I*c) + 6*d*e^(7*I*d*x + I*c) + 4*d*
e^(5*I*d*x - I*c) + d*e^(3*I*d*x - 3*I*c))
```

3.52 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=163

$$\frac{7ia^4\sec^3(c+dx)}{8d}+\frac{21a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d}+\frac{21i\sec^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{40d}$$

```
[Out] (21*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (((7*I)/8)*a^4*Sec[c + d*x]^3)/d +
(21*a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((I/6)*a*Sec[c + d*x]^3*(a + I*
a*Tan[c + d*x])^3)/d + (((3*I)/10)*Sec[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x]
)^2)/d + (((21*I)/40)*Sec[c + d*x]^3*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rubi [A] time = 0.161987, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3498, 3486, 3768, 3770}

$$\frac{7ia^4\sec^3(c+dx)}{8d}+\frac{21a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d}+\frac{21i\sec^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (21*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (((7*I)/8)*a^4*Sec[c + d*x]^3)/d +
(21*a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((I/6)*a*Sec[c + d*x]^3*(a + I*
a*Tan[c + d*x])^3)/d + (((3*I)/10)*Sec[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x]
)^2)/d + (((21*I)/40)*Sec[c + d*x]^3*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d} + \frac{1}{2}(3a)\int \sec^3(c+dx)(a+ia\tan(c+dx)$$

$$= \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d} + \frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d} +$$

$$= \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d} + \frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d} +$$

$$= \frac{7ia^4\sec^3(c+dx)}{8d} + \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d} + \frac{3i\sec^3(c+dx)\left(a^2+\right.}{10}$$

$$= \frac{7ia^4\sec^3(c+dx)}{8d} + \frac{21a^4\sec(c+dx)\tan(c+dx)}{16d} + \frac{ia\sec^3(c+dx)(a+ia\tan(}{6d}$$

$$= \frac{21a^4\tanh^{-1}(\sin(c+dx))}{16d} + \frac{7ia^4\sec^3(c+dx)}{8d} + \frac{21a^4\sec(c+dx)\tan(c+dx)}{16d} +$$

Mathematica [A] time = 1.78356, size = 171, normalized size = 1.05

$$-\frac{a^4(\cos(4c)-i\sin(4c))(\tan(c+dx)-i)^4\sec^2(c+dx)\left(-4608i\cos(c+dx)+5(90\sin(c+dx)+155\sin(3(c+dx))-63\,\mathrm{s}\right.}{38}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -(a^4*Sec[c + d*x]^2*(Cos[4*c] - I*Sin[4*c])*((-4608*I)*Cos[c + d*x] + 5040
*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + 5*((-512*I)*Cos[3*(c + d*x)] + 90*Sin[c + d*x]
 + 155*Sin[3*(c + d*x)] - 63*Sin[5*(c + d*x)]))*(-I + Tan[c + d*x])^4)/(384
0*d*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [B] time = 0.064, size = 324, normalized size = 2.

$$\frac{a^4(\sin(dx+c))^5}{6d(\cos(dx+c))^6} + \frac{a^4(\sin(dx+c))^5}{24d(\cos(dx+c))^4} - \frac{a^4(\sin(dx+c))^5}{48d(\cos(dx+c))^2} - \frac{a^4(\sin(dx+c))^3}{48d} - \frac{13a^4\sin(dx+c)}{16d} + \frac{21a^4\ln(\sec(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/6/d*a^4*sin(d*x+c)^5/cos(d*x+c)^6+1/24/d*a^4*sin(d*x+c)^5/cos(d*x+c)^4-1/
48/d*a^4*sin(d*x+c)^5/cos(d*x+c)^2-1/48*a^4*sin(d*x+c)^3/d-13/16*a^4*sin(d*
x+c)/d+21/16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+8/15*I/d*a^4*cos(d*x+c)+4/15*I
/d*a^4*sin(d*x+c)^4/cos(d*x+c)-4/15*I/d*a^4*sin(d*x+c)^4/cos(d*x+c)^3+4/15*
I/d*a^4*cos(d*x+c)*sin(d*x+c)^2-4/5*I/d*a^4*sin(d*x+c)^4/cos(d*x+c)^5-3/2/d
*a^4*sin(d*x+c)^3/cos(d*x+c)^4-3/4/d*a^4*sin(d*x+c)^3/cos(d*x+c)^2+4/3*I/d*
a^4/cos(d*x+c)^3+1/2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

Maxima [A] time = 1.10252, size = 332, normalized size = 2.04

$$-\frac{5\,a^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+180\,a^4\left(\frac{2\left(\sin(dx+c)^3+\sin\cdots\right)}{\sin(dx+c)^4-2\,\sin(\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(5*a^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) + 180*a^4*(2*(sin(d*x + c)^3 + sin(d*x +
c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(s
in(d*x + c) - 1)) + 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) - 640*I*a^4/cos(d*x + c)^3 - 128*I*(
5*cos(d*x + c)^2 - 3)*a^4/cos(d*x + c)^5)/d
```

Fricas [B] time = 1.21524, size = 1064, normalized size = 6.53

$$\frac{-630i\,a^4e^{(11i\,dx+11i\,c)}+6670i\,a^4e^{(9i\,dx+9i\,c)}+10116i\,a^4e^{(7i\,dx+7i\,c)}+8316i\,a^4e^{(5i\,dx+5i\,c)}+3570i\,a^4e^{(3i\,dx+3i\,c)}+630i\,a^4e^{(i\,dx\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(-630*I*a^4*e^(11*I*d*x + 11*I*c) + 6670*I*a^4*e^(9*I*d*x + 9*I*c) +
10116*I*a^4*e^(7*I*d*x + 7*I*c) + 8316*I*a^4*e^(5*I*d*x + 5*I*c) + 3570*I*a
^4*e^(3*I*d*x + 3*I*c) + 630*I*a^4*e^(I*d*x + I*c) + 315*(a^4*e^(12*I*d*x +
 12*I*c) + 6*a^4*e^(10*I*d*x + 10*I*c) + 15*a^4*e^(8*I*d*x + 8*I*c) + 20*a^
4*e^(6*I*d*x + 6*I*c) + 15*a^4*e^(4*I*d*x + 4*I*c) + 6*a^4*e^(2*I*d*x + 2*I
*c) + a^4)*log(e^(I*d*x + I*c) + I) - 315*(a^4*e^(12*I*d*x + 12*I*c) + 6*a^
4*e^(10*I*d*x + 10*I*c) + 15*a^4*e^(8*I*d*x + 8*I*c) + 20*a^4*e^(6*I*d*x +
6*I*c) + 15*a^4*e^(4*I*d*x + 4*I*c) + 6*a^4*e^(2*I*d*x + 2*I*c) + a^4)*log(
e^(I*d*x + I*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c)
+ 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4
*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int -6\tan^2\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int \tan^4\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int 4i\tan\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int -\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(-6*tan(c + d*x)**2*sec(c + d*x)**3, x) + Integral(tan(c + d*
x)**4*sec(c + d*x)**3, x) + Integral(4*I*tan(c + d*x)*sec(c + d*x)**3, x) +
 Integral(-4*I*tan(c + d*x)**3*sec(c + d*x)**3, x) + Integral(sec(c + d*x)*
*3, x))
```

Giac [A] time = 1.35276, size = 323, normalized size = 1.98

$$315\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-315\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(75\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+960i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+1175\,a^4\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(315*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 315*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(75*a^4*tan(1/2*d*x + 1/2*c)^11 + 960*I*a^4*tan(1/2
*d*x + 1/2*c)^10 + 1175*a^4*tan(1/2*d*x + 1/2*c)^9 - 4800*I*a^4*tan(1/2*d*x
 + 1/2*c)^8 - 1890*a^4*tan(1/2*d*x + 1/2*c)^7 + 4480*I*a^4*tan(1/2*d*x + 1/
2*c)^6 - 1890*a^4*tan(1/2*d*x + 1/2*c)^5 - 1920*I*a^4*tan(1/2*d*x + 1/2*c)^
4 + 1175*a^4*tan(1/2*d*x + 1/2*c)^3 + 1728*I*a^4*tan(1/2*d*x + 1/2*c)^2 + 7
5*a^4*tan(1/2*d*x + 1/2*c) - 448*I*a^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.53 $\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=133

$$\frac{35ia^4\sec(c+dx)}{8d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d}+\frac{35i\sec(c+dx)\left(a^4+ia^4\tan\right.}{24d}$$

```
[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (((35*I)/8)*a^4*Sec[c + d*x])/d + ((
I/4)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d + (((7*I)/12)*Sec[c + d*x]*
(a^2 + I*a^2*Tan[c + d*x])^2)/d + (((35*I)/24)*Sec[c + d*x]*(a^4 + I*a^4*Ta
n[c + d*x]))/d
```

Rubi [A] time = 0.0956145, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3498, 3486, 3770}

$$\frac{35ia^4\sec(c+dx)}{8d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d}+\frac{35i\sec(c+dx)\left(a^4+ia^4\tan\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (((35*I)/8)*a^4*Sec[c + d*x])/d + ((
I/4)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d + (((7*I)/12)*Sec[c + d*x]*
(a^2 + I*a^2*Tan[c + d*x])^2)/d + (((35*I)/24)*Sec[c + d*x]*(a^4 + I*a^4*Ta
n[c + d*x]))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{1}{4}(7a)\int \sec(c+dx)(a+ia\tan(c+dx))^3\,d$$
$$= \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d} + \frac{1}{12}$$
$$= \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d} + \frac{35}{}$$
$$= \frac{35ia^4\sec(c+dx)}{8d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{7i\sec(c+dx)\left(a^2+ia\right.}{12d}$$
$$= \frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d} + \frac{35ia^4\sec(c+dx)}{8d} + \frac{ia\sec(c+dx)(a+ia\tan(c+d}{4d}$$

Mathematica [A] time = 1.21507, size = 237, normalized size = 1.78

$$-\frac{a^4\sec^4(c+dx)\left(3\left(42\sin(c+dx)+58\sin(3(c+dx))-128i\cos(3(c+dx))+35\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\text{s}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -(a^4*Sec[c + d*x]^4*((-896*I)*Cos[c + d*x] + 3*((-128*I)*Cos[3*(c + d*x)]
+ 105*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 35*Cos[4*(c + d*x)]*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] + 140*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 105*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 35*Cos[4*(c + d*x)]*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 42*Sin[c + d*x] + 58*Sin[3*(c + d*x)]))))/(19
2*d)
```

Maple [A] time = 0.028, size = 231, normalized size = 1.7

$$\frac{a^4(\sin(dx+c))^5}{4d(\cos(dx+c))^4} - \frac{a^4(\sin(dx+c))^5}{8d(\cos(dx+c))^2} - \frac{a^4(\sin(dx+c))^3}{8d} - \frac{27a^4\sin(dx+c)}{8d} + \frac{35a^4\ln(\sec(dx+c)+\tan(dx+c))}{8d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/4/d*a^4*sin(d*x+c)^5/cos(d*x+c)^4-1/8/d*a^4*sin(d*x+c)^5/cos(d*x+c)^2-1/8
*a^4*sin(d*x+c)^3/d-27/8*a^4*sin(d*x+c)/d+35/8/d*a^4*ln(sec(d*x+c)+tan(d*x+
c))-4/3*I/d*a^4*sin(d*x+c)^4/cos(d*x+c)^3+4/3*I/d*a^4*sin(d*x+c)^4/cos(d*x+
c)+4/3*I/d*a^4*cos(d*x+c)*sin(d*x+c)^2+8/3*I/d*a^4*cos(d*x+c)-3/d*a^4*sin(d
*x+c)^3/cos(d*x+c)^2+4*I/d*a^4/cos(d*x+c)
```

Maxima [A] time = 1.1867, size = 243, normalized size = 1.83

$$\frac{3a^4\left(\frac{2\left(5\sin(dx+c)^3-3\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}+3\log(\sin(dx+c)+1)-3\log(\sin(dx+c)-1)\right)+72a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) + 72*a
^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) + 48*a^4*log(sec(d*x + c) + tan(d*x + c)) + 192*I*a^4/cos(d*x
 + c) + 64*I*(3*cos(d*x + c)^2 - 1)*a^4/cos(d*x + c)^3)/d
```

Fricas [B] time = 1.19609, size = 717, normalized size = 5.39

$$\frac{558i\,a^4e^{(7i\,dx+7i\,c)}+1022i\,a^4e^{(5i\,dx+5i\,c)}+770i\,a^4e^{(3i\,dx+3i\,c)}+210i\,a^4e^{(i\,dx+i\,c)}+105\left(a^4e^{(8i\,dx+8i\,c)}+4\,a^4e^{(6i\,dx+6i\,c)}+6\,a^4e^{(}\right.}{24\left(de^{(8i\,dx+8i\,c)}+4\,de^{(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(558*I*a^4*e^(7*I*d*x + 7*I*c) + 1022*I*a^4*e^(5*I*d*x + 5*I*c) + 770*
I*a^4*e^(3*I*d*x + 3*I*c) + 210*I*a^4*e^(I*d*x + I*c) + 105*(a^4*e^(8*I*d*x
 + 8*I*c) + 4*a^4*e^(6*I*d*x + 6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) + 4*a^4*e
^(2*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) + I) - 105*(a^4*e^(8*I*d*x +
8*I*c) + 4*a^4*e^(6*I*d*x + 6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) + 4*a^4*e^(2
*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*c) + 4
*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
+ d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int -6\tan^2(c+dx)\sec(c+dx)\,dx+\int \tan^4(c+dx)\sec(c+dx)\,dx+\int 4i\tan(c+dx)\sec(c+dx)\,dx+\int -4i\,\mathrm{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(-6*tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)*
*4*sec(c + d*x), x) + Integral(4*I*tan(c + d*x)*sec(c + d*x), x) + Integral
(-4*I*tan(c + d*x)**3*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

Giac [A] time = 1.33787, size = 236, normalized size = 1.77

$$\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(81\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+96i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-105\,a^4\,\mathrm{ta}\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(81*a^4*tan(1/2*d*x + 1/2*c)^7 + 96*I*a^4*tan(1/2*d*
x + 1/2*c)^6 - 105*a^4*tan(1/2*d*x + 1/2*c)^5 - 480*I*a^4*tan(1/2*d*x + 1/2
*c)^4 - 105*a^4*tan(1/2*d*x + 1/2*c)^3 + 544*I*a^4*tan(1/2*d*x + 1/2*c)^2 +
 81*a^4*tan(1/2*d*x + 1/2*c) - 160*I*a^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.54 $\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=97

$$-\frac{15ia^4\sec(c+dx)}{2d}-\frac{15a^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{2d}-\frac{2ia\cos(c+dx)(a+ia\tan(c$$

```
[Out] (-15*a^4*ArcTanh[Sin[c + d*x]])/(2*d) - (((15*I)/2)*a^4*Sec[c + d*x])/d - (
(2*I)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d - (((5*I)/2)*Sec[c + d*x]*
(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rubi [A] time = 0.0744401, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3496, 3498, 3486, 3770}

$$-\frac{15ia^4\sec(c+dx)}{2d}-\frac{15a^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{2d}-\frac{2ia\cos(c+dx)(a+ia\tan(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-15*a^4*ArcTanh[Sin[c + d*x]])/(2*d) - (((15*I)/2)*a^4*Sec[c + d*x])/d - (
(2*I)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d - (((5*I)/2)*Sec[c + d*x]*
(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d} - \left(5a^2\right)\int \sec(c+dx)(a+ia\tan(c+dx))$$
$$= -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d} - \frac{5i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{2d} -$$
$$= -\frac{15ia^4\sec(c+dx)}{2d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d} - \frac{5i\sec(c+dx)\left(a^4+\right.}{2d}$$
$$= -\frac{15a^4\tanh^{-1}(\sin(c+dx))}{2d} - \frac{15ia^4\sec(c+dx)}{2d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+}{d}$$

Mathematica [B] time = 6.43177, size = 906, normalized size = 9.34

$$\frac{\cos^4(c+dx)(8\cos(3c)-8i\sin(3c))\sin(dx)(i\tan(c+dx)a+a)^4}{d(\cos(dx)+i\sin(dx))^4} - \frac{i\cos^4(c+dx)(4\cos(4c)-4i\sin(4c))\sin\left(\frac{dx}{2}\right)(i\tan}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^4\left(\cos\left(\frac{c}{2}+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (15*Cos[4*c]*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*(a
 + I*a*Tan[c + d*x])^4)/(2*d*(Cos[d*x] + I*Sin[d*x])^4) - (15*Cos[4*c]*Cos[
c + d*x]^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c + d*
x])^4)/(2*d*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[d*x]*Cos[c + d*x]^4*((-8*I)*C
os[3*c] - 8*Sin[3*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^
4) + (Cos[c + d*x]^4*Sec[c]*((-4*I)*Cos[4*c] - 4*Sin[4*c])*(a + I*a*Tan[c +
 d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) - (((15*I)/2)*Cos[c + d*x]^4*Log[Co
s[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[4*c]*(a + I*a*Tan[c + d*x])^4)/(
d*(Cos[d*x] + I*Sin[d*x])^4) + (((15*I)/2)*Cos[c + d*x]^4*Log[Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2]]*Sin[4*c]*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x]
 + I*Sin[d*x])^4) + (Cos[c + d*x]^4*(8*Cos[3*c] - (8*I)*Sin[3*c])*Sin[d*x]*
(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[c + d*x]^4*(
Cos[4*c]/4 - (I/4)*Sin[4*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin
[d*x])^4*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) - (I*Cos[c + d*x]^4*(
4*Cos[4*c] - (4*I)*Sin[4*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^4)/(d*(Cos
[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*x)/2] - Sin[c/2 +
 (d*x)/2])) + (Cos[c + d*x]^4*(-Cos[4*c]/4 + (I/4)*Sin[4*c])*(a + I*a*Tan[c
 + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d
*x)/2])^2) + (I*Cos[c + d*x]^4*(4*Cos[4*c] - (4*I)*Sin[4*c])*Sin[(d*x)/2]*(
a + I*a*Tan[c + d*x])^4)/(d*(Cos[c/2] + Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^4
*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.052, size = 141, normalized size = 1.5

$$\frac{a^4(\sin(dx+c))^5}{2d(\cos(dx+c))^2} + \frac{a^4(\sin(dx+c))^3}{2d} + \frac{17a^4\sin(dx+c)}{2d} - \frac{15a^4\ln(\sec(dx+c)+\tan(dx+c))}{2d} - \frac{4ia^4(\sin(dx+c))^4}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/2/d*a^4*sin(d*x+c)^5/cos(d*x+c)^2+1/2*a^4*sin(d*x+c)^3/d+17/2*a^4*sin(d*x
+c)/d-15/2/d*a^4*ln(sec(d*x+c)+tan(d*x+c))-4*I/d*a^4*sin(d*x+c)^4/cos(d*x+c
)-4*I/d*a^4*sin(d*x+c)^2*cos(d*x+c)-12*I/d*a^4*cos(d*x+c)
```

Maxima [A] time = 1.12994, size = 185, normalized size = 1.91

$$-\frac{a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+3\log\left(\sin\left(dx+c\right)+1\right)-3\log\left(\sin\left(dx+c\right)-1\right)-4\sin\left(dx+c\right)\right)+16i\,a^4\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/4*(a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 16*I*a^4*(1/cos(d*x + c) + cos(
d*x + c)) + 12*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d
*x + c)) + 16*I*a^4*cos(d*x + c) - 4*a^4*sin(d*x + c))/d
```

Fricas [B] time = 1.20279, size = 444, normalized size = 4.58

$$\frac{-16i\,a^4e^{(5i\,dx+5i\,c)}-50i\,a^4e^{(3i\,dx+3i\,c)}-30i\,a^4e^{(i\,dx+i\,c)}-15\left(a^4e^{(4i\,dx+4i\,c)}+2\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(i\,dx+i\,c)}+i\right)+15\left(a^4e\right.}{2\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2*(-16*I*a^4*e^(5*I*d*x + 5*I*c) - 50*I*a^4*e^(3*I*d*x + 3*I*c) - 30*I*a^
4*e^(I*d*x + I*c) - 15*(a^4*e^(4*I*d*x + 4*I*c) + 2*a^4*e^(2*I*d*x + 2*I*c)
 + a^4)*log(e^(I*d*x + I*c) + I) + 15*(a^4*e^(4*I*d*x + 4*I*c) + 2*a^4*e^(2
*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2
*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 1.34373, size = 153, normalized size = 1.58

$$\frac{15a^4\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d}+\frac{-\frac{9ia^4e^{-ic}e^{3idx}}{d}-\frac{7ia^4e^{-3ic}e^{idx}}{d}}{e^{4idx}+2e^{-2ic}e^{2idx}+e^{-4ic}}+\begin{cases}-\frac{8ia^4e^{ic}e^{idx}}{d} & \text{for } d\neq 0\\ 8a^4xe^{ic} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] 15*a**4*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2)
/d + (-9*I*a**4*exp(-I*c)*exp(3*I*d*x)/d - 7*I*a**4*exp(-3*I*c)*exp(I*d*x)/
d)/(exp(4*I*d*x) + 2*exp(-2*I*c)*exp(2*I*d*x) + exp(-4*I*c)) + Piecewise((-
8*I*a**4*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (8*a**4*x*exp(I*c), True))
```

Giac [B] time = 1.41703, size = 502, normalized size = 5.18

$$\frac{235\,a^{4}e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+470\,a^{4}e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)-5\,a^{4}e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-10\,a^{4}e^{(2i\,dx+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/32*(235*a^4*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 470*a^4*e^(2
*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 5*a^4*e^(4*I*d*x + 4*I*c)*log(
I*e^(I*d*x + I*c) - 1) - 10*a^4*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) -
 1) - 235*a^4*e^(4*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 470*a^4*e^(
2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) + 5*a^4*e^(4*I*d*x + 4*I*c)*lo
g(-I*e^(I*d*x + I*c) - 1) + 10*a^4*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*
c) - 1) - 256*I*a^4*e^(5*I*d*x + 5*I*c) - 800*I*a^4*e^(3*I*d*x + 3*I*c) - 4
80*I*a^4*e^(I*d*x + I*c) + 235*a^4*log(I*e^(I*d*x + I*c) + 1) - 5*a^4*log(I
*e^(I*d*x + I*c) - 1) - 235*a^4*log(-I*e^(I*d*x + I*c) + 1) + 5*a^4*log(-I*
e^(I*d*x + I*c) - 1))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

3.55 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=78

$$\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2i\cos(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/d - (((2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c
+ d*x])^3)/d + ((2*I)*Cos[c + d*x]*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rubi [A] time = 0.075611, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3496, 3770}

$$\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2i\cos(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/d - (((2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c
+ d*x])^3)/d + ((2*I)*Cos[c + d*x]*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}-a^2\int \cos(c+dx)(a+ia\tan(c+dx)\\ &= -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}+\frac{2i\cos(c+dx)\left(a^4+ia^4\tan(c+dx\right.}{d}\\ &= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}+\frac{2i\cos(c+}{}\end{aligned}$$

Mathematica [B] time = 0.485552, size = 246, normalized size = 3.15

$$\frac{a^4(\cos(c+dx)+i\sin(c+dx))^4\left(6i\sin(3c)\sin(dx)-2i\sin(c)\sin(3dx)-2\sin(c)\cos(3dx)+6\sin(3c)\cos(dx)+\cos(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*(-3*Cos[4*c]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Cos[4*c]*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*Cos[3*d*x]*Sin[c] + 6*Cos[d*x]*Si
n[3*c] + (3*I)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[4*c] - (3*I)*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[4*c] + Cos[3*c]*((6*I)*Cos[d*x]
- 6*Sin[d*x]) + (6*I)*Sin[3*c]*Sin[d*x] - (2*I)*Sin[c]*Sin[3*d*x] + 2*Cos[c
]*((-I)*Cos[3*d*x] + Sin[3*d*x]))*(Cos[c + d*x] + I*Sin[c + d*x])^4)/(3*d*(
Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A] time = 0.052, size = 130, normalized size = 1.7

$$-\frac{7\,a^4\,(\sin(dx+c))^3}{3\,d}-\frac{a^4\sin(dx+c)}{3\,d}+\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{\frac{4i}{3}a^4\cos(dx+c)\,(\sin(dx+c))^2}{d}+\frac{\frac{8i}{3}a^4\cos}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] -7/3*a^4*sin(d*x+c)^3/d-1/3*a^4*sin(d*x+c)/d+1/d*a^4*ln(sec(d*x+c)+tan(d*x+
c))+4/3*I/d*a^4*cos(d*x+c)*sin(d*x+c)^2+8/3*I/d*a^4*cos(d*x+c)-4/3*I/d*a^4*
cos(d*x+c)^3+1/3/d*sin(d*x+c)*cos(d*x+c)^2*a^4
```

Maxima [A] time = 1.08211, size = 163, normalized size = 2.09

$$-\frac{8i\,a^4\cos(dx+c)^3+12\,a^4\sin(dx+c)^3+8i\left(\cos(dx+c)^3-3\,\cos(dx+c)\right)a^4+\left(2\,\sin(dx+c)^3-3\,\log(\sin(dx+c)+\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(8*I*a^4*cos(d*x + c)^3 + 12*a^4*sin(d*x + c)^3 + 8*I*(cos(d*x + c)^3
- 3*cos(d*x + c))*a^4 + (2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log
(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^4 + 2*(sin(d*x + c)^3 - 3*sin(d*x +
c))*a^4)/d
```

Fricas [A] time = 1.19146, size = 176, normalized size = 2.26

$$\frac{-2i\,a^4e^{(3i\,dx+3i\,c)}+6i\,a^4e^{(i\,dx+i\,c)}+3\,a^4\log\left(e^{(i\,dx+i\,c)}+i\right)-3\,a^4\log\left(e^{(i\,dx+i\,c)}-i\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(-2*I*a^4*e^(3*I*d*x + 3*I*c) + 6*I*a^4*e^(I*d*x + I*c) + 3*a^4*log(e^(
I*d*x + I*c) + I) - 3*a^4*log(e^(I*d*x + I*c) - I))/d
```

Sympy [A] time = 0.735001, size = 110, normalized size = 1.41

$$\frac{a^4\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}+\begin{cases}\frac{-2ia^4de^{3ic}e^{3idx}+6ia^4de^{ic}e^{idx}}{3d^2} & \text{for } 3d^2\neq 0\\ x\left(2a^4e^{3ic}-2a^4e^{ic}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(-log(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d + P
iecewise(((-2*I*a**4*d*exp(3*I*c)*exp(3*I*d*x) + 6*I*a**4*d*exp(I*c)*exp(I*
d*x))/(3*d**2), Ne(3*d**2, 0)), (x*(2*a**4*exp(3*I*c) - 2*a**4*exp(I*c)), T
rue))
```

Giac [B] time = 1.65592, size = 1754, normalized size = 22.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/768*(1110*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6660*a^4*
e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 16650*a^4*e^(8*I*d*x + 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 16650*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*
d*x + I*c) + 1) + 6660*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 22200*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1110*a^4*e^(-6*I*c)*log
(I*e^(I*d*x + I*c) + 1) + 1875*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 11250*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 28125
*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 28125*a^4*e^(4*I*d*x
- 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 11250*a^4*e^(2*I*d*x - 4*I*c)*log(I*e
^(I*d*x + I*c) - 1) + 37500*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 18
75*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1110*a^4*e^(12*I*d*x + 6*I*c
)*log(-I*e^(I*d*x + I*c) + 1) - 6660*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 16650*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 16650*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 6660*a^4*e^(2
*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 22200*a^4*e^(6*I*d*x)*log(-I*
e^(I*d*x + I*c) + 1) - 1110*a^4*e^(-6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 18
75*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11250*a^4*e^(10*I
*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 28125*a^4*e^(8*I*d*x + 2*I*c)*l
og(-I*e^(I*d*x + I*c) - 1) - 28125*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x
+ I*c) - 1) - 11250*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3
7500*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 1875*a^4*e^(-6*I*c)*log(
-I*e^(I*d*x + I*c) - 1) - 3*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-
I*c)) - 18*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 45*a^4*e^
(8*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 45*a^4*e^(4*I*d*x - 2*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) - 18*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x) +
e^(-I*c)) - 60*a^4*e^(6*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 3*a^4*e^(-6*I*
c)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x
) + e^(-I*c)) + 18*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) +
45*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 45*a^4*e^(4*I*d*x
 - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 18*a^4*e^(2*I*d*x - 4*I*c)*log(-I*
e^(I*d*x) + e^(-I*c)) + 60*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 3
*a^4*e^(-6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 512*I*a^4*e^(15*I*d*x + 9*I*
```

```
c) - 1536*I*a^4*e^(13*I*d*x + 7*I*c) + 1536*I*a^4*e^(11*I*d*x + 5*I*c) + 12
800*I*a^4*e^(9*I*d*x + 3*I*c) + 23040*I*a^4*e^(7*I*d*x + I*c) + 19968*I*a^4
*e^(5*I*d*x - I*c) + 8704*I*a^4*e^(3*I*d*x - 3*I*c) + 1536*I*a^4*e^(I*d*x -
 5*I*c))/(d*e^(12*I*d*x + 6*I*c) + 6*d*e^(10*I*d*x + 4*I*c) + 15*d*e^(8*I*d
*x + 2*I*c) + 15*d*e^(4*I*d*x - 2*I*c) + 6*d*e^(2*I*d*x - 4*I*c) + 20*d*e^(
6*I*d*x) + d*e^(-6*I*c))
```

3.56 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=66

$$-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}$$

```
[Out] ((-I/15)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - ((I/5)*Cos[c + d*x]
^5*(a + I*a*Tan[c + d*x])^4)/d
```

Rubi [A] time = 0.0736642, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3497, 3488}

$$-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-I/15)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - ((I/5)*Cos[c + d*x]
^5*(a + I*a*Tan[c + d*x])^4)/d
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}+\frac{1}{5}a\int \cos^3(c+dx)(a+ia\tan(c+dx)\\ &= -\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}\end{aligned}$$

Mathematica [A] time = 0.355633, size = 50, normalized size = 0.76

$$\frac{a^4(4\cos(c+dx)-i\sin(c+dx))(\sin(4(c+dx))-i\cos(4(c+dx)))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*(4*Cos[c + d*x] - I*Sin[c + d*x])*((-I)*Cos[4*(c + d*x)] + Sin[4*(c +
d*x)]))/(15*d)
```

Maple [B] time = 0.063, size = 139, normalized size = 2.1

$$\frac{1}{d}\left(\frac{a^4(\sin(dx+c))^5}{5} - 4\,ia^4\left(-\frac{(\cos(dx+c))^3(\sin(dx+c))^2}{5} - \frac{2(\cos(dx+c))^3}{15}\right) - 6\,a^4\left(-1/5\sin(dx+c)(\cos(dx+c))^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/d*(1/5*a^4*sin(d*x+c)^5-4*I*a^4*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(
d*x+c)^3)-6*a^4*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x
+c))-4/5*I*a^4*cos(d*x+c)^5+1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin
(d*x+c))
```

Maxima [B] time = 1.14519, size = 159, normalized size = 2.41

$$-\frac{12i\,a^4\cos(dx+c)^5 - 3\,a^4\sin(dx+c)^5 + 4i\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a^4 - 6\left(3\sin(dx+c)^5 - 5\sin(dx+c)^3\right)a}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/15*(12*I*a^4*cos(d*x + c)^5 - 3*a^4*sin(d*x + c)^5 + 4*I*(3*cos(d*x + c)
^5 - 5*cos(d*x + c)^3)*a^4 - 6*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^4 -
(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4)/d
```

Fricas [A] time = 1.00192, size = 93, normalized size = 1.41

$$\frac{-3i\,a^4e^{(5i\,dx+5i\,c)} - 5i\,a^4e^{(3i\,dx+3i\,c)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(-3*I*a^4*e^(5*I*d*x + 5*I*c) - 5*I*a^4*e^(3*I*d*x + 3*I*c))/d
```

Sympy [A] time = 0.652151, size = 82, normalized size = 1.24

$$\begin{cases} \frac{-6ia^4de^{5ic}e^{5idx}-10ia^4de^{3ic}e^{3idx}}{60d^2} & \text{for}\;60d^2 \neq 0 \\ x\left(\frac{a^4e^{5ic}}{2} + \frac{a^4e^{3ic}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-6*I*a**4*d*exp(5*I*c)*exp(5*I*d*x) - 10*I*a**4*d*exp(3*I*c)*ex
p(3*I*d*x))/(60*d**2), Ne(60*d**2, 0)), (x*(a**4*exp(5*I*c)/2 + a**4*exp(3*
I*c)/2), True))
```

Giac [B] time = 1.6316, size = 1235, normalized size = 18.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/7680*(9075*a^4*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 36300*a^4
*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 36300*a^4*e^(2*I*d*x - 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 54450*a^4*e^(4*I*d*x)*log(I*e^(I*d*x + I*
c) + 1) + 9075*a^4*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9000*a^4*e^(8*I*
d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 36000*a^4*e^(6*I*d*x + 2*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 36000*a^4*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 54000*a^4*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 9000*a^4*e^(-4
*I*c)*log(I*e^(I*d*x + I*c) - 1) - 9075*a^4*e^(8*I*d*x + 4*I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 36300*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1
) - 36300*a^4*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 54450*a^4*e
^(4*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 9075*a^4*e^(-4*I*c)*log(-I*e^(I*d*
x + I*c) + 1) - 9000*a^4*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) -
36000*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 36000*a^4*e^(2*
I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 54000*a^4*e^(4*I*d*x)*log(-I*e
^(I*d*x + I*c) - 1) - 9000*a^4*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 75*
a^4*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 300*a^4*e^(6*I*d*x +
2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 300*a^4*e^(2*I*d*x - 2*I*c)*log(I*e^(I
*d*x) + e^(-I*c)) - 450*a^4*e^(4*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 75*a^
4*e^(-4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 75*a^4*e^(8*I*d*x + 4*I*c)*log(-
I*e^(I*d*x) + e^(-I*c)) + 300*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^
(-I*c)) + 300*a^4*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 450*a^
4*e^(4*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 75*a^4*e^(-4*I*c)*log(-I*e^(I*
d*x) + e^(-I*c)) - 768*I*a^4*e^(13*I*d*x + 9*I*c) - 4352*I*a^4*e^(11*I*d*x
+ 7*I*c) - 9728*I*a^4*e^(9*I*d*x + 5*I*c) - 10752*I*a^4*e^(7*I*d*x + 3*I*c)
 - 5888*I*a^4*e^(5*I*d*x + I*c) - 1280*I*a^4*e^(3*I*d*x - I*c))/(d*e^(8*I*d
*x + 4*I*c) + 4*d*e^(6*I*d*x + 2*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*
I*d*x) + d*e^(-4*I*c))
```

3.57 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=102

$$-\frac{a^4\sin^3(c+dx)}{35d}+\frac{3a^4\sin(c+dx)}{35d}-\frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

```
[Out] (3*a^4*Sin[c + d*x])/(35*d) - (a^4*Sin[c + d*x]^3)/(35*d) - (((2*I)/7)*a*Co
s[c + d*x]^7*(a + I*a*Tan[c + d*x])^3)/d - (((2*I)/35)*Cos[c + d*x]^5*(a^4
+ I*a^4*Tan[c + d*x]))/d
```

Rubi [A] time = 0.0889054, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3496, 2633}

$$-\frac{a^4\sin^3(c+dx)}{35d}+\frac{3a^4\sin(c+dx)}{35d}-\frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (3*a^4*Sin[c + d*x])/(35*d) - (a^4*Sin[c + d*x]^3)/(35*d) - (((2*I)/7)*a*Co
s[c + d*x]^7*(a + I*a*Tan[c + d*x])^3)/d - (((2*I)/35)*Cos[c + d*x]^5*(a^4
+ I*a^4*Tan[c + d*x]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx=-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}+\frac{1}{7}a^2\int \cos^5(c+dx)(a+ia\tan(c+dx)$$

$$=-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}-\frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d}$$

$$=-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}-\frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d}$$

$$=\frac{3a^4\sin(c+dx)}{35d}-\frac{a^4\sin^3(c+dx)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}-\frac{2i}{}$$

Mathematica [A] time = 0.683372, size = 73, normalized size = 0.72

$$\frac{a^4(-i(7\sin(c+dx)+15\sin(3(c+dx)))+28\cos(c+dx)+20\cos(3(c+dx)))(\sin(4(c+dx))-i\cos(4(c+dx)))}{140d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*(28*Cos[c + d*x] + 20*Cos[3*(c + d*x)] - I*(7*Sin[c + d*x] + 15*Sin[3*
(c + d*x)]))*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)]))/(140*d)
```

Maple [B] time = 0.065, size = 203, normalized size = 2.

$$\frac{1}{d}\left(a^4\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^4}{7}-\frac{3\sin(dx+c)(\cos(dx+c))^4}{35}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{35}\right)-4ia^4\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*
(2+cos(d*x+c)^2)*sin(d*x+c))-4*I*a^4*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*c
os(d*x+c)^5)-6*a^4*(-1/7*cos(d*x+c)^6*sin(d*x+c)+1/35*(8/3+cos(d*x+c)^4+4/3
*cos(d*x+c)^2)*sin(d*x+c))-4/7*I*a^4*cos(d*x+c)^7+1/7*a^4*(16/5+cos(d*x+c)^
6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.10339, size = 201, normalized size = 1.97

$$-\frac{20ia^4\cos(dx+c)^7+4i\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^4+2\left(15\sin(dx+c)^7-42\sin(dx+c)^5+35\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/35*(20*I*a^4*cos(d*x + c)^7 + 4*I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*
a^4 + 2*(15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^4 + (
5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^4 + (5*sin(d*x + c)^7 - 21*sin(d*x +
 c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^4)/d
```

Fricas [A] time = 1.24938, size = 174, normalized size = 1.71

$$\frac{-5ia^4e^{(7idx+7ic)}-21ia^4e^{(5idx+5ic)}-35ia^4e^{(3idx+3ic)}-35ia^4e^{(idx+ic)}}{280d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/280*(-5*I*a^4*e^(7*I*d*x + 7*I*c) - 21*I*a^4*e^(5*I*d*x + 5*I*c) - 35*I*a
^4*e^(3*I*d*x + 3*I*c) - 35*I*a^4*e^(I*d*x + I*c))/d
```

Sympy [A] time = 1.06817, size = 158, normalized size = 1.55

$$\begin{cases} \frac{-2560ia^4d^3e^{7ic}e^{7idx}-10752ia^4d^3e^{5ic}e^{5idx}-17920ia^4d^3e^{3ic}e^{3idx}-17920ia^4d^3e^{ic}e^{idx}}{143360d^4} & \text{for } 143360d^4 \neq 0 \\ x\left(\frac{a^4e^{7ic}}{8}+\frac{3a^4e^{5ic}}{8}+\frac{3a^4e^{3ic}}{8}+\frac{a^4e^{ic}}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-2560*I*a**4*d**3*exp(7*I*c)*exp(7*I*d*x) - 10752*I*a**4*d**3*e
xp(5*I*c)*exp(5*I*d*x) - 17920*I*a**4*d**3*exp(3*I*c)*exp(3*I*d*x) - 17920*
I*a**4*d**3*exp(I*c)*exp(I*d*x))/(143360*d**4), Ne(143360*d**4, 0)), (x*(a*
*4*exp(7*I*c)/8 + 3*a**4*exp(5*I*c)/8 + 3*a**4*exp(3*I*c)/8 + a**4*exp(I*c)
/8), True))
```

Giac [B] time = 1.81071, size = 1791, normalized size = 17.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/143360*(89950*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 53970
0*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1349250*a^4*e^(8*I*
d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1349250*a^4*e^(4*I*d*x - 2*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 539700*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x +
 I*c) + 1) + 1799000*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 89950*a^4
*e^(-6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 86065*a^4*e^(12*I*d*x + 6*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 516390*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 1290975*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 12
90975*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 516390*a^4*e^(2*
I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1721300*a^4*e^(6*I*d*x)*log(I*e
^(I*d*x + I*c) - 1) + 86065*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c) - 1) - 899
50*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 539700*a^4*e^(10*
I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1349250*a^4*e^(8*I*d*x + 2*I*c
)*log(-I*e^(I*d*x + I*c) + 1) - 1349250*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 539700*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) +
1) - 1799000*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 89950*a^4*e^(-6*
I*c)*log(-I*e^(I*d*x + I*c) + 1) - 86065*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^
(I*d*x + I*c) - 1) - 516390*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c)
 - 1) - 1290975*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 12909
75*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 516390*a^4*e^(2*I*
d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1721300*a^4*e^(6*I*d*x)*log(-I*e
^(I*d*x + I*c) - 1) - 86065*a^4*e^(-6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 38
85*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23310*a^4*e^(10*I
*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 58275*a^4*e^(8*I*d*x + 2*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) - 58275*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*d*x)
+ e^(-I*c)) - 23310*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 7
7700*a^4*e^(6*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 3885*a^4*e^(-6*I*c)*log(
```

```
I*e^(I*d*x) + e^(-I*c)) + 3885*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x) +
e^(-I*c)) + 23310*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 5
8275*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 58275*a^4*e^(4*
I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 23310*a^4*e^(2*I*d*x - 4*I*c)
*log(-I*e^(I*d*x) + e^(-I*c)) + 77700*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x) + e^
(-I*c)) + 3885*a^4*e^(-6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 2560*I*a^4*e^(
19*I*d*x + 13*I*c) - 26112*I*a^4*e^(17*I*d*x + 11*I*c) - 120832*I*a^4*e^(15
*I*d*x + 9*I*c) - 337920*I*a^4*e^(13*I*d*x + 7*I*c) - 629760*I*a^4*e^(11*I*
d*x + 5*I*c) - 803840*I*a^4*e^(9*I*d*x + 3*I*c) - 694272*I*a^4*e^(7*I*d*x +
 I*c) - 387072*I*a^4*e^(5*I*d*x - I*c) - 125440*I*a^4*e^(3*I*d*x - 3*I*c) -
 17920*I*a^4*e^(I*d*x - 5*I*c))/(d*e^(12*I*d*x + 6*I*c) + 6*d*e^(10*I*d*x +
 4*I*c) + 15*d*e^(8*I*d*x + 2*I*c) + 15*d*e^(4*I*d*x - 2*I*c) + 6*d*e^(2*I*
d*x - 4*I*c) + 20*d*e^(6*I*d*x) + d*e^(-6*I*c))
```

3.58 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=120

$$\frac{a^4\sin^5(c+dx)}{21d}-\frac{10a^4\sin^3(c+dx)}{63d}+\frac{5a^4\sin(c+dx)}{21d}-\frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d}-\frac{2ia\cos^9(c+dx)(a+ia\tan$$

```
[Out] (5*a^4*Sin[c + d*x])/(21*d) - (10*a^4*Sin[c + d*x]^3)/(63*d) + (a^4*Sin[c +
 d*x]^5)/(21*d) - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3)/d -
 (((2*I)/21)*Cos[c + d*x]^7*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rubi [A] time = 0.100777, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3496, 2633}

$$\frac{a^4\sin^5(c+dx)}{21d}-\frac{10a^4\sin^3(c+dx)}{63d}+\frac{5a^4\sin(c+dx)}{21d}-\frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d}-\frac{2ia\cos^9(c+dx)(a+ia\tan$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (5*a^4*Sin[c + d*x])/(21*d) - (10*a^4*Sin[c + d*x]^3)/(63*d) + (a^4*Sin[c +
 d*x]^5)/(21*d) - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3)/d -
 (((2*I)/21)*Cos[c + d*x]^7*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx &= -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}+\frac{1}{3}a^2\int \cos^7(c+dx)(a+ia\tan(c+dx\\ &= -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}-\frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d}\\ &= -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}-\frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d}\\ &= \frac{5a^4\sin(c+dx)}{21d}-\frac{10a^4\sin^3(c+dx)}{63d}+\frac{a^4\sin^5(c+dx)}{21d}-\frac{2ia\cos^9(c+dx)(a+i\end{aligned}$$

Mathematica [A] time = 0.706612, size = 111, normalized size = 0.92

$$\frac{a^4(-42\sin(c+dx)-135\sin(3(c+dx))+35\sin(5(c+dx))-168i\cos(c+dx)-180i\cos(3(c+dx))+28i\cos(5(c+d}{1008d(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*((-168*I)*Cos[c + d*x] - (180*I)*Cos[3*(c + d*x)] + (28*I)*Cos[5*(c +
d*x)] - 42*Sin[c + d*x] - 135*Sin[3*(c + d*x)] + 35*Sin[5*(c + d*x)])*(Cos[
4*(c + 2*d*x)] + I*Sin[4*(c + 2*d*x)]))/(1008*d*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [B] time = 0.069, size = 233, normalized size = 1.9

$$\frac{1}{d}\left(a^4\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^6}{9}-\frac{(\cos(dx+c))^6\sin(dx+c)}{21}+\frac{\sin(dx+c)}{105}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx}{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*cos(d*x+c)^6*sin(d*x+c)+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-4*I*a^4*(-1/9*sin(d*x+c)^2
*cos(d*x+c)^7-2/63*cos(d*x+c)^7)-6*a^4*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(
16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-4/9*I*a^4*
cos(d*x+c)^9+1/9*a^4*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)
^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.06109, size = 244, normalized size = 2.03

$$-\frac{140i\,a^4\cos(dx+c)^9+20i\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^4-\left(35\sin(dx+c)^9-90\sin(dx+c)^7+63\sin(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/315*(140*I*a^4*cos(d*x + c)^9 + 20*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^
7)*a^4 - (35*sin(d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^4 -
6*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*
x + c)^3)*a^4 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)^
5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^4)/d
```

Fricas [A] time = 1.09222, size = 267, normalized size = 2.22

$$\frac{\left(-7i\,a^4e^{(10i\,dx+10i\,c)}-45i\,a^4e^{(8i\,dx+8i\,c)}-126i\,a^4e^{(6i\,dx+6i\,c)}-210i\,a^4e^{(4i\,dx+4i\,c)}-315i\,a^4e^{(2i\,dx+2i\,c)}+63i\,a^4\right)e^{(-i\,dx-i\,c)}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2016*(-7*I*a^4*e^(10*I*d*x + 10*I*c) - 45*I*a^4*e^(8*I*d*x + 8*I*c) - 126
*I*a^4*e^(6*I*d*x + 6*I*c) - 210*I*a^4*e^(4*I*d*x + 4*I*c) - 315*I*a^4*e^(2
*I*d*x + 2*I*c) + 63*I*a^4)*e^(-I*d*x - I*c)/d
```

Sympy [A] time = 1.26361, size = 230, normalized size = 1.92

$$\begin{cases} \frac{\left(-176160768ia^4d^5e^{10ic}e^{9idx}-1132462080ia^4d^5e^{8ic}e^{7idx}-3170893824ia^4d^5e^{6ic}e^{5idx}-5284823040ia^4d^5e^{4ic}e^{3idx}-7927234560ia^4d^5e^{2ic}e^{idx}+1585446912ia^4d^5e^{-idx}\right)}{50734301184d^6} \\ \frac{x\left(a^4e^{10ic}+5a^4e^{8ic}+10a^4e^{6ic}+10a^4e^{4ic}+5a^4e^{2ic}+a^4\right)e^{-ic}}{32} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-176160768*I*a**4*d**5*exp(10*I*c)*exp(9*I*d*x) - 1132462080*I*
a**4*d**5*exp(8*I*c)*exp(7*I*d*x) - 3170893824*I*a**4*d**5*exp(6*I*c)*exp(5
*I*d*x) - 5284823040*I*a**4*d**5*exp(4*I*c)*exp(3*I*d*x) - 7927234560*I*a**
4*d**5*exp(2*I*c)*exp(I*d*x) + 1585446912*I*a**4*d**5*exp(-I*d*x))*exp(-I*c
)/(50734301184*d**6), Ne(50734301184*d**6*exp(I*c), 0)), (x*(a**4*exp(10*I*
c) + 5*a**4*exp(8*I*c) + 10*a**4*exp(6*I*c) + 10*a**4*exp(4*I*c) + 5*a**4*e
xp(2*I*c) + a**4)*exp(-I*c)/32, True))
```

Giac [B] time = 1.88701, size = 1902, normalized size = 15.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/516096*(435267*a^4*e^(13*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2611
602*a^4*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6529005*a^4*e^(9*
I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 8705340*a^4*e^(7*I*d*x + I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 6529005*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*x +
I*c) + 1) + 2611602*a^4*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 43
5267*a^4*e^(I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 427896*a^4*e^(13*I*
d*x + 7*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2567376*a^4*e^(11*I*d*x + 5*I*c)*
log(I*e^(I*d*x + I*c) - 1) + 6418440*a^4*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x
 + I*c) - 1) + 8557920*a^4*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 6
418440*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) + 2567376*a^4*e^(3*
I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 427896*a^4*e^(I*d*x - 5*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) - 435267*a^4*e^(13*I*d*x + 7*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 2611602*a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 6529005*a^4*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 8705340*a^4
*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 6529005*a^4*e^(5*I*d*x - I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 2611602*a^4*e^(3*I*d*x - 3*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 435267*a^4*e^(I*d*x - 5*I*c)*log(-I*e^(I*d*x + I*c) +
1) - 427896*a^4*e^(13*I*d*x + 7*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2567376*
a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6418440*a^4*e^(9*I*d
*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 8557920*a^4*e^(7*I*d*x + I*c)*log
(-I*e^(I*d*x + I*c) - 1) - 6418440*a^4*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 2567376*a^4*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 4
27896*a^4*e^(I*d*x - 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7371*a^4*e^(13*I*
d*x + 7*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 44226*a^4*e^(11*I*d*x + 5*I*c)*l
```

```
og(I*e^(I*d*x) + e^(-I*c)) - 110565*a^4*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x)
 + e^(-I*c)) - 147420*a^4*e^(7*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1
10565*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 44226*a^4*e^(3*I*
d*x - 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 7371*a^4*e^(I*d*x - 5*I*c)*log(I
*e^(I*d*x) + e^(-I*c)) + 7371*a^4*e^(13*I*d*x + 7*I*c)*log(-I*e^(I*d*x) + e
^(-I*c)) + 44226*a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 11
0565*a^4*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 147420*a^4*e^(7
*I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 110565*a^4*e^(5*I*d*x - I*c)*l
og(-I*e^(I*d*x) + e^(-I*c)) + 44226*a^4*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x
) + e^(-I*c)) + 7371*a^4*e^(I*d*x - 5*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 1
792*I*a^4*e^(22*I*d*x + 16*I*c) - 22272*I*a^4*e^(20*I*d*x + 14*I*c) - 12825
6*I*a^4*e^(18*I*d*x + 12*I*c) - 455936*I*a^4*e^(16*I*d*x + 10*I*c) - 114432
0*I*a^4*e^(14*I*d*x + 8*I*c) - 2102784*I*a^4*e^(12*I*d*x + 6*I*c) - 2742784
*I*a^4*e^(10*I*d*x + 4*I*c) - 2382336*I*a^4*e^(8*I*d*x + 2*I*c) - 295680*I*
a^4*e^(4*I*d*x - 2*I*c) + 16128*I*a^4*e^(2*I*d*x - 4*I*c) - 1241856*I*a^4*e
^(6*I*d*x) + 16128*I*a^4*e^(-6*I*c))/(d*e^(13*I*d*x + 7*I*c) + 6*d*e^(11*I*
d*x + 5*I*c) + 15*d*e^(9*I*d*x + 3*I*c) + 20*d*e^(7*I*d*x + I*c) + 15*d*e^(
5*I*d*x - I*c) + 6*d*e^(3*I*d*x - 3*I*c) + d*e^(I*d*x - 5*I*c))
```

3.59 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=109

$$\frac{i(a+ia\tan(c+dx))^{12}}{12a^7d}-\frac{6i(a+ia\tan(c+dx))^{11}}{11a^6d}+\frac{6i(a+ia\tan(c+dx))^{10}}{5a^5d}-\frac{8i(a+ia\tan(c+dx))^9}{9a^4d}$$

```
[Out] (((-8*I)/9)*(a + I*a*Tan[c + d*x])^9)/(a^4*d) + (((6*I)/5)*(a + I*a*Tan[c +
 d*x])^10)/(a^5*d) - (((6*I)/11)*(a + I*a*Tan[c + d*x])^11)/(a^6*d) + ((I/1
2)*(a + I*a*Tan[c + d*x])^12)/(a^7*d)
```

Rubi [A] time = 0.073068, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a+ia\tan(c+dx))^{12}}{12a^7d}-\frac{6i(a+ia\tan(c+dx))^{11}}{11a^6d}+\frac{6i(a+ia\tan(c+dx))^{10}}{5a^5d}-\frac{8i(a+ia\tan(c+dx))^9}{9a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-8*I)/9)*(a + I*a*Tan[c + d*x])^9)/(a^4*d) + (((6*I)/5)*(a + I*a*Tan[c +
 d*x])^10)/(a^5*d) - (((6*I)/11)*(a + I*a*Tan[c + d*x])^11)/(a^6*d) + ((I/1
2)*(a + I*a*Tan[c + d*x])^12)/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^3(a+x)^8\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(8a^3(a+x)^8-12a^2(a+x)^9+6a(a+x)^{10}-(a+x)^{11}\right)dx,x,ia\tan\right.}{a^7d}\\ &= -\frac{8i(a+ia\tan(c+dx))^9}{9a^4d}+\frac{6i(a+ia\tan(c+dx))^{10}}{5a^5d}-\frac{6i(a+ia\tan(c+dx))^{11}}{11a^6d}\end{aligned}$$

Mathematica [A] time = 3.58874, size = 167, normalized size = 1.53

$$a^5\sec(c)\sec^{12}(c+dx)(792\sin(c+2dx)-792\sin(3c+2dx)+495\sin(3c+4dx)-495\sin(5c+4dx)+440\sin(5c+6dx$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c]*Sec[c + d*x]^12*((924*I)*Cos[c] + (792*I)*Cos[c + 2*d*x] + (792
*I)*Cos[3*c + 2*d*x] + (495*I)*Cos[3*c + 4*d*x] + (495*I)*Cos[5*c + 4*d*x]
- 924*Sin[c] + 792*Sin[c + 2*d*x] - 792*Sin[3*c + 2*d*x] + 495*Sin[3*c + 4*
d*x] - 495*Sin[5*c + 4*d*x] + 440*Sin[5*c + 6*d*x] + 132*Sin[7*c + 8*d*x] +
 24*Sin[9*c + 10*d*x] + 2*Sin[11*c + 12*d*x]))/(3960*d)
```

Maple [B] time = 0.083, size = 377, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(1/12*sin(d*x+c)^6/cos(d*x+c)^12+1/20*sin(d*x+c)^6/cos(d*x+c)^10
+1/40*sin(d*x+c)^6/cos(d*x+c)^8+1/120*sin(d*x+c)^6/cos(d*x+c)^6)+5*a^5*(1/1
1*sin(d*x+c)^5/cos(d*x+c)^11+2/33*sin(d*x+c)^5/cos(d*x+c)^9+8/231*sin(d*x+c
)^5/cos(d*x+c)^7+16/1155*sin(d*x+c)^5/cos(d*x+c)^5)-10*I*a^5*(1/10*sin(d*x+
c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x
+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4)-10*a^5*(1/9*sin(d*x+c)^3/cos(d*x+c)^9
+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(
d*x+c)^3/cos(d*x+c)^3)+5/8*I*a^5/cos(d*x+c)^8-a^5*(-16/35-1/7*sec(d*x+c)^6-
6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.12479, size = 216, normalized size = 1.98

$$\frac{2310i\,a^5\tan(dx+c)^{12} + 12600\,a^5\tan(dx+c)^{11} - 19404i\,a^5\tan(dx+c)^{10} + 15400\,a^5\tan(dx+c)^9 - 76230i\,a^5\tan(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/27720*(2310*I*a^5*tan(d*x + c)^12 + 12600*a^5*tan(d*x + c)^11 - 19404*I*a
^5*tan(d*x + c)^10 + 15400*a^5*tan(d*x + c)^9 - 76230*I*a^5*tan(d*x + c)^8
- 55440*a^5*tan(d*x + c)^7 - 64680*I*a^5*tan(d*x + c)^6 - 121968*a^5*tan(d*
x + c)^5 + 34650*I*a^5*tan(d*x + c)^4 - 64680*a^5*tan(d*x + c)^3 + 69300*I*
a^5*tan(d*x + c)^2 + 27720*a^5*tan(d*x + c))/d
```

Fricas [B] time = 1.19618, size = 883, normalized size = 8.1

$$\frac{506880i\,a^5e^{(16i\,dx+16i\,c)} + 811008i\,a^5e^{(14i\,dx+14i\,c)} + 946176i\,a^5e^{(12i\,dx+12i\,c)} + 811008i\,a^5e^{(10i\,dx+10i\,c)} + 5068\ldots}{495\left(de^{(24i\,dx+24i\,c)} + 12\,de^{(22i\,dx+22i\,c)} + 66\,de^{(20i\,dx+20i\,c)} + 220\,de^{(18i\,dx+18i\,c)} + 495\,de^{(16i\,dx+16i\,c)} + 792\,de^{(14i\,dx+14i\,c)} + 92\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/495*(506880*I*a^5*e^(16*I*d*x + 16*I*c) + 811008*I*a^5*e^(14*I*d*x + 14*I
*c) + 946176*I*a^5*e^(12*I*d*x + 12*I*c) + 811008*I*a^5*e^(10*I*d*x + 10*I*
c) + 506880*I*a^5*e^(8*I*d*x + 8*I*c) + 225280*I*a^5*e^(6*I*d*x + 6*I*c) +
67584*I*a^5*e^(4*I*d*x + 4*I*c) + 12288*I*a^5*e^(2*I*d*x + 2*I*c) + 1024*I*
a^5)/(d*e^(24*I*d*x + 24*I*c) + 12*d*e^(22*I*d*x + 22*I*c) + 66*d*e^(20*I*d
*x + 20*I*c) + 220*d*e^(18*I*d*x + 18*I*c) + 495*d*e^(16*I*d*x + 16*I*c) +
792*d*e^(14*I*d*x + 14*I*c) + 924*d*e^(12*I*d*x + 12*I*c) + 792*d*e^(10*I*d
*x + 10*I*c) + 495*d*e^(8*I*d*x + 8*I*c) + 220*d*e^(6*I*d*x + 6*I*c) + 66*d
*e^(4*I*d*x + 4*I*c) + 12*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43765, size = 216, normalized size = 1.98

$$-\frac{-165i\,a^5\tan(dx+c)^{12} - 900\,a^5\tan(dx+c)^{11} + 1386i\,a^5\tan(dx+c)^{10} - 1100\,a^5\tan(dx+c)^9 + 5445i\,a^5\tan(dx+c)^{8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1980*(-165*I*a^5*tan(d*x + c)^12 - 900*a^5*tan(d*x + c)^11 + 1386*I*a^5*
tan(d*x + c)^10 - 1100*a^5*tan(d*x + c)^9 + 5445*I*a^5*tan(d*x + c)^8 + 396
0*a^5*tan(d*x + c)^7 + 4620*I*a^5*tan(d*x + c)^6 + 8712*a^5*tan(d*x + c)^5
- 2475*I*a^5*tan(d*x + c)^4 + 4620*a^5*tan(d*x + c)^3 - 4950*I*a^5*tan(d*x
+ c)^2 - 1980*a^5*tan(d*x + c))/d
```

3.60 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=82

$$-\frac{i(a+ia\tan(c+dx))^{10}}{10a^5d}+\frac{4i(a+ia\tan(c+dx))^9}{9a^4d}-\frac{i(a+ia\tan(c+dx))^8}{2a^3d}$$

```
[Out] ((-I/2)*(a + I*a*Tan[c + d*x])^8)/(a^3*d) + (((4*I)/9)*(a + I*a*Tan[c + d*x
])^9)/(a^4*d) - ((I/10)*(a + I*a*Tan[c + d*x])^10)/(a^5*d)
```

Rubi [A] time = 0.0566016, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{i(a+ia\tan(c+dx))^{10}}{10a^5d}+\frac{4i(a+ia\tan(c+dx))^9}{9a^4d}-\frac{i(a+ia\tan(c+dx))^8}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-I/2)*(a + I*a*Tan[c + d*x])^8)/(a^3*d) + (((4*I)/9)*(a + I*a*Tan[c + d*x
])^9)/(a^4*d) - ((I/10)*(a + I*a*Tan[c + d*x])^10)/(a^5*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^2(a+x)^7\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a+x)^7-4a(a+x)^8+(a+x)^9\right)dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i(a+ia\tan(c+dx))^8}{2a^3d}+\frac{4i(a+ia\tan(c+dx))^9}{9a^4d}-\frac{i(a+ia\tan(c+dx))^{10}}{10a^5d}\end{aligned}$$

Mathematica [A] time = 2.21879, size = 154, normalized size = 1.88

$$\frac{a^5\sec(c)\sec^{10}(c+dx)(105\sin(c+2dx)-105\sin(3c+2dx)+60\sin(3c+4dx)-60\sin(5c+4dx)+45\sin(5c+6dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c]*Sec[c + d*x]^10*((126*I)*Cos[c] + (105*I)*Cos[c + 2*d*x] + (105
*I)*Cos[3*c + 2*d*x] + (60*I)*Cos[3*c + 4*d*x] + (60*I)*Cos[5*c + 4*d*x] -
126*Sin[c] + 105*Sin[c + 2*d*x] - 105*Sin[3*c + 2*d*x] + 60*Sin[3*c + 4*d*x
] - 60*Sin[5*c + 4*d*x] + 45*Sin[5*c + 6*d*x] + 10*Sin[7*c + 8*d*x] + Sin[9
*c + 10*d*x]))/(360*d)
```

Maple [B] time = 0.079, size = 295, normalized size = 3.6

$$\frac{1}{d}\left(ia^5\left(\frac{(\sin(dx+c))^6}{10\,(\cos(dx+c))^{10}}+\frac{(\sin(dx+c))^6}{20\,(\cos(dx+c))^8}+\frac{(\sin(dx+c))^6}{60\,(\cos(dx+c))^6}\right)+5\,a^5\left(1/9\,\frac{(\sin(dx+c))^5}{(\cos(dx+c))^9}+\frac{4\,(\sin(dx+c))^5}{63\,(\cos(dx+c))^7}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(1/10*sin(d*x+c)^6/cos(d*x+c)^10+1/20*sin(d*x+c)^6/cos(d*x+c)^8+
1/60*sin(d*x+c)^6/cos(d*x+c)^6)+5*a^5*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*s
in(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5)-10*I*a^5*(1/8*sin
(d*x+c)^4/cos(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos
(d*x+c)^4)-10*a^5*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+
c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+5/6*I*a^5/cos(d*x+c)^6-a^5*(-8/15-1/5
*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.07371, size = 146, normalized size = 1.78

$$\frac{126i\,a^5\tan(dx+c)^{10}+700\,a^5\tan(dx+c)^9-1260i\,a^5\tan(dx+c)^8-2940i\,a^5\tan(dx+c)^6-3528\,a^5\tan(dx+c)^5-3}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1260*(126*I*a^5*tan(d*x + c)^10 + 700*a^5*tan(d*x + c)^9 - 1260*I*a^5*tan
(d*x + c)^8 - 2940*I*a^5*tan(d*x + c)^6 - 3528*a^5*tan(d*x + c)^5 - 3360*a^
5*tan(d*x + c)^3 + 3150*I*a^5*tan(d*x + c)^2 + 1260*a^5*tan(d*x + c))/d
```

Fricas [B] time = 1.14535, size = 740, normalized size = 9.02

$$\frac{15360i\,a^5e^{(14i\,dx+14i\,c)}+26880i\,a^5e^{(12i\,dx+12i\,c)}+32256i\,a^5e^{(10i\,dx+10i\,c)}+26880i\,a^5e^{(8i\,dx+8i\,c)}+15360i\,a^5e^{(6i\,dx+6i}}{45\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de^{(10i\,dx+10i\,c)}+210\,de^{(}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/45*(15360*I*a^5*e^(14*I*d*x + 14*I*c) + 26880*I*a^5*e^(12*I*d*x + 12*I*c)
 + 32256*I*a^5*e^(10*I*d*x + 10*I*c) + 26880*I*a^5*e^(8*I*d*x + 8*I*c) + 15
360*I*a^5*e^(6*I*d*x + 6*I*c) + 5760*I*a^5*e^(4*I*d*x + 4*I*c) + 1280*I*a^5
*e^(2*I*d*x + 2*I*c) + 128*I*a^5)/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(18*I*d
```

```
*x + 18*I*c) + 45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*c) + 2
10*d*e^(12*I*d*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8*I*d*x
 + 8*I*c) + 120*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(
2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48996, size = 146, normalized size = 1.78

$$-\frac{-9i\,a^5\tan\left(dx+c\right)^{10}-50\,a^5\tan\left(dx+c\right)^9+90i\,a^5\tan\left(dx+c\right)^8+210i\,a^5\tan\left(dx+c\right)^6+252\,a^5\tan\left(dx+c\right)^5+240}{90\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/90*(-9*I*a^5*tan(d*x + c)^10 - 50*a^5*tan(d*x + c)^9 + 90*I*a^5*tan(d*x
+ c)^8 + 210*I*a^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 240*a^5*tan(d*
x + c)^3 - 225*I*a^5*tan(d*x + c)^2 - 90*a^5*tan(d*x + c))/d
```

3.61 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=55

$$\frac{i(a+ia\tan(c+dx))^8}{8a^3d}-\frac{2i(a+ia\tan(c+dx))^7}{7a^2d}$$

```
[Out] (((-2*I)/7)*(a + I*a*Tan[c + d*x])^7)/(a^2*d) + ((I/8)*(a + I*a*Tan[c + d*x
])^8)/(a^3*d)
```

Rubi [A] time = 0.0429323, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a+ia\tan(c+dx))^8}{8a^3d}-\frac{2i(a+ia\tan(c+dx))^7}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-2*I)/7)*(a + I*a*Tan[c + d*x])^7)/(a^2*d) + ((I/8)*(a + I*a*Tan[c + d*x
])^8)/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{i\,\mathrm{Subst}\left(\int (a-x)(a+x)^6\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\mathrm{Subst}\left(\int \left(2a(a+x)^6-(a+x)^7\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{2i(a+ia\tan(c+dx))^7}{7a^2d}+\frac{i(a+ia\tan(c+dx))^8}{8a^3d}\end{aligned}$$

Mathematica [B] time = 1.6968, size = 143, normalized size = 2.6

$$\frac{a^5\sec(c)\sec^8(c+dx)(28\sin(c+2dx)-28\sin(3c+2dx)+14\sin(3c+4dx)-14\sin(5c+4dx)+8\sin(5c+6dx)+\sin(7}{56d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c]*Sec[c + d*x]^8*((35*I)*Cos[c] + (28*I)*Cos[c + 2*d*x] + (28*I)*
Cos[3*c + 2*d*x] + (14*I)*Cos[3*c + 4*d*x] + (14*I)*Cos[5*c + 4*d*x] - 35*S
in[c] + 28*Sin[c + 2*d*x] - 28*Sin[3*c + 2*d*x] + 14*Sin[3*c + 4*d*x] - 14*
Sin[5*c + 4*d*x] + 8*Sin[5*c + 6*d*x] + Sin[7*c + 8*d*x]))/(56*d)
```

Maple [B] time = 0.078, size = 213, normalized size = 3.9

$$\frac{1}{d}\left(ia^5\left(\frac{(\sin(dx+c))^6}{8\,(\cos(dx+c))^8}+\frac{(\sin(dx+c))^6}{24\,(\cos(dx+c))^6}\right)+5\,a^5\left(1/7\,\frac{(\sin(dx+c))^5}{(\cos(dx+c))^7}+\frac{2\,(\sin(dx+c))^5}{35\,(\cos(dx+c))^5}\right)-10\,ia^5\left(\frac{(\sin(dx+}{6\,(\cos(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(1/8*sin(d*x+c)^6/cos(d*x+c)^8+1/24*sin(d*x+c)^6/cos(d*x+c)^6)+5
*a^5*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5)-10*I*a^
5*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)-10*a^5*(1/
5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+5/4*I*a^5/cos(d
*x+c)^4-a^5*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [B] time = 1.13411, size = 146, normalized size = 2.65

$$\frac{21i\,a^5\tan(dx+c)^8+120\,a^5\tan(dx+c)^7-252i\,a^5\tan(dx+c)^6-168\,a^5\tan(dx+c)^5-210i\,a^5\tan(dx+c)^4-504}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/168*(21*I*a^5*tan(d*x + c)^8 + 120*a^5*tan(d*x + c)^7 - 252*I*a^5*tan(d*x
 + c)^6 - 168*a^5*tan(d*x + c)^5 - 210*I*a^5*tan(d*x + c)^4 - 504*a^5*tan(d
*x + c)^3 + 420*I*a^5*tan(d*x + c)^2 + 168*a^5*tan(d*x + c))/d
```

Fricas [B] time = 1.09312, size = 591, normalized size = 10.75

$$\frac{896i\,a^5e^{(12i\,dx+12i\,c)}+1792i\,a^5e^{(10i\,dx+10i\,c)}+2240i\,a^5e^{(8i\,dx+8i\,c)}+1792i\,a^5e^{(6i\,dx+6i\,c)}+896i\,a^5e^{(4i\,dx+4i\,c)}+256i\,a^5e^{(2}}{7\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i\,c)}+28\,de^{(4i\,dx+4i\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/7*(896*I*a^5*e^(12*I*d*x + 12*I*c) + 1792*I*a^5*e^(10*I*d*x + 10*I*c) + 2
240*I*a^5*e^(8*I*d*x + 8*I*c) + 1792*I*a^5*e^(6*I*d*x + 6*I*c) + 896*I*a^5*
e^(4*I*d*x + 4*I*c) + 256*I*a^5*e^(2*I*d*x + 2*I*c) + 32*I*a^5)/(d*e^(16*I*
d*x + 16*I*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56
*d*e^(10*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I
*c) + 28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^5\left(\int -10\tan^2(c+dx)\sec^4(c+dx)\,dx+\int 5\tan^4(c+dx)\sec^4(c+dx)\,dx+\int 5i\tan(c+dx)\sec^4(c+dx)\,dx+\int -1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] a**5*(Integral(-10*tan(c + d*x)**2*sec(c + d*x)**4, x) + Integral(5*tan(c +
 d*x)**4*sec(c + d*x)**4, x) + Integral(5*I*tan(c + d*x)*sec(c + d*x)**4, x
) + Integral(-10*I*tan(c + d*x)**3*sec(c + d*x)**4, x) + Integral(I*tan(c +
 d*x)**5*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

Giac [B] time = 1.53702, size = 146, normalized size = 2.65

$$-\frac{-7i\,a^5\tan(dx+c)^8-40\,a^5\tan(dx+c)^7+84i\,a^5\tan(dx+c)^6+56\,a^5\tan(dx+c)^5+70i\,a^5\tan(dx+c)^4+168\,a^5\tan}{56\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/56*(-7*I*a^5*tan(d*x + c)^8 - 40*a^5*tan(d*x + c)^7 + 84*I*a^5*tan(d*x +
 c)^6 + 56*a^5*tan(d*x + c)^5 + 70*I*a^5*tan(d*x + c)^4 + 168*a^5*tan(d*x +
 c)^3 - 140*I*a^5*tan(d*x + c)^2 - 56*a^5*tan(d*x + c))/d
```

3.62 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=27

$$-\frac{i(a+ia\tan(c+dx))^6}{6ad}$$

```
[Out] ((-I/6)*(a + I*a*Tan[c + d*x])^6)/(a*d)
```

Rubi [A] time = 0.0361545, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$-\frac{i(a+ia\tan(c+dx))^6}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-I/6)*(a + I*a*Tan[c + d*x])^6)/(a*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{i\,\text{Subst}\left(\int (a+x)^5\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{i(a+ia\tan(c+dx))^6}{6ad}\end{aligned}$$

Mathematica [B] time = 1.45354, size = 134, normalized size = 4.96

$$\frac{a^5\sec(c)\sec^6(c+dx)(15\sin(c+2dx)-15\sin(3c+2dx)+6\sin(3c+4dx)-6\sin(5c+4dx)+2\sin(5c+6dx)+15i\,c}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c]*Sec[c + d*x]^6*((20*I)*Cos[c] + (15*I)*Cos[c + 2*d*x] + (15*I)*
Cos[3*c + 2*d*x] + (6*I)*Cos[3*c + 4*d*x] + (6*I)*Cos[5*c + 4*d*x] - 20*Sin
[c] + 15*Sin[c + 2*d*x] - 15*Sin[3*c + 2*d*x] + 6*Sin[3*c + 4*d*x] - 6*Sin[
```

```
5*c + 4*d*x] + 2*Sin[5*c + 6*d*x]))/(12*d)
```

Maple [B] time = 0.076, size = 115, normalized size = 4.3

$$\frac{1}{d}\left(\frac{\frac{i}{6}a^5(\sin(dx+c))^6}{(\cos(dx+c))^6}+\frac{a^5(\sin(dx+c))^5}{(\cos(dx+c))^5}-\frac{\frac{5i}{2}a^5(\sin(dx+c))^4}{(\cos(dx+c))^4}-\frac{10a^5(\sin(dx+c))^3}{3(\cos(dx+c))^3}+\frac{\frac{5i}{2}a^5}{(\cos(dx+c))^2}+a^5\tan(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(1/6*I*a^5*sin(d*x+c)^6/cos(d*x+c)^6+a^5*sin(d*x+c)^5/cos(d*x+c)^5-5/2*
I*a^5*sin(d*x+c)^4/cos(d*x+c)^4-10/3*a^5*sin(d*x+c)^3/cos(d*x+c)^3+5/2*I*a^
5/cos(d*x+c)^2+a^5*tan(d*x+c))
```

Maxima [A] time = 1.1303, size = 28, normalized size = 1.04

$$-\frac{i\,(i\,a\tan(dx+c)+a)^6}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/6*I*(I*a*tan(d*x + c) + a)^6/(a*d)
```

Fricas [B] time = 1.10871, size = 463, normalized size = 17.15

$$\frac{192i\,a^5e^{(10i\,dx+10i\,c)}+480i\,a^5e^{(8i\,dx+8i\,c)}+640i\,a^5e^{(6i\,dx+6i\,c)}+480i\,a^5e^{(4i\,dx+4i\,c)}+192i\,a^5e^{(2i\,dx+2i\,c)}+32i\,a^5}{3\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3*(192*I*a^5*e^(10*I*d*x + 10*I*c) + 480*I*a^5*e^(8*I*d*x + 8*I*c) + 640*
I*a^5*e^(6*I*d*x + 6*I*c) + 480*I*a^5*e^(4*I*d*x + 4*I*c) + 192*I*a^5*e^(2*
I*d*x + 2*I*c) + 32*I*a^5)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*
I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*
x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^5\left(\int -10\tan^2(c+dx)\sec^2(c+dx)\,dx+\int 5\tan^4(c+dx)\sec^2(c+dx)\,dx+\int 5i\tan(c+dx)\sec^2(c+dx)\,dx+\int -1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] a**5*(Integral(-10*tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(5*tan(c +
 d*x)**4*sec(c + d*x)**2, x) + Integral(5*I*tan(c + d*x)*sec(c + d*x)**2, x
) + Integral(-10*I*tan(c + d*x)**3*sec(c + d*x)**2, x) + Integral(I*tan(c +
 d*x)**5*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

Giac [B] time = 1.58311, size = 111, normalized size = 4.11

$$-\frac{-i\,a^5\tan\left(dx+c\right)^6-6\,a^5\tan\left(dx+c\right)^5+15i\,a^5\tan\left(dx+c\right)^4+20\,a^5\tan\left(dx+c\right)^3-15i\,a^5\tan\left(dx+c\right)^2-6\,a^5\tan\left(\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/6*(-I*a^5*tan(d*x + c)^6 - 6*a^5*tan(d*x + c)^5 + 15*I*a^5*tan(d*x + c)^
4 + 20*a^5*tan(d*x + c)^3 - 15*I*a^5*tan(d*x + c)^2 - 6*a^5*tan(d*x + c))/d
```

3.63 $\int (a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=117

$$-\frac{8a^5\tan(c+dx)}{d}+\frac{2ia^2(a+ia\tan(c+dx))^3}{3d}+\frac{2ia\left(a^2+ia^2\tan(c+dx)\right)^2}{d}-\frac{16ia^5\log(\cos(c+dx))}{d}+16a^5x+\frac{ia(a+ia}{}$$

```
[Out] 16*a^5*x - ((16*I)*a^5*Log[Cos[c + d*x]])/d - (8*a^5*Tan[c + d*x])/d + (((2
*I)/3)*a^2*(a + I*a*Tan[c + d*x])^3)/d + ((I/4)*a*(a + I*a*Tan[c + d*x])^4)
/d + ((2*I)*a*(a^2 + I*a^2*Tan[c + d*x])^2)/d
```

Rubi [A] time = 0.0658658, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3478, 3477, 3475}

$$-\frac{8a^5\tan(c+dx)}{d}+\frac{2ia^2(a+ia\tan(c+dx))^3}{3d}+\frac{2ia\left(a^2+ia^2\tan(c+dx)\right)^2}{d}-\frac{16ia^5\log(\cos(c+dx))}{d}+16a^5x+\frac{ia(a+ia}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] 16*a^5*x - ((16*I)*a^5*Log[Cos[c + d*x]])/d - (8*a^5*Tan[c + d*x])/d + (((2
*I)/3)*a^2*(a + I*a*Tan[c + d*x])^3)/d + ((I/4)*a*(a + I*a*Tan[c + d*x])^4)
/d + ((2*I)*a*(a^2 + I*a^2*Tan[c + d*x])^2)/d
```

Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+ia\tan(c+dx))^5\,dx = \frac{ia(a+ia\tan(c+dx))^4}{4d} + (2a)\int (a+ia\tan(c+dx))^4\,dx$$
$$= \frac{2ia^2(a+ia\tan(c+dx))^3}{3d} + \frac{ia(a+ia\tan(c+dx))^4}{4d} + \left(4a^2\right)\int (a+ia\tan(c+dx))^3\,dx$$
$$= \frac{2ia^3(a+ia\tan(c+dx))^2}{d} + \frac{2ia^2(a+ia\tan(c+dx))^3}{3d} + \frac{ia(a+ia\tan(c+dx))^4}{4d} + \left(8a^3\right)$$
$$= 16a^5x - \frac{8a^5\tan(c+dx)}{d} + \frac{2ia^3(a+ia\tan(c+dx))^2}{d} + \frac{2ia^2(a+ia\tan(c+dx))^3}{3d} + \frac{ia(a}{}$$
$$= 16a^5x - \frac{16ia^5\log(\cos(c+dx))}{d} - \frac{8a^5\tan(c+dx)}{d} + \frac{2ia^3(a+ia\tan(c+dx))^2}{d} + \frac{2ia^2(a}{}$$

Mathematica [A] time = 2.5629, size = 228, normalized size = 1.95

$$\frac{a^5\sec(c)\sec^4(c+dx)\left(-70\sin(c+2dx)+30\sin(3c+2dx)-25\sin(3c+4dx)+48dx\cos(3c+2dx)-18i\cos(3c+2d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c]*Sec[c + d*x]^4*((-18*I)*Cos[3*c + 2*d*x] + 48*d*x*Cos[3*c + 2*d
*x] + 12*d*x*Cos[3*c + 4*d*x] + 12*d*x*Cos[5*c + 4*d*x] + 6*Cos[c + 2*d*x]*
(-3*I + 8*d*x - (4*I)*Log[Cos[c + d*x]^2]) + Cos[c]*(-33*I + 72*d*x - (36*I
)*Log[Cos[c + d*x]^2]) - (24*I)*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]^2] - (6*I
)*Cos[3*c + 4*d*x]*Log[Cos[c + d*x]^2] - (6*I)*Cos[5*c + 4*d*x]*Log[Cos[c +
 d*x]^2] + 75*Sin[c] - 70*Sin[c + 2*d*x] + 30*Sin[3*c + 2*d*x] - 25*Sin[3*c
 + 4*d*x]))/(12*d)
```

Maple [A] time = 0.005, size = 101, normalized size = 0.9

$$-15\,\frac{a^5\tan(dx+c)}{d} + \frac{\frac{i}{4}a^5(\tan(dx+c))^4}{d} + \frac{5\,a^5(\tan(dx+c))^3}{3\,d} - \frac{\frac{11\,i}{2}a^5(\tan(dx+c))^2}{d} + \frac{8\,ia^5\ln\left(1+(\tan(dx+c))^2\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^5,x)
```

```
[Out] -15*a^5*tan(d*x+c)/d+1/4*I/d*a^5*tan(d*x+c)^4+5/3/d*a^5*tan(d*x+c)^3-11/2*I
/d*a^5*tan(d*x+c)^2+8*I/d*a^5*ln(1+tan(d*x+c)^2)+16/d*a^5*arctan(tan(d*x+c)
)
```

Maxima [A] time = 1.85101, size = 223, normalized size = 1.91

$$a^5x + \frac{5\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^5}{3\,d} + \frac{10\,(dx+c-\tan(dx+c))a^5}{d} + \frac{i\,a^5\left(\frac{4\,\sin(dx+c)^2-3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] a^5*x + 5/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^5/d + 10*(d*x
 + c - tan(d*x + c))*a^5/d + 1/4*I*a^5*((4*sin(d*x + c)^2 - 3)/(sin(d*x + c
)^4 - 2*sin(d*x + c)^2 + 1) - 2*log(sin(d*x + c)^2 - 1))/d + 5*I*a^5*(1/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d + 5*I*a^5*log(sec(d*x + c))/
d
```

Fricas [A] time = 1.06098, size = 527, normalized size = 4.5

$$\frac{-192i\,a^5e^{(6i\,dx+6i\,c)} - 432i\,a^5e^{(4i\,dx+4i\,c)} - 352i\,a^5e^{(2i\,dx+2i\,c)} - 100i\,a^5 + \left(-48i\,a^5e^{(8i\,dx+8i\,c)} - 192i\,a^5e^{(6i\,dx+6i\,c)} - 288i\,a^5e^{(4i\,dx-}\right.}{3\left(de^{(8i\,dx+8i\,c)} + 4\,de^{(6i\,dx+6i\,c)} + 6\,de^{(4i\,dx+4i\,c)} + 4\,de^{(2i\,dx+2i\,c)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3*(-192*I*a^5*e^(6*I*d*x + 6*I*c) - 432*I*a^5*e^(4*I*d*x + 4*I*c) - 352*I
*a^5*e^(2*I*d*x + 2*I*c) - 100*I*a^5 + (-48*I*a^5*e^(8*I*d*x + 8*I*c) - 192
*I*a^5*e^(6*I*d*x + 6*I*c) - 288*I*a^5*e^(4*I*d*x + 4*I*c) - 192*I*a^5*e^(2
*I*d*x + 2*I*c) - 48*I*a^5)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8
*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x
+ 2*I*c) + d)
```

Sympy [A] time = 3.30864, size = 185, normalized size = 1.58

$$-\frac{16ia^5\log\left(e^{2idx} + e^{-2ic}\right)}{d} + \frac{-\frac{64ia^5e^{-2ic}e^{6idx}}{d} - \frac{144ia^5e^{-4ic}e^{4idx}}{d} - \frac{352ia^5e^{-6ic}e^{2idx}}{3d} - \frac{100ia^5e^{-8ic}}{3d}}{e^{8idx} + 4e^{-2ic}e^{6idx} + 6e^{-4ic}e^{4idx} + 4e^{-6ic}e^{2idx} + e^{-8ic}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**5,x)
```

```
[Out] -16*I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-64*I*a**5*exp(-2*I*c)*exp(
6*I*d*x)/d - 144*I*a**5*exp(-4*I*c)*exp(4*I*d*x)/d - 352*I*a**5*exp(-6*I*c)
*exp(2*I*d*x)/(3*d) - 100*I*a**5*exp(-8*I*c)/(3*d))/(exp(8*I*d*x) + 4*exp(-
2*I*c)*exp(6*I*d*x) + 6*exp(-4*I*c)*exp(4*I*d*x) + 4*exp(-6*I*c)*exp(2*I*d*
x) + exp(-8*I*c))
```

Giac [B] time = 1.15091, size = 300, normalized size = 2.56

$$\frac{-48i\,a^5e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - 192i\,a^5e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - 288i\,a^5e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - 192i}{3\left(de^{(8i\,dx+8i\,c)} + 4\,de^{(6i\,dx+6i\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/3*(-48*I*a^5*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 192*I*a^5
*e^(6*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 288*I*a^5*e^(4*I*d*x +
4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 192*I*a^5*e^(2*I*d*x + 2*I*c)*log(e^(
2*I*d*x + 2*I*c) + 1) - 192*I*a^5*e^(6*I*d*x + 6*I*c) - 432*I*a^5*e^(4*I*d*
x + 4*I*c) - 352*I*a^5*e^(2*I*d*x + 2*I*c) - 48*I*a^5*log(e^(2*I*d*x + 2*I*
```

```
c) + 1) - 100*I*a^5)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d
*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

3.64 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=83

$$\frac{ia^5\tan^2(c+dx)}{2d}-\frac{8ia^6}{d(a-ia\tan(c+dx))}+\frac{5a^5\tan(c+dx)}{d}+\frac{12ia^5\log(\cos(c+dx))}{d}-12a^5x$$

```
[Out] -12*a^5*x + ((12*I)*a^5*Log[Cos[c + d*x]])/d + (5*a^5*Tan[c + d*x])/d + ((I
/2)*a^5*Tan[c + d*x]^2)/d - ((8*I)*a^6)/(d*(a - I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0585825, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{ia^5\tan^2(c+dx)}{2d}-\frac{8ia^6}{d(a-ia\tan(c+dx))}+\frac{5a^5\tan(c+dx)}{d}+\frac{12ia^5\log(\cos(c+dx))}{d}-12a^5x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] -12*a^5*x + ((12*I)*a^5*Log[Cos[c + d*x]])/d + (5*a^5*Tan[c + d*x])/d + ((I
/2)*a^5*Tan[c + d*x]^2)/d - ((8*I)*a^6)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{\left(ia^3\right)\operatorname{Subst}\left(\int \frac{(a+x)^3}{(a-x)^2}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{\left(ia^3\right)\operatorname{Subst}\left(\int \left(5a+\frac{8a^3}{(a-x)^2}-\frac{12a^2}{a-x}+x\right)dx,x,ia\tan(c+dx)\right)}{d}\\ &= -12a^5x+\frac{12ia^5\log(\cos(c+dx))}{d}+\frac{5a^5\tan(c+dx)}{d}+\frac{ia^5\tan^2(c+dx)}{2d}-\frac{}{d(a-}\end{aligned}$$

Mathematica [B] time = 6.55229, size = 649, normalized size = 7.82

$$x\cos^5(c+dx)\left(36i\sin^5(c)+24i\sin^3(c)-6\cos^5(c)+6\cos^3(c)+6\sin^5(c)\tan(c)+6\sin^3(c)\tan(c)+90\sin^2(c)\cos^3(c)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (-12*x*Cos[5*c]*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/(Cos[d*x] + I*Sin[
d*x])^5 + ((6*I)*Cos[5*c]*Cos[c + d*x]^5*Log[Cos[c + d*x]^2]*(a + I*a*Tan[c
 + d*x])^5)/(d*(Cos[d*x] + I*Sin[d*x])^5) + (Cos[2*d*x]*Cos[c + d*x]^5*((-4
*I)*Cos[3*c] - 4*Sin[3*c])*(a + I*a*Tan[c + d*x])^5)/(d*(Cos[d*x] + I*Sin[d
*x])^5) + (Cos[c + d*x]^3*((I/2)*Cos[5*c] + Sin[5*c]/2)*(a + I*a*Tan[c + d*
x])^5)/(d*(Cos[d*x] + I*Sin[d*x])^5) + ((12*I)*x*Cos[c + d*x]^5*Sin[5*c]*(a
 + I*a*Tan[c + d*x])^5)/(Cos[d*x] + I*Sin[d*x])^5 + (6*Cos[c + d*x]^5*Log[C
os[c + d*x]^2]*Sin[5*c]*(a + I*a*Tan[c + d*x])^5)/(d*(Cos[d*x] + I*Sin[d*x]
)^5) + (Cos[c + d*x]^4*(5*Cos[5*c] - (5*I)*Sin[5*c])*Sin[d*x]*(a + I*a*Tan[
c + d*x])^5)/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[d*x] + I*S
in[d*x])^5) + (Cos[c + d*x]^5*(4*Cos[3*c] - (4*I)*Sin[3*c])*Sin[2*d*x]*(a +
 I*a*Tan[c + d*x])^5)/(d*(Cos[d*x] + I*Sin[d*x])^5) + (x*Cos[c + d*x]^5*(6*
Cos[c]^3 - 6*Cos[c]^5 - (24*I)*Cos[c]^2*Sin[c] + (36*I)*Cos[c]^4*Sin[c] - 3
6*Cos[c]*Sin[c]^2 + 90*Cos[c]^3*Sin[c]^2 + (24*I)*Sin[c]^3 - (120*I)*Cos[c]
^2*Sin[c]^3 - 90*Cos[c]*Sin[c]^4 + (36*I)*Sin[c]^5 + 6*Sin[c]^3*Tan[c] + 6*
Sin[c]^5*Tan[c] - I*(12*Cos[5*c] - (12*I)*Sin[5*c])*Tan[c])*(a + I*a*Tan[c
+ d*x])^5)/(Cos[d*x] + I*Sin[d*x])^5
```

Maple [B] time = 0.107, size = 175, normalized size = 2.1

$$\frac{6\,ia^5\,(\sin(dx+c))^2}{d}+\frac{\frac{i}{2}a^5\,(\sin(dx+c))^4}{d}-\frac{\frac{5\,i}{2}a^5\,(\cos(dx+c))^2}{d}+\frac{\frac{i}{2}a^5\,(\sin(dx+c))^6}{d\,(\cos(dx+c))^2}+5\,\frac{a^5\,(\sin(dx+c))^5}{d\cos(dx+c)}+5\,\frac{a^5\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 6*I/d*a^5*sin(d*x+c)^2+1/2*I/d*a^5*sin(d*x+c)^4-5/2*I/d*a^5*cos(d*x+c)^2+1/
2*I/d*a^5*sin(d*x+c)^6/cos(d*x+c)^2+5/d*a^5*sin(d*x+c)^5/cos(d*x+c)+5/d*a^5
*cos(d*x+c)*sin(d*x+c)^3+13/d*a^5*sin(d*x+c)*cos(d*x+c)-12*a^5*x-12/d*a^5*c
+12*I*a^5*ln(cos(d*x+c))/d
```

Maxima [A] time = 1.70172, size = 116, normalized size = 1.4

$$-\frac{-i\,a^5\tan(dx+c)^2+24\,(dx+c)a^5+12i\,a^5\log\left(\tan(dx+c)^2+1\right)-10\,a^5\tan(dx+c)-\frac{16\left(a^5\tan(dx+c)-i\,a^5\right)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/2*(-I*a^5*tan(d*x + c)^2 + 24*(d*x + c)*a^5 + 12*I*a^5*log(tan(d*x + c)^
2 + 1) - 10*a^5*tan(d*x + c) - 16*(a^5*tan(d*x + c) - I*a^5)/(tan(d*x + c)^
2 + 1))/d
```

Fricas [A] time = 1.08967, size = 352, normalized size = 4.24

$$\frac{-4i\,a^5e^{(6i\,dx+6i\,c)}-8i\,a^5e^{(4i\,dx+4i\,c)}+8i\,a^5e^{(2i\,dx+2i\,c)}+10i\,a^5+\left(12i\,a^5e^{(4i\,dx+4i\,c)}+24i\,a^5e^{(2i\,dx+2i\,c)}+12i\,a^5\right)\log\left(e^{(2i\,dx+2i}\right.}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] (-4*I*a^5*e^(6*I*d*x + 6*I*c) - 8*I*a^5*e^(4*I*d*x + 4*I*c) + 8*I*a^5*e^(2*
I*d*x + 2*I*c) + 10*I*a^5 + (12*I*a^5*e^(4*I*d*x + 4*I*c) + 24*I*a^5*e^(2*I
*d*x + 2*I*c) + 12*I*a^5)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I
*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 1.38713, size = 128, normalized size = 1.54

$$8a^5\left(\begin{cases} -\frac{ie^{2idx}}{2d} & \text{for } d \neq 0 \\ x & \text{otherwise} \end{cases}\right)e^{2ic} + \frac{12ia^5\log\left(e^{2idx} + e^{-2ic}\right)}{d} + \frac{\frac{12ia^5e^{-2ic}e^{2idx}}{d} + \frac{10ia^5e^{-4ic}}{d}}{e^{4idx} + 2e^{-2ic}e^{2idx} + e^{-4ic}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 8*a**5*Piecewise((-I*exp(2*I*d*x)/(2*d), Ne(d, 0)), (x, True))*exp(2*I*c) +
 12*I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + (12*I*a**5*exp(-2*I*c)*exp(2
*I*d*x)/d + 10*I*a**5*exp(-4*I*c)/d)/(exp(4*I*d*x) + 2*exp(-2*I*c)*exp(2*I*
d*x) + exp(-4*I*c))
```

Giac [A] time = 1.43947, size = 196, normalized size = 2.36

$$\frac{12i\,a^5e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+24i\,a^5e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-4i\,a^5e^{(6i\,dx+6i\,c)}-8i\,a^5e^{(4i\,dx+4i\,c)}+8i\,a^5e^{(2i\,dx+2i}}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] (12*I*a^5*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 24*I*a^5*e^(2*
I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 4*I*a^5*e^(6*I*d*x + 6*I*c) -
 8*I*a^5*e^(4*I*d*x + 4*I*c) + 8*I*a^5*e^(2*I*d*x + 2*I*c) + 12*I*a^5*log(e
^(2*I*d*x + 2*I*c) + 1) + 10*I*a^5)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x
 + 2*I*c) + d)
```

3.65 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=73

$$-\frac{2ia^7}{d(a-ia\tan(c+dx))^2}+\frac{4ia^6}{d(a-ia\tan(c+dx))}-\frac{ia^5\log(\cos(c+dx))}{d}+a^5x$$

```
[Out] a^5*x - (I*a^5*Log[Cos[c + d*x]])/d - ((2*I)*a^7)/(d*(a - I*a*Tan[c + d*x])
^2) + ((4*I)*a^6)/(d*(a - I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0550322, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{2ia^7}{d(a-ia\tan(c+dx))^2}+\frac{4ia^6}{d(a-ia\tan(c+dx))}-\frac{ia^5\log(\cos(c+dx))}{d}+a^5x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] a^5*x - (I*a^5*Log[Cos[c + d*x]])/d - ((2*I)*a^7)/(d*(a - I*a*Tan[c + d*x])
^2) + ((4*I)*a^6)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{(a+x)^2}{(a-x)^3}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{4a^2}{(a-x)^3}-\frac{4a}{(a-x)^2}+\frac{1}{a-x}\right)dx,x,ia\tan(c+dx)\right)}{d}\\ &= a^5x-\frac{ia^5\log(\cos(c+dx))}{d}-\frac{2ia^7}{d(a-ia\tan(c+dx))^2}+\frac{4ia^6}{d(a-ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.625467, size = 110, normalized size = 1.51

$$\frac{a^5(\cos(2c+7dx)+i\sin(2c+7dx))\left(\cos(2(c+dx))\left(-i\log\left(\cos^2(c+dx)\right)+2dx-i\right)+\sin(2(c+dx))\left(-\log\left(\cos^2(c+\right.\right.\right.}{2d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(2*I + Cos[2*(c + d*x)]*(-I + 2*d*x - I*Log[Cos[c + d*x]^2]) + (1 - (2
*I)*d*x - Log[Cos[c + d*x]^2])*Sin[2*(c + d*x)])*(Cos[2*c + 7*d*x] + I*Sin[
2*c + 7*d*x]))/(2*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [B] time = 0.069, size = 146, normalized size = 2.

$$-\frac{\frac{5i}{4}a^5(\cos(dx+c))^4}{d}-\frac{ia^5\ln(\cos(dx+c))}{d}-\frac{\frac{i}{2}a^5(\sin(dx+c))^2}{d}-\frac{5a^5\cos(dx+c)(\sin(dx+c))^3}{4d}-\frac{11a^5\sin(dx+c)\cos}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] -5/4*I/d*a^5*cos(d*x+c)^4-I*a^5*ln(cos(d*x+c))/d-1/2*I/d*a^5*sin(d*x+c)^2-5
/4/d*a^5*cos(d*x+c)*sin(d*x+c)^3-11/4/d*a^5*sin(d*x+c)*cos(d*x+c)+a^5*x+1/d
*a^5*c-11/4*I/d*a^5*sin(d*x+c)^4+11/4/d*a^5*sin(d*x+c)*cos(d*x+c)^3
```

Maxima [A] time = 1.80468, size = 119, normalized size = 1.63

$$\frac{8(dx+c)a^5+4ia^5\log\left(\tan(dx+c)^2+1\right)-\frac{32a^5\tan(dx+c)^3-48ia^5\tan(dx+c)^2-16ia^5}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(d*x + c)*a^5 + 4*I*a^5*log(tan(d*x + c)^2 + 1) - (32*a^5*tan(d*x +
c)^3 - 48*I*a^5*tan(d*x + c)^2 - 16*I*a^5)/(tan(d*x + c)^4 + 2*tan(d*x + c)
^2 + 1))/d
```

Fricas [A] time = 1.15905, size = 142, normalized size = 1.95

$$\frac{-ia^5e^{(4idx+4ic)}+2ia^5e^{(2idx+2ic)}-2ia^5\log\left(e^{(2idx+2ic)}+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a^5*e^(4*I*d*x + 4*I*c) + 2*I*a^5*e^(2*I*d*x + 2*I*c) - 2*I*a^5*log
(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [A] time = 0.935863, size = 82, normalized size = 1.12

$$-2a^5\left(\begin{cases}-\frac{ie^{2idx}}{2d} & \text{for } d\neq 0\\ x & \text{otherwise}\end{cases}\right)e^{2ic}+2a^5\left(\begin{cases}-\frac{ie^{4idx}}{4d} & \text{for } d\neq 0\\ x & \text{otherwise}\end{cases}\right)e^{4ic}-\frac{ia^5\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] -2*a**5*Piecewise((-I*exp(2*I*d*x)/(2*d), Ne(d, 0)), (x, True))*exp(2*I*c)
+ 2*a**5*Piecewise((-I*exp(4*I*d*x)/(4*d), Ne(d, 0)), (x, True))*exp(4*I*c)
 - I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d
```

Giac [B] time = 1.55009, size = 608, normalized size = 8.33

$$-384i\,a^5e^{(16i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-3072i\,a^5e^{(14i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-10752i\,a^5e^{(12i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/384*(-384*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 3072*
I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 10752*I*a^5*e^(12
*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 21504*I*a^5*e^(10*I*d*x + 2*
I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 21504*I*a^5*e^(6*I*d*x - 2*I*c)*log(e^(
2*I*d*x + 2*I*c) + 1) - 10752*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*
I*c) + 1) - 3072*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2
6880*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 384*I*a^5*e^(-8*I*c)*
log(e^(2*I*d*x + 2*I*c) + 1) - 192*I*a^5*e^(20*I*d*x + 12*I*c) - 1152*I*a^5
*e^(18*I*d*x + 10*I*c) - 2304*I*a^5*e^(16*I*d*x + 8*I*c) + 8064*I*a^5*e^(12
*I*d*x + 4*I*c) + 16128*I*a^5*e^(10*I*d*x + 2*I*c) + 9216*I*a^5*e^(6*I*d*x
- 2*I*c) + 2880*I*a^5*e^(4*I*d*x - 4*I*c) + 384*I*a^5*e^(2*I*d*x - 6*I*c) +
 16128*I*a^5*e^(8*I*d*x))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*x + 6*I*c
) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6*I*d*x
 - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d*e^(8*
I*d*x) + d*e^(-8*I*c))
```

3.66 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=55

$$\frac{ia^7}{2d(a-ia\tan(c+dx))^2}-\frac{2ia^8}{3d(a-ia\tan(c+dx))^3}$$

```
[Out] (((-2*I)/3)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) + ((I/2)*a^7)/(d*(a - I*a*Tan
[c + d*x])^2)
```

Rubi [A] time = 0.0485425, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{ia^7}{2d(a-ia\tan(c+dx))^2}-\frac{2ia^8}{3d(a-ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-2*I)/3)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) + ((I/2)*a^7)/(d*(a - I*a*Tan
[c + d*x])^2)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{\left(ia^7\right)\operatorname{Subst}\left(\int \frac{a+x}{(a-x)^4}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{\left(ia^7\right)\operatorname{Subst}\left(\int \left(\frac{2a}{(a-x)^4}-\frac{1}{(a-x)^3}\right)dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{2ia^8}{3d(a-ia\tan(c+dx))^3}+\frac{ia^7}{2d(a-ia\tan(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.485156, size = 50, normalized size = 0.91

$$\frac{a^5(5\cos(c+dx)-i\sin(c+dx))(\sin(5(c+dx))-i\cos(5(c+dx)))}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(5*Cos[c + d*x] - I*Sin[c + d*x])*((-I)*Cos[5*(c + d*x)] + Sin[5*(c +
d*x)]))/(24*d)
```

Maple [B] time = 0.084, size = 231, normalized size = 4.2

$$\frac{1}{d}\left(\frac{i}{6}a^5(\sin(dx+c))^6 + 5a^5\left(-1/6\,(\sin(dx+c))^3(\cos(dx+c))^3 - 1/8\,(\cos(dx+c))^3\sin(dx+c) + 1/16\,\cos(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(1/6*I*a^5*sin(d*x+c)^6+5*a^5*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d
*x+c)^3*sin(d*x+c)+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)-10*I*a^5*(-1
/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)-10*a^5*(-1/6*cos(d*x+c)^5*s
in(d*x+c)+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-5/
6*I*a^5*cos(d*x+c)^6+a^5*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c
))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [B] time = 1.6701, size = 126, normalized size = 2.29

$$\frac{-24i\,a^5\tan(dx+c)^4 - 80\,a^5\tan(dx+c)^3 + 96i\,a^5\tan(dx+c)^2 + 48\,a^5\tan(dx+c) - 8i\,a^5}{48\left(\tan(dx+c)^6 + 3\,\tan(dx+c)^4 + 3\,\tan(dx+c)^2 + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/48*(-24*I*a^5*tan(d*x + c)^4 - 80*a^5*tan(d*x + c)^3 + 96*I*a^5*tan(d*x +
 c)^2 + 48*a^5*tan(d*x + c) - 8*I*a^5)/((tan(d*x + c)^6 + 3*tan(d*x + c)^4
+ 3*tan(d*x + c)^2 + 1)*d)
```

Fricas [A] time = 1.12146, size = 93, normalized size = 1.69

$$\frac{-2i\,a^5e^{(6i\,dx+6i\,c)} - 3i\,a^5e^{(4i\,dx+4i\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/24*(-2*I*a^5*e^(6*I*d*x + 6*I*c) - 3*I*a^5*e^(4*I*d*x + 4*I*c))/d
```

Sympy [A] time = 0.604502, size = 82, normalized size = 1.49

$$\begin{cases} \frac{-8ia^5de^{6ic}e^{6idx} - 12ia^5de^{4ic}e^{4idx}}{96d^2} & \text{for } 96d^2 \neq 0 \\ x\left(\frac{a^5e^{6ic}}{2} + \frac{a^5e^{4ic}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise((((-8*I*a**5*d*exp(6*I*c)*exp(6*I*d*x) - 12*I*a**5*d*exp(4*I*c)*ex
p(4*I*d*x))/(96*d**2), Ne(96*d**2, 0)), (x*(a**5*exp(6*I*c)/2 + a**5*exp(4*
I*c)/2), True))
```

Giac [B] time = 1.58836, size = 252, normalized size = 4.58

$$\frac{-32i\,a^5e^{(18i\,dx+12i\,c)} - 240i\,a^5e^{(16i\,dx+10i\,c)} - 768i\,a^5e^{(14i\,dx+8i\,c)} - 1360i\,a^5e^{(12i\,dx+6i\,c)} - 1440i\,a^5e^{(10i\,dx+4i\,c)} - 912i\,a^5e^{(8i\,dx+2i\,c)}}{384\left(de^{(12i\,dx+6i\,c)} + 6\,de^{(10i\,dx+4i\,c)} + 15\,de^{(8i\,dx+2i\,c)} + 15\,de^{(4i\,dx-2i\,c)} + 6\,de^{(2i\,dx-4i\,c)} + 20\,de^{(6i\,dx)} + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/384*(-32*I*a^5*e^(18*I*d*x + 12*I*c) - 240*I*a^5*e^(16*I*d*x + 10*I*c) -
768*I*a^5*e^(14*I*d*x + 8*I*c) - 1360*I*a^5*e^(12*I*d*x + 6*I*c) - 1440*I*a
^5*e^(10*I*d*x + 4*I*c) - 912*I*a^5*e^(8*I*d*x + 2*I*c) - 48*I*a^5*e^(4*I*d
*x - 2*I*c) - 320*I*a^5*e^(6*I*d*x))/(d*e^(12*I*d*x + 6*I*c) + 6*d*e^(10*I*
d*x + 4*I*c) + 15*d*e^(8*I*d*x + 2*I*c) + 15*d*e^(4*I*d*x - 2*I*c) + 6*d*e^
(2*I*d*x - 4*I*c) + 20*d*e^(6*I*d*x) + d*e^(-6*I*c))
```

3.67 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=27

$$-\frac{ia^9}{4d(a-ia\tan(c+dx))^4}$$

```
[Out] ((-I/4)*a^9)/(d*(a - I*a*Tan[c + d*x])^4)
```

Rubi [A] time = 0.0378692, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$-\frac{ia^9}{4d(a-ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-I/4)*a^9)/(d*(a - I*a*Tan[c + d*x])^4)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{\left(ia^9\right)\text{Subst}\left(\int \frac{1}{(a-x)^5}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{ia^9}{4d(a-ia\tan(c+dx))^4}\end{aligned}$$

Mathematica [B] time = 0.932438, size = 73, normalized size = 2.7

$$\frac{a^5(-i(2\sin(c+dx)+3\sin(3(c+dx)))+10\cos(c+dx)+5\cos(3(c+dx)))(\sin(5(c+dx))-i\cos(5(c+dx)))}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(10*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - I*(2*Sin[c + d*x] + 3*Sin[3*(c
 + d*x)]))*((-I)*Cos[5*(c + d*x)] + Sin[5*(c + d*x)]))/(64*d)
```

Maple [B] time = 0.091, size = 301, normalized size = 11.2

$$\frac{1}{d}\left(ia^5\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^4}{8}-\frac{(\sin(dx+c))^2(\cos(dx+c))^4}{12}-\frac{(\cos(dx+c))^4}{24}\right)+5a^5\left(-1/8\,(\sin(dx+c))^3(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(-1/8*sin(d*x+c)^4*cos(d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1
/24*cos(d*x+c)^4)+5*a^5*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*cos(d*x+c)^5*s
in(d*x+c)+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)-
10*I*a^5*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)-10*a^5*(-1/8*si
n(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*
sin(d*x+c)+5/128*d*x+5/128*c)-5/8*I*a^5*cos(d*x+c)^8+a^5*(1/8*(cos(d*x+c)^7
+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*
x+35/128*c))
```

Maxima [B] time = 1.64053, size = 139, normalized size = 5.15

$$\frac{-96i\,a^5\tan(dx+c)^4-384\,a^5\tan(dx+c)^3+576i\,a^5\tan(dx+c)^2+384\,a^5\tan(dx+c)-96i\,a^5}{384\left(\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/384*(-96*I*a^5*tan(d*x + c)^4 - 384*a^5*tan(d*x + c)^3 + 576*I*a^5*tan(d*
x + c)^2 + 384*a^5*tan(d*x + c) - 96*I*a^5)/((tan(d*x + c)^8 + 4*tan(d*x +
c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1)*d)
```

Fricas [B] time = 1.16028, size = 171, normalized size = 6.33

$$\frac{-i\,a^5e^{(8i\,dx+8i\,c)}-4i\,a^5e^{(6i\,dx+6i\,c)}-6i\,a^5e^{(4i\,dx+4i\,c)}-4i\,a^5e^{(2i\,dx+2i\,c)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/64*(-I*a^5*e^(8*I*d*x + 8*I*c) - 4*I*a^5*e^(6*I*d*x + 6*I*c) - 6*I*a^5*e^
(4*I*d*x + 4*I*c) - 4*I*a^5*e^(2*I*d*x + 2*I*c))/d
```

Sympy [B] time = 0.830825, size = 163, normalized size = 6.04

$$\begin{cases}\frac{-8192ia^5d^3e^{8ic}e^{8idx}-32768ia^5d^3e^{6ic}e^{6idx}-49152ia^5d^3e^{4ic}e^{4idx}-32768ia^5d^3e^{2ic}e^{2idx}}{524288d^4} & \text{for } 524288d^4\neq 0\\ x\left(\frac{a^5e^{8ic}}{8}+\frac{3a^5e^{6ic}}{8}+\frac{3a^5e^{4ic}}{8}+\frac{a^5e^{2ic}}{8}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-8192*I*a**5*d**3*exp(8*I*c)*exp(8*I*d*x) - 32768*I*a**5*d**3*e
xp(6*I*c)*exp(6*I*d*x) - 49152*I*a**5*d**3*exp(4*I*c)*exp(4*I*d*x) - 32768*
I*a**5*d**3*exp(2*I*c)*exp(2*I*d*x))/(524288*d**4), Ne(524288*d**4, 0)), (x
*(a**5*exp(8*I*c)/8 + 3*a**5*exp(6*I*c)/8 + 3*a**5*exp(4*I*c)/8 + a**5*exp(
2*I*c)/8), True))
```

Giac [B] time = 1.61835, size = 360, normalized size = 13.33

$$\frac{-24i\,a^5e^{(24i\,dx+16i\,c)} - 288i\,a^5e^{(22i\,dx+14i\,c)} - 1584i\,a^5e^{(20i\,dx+12i\,c)} - 5280i\,a^5e^{(18i\,dx+10i\,c)} - 11856i\,a^5e^{(16i\,dx+8i\,c)} - 18816i\,a^5}{1536\left(de^{(16i\,dx+8i\,c)} + 8\,de^{(14i\,dx+6i\,c)} + 28\,de^{(12i\,dx+4i\,c)} + 56\,de^{(10i\,d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1536*(-24*I*a^5*e^(24*I*d*x + 16*I*c) - 288*I*a^5*e^(22*I*d*x + 14*I*c) -
 1584*I*a^5*e^(20*I*d*x + 12*I*c) - 5280*I*a^5*e^(18*I*d*x + 10*I*c) - 1185
6*I*a^5*e^(16*I*d*x + 8*I*c) - 18816*I*a^5*e^(14*I*d*x + 6*I*c) - 21504*I*a
^5*e^(12*I*d*x + 4*I*c) - 17664*I*a^5*e^(10*I*d*x + 2*I*c) - 3936*I*a^5*e^(
6*I*d*x - 2*I*c) - 912*I*a^5*e^(4*I*d*x - 4*I*c) - 96*I*a^5*e^(2*I*d*x - 6*
I*c) - 10200*I*a^5*e^(8*I*d*x))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*x +
 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6
*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d
*e^(8*I*d*x) + d*e^(-8*I*c))
```

3.68 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=144

$$-\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5}-\frac{ia^9}{16d(a-ia\tan(c+dx))^4}-\frac{ia^8}{24d(a-ia\tan(c+dx))^3}-\frac{ia^7}{32d(a-ia\tan(c+dx))^2}-\frac{}{32d(a-ia}$$

```
[Out] (a^5*x)/32 - ((I/10)*a^10)/(d*(a - I*a*Tan[c + d*x])^5) - ((I/16)*a^9)/(d*(
a - I*a*Tan[c + d*x])^4) - ((I/24)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/
32)*a^7)/(d*(a - I*a*Tan[c + d*x])^2) - ((I/32)*a^6)/(d*(a - I*a*Tan[c + d*
x]))
```

Rubi [A] time = 0.0859356, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5}-\frac{ia^9}{16d(a-ia\tan(c+dx))^4}-\frac{ia^8}{24d(a-ia\tan(c+dx))^3}-\frac{ia^7}{32d(a-ia\tan(c+dx))^2}-\frac{}{32d(a-ia}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*x)/32 - ((I/10)*a^10)/(d*(a - I*a*Tan[c + d*x])^5) - ((I/16)*a^9)/(d*(
a - I*a*Tan[c + d*x])^4) - ((I/24)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/
32)*a^7)/(d*(a - I*a*Tan[c + d*x])^2) - ((I/32)*a^6)/(d*(a - I*a*Tan[c + d*
x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{(ia^{11})\,\text{Subst}\left(\int \frac{1}{(a-x)^6(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^{11})\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^6} + \frac{1}{4a^2(a-x)^5} + \frac{1}{8a^3(a-x)^4} + \frac{1}{16a^4(a-x)^3} + \frac{1}{32a^5(a-x)^2} + \right.\right.}{d}$$

$$= -\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5} - \frac{ia^9}{16d(a-ia\tan(c+dx))^4} - \frac{ia^8}{24d(a-ia\tan(c+}$$

$$= \frac{a^5x}{32} - \frac{ia^{10}}{10d(a-ia\tan(c+dx))^5} - \frac{ia^9}{16d(a-ia\tan(c+dx))^4} - \frac{ia^8}{24d(a-ia\tan}$$

Mathematica [A] time = 1.41093, size = 137, normalized size = 0.95

$$\frac{a^5(-100\sin(c+dx) - 225\sin(3(c+dx)) - 120idx\sin(5(c+dx)) + 12\sin(5(c+dx)) - 500i\cos(c+dx) - 375i\cos(3(c}{3840d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*((-500*I)*Cos[c + d*x] - (375*I)*Cos[3*(c + d*x)] - (12*I)*Cos[5*(c +
d*x)] + 120*d*x*Cos[5*(c + d*x)] - 100*Sin[c + d*x] - 225*Sin[3*(c + d*x)]
+ 12*Sin[5*(c + d*x)] - (120*I)*d*x*Sin[5*(c + d*x)])*(Cos[5*(c + 2*d*x)] +
 I*Sin[5*(c + 2*d*x)]))/(3840*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [B] time = 0.135, size = 331, normalized size = 2.3

$$\frac{1}{d}\left(ia^5\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^6}{10} - \frac{(\sin(dx+c))^2(\cos(dx+c))^6}{20} - \frac{(\cos(dx+c))^6}{60}\right) + 5\,a^5\left(-1/10\,(\sin(dx+c))\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-
1/60*cos(d*x+c)^6)+5*a^5*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*c
os(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c
)+3/256*d*x+3/256*c)-10*I*a^5*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d*x
+c)^8)-10*a^5*(-1/10*sin(d*x+c)*cos(d*x+c)^9+1/80*(cos(d*x+c)^7+7/6*cos(d*x
+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/256*d*x+7/256*c)-1/
2*I*a^5*cos(d*x+c)^10+a^5*(1/10*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*
x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+63/256*d*x+63/256
*c))
```

Maxima [A] time = 1.73067, size = 221, normalized size = 1.53

$$\frac{120\,(dx+c)a^5 + \frac{120\,a^5\tan(dx+c)^9 + 560\,a^5\tan(dx+c)^7 + 1024\,a^5\tan(dx+c)^5 - 640i\,a^5\tan(dx+c)^4 - 1840\,a^5\tan(dx+c)^3 + 4480i\,a^5\tan(dx+c)^2 + 3720\,a^5\,t}{\tan(dx+c)^{10} + 5\,\tan(dx+c)^8 + 10\,\tan(dx+c)^6 + 10\,\tan(dx+c)^4 + 5\,\tan(dx+c)^2 + 1}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/3840*(120*(d*x + c)*a^5 + (120*a^5*tan(d*x + c)^9 + 560*a^5*tan(d*x + c)^
7 + 1024*a^5*tan(d*x + c)^5 - 640*I*a^5*tan(d*x + c)^4 - 1840*a^5*tan(d*x +
 c)^3 + 4480*I*a^5*tan(d*x + c)^2 + 3720*a^5*tan(d*x + c) - 1024*I*a^5)/(ta
n(d*x + c)^10 + 5*tan(d*x + c)^8 + 10*tan(d*x + c)^6 + 10*tan(d*x + c)^4 +
5*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.08802, size = 248, normalized size = 1.72

$$\frac{120\,a^5dx - 12i\,a^5e^{(10i\,dx+10i\,c)} - 75i\,a^5e^{(8i\,dx+8i\,c)} - 200i\,a^5e^{(6i\,dx+6i\,c)} - 300i\,a^5e^{(4i\,dx+4i\,c)} - 300i\,a^5e^{(2i\,dx+2i\,c)}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3840*(120*a^5*d*x - 12*I*a^5*e^(10*I*d*x + 10*I*c) - 75*I*a^5*e^(8*I*d*x
+ 8*I*c) - 200*I*a^5*e^(6*I*d*x + 6*I*c) - 300*I*a^5*e^(4*I*d*x + 4*I*c) -
300*I*a^5*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A] time = 1.0506, size = 211, normalized size = 1.47

$$\frac{a^5x}{32} + \begin{cases} \frac{-100663296ia^5d^4e^{10ic}e^{10idx} - 629145600ia^5d^4e^{8ic}e^{8idx} - 1677721600ia^5d^4e^{6ic}e^{6idx} - 2516582400ia^5d^4e^{4ic}e^{4idx} - 2516582400ia^5d^4e^{2ic}e^{2idx}}{32212254720d^5} & \text{for } 32212 \\ x\left(\frac{a^5e^{10ic}}{32} + \frac{5a^5e^{8ic}}{32} + \frac{5a^5e^{6ic}}{16} + \frac{5a^5e^{4ic}}{16} + \frac{5a^5e^{2ic}}{32}\right) & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**10*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] a**5*x/32 + Piecewise(((-100663296*I*a**5*d**4*exp(10*I*c)*exp(10*I*d*x) -
629145600*I*a**5*d**4*exp(8*I*c)*exp(8*I*d*x) - 1677721600*I*a**5*d**4*exp(
6*I*c)*exp(6*I*d*x) - 2516582400*I*a**5*d**4*exp(4*I*c)*exp(4*I*d*x) - 2516
582400*I*a**5*d**4*exp(2*I*c)*exp(2*I*d*x))/(32212254720*d**5), Ne(32212254
720*d**5, 0)), (x*(a**5*exp(10*I*c)/32 + 5*a**5*exp(8*I*c)/32 + 5*a**5*exp(
6*I*c)/16 + 5*a**5*exp(4*I*c)/16 + 5*a**5*exp(2*I*c)/32), True))
```

Giac [B] time = 1.80842, size = 1157, normalized size = 8.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/30720*(960*a^5*d*x*e^(16*I*d*x + 8*I*c) + 7680*a^5*d*x*e^(14*I*d*x + 6*I*
c) + 26880*a^5*d*x*e^(12*I*d*x + 4*I*c) + 53760*a^5*d*x*e^(10*I*d*x + 2*I*c
) + 53760*a^5*d*x*e^(6*I*d*x - 2*I*c) + 26880*a^5*d*x*e^(4*I*d*x - 4*I*c) +
 7680*a^5*d*x*e^(2*I*d*x - 6*I*c) + 67200*a^5*d*x*e^(8*I*d*x) + 960*a^5*d*x
*e^(-8*I*c) - 390*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) -
```

```
3120*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 10920*I*a^5
*e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 21840*I*a^5*e^(10*I*d*
x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 21840*I*a^5*e^(6*I*d*x - 2*I*c)*l
og(e^(2*I*d*x + 2*I*c) + 1) - 10920*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2*I*d*
x + 2*I*c) + 1) - 3120*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) +
1) - 27300*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 390*I*a^5*e^(-8
*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 390*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(
2*I*d*x) + e^(-2*I*c)) + 3120*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x) +
e^(-2*I*c)) + 10920*I*a^5*e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)
) + 21840*I*a^5*e^(10*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 21840*
I*a^5*e^(6*I*d*x - 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 10920*I*a^5*e^(4*
I*d*x - 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 3120*I*a^5*e^(2*I*d*x - 6*I*
c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 27300*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x)
+ e^(-2*I*c)) + 390*I*a^5*e^(-8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) - 96*I*a
^5*e^(26*I*d*x + 18*I*c) - 1368*I*a^5*e^(24*I*d*x + 16*I*c) - 9088*I*a^5*e^
(22*I*d*x + 14*I*c) - 37376*I*a^5*e^(20*I*d*x + 12*I*c) - 106720*I*a^5*e^(1
8*I*d*x + 10*I*c) - 223376*I*a^5*e^(16*I*d*x + 8*I*c) - 349888*I*a^5*e^(14*
I*d*x + 6*I*c) - 409568*I*a^5*e^(12*I*d*x + 4*I*c) - 352096*I*a^5*e^(10*I*d
*x + 2*I*c) - 88000*I*a^5*e^(6*I*d*x - 2*I*c) - 21600*I*a^5*e^(4*I*d*x - 4*
I*c) - 2400*I*a^5*e^(2*I*d*x - 6*I*c) - 215000*I*a^5*e^(8*I*d*x))/(d*e^(16*
I*d*x + 8*I*c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*
d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c
) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

3.69 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=198

$$-\frac{ia^{11}}{24d(a-ia\tan(c+dx))^6}-\frac{ia^{10}}{20d(a-ia\tan(c+dx))^5}-\frac{3ia^9}{64d(a-ia\tan(c+dx))^4}-\frac{ia^8}{24d(a-ia\tan(c+dx))^3}-\frac{}{128d(a-}$$

```
[Out] (7*a^5*x)/128 - ((I/24)*a^11)/(d*(a - I*a*Tan[c + d*x])^6) - ((I/20)*a^10)/
(d*(a - I*a*Tan[c + d*x])^5) - (((3*I)/64)*a^9)/(d*(a - I*a*Tan[c + d*x])^4
) - ((I/24)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) - (((5*I)/128)*a^7)/(d*(a - I
*a*Tan[c + d*x])^2) - (((3*I)/64)*a^6)/(d*(a - I*a*Tan[c + d*x])) + ((I/128
)*a^6)/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.11411, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{ia^{11}}{24d(a-ia\tan(c+dx))^6}-\frac{ia^{10}}{20d(a-ia\tan(c+dx))^5}-\frac{3ia^9}{64d(a-ia\tan(c+dx))^4}-\frac{ia^8}{24d(a-ia\tan(c+dx))^3}-\frac{}{128d(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^5,x]
```


```
[Out] (7*a^5*x)/128 - ((I/24)*a^11)/(d*(a - I*a*Tan[c + d*x])^6) - ((I/20)*a^10)/
(d*(a - I*a*Tan[c + d*x])^5) - (((3*I)/64)*a^9)/(d*(a - I*a*Tan[c + d*x])^4
) - ((I/24)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) - (((5*I)/128)*a^7)/(d*(a - I
*a*Tan[c + d*x])^2) - (((3*I)/64)*a^6)/(d*(a - I*a*Tan[c + d*x])) + ((I/128
)*a^6)/(d*(a + I*a*Tan[c + d*x]))
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{(ia^{13})\,\text{Subst}\left(\int \frac{1}{(a-x)^7(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^{13})\,\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^7} + \frac{1}{4a^3(a-x)^6} + \frac{3}{16a^4(a-x)^5} + \frac{1}{8a^5(a-x)^4} + \frac{5}{64a^6(a-x)^3} + \right.\right.}{d}$$

$$= -\frac{ia^{11}}{24d(a-ia\tan(c+dx))^6} - \frac{ia^{10}}{20d(a-ia\tan(c+dx))^5} - \frac{3ia^9}{64d(a-ia\tan(c+}$$

$$= \frac{7a^5x}{128} - \frac{ia^{11}}{24d(a-ia\tan(c+dx))^6} - \frac{ia^{10}}{20d(a-ia\tan(c+dx))^5} - \frac{3i}{64d(a-ia\,\text{t}}$$

Mathematica [A] time = 2.64195, size = 159, normalized size = 0.8

$$\frac{a^5(-350\sin(c+dx) - 945\sin(3(c+dx)) - 840idx\sin(5(c+dx)) + 84\sin(5(c+dx)) + 70\sin(7(c+dx)) - 1750i\cos(}{15360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*((-1750*I)*Cos[c + d*x] - (1575*I)*Cos[3*(c + d*x)] - (84*I)*Cos[5*(c
+ d*x)] + 840*d*x*Cos[5*(c + d*x)] + (50*I)*Cos[7*(c + d*x)] - 350*Sin[c +
d*x] - 945*Sin[3*(c + d*x)] + 84*Sin[5*(c + d*x)] - (840*I)*d*x*Sin[5*(c +
d*x)] + 70*Sin[7*(c + d*x)])*(Cos[5*(c + 2*d*x)] + I*Sin[5*(c + 2*d*x)]))/(
15360*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [B] time = 0.133, size = 361, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(-1/12*sin(d*x+c)^4*cos(d*x+c)^8-1/30*sin(d*x+c)^2*cos(d*x+c)^8-
1/120*cos(d*x+c)^8)+5*a^5*(-1/12*sin(d*x+c)^3*cos(d*x+c)^9-1/40*sin(d*x+c)*
cos(d*x+c)^9+1/320*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*
cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)-10*I*a^5*(-1/12*sin(d*x+c)^2*co
s(d*x+c)^10-1/60*cos(d*x+c)^10)-10*a^5*(-1/12*sin(d*x+c)*cos(d*x+c)^11+1/12
0*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315
/128*cos(d*x+c))*sin(d*x+c)+21/1024*d*x+21/1024*c)-5/12*I*a^5*cos(d*x+c)^12
+a^5*(1/12*(cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos
(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+231/1024*d*x+
231/1024*c))
```

Maxima [A] time = 1.67657, size = 252, normalized size = 1.27

$$\frac{840\,(dx+c)a^5 + \frac{840\,a^5\tan(dx+c)^{11} + 4760\,a^5\tan(dx+c)^9 + 11088\,a^5\tan(dx+c)^7 + 13488\,a^5\tan(dx+c)^5 - 1920i\,a^5\tan(dx+c)^4 + 360\,a^5\tan(dx+c)^3 + 1459}{\tan(dx+c)^{12} + 6\,\tan(dx+c)^{10} + 15\,\tan(dx+c)^8 + 20\,\tan(dx+c)^6 + 15\,\tan(dx+c)^4 + 6\,\tan(dx+c}}{15360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/15360*(840*(d*x + c)*a^5 + (840*a^5*tan(d*x + c)^11 + 4760*a^5*tan(d*x +
c)^9 + 11088*a^5*tan(d*x + c)^7 + 13488*a^5*tan(d*x + c)^5 - 1920*I*a^5*tan
(d*x + c)^4 + 360*a^5*tan(d*x + c)^3 + 14592*I*a^5*tan(d*x + c)^2 + 14520*a
^5*tan(d*x + c) - 3968*I*a^5)/(tan(d*x + c)^12 + 6*tan(d*x + c)^10 + 15*tan
(d*x + c)^8 + 20*tan(d*x + c)^6 + 15*tan(d*x + c)^4 + 6*tan(d*x + c)^2 + 1)
)/d
```

Fricas [A] time = 1.31561, size = 371, normalized size = 1.87

$$\frac{\left(840\,a^{5}dxe^{(2i\,dx+2i\,c)} - 10i\,a^{5}e^{(14i\,dx+14i\,c)} - 84i\,a^{5}e^{(12i\,dx+12i\,c)} - 315i\,a^{5}e^{(10i\,dx+10i\,c)} - 700i\,a^{5}e^{(8i\,dx+8i\,c)} - 1050i\,a^{5}e^{(6i\,dx+6i\,c)} - \cdots}{15360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/15360*(840*a^5*d*x*e^(2*I*d*x + 2*I*c) - 10*I*a^5*e^(14*I*d*x + 14*I*c) -
 84*I*a^5*e^(12*I*d*x + 12*I*c) - 315*I*a^5*e^(10*I*d*x + 10*I*c) - 700*I*a
^5*e^(8*I*d*x + 8*I*c) - 1050*I*a^5*e^(6*I*d*x + 6*I*c) - 1260*I*a^5*e^(4*I
*d*x + 4*I*c) + 60*I*a^5)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A] time = 1.32525, size = 303, normalized size = 1.53

$$\frac{7a^5x}{128} + \begin{cases} \frac{\left(-33776997205278720ia^5d^6e^{14ic}e^{12idx} - 283726776524341248ia^5d^6e^{12ic}e^{10idx} - 1063975411966279680ia^5d^6e^{10ic}e^{8idx} - 2364389804369510400ia^5d^6e^{8ic}\cdots}{51881467707308113920d^7} \\ x\left(-\frac{7a^5}{128} + \frac{\left(a^5e^{14ic}+7a^5e^{12ic}+21a^5e^{10ic}+35a^5e^{8ic}+35a^5e^{6ic}+21a^5e^{4ic}+7a^5e^{2ic}+a^5\right)e^{-2ic}}{128}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**12*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 7*a**5*x/128 + Piecewise(((-33776997205278720*I*a**5*d**6*exp(14*I*c)*exp(1
2*I*d*x) - 283726776524341248*I*a**5*d**6*exp(12*I*c)*exp(10*I*d*x) - 10639
75411966279680*I*a**5*d**6*exp(10*I*c)*exp(8*I*d*x) - 2364389804369510400*I
*a**5*d**6*exp(8*I*c)*exp(6*I*d*x) - 3546584706554265600*I*a**5*d**6*exp(6*
I*c)*exp(4*I*d*x) - 4255901647865118720*I*a**5*d**6*exp(4*I*c)*exp(2*I*d*x)
 + 202661983231672320*I*a**5*d**6*exp(-2*I*d*x))*exp(-2*I*c)/(5188146770730
8113920*d**7), Ne(51881467707308113920*d**7*exp(2*I*c), 0)), (x*(-7*a**5/12
8 + (a**5*exp(14*I*c) + 7*a**5*exp(12*I*c) + 21*a**5*exp(10*I*c) + 35*a**5*
exp(8*I*c) + 35*a**5*exp(6*I*c) + 21*a**5*exp(4*I*c) + 7*a**5*exp(2*I*c) +
a**5)*exp(-2*I*c)/128), True))
```

Giac [B] time = 1.86856, size = 1234, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/245760*(13440*a^5*d*x*e^(18*I*d*x + 10*I*c) + 107520*a^5*d*x*e^(16*I*d*x
+ 8*I*c) + 376320*a^5*d*x*e^(14*I*d*x + 6*I*c) + 752640*a^5*d*x*e^(12*I*d*x
 + 4*I*c) + 940800*a^5*d*x*e^(10*I*d*x + 2*I*c) + 376320*a^5*d*x*e^(6*I*d*x
 - 2*I*c) + 107520*a^5*d*x*e^(4*I*d*x - 4*I*c) + 13440*a^5*d*x*e^(2*I*d*x -
 6*I*c) + 752640*a^5*d*x*e^(8*I*d*x) - 4710*I*a^5*e^(18*I*d*x + 10*I*c)*log
(e^(2*I*d*x + 2*I*c) + 1) - 37680*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) - 131880*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c)
+ 1) - 263760*I*a^5*e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 329
700*I*a^5*e^(10*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 131880*I*a^5*
e^(6*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 37680*I*a^5*e^(4*I*d*x -
 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 4710*I*a^5*e^(2*I*d*x - 6*I*c)*log(e
^(2*I*d*x + 2*I*c) + 1) - 263760*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x + 2*I*c)
+ 1) + 4710*I*a^5*e^(18*I*d*x + 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 376
80*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 131880*I*a^5*
e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 263760*I*a^5*e^(12*I*d
*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 329700*I*a^5*e^(10*I*d*x + 2*I*
c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 131880*I*a^5*e^(6*I*d*x - 2*I*c)*log(e^(
2*I*d*x) + e^(-2*I*c)) + 37680*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2*I*d*x) +
e^(-2*I*c)) + 4710*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c))
+ 263760*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) - 160*I*a^5*e^(30*
I*d*x + 22*I*c) - 2624*I*a^5*e^(28*I*d*x + 20*I*c) - 20272*I*a^5*e^(26*I*d*
x + 18*I*c) - 98112*I*a^5*e^(24*I*d*x + 16*I*c) - 333984*I*a^5*e^(22*I*d*x
+ 14*I*c) - 853440*I*a^5*e^(20*I*d*x + 12*I*c) - 1691424*I*a^5*e^(18*I*d*x
+ 10*I*c) - 2609472*I*a^5*e^(16*I*d*x + 8*I*c) - 3076512*I*a^5*e^(14*I*d*x
+ 6*I*c) - 2680384*I*a^5*e^(12*I*d*x + 4*I*c) - 1640240*I*a^5*e^(10*I*d*x +
 2*I*c) - 124320*I*a^5*e^(6*I*d*x - 2*I*c) + 6720*I*a^5*e^(4*I*d*x - 4*I*c)
 + 7680*I*a^5*e^(2*I*d*x - 6*I*c) - 642880*I*a^5*e^(8*I*d*x) + 960*I*a^5*e^
(-8*I*c))/(d*e^(18*I*d*x + 10*I*c) + 8*d*e^(16*I*d*x + 8*I*c) + 28*d*e^(14*
I*d*x + 6*I*c) + 56*d*e^(12*I*d*x + 4*I*c) + 70*d*e^(10*I*d*x + 2*I*c) + 28
*d*e^(6*I*d*x - 2*I*c) + 8*d*e^(4*I*d*x - 4*I*c) + d*e^(2*I*d*x - 6*I*c) +
56*d*e^(8*I*d*x))
```

3.70 $\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=167

$$\frac{63ia^5\sec(c+dx)}{8d}+\frac{63a^5\tanh^{-1}(\sin(c+dx))}{8d}+\frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d}+\frac{21ia\sec(c+dx)\left(a^2+ia^2\tan(c\right.}{20d}$$

```
[Out] (63*a^5*ArcTanh[Sin[c + d*x]])/(8*d) + (((63*I)/8)*a^5*Sec[c + d*x])/d + ((
(9*I)/20)*a^2*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d + ((I/5)*a*Sec[c + d
*x]*(a + I*a*Tan[c + d*x])^4)/d + (((21*I)/20)*a*Sec[c + d*x]*(a^2 + I*a^2*
Tan[c + d*x])^2)/d + (((21*I)/8)*Sec[c + d*x]*(a^5 + I*a^5*Tan[c + d*x]))/d
```

Rubi [A] time = 0.126343, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3498, 3486, 3770}

$$\frac{63ia^5\sec(c+dx)}{8d}+\frac{63a^5\tanh^{-1}(\sin(c+dx))}{8d}+\frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d}+\frac{21ia\sec(c+dx)\left(a^2+ia^2\tan(c\right.}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (63*a^5*ArcTanh[Sin[c + d*x]])/(8*d) + (((63*I)/8)*a^5*Sec[c + d*x])/d + ((
(9*I)/20)*a^2*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d + ((I/5)*a*Sec[c + d
*x]*(a + I*a*Tan[c + d*x])^4)/d + (((21*I)/20)*a*Sec[c + d*x]*(a^2 + I*a^2*
Tan[c + d*x])^2)/d + (((21*I)/8)*Sec[c + d*x]*(a^5 + I*a^5*Tan[c + d*x]))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d} + \frac{1}{5}(9a)\int \sec(c+dx)(a+ia\tan(c+dx)\cdots$$
$$= \frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d} + \cdots$$
$$= \frac{21ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{20d} + \frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))\cdots}{20d}$$
$$= \frac{21ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{20d} + \frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))\cdots}{20d}$$
$$= \frac{63ia^5\sec(c+dx)}{8d} + \frac{21ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{20d} + \frac{9ia^2\sec(c+dx\cdots}{}$$
$$= \frac{63a^5\tanh^{-1}(\sin(c+dx))}{8d} + \frac{63ia^5\sec(c+dx)}{8d} + \frac{21ia^3\sec(c+dx)(a+ia\tan\cdots}{20d}$$

Mathematica [A] time = 1.01146, size = 115, normalized size = 0.69

$$\frac{a^5(\cos(5dx)+i\sin(5dx))\left(5040\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)+i\sec^5(c+dx)(450i\sin(2(c+dx))+325i\sin(4(c\cdots\right.}{320d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(Cos[5*d*x] + I*Sin[5*d*x])*(5040*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]
] + I*Sec[c + d*x]^5*(1344 + 1920*Cos[2*(c + d*x)] + 640*Cos[4*(c + d*x)] +
 (450*I)*Sin[2*(c + d*x)] + (325*I)*Sin[4*(c + d*x)])))/(320*d*(Cos[d*x] +
I*Sin[d*x])^5)
```

Maple [B] time = 0.072, size = 329, normalized size = 2.

$$\frac{\frac{36i}{5}a^5\cos(dx+c)}{d} + \frac{\frac{10i}{3}a^5(\sin(dx+c))^4}{d\cos(dx+c)} - \frac{\frac{10i}{3}a^5(\sin(dx+c))^4}{d(\cos(dx+c))^3} + \frac{\frac{i}{5}a^5(\sin(dx+c))^6}{d(\cos(dx+c))^5} + \frac{\frac{18i}{5}a^5\cos(dx+c)(\sin(dx+\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 36/5*I/d*a^5*cos(d*x+c)+10/3*I/d*a^5*sin(d*x+c)^4/cos(d*x+c)-10/3*I/d*a^5*s
in(d*x+c)^4/cos(d*x+c)^3+1/5*I/d*a^5*sin(d*x+c)^6/cos(d*x+c)^5+18/5*I/d*a^5
*cos(d*x+c)*sin(d*x+c)^2+5*I/d*a^5/cos(d*x+c)+5/4/d*a^5*sin(d*x+c)^5/cos(d*
x+c)^4-5/8/d*a^5*sin(d*x+c)^5/cos(d*x+c)^2-5/8*a^5*sin(d*x+c)^3/d-55/8*a^5*
sin(d*x+c)/d+63/8/d*a^5*ln(sec(d*x+c)+tan(d*x+c))-1/15*I/d*a^5*sin(d*x+c)^6
/cos(d*x+c)^3+1/5*I/d*a^5*sin(d*x+c)^6/cos(d*x+c)+1/5*I/d*a^5*cos(d*x+c)*si
n(d*x+c)^4-5/d*a^5*sin(d*x+c)^3/cos(d*x+c)^2
```

Maxima [A] time = 1.11655, size = 290, normalized size = 1.74

$$75a^5\left(\frac{2\left(5\sin(dx+c)^3-3\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}+3\log(\sin(dx+c)+1)-3\log(\sin(dx+c)-1)\right)+600a^5\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/240*(75*a^5*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) + 60
0*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(si
n(d*x + c) - 1)) + 240*a^5*log(sec(d*x + c) + tan(d*x + c)) + 1200*I*a^5/co
s(d*x + c) + 800*I*(3*cos(d*x + c)^2 - 1)*a^5/cos(d*x + c)^3 + 16*I*(15*cos
(d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*a^5/cos(d*x + c)^5)/d
```

Fricas [B] time = 1.13587, size = 891, normalized size = 5.34

$$\frac{1930i\,a^5e^{(9i\,dx+9i\,c)} + 4740i\,a^5e^{(7i\,dx+7i\,c)} + 5376i\,a^5e^{(5i\,dx+5i\,c)} + 2940i\,a^5e^{(3i\,dx+3i\,c)} + 630i\,a^5e^{(i\,dx+i\,c)} + 315\left(a^5e^{(10i\,dx+10i\,c)} + \cdots\right.}{40\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/40*(1930*I*a^5*e^(9*I*d*x + 9*I*c) + 4740*I*a^5*e^(7*I*d*x + 7*I*c) + 537
6*I*a^5*e^(5*I*d*x + 5*I*c) + 2940*I*a^5*e^(3*I*d*x + 3*I*c) + 630*I*a^5*e^
(I*d*x + I*c) + 315*(a^5*e^(10*I*d*x + 10*I*c) + 5*a^5*e^(8*I*d*x + 8*I*c)
+ 10*a^5*e^(6*I*d*x + 6*I*c) + 10*a^5*e^(4*I*d*x + 4*I*c) + 5*a^5*e^(2*I*d*
x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) + I) - 315*(a^5*e^(10*I*d*x + 10*I*c)
 + 5*a^5*e^(8*I*d*x + 8*I*c) + 10*a^5*e^(6*I*d*x + 6*I*c) + 10*a^5*e^(4*I*d
*x + 4*I*c) + 5*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d
*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c)
 + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^5\left(\int -10\tan^2(c+dx)\sec(c+dx)\,dx + \int 5\tan^4(c+dx)\sec(c+dx)\,dx + \int 5i\tan(c+dx)\sec(c+dx)\,dx + \int -10i\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] a**5*(Integral(-10*tan(c + d*x)**2*sec(c + d*x), x) + Integral(5*tan(c + d*
x)**4*sec(c + d*x), x) + Integral(5*I*tan(c + d*x)*sec(c + d*x), x) + Integ
ral(-10*I*tan(c + d*x)**3*sec(c + d*x), x) + Integral(I*tan(c + d*x)**5*sec
(c + d*x), x) + Integral(sec(c + d*x), x))
```

Giac [A] time = 1.46856, size = 258, normalized size = 1.54

$$\frac{315\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 315\,a^5\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(275\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+200i\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-750\,a^5\tan\cdots\right.}{\cdots}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/40*(315*a^5*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 315*a^5*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(275*a^5*tan(1/2*d*x + 1/2*c)^9 + 200*I*a^5*tan(1/2*
d*x + 1/2*c)^8 - 750*a^5*tan(1/2*d*x + 1/2*c)^7 - 1600*I*a^5*tan(1/2*d*x +
1/2*c)^6 + 3280*I*a^5*tan(1/2*d*x + 1/2*c)^4 + 750*a^5*tan(1/2*d*x + 1/2*c)
^3 - 2240*I*a^5*tan(1/2*d*x + 1/2*c)^2 - 275*a^5*tan(1/2*d*x + 1/2*c) + 488
*I*a^5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.71 $\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=130

$$-\frac{35ia^5\sec(c+dx)}{2d}-\frac{35a^5\tanh^{-1}(\sin(c+dx))}{2d}-\frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{35i\sec(c+dx)\left(a^5+ia^5\tan(c\right.}{6d}$$

```
[Out] (-35*a^5*ArcTanh[Sin[c + d*x]])/(2*d) - (((35*I)/2)*a^5*Sec[c + d*x])/d - (
((7*I)/3)*a^3*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d - ((2*I)*a*Cos[c + d
*x]*(a + I*a*Tan[c + d*x])^4)/d - (((35*I)/6)*Sec[c + d*x]*(a^5 + I*a^5*Tan
[c + d*x]))/d
```

Rubi [A] time = 0.10423, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3496, 3498, 3486, 3770}

$$-\frac{35ia^5\sec(c+dx)}{2d}-\frac{35a^5\tanh^{-1}(\sin(c+dx))}{2d}-\frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{35i\sec(c+dx)\left(a^5+ia^5\tan(c\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (-35*a^5*ArcTanh[Sin[c + d*x]])/(2*d) - (((35*I)/2)*a^5*Sec[c + d*x])/d - (
((7*I)/3)*a^3*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d - ((2*I)*a*Cos[c + d
*x]*(a + I*a*Tan[c + d*x])^4)/d - (((35*I)/6)*Sec[c + d*x]*(a^5 + I*a^5*Tan
[c + d*x]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d} - \left(7a^2\right)\int \sec(c+dx)(a+ia\tan(c+a$$
$$= -\frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$
$$= -\frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$
$$= -\frac{35ia^5\sec(c+dx)}{2d} - \frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos(c+dx)}{}$$
$$= -\frac{35a^5\tanh^{-1}(\sin(c+dx))}{2d} - \frac{35ia^5\sec(c+dx)}{2d} - \frac{7ia^3\sec(c+dx)(a+ia\tan}{3d}$$

Mathematica [A] time = 1.54654, size = 151, normalized size = 1.16

$$\frac{a^5\cos^2(c+dx)(\tan(c+dx)-i)^5\left((\cos(4c-dx)-i\sin(4c-dx))(-i(49\sin(c+dx)+57\sin(3(c+dx)))+511\cos(c+\right.}{24d(\cos(dx)+i\sin(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Cos[c + d*x]^2*((-840*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c +
 d*x]^3*(Cos[5*c] - I*Sin[5*c]) + (Cos[4*c - d*x] - I*Sin[4*c - d*x])*(511*
Cos[c + d*x] + 153*Cos[3*(c + d*x)] - I*(49*Sin[c + d*x] + 57*Sin[3*(c + d*
x)])))*(-I + Tan[c + d*x])^5)/(24*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [A] time = 0.064, size = 214, normalized size = 1.7

$$\frac{-ia^5(\sin(dx+c))^6}{d\cos(dx+c)} + \frac{\frac{i}{3}a^5(\sin(dx+c))^6}{d(\cos(dx+c))^3} - \frac{10\,ia^5(\sin(dx+c))^4}{d\cos(dx+c)} - \frac{ia^5\cos(dx+c)(\sin(dx+c))^4}{d} - \frac{\frac{34\,i}{3}a^5\cos(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] -I/d*a^5*sin(d*x+c)^6/cos(d*x+c)+1/3*I/d*a^5*sin(d*x+c)^6/cos(d*x+c)^3-10*I
/d*a^5*sin(d*x+c)^4/cos(d*x+c)-I/d*a^5*cos(d*x+c)*sin(d*x+c)^4-34/3*I/d*a^5
*cos(d*x+c)*sin(d*x+c)^2+5/2/d*a^5*sin(d*x+c)^5/cos(d*x+c)^2+5/2*a^5*sin(d*
x+c)^3/d+37/2*a^5*sin(d*x+c)/d-35/2/d*a^5*ln(sec(d*x+c)+tan(d*x+c))-83/3*I/
d*a^5*cos(d*x+c)
```

Maxima [A] time = 1.13268, size = 234, normalized size = 1.8

$$-\frac{15\,a^5\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + 3\log(\sin(dx+c)+1) - 3\log(\sin(dx+c)-1) - 4\sin(dx+c)\right) + 120i\,a^5\left(\frac{1}{\cos(dx+c)} + \cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/12*(15*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1
) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 120*I*a^5*(1/cos(d*x + c) +
 cos(d*x + c)) + 4*I*a^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x
 + c)) + 60*a^5*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x
+ c)) + 60*I*a^5*cos(d*x + c) - 12*a^5*sin(d*x + c))/d
```

Fricas [A] time = 1.18931, size = 603, normalized size = 4.64

$$\frac{-96i\,a^5e^{(7i\,dx+7i\,c)} - 462i\,a^5e^{(5i\,dx+5i\,c)} - 560i\,a^5e^{(3i\,dx+3i\,c)} - 210i\,a^5e^{(i\,dx+i\,c)} - 105\left(a^5e^{(6i\,dx+6i\,c)} + 3\,a^5e^{(4i\,dx+4i\,c)} + 3\,a^5e^{(2i\,dx+}\right.}{6\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4i\,dx+4i\,c)} + 3\,d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(-96*I*a^5*e^(7*I*d*x + 7*I*c) - 462*I*a^5*e^(5*I*d*x + 5*I*c) - 560*I*
a^5*e^(3*I*d*x + 3*I*c) - 210*I*a^5*e^(I*d*x + I*c) - 105*(a^5*e^(6*I*d*x +
 6*I*c) + 3*a^5*e^(4*I*d*x + 4*I*c) + 3*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(
e^(I*d*x + I*c) + I) + 105*(a^5*e^(6*I*d*x + 6*I*c) + 3*a^5*e^(4*I*d*x + 4*
I*c) + 3*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d*e^(6*I
*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 2.24636, size = 196, normalized size = 1.51

$$\frac{35a^5\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2} - \frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d} + \frac{-\frac{29ia^5e^{-ic}e^{5idx}}{d} - \frac{136ia^5e^{-3ic}e^{3idx}}{3d} - \frac{19ia^5e^{-5ic}e^{idx}}{d}}{e^{6idx} + 3e^{-2ic}e^{4idx} + 3e^{-4ic}e^{2idx} + e^{-6ic}} + \begin{cases} -\frac{16ia^5e^{ic}e^{idx}}{d} & \text{for}\ d \neq 0 \\ 16a^5xe^{ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 35*a**5*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2)
/d + (-29*I*a**5*exp(-I*c)*exp(5*I*d*x)/d - 136*I*a**5*exp(-3*I*c)*exp(3*I*
d*x)/(3*d) - 19*I*a**5*exp(-5*I*c)*exp(I*d*x)/d)/(exp(6*I*d*x) + 3*exp(-2*I
*c)*exp(4*I*d*x) + 3*exp(-4*I*c)*exp(2*I*d*x) + exp(-6*I*c)) + Piecewise((-
16*I*a**5*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (16*a**5*x*exp(I*c), True))
```

Giac [B] time = 1.7035, size = 689, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1536*(8295*a^5*e^(6*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 24885*a^5
*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 24885*a^5*e^(2*I*d*x + 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) - 18585*a^5*e^(6*I*d*x + 6*I*c)*log(I*e^(I*
d*x + I*c) - 1) - 55755*a^5*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1)
- 55755*a^5*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) - 8295*a^5*e^(6*
I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 24885*a^5*e^(4*I*d*x + 4*I*c)*
log(-I*e^(I*d*x + I*c) + 1) - 24885*a^5*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) + 18585*a^5*e^(6*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) +
55755*a^5*e^(4*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 55755*a^5*e^(2*
I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 24576*I*a^5*e^(7*I*d*x + 7*I*c
) - 118272*I*a^5*e^(5*I*d*x + 5*I*c) - 143360*I*a^5*e^(3*I*d*x + 3*I*c) - 5
3760*I*a^5*e^(I*d*x + I*c) + 8295*a^5*log(I*e^(I*d*x + I*c) + 1) - 18585*a^
5*log(I*e^(I*d*x + I*c) - 1) - 8295*a^5*log(-I*e^(I*d*x + I*c) + 1) + 18585
*a^5*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x +
 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

3.72 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=98

$$\frac{5ia^5\sec(c+dx)}{d}+\frac{5a^5\tanh^{-1}(\sin(c+dx))}{d}+\frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+d}{3d}$$

```
[Out] (5*a^5*ArcTanh[Sin[c + d*x]])/d + ((5*I)*a^5*Sec[c + d*x])/d + (((10*I)/3)*
a^3*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d - (((2*I)/3)*a*Cos[c + d*x]^3*
(a + I*a*Tan[c + d*x])^4)/d
```

Rubi [A] time = 0.0892792, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3496, 3486, 3770}

$$\frac{5ia^5\sec(c+dx)}{d}+\frac{5a^5\tanh^{-1}(\sin(c+dx))}{d}+\frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+d}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (5*a^5*ArcTanh[Sin[c + d*x]])/d + ((5*I)*a^5*Sec[c + d*x])/d + (((10*I)/3)*
a^3*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d - (((2*I)/3)*a*Cos[c + d*x]^3*
(a + I*a*Tan[c + d*x])^4)/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d} - \frac{1}{3}\left(5a^2\right)\int \cos(c+dx)(a+ia\tan($$
$$= \frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx}{3d}$$
$$= \frac{5ia^5\sec(c+dx)}{d} + \frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos^3(c+dx}{}$$
$$= \frac{5a^5\tanh^{-1}(\sin(c+dx))}{d} + \frac{5ia^5\sec(c+dx)}{d} + \frac{10ia^3\cos(c+dx)(a+ia\tan(}{3d}$$

Mathematica [A] time = 1.63698, size = 130, normalized size = 1.33

$$\frac{a^5\cos^4(c+dx)(\tan(c+dx)-i)^5\left(30(\sin(5c)+i\cos(5c))\cos(c+dx)\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)-(\cos(3c-2d}{3d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Cos[c + d*x]^4*(30*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]*
(I*Cos[5*c] + Sin[5*c]) - (Cos[3*c - 2*d*x] - I*Sin[3*c - 2*d*x])*(10 + 13*
Cos[2*(c + d*x)] - (17*I)*Sin[2*(c + d*x)]))*(-I + Tan[c + d*x])^5)/(3*d*(C
os[d*x] + I*Sin[d*x])^5)
```

Maple [A] time = 0.062, size = 179, normalized size = 1.8

$$\frac{ia^5(\sin(dx+c))^6}{d\cos(dx+c)} + \frac{\frac{28i}{3}a^5\cos(dx+c)}{d} + \frac{ia^5\cos(dx+c)(\sin(dx+c))^4}{d} + \frac{\frac{14i}{3}a^5\cos(dx+c)(\sin(dx+c))^2}{d} - 5\frac{a^5(\mathrm{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] I/d*a^5*sin(d*x+c)^6/cos(d*x+c)+28/3*I/d*a^5*cos(d*x+c)+I/d*a^5*cos(d*x+c)*
sin(d*x+c)^4+14/3*I/d*a^5*cos(d*x+c)*sin(d*x+c)^2-5*a^5*sin(d*x+c)^3/d-13/3
*a^5*sin(d*x+c)/d+5/d*a^5*ln(sec(d*x+c)+tan(d*x+c))-5/3*I/d*a^5*cos(d*x+c)^
3+1/3/d*sin(d*x+c)*cos(d*x+c)^2*a^5
```

Maxima [A] time = 1.13882, size = 208, normalized size = 2.12

$$-\frac{10i\,a^5\cos(dx+c)^3+20\,a^5\sin(dx+c)^3+2i\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\cos(dx+c)\right)a^5+20i\left(\cos(dx+c)^3-3\,\mathrm{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/6*(10*I*a^5*cos(d*x + c)^3 + 20*a^5*sin(d*x + c)^3 + 2*I*(cos(d*x + c)^3
 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^5 + 20*I*(cos(d*x + c)^3 - 3*cos(d*x
+ c))*a^5 + 5*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x +
```

```
c) - 1) + 6*sin(d*x + c))*a^5 + 2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^5)/d
```

Fricas [A] time = 1.28215, size = 332, normalized size = 3.39

$$\frac{-4i\,a^5e^{(5i\,dx+5i\,c)}+20i\,a^5e^{(3i\,dx+3i\,c)}+30i\,a^5e^{(i\,dx+i\,c)}+15\left(a^5e^{(2i\,dx+2i\,c)}+a^5\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(a^5e^{(2i\,dx+2i\,c)}+a^5\right)\log\left(\right.}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3*(-4*I*a^5*e^(5*I*d*x + 5*I*c) + 20*I*a^5*e^(3*I*d*x + 3*I*c) + 30*I*a^5
*e^(I*d*x + I*c) + 15*(a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) +
 I) - 15*(a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d*e^(2*
I*d*x + 2*I*c) + d)
```

Sympy [A] time = 0.977797, size = 150, normalized size = 1.53

$$\frac{5a^5\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d}+\frac{2ia^5e^{-ic}e^{idx}}{d\left(e^{2idx}+e^{-2ic}\right)}+\begin{cases}\frac{-4ia^5de^{3ic}e^{3idx}+24ia^5de^{ic}e^{idx}}{3d^2} & \text{for } 3d^2\neq 0\\ x\left(4a^5e^{3ic}-8a^5e^{ic}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 5*a**5*(-log(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d +
 2*I*a**5*exp(-I*c)*exp(I*d*x)/(d*(exp(2*I*d*x) + exp(-2*I*c))) + Piecewise
(((-4*I*a**5*d*exp(3*I*c)*exp(3*I*d*x) + 24*I*a**5*d*exp(I*c)*exp(I*d*x))/(
3*d**2), Ne(3*d**2, 0)), (x*(4*a**5*exp(3*I*c) - 8*a**5*exp(I*c)), True))
```

Giac [B] time = 2.05691, size = 2272, normalized size = 23.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/6144*(39225*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 313800
*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1098300*a^5*e^(12*I*
d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2196600*a^5*e^(10*I*d*x + 2*I*c)*
log(I*e^(I*d*x + I*c) + 1) + 2196600*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x
 + I*c) + 1) + 1098300*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 313800*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2745750*a^5*e^
(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 39225*a^5*e^(-8*I*c)*log(I*e^(I*d*x
+ I*c) + 1) + 8520*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 68
160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 238560*a^5*e^(12*
I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 477120*a^5*e^(10*I*d*x + 2*I*c)
*log(I*e^(I*d*x + I*c) - 1) + 477120*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x
 + I*c) - 1) + 238560*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) +
68160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 596400*a^5*e^(8*
```

```
I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 8520*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*
c) - 1) - 39225*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3138
00*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1098300*a^5*e^(12
*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2196600*a^5*e^(10*I*d*x + 2*I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 2196600*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 1098300*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c)
 + 1) - 313800*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 274575
0*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 39225*a^5*e^(-8*I*c)*log(-I
*e^(I*d*x + I*c) + 1) - 8520*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c
) - 1) - 68160*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 23856
0*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 477120*a^5*e^(10*I
*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 477120*a^5*e^(6*I*d*x - 2*I*c)*
log(-I*e^(I*d*x + I*c) - 1) - 238560*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*
x + I*c) - 1) - 68160*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) -
 596400*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 8520*a^5*e^(-8*I*c)*l
og(-I*e^(I*d*x + I*c) - 1) + 15*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x) +
e^(-I*c)) + 120*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 420*
a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 840*a^5*e^(10*I*d*x
+ 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 840*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^
(I*d*x) + e^(-I*c)) + 420*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c
)) + 120*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 1050*a^5*e^(
8*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 15*a^5*e^(-8*I*c)*log(I*e^(I*d*x) +
e^(-I*c)) - 15*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 120*
a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 420*a^5*e^(12*I*d*x
 + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 840*a^5*e^(10*I*d*x + 2*I*c)*log(-
I*e^(I*d*x) + e^(-I*c)) - 840*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^
(-I*c)) - 420*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 120*a^
5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 1050*a^5*e^(8*I*d*x)*l
og(-I*e^(I*d*x) + e^(-I*c)) - 15*a^5*e^(-8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)
) + 8192*I*a^5*e^(19*I*d*x + 11*I*c) + 16384*I*a^5*e^(17*I*d*x + 9*I*c) - 1
76128*I*a^5*e^(15*I*d*x + 7*I*c) - 1003520*I*a^5*e^(13*I*d*x + 5*I*c) - 243
7120*I*a^5*e^(11*I*d*x + 3*I*c) - 3411968*I*a^5*e^(9*I*d*x + I*c) - 2953216
*I*a^5*e^(7*I*d*x - I*c) - 1568768*I*a^5*e^(5*I*d*x - 3*I*c) - 471040*I*a^5
*e^(3*I*d*x - 5*I*c) - 61440*I*a^5*e^(I*d*x - 7*I*c))/(d*e^(16*I*d*x + 8*I*
c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*
x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2
*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

3.73 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=32

$$-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

```
[Out] ((-I/5)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d
```

Rubi [A] time = 0.0356342, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3488}

$$-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-I/5)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

Mathematica [A] time = 0.155479, size = 31, normalized size = 0.97

$$-\frac{ia^5(\cos(c+dx)+i\sin(c+dx))^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-I/5)*a^5*(Cos[c + d*x] + I*Sin[c + d*x])^5)/d
```

Maple [B] time = 0.074, size = 170, normalized size = 5.3

$$\frac{1}{d}\left(-\frac{i}{5}a^5\left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4(\sin(dx+c))^2}{3}\right)\cos(dx+c)+a^5(\sin(dx+c))^5-10\,ia^5\left(-\frac{(\cos(dx+c))^3(\sin(dx+c)}{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(-1/5*I*a^5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+a^5*sin(d*x+
c)^5-10*I*a^5*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(d*x+c)^3)-10*a^5*(-1
/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-I*a^5*cos(d*x+
c)^5+1/5*a^5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 1.19405, size = 205, normalized size = 6.41

$$-\frac{15i\,a^5\cos(dx+c)^5 - 15\,a^5\sin(dx+c)^5 + 10i\left(3\,\cos(dx+c)^5 - 5\,\cos(dx+c)^3\right)a^5 + i\left(3\,\cos(dx+c)^5 - 10\,\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/15*(15*I*a^5*cos(d*x + c)^5 - 15*a^5*sin(d*x + c)^5 + 10*I*(3*cos(d*x +
c)^5 - 5*cos(d*x + c)^3)*a^5 + I*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15
*cos(d*x + c))*a^5 - 10*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^5 - (3*sin(
d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5)/d
```

Fricas [A] time = 1.4343, size = 46, normalized size = 1.44

$$-\frac{i\,a^5e^{(5i\,dx+5i\,c)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/5*I*a^5*e^(5*I*d*x + 5*I*c)/d
```

Sympy [A] time = 0.471221, size = 37, normalized size = 1.16

$$\begin{cases} -\frac{ia^5e^{5ic}e^{5idx}}{5d} & \text{for } 5d \neq 0 \\ a^5xe^{5ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise((-I*a**5*exp(5*I*c)*exp(5*I*d*x)/(5*d), Ne(5*d, 0)), (a**5*x*exp(
5*I*c), True))
```

Giac [B] time = 1.95588, size = 2253, normalized size = 70.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/122880*(34125*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2730
00*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 955500*a^5*e^(12*I
*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1911000*a^5*e^(10*I*d*x + 2*I*c)
*log(I*e^(I*d*x + I*c) + 1) + 1911000*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*
x + I*c) + 1) + 955500*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 273000*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2388750*a^5*e^
(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 34125*a^5*e^(-8*I*c)*log(I*e^(I*d*x
+ I*c) + 1) + 34770*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2
78160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 973560*a^5*e^(1
2*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1947120*a^5*e^(10*I*d*x + 2*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 1947120*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I
*d*x + I*c) - 1) + 973560*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1
) + 278160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2433900*a^5
*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 34770*a^5*e^(-8*I*c)*log(I*e^(I*d
*x + I*c) - 1) - 34125*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1)
 - 273000*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 955500*a^5
*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1911000*a^5*e^(10*I*d*x
 + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1911000*a^5*e^(6*I*d*x - 2*I*c)*log
(-I*e^(I*d*x + I*c) + 1) - 955500*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x +
 I*c) + 1) - 273000*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2
388750*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 34125*a^5*e^(-8*I*c)*l
og(-I*e^(I*d*x + I*c) + 1) - 34770*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x
 + I*c) - 1) - 278160*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 973560*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1947120*a^5
*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1947120*a^5*e^(6*I*d*x
- 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 973560*a^5*e^(4*I*d*x - 4*I*c)*log(-
I*e^(I*d*x + I*c) - 1) - 278160*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I
*c) - 1) - 2433900*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 34770*a^5*
e^(-8*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 645*a^5*e^(16*I*d*x + 8*I*c)*log(I
*e^(I*d*x) + e^(-I*c)) + 5160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^
(-I*c)) + 18060*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 3612
0*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 36120*a^5*e^(6*I*d
*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 18060*a^5*e^(4*I*d*x - 4*I*c)*log
(I*e^(I*d*x) + e^(-I*c)) + 5160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e
^(-I*c)) + 45150*a^5*e^(8*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 645*a^5*e^(-
8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 645*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) - 5160*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-
I*c)) - 18060*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 36120
*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 36120*a^5*e^(6*I*d
*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 18060*a^5*e^(4*I*d*x - 4*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) - 5160*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x)
+ e^(-I*c)) - 45150*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) - 645*a^5*
e^(-8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 24576*I*a^5*e^(21*I*d*x + 13*I*c)
 + 196608*I*a^5*e^(19*I*d*x + 11*I*c) + 688128*I*a^5*e^(17*I*d*x + 9*I*c) +
 1376256*I*a^5*e^(15*I*d*x + 7*I*c) + 1720320*I*a^5*e^(13*I*d*x + 5*I*c) +
1376256*I*a^5*e^(11*I*d*x + 3*I*c) + 688128*I*a^5*e^(9*I*d*x + I*c) + 19660
8*I*a^5*e^(7*I*d*x - I*c) + 24576*I*a^5*e^(5*I*d*x - 3*I*c))/(d*e^(16*I*d*x
 + 8*I*c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(
10*I*d*x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8
*d*e^(2*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

3.74 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=101

$$-\frac{2ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^3}{105d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d}$$

```
[Out] (((-2*I)/105)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - (((2*I)/35)*
a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4)/d - ((I/7)*Cos[c + d*x]^7*(a + I
*a*Tan[c + d*x])^5)/d
```

Rubi [A] time = 0.113655, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3497, 3488}

$$-\frac{2ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^3}{105d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-2*I)/105)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - (((2*I)/35)*
a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4)/d - ((I/7)*Cos[c + d*x]^7*(a + I
*a*Tan[c + d*x])^5)/d
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx &= -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}+\frac{1}{7}(2a)\int \cos^5(c+dx)(a+ia\tan(c+\\ &= -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}\\ &= -\frac{2ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^3}{105d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+d}{35d}\end{aligned}$$

Mathematica [A] time = 0.771674, size = 55, normalized size = 0.54

$$\frac{a^5(-10i\sin(2(c+dx))+25\cos(2(c+dx))+21)(\sin(5(c+dx))-i\cos(5(c+dx)))}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(21 + 25*Cos[2*(c + d*x)] - (10*I)*Sin[2*(c + d*x)])*((-I)*Cos[5*(c +
d*x)] + Sin[5*(c + d*x)]))/(210*d)
```

Maple [B] time = 0.091, size = 257, normalized size = 2.5

$$\frac{1}{d}\left(ia^5\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^3}{7}-\frac{4(\cos(dx+c))^3(\sin(dx+c))^2}{35}-\frac{8(\cos(dx+c))^3}{105}\right)+5a^5\left(-1/7(\sin(dx+c))\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*cos(d*x+c)^3*sin(d*x+c)^2-8
/105*cos(d*x+c)^3)+5*a^5*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*co
s(d*x+c)^4+1/35*(2+cos(d*x+c)^2)*sin(d*x+c))-10*I*a^5*(-1/7*sin(d*x+c)^2*co
s(d*x+c)^5-2/35*cos(d*x+c)^5)-10*a^5*(-1/7*cos(d*x+c)^6*sin(d*x+c)+1/35*(8/
3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-5/7*I*a^5*cos(d*x+c)^7+1/7*a^5
*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 1.15952, size = 252, normalized size = 2.5

$$-\frac{75ia^5\cos(dx+c)^7+i\left(15\cos(dx+c)^7-42\cos(dx+c)^5+35\cos(dx+c)^3\right)a^5+30i\left(5\cos(dx+c)^7-7\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/105*(75*I*a^5*cos(d*x + c)^7 + I*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5
+ 35*cos(d*x + c)^3)*a^5 + 30*I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^5 +
 10*(15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^5 + 15*(5
*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^5 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x
+ c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^5)/d
```

Fricas [A] time = 1.39852, size = 139, normalized size = 1.38

$$\frac{-15ia^5e^{(7idx+7ic)}-42ia^5e^{(5idx+5ic)}-35ia^5e^{(3idx+3ic)}}{420d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/420*(-15*I*a^5*e^(7*I*d*x + 7*I*c) - 42*I*a^5*e^(5*I*d*x + 5*I*c) - 35*I*
a^5*e^(3*I*d*x + 3*I*c))/d
```

Sympy [A] time = 0.757066, size = 122, normalized size = 1.21

$$\begin{cases} \frac{-120ia^5d^2e^{7ic}e^{7idx}-336ia^5d^2e^{5ic}e^{5idx}-280ia^5d^2e^{3ic}e^{3idx}}{3360d^3} & \text{for } 3360d^3 \neq 0 \\ x\left(\frac{a^5e^{7ic}}{4}+\frac{a^5e^{5ic}}{2}+\frac{a^5e^{3ic}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-120*I*a**5*d**2*exp(7*I*c)*exp(7*I*d*x) - 336*I*a**5*d**2*exp(
5*I*c)*exp(5*I*d*x) - 280*I*a**5*d**2*exp(3*I*c)*exp(3*I*d*x))/(3360*d**3),
 Ne(3360*d**3, 0)), (x*(a**5*exp(7*I*c)/4 + a**5*exp(5*I*c)/2 + a**5*exp(3*
I*c)/4), True))
```

Giac [B] time = 2.1073, size = 2291, normalized size = 22.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/3440640*(7357770*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5
8862160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 206017560*a^5
*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 412035120*a^5*e^(10*I*d*
x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 412035120*a^5*e^(6*I*d*x - 2*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 206017560*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*
x + I*c) + 1) + 58862160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 515043900*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 7357770*a^5*e^(-8
*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7390425*a^5*e^(16*I*d*x + 8*I*c)*log(I*e
^(I*d*x + I*c) - 1) + 59123400*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 206931900*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4
13863800*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 413863800*a^
5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 206931900*a^5*e^(4*I*d*x
 - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 59123400*a^5*e^(2*I*d*x - 6*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 517329750*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) -
 1) + 7390425*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 7357770*a^5*e^(16
*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 58862160*a^5*e^(14*I*d*x + 6*
I*c)*log(-I*e^(I*d*x + I*c) + 1) - 206017560*a^5*e^(12*I*d*x + 4*I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 412035120*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 412035120*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1
) - 206017560*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5886216
0*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 515043900*a^5*e^(8*
I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 7357770*a^5*e^(-8*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 7390425*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1)
 - 59123400*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 20693190
0*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 413863800*a^5*e^(1
0*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 413863800*a^5*e^(6*I*d*x - 2
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 206931900*a^5*e^(4*I*d*x - 4*I*c)*log(-
I*e^(I*d*x + I*c) - 1) - 59123400*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x +
 I*c) - 1) - 517329750*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 739042
5*a^5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 32655*a^5*e^(16*I*d*x + 8*I*
c)*log(I*e^(I*d*x) + e^(-I*c)) + 261240*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I
*d*x) + e^(-I*c)) + 914340*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I
*c)) + 1828680*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 18286
```

```
80*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 914340*a^5*e^(4*I*
d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 261240*a^5*e^(2*I*d*x - 6*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) + 2285850*a^5*e^(8*I*d*x)*log(I*e^(I*d*x) + e^(-
I*c)) + 32655*a^5*e^(-8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 32655*a^5*e^(16*
I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 261240*a^5*e^(14*I*d*x + 6*I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) - 914340*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) - 1828680*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e
^(-I*c)) - 1828680*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 9
14340*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 261240*a^5*e^(
2*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 2285850*a^5*e^(8*I*d*x)*log
(-I*e^(I*d*x) + e^(-I*c)) - 32655*a^5*e^(-8*I*c)*log(-I*e^(I*d*x) + e^(-I*c
)) + 122880*I*a^5*e^(23*I*d*x + 15*I*c) + 1327104*I*a^5*e^(21*I*d*x + 13*I*
c) + 6479872*I*a^5*e^(19*I*d*x + 11*I*c) + 18808832*I*a^5*e^(17*I*d*x + 9*I
*c) + 35897344*I*a^5*e^(15*I*d*x + 7*I*c) + 47022080*I*a^5*e^(13*I*d*x + 5*
I*c) + 42778624*I*a^5*e^(11*I*d*x + 3*I*c) + 26673152*I*a^5*e^(9*I*d*x + I*
c) + 10903552*I*a^5*e^(7*I*d*x - I*c) + 2637824*I*a^5*e^(5*I*d*x - 3*I*c) +
 286720*I*a^5*e^(3*I*d*x - 5*I*c))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*
x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e
^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 7
0*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

3.75 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=141

$$\frac{a^5\sin^5(c+dx)}{105d}-\frac{2a^5\sin^3(c+dx)}{63d}+\frac{a^5\sin(c+dx)}{21d}-\frac{ia^5\cos^5(c+dx)}{105d}-\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d}-\frac{2ia}{}$$

```
[Out] ((-I/105)*a^5*Cos[c + d*x]^5)/d + (a^5*Sin[c + d*x])/(21*d) - (2*a^5*Sin[c
+ d*x]^3)/(63*d) + (a^5*Sin[c + d*x]^5)/(105*d) - (((2*I)/63)*a^3*Cos[c + d
*x]^7*(a + I*a*Tan[c + d*x])^2)/d - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Ta
n[c + d*x])^4)/d
```

Rubi [A] time = 0.121577, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3496, 3486, 2633}

$$\frac{a^5\sin^5(c+dx)}{105d}-\frac{2a^5\sin^3(c+dx)}{63d}+\frac{a^5\sin(c+dx)}{21d}-\frac{ia^5\cos^5(c+dx)}{105d}-\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d}-\frac{2ia}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^5,x]
```


```
[Out] ((-I/105)*a^5*Cos[c + d*x]^5)/d + (a^5*Sin[c + d*x])/(21*d) - (2*a^5*Sin[c
+ d*x]^3)/(63*d) + (a^5*Sin[c + d*x]^5)/(105*d) - (((2*I)/63)*a^3*Cos[c + d
*x]^7*(a + I*a*Tan[c + d*x])^2)/d - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Ta
n[c + d*x])^4)/d
```


Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d} + \frac{1}{9}a^2\int \cos^7(c+dx)(a+ia\tan(c+dx$$
$$= -\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d} - \frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$
$$= -\frac{ia^5\cos^5(c+dx)}{105d} - \frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d} - \frac{2ia\cos^9(c+dx)(}{}$$
$$= -\frac{ia^5\cos^5(c+dx)}{105d} - \frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d} - \frac{2ia\cos^9(c+dx)(}{}$$
$$= -\frac{ia^5\cos^5(c+dx)}{105d} + \frac{a^5\sin(c+dx)}{21d} - \frac{2a^5\sin^3(c+dx)}{63d} + \frac{a^5\sin^5(c+dx)}{105d} - \frac{2ia^3}{}$$

Mathematica [A] time = 0.833622, size = 94, normalized size = 0.67

$$\frac{a^5(-120i\sin(2(c+dx)) - 140i\sin(4(c+dx)) + 300\cos(2(c+dx)) + 175\cos(4(c+dx)) + 189)(\sin(5(c+2dx)) - i\cos(5(}{2520d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(189 + 300*Cos[2*(c + d*x)] + 175*Cos[4*(c + d*x)] - (120*I)*Sin[2*(c
+ d*x)] - (140*I)*Sin[4*(c + d*x)])*((-I)*Cos[5*(c + 2*d*x)] + Sin[5*(c + 2
*d*x)]))/(2520*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [B] time = 0.09, size = 287, normalized size = 2.

$$\frac{1}{d}\left(ia^5\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^5}{9} - \frac{4(\sin(dx+c))^2(\cos(dx+c))^5}{63} - \frac{8(\cos(dx+c))^5}{315}\right) + 5a^5\left(-1/9(\sin(dx+c))\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8
/315*cos(d*x+c)^5)+5*a^5*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*cos(d*x+c)^6*
sin(d*x+c)+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-10*I*a^5*(
-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)-10*a^5*(-1/9*sin(d*x+c)*c
os(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(
d*x+c))-5/9*I*a^5*cos(d*x+c)^9+1/9*a^5*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^
6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.19079, size = 293, normalized size = 2.08

$$-\frac{175ia^5\cos(dx+c)^9 + i\left(35\cos(dx+c)^9 - 90\cos(dx+c)^7 + 63\cos(dx+c)^5\right)a^5 + 50i\left(7\cos(dx+c)^9 - 9\cos(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/315*(175*I*a^5*cos(d*x + c)^9 + I*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7
 + 63*cos(d*x + c)^5)*a^5 + 50*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^5
- 5*(35*sin(d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^5 - 10*(3
5*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x +
c)^3)*a^5 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)^5 -
420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^5)/d
```

Fricas [A] time = 1.58065, size = 224, normalized size = 1.59

$$\frac{-35i\,a^5e^{(9i\,dx+9i\,c)} - 180i\,a^5e^{(7i\,dx+7i\,c)} - 378i\,a^5e^{(5i\,dx+5i\,c)} - 420i\,a^5e^{(3i\,dx+3i\,c)} - 315i\,a^5e^{(i\,dx+i\,c)}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/5040*(-35*I*a^5*e^(9*I*d*x + 9*I*c) - 180*I*a^5*e^(7*I*d*x + 7*I*c) - 378
*I*a^5*e^(5*I*d*x + 5*I*c) - 420*I*a^5*e^(3*I*d*x + 3*I*c) - 315*I*a^5*e^(I
*d*x + I*c))/d
```

Sympy [A] time = 1.11539, size = 194, normalized size = 1.38

$$\begin{cases} \frac{-215040ia^5d^4e^{9ic}e^{9idx}-1105920ia^5d^4e^{7ic}e^{7idx}-2322432ia^5d^4e^{5ic}e^{5idx}-2580480ia^5d^4e^{3ic}e^{3idx}-1935360ia^5d^4e^{ic}e^{idx}}{30965760d^5} & \text{for}\ 30965760d^5 \neq 0 \\ x\left(\frac{a^5e^{9ic}}{16}+\frac{a^5e^{7ic}}{4}+\frac{3a^5e^{5ic}}{8}+\frac{a^5e^{3ic}}{4}+\frac{a^5e^{ic}}{16}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-215040*I*a**5*d**4*exp(9*I*c)*exp(9*I*d*x) - 1105920*I*a**5*d*
*4*exp(7*I*c)*exp(7*I*d*x) - 2322432*I*a**5*d**4*exp(5*I*c)*exp(5*I*d*x) -
2580480*I*a**5*d**4*exp(3*I*c)*exp(3*I*d*x) - 1935360*I*a**5*d**4*exp(I*c)*
exp(I*d*x))/(30965760*d**5), Ne(30965760*d**5, 0)), (x*(a**5*exp(9*I*c)/16
+ a**5*exp(7*I*c)/4 + 3*a**5*exp(5*I*c)/8 + a**5*exp(3*I*c)/4 + a**5*exp(I*
c)/16), True))
```

Giac [B] time = 2.18092, size = 2329, normalized size = 16.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/41287680*(69853455*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 558827640*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1955896740
*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3911793480*a^5*e^(10
*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3911793480*a^5*e^(6*I*d*x - 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 1955896740*a^5*e^(4*I*d*x - 4*I*c)*log(I*
e^(I*d*x + I*c) + 1) + 558827640*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I
*c) + 1) + 4889741850*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 69853455
```

```
*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 70703325*a^5*e^(16*I*d*x + 8*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 565626600*a^5*e^(14*I*d*x + 6*I*c)*log(I*e
^(I*d*x + I*c) - 1) + 1979693100*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 3959386200*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1)
+ 3959386200*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 197969310
0*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 565626600*a^5*e^(2*I
*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4949232750*a^5*e^(8*I*d*x)*log(I
*e^(I*d*x + I*c) - 1) + 70703325*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) - 1)
- 69853455*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 558827640
*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1955896740*a^5*e^(1
2*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3911793480*a^5*e^(10*I*d*x +
 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3911793480*a^5*e^(6*I*d*x - 2*I*c)*lo
g(-I*e^(I*d*x + I*c) + 1) - 1955896740*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*
d*x + I*c) + 1) - 558827640*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 4889741850*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 69853455*a^
5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 70703325*a^5*e^(16*I*d*x + 8*I*c
)*log(-I*e^(I*d*x + I*c) - 1) - 565626600*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 1979693100*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x +
 I*c) - 1) - 3959386200*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1
) - 3959386200*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 197969
3100*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 565626600*a^5*e^
(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 4949232750*a^5*e^(8*I*d*x)*
log(-I*e^(I*d*x + I*c) - 1) - 70703325*a^5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c
) - 1) + 849870*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 6798
960*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 23796360*a^5*e^(
12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 47592720*a^5*e^(10*I*d*x +
2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 47592720*a^5*e^(6*I*d*x - 2*I*c)*log(I
*e^(I*d*x) + e^(-I*c)) + 23796360*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x) +
 e^(-I*c)) + 6798960*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) +
59490900*a^5*e^(8*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 849870*a^5*e^(-8*I*c
)*log(I*e^(I*d*x) + e^(-I*c)) - 849870*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I
*d*x) + e^(-I*c)) - 6798960*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(
-I*c)) - 23796360*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 4
7592720*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 47592720*a^
5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 23796360*a^5*e^(4*I*d*
x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 6798960*a^5*e^(2*I*d*x - 6*I*c)*l
og(-I*e^(I*d*x) + e^(-I*c)) - 59490900*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x) + e
^(-I*c)) - 849870*a^5*e^(-8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 286720*I*a^
5*e^(25*I*d*x + 17*I*c) + 3768320*I*a^5*e^(23*I*d*x + 15*I*c) + 22921216*I*
a^5*e^(21*I*d*x + 13*I*c) + 85557248*I*a^5*e^(19*I*d*x + 11*I*c) + 21945548
8*I*a^5*e^(17*I*d*x + 9*I*c) + 409665536*I*a^5*e^(15*I*d*x + 7*I*c) + 57229
3120*I*a^5*e^(13*I*d*x + 5*I*c) + 602341376*I*a^5*e^(11*I*d*x + 3*I*c) + 47
2096768*I*a^5*e^(9*I*d*x + I*c) + 267091968*I*a^5*e^(7*I*d*x - I*c) + 10287
5136*I*a^5*e^(5*I*d*x - 3*I*c) + 24084480*I*a^5*e^(3*I*d*x - 5*I*c) + 25804
80*I*a^5*e^(I*d*x - 7*I*c))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*x + 6*I
*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6*I*d
*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d*e^(
8*I*d*x) + d*e^(-8*I*c))
```

3.76 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=159

$$-\frac{5a^5\sin^7(c+dx)}{231d}+\frac{a^5\sin^5(c+dx)}{11d}-\frac{5a^5\sin^3(c+dx)}{33d}+\frac{5a^5\sin(c+dx)}{33d}-\frac{5ia^5\cos^7(c+dx)}{231d}-\frac{2ia^3\cos^9(c+dx)(a+}{33d}$$

```
[Out] (((-5*I)/231)*a^5*Cos[c + d*x]^7)/d + (5*a^5*Sin[c + d*x])/(33*d) - (5*a^5*
Sin[c + d*x]^3)/(33*d) + (a^5*Sin[c + d*x]^5)/(11*d) - (5*a^5*Sin[c + d*x]^
7)/(231*d) - (((2*I)/33)*a^3*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2)/d - (
((2*I)/11)*a*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^4)/d
```

Rubi [A] time = 0.125985, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3496, 3486, 2633}

$$-\frac{5a^5\sin^7(c+dx)}{231d}+\frac{a^5\sin^5(c+dx)}{11d}-\frac{5a^5\sin^3(c+dx)}{33d}+\frac{5a^5\sin(c+dx)}{33d}-\frac{5ia^5\cos^7(c+dx)}{231d}-\frac{2ia^3\cos^9(c+dx)(a+}{33d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-5*I)/231)*a^5*Cos[c + d*x]^7)/d + (5*a^5*Sin[c + d*x])/(33*d) - (5*a^5*
Sin[c + d*x]^3)/(33*d) + (a^5*Sin[c + d*x]^5)/(11*d) - (5*a^5*Sin[c + d*x]^
7)/(231*d) - (((2*I)/33)*a^3*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2)/d - (
((2*I)/11)*a*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^4)/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d} + \frac{1}{11}\left(3a^2\right)\int \cos^9(c+dx)(a+ia\tan(c+dx)$$
$$= -\frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} - \frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx)}{11d}$$
$$= -\frac{5ia^5\cos^7(c+dx)}{231d} - \frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} - \frac{2ia\cos^{11}(c+d}{}$$
$$= -\frac{5ia^5\cos^7(c+dx)}{231d} - \frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} - \frac{2ia\cos^{11}(c+d}{}$$
$$= -\frac{5ia^5\cos^7(c+dx)}{231d} + \frac{5a^5\sin(c+dx)}{33d} - \frac{5a^5\sin^3(c+dx)}{33d} + \frac{a^5\sin^5(c+dx)}{11d} -$$

Mathematica [A] time = 1.19917, size = 118, normalized size = 0.74

$$\frac{ia^5(330i\sin(2(c+dx)) + 616i\sin(4(c+dx)) - 126i\sin(6(c+dx)) - 825\cos(2(c+dx)) - 770\cos(4(c+dx)) + 105\cos(6(}{7392d(\cos(dx)+i\sin(dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((I/7392)*a^5*(-462 - 825*Cos[2*(c + d*x)] - 770*Cos[4*(c + d*x)] + 105*Cos
[6*(c + d*x)] + (330*I)*Sin[2*(c + d*x)] + (616*I)*Sin[4*(c + d*x)] - (126*
I)*Sin[6*(c + d*x)])*(Cos[5*(c + 2*d*x)] + I*Sin[5*(c + 2*d*x)]))/(d*(Cos[d
*x] + I*Sin[d*x])^5)
```

Maple [B] time = 0.121, size = 317, normalized size = 2.

$$\frac{1}{d}\left(ia^5\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^7}{11} - \frac{4(\sin(dx+c))^2(\cos(dx+c))^7}{99} - \frac{8(\cos(dx+c))^7}{693}\right) + 5a^5\left(-1/11(\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-
8/693*cos(d*x+c)^7)+5*a^5*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*sin(d*x+c)*
cos(d*x+c)^8+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*si
n(d*x+c))-10*I*a^5*(-1/11*sin(d*x+c)^2*cos(d*x+c)^9-2/99*cos(d*x+c)^9)-10*a
^5*(-1/11*sin(d*x+c)*cos(d*x+c)^10+1/99*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)
^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-5/11*I*a^5*cos(d*x+c)
^11+1/11*a^5*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c)^6+32/
21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.13572, size = 332, normalized size = 2.09

$$-\frac{315ia^5\cos(dx+c)^{11} + i\left(63\cos(dx+c)^{11} - 154\cos(dx+c)^9 + 99\cos(dx+c)^7\right)a^5 + 70i\left(9\cos(dx+c)^{11} - 11\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/693*(315*I*a^5*cos(d*x + c)^11 + I*(63*cos(d*x + c)^11 - 154*cos(d*x + c
)^9 + 99*cos(d*x + c)^7)*a^5 + 70*I*(9*cos(d*x + c)^11 - 11*cos(d*x + c)^9)
*a^5 + 2*(315*sin(d*x + c)^11 - 1540*sin(d*x + c)^9 + 2970*sin(d*x + c)^7 -
 2772*sin(d*x + c)^5 + 1155*sin(d*x + c)^3)*a^5 + 3*(105*sin(d*x + c)^11 -
385*sin(d*x + c)^9 + 495*sin(d*x + c)^7 - 231*sin(d*x + c)^5)*a^5 + (63*sin
(d*x + c)^11 - 385*sin(d*x + c)^9 + 990*sin(d*x + c)^7 - 1386*sin(d*x + c)^
5 + 1155*sin(d*x + c)^3 - 693*sin(d*x + c))*a^5)/d
```

Fricas [A] time = 1.55928, size = 321, normalized size = 2.02

$$\frac{\left(-21 i a^5 e^{(12 i dx+12 i c)} - 154 i a^5 e^{(10 i dx+10 i c)} - 495 i a^5 e^{(8 i dx+8 i c)} - 924 i a^5 e^{(6 i dx+6 i c)} - 1155 i a^5 e^{(4 i dx+4 i c)} - 1386 i a^5 e^{(2 i dx+2 i}\right.}{14784\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/14784*(-21*I*a^5*e^(12*I*d*x + 12*I*c) - 154*I*a^5*e^(10*I*d*x + 10*I*c)
- 495*I*a^5*e^(8*I*d*x + 8*I*c) - 924*I*a^5*e^(6*I*d*x + 6*I*c) - 1155*I*a^
5*e^(4*I*d*x + 4*I*c) - 1386*I*a^5*e^(2*I*d*x + 2*I*c) + 231*I*a^5)*e^(-I*d
*x - I*c)/d
```

Sympy [A] time = 1.45324, size = 267, normalized size = 1.68

$$\begin{cases} \frac{\left(-90194313216 i a^5 d^6 e^{12 i c} e^{11 i dx} - 661424963584 i a^5 d^6 e^{10 i c} e^{9 i dx} - 2126008811520 i a^5 d^6 e^{8 i c} e^{7 i dx} - 3968549781504 i a^5 d^6 e^{6 i c} e^{5 i dx} - 4960687226880 i a^5 d^6 e^{4 i c} e^{3 i dx} - \right.}{63496796504064 d^7} \\ \frac{x \left(a^5 e^{12 i c} + 6 a^5 e^{10 i c} + 15 a^5 e^{8 i c} + 20 a^5 e^{6 i c} + 15 a^5 e^{4 i c} + 6 a^5 e^{2 i c} + a^5\right) e^{-i c}}{64} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-90194313216*I*a**5*d**6*exp(12*I*c)*exp(11*I*d*x) - 6614249635
84*I*a**5*d**6*exp(10*I*c)*exp(9*I*d*x) - 2126008811520*I*a**5*d**6*exp(8*I
*c)*exp(7*I*d*x) - 3968549781504*I*a**5*d**6*exp(6*I*c)*exp(5*I*d*x) - 4960
687226880*I*a**5*d**6*exp(4*I*c)*exp(3*I*d*x) - 5952824672256*I*a**5*d**6*e
xp(2*I*c)*exp(I*d*x) + 992137445376*I*a**5*d**6*exp(-I*d*x))*exp(-I*c)/(634
96796504064*d**7), Ne(63496796504064*d**7*exp(I*c), 0)), (x*(a**5*exp(12*I*
c) + 6*a**5*exp(10*I*c) + 15*a**5*exp(8*I*c) + 20*a**5*exp(6*I*c) + 15*a**5
*exp(4*I*c) + 6*a**5*exp(2*I*c) + a**5)*exp(-I*c)/64, True))
```

Giac [B] time = 2.30674, size = 2439, normalized size = 15.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/121110528*(168111405*a^5*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 1344891240*a^5*e^(15*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4707119
340*a^5*e^(13*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9414238680*a^5*e^
```

```
(11*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11767798350*a^5*e^(9*I*d*x
+ I*c)*log(I*e^(I*d*x + I*c) + 1) + 9414238680*a^5*e^(7*I*d*x - I*c)*log(I*
e^(I*d*x + I*c) + 1) + 4707119340*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*d*x +
I*c) + 1) + 1344891240*a^5*e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 168111405*a^5*e^(I*d*x - 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 170251620*a^5
*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1362012960*a^5*e^(15*I*d
*x + 7*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4767045360*a^5*e^(13*I*d*x + 5*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 9534090720*a^5*e^(11*I*d*x + 3*I*c)*log(I*e^
(I*d*x + I*c) - 1) + 11917613400*a^5*e^(9*I*d*x + I*c)*log(I*e^(I*d*x + I*c
) - 1) + 9534090720*a^5*e^(7*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) + 4767
045360*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1362012960*a^5*
e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 170251620*a^5*e^(I*d*x - 7
*I*c)*log(I*e^(I*d*x + I*c) - 1) - 168111405*a^5*e^(17*I*d*x + 9*I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 1344891240*a^5*e^(15*I*d*x + 7*I*c)*log(-I*e^(I*d*
x + I*c) + 1) - 4707119340*a^5*e^(13*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 9414238680*a^5*e^(11*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 11
767798350*a^5*e^(9*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 9414238680*a^
5*e^(7*I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 4707119340*a^5*e^(5*I*d*x
 - 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1344891240*a^5*e^(3*I*d*x - 5*I*c)*
log(-I*e^(I*d*x + I*c) + 1) - 168111405*a^5*e^(I*d*x - 7*I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 170251620*a^5*e^(17*I*d*x + 9*I*c)*log(-I*e^(I*d*x + I*c)
- 1) - 1362012960*a^5*e^(15*I*d*x + 7*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 47
67045360*a^5*e^(13*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9534090720*
a^5*e^(11*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11917613400*a^5*e^(9
*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9534090720*a^5*e^(7*I*d*x - I*c
)*log(-I*e^(I*d*x + I*c) - 1) - 4767045360*a^5*e^(5*I*d*x - 3*I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 1362012960*a^5*e^(3*I*d*x - 5*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 170251620*a^5*e^(I*d*x - 7*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 2
140215*a^5*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 17121720*a^5*
e^(15*I*d*x + 7*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 59926020*a^5*e^(13*I*d*x
 + 5*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 119852040*a^5*e^(11*I*d*x + 3*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) + 149815050*a^5*e^(9*I*d*x + I*c)*log(I*e^(I*d*
x) + e^(-I*c)) + 119852040*a^5*e^(7*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)
) + 59926020*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 17121720
*a^5*e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 2140215*a^5*e^(I*d*x
 - 7*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2140215*a^5*e^(17*I*d*x + 9*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) - 17121720*a^5*e^(15*I*d*x + 7*I*c)*log(-I*e^(I*
d*x) + e^(-I*c)) - 59926020*a^5*e^(13*I*d*x + 5*I*c)*log(-I*e^(I*d*x) + e^(
-I*c)) - 119852040*a^5*e^(11*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) -
149815050*a^5*e^(9*I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 119852040*a^
5*e^(7*I*d*x - I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 59926020*a^5*e^(5*I*d*x
- 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 17121720*a^5*e^(3*I*d*x - 5*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) - 2140215*a^5*e^(I*d*x - 7*I*c)*log(-I*e^(I*d*x)
 + e^(-I*c)) + 172032*I*a^5*e^(28*I*d*x + 20*I*c) + 2637824*I*a^5*e^(26*I*d
*x + 18*I*c) + 18964480*I*a^5*e^(24*I*d*x + 16*I*c) + 84967424*I*a^5*e^(22*
I*d*x + 14*I*c) + 266248192*I*a^5*e^(20*I*d*x + 12*I*c) + 624017408*I*a^5*e
^(18*I*d*x + 10*I*c) + 1137074176*I*a^5*e^(16*I*d*x + 8*I*c) + 1626275840*I
*a^5*e^(14*I*d*x + 6*I*c) + 1792860160*I*a^5*e^(12*I*d*x + 4*I*c) + 1464320
000*I*a^5*e^(10*I*d*x + 2*I*c) + 295206912*I*a^5*e^(6*I*d*x - 2*I*c) + 4730
8800*I*a^5*e^(4*I*d*x - 4*I*c) - 3784704*I*a^5*e^(2*I*d*x - 6*I*c) + 832905
216*I*a^5*e^(8*I*d*x) - 1892352*I*a^5*e^(-8*I*c))/(d*e^(17*I*d*x + 9*I*c) +
 8*d*e^(15*I*d*x + 7*I*c) + 28*d*e^(13*I*d*x + 5*I*c) + 56*d*e^(11*I*d*x +
3*I*c) + 70*d*e^(9*I*d*x + I*c) + 56*d*e^(7*I*d*x - I*c) + 28*d*e^(5*I*d*x
- 3*I*c) + 8*d*e^(3*I*d*x - 5*I*c) + d*e^(I*d*x - 7*I*c))
```

3.77 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=109

$$\frac{i(a+ia\tan(c+dx))^{15}}{15a^7d}-\frac{3i(a+ia\tan(c+dx))^{14}}{7a^6d}+\frac{12i(a+ia\tan(c+dx))^{13}}{13a^5d}-\frac{2i(a+ia\tan(c+dx))^{12}}{3a^4d}$$

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^12)/(a^4*d) + (((12*I)/13)*(a + I*a*Tan[
c + d*x])^13)/(a^5*d) - (((3*I)/7)*(a + I*a*Tan[c + d*x])^14)/(a^6*d) + ((I
/15)*(a + I*a*Tan[c + d*x])^15)/(a^7*d)
```

Rubi [A] time = 0.0808121, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a+ia\tan(c+dx))^{15}}{15a^7d}-\frac{3i(a+ia\tan(c+dx))^{14}}{7a^6d}+\frac{12i(a+ia\tan(c+dx))^{13}}{13a^5d}-\frac{2i(a+ia\tan(c+dx))^{12}}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^12)/(a^4*d) + (((12*I)/13)*(a + I*a*Tan[
c + d*x])^13)/(a^5*d) - (((3*I)/7)*(a + I*a*Tan[c + d*x])^14)/(a^6*d) + ((I
/15)*(a + I*a*Tan[c + d*x])^15)/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{i\,\mathrm{Subst}\left(\int (a-x)^3(a+x)^{11}\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\mathrm{Subst}\left(\int \left(8a^3(a+x)^{11}-12a^2(a+x)^{12}+6a(a+x)^{13}-(a+x)^{14}\right)dx,x,i\right.}{a^7d}\\ &= -\frac{2i(a+ia\tan(c+dx))^{12}}{3a^4d}+\frac{12i(a+ia\tan(c+dx))^{13}}{13a^5d}-\frac{3i(a+ia\tan(c+dx}{7a^6d}\end{aligned}$$

Mathematica [B] time = 8.61341, size = 245, normalized size = 2.25

$$a^8\sec(c)\sec^{15}(c+dx)(-6435\sin(2c+dx)+5005\sin(2c+3dx)-5005\sin(4c+3dx)+3003\sin(4c+5dx)-3003\,\mathrm{si}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^15*((6435*I)*Cos[d*x] + (6435*I)*Cos[2*c + d*x] +
(5005*I)*Cos[2*c + 3*d*x] + (5005*I)*Cos[4*c + 3*d*x] + (3003*I)*Cos[4*c +
5*d*x] + (3003*I)*Cos[6*c + 5*d*x] + (1365*I)*Cos[6*c + 7*d*x] + (1365*I)*C
os[8*c + 7*d*x] + 6435*Sin[d*x] - 6435*Sin[2*c + d*x] + 5005*Sin[2*c + 3*d*
x] - 5005*Sin[4*c + 3*d*x] + 3003*Sin[4*c + 5*d*x] - 3003*Sin[6*c + 5*d*x]
+ 1365*Sin[6*c + 7*d*x] - 1365*Sin[8*c + 7*d*x] + 910*Sin[8*c + 9*d*x] + 21
0*Sin[10*c + 11*d*x] + 30*Sin[12*c + 13*d*x] + 2*Sin[14*c + 15*d*x]))/(1092
0*d)
```

Maple [B] time = 0.1, size = 611, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(1/15*sin(d*x+c)^9/cos(d*x+c)^15+2/65*sin(d*x+c)^9/cos(d*x+c)^13+8
/715*sin(d*x+c)^9/cos(d*x+c)^11+16/6435*sin(d*x+c)^9/cos(d*x+c)^9)-56*I*a^8
*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/20*sin(d
*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4)-28*a^8*(1/13*sin(d*x+c
)^7/cos(d*x+c)^13+6/143*sin(d*x+c)^7/cos(d*x+c)^11+8/429*sin(d*x+c)^7/cos(d
*x+c)^9+16/3003*sin(d*x+c)^7/cos(d*x+c)^7)+56*I*a^8*(1/12*sin(d*x+c)^6/cos(
d*x+c)^12+1/20*sin(d*x+c)^6/cos(d*x+c)^10+1/40*sin(d*x+c)^6/cos(d*x+c)^8+1/
120*sin(d*x+c)^6/cos(d*x+c)^6)+70*a^8*(1/11*sin(d*x+c)^5/cos(d*x+c)^11+2/33
*sin(d*x+c)^5/cos(d*x+c)^9+8/231*sin(d*x+c)^5/cos(d*x+c)^7+16/1155*sin(d*x+
c)^5/cos(d*x+c)^5)+I*a^8/cos(d*x+c)^8-28*a^8*(1/9*sin(d*x+c)^3/cos(d*x+c)^9
+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(
d*x+c)^3/cos(d*x+c)^3)-8*I*a^8*(1/14*sin(d*x+c)^8/cos(d*x+c)^14+1/28*sin(d*
x+c)^8/cos(d*x+c)^12+1/70*sin(d*x+c)^8/cos(d*x+c)^10+1/280*sin(d*x+c)^8/cos
(d*x+c)^8)-a^8*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2
)*tan(d*x+c))
```

Maxima [B] time = 1.47601, size = 251, normalized size = 2.3

$$3003\,a^8\tan\left(dx+c\right)^{15} - 25740i\,a^8\tan\left(dx+c\right)^{14} - 86625\,a^8\tan\left(dx+c\right)^{13} + 120120i\,a^8\tan\left(dx+c\right)^{12} - 45045\,a^8\tan\left(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/45045*(3003*a^8*tan(d*x + c)^15 - 25740*I*a^8*tan(d*x + c)^14 - 86625*a^8
*tan(d*x + c)^13 + 120120*I*a^8*tan(d*x + c)^12 - 45045*a^8*tan(d*x + c)^11
 + 396396*I*a^8*tan(d*x + c)^10 + 495495*a^8*tan(d*x + c)^9 + 637065*a^8*ta
n(d*x + c)^7 - 660660*I*a^8*tan(d*x + c)^6 - 99099*a^8*tan(d*x + c)^5 - 360
360*I*a^8*tan(d*x + c)^4 - 375375*a^8*tan(d*x + c)^3 + 180180*I*a^8*tan(d*x
 + c)^2 + 45045*a^8*tan(d*x + c))/d
```

Fricas [B] time = 1.44153, size = 1195, normalized size = 10.96

$$\frac{11182080i\,a^8e^{(22i\,dx+22i\,c)} + 24600576i\,a^8e^{(20i\,dx+20i\,c)} + 41000960i\,a^8e^{(18i\,dx+18i\,c)} + 52715520i\,a^8e^{(16i\,dx+16i\,c)} + 527155}{1365\left(de^{(30i\,dx+30i\,c)} + 15\,de^{(28i\,dx+28i\,c)} + 105\,de^{(26i\,dx+26i\,c)} + 455\,de^{(24i\,dx+24i\,c)} + 1365\,de^{(22i\,dx+22i\,c)} + 3003\,de^{(20i\,dx+20i\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/1365*(11182080*I*a^8*e^(22*I*d*x + 22*I*c) + 24600576*I*a^8*e^(20*I*d*x +
 20*I*c) + 41000960*I*a^8*e^(18*I*d*x + 18*I*c) + 52715520*I*a^8*e^(16*I*d*
x + 16*I*c) + 52715520*I*a^8*e^(14*I*d*x + 14*I*c) + 41000960*I*a^8*e^(12*I
*d*x + 12*I*c) + 24600576*I*a^8*e^(10*I*d*x + 10*I*c) + 11182080*I*a^8*e^(8
*I*d*x + 8*I*c) + 3727360*I*a^8*e^(6*I*d*x + 6*I*c) + 860160*I*a^8*e^(4*I*d
*x + 4*I*c) + 122880*I*a^8*e^(2*I*d*x + 2*I*c) + 8192*I*a^8)/(d*e^(30*I*d*x
 + 30*I*c) + 15*d*e^(28*I*d*x + 28*I*c) + 105*d*e^(26*I*d*x + 26*I*c) + 455
*d*e^(24*I*d*x + 24*I*c) + 1365*d*e^(22*I*d*x + 22*I*c) + 3003*d*e^(20*I*d*
x + 20*I*c) + 5005*d*e^(18*I*d*x + 18*I*c) + 6435*d*e^(16*I*d*x + 16*I*c) +
 6435*d*e^(14*I*d*x + 14*I*c) + 5005*d*e^(12*I*d*x + 12*I*c) + 3003*d*e^(10
*I*d*x + 10*I*c) + 1365*d*e^(8*I*d*x + 8*I*c) + 455*d*e^(6*I*d*x + 6*I*c) +
 105*d*e^(4*I*d*x + 4*I*c) + 15*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.81532, size = 251, normalized size = 2.3

$$\frac{91\,a^8\tan\left(dx+c\right)^{15} - 780i\,a^8\tan\left(dx+c\right)^{14} - 2625\,a^8\tan\left(dx+c\right)^{13} + 3640i\,a^8\tan\left(dx+c\right)^{12} - 1365\,a^8\tan\left(dx+c\right)^{1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/1365*(91*a^8*tan(d*x + c)^15 - 780*I*a^8*tan(d*x + c)^14 - 2625*a^8*tan(d
*x + c)^13 + 3640*I*a^8*tan(d*x + c)^12 - 1365*a^8*tan(d*x + c)^11 + 12012*
I*a^8*tan(d*x + c)^10 + 15015*a^8*tan(d*x + c)^9 + 19305*a^8*tan(d*x + c)^7
 - 20020*I*a^8*tan(d*x + c)^6 - 3003*a^8*tan(d*x + c)^5 - 10920*I*a^8*tan(d
*x + c)^4 - 11375*a^8*tan(d*x + c)^3 + 5460*I*a^8*tan(d*x + c)^2 + 1365*a^8
*tan(d*x + c))/d
```

3.78 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=82

$$-\frac{i(a+ia\tan(c+dx))^{13}}{13a^5d}+\frac{i(a+ia\tan(c+dx))^{12}}{3a^4d}-\frac{4i(a+ia\tan(c+dx))^{11}}{11a^3d}$$

```
[Out] (((-4*I)/11)*(a + I*a*Tan[c + d*x])^11)/(a^3*d) + ((I/3)*(a + I*a*Tan[c + d
*x])^12)/(a^4*d) - ((I/13)*(a + I*a*Tan[c + d*x])^13)/(a^5*d)
```

Rubi [A] time = 0.0705574, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{i(a+ia\tan(c+dx))^{13}}{13a^5d}+\frac{i(a+ia\tan(c+dx))^{12}}{3a^4d}-\frac{4i(a+ia\tan(c+dx))^{11}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-4*I)/11)*(a + I*a*Tan[c + d*x])^11)/(a^3*d) + ((I/3)*(a + I*a*Tan[c + d
*x])^12)/(a^4*d) - ((I/13)*(a + I*a*Tan[c + d*x])^13)/(a^5*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^2(a+x)^{10}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a+x)^{10}-4a(a+x)^{11}+(a+x)^{12}\right)dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{11}}{11a^3d}+\frac{i(a+ia\tan(c+dx))^{12}}{3a^4d}-\frac{i(a+ia\tan(c+dx))^{13}}{13a^5d}\end{aligned}$$

Mathematica [B] time = 6.22252, size = 234, normalized size = 2.85

$a^8\sec(c)\sec^{13}(c+dx)(-1716\sin(2c+dx)+1287\sin(2c+3dx)-1287\sin(4c+3dx)+715\sin(4c+5dx)-715\sin(6c+$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^13*((1716*I)*Cos[d*x] + (1716*I)*Cos[2*c + d*x] +
(1287*I)*Cos[2*c + 3*d*x] + (1287*I)*Cos[4*c + 3*d*x] + (715*I)*Cos[4*c + 5
*d*x] + (715*I)*Cos[6*c + 5*d*x] + (286*I)*Cos[6*c + 7*d*x] + (286*I)*Cos[8
*c + 7*d*x] + 1716*Sin[d*x] - 1716*Sin[2*c + d*x] + 1287*Sin[2*c + 3*d*x] -
 1287*Sin[4*c + 3*d*x] + 715*Sin[4*c + 5*d*x] - 715*Sin[6*c + 5*d*x] + 286*
Sin[6*c + 7*d*x] - 286*Sin[8*c + 7*d*x] + 156*Sin[8*c + 9*d*x] + 26*Sin[10*
c + 11*d*x] + 2*Sin[12*c + 13*d*x]))/(1716*d)
```

Maple [B] time = 0.097, size = 475, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(1/13*sin(d*x+c)^9/cos(d*x+c)^13+4/143*sin(d*x+c)^9/cos(d*x+c)^11+
8/1287*sin(d*x+c)^9/cos(d*x+c)^9)+4/3*I*a^8/cos(d*x+c)^6-28*a^8*(1/11*sin(d
*x+c)^7/cos(d*x+c)^11+4/99*sin(d*x+c)^7/cos(d*x+c)^9+8/693*sin(d*x+c)^7/cos
(d*x+c)^7)+56*I*a^8*(1/10*sin(d*x+c)^6/cos(d*x+c)^10+1/20*sin(d*x+c)^6/cos(
d*x+c)^8+1/60*sin(d*x+c)^6/cos(d*x+c)^6)+70*a^8*(1/9*sin(d*x+c)^5/cos(d*x+c
)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5)-8*I*a^8
*(1/12*sin(d*x+c)^8/cos(d*x+c)^12+1/30*sin(d*x+c)^8/cos(d*x+c)^10+1/120*sin
(d*x+c)^8/cos(d*x+c)^8)-28*a^8*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+
c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)-56*I*a^8*(1/8*sin(d*x+c)
^4/cos(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)
^4)-a^8*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [B] time = 1.1192, size = 234, normalized size = 2.85

$$\frac{495\, a^8 \tan\left(dx+c\right)^{13} - 4290i\, a^8 \tan\left(dx+c\right)^{12} - 15210\, a^8 \tan\left(dx+c\right)^{11} + 25740i\, a^8 \tan\left(dx+c\right)^{10} + 10725\, a^8 \tan\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/6435*(495*a^8*tan(d*x + c)^13 - 4290*I*a^8*tan(d*x + c)^12 - 15210*a^8*ta
n(d*x + c)^11 + 25740*I*a^8*tan(d*x + c)^10 + 10725*a^8*tan(d*x + c)^9 + 38
610*I*a^8*tan(d*x + c)^8 + 77220*a^8*tan(d*x + c)^7 - 51480*I*a^8*tan(d*x +
 c)^6 + 19305*a^8*tan(d*x + c)^5 - 64350*I*a^8*tan(d*x + c)^4 - 55770*a^8*t
an(d*x + c)^3 + 25740*I*a^8*tan(d*x + c)^2 + 6435*a^8*tan(d*x + c))/d
```

Fricas [B] time = 1.39586, size = 1041, normalized size = 12.7

$$\frac{1171456i\, a^8 e^{(20i\,dx+20i\,c)} + 2928640i\, a^8 e^{(18i\,dx+18i\,c)} + 5271552i\, a^8 e^{(16i\,dx+16i\,c)} + 7028736i\, a^8 e^{(14i\,dx+14i\,c)} + 7028736i\, a^8 e^{(}}{429\left(de^{(26i\,dx+26i\,c)} + 13\, de^{(24i\,dx+24i\,c)} + 78\, de^{(22i\,dx+22i\,c)} + 286\, de^{(20i\,dx+20i\,c)} + 715\, de^{(18i\,dx+18i\,c)} + 1287\, de^{(16i\,dx+16i\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/429*(1171456*I*a^8*e^(20*I*d*x + 20*I*c) + 2928640*I*a^8*e^(18*I*d*x + 18
*I*c) + 5271552*I*a^8*e^(16*I*d*x + 16*I*c) + 7028736*I*a^8*e^(14*I*d*x + 1
4*I*c) + 7028736*I*a^8*e^(12*I*d*x + 12*I*c) + 5271552*I*a^8*e^(10*I*d*x +
10*I*c) + 2928640*I*a^8*e^(8*I*d*x + 8*I*c) + 1171456*I*a^8*e^(6*I*d*x + 6*
I*c) + 319488*I*a^8*e^(4*I*d*x + 4*I*c) + 53248*I*a^8*e^(2*I*d*x + 2*I*c) +
 4096*I*a^8)/(d*e^(26*I*d*x + 26*I*c) + 13*d*e^(24*I*d*x + 24*I*c) + 78*d*e
^(22*I*d*x + 22*I*c) + 286*d*e^(20*I*d*x + 20*I*c) + 715*d*e^(18*I*d*x + 18
*I*c) + 1287*d*e^(16*I*d*x + 16*I*c) + 1716*d*e^(14*I*d*x + 14*I*c) + 1716*
d*e^(12*I*d*x + 12*I*c) + 1287*d*e^(10*I*d*x + 10*I*c) + 715*d*e^(8*I*d*x +
 8*I*c) + 286*d*e^(6*I*d*x + 6*I*c) + 78*d*e^(4*I*d*x + 4*I*c) + 13*d*e^(2*
I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.76486, size = 234, normalized size = 2.85

$$33\,a^8\tan\left(dx+c\right)^{13}-286i\,a^8\tan\left(dx+c\right)^{12}-1014\,a^8\tan\left(dx+c\right)^{11}+1716i\,a^8\tan\left(dx+c\right)^{10}+715\,a^8\tan\left(dx+c\right)^{9}+25$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/429*(33*a^8*tan(d*x + c)^13 - 286*I*a^8*tan(d*x + c)^12 - 1014*a^8*tan(d*
x + c)^11 + 1716*I*a^8*tan(d*x + c)^10 + 715*a^8*tan(d*x + c)^9 + 2574*I*a^
8*tan(d*x + c)^8 + 5148*a^8*tan(d*x + c)^7 - 3432*I*a^8*tan(d*x + c)^6 + 12
87*a^8*tan(d*x + c)^5 - 4290*I*a^8*tan(d*x + c)^4 - 3718*a^8*tan(d*x + c)^3
 + 1716*I*a^8*tan(d*x + c)^2 + 429*a^8*tan(d*x + c))/d
```

3.79 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=55

$$\frac{i(a+ia\tan(c+dx))^{11}}{11a^3d}-\frac{i(a+ia\tan(c+dx))^{10}}{5a^2d}$$

```
[Out] ((-I/5)*(a + I*a*Tan[c + d*x])^10)/(a^2*d) + ((I/11)*(a + I*a*Tan[c + d*x])
^11)/(a^3*d)
```

Rubi [A] time = 0.0454868, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a+ia\tan(c+dx))^{11}}{11a^3d}-\frac{i(a+ia\tan(c+dx))^{10}}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-I/5)*(a + I*a*Tan[c + d*x])^10)/(a^2*d) + ((I/11)*(a + I*a*Tan[c + d*x])
^11)/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)(a+x)^9\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(2a(a+x)^9-(a+x)^{10}\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i(a+ia\tan(c+dx))^{10}}{5a^2d}+\frac{i(a+ia\tan(c+dx))^{11}}{11a^3d}\end{aligned}$$

Mathematica [B] time = 4.20502, size = 223, normalized size = 4.05

$$\frac{a^8\sec(c)\sec^{11}(c+dx)(-462\sin(2c+dx)+330\sin(2c+3dx)-330\sin(4c+3dx)+165\sin(4c+5dx)-165\sin(6c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^11*((462*I)*Cos[d*x] + (462*I)*Cos[2*c + d*x] + (3
30*I)*Cos[2*c + 3*d*x] + (330*I)*Cos[4*c + 3*d*x] + (165*I)*Cos[4*c + 5*d*x
] + (165*I)*Cos[6*c + 5*d*x] + (55*I)*Cos[6*c + 7*d*x] + (55*I)*Cos[8*c + 7
*d*x] + 462*Sin[d*x] - 462*Sin[2*c + d*x] + 330*Sin[2*c + 3*d*x] - 330*Sin[
4*c + 3*d*x] + 165*Sin[4*c + 5*d*x] - 165*Sin[6*c + 5*d*x] + 55*Sin[6*c + 7
*d*x] - 55*Sin[8*c + 7*d*x] + 22*Sin[8*c + 9*d*x] + 2*Sin[10*c + 11*d*x]))/
(220*d)
```

Maple [B] time = 0.095, size = 339, normalized size = 6.2

$$\frac{1}{d}\left(a^8\left(\frac{(\sin(dx+c))^9}{11\,(\cos(dx+c))^{11}}+\frac{2\,(\sin(dx+c))^9}{99\,(\cos(dx+c))^9}\right)-56\,ia^8\left(\frac{(\sin(dx+c))^4}{6\,(\cos(dx+c))^6}+\frac{(\sin(dx+c))^4}{12\,(\cos(dx+c))^4}\right)-28\,a^8\left(1/9\,\frac{(\sin(dx+}{(\cos(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(1/11*sin(d*x+c)^9/cos(d*x+c)^11+2/99*sin(d*x+c)^9/cos(d*x+c)^9)-5
6*I*a^8*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)-28*a
^8*(1/9*sin(d*x+c)^7/cos(d*x+c)^9+2/63*sin(d*x+c)^7/cos(d*x+c)^7)+56*I*a^8*
(1/8*sin(d*x+c)^6/cos(d*x+c)^8+1/24*sin(d*x+c)^6/cos(d*x+c)^6)+70*a^8*(1/7*
sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5)-8*I*a^8*(1/10*sin
(d*x+c)^8/cos(d*x+c)^10+1/40*sin(d*x+c)^8/cos(d*x+c)^8)-28*a^8*(1/5*sin(d*x
+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+2*I*a^8/cos(d*x+c)^4-a^8
*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [B] time = 1.34338, size = 181, normalized size = 3.29

$$\frac{45\,a^8\tan(dx+c)^{11}-396i\,a^8\tan(dx+c)^{10}-1485\,a^8\tan(dx+c)^9+2970i\,a^8\tan(dx+c)^8+2970\,a^8\tan(dx+c)^7+41}{495\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/495*(45*a^8*tan(d*x + c)^11 - 396*I*a^8*tan(d*x + c)^10 - 1485*a^8*tan(d*
x + c)^9 + 2970*I*a^8*tan(d*x + c)^8 + 2970*a^8*tan(d*x + c)^7 + 4158*a^8*t
an(d*x + c)^5 - 5940*I*a^8*tan(d*x + c)^4 - 4455*a^8*tan(d*x + c)^3 + 1980*
I*a^8*tan(d*x + c)^2 + 495*a^8*tan(d*x + c))/d
```

Fricas [B] time = 1.54089, size = 890, normalized size = 16.18

$$\frac{56320i\,a^8e^{(18i\,dx+18i\,c)}+168960i\,a^8e^{(16i\,dx+16i\,c)}+337920i\,a^8e^{(14i\,dx+14i\,c)}+473088i\,a^8e^{(12i\,dx+12i\,c)}+473088i\,a^8e^{(10i\,dx+10i\,c)}}{55\left(de^{(22i\,dx+22i\,c)}+11\,de^{(20i\,dx+20i\,c)}+55\,de^{(18i\,dx+18i\,c)}+165\,de^{(16i\,dx+16i\,c)}+330\,de^{(14i\,dx+14i\,c)}+462\,de^{(12i\,dx+12i\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/55*(56320*I*a^8*e^(18*I*d*x + 18*I*c) + 168960*I*a^8*e^(16*I*d*x + 16*I*c
) + 337920*I*a^8*e^(14*I*d*x + 14*I*c) + 473088*I*a^8*e^(12*I*d*x + 12*I*c)
 + 473088*I*a^8*e^(10*I*d*x + 10*I*c) + 337920*I*a^8*e^(8*I*d*x + 8*I*c) +
168960*I*a^8*e^(6*I*d*x + 6*I*c) + 56320*I*a^8*e^(4*I*d*x + 4*I*c) + 11264*
I*a^8*e^(2*I*d*x + 2*I*c) + 1024*I*a^8)/(d*e^(22*I*d*x + 22*I*c) + 11*d*e^(
20*I*d*x + 20*I*c) + 55*d*e^(18*I*d*x + 18*I*c) + 165*d*e^(16*I*d*x + 16*I*
c) + 330*d*e^(14*I*d*x + 14*I*c) + 462*d*e^(12*I*d*x + 12*I*c) + 462*d*e^(1
0*I*d*x + 10*I*c) + 330*d*e^(8*I*d*x + 8*I*c) + 165*d*e^(6*I*d*x + 6*I*c) +
 55*d*e^(4*I*d*x + 4*I*c) + 11*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.81662, size = 181, normalized size = 3.29

$$\frac{5\,a^8\tan\left(dx+c\right)^{11}-44i\,a^8\tan\left(dx+c\right)^{10}-165\,a^8\tan\left(dx+c\right)^9+330i\,a^8\tan\left(dx+c\right)^8+330\,a^8\tan\left(dx+c\right)^7+462\,a}{55\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/55*(5*a^8*tan(d*x + c)^11 - 44*I*a^8*tan(d*x + c)^10 - 165*a^8*tan(d*x +
c)^9 + 330*I*a^8*tan(d*x + c)^8 + 330*a^8*tan(d*x + c)^7 + 462*a^8*tan(d*x
+ c)^5 - 660*I*a^8*tan(d*x + c)^4 - 495*a^8*tan(d*x + c)^3 + 220*I*a^8*tan(
d*x + c)^2 + 55*a^8*tan(d*x + c))/d
```

3.80 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=27

$$-\frac{i(a+ia\tan(c+dx))^9}{9ad}$$

```
[Out] ((-I/9)*(a + I*a*Tan[c + d*x])^9)/(a*d)
```

Rubi [A] time = 0.0377482, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$-\frac{i(a+ia\tan(c+dx))^9}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-I/9)*(a + I*a*Tan[c + d*x])^9)/(a*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{i\,\text{Subst}\left(\int (a+x)^8\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{i(a+ia\tan(c+dx))^9}{9ad}\end{aligned}$$

Mathematica [B] time = 2.86396, size = 212, normalized size = 7.85

$$a^8\sec(c)\sec^9(c+dx)(-126\sin(2c+dx)+84\sin(2c+3dx)-84\sin(4c+3dx)+36\sin(4c+5dx)-36\sin(6c+5dx)+9$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^9*((126*I)*Cos[d*x] + (126*I)*Cos[2*c + d*x] + (84
*I)*Cos[2*c + 3*d*x] + (84*I)*Cos[4*c + 3*d*x] + (36*I)*Cos[4*c + 5*d*x] +
(36*I)*Cos[6*c + 5*d*x] + (9*I)*Cos[6*c + 7*d*x] + (9*I)*Cos[8*c + 7*d*x] +
 126*Sin[d*x] - 126*Sin[2*c + d*x] + 84*Sin[2*c + 3*d*x] - 84*Sin[4*c + 3*d
```

```
*x] + 36*Sin[4*c + 5*d*x] - 36*Sin[6*c + 5*d*x] + 9*Sin[6*c + 7*d*x] - 9*Si
n[8*c + 7*d*x] + 2*Sin[8*c + 9*d*x]))/(18*d)
```

Maple [B] time = 0.089, size = 180, normalized size = 6.7

$$\frac{1}{d}\left(\frac{a^8(\sin(dx+c))^9}{9(\cos(dx+c))^9}-\frac{14ia^8(\sin(dx+c))^4}{(\cos(dx+c))^4}-4\frac{a^8(\sin(dx+c))^7}{(\cos(dx+c))^7}+\frac{\frac{28i}{3}a^8(\sin(dx+c))^6}{(\cos(dx+c))^6}+14\frac{a^8(\sin(dx+c))^5}{(\cos(dx+c))^5}-\frac{ia^8}{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(1/9*a^8*sin(d*x+c)^9/cos(d*x+c)^9-14*I*a^8*sin(d*x+c)^4/cos(d*x+c)^4-4
*a^8*sin(d*x+c)^7/cos(d*x+c)^7+28/3*I*a^8*sin(d*x+c)^6/cos(d*x+c)^6+14*a^8*
sin(d*x+c)^5/cos(d*x+c)^5-I*a^8*sin(d*x+c)^8/cos(d*x+c)^8-28/3*a^8*sin(d*x+
c)^3/cos(d*x+c)^3+4*I*a^8/cos(d*x+c)^2+a^8*tan(d*x+c))
```

Maxima [A] time = 1.16394, size = 28, normalized size = 1.04

$$-\frac{i(ia\tan(dx+c)+a)^9}{9ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/9*I*(I*a*tan(d*x + c) + a)^9/(a*d)
```

Fricas [B] time = 1.29112, size = 741, normalized size = 27.44

$$\frac{4608ia^8e^{(16idx+16ic)}+18432ia^8e^{(14idx+14ic)}+43008ia^8e^{(12idx+12ic)}+64512ia^8e^{(10idx+10ic)}+64512ia^8e^{(8idx+8ic)}+43}{9\left(de^{(18idx+18ic)}+9de^{(16idx+16ic)}+36de^{(14idx+14ic)}+84de^{(12idx+12ic)}+126de^{(10idx+10ic)}+126de^{(8idx+8i}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9*(4608*I*a^8*e^(16*I*d*x + 16*I*c) + 18432*I*a^8*e^(14*I*d*x + 14*I*c) +
 43008*I*a^8*e^(12*I*d*x + 12*I*c) + 64512*I*a^8*e^(10*I*d*x + 10*I*c) + 64
512*I*a^8*e^(8*I*d*x + 8*I*c) + 43008*I*a^8*e^(6*I*d*x + 6*I*c) + 18432*I*a
^8*e^(4*I*d*x + 4*I*c) + 4608*I*a^8*e^(2*I*d*x + 2*I*c) + 512*I*a^8)/(d*e^(
18*I*d*x + 18*I*c) + 9*d*e^(16*I*d*x + 16*I*c) + 36*d*e^(14*I*d*x + 14*I*c)
 + 84*d*e^(12*I*d*x + 12*I*c) + 126*d*e^(10*I*d*x + 10*I*c) + 126*d*e^(8*I*
d*x + 8*I*c) + 84*d*e^(6*I*d*x + 6*I*c) + 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^
(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^8\left(\int -28\tan^2(c+dx)\sec^2(c+dx)\,dx+\int 70\tan^4(c+dx)\sec^2(c+dx)\,dx+\int -28\tan^6(c+dx)\sec^2(c+dx)\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(70*tan(c
+ d*x)**4*sec(c + d*x)**2, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)**
2, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**2, x) + Integral(8*I*tan(c +
 d*x)*sec(c + d*x)**2, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)**2,
 x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**2, x) + Integral(-8*I*tan
(c + d*x)**7*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

Giac [B] time = 1.79316, size = 162, normalized size = 6.

$$\frac{a^8\tan(dx+c)^9 - 9i\,a^8\tan(dx+c)^8 - 36\,a^8\tan(dx+c)^7 + 84i\,a^8\tan(dx+c)^6 + 126\,a^8\tan(dx+c)^5 - 126i\,a^8\tan(dx}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/9*(a^8*tan(d*x + c)^9 - 9*I*a^8*tan(d*x + c)^8 - 36*a^8*tan(d*x + c)^7 +
84*I*a^8*tan(d*x + c)^6 + 126*a^8*tan(d*x + c)^5 - 126*I*a^8*tan(d*x + c)^4
 - 84*a^8*tan(d*x + c)^3 + 36*I*a^8*tan(d*x + c)^2 + 9*a^8*tan(d*x + c))/d
```

3.81 $\int (a + ia\tan(c + dx))^8 \, dx$

Optimal. Leaf size=200

$$-\frac{64a^8\tan(c+dx)}{d}+\frac{4ia^3(a+ia\tan(c+dx))^5}{5d}+\frac{ia^2(a+ia\tan(c+dx))^6}{3d}+\frac{16ia^2\left(a^2+ia^2\tan(c+dx)\right)^3}{3d}+\frac{2i\left(a^2+i\right.}{}$$

```
[Out] 128*a^8*x - ((128*I)*a^8*Log[Cos[c + d*x]])/d - (64*a^8*Tan[c + d*x])/d + (
((4*I)/5)*a^3*(a + I*a*Tan[c + d*x])^5)/d + ((I/3)*a^2*(a + I*a*Tan[c + d*x
])^6)/d + ((I/7)*a*(a + I*a*Tan[c + d*x])^7)/d + (((16*I)/3)*a^2*(a^2 + I*a
^2*Tan[c + d*x])^3)/d + ((2*I)*(a^2 + I*a^2*Tan[c + d*x])^4)/d + ((16*I)*(a
^4 + I*a^4*Tan[c + d*x])^2)/d
```

Rubi [A] time = 0.138817, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3478, 3477, 3475}

$$-\frac{64a^8\tan(c+dx)}{d}+\frac{4ia^3(a+ia\tan(c+dx))^5}{5d}+\frac{ia^2(a+ia\tan(c+dx))^6}{3d}+\frac{16ia^2\left(a^2+ia^2\tan(c+dx)\right)^3}{3d}+\frac{2i\left(a^2+i\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^8,x]
```


```
[Out] 128*a^8*x - ((128*I)*a^8*Log[Cos[c + d*x]])/d - (64*a^8*Tan[c + d*x])/d + (
((4*I)/5)*a^3*(a + I*a*Tan[c + d*x])^5)/d + ((I/3)*a^2*(a + I*a*Tan[c + d*x
])^6)/d + ((I/7)*a*(a + I*a*Tan[c + d*x])^7)/d + (((16*I)/3)*a^2*(a^2 + I*a
^2*Tan[c + d*x])^3)/d + ((2*I)*(a^2 + I*a^2*Tan[c + d*x])^4)/d + ((16*I)*(a
^4 + I*a^4*Tan[c + d*x])^2)/d
```


Rule 3478

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+ia\tan(c+dx))^8\,dx = \frac{ia(a+ia\tan(c+dx))^7}{7d} + (2a)\int (a+ia\tan(c+dx))^7\,dx$$
$$= \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{ia(a+ia\tan(c+dx))^7}{7d} + \left(4a^2\right)\int (a+ia\tan(c+dx))^6\,dx$$
$$= \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{ia(a+ia\tan(c+dx))^7}{7d} + \left(8a^3\right)\int (a$$
$$= \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{ia(a+ia\tan(c+dx))^7}{7d} + \frac{2i\left(a^2+ia^2\right.}{}$$
$$= \frac{16ia^5(a+ia\tan(c+dx))^3}{3d} + \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{ia(a+i}{}$$
$$= \frac{16ia^5(a+ia\tan(c+dx))^3}{3d} + \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{ia(a+i}{}$$
$$= 128a^8x - \frac{64a^8\tan(c+dx)}{d} + \frac{16ia^5(a+ia\tan(c+dx))^3}{3d} + \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} + \frac{ia^2(}{}$$
$$= 128a^8x - \frac{128ia^8\log(\cos(c+dx))}{d} - \frac{64a^8\tan(c+dx)}{d} + \frac{16ia^5(a+ia\tan(c+dx))^3}{3d} + \frac{4ia^3(a}{}$$

Mathematica [A] time = 4.13016, size = 383, normalized size = 1.92

$$\frac{a^8\sec(c)\sec^7(c+dx)\left(70\cos(dx)\left(-105i\log\left(\cos^2(c+dx)\right)+210dx-139i\right)+70\cos(2c+dx)\left(-105i\log\left(\cos^2(c+dx)\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^7*(70*Cos[d*x]*(-139*I + 210*d*x - (105*I)*Log[Cos
[c + d*x]^2]) + 70*Cos[2*c + d*x]*(-139*I + 210*d*x - (105*I)*Log[Cos[c + d
*x]^2]) + 3*((-420*I)*Cos[4*c + 5*d*x] + 980*d*x*Cos[4*c + 5*d*x] - (420*I)
*Cos[6*c + 5*d*x] + 980*d*x*Cos[6*c + 5*d*x] + 140*d*x*Cos[6*c + 7*d*x] + 1
40*d*x*Cos[8*c + 7*d*x] + 70*Cos[2*c + 3*d*x]*(-25*I + 42*d*x - (21*I)*Log[
Cos[c + d*x]^2]) + 70*Cos[4*c + 3*d*x]*(-25*I + 42*d*x - (21*I)*Log[Cos[c +
 d*x]^2]) - (490*I)*Cos[4*c + 5*d*x]*Log[Cos[c + d*x]^2] - (490*I)*Cos[6*c
+ 5*d*x]*Log[Cos[c + d*x]^2] - (70*I)*Cos[6*c + 7*d*x]*Log[Cos[c + d*x]^2]
- (70*I)*Cos[8*c + 7*d*x]*Log[Cos[c + d*x]^2] - 6965*Sin[d*x] + 5740*Sin[2*
c + d*x] - 4963*Sin[2*c + 3*d*x] + 2660*Sin[4*c + 3*d*x] - 1981*Sin[4*c + 5
*d*x] + 560*Sin[6*c + 5*d*x] - 363*Sin[6*c + 7*d*x])))/(420*d)
```

Maple [A] time = 0.006, size = 150, normalized size = 0.8

$$-127\,\frac{a^8\tan(dx+c)}{d} + \frac{a^8(\tan(dx+c))^7}{7\,d} - \frac{\frac{4\,i}{3}a^8(\tan(dx+c))^6}{d} - \frac{29\,a^8(\tan(dx+c))^5}{5\,d} + \frac{16\,ia^8(\tan(dx+c))^4}{d} + 33\,\frac{a^8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^8,x)
```

```
[Out] -127*a^8*tan(d*x+c)/d+1/7/d*a^8*tan(d*x+c)^7-4/3*I/d*a^8*tan(d*x+c)^6-29/5*
a^8*tan(d*x+c)^5/d+16*I/d*a^8*tan(d*x+c)^4+33*a^8*tan(d*x+c)^3/d-60*I/d*a^8
*tan(d*x+c)^2+64*I/d*a^8*ln(1+tan(d*x+c)^2)+128/d*a^8*arctan(tan(d*x+c))
```

Maxima [A] time = 1.62787, size = 163, normalized size = 0.82

$$\frac{15\,a^8\tan\left(dx+c\right)^7-140i\,a^8\tan\left(dx+c\right)^6-609\,a^8\tan\left(dx+c\right)^5+1680i\,a^8\tan\left(dx+c\right)^4+3465\,a^8\tan\left(dx+c\right)^3-63}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/105*(15*a^8*tan(d*x + c)^7 - 140*I*a^8*tan(d*x + c)^6 - 609*a^8*tan(d*x +
 c)^5 + 1680*I*a^8*tan(d*x + c)^4 + 3465*a^8*tan(d*x + c)^3 - 6300*I*a^8*ta
n(d*x + c)^2 + 13440*(d*x + c)*a^8 + 6720*I*a^8*log(tan(d*x + c)^2 + 1) - 1
3335*a^8*tan(d*x + c))/d
```

Fricas [A] time = 1.4577, size = 976, normalized size = 4.88

$$\frac{-94080i\,a^8e^{(12i\,dx+12i\,c)}-423360i\,a^8e^{(10i\,dx+10i\,c)}-862400i\,a^8e^{(8i\,dx+8i\,c)}-980000i\,a^8e^{(6i\,dx+6i\,c)}-644448i\,a^8e^{(4i\,dx+4i\,c)}-}{105\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(-94080*I*a^8*e^(12*I*d*x + 12*I*c) - 423360*I*a^8*e^(10*I*d*x + 10*I
*c) - 862400*I*a^8*e^(8*I*d*x + 8*I*c) - 980000*I*a^8*e^(6*I*d*x + 6*I*c) -
 644448*I*a^8*e^(4*I*d*x + 4*I*c) - 230496*I*a^8*e^(2*I*d*x + 2*I*c) - 3484
8*I*a^8 + (-13440*I*a^8*e^(14*I*d*x + 14*I*c) - 94080*I*a^8*e^(12*I*d*x + 1
2*I*c) - 282240*I*a^8*e^(10*I*d*x + 10*I*c) - 470400*I*a^8*e^(8*I*d*x + 8*I
*c) - 470400*I*a^8*e^(6*I*d*x + 6*I*c) - 282240*I*a^8*e^(4*I*d*x + 4*I*c) -
 94080*I*a^8*e^(2*I*d*x + 2*I*c) - 13440*I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1
))/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x
+ 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4
*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 13.3389, size = 313, normalized size = 1.56

$$-\frac{128ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-\frac{896ia^8e^{-2ic}e^{12idx}}{d}-\frac{4032ia^8e^{-4ic}e^{10idx}}{d}-\frac{24640ia^8e^{-6ic}e^{8idx}}{3d}-\frac{28000ia^8e^{-8ic}e^{6idx}}{3d}-\frac{30688ia^8e^{-10ic}e^{4idx}}{5d}-\frac{1097}{}}{e^{14idx}+7e^{-2ic}e^{12idx}+21e^{-4ic}e^{10idx}+35e^{-6ic}e^{8idx}+35e^{-8ic}e^{6idx}+21e^{-10ic}e^{4idx}+7e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**8,x)
```

```
[Out] -128*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-896*I*a**8*exp(-2*I*c)*ex
p(12*I*d*x)/d - 4032*I*a**8*exp(-4*I*c)*exp(10*I*d*x)/d - 24640*I*a**8*exp(
-6*I*c)*exp(8*I*d*x)/(3*d) - 28000*I*a**8*exp(-8*I*c)*exp(6*I*d*x)/(3*d) -
30688*I*a**8*exp(-10*I*c)*exp(4*I*d*x)/(5*d) - 10976*I*a**8*exp(-12*I*c)*ex
p(2*I*d*x)/(5*d) - 11616*I*a**8*exp(-14*I*c)/(35*d))/(exp(14*I*d*x) + 7*exp
(-2*I*c)*exp(12*I*d*x) + 21*exp(-4*I*c)*exp(10*I*d*x) + 35*exp(-6*I*c)*exp(
8*I*d*x) + 35*exp(-8*I*c)*exp(6*I*d*x) + 21*exp(-10*I*c)*exp(4*I*d*x) + 7*e
xp(-12*I*c)*exp(2*I*d*x) + exp(-14*I*c))
```

Giac [B] time = 1.212, size = 510, normalized size = 2.55

$$\frac{-13440i\,a^8e^{(14i\,dx+14i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-94080i\,a^8e^{(12i\,dx+12i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-282240i\,a^8e^{(10i\,dx+10i\,c)}\log\left(e^{(2i\,dx+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/105*(-13440*I*a^8*e^(14*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 94
080*I*a^8*e^(12*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 282240*I*a^8
*e^(10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 470400*I*a^8*e^(8*I*d
*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 470400*I*a^8*e^(6*I*d*x + 6*I*c)
*log(e^(2*I*d*x + 2*I*c) + 1) - 282240*I*a^8*e^(4*I*d*x + 4*I*c)*log(e^(2*I
*d*x + 2*I*c) + 1) - 94080*I*a^8*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c
) + 1) - 94080*I*a^8*e^(12*I*d*x + 12*I*c) - 423360*I*a^8*e^(10*I*d*x + 10*
I*c) - 862400*I*a^8*e^(8*I*d*x + 8*I*c) - 980000*I*a^8*e^(6*I*d*x + 6*I*c)
- 644448*I*a^8*e^(4*I*d*x + 4*I*c) - 230496*I*a^8*e^(2*I*d*x + 2*I*c) - 134
40*I*a^8*log(e^(2*I*d*x + 2*I*c) + 1) - 34848*I*a^8)/(d*e^(14*I*d*x + 14*I*
c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d
*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(
2*I*d*x + 2*I*c) + d)
```

3.82 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=133

$$\frac{a^8\tan^5(c+dx)}{5d}-\frac{2ia^8\tan^4(c+dx)}{d}-\frac{10a^8\tan^3(c+dx)}{d}+\frac{36ia^8\tan^2(c+dx)}{d}-\frac{64ia^9}{d(a-ia\tan(c+dx))}+\frac{129a^8\tan(c}{d}$$

```
[Out] -192*a^8*x + ((192*I)*a^8*Log[Cos[c + d*x]])/d + (129*a^8*Tan[c + d*x])/d +
 ((36*I)*a^8*Tan[c + d*x]^2)/d - (10*a^8*Tan[c + d*x]^3)/d - ((2*I)*a^8*Tan
[c + d*x]^4)/d + (a^8*Tan[c + d*x]^5)/(5*d) - ((64*I)*a^9)/(d*(a - I*a*Tan[
c + d*x]))
```

Rubi [A] time = 0.0816082, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{a^8\tan^5(c+dx)}{5d}-\frac{2ia^8\tan^4(c+dx)}{d}-\frac{10a^8\tan^3(c+dx)}{d}+\frac{36ia^8\tan^2(c+dx)}{d}-\frac{64ia^9}{d(a-ia\tan(c+dx))}+\frac{129a^8\tan(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] -192*a^8*x + ((192*I)*a^8*Log[Cos[c + d*x]])/d + (129*a^8*Tan[c + d*x])/d +
 ((36*I)*a^8*Tan[c + d*x]^2)/d - (10*a^8*Tan[c + d*x]^3)/d - ((2*I)*a^8*Tan
[c + d*x]^4)/d + (a^8*Tan[c + d*x]^5)/(5*d) - ((64*I)*a^9)/(d*(a - I*a*Tan[
c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{\left(ia^3\right)\text{Subst}\left(\int \frac{(a+x)^6}{(a-x)^2}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{\left(ia^3\right)\text{Subst}\left(\int \left(129a^4+\frac{64a^6}{(a-x)^2}-\frac{192a^5}{a-x}+72a^3x+30a^2x^2+8ax^3+x^4\right)dx,\right.}{d}\\ &= -192a^8x+\frac{192ia^8\log(\cos(c+dx))}{d}+\frac{129a^8\tan(c+dx)}{d}+\frac{36ia^8\tan^2(c+d}{d}\end{aligned}$$

Mathematica [B] time = 6.05142, size = 321, normalized size = 2.41

$$\cos^3(c+dx)(a+ia\tan(c+dx))^8\left(-960dx\cos(8c)\cos^5(c+dx)+480i\cos(8c)\cos^5(c+dx)\log\left(\cos^2(c+dx)\right)-160i(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Cos[c + d*x]^3*(-960*d*x*Cos[8*c]*Cos[c + d*x]^5 + (480*I)*Cos[8*c]*Cos[c
+ d*x]^5*Log[Cos[c + d*x]^2] - (160*I)*Cos[2*d*x]*Cos[c + d*x]^5*(Cos[6*c]
- I*Sin[6*c]) + (960*I)*d*x*Cos[c + d*x]^5*Sin[8*c] + 480*Cos[c + d*x]^5*Lo
g[Cos[c + d*x]^2]*Sin[8*c] + Sec[c]*(Cos[8*c] - I*Sin[8*c])*Sin[d*x] - 52*C
os[c + d*x]^2*Sec[c]*(Cos[8*c] - I*Sin[8*c])*Sin[d*x] + 696*Cos[c + d*x]^4*
Sec[c]*(Cos[8*c] - I*Sin[8*c])*Sin[d*x] + 160*Cos[c + d*x]^5*(Cos[6*c] - I*
Sin[6*c])*Sin[2*d*x] + Cos[c + d*x]*(Cos[8*c] - I*Sin[8*c])*(-10*I + Tan[c]
) - 4*Cos[c + d*x]^3*(Cos[8*c] - I*Sin[8*c])*(-50*I + 13*Tan[c]))*(a + I*a*
Tan[c + d*x])^8)/(5*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] time = 0.156, size = 406, normalized size = 3.1

$$\frac{8\,a^8\cos(dx+c)(\sin(dx+c))^7}{5\,d} - 192\,\frac{a^8c}{d} + \frac{34\,ia^8(\sin(dx+c))^4}{d} + \frac{4\,ia^8(\sin(dx+c))^8}{d(\cos(dx+c))^2} + \frac{28\,ia^8(\sin(dx+c))^6}{d(\cos(dx+c))^2} + \frac{96\,ia^8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 8/5/d*a^8*cos(d*x+c)*sin(d*x+c)^7-192/d*a^8*c+34*I/d*a^8*sin(d*x+c)^4+4*I/d
*a^8*sin(d*x+c)^8/cos(d*x+c)^2+28*I/d*a^8*sin(d*x+c)^6/cos(d*x+c)^2+96*I/d*
a^8*sin(d*x+c)^2-4*I/d*a^8*cos(d*x+c)^2-2*I/d*a^8*sin(d*x+c)^8/cos(d*x+c)^4
+192*I*a^8*ln(cos(d*x+c))/d+4*I/d*a^8*sin(d*x+c)^6+70/d*a^8*sin(d*x+c)^5/co
s(d*x+c)+193/d*a^8*sin(d*x+c)*cos(d*x+c)-28/3/d*a^8*sin(d*x+c)^7/cos(d*x+c)
^3+112/3/d*a^8*sin(d*x+c)^7/cos(d*x+c)-192*a^8*x+1/5/d*a^8*sin(d*x+c)^9/cos
(d*x+c)^5-4/15/d*a^8*sin(d*x+c)^9/cos(d*x+c)^3+8/5/d*a^8*sin(d*x+c)^9/cos(d
*x+c)+196/5/d*a^8*cos(d*x+c)*sin(d*x+c)^5+119/d*a^8*cos(d*x+c)*sin(d*x+c)^3
```

Maxima [A] time = 1.51626, size = 167, normalized size = 1.26

$$\frac{a^8\tan(dx+c)^5 - 10i\,a^8\tan(dx+c)^4 - 50\,a^8\tan(dx+c)^3 + 180i\,a^8\tan(dx+c)^2 - 960\,(dx+c)a^8 - 480i\,a^8\log\left(\tan(d\right.}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(a^8*tan(d*x + c)^5 - 10*I*a^8*tan(d*x + c)^4 - 50*a^8*tan(d*x + c)^3 +
 180*I*a^8*tan(d*x + c)^2 - 960*(d*x + c)*a^8 - 480*I*a^8*log(tan(d*x + c)^
2 + 1) + 645*a^8*tan(d*x + c) + 320*(a^8*tan(d*x + c) - I*a^8)/(tan(d*x + c
)^2 + 1))/d
```

Fricas [B] time = 1.5429, size = 760, normalized size = 5.71

$$\frac{-160i\,a^8e^{(12i\,dx+12i\,c)} - 800i\,a^8e^{(10i\,dx+10i\,c)} + 800i\,a^8e^{(8i\,dx+8i\,c)} + 6400i\,a^8e^{(6i\,dx+6i\,c)} + 9600i\,a^8e^{(4i\,dx+4i\,c)} + 6000i\,a^8e^{(2i\,dx+2i\,c}}{5\left(de^{(10i\,dx+10i\,c)} + 5\,de^{(8i\,dx+8i}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/5*(-160*I*a^8*e^(12*I*d*x + 12*I*c) - 800*I*a^8*e^(10*I*d*x + 10*I*c) + 8
00*I*a^8*e^(8*I*d*x + 8*I*c) + 6400*I*a^8*e^(6*I*d*x + 6*I*c) + 9600*I*a^8*
e^(4*I*d*x + 4*I*c) + 6000*I*a^8*e^(2*I*d*x + 2*I*c) + 1392*I*a^8 + (960*I*
a^8*e^(10*I*d*x + 10*I*c) + 4800*I*a^8*e^(8*I*d*x + 8*I*c) + 9600*I*a^8*e^(
6*I*d*x + 6*I*c) + 9600*I*a^8*e^(4*I*d*x + 4*I*c) + 4800*I*a^8*e^(2*I*d*x +
 2*I*c) + 960*I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c)
 + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4
*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 5.69276, size = 252, normalized size = 1.89

$$64a^8\left(\begin{cases} -\frac{ie^{2idx}}{2d} & \text{for } d \neq 0 \\ x & \text{otherwise} \end{cases}\right)e^{2ic} + \frac{192ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{\frac{480ia^8e^{-2ic}e^{8idx}}{d} + \frac{1600ia^8e^{-4ic}e^{6idx}}{d} + \frac{2080ia^8e^{-6ic}e^{4idx}}{d} + \frac{1232ia^8e^{-}}{d}}{e^{10idx} + 5e^{-2ic}e^{8idx} + 10e^{-4ic}e^{6idx} + 10e^{-6ic}e^{4idx} + 5e^{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 64*a**8*Piecewise((-I*exp(2*I*d*x)/(2*d), Ne(d, 0)), (x, True))*exp(2*I*c)
+ 192*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (480*I*a**8*exp(-2*I*c)*ex
p(8*I*d*x)/d + 1600*I*a**8*exp(-4*I*c)*exp(6*I*d*x)/d + 2080*I*a**8*exp(-6*
I*c)*exp(4*I*d*x)/d + 1232*I*a**8*exp(-8*I*c)*exp(2*I*d*x)/d + 1392*I*a**8*
exp(-10*I*c)/(5*d))/(exp(10*I*d*x) + 5*exp(-2*I*c)*exp(8*I*d*x) + 10*exp(-4
*I*c)*exp(6*I*d*x) + 10*exp(-6*I*c)*exp(4*I*d*x) + 5*exp(-8*I*c)*exp(2*I*d*
x) + exp(-10*I*c))
```

Giac [B] time = 1.91307, size = 408, normalized size = 3.07

$$\frac{960i\,a^8e^{(10i\,dx+10i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) + 4800i\,a^8e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) + 9600i\,a^8e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/5*(960*I*a^8*e^(10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 4800*I*
a^8*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 9600*I*a^8*e^(6*I*d*
x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 9600*I*a^8*e^(4*I*d*x + 4*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) + 4800*I*a^8*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x
+ 2*I*c) + 1) - 160*I*a^8*e^(12*I*d*x + 12*I*c) - 800*I*a^8*e^(10*I*d*x + 1
0*I*c) + 800*I*a^8*e^(8*I*d*x + 8*I*c) + 6400*I*a^8*e^(6*I*d*x + 6*I*c) + 9
600*I*a^8*e^(4*I*d*x + 4*I*c) + 6000*I*a^8*e^(2*I*d*x + 2*I*c) + 960*I*a^8*
log(e^(2*I*d*x + 2*I*c) + 1) + 1392*I*a^8)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e
^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) +
5*d*e^(2*I*d*x + 2*I*c) + d)
```

3.83 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=124

$$\frac{a^8\tan^3(c+dx)}{3d}-\frac{4ia^8\tan^2(c+dx)}{d}-\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}+\frac{80ia^9}{d(a-ia\tan(c+dx))}-\frac{31a^8\tan(c+dx)}{d}-\frac{80ia^8\log(\cos}{d}$$

```
[Out] 80*a^8*x - ((80*I)*a^8*Log[Cos[c + d*x]])/d - (31*a^8*Tan[c + d*x])/d - ((4
*I)*a^8*Tan[c + d*x]^2)/d + (a^8*Tan[c + d*x]^3)/(3*d) - ((16*I)*a^10)/(d*(
a - I*a*Tan[c + d*x])^2) + ((80*I)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.077228, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{a^8\tan^3(c+dx)}{3d}-\frac{4ia^8\tan^2(c+dx)}{d}-\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}+\frac{80ia^9}{d(a-ia\tan(c+dx))}-\frac{31a^8\tan(c+dx)}{d}-\frac{80ia^8\log(\cos}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] 80*a^8*x - ((80*I)*a^8*Log[Cos[c + d*x]])/d - (31*a^8*Tan[c + d*x])/d - ((4
*I)*a^8*Tan[c + d*x]^2)/d + (a^8*Tan[c + d*x]^3)/(3*d) - ((16*I)*a^10)/(d*(
a - I*a*Tan[c + d*x])^2) + ((80*I)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{\left(ia^5\right)\text{Subst}\left(\int \frac{(a+x)^5}{(a-x)^3}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{\left(ia^5\right)\text{Subst}\left(\int \left(-31a^2+\frac{32a^5}{(a-x)^3}-\frac{80a^4}{(a-x)^2}+\frac{80a^3}{a-x}-8ax-x^2\right)dx,x,ia\tan(c+d\right.}{d}\\ &= 80a^8x-\frac{80ia^8\log(\cos(c+dx))}{d}-\frac{31a^8\tan(c+dx)}{d}-\frac{4ia^8\tan^2(c+dx)}{d}+\frac{a^8\,\mathrm{ta}}{}\end{aligned}$$

Mathematica [B] time = 2.2309, size = 566, normalized size = 4.56

$$\frac{a^8\sec(c)\sec^3(c+dx)(\cos(2(c+5dx))+i\sin(2(c+5dx)))\left(-120idx\sin(2c+dx)+87\sin(2c+dx)-180idx\sin(2c+3dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c]*Sec[c + d*x]^3*(Cos[2*(c + 5*d*x)] + I*Sin[2*(c + 5*d*x)])*((-6
6*I)*Cos[2*c + 3*d*x] + 180*d*x*Cos[2*c + 3*d*x] + (75*I)*Cos[4*c + 3*d*x]
+ 180*d*x*Cos[4*c + 3*d*x] - (50*I)*Cos[4*c + 5*d*x] + 60*d*x*Cos[4*c + 5*d
*x] - (3*I)*Cos[6*c + 5*d*x] + 60*d*x*Cos[6*c + 5*d*x] + 3*Cos[2*c + d*x]*(
71*I + 80*d*x - (40*I)*Log[Cos[c + d*x]^2]) + Cos[d*x]*(119*I + 240*d*x - (
120*I)*Log[Cos[c + d*x]^2]) - (90*I)*Cos[2*c + 3*d*x]*Log[Cos[c + d*x]^2] -
 (90*I)*Cos[4*c + 3*d*x]*Log[Cos[c + d*x]^2] - (30*I)*Cos[4*c + 5*d*x]*Log[
Cos[c + d*x]^2] - (30*I)*Cos[6*c + 5*d*x]*Log[Cos[c + d*x]^2] - 101*Sin[d*x
] - (120*I)*d*x*Sin[d*x] - 60*Log[Cos[c + d*x]^2]*Sin[d*x] + 87*Sin[2*c + d
*x] - (120*I)*d*x*Sin[2*c + d*x] - 60*Log[Cos[c + d*x]^2]*Sin[2*c + d*x] -
96*Sin[2*c + 3*d*x] - (180*I)*d*x*Sin[2*c + 3*d*x] - 90*Log[Cos[c + d*x]^2]
*Sin[2*c + 3*d*x] + 45*Sin[4*c + 3*d*x] - (180*I)*d*x*Sin[4*c + 3*d*x] - 90
*Log[Cos[c + d*x]^2]*Sin[4*c + 3*d*x] - 44*Sin[4*c + 5*d*x] - (60*I)*d*x*Si
n[4*c + 5*d*x] - 30*Log[Cos[c + d*x]^2]*Sin[4*c + 5*d*x] + 3*Sin[6*c + 5*d*
x] - (60*I)*d*x*Sin[6*c + 5*d*x] - 30*Log[Cos[c + d*x]^2]*Sin[6*c + 5*d*x])
)/(12*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] time = 0.075, size = 306, normalized size = 2.5

$$-2\,\frac{a^8\cos(dx+c)(\sin(dx+c))^7}{d}-\frac{40\,ia^8(\sin(dx+c))^2}{d}-\frac{4\,ia^8(\sin(dx+c))^6}{d}-\frac{4\,ia^8(\sin(dx+c))^8}{d(\cos(dx+c))^2}+\frac{29\,a^8(\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] -2/d*a^8*cos(d*x+c)*sin(d*x+c)^7-40*I/d*a^8*sin(d*x+c)^2-4*I/d*a^8*sin(d*x+
c)^6-4*I/d*a^8*sin(d*x+c)^8/cos(d*x+c)^2+29/4/d*a^8*cos(d*x+c)^3*sin(d*x+c)
-28/d*a^8*sin(d*x+c)^7/cos(d*x+c)-2*I/d*a^8*cos(d*x+c)^4-34*I/d*a^8*sin(d*x
+c)^4+80*a^8*x+1/3/d*a^8*sin(d*x+c)^9/cos(d*x+c)^3-2/d*a^8*sin(d*x+c)^9/cos
(d*x+c)-91/3/d*a^8*cos(d*x+c)*sin(d*x+c)^5-665/12/d*a^8*cos(d*x+c)*sin(d*x+
c)^3-345/4/d*a^8*sin(d*x+c)*cos(d*x+c)+80/d*a^8*c-80*I*a^8*ln(cos(d*x+c))/d
```

Maxima [A] time = 1.57149, size = 184, normalized size = 1.48

$$\frac{8\,a^8\tan(dx+c)^3-96i\,a^8\tan(dx+c)^2+1920\,(dx+c)a^8+960i\,a^8\log\left(\tan(dx+c)^2+1\right)-744\,a^8\tan(dx+c)-\frac{3(}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/24*(8*a^8*tan(d*x + c)^3 - 96*I*a^8*tan(d*x + c)^2 + 1920*(d*x + c)*a^8 +
 960*I*a^8*log(tan(d*x + c)^2 + 1) - 744*a^8*tan(d*x + c) - 3*(640*a^8*tan(
d*x + c)^3 - 768*I*a^8*tan(d*x + c)^2 + 384*a^8*tan(d*x + c) - 512*I*a^8)/(
tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.53678, size = 536, normalized size = 4.32

$$\frac{-12i\,a^8e^{(10i\,dx+10i\,c)} + 60i\,a^8e^{(8i\,dx+8i\,c)} + 252i\,a^8e^{(6i\,dx+6i\,c)} + 36i\,a^8e^{(4i\,dx+4i\,c)} - 324i\,a^8e^{(2i\,dx+2i\,c)} - 188i\,a^8 + \left(-240i\,a^8e^{(6i\,dx}\right.}{3\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4i\,dx+4i\,c)} + 3\,de^{(2i\,dx+2i\,c)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/3*(-12*I*a^8*e^(10*I*d*x + 10*I*c) + 60*I*a^8*e^(8*I*d*x + 8*I*c) + 252*I
*a^8*e^(6*I*d*x + 6*I*c) + 36*I*a^8*e^(4*I*d*x + 4*I*c) - 324*I*a^8*e^(2*I*
d*x + 2*I*c) - 188*I*a^8 + (-240*I*a^8*e^(6*I*d*x + 6*I*c) - 720*I*a^8*e^(4
*I*d*x + 4*I*c) - 720*I*a^8*e^(2*I*d*x + 2*I*c) - 240*I*a^8)*log(e^(2*I*d*x
 + 2*I*c) + 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2
*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 2.8681, size = 201, normalized size = 1.62

$$-64a^8\left(\begin{cases}-\frac{ie^{2idx}}{2d} & \text{for}\ d\neq 0\\ x & \text{otherwise}\end{cases}\right)e^{2ic} + 16a^8\left(\begin{cases}-\frac{ie^{4idx}}{4d} & \text{for}\ d\neq 0\\ x & \text{otherwise}\end{cases}\right)e^{4ic} - \frac{80ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-\frac{80ia^8e^{-2ic}e^{4idx}}{d} - \frac{140ia^8e^{-4}}{d}}{e^{6idx}+3e^{-2ic}e^{4idx}+3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] -64*a**8*Piecewise((-I*exp(2*I*d*x)/(2*d), Ne(d, 0)), (x, True))*exp(2*I*c)
 + 16*a**8*Piecewise((-I*exp(4*I*d*x)/(4*d), Ne(d, 0)), (x, True))*exp(4*I*
c) - 80*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-80*I*a**8*exp(-2*I*c)*
exp(4*I*d*x)/d - 140*I*a**8*exp(-4*I*c)*exp(2*I*d*x)/d - 188*I*a**8*exp(-6*
I*c)/(3*d))/(exp(6*I*d*x) + 3*exp(-2*I*c)*exp(4*I*d*x) + 3*exp(-4*I*c)*exp(
2*I*d*x) + exp(-6*I*c))
```

Giac [B] time = 2.19591, size = 1060, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/1344*(-107520*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) -
1505280*I*a^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9784320*
I*a^8*e^(24*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 39137280*I*a^8*e
^(22*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 107627520*I*a^8*e^(20*I*
d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 215255040*I*a^8*e^(18*I*d*x + 4
*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 322882560*I*a^8*e^(16*I*d*x + 2*I*c)*l
og(e^(2*I*d*x + 2*I*c) + 1) - 322882560*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2
*I*d*x + 2*I*c) + 1) - 215255040*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x
+ 2*I*c) + 1) - 107627520*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c)
 + 1) - 39137280*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9
784320*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 1505280*I*
a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 369008640*I*a^8*e^(
14*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 107520*I*a^8*e^(-14*I*c)*log(e^(2*
```

```
I*d*x + 2*I*c) + 1) - 5376*I*a^8*e^(32*I*d*x + 18*I*c) - 32256*I*a^8*e^(30*
I*d*x + 16*I*c) + 112896*I*a^8*e^(28*I*d*x + 14*I*c) + 1849344*I*a^8*e^(26*
I*d*x + 12*I*c) + 8902656*I*a^8*e^(24*I*d*x + 10*I*c) + 24220672*I*a^8*e^(2
2*I*d*x + 8*I*c) + 40941824*I*a^8*e^(20*I*d*x + 6*I*c) + 39542272*I*a^8*e^(
18*I*d*x + 4*I*c) + 5795328*I*a^8*e^(16*I*d*x + 2*I*c) - 80602368*I*a^8*e^(
12*I*d*x - 2*I*c) - 77650944*I*a^8*e^(10*I*d*x - 4*I*c) - 49588224*I*a^8*e^
(8*I*d*x - 6*I*c) - 21590016*I*a^8*e^(6*I*d*x - 8*I*c) - 6212864*I*a^8*e^(4
*I*d*x - 10*I*c) - 1071616*I*a^8*e^(2*I*d*x - 12*I*c) - 46007808*I*a^8*e^(1
4*I*d*x) - 84224*I*a^8*e^(-14*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I
*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) +
1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*
d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) +
1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x -
10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

3.84 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=114

$$-\frac{16ia^{11}}{3d(a-ia\tan(c+dx))^3}+\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}-\frac{24ia^9}{d(a-ia\tan(c+dx))}+\frac{a^8\tan(c+dx)}{d}+\frac{8ia^8\log(\cos(c+dx))}{d}-8a$$

```
[Out] -8*a^8*x + ((8*I)*a^8*Log[Cos[c + d*x]])/d + (a^8*Tan[c + d*x])/d - (((16*I
)/3)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) + ((16*I)*a^10)/(d*(a - I*a*Tan[c +
 d*x])^2) - ((24*I)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0709375, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{16ia^{11}}{3d(a-ia\tan(c+dx))^3}+\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}-\frac{24ia^9}{d(a-ia\tan(c+dx))}+\frac{a^8\tan(c+dx)}{d}+\frac{8ia^8\log(\cos(c+dx))}{d}-8a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] -8*a^8*x + ((8*I)*a^8*Log[Cos[c + d*x]])/d + (a^8*Tan[c + d*x])/d - (((16*I
)/3)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) + ((16*I)*a^10)/(d*(a - I*a*Tan[c +
 d*x])^2) - ((24*I)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{\left(ia^7\right)\text{Subst}\left(\int \frac{(a+x)^4}{(a-x)^4}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{\left(ia^7\right)\text{Subst}\left(\int \left(1+\frac{16a^4}{(a-x)^4}-\frac{32a^3}{(a-x)^3}+\frac{24a^2}{(a-x)^2}-\frac{8a}{a-x}\right)dx,x,ia\tan(c+dx)\right)}{d}\\ &= -8a^8x+\frac{8ia^8\log(\cos(c+dx))}{d}+\frac{a^8\tan(c+dx)}{d}-\frac{16ia^{11}}{3d(a-ia\tan(c+dx))^3}+\frac{}{d}\end{aligned}$$

Mathematica [B] time = 2.09391, size = 414, normalized size = 3.63

$$-\frac{a^8\sec(c)\sec(c+dx)(\cos(3c+11dx)+i\sin(3c+11dx))\left(-12idx\sin(c+2dx)+11\sin(c+2dx)-12idx\sin(3c+2dx)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] -(a^8*Sec[c]*Sec[c + d*x]*((12*I)*Cos[c] + (10*I)*Cos[3*c + 2*d*x] + 12*d*x
*Cos[3*c + 2*d*x] - (2*I)*Cos[3*c + 4*d*x] + 12*d*x*Cos[3*c + 4*d*x] + I*Co
s[5*c + 4*d*x] + 12*d*x*Cos[5*c + 4*d*x] + Cos[c + 2*d*x]*(7*I + 12*d*x - (
6*I)*Log[Cos[c + d*x]^2]) - (6*I)*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]^2] - (6
*I)*Cos[3*c + 4*d*x]*Log[Cos[c + d*x]^2] - (6*I)*Cos[5*c + 4*d*x]*Log[Cos[c
 + d*x]^2] + 11*Sin[c + 2*d*x] - (12*I)*d*x*Sin[c + 2*d*x] - 6*Log[Cos[c +
d*x]^2]*Sin[c + 2*d*x] + 14*Sin[3*c + 2*d*x] - (12*I)*d*x*Sin[3*c + 2*d*x]
- 6*Log[Cos[c + d*x]^2]*Sin[3*c + 2*d*x] - 4*Sin[3*c + 4*d*x] - (12*I)*d*x*
Sin[3*c + 4*d*x] - 6*Log[Cos[c + d*x]^2]*Sin[3*c + 4*d*x] - Sin[5*c + 4*d*x
] - (12*I)*d*x*Sin[5*c + 4*d*x] - 6*Log[Cos[c + d*x]^2]*Sin[5*c + 4*d*x])*(
Cos[3*c + 11*d*x] + I*Sin[3*c + 11*d*x]))/(6*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] time = 0.07, size = 319, normalized size = 2.8

$$\frac{a^8\cos(dx+c)(\sin(dx+c))^7}{d}+\frac{8ia^8\ln(\cos(dx+c))}{d}-\frac{35a^8(\sin(dx+c))^3(\cos(dx+c))^3}{3d}+\frac{\frac{14i}{3}a^8(\cos(dx+c))^4}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*a^8*cos(d*x+c)*sin(d*x+c)^7+8*I*a^8*ln(cos(d*x+c))/d-35/3/d*a^8*sin(d*x
+c)^3*cos(d*x+c)^3+14/3*I/d*a^8*cos(d*x+c)^4+28/3*I/d*a^8*sin(d*x+c)^2*cos(
d*x+c)^4+2*I/d*a^8*sin(d*x+c)^4+32/3*I/d*a^8*sin(d*x+c)^6-8*a^8*x+1/d*a^8*s
in(d*x+c)^9/cos(d*x+c)+35/6/d*a^8*cos(d*x+c)*sin(d*x+c)^5+175/24/d*a^8*cos(
d*x+c)*sin(d*x+c)^3+4*I/d*a^8*sin(d*x+c)^2+29/6/d*a^8*cos(d*x+c)^5*sin(d*x+
c)-233/24/d*a^8*cos(d*x+c)^3*sin(d*x+c)+111/8/d*a^8*sin(d*x+c)*cos(d*x+c)-8
/d*a^8*c-4/3*I/d*a^8*cos(d*x+c)^6
```

Maxima [A] time = 1.63764, size = 197, normalized size = 1.73

$$-\frac{384(dx+c)a^8+192i\,a^8\log\left(\tan(dx+c)^2+1\right)-48\,a^8\tan(dx+c)-\frac{1152\,a^8\tan(dx+c)^5-1920i\,a^8\tan(dx+c)^4+512\,a^8\tan(dx+c)^3}{\tan(dx+c)^6+3\tan(dx+c)^4+}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/48*(384*(d*x + c)*a^8 + 192*I*a^8*log(tan(d*x + c)^2 + 1) - 48*a^8*tan(d
*x + c) - (1152*a^8*tan(d*x + c)^5 - 1920*I*a^8*tan(d*x + c)^4 + 512*a^8*ta
n(d*x + c)^3 - 1536*I*a^8*tan(d*x + c)^2 + 384*a^8*tan(d*x + c) - 640*I*a^8
)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.4936, size = 323, normalized size = 2.83

$$\frac{-2i\,a^8e^{(8i\,dx+8i\,c)}+4i\,a^8e^{(6i\,dx+6i\,c)}-12i\,a^8e^{(4i\,dx+4i\,c)}-18i\,a^8e^{(2i\,dx+2i\,c)}+6i\,a^8+\left(24i\,a^8e^{(2i\,dx+2i\,c)}+24i\,a^8\right)\log\left(e^{(2i\,dx+2i\,}\right.}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/3*(-2*I*a^8*e^(8*I*d*x + 8*I*c) + 4*I*a^8*e^(6*I*d*x + 6*I*c) - 12*I*a^8*
e^(4*I*d*x + 4*I*c) - 18*I*a^8*e^(2*I*d*x + 2*I*c) + 6*I*a^8 + (24*I*a^8*e^
(2*I*d*x + 2*I*c) + 24*I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x +
 2*I*c) + d)
```

Sympy [A] time = 1.62002, size = 144, normalized size = 1.26

$$12a^8\left(\begin{cases}-\frac{ie^{2idx}}{2d} & \text{for } d\neq 0\\ x & \text{otherwise}\end{cases}\right)e^{2ic}-8a^8\left(\begin{cases}-\frac{ie^{4idx}}{4d} & \text{for } d\neq 0\\ x & \text{otherwise}\end{cases}\right)e^{4ic}+4a^8\left(\begin{cases}-\frac{ie^{6idx}}{6d} & \text{for } d\neq 0\\ x & \text{otherwise}\end{cases}\right)e^{6ic}+\frac{8ia^8\log\left(e^{2idx}+e^{-}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 12*a**8*Piecewise((-I*exp(2*I*d*x)/(2*d), Ne(d, 0)), (x, True))*exp(2*I*c)
- 8*a**8*Piecewise((-I*exp(4*I*d*x)/(4*d), Ne(d, 0)), (x, True))*exp(4*I*c)
 + 4*a**8*Piecewise((-I*exp(6*I*d*x)/(6*d), Ne(d, 0)), (x, True))*exp(6*I*c
) + 8*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + 2*I*a**8*exp(-2*I*c)/(d*(e
xp(2*I*d*x) + exp(-2*I*c)))
```

Giac [B] time = 2.33369, size = 1079, normalized size = 9.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/13440*(107520*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) +
1505280*I*a^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 9784320*
I*a^8*e^(24*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 39137280*I*a^8*e
^(22*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 107627520*I*a^8*e^(20*I*
d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 215255040*I*a^8*e^(18*I*d*x + 4
*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 322882560*I*a^8*e^(16*I*d*x + 2*I*c)*l
og(e^(2*I*d*x + 2*I*c) + 1) + 322882560*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2
*I*d*x + 2*I*c) + 1) + 215255040*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x
+ 2*I*c) + 1) + 107627520*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c)
 + 1) + 39137280*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 9
784320*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 1505280*I*
a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 369008640*I*a^8*e^(
14*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) + 107520*I*a^8*e^(-14*I*c)*log(e^(2*
I*d*x + 2*I*c) + 1) - 8960*I*a^8*e^(34*I*d*x + 20*I*c) - 98560*I*a^8*e^(32*
I*d*x + 18*I*c) - 519680*I*a^8*e^(30*I*d*x + 16*I*c) - 1944320*I*a^8*e^(28*
I*d*x + 14*I*c) - 6496000*I*a^8*e^(26*I*d*x + 12*I*c) - 20034560*I*a^8*e^(2
4*I*d*x + 10*I*c) - 51717120*I*a^8*e^(22*I*d*x + 8*I*c) - 103783680*I*a^8*e
^(20*I*d*x + 6*I*c) - 157597440*I*a^8*e^(18*I*d*x + 4*I*c) - 179379200*I*a^
8*e^(16*I*d*x + 2*I*c) - 91669760*I*a^8*e^(12*I*d*x - 2*I*c) - 37157120*I*a
^8*e^(10*I*d*x - 4*I*c) - 7813120*I*a^8*e^(8*I*d*x - 6*I*c) + 716800*I*a^8*
e^(6*I*d*x - 8*I*c) + 994560*I*a^8*e^(4*I*d*x - 10*I*c) + 268800*I*a^8*e^(2
```

```
*I*d*x - 12*I*c) - 151191040*I*a^8*e^(14*I*d*x) + 26880*I*a^8*e^(-14*I*c))/
(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x +
10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d
*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x -
2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e
^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c)
+ 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

3.85 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=43

$$-\frac{i\left(a^3+ia^3\tan(c+dx)\right)^4}{8d(a-ia\tan(c+dx))^4}$$

```
[Out] ((-I/8)*(a^3 + I*a^3*Tan[c + d*x])^4)/(d*(a - I*a*Tan[c + d*x])^4)
```

Rubi [A] time = 0.0422143, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 37}

$$-\frac{i\left(a^3+ia^3\tan(c+dx)\right)^4}{8d(a-ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-I/8)*(a^3 + I*a^3*Tan[c + d*x])^4)/(d*(a - I*a*Tan[c + d*x])^4)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{\left(ia^9\right)\operatorname{Subst}\left(\int \frac{(a+x)^3}{(a-x)^5}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{i\left(a^3+ia^3\tan(c+dx)\right)^4}{8d(a-ia\tan(c+dx))^4}\end{aligned}$$

Mathematica [A] time = 0.295275, size = 31, normalized size = 0.72

$$-\frac{ia^8(\cos(c+dx)+i\sin(c+dx))^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-I/8)*a^8*(Cos[c + d*x] + I*Sin[c + d*x])^8)/d
```

Maple [B] time = 0.077, size = 451, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/8*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(
d*x+c))*cos(d*x+c)+35/128*d*x+35/128*c)-I*a^8*sin(d*x+c)^8-28*a^8*(-1/8*sin
(d*x+c)^5*cos(d*x+c)^3-5/48*sin(d*x+c)^3*cos(d*x+c)^3-5/64*cos(d*x+c)^3*sin
(d*x+c)+5/128*cos(d*x+c)*sin(d*x+c)+5/128*d*x+5/128*c)+56*I*a^8*(-1/8*sin(d
*x+c)^4*cos(d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1/24*cos(d*x+c)^4)+70*a
^8*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*cos(d*x+c)^5*sin(d*x+c)+1/64*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)-56*I*a^8*(-1/8*sin(d*
x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)-28*a^8*(-1/8*sin(d*x+c)*cos(d*x+c)^7
+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+
5/128*c)-I*a^8*cos(d*x+c)^8+a^8*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*c
os(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [B] time = 2.41403, size = 185, normalized size = 4.3

$$-\frac{384\,a^8\tan\left(dx+c\right)^7-1536i\,a^8\tan\left(dx+c\right)^6-2688\,a^8\tan\left(dx+c\right)^5+3072i\,a^8\tan\left(dx+c\right)^4+2688\,a^8\tan\left(dx+c\right)^{3}}{384\left(\tan\left(dx+c\right)^8+4\,\tan\left(dx+c\right)^6+6\,\tan\left(dx+c\right)^4+4\,\tan\left(dx+c\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/384*(384*a^8*tan(d*x + c)^7 - 1536*I*a^8*tan(d*x + c)^6 - 2688*a^8*tan(d
*x + c)^5 + 3072*I*a^8*tan(d*x + c)^4 + 2688*a^8*tan(d*x + c)^3 - 1536*I*a^
8*tan(d*x + c)^2 - 384*a^8*tan(d*x + c))/((tan(d*x + c)^8 + 4*tan(d*x + c)^
6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1)*d)
```

Fricas [A] time = 1.42455, size = 46, normalized size = 1.07

$$-\frac{i\,a^8e^{(8i\,dx+8i\,c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/8*I*a^8*e^(8*I*d*x + 8*I*c)/d
```

Sympy [A] time = 0.948915, size = 37, normalized size = 0.86

$$\begin{cases} -\frac{ia^8e^{8ic}e^{8idx}}{8d} & \text{for } 8d \neq 0 \\ a^8xe^{8ic} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((-I*a**8*exp(8*I*c)*exp(8*I*d*x)/(8*d), Ne(8*d, 0)), (a**8*x*exp(
8*I*c), True))
```

Giac [B] time = 2.31477, size = 514, normalized size = 11.95

$$\frac{-3840i\,a^8e^{(36i\,dx+22i\,c)} - 53760i\,a^8e^{(34i\,dx+20i\,c)} - 349440i\,a^8e^{(32i\,dx+18i\,c)} - 1397760i\,a^8e^{(30i\,dx+16i\,c)} - 3843840i\,a^8e^{(28i\,dx+14i\,c)}}{30720\left(de^{(28i\,dx+14i\,c)} + 14\,de^{(26i\,dx+12i\,c)} + 91\,de^{(24i\,dx+10i\,c)} + 364\,de^{(22i\,dx+8i\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/30720*(-3840*I*a^8*e^(36*I*d*x + 22*I*c) - 53760*I*a^8*e^(34*I*d*x + 20*I
*c) - 349440*I*a^8*e^(32*I*d*x + 18*I*c) - 1397760*I*a^8*e^(30*I*d*x + 16*I
*c) - 3843840*I*a^8*e^(28*I*d*x + 14*I*c) - 7687680*I*a^8*e^(26*I*d*x + 12*
I*c) - 11531520*I*a^8*e^(24*I*d*x + 10*I*c) - 13178880*I*a^8*e^(22*I*d*x +
8*I*c) - 11531520*I*a^8*e^(20*I*d*x + 6*I*c) - 7687680*I*a^8*e^(18*I*d*x +
4*I*c) - 3843840*I*a^8*e^(16*I*d*x + 2*I*c) - 349440*I*a^8*e^(12*I*d*x - 2*
I*c) - 53760*I*a^8*e^(10*I*d*x - 4*I*c) - 3840*I*a^8*e^(8*I*d*x - 6*I*c) -
1397760*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 1
2*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e
^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*
I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e
^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c)
+ 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

3.86 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=80

$$-\frac{4ia^{13}}{5d(a-ia\tan(c+dx))^5}+\frac{ia^{12}}{d(a-ia\tan(c+dx))^4}-\frac{ia^{11}}{3d(a-ia\tan(c+dx))^3}$$

```
[Out] (((-4*I)/5)*a^13)/(d*(a - I*a*Tan[c + d*x])^5) + (I*a^12)/(d*(a - I*a*Tan[c
 + d*x])^4) - ((I/3)*a^11)/(d*(a - I*a*Tan[c + d*x])^3)
```

Rubi [A] time = 0.0562357, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{4ia^{13}}{5d(a-ia\tan(c+dx))^5}+\frac{ia^{12}}{d(a-ia\tan(c+dx))^4}-\frac{ia^{11}}{3d(a-ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-4*I)/5)*a^13)/(d*(a - I*a*Tan[c + d*x])^5) + (I*a^12)/(d*(a - I*a*Tan[c
 + d*x])^4) - ((I/3)*a^11)/(d*(a - I*a*Tan[c + d*x])^3)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{\left(ia^{11}\right)\operatorname{Subst}\left(\int \frac{(a+x)^2}{(a-x)^6}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{\left(ia^{11}\right)\operatorname{Subst}\left(\int \left(\frac{4a^2}{(a-x)^6}-\frac{4a}{(a-x)^5}+\frac{1}{(a-x)^4}\right)dx,x,ia\tan(c+dx)\right)}{d}\\ &= -\frac{4ia^{13}}{5d(a-ia\tan(c+dx))^5}+\frac{ia^{12}}{d(a-ia\tan(c+dx))^4}-\frac{ia^{11}}{3d(a-ia\tan(c+dx))^3}\end{aligned}$$

Mathematica [A] time = 1.15056, size = 55, normalized size = 0.69

$$\frac{a^8(-4i\sin(2(c+dx))+16\cos(2(c+dx))+15)(\sin(8(c+dx))-i\cos(8(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(15 + 16*Cos[2*(c + d*x)] - (4*I)*Sin[2*(c + d*x)])*((-I)*Cos[8*(c + d
*x)] + Sin[8*(c + d*x)]))/(240*d)
```

Maple [B] time = 0.092, size = 588, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/10*sin(d*x+c)^7*cos(d*x+c)^3-7/80*sin(d*x+c)^5*cos(d*x+c)^3-7/
96*sin(d*x+c)^3*cos(d*x+c)^3-7/128*cos(d*x+c)^3*sin(d*x+c)+7/256*cos(d*x+c)
*sin(d*x+c)+7/256*d*x+7/256*c)-8*I*a^8*(-1/10*sin(d*x+c)^6*cos(d*x+c)^4-3/4
0*sin(d*x+c)^4*cos(d*x+c)^4-1/20*sin(d*x+c)^2*cos(d*x+c)^4-1/40*cos(d*x+c)^
4)-28*a^8*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1
/32*cos(d*x+c)^5*sin(d*x+c)+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+
3/256*d*x+3/256*c)+56*I*a^8*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c
)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6)+70*a^8*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7
-3/80*sin(d*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos
(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)-56*I*a^8*(-1/10*sin(d*x+c)^2*cos(d*x
+c)^8-1/40*cos(d*x+c)^8)-28*a^8*(-1/10*sin(d*x+c)*cos(d*x+c)^9+1/80*(cos(d*
x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/2
56*d*x+7/256*c)-4/5*I*a^8*cos(d*x+c)^10+a^8*(1/10*(cos(d*x+c)^9+9/8*cos(d*x
+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)
+63/256*d*x+63/256*c))
```

Maxima [B] time = 1.59788, size = 205, normalized size = 2.56

$$-\frac{1280\,a^{8}\tan\left(dx+c\right)^{7}-7680i\,a^{8}\tan\left(dx+c\right)^{6}-19712\,a^{8}\tan\left(dx+c\right)^{5}+28160i\,a^{8}\tan\left(dx+c\right)^{4}+24320\,a^{8}\tan\left(dx+c\right.}{3840\left(\tan\left(dx+c\right)^{10}+5\,\tan\left(dx+c\right)^{8}+10\,\tan\left(dx+c\right)^{6}+10\,\tan\left(dx+c\right)^{4}+5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/3840*(1280*a^8*tan(d*x + c)^7 - 7680*I*a^8*tan(d*x + c)^6 - 19712*a^8*ta
n(d*x + c)^5 + 28160*I*a^8*tan(d*x + c)^4 + 24320*a^8*tan(d*x + c)^3 - 1280
0*I*a^8*tan(d*x + c)^2 - 3840*a^8*tan(d*x + c) + 512*I*a^8)/((tan(d*x + c)^
10 + 5*tan(d*x + c)^8 + 10*tan(d*x + c)^6 + 10*tan(d*x + c)^4 + 5*tan(d*x +
 c)^2 + 1)*d)
```

Fricas [A] time = 1.65638, size = 140, normalized size = 1.75

$$\frac{-6i\,a^{8}e^{(10i\,dx+10i\,c)}-15i\,a^{8}e^{(8i\,dx+8i\,c)}-10i\,a^{8}e^{(6i\,dx+6i\,c)}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(-6*I*a^8*e^(10*I*d*x + 10*I*c) - 15*I*a^8*e^(8*I*d*x + 8*I*c) - 10*I
*a^8*e^(6*I*d*x + 6*I*c))/d
```

Sympy [A] time = 1.32546, size = 122, normalized size = 1.52

$$\begin{cases} \frac{-384ia^8d^2e^{10ic}e^{10idx}-960ia^8d^2e^{8ic}e^{8idx}-640ia^8d^2e^{6ic}e^{6idx}}{15360d^3} & \text{for } 15360d^3 \neq 0 \\ x\left(\frac{a^8e^{10ic}}{4}+\frac{a^8e^{8ic}}{2}+\frac{a^8e^{6ic}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**10*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-384*I*a**8*d**2*exp(10*I*c)*exp(10*I*d*x) - 960*I*a**8*d**2*ex
p(8*I*c)*exp(8*I*d*x) - 640*I*a**8*d**2*exp(6*I*c)*exp(6*I*d*x))/(15360*d**
3), Ne(15360*d**3, 0)), (x*(a**8*exp(10*I*c)/4 + a**8*exp(8*I*c)/2 + a**8*e
xp(6*I*c)/4), True))
```

Giac [B] time = 2.42252, size = 552, normalized size = 6.9

$$\frac{-10752i\,a^8e^{(38i\,dx+24i\,c)}-177408i\,a^8e^{(36i\,dx+22i\,c)}-1372672i\,a^8e^{(34i\,dx+20i\,c)}-6610688i\,a^8e^{(32i\,dx+18i\,c)}-22177792i\,a^8e^{(30}}{430080\left(de^{(28i\,dx+14i\,c)}+14\,de^{(26i\,dx+12}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/430080*(-10752*I*a^8*e^(38*I*d*x + 24*I*c) - 177408*I*a^8*e^(36*I*d*x + 2
2*I*c) - 1372672*I*a^8*e^(34*I*d*x + 20*I*c) - 6610688*I*a^8*e^(32*I*d*x +
18*I*c) - 22177792*I*a^8*e^(30*I*d*x + 16*I*c) - 54955264*I*a^8*e^(28*I*d*x
 + 14*I*c) - 104039936*I*a^8*e^(26*I*d*x + 12*I*c) - 153497344*I*a^8*e^(24*
I*d*x + 10*I*c) - 178354176*I*a^8*e^(22*I*d*x + 8*I*c) - 163747584*I*a^8*e^
(20*I*d*x + 6*I*c) - 118390272*I*a^8*e^(18*I*d*x + 4*I*c) - 66696448*I*a^8*
e^(16*I*d*x + 2*I*c) - 9119488*I*a^8*e^(12*I*d*x - 2*I*c) - 2017792*I*a^8*e
^(10*I*d*x - 4*I*c) - 277760*I*a^8*e^(8*I*d*x - 6*I*c) - 17920*I*a^8*e^(6*I
*d*x - 8*I*c) - 28700672*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x + 14*I*c) + 14*
d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x +
8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d
*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x -
4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4
*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-
14*I*c))
```

3.87 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=55

$$\frac{ia^{13}}{5d(a-ia\tan(c+dx))^5}-\frac{ia^{14}}{3d(a-ia\tan(c+dx))^6}$$

```
[Out] ((-I/3)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) + ((I/5)*a^13)/(d*(a - I*a*Tan[c
 + d*x])^5)
```

Rubi [A] time = 0.0476216, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{ia^{13}}{5d(a-ia\tan(c+dx))^5}-\frac{ia^{14}}{3d(a-ia\tan(c+dx))^6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-I/3)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) + ((I/5)*a^13)/(d*(a - I*a*Tan[c
 + d*x])^5)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{\left(ia^{13}\right)\operatorname{Subst}\left(\int \frac{a+x}{(a-x)^7}\,dx,x,ia\tan(c+dx)\right)}{d}\\
&= -\frac{\left(ia^{13}\right)\operatorname{Subst}\left(\int \left(\frac{2a}{(a-x)^7}-\frac{1}{(a-x)^6}\right)dx,x,ia\tan(c+dx)\right)}{d}\\
&= -\frac{ia^{14}}{3d(a-ia\tan(c+dx))^6}+\frac{ia^{13}}{5d(a-ia\tan(c+dx))^5}
\end{aligned}$$

Mathematica [A] time = 1.16954, size = 77, normalized size = 1.4

$$\frac{a^8(-16i\sin(2(c+dx))-10i\sin(4(c+dx))+64\cos(2(c+dx))+20\cos(4(c+dx))+45)(\sin(8(c+dx))-i\cos(8(c+dx)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(45 + 64*Cos[2*(c + d*x)] + 20*Cos[4*(c + d*x)] - (16*I)*Sin[2*(c + d*
x)] - (10*I)*Sin[4*(c + d*x)])*((-I)*Cos[8*(c + d*x)] + Sin[8*(c + d*x)]))/
(960*d)
```

Maple [B] time = 0.092, size = 639, normalized size = 11.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/12*sin(d*x+c)^7*cos(d*x+c)^5-7/120*sin(d*x+c)^5*cos(d*x+c)^5-7
/192*sin(d*x+c)^3*cos(d*x+c)^5-7/384*cos(d*x+c)^5*sin(d*x+c)+7/1536*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)-8*I*a^8*(-1/12*sin(d
*x+c)^6*cos(d*x+c)^6-1/20*sin(d*x+c)^4*cos(d*x+c)^6-1/40*sin(d*x+c)^2*cos(d
*x+c)^6-1/120*cos(d*x+c)^6)-28*a^8*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*si
n(d*x+c)^3*cos(d*x+c)^7-1/64*sin(d*x+c)*cos(d*x+c)^7+1/384*(cos(d*x+c)^5+5/
4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+56*I*a^8*(-
1/12*sin(d*x+c)^4*cos(d*x+c)^8-1/30*sin(d*x+c)^2*cos(d*x+c)^8-1/120*cos(d*x
+c)^8)+70*a^8*(-1/12*sin(d*x+c)^3*cos(d*x+c)^9-1/40*sin(d*x+c)*cos(d*x+c)^9
+1/320*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*
sin(d*x+c)+7/1024*d*x+7/1024*c)-56*I*a^8*(-1/12*sin(d*x+c)^2*cos(d*x+c)^10-
1/60*cos(d*x+c)^10)-28*a^8*(-1/12*sin(d*x+c)*cos(d*x+c)^11+1/120*(cos(d*x+c
)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x
+c))*sin(d*x+c)+21/1024*d*x+21/1024*c)-2/3*I*a^8*cos(d*x+c)^12+a^8*(1/12*(c
os(d*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231
/128*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+231/1024*d*x+231/1024*c))
```

Maxima [B] time = 1.52452, size = 219, normalized size = 3.98

$$-\frac{3072\,a^8\tan\left(dx+c\right)^7-20480i\,a^8\tan\left(dx+c\right)^6-58368\,a^8\tan\left(dx+c\right)^5+92160i\,a^8\tan\left(dx+c\right)^4+87040\,a^8\tan\left(d\right.}{15360\left(\tan\left(dx+c\right)^{12}+6\,\tan\left(dx+c\right)^{10}+15\,\tan\left(dx+c\right)^8+20\,\tan\left(dx+c\right)^6+15\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15360*(3072*a^8*tan(d*x + c)^7 - 20480*I*a^8*tan(d*x + c)^6 - 58368*a^8*
tan(d*x + c)^5 + 92160*I*a^8*tan(d*x + c)^4 + 87040*a^8*tan(d*x + c)^3 - 49
152*I*a^8*tan(d*x + c)^2 - 15360*a^8*tan(d*x + c) + 2048*I*a^8)/((tan(d*x +
 c)^12 + 6*tan(d*x + c)^10 + 15*tan(d*x + c)^8 + 20*tan(d*x + c)^6 + 15*tan
(d*x + c)^4 + 6*tan(d*x + c)^2 + 1)*d)
```

Fricas [A] time = 1.68441, size = 227, normalized size = 4.13

$$\frac{-5i\,a^8e^{(12i\,dx+12i\,c)}-24i\,a^8e^{(10i\,dx+10i\,c)}-45i\,a^8e^{(8i\,dx+8i\,c)}-40i\,a^8e^{(6i\,dx+6i\,c)}-15i\,a^8e^{(4i\,dx+4i\,c)}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/960*(-5*I*a^8*e^(12*I*d*x + 12*I*c) - 24*I*a^8*e^(10*I*d*x + 10*I*c) - 45
*I*a^8*e^(8*I*d*x + 8*I*c) - 40*I*a^8*e^(6*I*d*x + 6*I*c) - 15*I*a^8*e^(4*I
*d*x + 4*I*c))/d
```

Sympy [A] time = 1.61318, size = 199, normalized size = 3.62

$$\begin{cases} \frac{-3932160ia^8d^4e^{12ic}e^{12idx}-18874368ia^8d^4e^{10ic}e^{10idx}-35389440ia^8d^4e^{8ic}e^{8idx}-31457280ia^8d^4e^{6ic}e^{6idx}-11796480ia^8d^4e^{4ic}e^{4idx}}{754974720d^5} & \text{for } 754974720d^5 \neq 0 \\ x\left(\frac{a^8e^{12ic}}{16}+\frac{a^8e^{10ic}}{4}+\frac{3a^8e^{8ic}}{8}+\frac{a^8e^{6ic}}{4}+\frac{a^8e^{4ic}}{16}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**12*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-3932160*I*a**8*d**4*exp(12*I*c)*exp(12*I*d*x) - 18874368*I*a**
8*d**4*exp(10*I*c)*exp(10*I*d*x) - 35389440*I*a**8*d**4*exp(8*I*c)*exp(8*I*
d*x) - 31457280*I*a**8*d**4*exp(6*I*c)*exp(6*I*d*x) - 11796480*I*a**8*d**4*
exp(4*I*c)*exp(4*I*d*x))/(754974720*d**5), Ne(754974720*d**5, 0)), (x*(a**8
*exp(12*I*c)/16 + a**8*exp(10*I*c)/4 + 3*a**8*exp(8*I*c)/8 + a**8*exp(6*I*c
)/4 + a**8*exp(4*I*c)/16), True))
```

Giac [B] time = 2.54573, size = 590, normalized size = 10.73

$$\frac{-8960i\,a^8e^{(40i\,dx+26i\,c)} - 168448i\,a^8e^{(38i\,dx+24i\,c)} - 1498112i\,a^8e^{(36i\,dx+22i\,c)} - 8375808i\,a^8e^{(34i\,dx+20i\,c)} - 32992512i\,a^8e^{(32i\,dx+1}}{1720320\left(de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/1720320*(-8960*I*a^8*e^(40*I*d*x + 26*I*c) - 168448*I*a^8*e^(38*I*d*x + 2
4*I*c) - 1498112*I*a^8*e^(36*I*d*x + 22*I*c) - 8375808*I*a^8*e^(34*I*d*x +
20*I*c) - 32992512*I*a^8*e^(32*I*d*x + 18*I*c) - 97241088*I*a^8*e^(30*I*d*x
 + 16*I*c) - 222267136*I*a^8*e^(28*I*d*x + 14*I*c) - 402881024*I*a^8*e^(26*
I*d*x + 12*I*c) - 587082496*I*a^8*e^(24*I*d*x + 10*I*c) - 692916224*I*a^8*e
^(22*I*d*x + 8*I*c) - 663959296*I*a^8*e^(20*I*d*x + 6*I*c) - 515260928*I*a^
8*e^(18*I*d*x + 4*I*c) - 321414912*I*a^8*e^(16*I*d*x + 2*I*c) - 60947712*I*
a^8*e^(12*I*d*x - 2*I*c) - 17479168*I*a^8*e^(10*I*d*x - 4*I*c) - 3530240*I*
a^8*e^(8*I*d*x - 6*I*c) - 448000*I*a^8*e^(6*I*d*x - 8*I*c) - 26880*I*a^8*e^
(4*I*d*x - 10*I*c) - 158957568*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x + 14*I*c)
 + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*
d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) +
3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*
d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*
d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) +
d*e^(-14*I*c))
```

3.88 $\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=27

$$-\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}$$

```
[Out] ((-I/7)*a^15)/(d*(a - I*a*Tan[c + d*x])^7)
```

Rubi [A] time = 0.0378439, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$-\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^14*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-I/7)*a^15)/(d*(a - I*a*Tan[c + d*x])^7)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{\left(ia^{15}\right)\text{Subst}\left(\int \frac{1}{(a-x)^8}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}\end{aligned}$$

Mathematica [B] time = 1.63517, size = 116, normalized size = 4.3

$$\frac{a^8(-14i\sin(2(c+dx))-14i\sin(4(c+dx))-6i\sin(6(c+dx))+56\cos(2(c+dx))+28\cos(4(c+dx))+8\cos(6(c+d}{896d(\cos(dx)+i\sin(dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^14*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(35 + 56*Cos[2*(c + d*x)] + 28*Cos[4*(c + d*x)] + 8*Cos[6*(c + d*x)] -
 (14*I)*Sin[2*(c + d*x)] - (14*I)*Sin[4*(c + d*x)] - (6*I)*Sin[6*(c + d*x)]
)*((-I)*Cos[8*(c + 2*d*x)] + Sin[8*(c + 2*d*x)]))/(896*d*(Cos[d*x] + I*Sin[
```

```
d*x])^8)
```

Maple [B] time = 0.136, size = 689, normalized size = 25.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/14*sin(d*x+c)^7*cos(d*x+c)^7-1/24*sin(d*x+c)^5*cos(d*x+c)^7-1/
48*sin(d*x+c)^3*cos(d*x+c)^7-1/128*sin(d*x+c)*cos(d*x+c)^7+1/768*(cos(d*x+c
)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2048*d*x+5/2048*c)-8*I*a
^8*(-1/14*sin(d*x+c)^6*cos(d*x+c)^8-1/28*sin(d*x+c)^4*cos(d*x+c)^8-1/70*sin
(d*x+c)^2*cos(d*x+c)^8-1/280*cos(d*x+c)^8)-28*a^8*(-1/14*sin(d*x+c)^5*cos(d
*x+c)^9-5/168*sin(d*x+c)^3*cos(d*x+c)^9-1/112*sin(d*x+c)*cos(d*x+c)^9+1/896
*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*
x+c)+5/2048*d*x+5/2048*c)+56*I*a^8*(-1/14*sin(d*x+c)^4*cos(d*x+c)^10-1/42*s
in(d*x+c)^2*cos(d*x+c)^10-1/210*cos(d*x+c)^10)+70*a^8*(-1/14*sin(d*x+c)^3*c
os(d*x+c)^11-1/56*sin(d*x+c)*cos(d*x+c)^11+1/560*(cos(d*x+c)^9+9/8*cos(d*x+
c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+
9/2048*d*x+9/2048*c)-56*I*a^8*(-1/14*sin(d*x+c)^2*cos(d*x+c)^12-1/84*cos(d*
x+c)^12)-28*a^8*(-1/14*sin(d*x+c)*cos(d*x+c)^13+1/168*(cos(d*x+c)^11+11/10*
cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*cos(d*x+c)^3+6
93/256*cos(d*x+c))*sin(d*x+c)+33/2048*d*x+33/2048*c)-4/7*I*a^8*cos(d*x+c)^1
4+a^8*(1/14*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos(d*x+c)^9+429/320
*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3+3003/1024*cos(d*x
+c))*sin(d*x+c)+429/2048*d*x+429/2048*c))
```

Maxima [B] time = 1.73427, size = 232, normalized size = 8.59

$$-\frac{30720\,a^8\tan(dx+c)^7-215040i\,a^8\tan(dx+c)^6-645120\,a^8\tan(dx+c)^5+1075200i\,a^8\tan(dx+c)^4+1075200\,a^8\,\mathrm{t}}{215040\left(\tan(dx+c)^{14}+7\,\tan(dx+c)^{12}+21\,\tan(dx+c)^{10}+35\,\tan(dx+c)^8+35\,\tan(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/215040*(30720*a^8*tan(d*x + c)^7 - 215040*I*a^8*tan(d*x + c)^6 - 645120*
a^8*tan(d*x + c)^5 + 1075200*I*a^8*tan(d*x + c)^4 + 1075200*a^8*tan(d*x + c
)^3 - 645120*I*a^8*tan(d*x + c)^2 - 215040*a^8*tan(d*x + c) + 30720*I*a^8)/
((tan(d*x + c)^14 + 7*tan(d*x + c)^12 + 21*tan(d*x + c)^10 + 35*tan(d*x + c
)^8 + 35*tan(d*x + c)^6 + 21*tan(d*x + c)^4 + 7*tan(d*x + c)^2 + 1)*d)
```

Fricas [B] time = 1.91081, size = 308, normalized size = 11.41

$$\frac{-i\,a^8e^{(14i\,dx+14i\,c)}-7i\,a^8e^{(12i\,dx+12i\,c)}-21i\,a^8e^{(10i\,dx+10i\,c)}-35i\,a^8e^{(8i\,dx+8i\,c)}-35i\,a^8e^{(6i\,dx+6i\,c)}-21i\,a^8e^{(4i\,dx+4i\,c)}-7i\,a^8e^{(2i\,dx}}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/896*(-I*a^8*e^(14*I*d*x + 14*I*c) - 7*I*a^8*e^(12*I*d*x + 12*I*c) - 21*I*
a^8*e^(10*I*d*x + 10*I*c) - 35*I*a^8*e^(8*I*d*x + 8*I*c) - 35*I*a^8*e^(6*I*
d*x + 6*I*c) - 21*I*a^8*e^(4*I*d*x + 4*I*c) - 7*I*a^8*e^(2*I*d*x + 2*I*c))/
d
```

Sympy [B] time = 2.02483, size = 280, normalized size = 10.37

$$\begin{cases} \frac{-4398046511104ia^8d^6e^{14ic}e^{14idx}-30786325577728ia^8d^6e^{12ic}e^{12idx}-92358976733184ia^8d^6e^{10ic}e^{10idx}-153931627888640ia^8d^6e^{8ic}e^{8idx}-153931627888640ia^8}{3940649673949184d^7} \\ x\left(\frac{a^8e^{14ic}}{64}+\frac{3a^8e^{12ic}}{32}+\frac{15a^8e^{10ic}}{64}+\frac{5a^8e^{8ic}}{16}+\frac{15a^8e^{6ic}}{64}+\frac{3a^8e^{4ic}}{32}+\frac{a^8e^{2ic}}{64}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**14*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-4398046511104*I*a**8*d**6*exp(14*I*c)*exp(14*I*d*x) - 30786325
577728*I*a**8*d**6*exp(12*I*c)*exp(12*I*d*x) - 92358976733184*I*a**8*d**6*e
xp(10*I*c)*exp(10*I*d*x) - 153931627888640*I*a**8*d**6*exp(8*I*c)*exp(8*I*d
*x) - 153931627888640*I*a**8*d**6*exp(6*I*c)*exp(6*I*d*x) - 92358976733184*
I*a**8*d**6*exp(4*I*c)*exp(4*I*d*x) - 30786325577728*I*a**8*d**6*exp(2*I*c)
*exp(2*I*d*x))/(3940649673949184*d**7), Ne(3940649673949184*d**7, 0)), (x*(
a**8*exp(14*I*c)/64 + 3*a**8*exp(12*I*c)/32 + 15*a**8*exp(10*I*c)/64 + 5*a*
*8*exp(8*I*c)/16 + 15*a**8*exp(6*I*c)/64 + 3*a**8*exp(4*I*c)/32 + a**8*exp(
2*I*c)/64), True))
```

Giac [B] time = 2.66231, size = 628, normalized size = 23.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3440640*(-3840*I*a^8*e^(42*I*d*x + 28*I*c) - 80640*I*a^8*e^(40*I*d*x + 26
*I*c) - 806400*I*a^8*e^(38*I*d*x + 24*I*c) - 5107200*I*a^8*e^(36*I*d*x + 22
*I*c) - 22982400*I*a^8*e^(34*I*d*x + 20*I*c) - 78140160*I*a^8*e^(32*I*d*x +
 18*I*c) - 208373760*I*a^8*e^(30*I*d*x + 16*I*c) - 446511360*I*a^8*e^(28*I*
d*x + 14*I*c) - 781347840*I*a^8*e^(26*I*d*x + 12*I*c) - 1128341760*I*a^8*e^
(24*I*d*x + 10*I*c) - 1353031680*I*a^8*e^(22*I*d*x + 8*I*c) - 1350585600*I*
a^8*e^(20*I*d*x + 6*I*c) - 1121003520*I*a^8*e^(18*I*d*x + 4*I*c) - 76987008
0*I*a^8*e^(16*I*d*x + 2*I*c) - 196842240*I*a^8*e^(12*I*d*x - 2*I*c) - 70452
480*I*a^8*e^(10*I*d*x - 4*I*c) - 19138560*I*a^8*e^(8*I*d*x - 6*I*c) - 37094
40*I*a^8*e^(6*I*d*x - 8*I*c) - 456960*I*a^8*e^(4*I*d*x - 10*I*c) - 26880*I*
a^8*e^(2*I*d*x - 12*I*c) - 433336320*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x + 1
4*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^
(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I
*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^
(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c)
 + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d
*x) + d*e^(-14*I*c))
```

3.89 $\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=225

$$-\frac{ia^{16}}{16d(a-ia\tan(c+dx))^8}-\frac{ia^{15}}{28d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+dx))^6}-\frac{ia^{13}}{80d(a-ia\tan(c+dx))^5}-\frac{}{128d(a-}$$

```
[Out] (a^8*x)/256 - ((I/16)*a^16)/(d*(a - I*a*Tan[c + d*x])^8) - ((I/28)*a^15)/(d
*(a - I*a*Tan[c + d*x])^7) - ((I/48)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) - (
(I/80)*a^13)/(d*(a - I*a*Tan[c + d*x])^5) - ((I/128)*a^12)/(d*(a - I*a*Tan[
c + d*x])^4) - ((I/192)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/256)*a^10)
/(d*(a - I*a*Tan[c + d*x])^2) - ((I/256)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.115079, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{ia^{16}}{16d(a-ia\tan(c+dx))^8}-\frac{ia^{15}}{28d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+dx))^6}-\frac{ia^{13}}{80d(a-ia\tan(c+dx))^5}-\frac{}{128d(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^16*(a + I*a*Tan[c + d*x])^8,x]
```


```
[Out] (a^8*x)/256 - ((I/16)*a^16)/(d*(a - I*a*Tan[c + d*x])^8) - ((I/28)*a^15)/(d
*(a - I*a*Tan[c + d*x])^7) - ((I/48)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) - (
(I/80)*a^13)/(d*(a - I*a*Tan[c + d*x])^5) - ((I/128)*a^12)/(d*(a - I*a*Tan[
c + d*x])^4) - ((I/192)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/256)*a^10)
/(d*(a - I*a*Tan[c + d*x])^2) - ((I/256)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{(ia^{17})\,\text{Subst}\left(\int \frac{1}{(a-x)^9(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^{17})\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^9}+\frac{1}{4a^2(a-x)^8}+\frac{1}{8a^3(a-x)^7}+\frac{1}{16a^4(a-x)^6}+\frac{1}{32a^5(a-x)^5}+\right.\right.}{d}$$

$$= -\frac{ia^{16}}{16d(a-ia\tan(c+dx))^8}-\frac{ia^{15}}{28d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+}$$

$$= \frac{a^8x}{256}-\frac{ia^{16}}{16d(a-ia\tan(c+dx))^8}-\frac{ia^{15}}{28d(a-ia\tan(c+dx))^7}-\frac{ia^1}{48d(a-ia\,\text{ta}}$$

Mathematica [A] time = 5.3514, size = 166, normalized size = 0.74

$$a^8(-6272\sin(2(c+dx))-7840\sin(4(c+dx))-5760\sin(6(c+dx))-1680idx\sin(8(c+dx))+105\sin(8(c+dx))-2$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^16*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(-14700*I - (25088*I)*Cos[2*(c + d*x)] - (15680*I)*Cos[4*(c + d*x)] -
(7680*I)*Cos[6*(c + d*x)] - (105*I)*Cos[8*(c + d*x)] + 1680*d*x*Cos[8*(c +
d*x)] - 6272*Sin[2*(c + d*x)] - 7840*Sin[4*(c + d*x)] - 5760*Sin[6*(c + d*x
)] + 105*Sin[8*(c + d*x)] - (1680*I)*d*x*Sin[8*(c + d*x)])*(Cos[8*(c + 2*d*
x)] + I*Sin[8*(c + 2*d*x)]))/(430080*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] time = 0.141, size = 739, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/16*sin(d*x+c)^7*cos(d*x+c)^9-1/32*sin(d*x+c)^5*cos(d*x+c)^9-5/
384*sin(d*x+c)^3*cos(d*x+c)^9-1/256*sin(d*x+c)*cos(d*x+c)^9+1/2048*(cos(d*x
+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/3
2768*d*x+35/32768*c)-8*I*a^8*(-1/16*sin(d*x+c)^6*cos(d*x+c)^10-3/112*sin(d*
x+c)^4*cos(d*x+c)^10-1/112*sin(d*x+c)^2*cos(d*x+c)^10-1/560*cos(d*x+c)^10)-
28*a^8*(-1/16*sin(d*x+c)^5*cos(d*x+c)^11-5/224*sin(d*x+c)^3*cos(d*x+c)^11-5
/896*sin(d*x+c)*cos(d*x+c)^11+1/1792*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*c
os(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+45/32768*d*x
+45/32768*c)+56*I*a^8*(-1/16*sin(d*x+c)^4*cos(d*x+c)^12-1/56*sin(d*x+c)^2*c
os(d*x+c)^12-1/336*cos(d*x+c)^12)+70*a^8*(-1/16*sin(d*x+c)^3*cos(d*x+c)^13-
3/224*sin(d*x+c)*cos(d*x+c)^13+1/896*(cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/8
0*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos(d*x+c)
)*sin(d*x+c)+99/32768*d*x+99/32768*c)-56*I*a^8*(-1/16*sin(d*x+c)^2*cos(d*x+
c)^14-1/112*cos(d*x+c)^14)-28*a^8*(-1/16*sin(d*x+c)*cos(d*x+c)^15+1/224*(co
s(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos(d*x+c)^9+429/320*cos(d*x+c)^7+1
001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3+3003/1024*cos(d*x+c))*sin(d*x+c)
+429/32768*d*x+429/32768*c)-1/2*I*a^8*cos(d*x+c)^16+a^8*(1/16*(cos(d*x+c)^1
5+15/14*cos(d*x+c)^13+65/56*cos(d*x+c)^11+143/112*cos(d*x+c)^9+1287/896*cos
(d*x+c)^7+429/256*cos(d*x+c)^5+2145/1024*cos(d*x+c)^3+6435/2048*cos(d*x+c))
```

```
*sin(d*x+c)+6435/32768*d*x+6435/32768*c))
```

Maxima [A] time = 1.81627, size = 332, normalized size = 1.48

$$\frac{13440\,(dx+c)a^8 + \frac{13440\,a^8\tan(dx+c)^{15}+103040\,a^8\tan(dx+c)^{13}+343168\,a^8\tan(dx+c)^{11}+646784\,a^8\tan(dx+c)^{9}+369024\,a^8\tan(dx+c)^{7}+2752512i\,a^8\tan\cdots}{\tan(dx+c)^{16}+8\tan(dx+c)^{14}+28\tan(dx+c)^{12}+56\tan(dx+\cdots}}{3440640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/3440640*(13440*(d*x + c)*a^8 + (13440*a^8*tan(d*x + c)^15 + 103040*a^8*ta
n(d*x + c)^13 + 343168*a^8*tan(d*x + c)^11 + 646784*a^8*tan(d*x + c)^9 + 36
9024*a^8*tan(d*x + c)^7 + 2752512*I*a^8*tan(d*x + c)^6 + 9061248*a^8*tan(d*
x + c)^5 - 14680064*I*a^8*tan(d*x + c)^4 - 15012480*a^8*tan(d*x + c)^3 + 95
68256*I*a^8*tan(d*x + c)^2 + 3427200*a^8*tan(d*x + c) - 524288*I*a^8)/(tan(
d*x + c)^16 + 8*tan(d*x + c)^14 + 28*tan(d*x + c)^12 + 56*tan(d*x + c)^10 +
 70*tan(d*x + c)^8 + 56*tan(d*x + c)^6 + 28*tan(d*x + c)^4 + 8*tan(d*x + c)
^2 + 1))/d
```

Fricas [A] time = 2.60139, size = 405, normalized size = 1.8

$$\frac{1680\,a^8dx - 105i\,a^8e^{(16i\,dx+16i\,c)} - 960i\,a^8e^{(14i\,dx+14i\,c)} - 3920i\,a^8e^{(12i\,dx+12i\,c)} - 9408i\,a^8e^{(10i\,dx+10i\,c)} - 14700i\,a^8e^{(8i\,dx+8i\,c)} - \cdots}{430080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/430080*(1680*a^8*d*x - 105*I*a^8*e^(16*I*d*x + 16*I*c) - 960*I*a^8*e^(14*
I*d*x + 14*I*c) - 3920*I*a^8*e^(12*I*d*x + 12*I*c) - 9408*I*a^8*e^(10*I*d*x
 + 10*I*c) - 14700*I*a^8*e^(8*I*d*x + 8*I*c) - 15680*I*a^8*e^(6*I*d*x + 6*I
*c) - 11760*I*a^8*e^(4*I*d*x + 4*I*c) - 6720*I*a^8*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A] time = 2.35049, size = 325, normalized size = 1.44

$$\frac{a^8x}{256} + \begin{cases} \frac{-354658470655426560ia^8d^7e^{16ic}e^{16idx} - 324259173170675712 0ia^8d^7e^{14ic}e^{14idx} - 13240582904469258240ia^8d^7e^{12ic}e^{12idx} - 31777398970726219776ia^8d^7\cdots}{14526\cdots} \\ x\left(\frac{a^8e^{16ic}}{256} + \frac{a^8e^{14ic}}{32} + \frac{7a^8e^{12ic}}{64} + \frac{7a^8e^{10ic}}{32} + \frac{35a^8e^{8ic}}{128} + \frac{7a^8e^{6ic}}{32} + \frac{7a^8e^{4ic}}{64} + \frac{a^8e^{2ic}}{32}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**16*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*x/256 + Piecewise(((-354658470655426560*I*a**8*d**7*exp(16*I*c)*exp(16
*I*d*x) - 3242591731706757120*I*a**8*d**7*exp(14*I*c)*exp(14*I*d*x) - 13240
582904469258240*I*a**8*d**7*exp(12*I*c)*exp(12*I*d*x) - 3177739897072621977
6*I*a**8*d**7*exp(10*I*c)*exp(10*I*d*x) - 49652185891759718400*I*a**8*d**7*
exp(8*I*c)*exp(8*I*d*x) - 52962331617877032960*I*a**8*d**7*exp(6*I*c)*exp(6
*I*d*x) - 39721748713407774720*I*a**8*d**7*exp(4*I*c)*exp(4*I*d*x) - 226981
42121947299840*I*a**8*d**7*exp(2*I*c)*exp(2*I*d*x))/(145268109580462718 9760
```

```
*d**8), Ne(1452681095804627189760*d**8, 0)), (x*(a**8*exp(16*I*c)/256 + a**
8*exp(14*I*c)/32 + 7*a**8*exp(12*I*c)/64 + 7*a**8*exp(10*I*c)/32 + 35*a**8*
exp(8*I*c)/128 + 7*a**8*exp(6*I*c)/32 + 7*a**8*exp(4*I*c)/64 + a**8*exp(2*I
*c)/32), True))
```

Giac [B] time = 3.07861, size = 1967, normalized size = 8.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/55050240*(215040*a^8*d*x*e^(28*I*d*x + 14*I*c) + 3010560*a^8*d*x*e^(26*I*
d*x + 12*I*c) + 19568640*a^8*d*x*e^(24*I*d*x + 10*I*c) + 78274560*a^8*d*x*e
^(22*I*d*x + 8*I*c) + 215255040*a^8*d*x*e^(20*I*d*x + 6*I*c) + 430510080*a^
8*d*x*e^(18*I*d*x + 4*I*c) + 645765120*a^8*d*x*e^(16*I*d*x + 2*I*c) + 64576
5120*a^8*d*x*e^(12*I*d*x - 2*I*c) + 430510080*a^8*d*x*e^(10*I*d*x - 4*I*c)
+ 215255040*a^8*d*x*e^(8*I*d*x - 6*I*c) + 78274560*a^8*d*x*e^(6*I*d*x - 8*I
*c) + 19568640*a^8*d*x*e^(4*I*d*x - 10*I*c) + 3010560*a^8*d*x*e^(2*I*d*x -
12*I*c) + 738017280*a^8*d*x*e^(14*I*d*x) + 215040*a^8*d*x*e^(-14*I*c) - 103
740*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 1452360*I*a^
8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9440340*I*a^8*e^(24*
I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 37761360*I*a^8*e^(22*I*d*x +
 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 103843740*I*a^8*e^(20*I*d*x + 6*I*c)
*log(e^(2*I*d*x + 2*I*c) + 1) - 207687480*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^
(2*I*d*x + 2*I*c) + 1) - 311531220*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*
x + 2*I*c) + 1) - 311531220*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I
*c) + 1) - 207687480*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1
) - 103843740*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 3776
1360*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9440340*I*a^8
*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 1452360*I*a^8*e^(2*I*d
*x - 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 356035680*I*a^8*e^(14*I*d*x)*lo
g(e^(2*I*d*x + 2*I*c) + 1) - 103740*I*a^8*e^(-14*I*c)*log(e^(2*I*d*x + 2*I*
c) + 1) + 103740*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x) + e^(-2*I*c))
+ 1452360*I*a^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 94403
40*I*a^8*e^(24*I*d*x + 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 37761360*I*a
^8*e^(22*I*d*x + 8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 103843740*I*a^8*e^(
20*I*d*x + 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 207687480*I*a^8*e^(18*I*d
*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 311531220*I*a^8*e^(16*I*d*x + 2
*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 311531220*I*a^8*e^(12*I*d*x - 2*I*c)*
log(e^(2*I*d*x) + e^(-2*I*c)) + 207687480*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^
(2*I*d*x) + e^(-2*I*c)) + 103843740*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*
x) + e^(-2*I*c)) + 37761360*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x) + e^(
-2*I*c)) + 9440340*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c))
 + 1452360*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 35603
5680*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) + 103740*I*a^8*e^(-14
*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) - 13440*I*a^8*e^(44*I*d*x + 30*I*c) - 3
11040*I*a^8*e^(42*I*d*x + 28*I*c) - 3445120*I*a^8*e^(40*I*d*x + 26*I*c) - 2
4303104*I*a^8*e^(38*I*d*x + 24*I*c) - 122582656*I*a^8*e^(36*I*d*x + 22*I*c)
 - 470484224*I*a^8*e^(34*I*d*x + 20*I*c) - 1427794816*I*a^8*e^(32*I*d*x + 1
8*I*c) - 3514563584*I*a^8*e^(30*I*d*x + 16*I*c) - 7142793088*I*a^8*e^(28*I*
d*x + 14*I*c) - 12136447232*I*a^8*e^(26*I*d*x + 12*I*c) - 17387563648*I*a^8
*e^(24*I*d*x + 10*I*c) - 21108086272*I*a^8*e^(22*I*d*x + 8*I*c) - 217400718
08*I*a^8*e^(20*I*d*x + 6*I*c) - 18942724864*I*a^8*e^(18*I*d*x + 4*I*c) - 13
859732096*I*a^8*e^(16*I*d*x + 2*I*c) - 4147974656*I*a^8*e^(12*I*d*x - 2*I*c
) - 1619129344*I*a^8*e^(10*I*d*x - 4*I*c) - 480058880*I*a^8*e^(8*I*d*x - 6*
```

```
I*c) - 101355520*I*a^8*e^(6*I*d*x - 8*I*c) - 13547520*I*a^8*e^(4*I*d*x - 10
*I*c) - 860160*I*a^8*e^(2*I*d*x - 12*I*c) - 8407312384*I*a^8*e^(14*I*d*x))/
(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x +
10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d
*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x -
2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e
^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c)
+ 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

3.90 $\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=279

$$-\frac{ia^{17}}{36d(a-ia\tan(c+dx))^9}-\frac{ia^{16}}{32d(a-ia\tan(c+dx))^8}-\frac{3ia^{15}}{112d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+dx))^6}-\frac{}{64d(a}$$

```
[Out] (5*a^8*x)/512 - ((I/36)*a^17)/(d*(a - I*a*Tan[c + d*x])^9) - ((I/32)*a^16)/
(d*(a - I*a*Tan[c + d*x])^8) - (((3*I)/112)*a^15)/(d*(a - I*a*Tan[c + d*x])
^7) - ((I/48)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) - ((I/64)*a^13)/(d*(a - I*
a*Tan[c + d*x])^5) - (((3*I)/256)*a^12)/(d*(a - I*a*Tan[c + d*x])^4) - (((7
*I)/768)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/128)*a^10)/(d*(a - I*a*Ta
n[c + d*x])^2) - (((9*I)/1024)*a^9)/(d*(a - I*a*Tan[c + d*x])) + ((I/1024)*
a^9)/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.155992, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{ia^{17}}{36d(a-ia\tan(c+dx))^9}-\frac{ia^{16}}{32d(a-ia\tan(c+dx))^8}-\frac{3ia^{15}}{112d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+dx))^6}-\frac{}{64d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^18*(a + I*a*Tan[c + d*x])^8,x]
```


```
[Out] (5*a^8*x)/512 - ((I/36)*a^17)/(d*(a - I*a*Tan[c + d*x])^9) - ((I/32)*a^16)/
(d*(a - I*a*Tan[c + d*x])^8) - (((3*I)/112)*a^15)/(d*(a - I*a*Tan[c + d*x])
^7) - ((I/48)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) - ((I/64)*a^13)/(d*(a - I*
a*Tan[c + d*x])^5) - (((3*I)/256)*a^12)/(d*(a - I*a*Tan[c + d*x])^4) - (((7
*I)/768)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/128)*a^10)/(d*(a - I*a*Ta
n[c + d*x])^2) - (((9*I)/1024)*a^9)/(d*(a - I*a*Tan[c + d*x])) + ((I/1024)*
a^9)/(d*(a + I*a*Tan[c + d*x]))
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{(ia^{19})\,\text{Subst}\left(\int \frac{1}{(a-x)^{10}(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^{19})\,\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^{10}} + \frac{1}{4a^3(a-x)^9} + \frac{3}{16a^4(a-x)^8} + \frac{1}{8a^5(a-x)^7} + \frac{5}{64a^6(a-x)^6} + \frac{}{64}\right.\right.}{}$$

$$= -\frac{ia^{17}}{36d(a-ia\tan(c+dx))^9} - \frac{ia^{16}}{32d(a-ia\tan(c+dx))^8} - \frac{3ia^{15}}{112d(a-ia\tan(c+d}$$

$$= \frac{5a^8x}{512} - \frac{ia^{17}}{36d(a-ia\tan(c+dx))^9} - \frac{ia^{16}}{32d(a-ia\tan(c+dx))^8} - \frac{3ia^{15}}{112d(a-ia\tan}$$

Mathematica [A] time = 6.71641, size = 188, normalized size = 0.67

$$\frac{a^8(-7056\sin(2(c+dx)) - 10080\sin(4(c+dx)) - 9720\sin(6(c+dx)) - 5040idx\sin(8(c+dx)) + 315\sin(8(c+dx)) + 28}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^18*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(-15876*I - (28224*I)*Cos[2*(c + d*x)] - (20160*I)*Cos[4*(c + d*x)] -
(12960*I)*Cos[6*(c + d*x)] - (315*I)*Cos[8*(c + d*x)] + 5040*d*x*Cos[8*(c +
 d*x)] + (224*I)*Cos[10*(c + d*x)] - 7056*Sin[2*(c + d*x)] - 10080*Sin[4*(c
 + d*x)] - 9720*Sin[6*(c + d*x)] + 315*Sin[8*(c + d*x)] - (5040*I)*d*x*Sin[
8*(c + d*x)] + 280*Sin[10*(c + d*x)])*(Cos[8*(c + 2*d*x)] + I*Sin[8*(c + 2*
d*x)]))/(516096*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] time = 0.13, size = 789, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/18*sin(d*x+c)^7*cos(d*x+c)^11-7/288*sin(d*x+c)^5*cos(d*x+c)^11
-5/576*sin(d*x+c)^3*cos(d*x+c)^11-5/2304*sin(d*x+c)*cos(d*x+c)^11+1/4608*(c
os(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128
*cos(d*x+c))*sin(d*x+c)+35/65536*d*x+35/65536*c)-8*I*a^8*(-1/18*sin(d*x+c)^
6*cos(d*x+c)^12-1/48*sin(d*x+c)^4*cos(d*x+c)^12-1/168*sin(d*x+c)^2*cos(d*x+
c)^12-1/1008*cos(d*x+c)^12)-28*a^8*(-1/18*sin(d*x+c)^5*cos(d*x+c)^13-5/288*
sin(d*x+c)^3*cos(d*x+c)^13-5/1344*sin(d*x+c)*cos(d*x+c)^13+5/16128*(cos(d*x
+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*c
os(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+55/65536*d*x+55/65536*c)+56*I*a^
8*(-1/18*sin(d*x+c)^4*cos(d*x+c)^14-1/72*sin(d*x+c)^2*cos(d*x+c)^14-1/504*c
os(d*x+c)^14)+70*a^8*(-1/18*sin(d*x+c)^3*cos(d*x+c)^15-1/96*sin(d*x+c)*cos(
d*x+c)^15+1/1344*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos(d*x+c)^9+42
9/320*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3+3003/1024*co
s(d*x+c))*sin(d*x+c)+143/65536*d*x+143/65536*c)-56*I*a^8*(-1/18*sin(d*x+c)^
2*cos(d*x+c)^16-1/144*cos(d*x+c)^16)-28*a^8*(-1/18*sin(d*x+c)*cos(d*x+c)^17
+1/288*(cos(d*x+c)^15+15/14*cos(d*x+c)^13+65/56*cos(d*x+c)^11+143/112*cos(d
*x+c)^9+1287/896*cos(d*x+c)^7+429/256*cos(d*x+c)^5+2145/1024*cos(d*x+c)^3+6
435/2048*cos(d*x+c))*sin(d*x+c)+715/65536*d*x+715/65536*c)-4/9*I*a^8*cos(d*
```

```
x+c)^18+a^8*(1/18*(cos(d*x+c)^17+17/16*cos(d*x+c)^15+255/224*cos(d*x+c)^13+
1105/896*cos(d*x+c)^11+2431/1792*cos(d*x+c)^9+21879/14336*cos(d*x+c)^7+7293
/4096*cos(d*x+c)^5+36465/16384*cos(d*x+c)^3+109395/32768*cos(d*x+c))*sin(d*
x+c)+12155/65536*d*x+12155/65536*c))
```

Maxima [A] time = 1.76884, size = 363, normalized size = 1.3

$$40320\,(dx+c)a^8 + \frac{40320\,a^8\tan(dx+c)^{17}+349440\,a^8\tan(dx+c)^{15}+1338624\,a^8\tan(dx+c)^{13}+2969856\,a^8\tan(dx+c)^{11}+4194304\,a^8\tan(dx+c)^9+35182}{\tan(dx+c)^{18}+9\tan(dx+c)^{16}+36\tan(dx+c)^{14}+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/4128768*(40320*(d*x + c)*a^8 + (40320*a^8*tan(d*x + c)^17 + 349440*a^8*ta
n(d*x + c)^15 + 1338624*a^8*tan(d*x + c)^13 + 2969856*a^8*tan(d*x + c)^11 +
 4194304*a^8*tan(d*x + c)^9 + 3518208*a^8*tan(d*x + c)^7 + 2752512*I*a^8*ta
n(d*x + c)^6 + 11047680*a^8*tan(d*x + c)^5 - 15335424*I*a^8*tan(d*x + c)^4
- 15488256*a^8*tan(d*x + c)^3 + 10616832*I*a^8*tan(d*x + c)^2 + 4088448*a^8
*tan(d*x + c) - 655360*I*a^8)/(tan(d*x + c)^18 + 9*tan(d*x + c)^16 + 36*tan
(d*x + c)^14 + 84*tan(d*x + c)^12 + 126*tan(d*x + c)^10 + 126*tan(d*x + c)^
8 + 84*tan(d*x + c)^6 + 36*tan(d*x + c)^4 + 9*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 2.82542, size = 528, normalized size = 1.89

$$\frac{\left(5040\,a^8dxe^{(2i\,dx+2i\,c)} - 28i\,a^8e^{(20i\,dx+20i\,c)} - 315i\,a^8e^{(18i\,dx+18i\,c)} - 1620i\,a^8e^{(16i\,dx+16i\,c)} - 5040i\,a^8e^{(14i\,dx+14i\,c)} - 10584i\,a^8}{516}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/516096*(5040*a^8*d*x*e^(2*I*d*x + 2*I*c) - 28*I*a^8*e^(20*I*d*x + 20*I*c)
 - 315*I*a^8*e^(18*I*d*x + 18*I*c) - 1620*I*a^8*e^(16*I*d*x + 16*I*c) - 504
0*I*a^8*e^(14*I*d*x + 14*I*c) - 10584*I*a^8*e^(12*I*d*x + 12*I*c) - 15876*I
*a^8*e^(10*I*d*x + 10*I*c) - 17640*I*a^8*e^(8*I*d*x + 8*I*c) - 15120*I*a^8*
e^(6*I*d*x + 6*I*c) - 11340*I*a^8*e^(4*I*d*x + 4*I*c) + 252*I*a^8)*e^(-2*I*
d*x - 2*I*c)/d
```

Sympy [A] time = 2.87144, size = 415, normalized size = 1.49

$$\frac{5a^8x}{512} + \begin{cases} \frac{(-277298568799925181577403826176ia^8d^9e^{20ic}e^{18idx} - 311960888999915829274579304448 0ia^8d^9e^{18ic}e^{16idx} - 1604370290913852836269264994}{} \\ x\left(-\frac{5a^8}{512} + \frac{\left(a^8e^{20ic}+10a^8e^{18ic}+45a^8e^{16ic}+120a^8e^{14ic}+210a^8e^{12ic}+252a^8e^{10ic}+210a^8e^{8ic}+120a^8e^{6ic}+45a^8e^{4ic}+10a^8e^{2ic}+a^8\right)e^{-2ic}}{1024}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**18*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 5*a**8*x/512 + Piecewise(((-277298568799925181577403826176*I*a**8*d**9*exp(
20*I*c)*exp(18*I*d*x) - 3119608889999158292745793044480*I*a**8*d**9*exp(18*
```

```
I*c)*exp(16*I*d*x) - 160437029091385283626926499430 40*I*a**8*d**9*exp(16*I*
c)*exp(14*I*d*x) - 499137423839865326839326887116 80*I*a**8*d**9*exp(14*I*c)
*exp(12*I*d*x) - 1048188590063717186362586462945 28*I*a**8*d**9*exp(12*I*c)*
exp(10*I*d*x) - 1572282885095575779543879694417 92*I*a**8*d**9*exp(10*I*c)*e
xp(8*I*d*x) - 17469809834395286439376441049088 0*I*a**8*d**9*exp(8*I*c)*exp(
6*I*d*x) - 149741227151959598051798066135040*I*a**8*d**9*exp(6*I*c)*exp(4*I
*d*x) - 112305920363969698538848549601280*I*a**8*d**9*exp(4*I*c)*exp(2*I*d*
x) + 24956871191993266341966344355 84*I*a**8*d**9*exp(-2*I*d*x))*exp(-2*I*c)
/(511116722012022094683470732 4076032*d**10), Ne(511116722012022094683470732
4076032*d**10*exp(2*I*c), 0)), (x*(-5*a**8/512 + (a**8*exp(20*I*c) + 10*a**
8*exp(18*I*c) + 45*a**8*exp(16*I*c) + 120*a**8*exp(14*I*c) + 210*a**8*exp(1
2*I*c) + 252*a**8*exp(10*I*c) + 210*a**8*exp(8*I*c) + 120*a**8*exp(6*I*c) +
 45*a**8*exp(4*I*c) + 10*a**8*exp(2*I*c) + a**8)*exp(-2*I*c)/1024), True))
```

Giac [B] time = 3.35362, size = 2044, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/330301440*(3225600*a^8*d*x*e^(30*I*d*x + 16*I*c) + 45158400*a^8*d*x*e^(28
*I*d*x + 14*I*c) + 293529600*a^8*d*x*e^(26*I*d*x + 12*I*c) + 1174118400*a^8
*d*x*e^(24*I*d*x + 10*I*c) + 3228825600*a^8*d*x*e^(22*I*d*x + 8*I*c) + 6457
651200*a^8*d*x*e^(20*I*d*x + 6*I*c) + 9686476800*a^8*d*x*e^(18*I*d*x + 4*I*
c) + 11070259200*a^8*d*x*e^(16*I*d*x + 2*I*c) + 6457651200*a^8*d*x*e^(12*I*
d*x - 2*I*c) + 3228825600*a^8*d*x*e^(10*I*d*x - 4*I*c) + 1174118400*a^8*d*x
*e^(8*I*d*x - 6*I*c) + 293529600*a^8*d*x*e^(6*I*d*x - 8*I*c) + 45158400*a^8
*d*x*e^(4*I*d*x - 10*I*c) + 3225600*a^8*d*x*e^(2*I*d*x - 12*I*c) + 96864768
00*a^8*d*x*e^(14*I*d*x) - 1515780*I*a^8*e^(30*I*d*x + 16*I*c)*log(e^(2*I*d*
x + 2*I*c) + 1) - 21220920*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I
*c) + 1) - 137935980*I*a^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) +
1) - 551743920*I*a^8*e^(24*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 1
517295780*I*a^8*e^(22*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 3034591
560*I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 4551887340*I*
a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 5202156960*I*a^8*e^
(16*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 3034591560*I*a^8*e^(12*I*
d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 1517295780*I*a^8*e^(10*I*d*x -
4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 551743920*I*a^8*e^(8*I*d*x - 6*I*c)*l
og(e^(2*I*d*x + 2*I*c) + 1) - 137935980*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*
I*d*x + 2*I*c) + 1) - 21220920*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x +
2*I*c) + 1) - 1515780*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x + 2*I*c) +
1) - 4551887340*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) + 1515780*I
*a^8*e^(30*I*d*x + 16*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 21220920*I*a^8*e
^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 137935980*I*a^8*e^(26*
I*d*x + 12*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 551743920*I*a^8*e^(24*I*d*x
 + 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 1517295780*I*a^8*e^(22*I*d*x + 8
*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 3034591560*I*a^8*e^(20*I*d*x + 6*I*c)
*log(e^(2*I*d*x) + e^(-2*I*c)) + 4551887340*I*a^8*e^(18*I*d*x + 4*I*c)*log(
e^(2*I*d*x) + e^(-2*I*c)) + 5202156960*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*
I*d*x) + e^(-2*I*c)) + 3034591560*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x
) + e^(-2*I*c)) + 1517295780*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x) + e
^(-2*I*c)) + 551743920*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*
c)) + 137935980*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 2
1220920*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 1515780*
I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 4551887340*I*a^8
```

```
*e^(14*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) - 17920*I*a^8*e^(48*I*d*x + 34*
I*c) - 452480*I*a^8*e^(46*I*d*x + 32*I*c) - 5489920*I*a^8*e^(44*I*d*x + 30*
I*c) - 42609280*I*a^8*e^(42*I*d*x + 28*I*c) - 237601280*I*a^8*e^(40*I*d*x +
 26*I*c) - 1013595520*I*a^8*e^(38*I*d*x + 24*I*c) - 3439322880*I*a^8*e^(36*
I*d*x + 22*I*c) - 9529403520*I*a^8*e^(34*I*d*x + 20*I*c) - 21965959680*I*a^
8*e^(32*I*d*x + 18*I*c) - 42709532800*I*a^8*e^(30*I*d*x + 16*I*c) - 7077203
8400*I*a^8*e^(28*I*d*x + 14*I*c) - 100658185600*I*a^8*e^(26*I*d*x + 12*I*c)
 - 123309222400*I*a^8*e^(24*I*d*x + 10*I*c) - 129974633600*I*a^8*e^(22*I*d*
x + 8*I*c) - 117140020480*I*a^8*e^(20*I*d*x + 6*I*c) - 89191105920*I*a^8*e^
(18*I*d*x + 4*I*c) - 56345172480*I*a^8*e^(16*I*d*x + 2*I*c) - 11479265280*I
*a^8*e^(12*I*d*x - 2*I*c) - 3367687680*I*a^8*e^(10*I*d*x - 4*I*c) - 6457651
20*I*a^8*e^(8*I*d*x - 6*I*c) - 52577280*I*a^8*e^(6*I*d*x - 8*I*c) + 7418880
*I*a^8*e^(4*I*d*x - 10*I*c) + 2257920*I*a^8*e^(2*I*d*x - 12*I*c) - 28794769
920*I*a^8*e^(14*I*d*x) + 161280*I*a^8*e^(-14*I*c))/(d*e^(30*I*d*x + 16*I*c)
 + 14*d*e^(28*I*d*x + 14*I*c) + 91*d*e^(26*I*d*x + 12*I*c) + 364*d*e^(24*I*
d*x + 10*I*c) + 1001*d*e^(22*I*d*x + 8*I*c) + 2002*d*e^(20*I*d*x + 6*I*c) +
 3003*d*e^(18*I*d*x + 4*I*c) + 3432*d*e^(16*I*d*x + 2*I*c) + 2002*d*e^(12*I
*d*x - 2*I*c) + 1001*d*e^(10*I*d*x - 4*I*c) + 364*d*e^(8*I*d*x - 6*I*c) + 9
1*d*e^(6*I*d*x - 8*I*c) + 14*d*e^(4*I*d*x - 10*I*c) + d*e^(2*I*d*x - 12*I*c
) + 3003*d*e^(14*I*d*x))
```

3.91 $\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=235

$$-\frac{3003ia^8\sec(c+dx)}{16d}-\frac{3003a^8\tanh^{-1}(\sin(c+dx))}{16d}-\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}-\frac{429ia^2\sec(c+dx)\left(a^2+\right.}{40d}$$

```
[Out] (-3003*a^8*ArcTanh[Sin[c + d*x]])/(16*d) - (((3003*I)/16)*a^8*Sec[c + d*x])
/d - (((13*I)/6)*a^3*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5)/d - ((2*I)*a*Co
s[c + d*x]*(a + I*a*Tan[c + d*x])^7)/d - (((429*I)/40)*a^2*Sec[c + d*x]*(a^
2 + I*a^2*Tan[c + d*x])^3)/d - (((143*I)/30)*Sec[c + d*x]*(a^2 + I*a^2*Tan[
c + d*x])^4)/d - (((1001*I)/40)*Sec[c + d*x]*(a^4 + I*a^4*Tan[c + d*x])^2)/
d - (((1001*I)/16)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```

Rubi [A] time = 0.20371, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3496, 3498, 3486, 3770}

$$-\frac{3003ia^8\sec(c+dx)}{16d}-\frac{3003a^8\tanh^{-1}(\sin(c+dx))}{16d}-\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}-\frac{429ia^2\sec(c+dx)\left(a^2+\right.}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^8,x]
```


```
[Out] (-3003*a^8*ArcTanh[Sin[c + d*x]])/(16*d) - (((3003*I)/16)*a^8*Sec[c + d*x])
/d - (((13*I)/6)*a^3*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5)/d - ((2*I)*a*Co
s[c + d*x]*(a + I*a*Tan[c + d*x])^7)/d - (((429*I)/40)*a^2*Sec[c + d*x]*(a^
2 + I*a^2*Tan[c + d*x])^3)/d - (((143*I)/30)*Sec[c + d*x]*(a^2 + I*a^2*Tan[
c + d*x])^4)/d - (((1001*I)/40)*Sec[c + d*x]*(a^4 + I*a^4*Tan[c + d*x])^2)/
d - (((1001*I)/16)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```


Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
```

```
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d} - \left(13a^2\right)\int \sec(c+dx)(a+ia\tan(c+$$

$$= -\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx)}{d}$$

$$= -\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx)}{d}$$

$$= -\frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d} - \frac{13ia^3\sec(c+dx)(a+ia\tan(c+}{6d}$$

$$= -\frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d} - \frac{13ia^3\sec(c+dx)(a+ia\tan(c+}{6d}$$

$$= -\frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d} - \frac{13ia^3\sec(c+dx)(a+ia\tan(c+}{6d}$$

$$= -\frac{3003ia^8\sec(c+dx)}{16d} - \frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d} - \frac{13ia^3\sec(}{}$$

$$= -\frac{3003a^8\tanh^{-1}(\sin(c+dx))}{16d} - \frac{3003ia^8\sec(c+dx)}{16d} - \frac{429ia^5\sec(c+dx)(a}{40d}$$

Mathematica [A] time = 2.6415, size = 205, normalized size = 0.87

$$\frac{a^8(\cos(8c)-i\sin(8c))\cos^2(c+dx)(\tan(c+dx)-i)^8\left(-658944i\cos(c+dx)+5(12870\sin(c+dx)+22165\sin(3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Cos[c + d*x]^2*(Cos[8*c] - I*Sin[8*c])*((-658944*I)*Cos[c + d*x] + 720
720*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) + 5*((-73216*I)*Cos[3*(c + d*x)] - (19968*I)*
Cos[5*(c + d*x)] - (1536*I)*Cos[7*(c + d*x)] + 12870*Sin[c + d*x] + 22165*S
in[3*(c + d*x)] + 10959*Sin[5*(c + d*x)] + 1536*Sin[7*(c + d*x)]))*(-I + Ta
n[c + d*x])^8)/(3840*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] time = 0.104, size = 464, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 5/16*a^8*sin(d*x+c)^7/d+175/16*a^8*sin(d*x+c)^5/d+2555/48*a^8*sin(d*x+c)^3/
d-3003/16/d*a^8*ln(sec(d*x+c)+tan(d*x+c))+3019/16*a^8*sin(d*x+c)/d+1/6/d*a^
8*sin(d*x+c)^9/cos(d*x+c)^6-1/8/d*a^8*sin(d*x+c)^9/cos(d*x+c)^4+5/16/d*a^8*
sin(d*x+c)^9/cos(d*x+c)^2+56/3*I/d*a^8*sin(d*x+c)^6/cos(d*x+c)^3-8/5*I/d*a^
8*sin(d*x+c)^8/cos(d*x+c)^5+8/5*I/d*a^8*sin(d*x+c)^8/cos(d*x+c)^3-4424/15*I
/d*a^8*cos(d*x+c)-328/5*I/d*a^8*cos(d*x+c)*sin(d*x+c)^4-8*I/d*a^8*cos(d*x+c
)*sin(d*x+c)^6-8*I/d*a^8*sin(d*x+c)^8/cos(d*x+c)-2152/15*I/d*a^8*cos(d*x+c)
*sin(d*x+c)^2-56*I/d*a^8*sin(d*x+c)^6/cos(d*x+c)-56*I/d*a^8*sin(d*x+c)^4/co
s(d*x+c)-7/d*a^8*sin(d*x+c)^7/cos(d*x+c)^4+21/2/d*a^8*sin(d*x+c)^7/cos(d*x+
c)^2+35/d*a^8*sin(d*x+c)^5/cos(d*x+c)^2
```

Maxima [B] time = 1.18915, size = 535, normalized size = 2.28

$$-\frac{5a^8\left(\frac{2\left(87\sin(dx+c)^5-136\sin(dx+c)^3+57\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}+105\log(\sin(dx+c)+1)-105\log(\sin(dx+c)-1)-96\sin(dx+c)\right)+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/480*(5*a^8*(2*(87*sin(d*x + c)^5 - 136*sin(d*x + c)^3 + 57*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 105*log(sin(d
*x + c) + 1) - 105*log(sin(d*x + c) - 1) - 96*sin(d*x + c)) + 840*a^8*(2*(9
*sin(d*x + c)^3 - 7*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) +
 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1) - 16*sin(d*x + c)) + 8
400*a^8*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) - 3*
log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 26880*I*a^8*(1/cos(d*x + c) + cos
(d*x + c)) + 8960*I*a^8*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x
+ c)) + 768*I*a^8*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^
5 + 5*cos(d*x + c)) + 6720*a^8*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1) - 2*sin(d*x + c)) + 3840*I*a^8*cos(d*x + c) - 480*a^8*sin(d*x + c))/d
```

Fricas [A] time = 1.79936, size = 1137, normalized size = 4.84

$$\frac{-30720i\,a^8e^{(13i\,dx+13i\,c)}-309270i\,a^8e^{(11i\,dx+11i\,c)}-953810i\,a^8e^{(9i\,dx+9i\,c)}-1446588i\,a^8e^{(7i\,dx+7i\,c)}-1189188i\,a^8e^{(5i\,dx+5i\,c)}-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(-30720*I*a^8*e^(13*I*d*x + 13*I*c) - 309270*I*a^8*e^(11*I*d*x + 11*I
*c) - 953810*I*a^8*e^(9*I*d*x + 9*I*c) - 1446588*I*a^8*e^(7*I*d*x + 7*I*c)
- 1189188*I*a^8*e^(5*I*d*x + 5*I*c) - 510510*I*a^8*e^(3*I*d*x + 3*I*c) - 90
090*I*a^8*e^(I*d*x + I*c) - 45045*(a^8*e^(12*I*d*x + 12*I*c) + 6*a^8*e^(10*
I*d*x + 10*I*c) + 15*a^8*e^(8*I*d*x + 8*I*c) + 20*a^8*e^(6*I*d*x + 6*I*c) +
 15*a^8*e^(4*I*d*x + 4*I*c) + 6*a^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x
 + I*c) + I) + 45045*(a^8*e^(12*I*d*x + 12*I*c) + 6*a^8*e^(10*I*d*x + 10*I*
c) + 15*a^8*e^(8*I*d*x + 8*I*c) + 20*a^8*e^(6*I*d*x + 6*I*c) + 15*a^8*e^(4*
I*d*x + 4*I*c) + 6*a^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))
/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8
*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*
x + 2*I*c) + d)
```

Sympy [A] time = 12.3875, size = 326, normalized size = 1.39

$$\frac{3003a^{8}\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{16}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{16}\right)}{d}+\frac{-\frac{4165ia^{8}e^{-ic}e^{11idx}}{8d}-\frac{49301ia^{8}e^{-3ic}e^{9idx}}{24d}-\frac{69349ia^{8}e^{-5ic}e^{7idx}}{20d}-\frac{60699ia^{8}e^{-7ic}e^{5idx}}{20d}-\frac{10873ia^{8}e^{-9ic}}{8d}}{e^{12idx}+6e^{-2ic}e^{10idx}+15e^{-4ic}e^{8idx}+20e^{-6ic}e^{6idx}+15e^{-8ic}e^{4idx}+6e^{-10ic}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 3003*a**8*(log(exp(I*d*x) - I*exp(-I*c))/16 - log(exp(I*d*x) + I*exp(-I*c))
/16)/d + (-4165*I*a**8*exp(-I*c)*exp(11*I*d*x)/(8*d) - 49301*I*a**8*exp(-3*
I*c)*exp(9*I*d*x)/(24*d) - 69349*I*a**8*exp(-5*I*c)*exp(7*I*d*x)/(20*d) - 6
0699*I*a**8*exp(-7*I*c)*exp(5*I*d*x)/(20*d) - 10873*I*a**8*exp(-9*I*c)*exp(
3*I*d*x)/(8*d) - 1979*I*a**8*exp(-11*I*c)*exp(I*d*x)/(8*d))/(exp(12*I*d*x)
+ 6*exp(-2*I*c)*exp(10*I*d*x) + 15*exp(-4*I*c)*exp(8*I*d*x) + 20*exp(-6*I*c
)*exp(6*I*d*x) + 15*exp(-8*I*c)*exp(4*I*d*x) + 6*exp(-10*I*c)*exp(2*I*d*x)
+ exp(-12*I*c)) + Piecewise((-128*I*a**8*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)),
(128*a**8*x*exp(I*c), True))
```

Giac [B] time = 2.30031, size = 1247, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/61440*(11512215*a^8*e^(12*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 69
073290*a^8*e^(10*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 172683225*a^8
*e^(8*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 230244300*a^8*e^(6*I*d*x
+ 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 172683225*a^8*e^(4*I*d*x + 4*I*c)*log
(I*e^(I*d*x + I*c) + 1) + 69073290*a^8*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x +
 I*c) + 1) - 19305*a^8*e^(12*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1
15830*a^8*e^(10*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) - 289575*a^8*e^(
8*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 386100*a^8*e^(6*I*d*x + 6*I*c
)*log(I*e^(I*d*x + I*c) - 1) - 289575*a^8*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*
x + I*c) - 1) - 115830*a^8*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) -
 11512215*a^8*e^(12*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 69073290*
a^8*e^(10*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 172683225*a^8*e^(8*
I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 230244300*a^8*e^(6*I*d*x + 6*I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 172683225*a^8*e^(4*I*d*x + 4*I*c)*log(-I*
e^(I*d*x + I*c) + 1) - 69073290*a^8*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I
*c) + 1) + 19305*a^8*e^(12*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 11
5830*a^8*e^(10*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 289575*a^8*e^(
8*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 386100*a^8*e^(6*I*d*x + 6*I*
c)*log(-I*e^(I*d*x + I*c) - 1) + 289575*a^8*e^(4*I*d*x + 4*I*c)*log(-I*e^(I
*d*x + I*c) - 1) + 115830*a^8*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) -
1) - 7864320*I*a^8*e^(13*I*d*x + 13*I*c) - 79173120*I*a^8*e^(11*I*d*x + 11*
I*c) - 244175360*I*a^8*e^(9*I*d*x + 9*I*c) - 370326528*I*a^8*e^(7*I*d*x + 7
*I*c) - 304432128*I*a^8*e^(5*I*d*x + 5*I*c) - 130690560*I*a^8*e^(3*I*d*x +
3*I*c) - 23063040*I*a^8*e^(I*d*x + I*c) + 11512215*a^8*log(I*e^(I*d*x + I*c
) + 1) - 19305*a^8*log(I*e^(I*d*x + I*c) - 1) - 11512215*a^8*log(-I*e^(I*d*
x + I*c) + 1) + 19305*a^8*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(12*I*d*x + 12*
I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d
```

```
*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

3.92 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=205

$$\frac{1155ia^8\sec(c+dx)}{8d}+\frac{1155a^8\tanh^{-1}(\sin(c+dx))}{8d}+\frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d}+\frac{33ia^2\sec(c+dx)\left(a^2+\right.}{4d}$$

```
[Out] (1155*a^8*ArcTanh[Sin[c + d*x]])/(8*d) + (((1155*I)/8)*a^8*Sec[c + d*x])/d
+ (((22*I)/3)*a^3*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/3)*a*C
os[c + d*x]^3*(a + I*a*Tan[c + d*x])^7)/d + (((33*I)/4)*a^2*Sec[c + d*x]*(a
^2 + I*a^2*Tan[c + d*x])^3)/d + (((77*I)/4)*Sec[c + d*x]*(a^4 + I*a^4*Tan[c
 + d*x])^2)/d + (((385*I)/8)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```

Rubi [A] time = 0.193307, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3496, 3498, 3486, 3770}

$$\frac{1155ia^8\sec(c+dx)}{8d}+\frac{1155a^8\tanh^{-1}(\sin(c+dx))}{8d}+\frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d}+\frac{33ia^2\sec(c+dx)\left(a^2+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^8,x]
```


```
[Out] (1155*a^8*ArcTanh[Sin[c + d*x]])/(8*d) + (((1155*I)/8)*a^8*Sec[c + d*x])/d
+ (((22*I)/3)*a^3*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/3)*a*C
os[c + d*x]^3*(a + I*a*Tan[c + d*x])^7)/d + (((33*I)/4)*a^2*Sec[c + d*x]*(a
^2 + I*a^2*Tan[c + d*x])^3)/d + (((77*I)/4)*Sec[c + d*x]*(a^4 + I*a^4*Tan[c
 + d*x])^2)/d + (((385*I)/8)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```


Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d} - \frac{1}{3}\left(11a^2\right)\int \cos(c+dx)(a+ia\tan(c+$$
$$= \frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$
$$= \frac{33ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d}$$
$$= \frac{33ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d}$$
$$= \frac{33ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d}$$
$$= \frac{1155ia^8\sec(c+dx)}{8d} + \frac{33ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3\cos(c+d}{}$$
$$= \frac{1155a^8\tanh^{-1}(\sin(c+dx))}{8d} + \frac{1155ia^8\sec(c+dx)}{8d} + \frac{33ia^5\sec(c+dx)(a+ia\,\mathrm{t}}{4d}$$

Mathematica [B] time = 6.94833, size = 1540, normalized size = 7.51

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-1155*Cos[8*c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]
*(a + I*a*Tan[c + d*x])^8)/(8*d*(Cos[d*x] + I*Sin[d*x])^8) + (1155*Cos[8*c]
*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c
 + d*x])^8)/(8*d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[3*d*x]*Cos[c + d*x]^8*((
(-32*I)/3)*Cos[5*c] - (32*Sin[5*c])/3)*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*
x] + I*Sin[d*x])^8) + (Cos[d*x]*Cos[c + d*x]^8*((160*I)*Cos[7*c] + 160*Sin[
7*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (((1155*I)/
8)*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a
+ I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) - (((1155*I)/8)*Cos[c
+ d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a + I*a*Tan
[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*Sec[c]*(((236
*I)/3)*Cos[8*c] + (236*Sin[8*c])/3)*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x]
+ I*Sin[d*x])^8) + (Cos[c + d*x]^8*(-160*Cos[7*c] + (160*I)*Sin[7*c])*Sin[d
*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]
^8*((32*Cos[5*c])/3 - ((32*I)/3)*Sin[5*c])*Sin[3*d*x]*(a + I*a*Tan[c + d*x]
)^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(Cos[8*c]/16 - (I/16)*
Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 +
 (d*x)/2] - Sin[c/2 + (d*x)/2])^4) - (I*Cos[c + d*x]^8*((4*Cos[8*c])/3 - ((
4*I)/3)*Sin[8*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin
[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^
3) + (Cos[c + d*x]^8*((-375 - 32*I)*Cos[c/2] + (375 - 32*I)*Sin[c/2])*(Cos[
8*c]/48 - (I/48)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin[c/2
])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) +
 (I*Cos[c + d*x]^8*((236*Cos[8*c])/3 - ((236*I)/3)*Sin[8*c])*Sin[(d*x)/2]*(
a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8
```

```
*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^8*(-Cos[8*c]/16
 + (I/16)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*
(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) + (I*Cos[c + d*x]^8*((4*Cos[8*
c])/3 - ((4*I)/3)*Sin[8*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[
c/2] + Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2 +
(d*x)/2])^3) + (Cos[c + d*x]^8*((375 - 32*I)*Cos[c/2] + (375 + 32*I)*Sin[c/
2])*(Cos[8*c]/48 - (I/48)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2]
+ Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)
/2])^2) - (I*Cos[c + d*x]^8*((236*Cos[8*c])/3 - ((236*I)/3)*Sin[8*c])*Sin[(
d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] + Sin[c/2])*(Cos[d*x] + I*Si
n[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.076, size = 356, normalized size = 1.7

$$-\frac{3449\,a^8\sin(dx+c)}{24\,d}+\frac{\frac{688\,i}{3}a^8\cos(dx+c)}{d}-14\,\frac{a^8(\sin(dx+c))^7}{d(\cos(dx+c))^2}-\frac{\frac{8\,i}{3}a^8(\cos(dx+c))^3}{d}+\frac{\sin(dx+c)(\cos(dx+c))^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] -3449/24*a^8*sin(d*x+c)/d+688/3*I/d*a^8*cos(d*x+c)-14/d*a^8*sin(d*x+c)^7/co
s(d*x+c)^2-8/3*I/d*a^8*cos(d*x+c)^3+1/3/d*sin(d*x+c)*cos(d*x+c)^2*a^8+1/4/d
*a^8*sin(d*x+c)^9/cos(d*x+c)^4-5/8/d*a^8*sin(d*x+c)^9/cos(d*x+c)^2-1379/24*
a^8*sin(d*x+c)^3/d-5/8*a^8*sin(d*x+c)^7/d-119/8*a^8*sin(d*x+c)^5/d+1155/8/d
*a^8*ln(sec(d*x+c)+tan(d*x+c))-8/3*I/d*a^8*sin(d*x+c)^8/cos(d*x+c)^3+56*I/d
*a^8*sin(d*x+c)^6/cos(d*x+c)+344/3*I/d*a^8*cos(d*x+c)*sin(d*x+c)^2+72*I/d*a
^8*cos(d*x+c)*sin(d*x+c)^4+40/3*I/d*a^8*cos(d*x+c)*sin(d*x+c)^6+40/3*I/d*a^
8*sin(d*x+c)^8/cos(d*x+c)
```

Maxima [B] time = 1.19282, size = 475, normalized size = 2.32

$$-\frac{128i\,a^8\cos(dx+c)^3+448\,a^8\sin(dx+c)^3+896i\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\cos(dx+c)\right)a^8+128i\left(\cos(dx+c)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/48*(128*I*a^8*cos(d*x + c)^3 + 448*a^8*sin(d*x + c)^3 + 896*I*(cos(d*x +
 c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^8 + 128*I*(cos(d*x + c)^3 - (9*c
os(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*a^8 + 896*I*(cos(d*x +
c)^3 - 3*cos(d*x + c))*a^8 + (16*sin(d*x + c)^3 - 6*(13*sin(d*x + c)^3 - 11
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 105*log(sin(d*x +
c) + 1) + 105*log(sin(d*x + c) - 1) + 144*sin(d*x + c))*a^8 + 112*(4*sin(d*
x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) +
 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*a^8 + 560*(2*sin(d*x + c)^3 -
3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^8 + 1
6*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^8)/d
```

Fricas [A] time = 1.86591, size = 819, normalized size = 4.

$$\frac{-256i\,a^8e^{(11i\,dx+11i\,c)}+2816i\,a^8e^{(9i\,dx+9i\,c)}+18414i\,a^8e^{(7i\,dx+7i\,c)}+33726i\,a^8e^{(5i\,dx+5i\,c)}+25410i\,a^8e^{(3i\,dx+3i\,c)}+6930i\,a^8e^{(i\,dx}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/24*(-256*I*a^8*e^(11*I*d*x + 11*I*c) + 2816*I*a^8*e^(9*I*d*x + 9*I*c) + 1
8414*I*a^8*e^(7*I*d*x + 7*I*c) + 33726*I*a^8*e^(5*I*d*x + 5*I*c) + 25410*I*
a^8*e^(3*I*d*x + 3*I*c) + 6930*I*a^8*e^(I*d*x + I*c) + 3465*(a^8*e^(8*I*d*x
 + 8*I*c) + 4*a^8*e^(6*I*d*x + 6*I*c) + 6*a^8*e^(4*I*d*x + 4*I*c) + 4*a^8*e
^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) + I) - 3465*(a^8*e^(8*I*d*x +
 8*I*c) + 4*a^8*e^(6*I*d*x + 6*I*c) + 6*a^8*e^(4*I*d*x + 4*I*c) + 4*a^8*e^(
2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*c) +
4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [A] time = 4.80562, size = 282, normalized size = 1.38

$$\frac{1155a^8\left(-\frac{\log\left(e^{idx}-ie^{-ic}\right)}{8}+\frac{\log\left(e^{idx}+ie^{-ic}\right)}{8}\right)}{d}+\frac{\frac{765ia^8e^{-ic}e^{7idx}}{4d}+\frac{5855ia^8e^{-3ic}e^{5idx}}{12d}+\frac{5153ia^8e^{-5ic}e^{3idx}}{12d}+\frac{515ia^8e^{-7ic}e^{idx}}{4d}}{e^{8idx}+4e^{-2ic}e^{6idx}+6e^{-4ic}e^{4idx}+4e^{-6ic}e^{2idx}+e^{-8ic}}+\begin{cases}\frac{-32ia^8de^{3ic}e^{3idx}+480i}{3d^2}\\ x\left(32a^8e^{3ic}-160\right.\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 1155*a**8*(-log(exp(I*d*x) - I*exp(-I*c))/8 + log(exp(I*d*x) + I*exp(-I*c))
/8)/d + (765*I*a**8*exp(-I*c)*exp(7*I*d*x)/(4*d) + 5855*I*a**8*exp(-3*I*c)*
exp(5*I*d*x)/(12*d) + 5153*I*a**8*exp(-5*I*c)*exp(3*I*d*x)/(12*d) + 515*I*a
**8*exp(-7*I*c)*exp(I*d*x)/(4*d))/(exp(8*I*d*x) + 4*exp(-2*I*c)*exp(6*I*d*x
) + 6*exp(-4*I*c)*exp(4*I*d*x) + 4*exp(-6*I*c)*exp(2*I*d*x) + exp(-8*I*c))
+ Piecewise(((-32*I*a**8*d*exp(3*I*c)*exp(3*I*d*x) + 480*I*a**8*d*exp(I*c)*
exp(I*d*x))/(3*d**2), Ne(3*d**2, 0)), (x*(32*a**8*exp(3*I*c) - 160*a**8*exp
(I*c)), True))
```

Giac [B] time = 2.99755, size = 3827, normalized size = 18.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3440640*(26725545*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1) +
374157630*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2432024595
*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9728098380*a^8*e^(2
2*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 26752270545*a^8*e^(20*I*d*x +
 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 53504541090*a^8*e^(18*I*d*x + 4*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 80256811635*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I
*d*x + I*c) + 1) + 80256811635*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*
```

c) + 1) + 53504541090*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 26752270545*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9728098380*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2432024595*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 374157630*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 91722070440*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 26725545*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 523464480*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7328502720*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 47635267680*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 190541070720*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 523987944480*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1047975888960*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1571963833440*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1571963833440*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1047975888960*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 523987944480*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 190541070720*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 47635267680*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7328502720*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1796530095360*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 523464480*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) - 26725545*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 374157630*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2432024595*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 9728098380*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 26752270545*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 53504541090*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 80256811635*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 80256811635*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 53504541090*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 26752270545*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 9728098380*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2432024595*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 374157630*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 91722070440*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 26725545*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 523464480*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7328502720*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 47635267680*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 190541070720*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 523987944480*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1047975888960*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1571963833440*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1571963833440*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1047975888960*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 523987944480*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 190541070720*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 47635267680*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7328502720*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1796530095360*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 523464480*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3465*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 48510*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 315315*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1261260*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 3468465*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 6936930*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 10405395*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 10405395*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 6936930*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 3468465*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1261260*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 315315*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 48510*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 11891880*a^8*e^(14*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 3465*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 3465*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 48510*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I

```
*d*x) + e^(-I*c)) + 315315*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x) + e^(
-I*c)) + 1261260*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 34
68465*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 6936930*a^8*e
^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 10405395*a^8*e^(16*I*d*x
 + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 10405395*a^8*e^(12*I*d*x - 2*I*c)*
log(-I*e^(I*d*x) + e^(-I*c)) + 6936930*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I
*d*x) + e^(-I*c)) + 3468465*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-
I*c)) + 1261260*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 3153
15*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 48510*a^8*e^(2*I
*d*x - 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 11891880*a^8*e^(14*I*d*x)*log
(-I*e^(I*d*x) + e^(-I*c)) + 3465*a^8*e^(-14*I*c)*log(-I*e^(I*d*x) + e^(-I*c
)) - 36700160*I*a^8*e^(31*I*d*x + 17*I*c) + 36700160*I*a^8*e^(29*I*d*x + 15
*I*c) + 5025341440*I*a^8*e^(27*I*d*x + 13*I*c) + 44995829760*I*a^8*e^(25*I*
d*x + 11*I*c) + 211521945600*I*a^8*e^(23*I*d*x + 9*I*c) + 647303086080*I*a^
8*e^(21*I*d*x + 7*I*c) + 1402445291520*I*a^8*e^(19*I*d*x + 5*I*c) + 2242792
366080*I*a^8*e^(17*I*d*x + 3*I*c) + 2703768453120*I*a^8*e^(15*I*d*x + I*c)
+ 2476532531200*I*a^8*e^(13*I*d*x - I*c) + 1718329303040*I*a^8*e^(11*I*d*x
- 3*I*c) + 890140303360*I*a^8*e^(9*I*d*x - 5*I*c) + 334132592640*I*a^8*e^(7
*I*d*x - 7*I*c) + 85969551360*I*a^8*e^(5*I*d*x - 9*I*c) + 13577625600*I*a^8
*e^(3*I*d*x - 11*I*c) + 993484800*I*a^8*e^(I*d*x - 13*I*c))/(d*e^(28*I*d*x
+ 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d
*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x +
4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d
*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I
*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*
I*d*x) + d*e^(-14*I*c))
```

3.93 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=173

$$-\frac{63ia^8\sec(c+dx)}{2d}-\frac{63a^8\tanh^{-1}(\sin(c+dx))}{2d}+\frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d}-\frac{42ia^2\cos(c+dx)\left(a^2+ia^2\right.}{5d}$$

```
[Out] (-63*a^8*ArcTanh[Sin[c + d*x]])/(2*d) - (((63*I)/2)*a^8*Sec[c + d*x])/d + (
((6*I)/5)*a^3*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/5)*a*Cos
[c + d*x]^5*(a + I*a*Tan[c + d*x])^7)/d - (((42*I)/5)*a^2*Cos[c + d*x]*(a^2
 + I*a^2*Tan[c + d*x])^3)/d - (((21*I)/2)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c +
 d*x]))/d
```

Rubi [A] time = 0.167414, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3496, 3498, 3486, 3770}

$$-\frac{63ia^8\sec(c+dx)}{2d}-\frac{63a^8\tanh^{-1}(\sin(c+dx))}{2d}+\frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d}-\frac{42ia^2\cos(c+dx)\left(a^2+ia^2\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^8,x]
```


```
[Out] (-63*a^8*ArcTanh[Sin[c + d*x]])/(2*d) - (((63*I)/2)*a^8*Sec[c + d*x])/d + (
((6*I)/5)*a^3*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/5)*a*Cos
[c + d*x]^5*(a + I*a*Tan[c + d*x])^7)/d - (((42*I)/5)*a^2*Cos[c + d*x]*(a^2
 + I*a^2*Tan[c + d*x])^3)/d - (((21*I)/2)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c +
 d*x]))/d
```


Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d} - \frac{1}{5}\left(9a^2\right)\int \cos^3(c+dx)(a+ia\tan(c+$$
$$= \frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$
$$= -\frac{42ia^5\cos(c+dx)(a+ia\tan(c+dx))^3}{5d} + \frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx)}{5d}$$
$$= -\frac{42ia^5\cos(c+dx)(a+ia\tan(c+dx))^3}{5d} + \frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx)}{5d}$$
$$= -\frac{63ia^8\sec(c+dx)}{2d} - \frac{42ia^5\cos(c+dx)(a+ia\tan(c+dx))^3}{5d} + \frac{6ia^3\cos^3(c+dx}{}$$
$$= -\frac{63a^8\tanh^{-1}(\sin(c+dx))}{2d} - \frac{63ia^8\sec(c+dx)}{2d} - \frac{42ia^5\cos(c+dx)(a+ia\tan}{5d}$$

Mathematica [B] time = 6.88599, size = 1162, normalized size = 6.72

$$\frac{\cos^8(c+dx)(48\cos(7c)-48i\sin(7c))\sin(dx)(i\tan(c+dx)a+a)^8}{d(\cos(dx)+i\sin(dx))^8} + \frac{\cos^8(c+dx)(8i\sin(5c)-8\cos(5c))\sin(3dx)(i\tan(}{d(\cos(dx)+i\sin(dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (63*Cos[8*c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*(a
 + I*a*Tan[c + d*x])^8)/(2*d*(Cos[d*x] + I*Sin[d*x])^8) - (63*Cos[8*c]*Cos[
c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c + d*
x])^8)/(2*d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[5*d*x]*Cos[c + d*x]^8*(((-8*I
)/5)*Cos[3*c] - (8*Sin[3*c])/5)*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*
Sin[d*x])^8) + (Cos[3*d*x]*Cos[c + d*x]^8*((8*I)*Cos[5*c] + 8*Sin[5*c])*(a
+ I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[d*x]*Cos[c + d*
x]^8*((-48*I)*Cos[7*c] - 48*Sin[7*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x
] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*Sec[c]*((-8*I)*Cos[8*c] - 8*Sin[8*c])*
(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) - (((63*I)/2)*Cos[c
 + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a + I*a*Ta
n[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (((63*I)/2)*Cos[c + d*x]^8*L
og[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a + I*a*Tan[c + d*x])
^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(48*Cos[7*c] - (48*I)*S
in[7*c])*Sin[d*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) +
 (Cos[c + d*x]^8*(-8*Cos[5*c] + (8*I)*Sin[5*c])*Sin[3*d*x]*(a + I*a*Tan[c +
 d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*((8*Cos[3*c])/5 -
 ((8*I)/5)*Sin[3*c])*Sin[5*d*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*
Sin[d*x])^8) + (Cos[c + d*x]^8*(Cos[8*c]/4 - (I/4)*Sin[8*c])*(a + I*a*Tan[c
 + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2])^2) - (I*Cos[c + d*x]^8*(8*Cos[8*c] - (8*I)*Sin[8*c])*Sin[(d*x)/2]*(
a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8
*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^8*(-Cos[8*c]/4
+ (I/4)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*(C
os[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (I*Cos[c + d*x]^8*(8*Cos[8*c]
```

```
- (8*I)*Sin[8*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] + Sin
[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 0.074, size = 322, normalized size = 1.9

$$\frac{a^8 (\sin (dx+c))^9}{2d (\cos (dx+c))^2} + \frac{a^8 (\sin (dx+c))^7}{2d} + \frac{203 a^8 (\sin (dx+c))^5}{10d} + \frac{21 a^8 (\sin (dx+c))^3}{2d} + \frac{283 a^8 \sin (dx+c)}{10d} - 63 a^8 \ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/2/d*a^8*sin(d*x+c)^9/cos(d*x+c)^2+1/2*a^8*sin(d*x+c)^7/d+203/10*a^8*sin(d
*x+c)^5/d+21/2*a^8*sin(d*x+c)^3/d+283/10*a^8*sin(d*x+c)/d-63/2/d*a^8*ln(sec
(d*x+c)+tan(d*x+c))-832/15*I/d*a^8*cos(d*x+c)-8*I/d*a^8*sin(d*x+c)^8/cos(d*
x+c)+56/5*I/d*a^8*cos(d*x+c)^3*sin(d*x+c)^2-104/5*I/d*a^8*cos(d*x+c)*sin(d*
x+c)^4+112/15*I/d*a^8*cos(d*x+c)^3-8*I/d*a^8*cos(d*x+c)*sin(d*x+c)^6-416/15
*I/d*a^8*sin(d*x+c)^2*cos(d*x+c)-8/5*I/d*a^8*cos(d*x+c)^5+29/5/d*a^8*sin(d*
x+c)*cos(d*x+c)^4-8/5/d*sin(d*x+c)*cos(d*x+c)^2*a^8
```

Maxima [B] time = 1.12216, size = 440, normalized size = 2.54

$$-\frac{96i a^8 \cos (dx+c)^5 - 840 a^8 \sin (dx+c)^5 + 224i \left(3 \cos (dx+c)^5 - 5 \cos (dx+c)^3\right) a^8 + 224i \left(3 \cos (dx+c)^5 - 10 \mathrm{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/60*(96*I*a^8*cos(d*x + c)^5 - 840*a^8*sin(d*x + c)^5 + 224*I*(3*cos(d*x
+ c)^5 - 5*cos(d*x + c)^3)*a^8 + 224*I*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^
3 + 15*cos(d*x + c))*a^8 + 96*I*(cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 5/cos(
d*x + c) + 15*cos(d*x + c))*a^8 - (12*sin(d*x + c)^5 + 40*sin(d*x + c)^3 -
30*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 105*log(sin(d*x + c) + 1) + 105*log(
sin(d*x + c) - 1) + 180*sin(d*x + c))*a^8 - 56*(6*sin(d*x + c)^5 + 10*sin(d
*x + c)^3 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*
x + c))*a^8 - 112*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^8 - 4*(3*sin(d*x
+ c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^8)/d
```

Fricas [A] time = 1.85001, size = 537, normalized size = 3.1

$$\frac{-16i a^8 e^{(9i dx+9i c)} + 48i a^8 e^{(7i dx+7i c)} - 336i a^8 e^{(5i dx+5i c)} - 1050i a^8 e^{(3i dx+3i c)} - 630i a^8 e^{(i dx+i c)} - 315 \left(a^8 e^{(4i dx+4i c)} + 2 a^8\right.}{10 \left(de^{(4i dx+4i c)} + 2 de^{(2i dx+2i c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/10*(-16*I*a^8*e^(9*I*d*x + 9*I*c) + 48*I*a^8*e^(7*I*d*x + 7*I*c) - 336*I*
a^8*e^(5*I*d*x + 5*I*c) - 1050*I*a^8*e^(3*I*d*x + 3*I*c) - 630*I*a^8*e^(I*d
```

```
*x + I*c) - 315*(a^8*e^(4*I*d*x + 4*I*c) + 2*a^8*e^(2*I*d*x + 2*I*c) + a^8)
*log(e^(I*d*x + I*c) + I) + 315*(a^8*e^(4*I*d*x + 4*I*c) + 2*a^8*e^(2*I*d*x
 + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(
2*I*d*x + 2*I*c) + d)
```

Sympy [A] time = 1.99023, size = 236, normalized size = 1.36

$$\frac{63a^8\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d}+\frac{-\frac{17ia^8e^{-ic}e^{3idx}}{d}-\frac{15ia^8e^{-3ic}e^{idx}}{d}}{e^{4idx}+2e^{-2ic}e^{2idx}+e^{-4ic}}+\begin{cases}\frac{-8ia^8d^2e^{5ic}e^{5idx}+40ia^8d^2e^{3ic}e^{3idx}-240ia^8d^2e^{ic}e^{idx}}{5d^3} & \text{for } 5d^3\neq 0\\ x\left(8a^8e^{5ic}-24a^8e^{3ic}+48a^8e^{ic}\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 63*a**8*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2)
/d + (-17*I*a**8*exp(-I*c)*exp(3*I*d*x)/d - 15*I*a**8*exp(-3*I*c)*exp(I*d*x
)/d)/(exp(4*I*d*x) + 2*exp(-2*I*c)*exp(2*I*d*x) + exp(-4*I*c)) + Piecewise(
((-8*I*a**8*d**2*exp(5*I*c)*exp(5*I*d*x) + 40*I*a**8*d**2*exp(3*I*c)*exp(3*
I*d*x) - 240*I*a**8*d**2*exp(I*c)*exp(I*d*x))/(5*d**3), Ne(5*d**3, 0)), (x*
(8*a**8*exp(5*I*c) - 24*a**8*exp(3*I*c) + 48*a**8*exp(I*c)), True))
```

Giac [B] time = 3.20439, size = 3846, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/13762560*(882454545*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1)
+ 12354363630*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 803033
63595*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 321213454380*a
^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 883336999545*a^8*e^(20
*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1766673999090*a^8*e^(18*I*d*x
+ 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2650010998635*a^8*e^(16*I*d*x + 2*I*c
)*log(I*e^(I*d*x + I*c) + 1) + 2650010998635*a^8*e^(12*I*d*x - 2*I*c)*log(I
*e^(I*d*x + I*c) + 1) + 1766673999090*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d
*x + I*c) + 1) + 883336999545*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c)
 + 1) + 321213454380*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 8
0303363595*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1235436363
0*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3028583998440*a^8*e
^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 882454545*a^8*e^(-14*I*c)*log(I*e^
(I*d*x + I*c) + 1) + 448908075*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I
*c) - 1) + 6284713050*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1)
+ 40850634825*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 163402
539300*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 449356983075*a
^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 898713966150*a^8*e^(18
*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1348070949225*a^8*e^(16*I*d*x
+ 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1348070949225*a^8*e^(12*I*d*x - 2*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 898713966150*a^8*e^(10*I*d*x - 4*I*c)*log(I*
e^(I*d*x + I*c) - 1) + 449356983075*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x
+ I*c) - 1) + 163402539300*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) -
1) + 40850634825*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 6284
```

713050*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1540652513400*
a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 448908075*a^8*e^(-14*I*c)*log
(I*e^(I*d*x + I*c) - 1) - 882454545*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 12354363630*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*
c) + 1) - 80303363595*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1)
 - 321213454380*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 8833
36999545*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 17666739990
90*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2650010998635*a^8
*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2650010998635*a^8*e^(12
*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1766673999090*a^8*e^(10*I*d*x
 - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 883336999545*a^8*e^(8*I*d*x - 6*I*c
)*log(-I*e^(I*d*x + I*c) + 1) - 321213454380*a^8*e^(6*I*d*x - 8*I*c)*log(-I
*e^(I*d*x + I*c) + 1) - 80303363595*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*
x + I*c) + 1) - 12354363630*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c)
 + 1) - 3028583998440*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 882454
545*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 448908075*a^8*e^(28*I*d*x
 + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6284713050*a^8*e^(26*I*d*x + 12*I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 40850634825*a^8*e^(24*I*d*x + 10*I*c)*log(
-I*e^(I*d*x + I*c) - 1) - 163402539300*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I
*d*x + I*c) - 1) - 449356983075*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 898713966150*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) -
1) - 1348070949225*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1
348070949225*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 8987139
66150*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 449356983075*a
^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 163402539300*a^8*e^(6*
I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 40850634825*a^8*e^(4*I*d*x - 1
0*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6284713050*a^8*e^(2*I*d*x - 12*I*c)*lo
g(-I*e^(I*d*x + I*c) - 1) - 1540652513400*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x
+ I*c) - 1) - 448908075*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 25830
*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 361620*a^8*e^(26*I
*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2350530*a^8*e^(24*I*d*x + 10*I
*c)*log(I*e^(I*d*x) + e^(-I*c)) - 9402120*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^
(I*d*x) + e^(-I*c)) - 25855830*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e
^(-I*c)) - 51711660*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) -
77567490*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 77567490*a^
8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 51711660*a^8*e^(10*I*d
*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 25855830*a^8*e^(8*I*d*x - 6*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) - 9402120*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*
x) + e^(-I*c)) - 2350530*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c
)) - 361620*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 88648560
*a^8*e^(14*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 25830*a^8*e^(-14*I*c)*log(I
*e^(I*d*x) + e^(-I*c)) + 25830*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x) +
 e^(-I*c)) + 361620*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c))
+ 2350530*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 9402120*
a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 25855830*a^8*e^(20*
I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 51711660*a^8*e^(18*I*d*x + 4*
I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 77567490*a^8*e^(16*I*d*x + 2*I*c)*log(-
I*e^(I*d*x) + e^(-I*c)) + 77567490*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x
) + e^(-I*c)) + 51711660*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*
c)) + 25855830*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 94021
20*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2350530*a^8*e^(4*
I*d*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 361620*a^8*e^(2*I*d*x - 12*I
*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 88648560*a^8*e^(14*I*d*x)*log(-I*e^(I*d*
x) + e^(-I*c)) + 25830*a^8*e^(-14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 22020
096*I*a^8*e^(33*I*d*x + 19*I*c) - 198180864*I*a^8*e^(31*I*d*x + 17*I*c) - 1
123024896*I*a^8*e^(29*I*d*x + 15*I*c) - 7478575104*I*a^8*e^(27*I*d*x + 13*I
*c) - 45094404096*I*a^8*e^(25*I*d*x + 11*I*c) - 192251953152*I*a^8*e^(23*I*
d*x + 9*I*c) - 572065579008*I*a^8*e^(21*I*d*x + 7*I*c) - 1228696584192*I*a^
8*e^(19*I*d*x + 5*I*c) - 1959538065408*I*a^8*e^(17*I*d*x + 3*I*c) - 2360323

```
080192*I*a^8*e^(15*I*d*x + I*c) - 2161459593216*I*a^8*e^(13*I*d*x - I*c) -
1499642855424*I*a^8*e^(11*I*d*x - 3*I*c) - 776849719296*I*a^8*e^(9*I*d*x -
5*I*c) - 291606626304*I*a^8*e^(7*I*d*x - 7*I*c) - 75027972096*I*a^8*e^(5*I*
d*x - 9*I*c) - 11849564160*I*a^8*e^(3*I*d*x - 11*I*c) - 867041280*I*a^8*e^(
I*d*x - 13*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91
*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x
+ 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003
*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x -
 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2
*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

3.94 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=152

$$\frac{a^8\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}-\frac{2ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{3d}+\frac{2i\cos(}{}$$

```
[Out] (a^8*ArcTanh[Sin[c + d*x]])/d + (((2*I)/5)*a^3*Cos[c + d*x]^5*(a + I*a*Tan[
c + d*x])^5)/d - (((2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d -
(((2*I)/3)*a^2*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x])^3)/d + ((2*I)*Cos[
c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```

Rubi [A] time = 0.159918, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3496, 3770}

$$\frac{a^8\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}-\frac{2ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{3d}+\frac{2i\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*ArcTanh[Sin[c + d*x]])/d + (((2*I)/5)*a^3*Cos[c + d*x]^5*(a + I*a*Tan[
c + d*x])^5)/d - (((2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d -
(((2*I)/3)*a^2*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x])^3)/d + ((2*I)*Cos[
c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d} - a^2\int \cos^5(c+dx)(a+ia\tan(c+dx))$$
$$= \frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$
$$= -\frac{2ia^5\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} + \frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx)}{5d}$$
$$= -\frac{2ia^5\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} + \frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx)}{5d}$$
$$= \frac{a^8\tanh^{-1}(\sin(c+dx))}{d} - \frac{2ia^5\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} + \frac{2ia^3\cos^5(c}{}$$

Mathematica [B] time = 2.23424, size = 305, normalized size = 2.01

$$\frac{a^8\left(\cos\left(\frac{1}{2}(7c+23dx)\right)+i\sin\left(\frac{1}{2}(7c+23dx)\right)\right)\left(-70\sin\left(\frac{1}{2}(c+dx)\right)-42\sin\left(\frac{3}{2}(c+dx)\right)+210\sin\left(\frac{5}{2}(c+dx)\right)+30\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*((-70*I)*Cos[(c + d*x)/2] + (42*I)*Cos[(3*(c + d*x))/2] + (210*I)*Cos[
(5*(c + d*x))/2] - (30*I)*Cos[(7*(c + d*x))/2] - 105*Cos[(7*(c + d*x))/2]*L
og[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*Cos[(7*(c + d*x))/2]*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] - 70*Sin[(c + d*x)/2] - 42*Sin[(3*(c + d*x
))/2] + 210*Sin[(5*(c + d*x))/2] + 30*Sin[(7*(c + d*x))/2] + (105*I)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] - (105*I)*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2])*(Cos[(7*c + 23*d*x)/2
] + I*Sin[(7*c + 23*d*x)/2]))/(105*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] time = 0.082, size = 385, normalized size = 2.5

$$-\frac{29\,a^8(\sin(dx+c))^7}{7\,d} - \frac{a^8(\sin(dx+c))^5}{5\,d} - \frac{a^8(\sin(dx+c))^3}{3\,d} + \frac{139\,a^8\sin(dx+c)}{105\,d} + \frac{a^8\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] -29/7*a^8*sin(d*x+c)^7/d-1/5*a^8*sin(d*x+c)^5/d-1/3*a^8*sin(d*x+c)^3/d+139/
105*a^8*sin(d*x+c)/d+1/d*a^8*ln(sec(d*x+c)+tan(d*x+c))+48/35*I/d*a^8*cos(d*
x+c)*sin(d*x+c)^4+16/5*I/d*a^8*cos(d*x+c)^5-8*I/d*a^8*sin(d*x+c)^4*cos(d*x+
c)^3-64/15*I/d*a^8*cos(d*x+c)^3-32/5*I/d*a^8*cos(d*x+c)^3*sin(d*x+c)^2+64/3
5*I/d*a^8*cos(d*x+c)*sin(d*x+c)^2+8*I/d*a^8*sin(d*x+c)^2*cos(d*x+c)^5+8/7*I
/d*a^8*cos(d*x+c)*sin(d*x+c)^6-10/d*a^8*sin(d*x+c)^3*cos(d*x+c)^4-232/35/d*
a^8*sin(d*x+c)*cos(d*x+c)^4+122/105/d*sin(d*x+c)*cos(d*x+c)^2*a^8-8/7*I/d*a
^8*cos(d*x+c)^7+29/7/d*a^8*sin(d*x+c)*cos(d*x+c)^6+128/35*I/d*a^8*cos(d*x+c
)
```

Maxima [B] time = 1.03751, size = 417, normalized size = 2.74

$$-\frac{240i\,a^8\cos(dx+c)^7 + 840\,a^8\sin(dx+c)^7 + 112i\left(15\cos(dx+c)^7 - 42\cos(dx+c)^5 + 35\cos(dx+c)^3\right)a^8 + 336i\left(5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/210*(240*I*a^8*cos(d*x + c)^7 + 840*a^8*sin(d*x + c)^7 + 112*I*(15*cos(d
*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^8 + 336*I*(5*cos(d*x +
 c)^7 - 7*cos(d*x + c)^5)*a^8 + 48*I*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5
+ 35*cos(d*x + c)^3 - 35*cos(d*x + c))*a^8 + (30*sin(d*x + c)^7 + 42*sin(d*
x + c)^5 + 70*sin(d*x + c)^3 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x
+ c) - 1) + 210*sin(d*x + c))*a^8 + 56*(15*sin(d*x + c)^7 - 42*sin(d*x + c)
^5 + 35*sin(d*x + c)^3)*a^8 + 420*(5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^8
 + 6*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x
 + c))*a^8)/d
```

Fricas [A] time = 1.91751, size = 271, normalized size = 1.78

$$\frac{-30 i\, a^{8} e^{(7 i\, dx+7 i\, c)} + 42 i\, a^{8} e^{(5 i\, dx+5 i\, c)} - 70 i\, a^{8} e^{(3 i\, dx+3 i\, c)} + 210 i\, a^{8} e^{(i\, dx+i\, c)} + 105\, a^{8} \log\left(e^{(i\, dx+i\, c)} + i\right) - 105\, a^{8} \log\left(e^{(i\, dx+i\, c}\right.}{105\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(-30*I*a^8*e^(7*I*d*x + 7*I*c) + 42*I*a^8*e^(5*I*d*x + 5*I*c) - 70*I*
a^8*e^(3*I*d*x + 3*I*c) + 210*I*a^8*e^(I*d*x + I*c) + 105*a^8*log(e^(I*d*x
+ I*c) + I) - 105*a^8*log(e^(I*d*x + I*c) - I))/d
```

Sympy [A] time = 1.51796, size = 189, normalized size = 1.24

$$\frac{a^{8}\left(-\log\left(e^{idx} - ie^{-ic}\right) + \log\left(e^{idx} + ie^{-ic}\right)\right)}{d} + \begin{cases} \frac{-30ia^{8}d^{3}e^{7ic}e^{7idx} + 42ia^{8}d^{3}e^{5ic}e^{5idx} - 70ia^{8}d^{3}e^{3ic}e^{3idx} + 210ia^{8}d^{3}e^{ic}e^{idx}}{105d^{4}} & \text{for}\: 105d^{4} \neq 0 \\ x\left(2a^{8}e^{7ic} - 2a^{8}e^{5ic} + 2a^{8}e^{3ic} - 2a^{8}e^{ic}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(-log(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d + P
iecewise(((-30*I*a**8*d**3*exp(7*I*c)*exp(7*I*d*x) + 42*I*a**8*d**3*exp(5*I
*c)*exp(5*I*d*x) - 70*I*a**8*d**3*exp(3*I*c)*exp(3*I*d*x) + 210*I*a**8*d**3
*exp(I*c)*exp(I*d*x))/(105*d**4), Ne(105*d**4, 0)), (x*(2*a**8*exp(7*I*c) -
 2*a**8*exp(5*I*c) + 2*a**8*exp(3*I*c) - 2*a**8*exp(I*c)), True))
```

Giac [B] time = 3.46648, size = 3865, normalized size = 25.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/55050240*(1635552135*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 22897729890*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 14883
5244285*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 595340977140
*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1637187687135*a^8*e^
(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3274375374270*a^8*e^(18*I*d
*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4911563061405*a^8*e^(16*I*d*x + 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 4911563061405*a^8*e^(12*I*d*x - 2*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 3274375374270*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(
I*d*x + I*c) + 1) + 1637187687135*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x +
I*c) + 1) + 595340977140*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 148835244285*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 22897
729890*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5613214927320*
a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1635552135*a^8*e^(-14*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 1690450650*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*
d*x + I*c) - 1) + 23666309100*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 153831009150*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 615324036600*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 16921
41100650*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 338428220130
0*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5076423301950*a^8*e
^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5076423301950*a^8*e^(12*I*
d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 3384282201300*a^8*e^(10*I*d*x - 4
*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1692141100650*a^8*e^(8*I*d*x - 6*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) + 615324036600*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*
d*x + I*c) - 1) + 153831009150*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 23666309100*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 5801626630800*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 1690450650*a^8
*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1635552135*a^8*e^(28*I*d*x + 14*I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 22897729890*a^8*e^(26*I*d*x + 12*I*c)*log
(-I*e^(I*d*x + I*c) + 1) - 148835244285*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 595340977140*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 1637187687135*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c)
 + 1) - 3274375374270*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 4911563061405*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 4911
563061405*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3274375374
270*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1637187687135*a^
8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 595340977140*a^8*e^(6*I
*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 148835244285*a^8*e^(4*I*d*x - 1
0*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 22897729890*a^8*e^(2*I*d*x - 12*I*c)*l
og(-I*e^(I*d*x + I*c) + 1) - 5613214927320*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x
 + I*c) + 1) - 1635552135*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 169
0450650*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 23666309100
*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 153831009150*a^8*e
^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 615324036600*a^8*e^(22*I
*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1692141100650*a^8*e^(20*I*d*x +
 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3384282201300*a^8*e^(18*I*d*x + 4*I*c
)*log(-I*e^(I*d*x + I*c) - 1) - 5076423301950*a^8*e^(16*I*d*x + 2*I*c)*log(
-I*e^(I*d*x + I*c) - 1) - 5076423301950*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(
I*d*x + I*c) - 1) - 3384282201300*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x
+ I*c) - 1) - 1692141100650*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c)
- 1) - 615324036600*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1
53831009150*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 23666309
100*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5801626630800*a^
8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 1690450650*a^8*e^(-14*I*c)*log
(-I*e^(I*d*x + I*c) - 1) - 151725*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x)
 + e^(-I*c)) - 2124150*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)
) - 13806975*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 552279
00*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 151876725*a^8*e^(
20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 303753450*a^8*e^(18*I*d*x +
 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 455630175*a^8*e^(16*I*d*x + 2*I*c)*lo
```

```
g(I*e^(I*d*x) + e^(-I*c)) - 455630175*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d
*x) + e^(-I*c)) - 303753450*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-
I*c)) - 151876725*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 552
27900*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 13806975*a^8*e^
(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2124150*a^8*e^(2*I*d*x - 1
2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 520720200*a^8*e^(14*I*d*x)*log(I*e^(I*
d*x) + e^(-I*c)) - 151725*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 151
725*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2124150*a^8*e^
(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 13806975*a^8*e^(24*I*d*x
 + 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 55227900*a^8*e^(22*I*d*x + 8*I*c)
*log(-I*e^(I*d*x) + e^(-I*c)) + 151876725*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e
^(I*d*x) + e^(-I*c)) + 303753450*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x)
+ e^(-I*c)) + 455630175*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c
)) + 455630175*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 3037
53450*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 151876725*a^8
*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 55227900*a^8*e^(6*I*d*x
 - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 13806975*a^8*e^(4*I*d*x - 10*I*c)*
log(-I*e^(I*d*x) + e^(-I*c)) + 2124150*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I
*d*x) + e^(-I*c)) + 520720200*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c))
 + 151725*a^8*e^(-14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 15728640*I*a^8*e^(
35*I*d*x + 21*I*c) - 198180864*I*a^8*e^(33*I*d*x + 19*I*c) - 1159725056*I*a
^8*e^(31*I*d*x + 17*I*c) - 4125097984*I*a^8*e^(29*I*d*x + 15*I*c) - 9527361
536*I*a^8*e^(27*I*d*x + 13*I*c) - 12786335744*I*a^8*e^(25*I*d*x + 11*I*c) +
 190840832*I*a^8*e^(23*I*d*x + 9*I*c) + 48882515968*I*a^8*e^(21*I*d*x + 7*I
*c) + 138550444032*I*a^8*e^(19*I*d*x + 5*I*c) + 239314403328*I*a^8*e^(17*I*
d*x + 3*I*c) + 295994130432*I*a^8*e^(15*I*d*x + I*c) + 273474912256*I*a^8*e
^(13*I*d*x - I*c) + 190268309504*I*a^8*e^(11*I*d*x - 3*I*c) + 98635350016*I
*a^8*e^(9*I*d*x - 5*I*c) + 37029412864*I*a^8*e^(7*I*d*x - 7*I*c) + 95273615
36*I*a^8*e^(5*I*d*x - 9*I*c) + 1504706560*I*a^8*e^(3*I*d*x - 11*I*c) + 1101
00480*I*a^8*e^(I*d*x - 13*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x
 + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001
*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x
+ 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001
*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I
*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

3.95 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=66

$$-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}-\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d}$$

```
[Out] ((-I/63)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d - ((I/9)*Cos[c + d*x]
^9*(a + I*a*Tan[c + d*x])^8)/d
```

Rubi [A] time = 0.073501, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3497, 3488}

$$-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}-\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-I/63)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d - ((I/9)*Cos[c + d*x]
^9*(a + I*a*Tan[c + d*x])^8)/d
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}+\frac{1}{9}a\int \cos^7(c+dx)(a+ia\tan(c+dx))^7 \\ &= -\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d}-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}\end{aligned}$$

Mathematica [A] time = 0.565298, size = 50, normalized size = 0.76

$$\frac{a^8(8\cos(c+dx)-i\sin(c+dx))(\sin(8(c+dx))-i\cos(8(c+dx)))}{63d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(8*Cos[c + d*x] - I*Sin[c + d*x])*((-I)*Cos[8*(c + d*x)] + Sin[8*(c +
d*x)]))/(63*d)
```

Maple [B] time = 0.094, size = 447, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(1/9*a^8*sin(d*x+c)^9-8*I*a^8*(-1/9*sin(d*x+c)^6*cos(d*x+c)^3-2/21*sin(
d*x+c)^4*cos(d*x+c)^3-8/105*cos(d*x+c)^3*sin(d*x+c)^2-16/315*cos(d*x+c)^3)-
28*a^8*(-1/9*sin(d*x+c)^5*cos(d*x+c)^4-5/63*sin(d*x+c)^3*cos(d*x+c)^4-1/21*
sin(d*x+c)*cos(d*x+c)^4+1/63*(2+cos(d*x+c)^2)*sin(d*x+c))+56*I*a^8*(-1/9*si
n(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5)+
70*a^8*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*cos(d*x+c)^6*sin(d*x+c)+1/105*(
8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-56*I*a^8*(-1/9*sin(d*x+c)^2*
cos(d*x+c)^7-2/63*cos(d*x+c)^7)-28*a^8*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(
16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-8/9*I*a^8*
cos(d*x+c)^9+1/9*a^8*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)
^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 1.14898, size = 408, normalized size = 6.18

$$-\frac{280i\,a^{8}\cos\left(dx+c\right)^{9}-35\,a^{8}\sin\left(dx+c\right)^{9}+56i\left(35\,\cos\left(dx+c\right)^{9}-90\,\cos\left(dx+c\right)^{7}+63\,\cos\left(dx+c\right)^{5}\right)a^{8}+8i\left(35\,\mathrm{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/315*(280*I*a^8*cos(d*x + c)^9 - 35*a^8*sin(d*x + c)^9 + 56*I*(35*cos(d*x
 + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*a^8 + 8*I*(35*cos(d*x + c)
^9 - 135*cos(d*x + c)^7 + 189*cos(d*x + c)^5 - 105*cos(d*x + c)^3)*a^8 + 28
0*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^8 - 70*(35*sin(d*x + c)^9 - 90*
sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^8 - 28*(35*sin(d*x + c)^9 - 135*sin(d
*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)^3)*a^8 - (35*sin(d*x + c)
^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin
(d*x + c))*a^8 - 140*(7*sin(d*x + c)^9 - 9*sin(d*x + c)^7)*a^8)/d
```

Fricas [A] time = 1.90622, size = 95, normalized size = 1.44

$$\frac{-7i\,a^{8}e^{\left(9i\,dx+9i\,c\right)}-9i\,a^{8}e^{\left(7i\,dx+7i\,c\right)}}{126\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/126*(-7*I*a^8*e^(9*I*d*x + 9*I*c) - 9*I*a^8*e^(7*I*d*x + 7*I*c))/d
```

Sympy [A] time = 1.21285, size = 82, normalized size = 1.24

$$\begin{cases} \frac{-14ia^8de^{9ic}e^{9idx}-18ia^8de^{7ic}e^{7idx}}{252d^2} & \text{for } 252d^2 \neq 0 \\ x\left(\frac{a^8e^{9ic}}{2}+\frac{a^8e^{7ic}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-14*I*a**8*d*exp(9*I*c)*exp(9*I*d*x) - 18*I*a**8*d*exp(7*I*c)*e
xp(7*I*d*x))/(252*d**2), Ne(252*d**2, 0)), (x*(a**8*exp(9*I*c)/2 + a**8*exp
(7*I*c)/2), True))
```

Giac [B] time = 3.54493, size = 3309, normalized size = 50.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/330301440*(7096716585*a^8*e^(24*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1
) + 85160599020*a^8*e^(22*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4683
83294610*a^8*e^(20*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 156127764870
0*a^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3512874709575*a^8*e
^(16*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5620599535320*a^8*e^(14*I*
d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5620599535320*a^8*e^(10*I*d*x - 2
*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3512874709575*a^8*e^(8*I*d*x - 4*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 1561277648700*a^8*e^(6*I*d*x - 6*I*c)*log(I*e^(I
*d*x + I*c) + 1) + 468383294610*a^8*e^(4*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*
c) + 1) + 85160599020*a^8*e^(2*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 6557366124540*a^8*e^(12*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 7096716585*a^8
*e^(-12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7095485250*a^8*e^(24*I*d*x + 12*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 85145823000*a^8*e^(22*I*d*x + 10*I*c)*log(
I*e^(I*d*x + I*c) - 1) + 468302026500*a^8*e^(20*I*d*x + 8*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 1561006755000*a^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 3512265198750*a^8*e^(16*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 5619624318000*a^8*e^(14*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5619
624318000*a^8*e^(10*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 35122651987
50*a^8*e^(8*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1561006755000*a^8*e
^(6*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 468302026500*a^8*e^(4*I*d*x
 - 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 85145823000*a^8*e^(2*I*d*x - 10*I*c)
*log(I*e^(I*d*x + I*c) - 1) + 6556228371000*a^8*e^(12*I*d*x)*log(I*e^(I*d*x
 + I*c) - 1) + 7095485250*a^8*e^(-12*I*c)*log(I*e^(I*d*x + I*c) - 1) - 7096
716585*a^8*e^(24*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 85160599020*
a^8*e^(22*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 468383294610*a^8*e^
(20*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1561277648700*a^8*e^(18*I*
d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3512874709575*a^8*e^(16*I*d*x +
4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5620599535320*a^8*e^(14*I*d*x + 2*I*c)
*log(-I*e^(I*d*x + I*c) + 1) - 5620599535320*a^8*e^(10*I*d*x - 2*I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 3512874709575*a^8*e^(8*I*d*x - 4*I*c)*log(-I*e^(I*
d*x + I*c) + 1) - 1561277648700*a^8*e^(6*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I
*c) + 1) - 468383294610*a^8*e^(4*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1)
 - 85160599020*a^8*e^(2*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 65573
```

66124540*a^8*e^(12*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 7096716585*a^8*e^(-
12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 7095485250*a^8*e^(24*I*d*x + 12*I*c)*
log(-I*e^(I*d*x + I*c) - 1) - 85145823000*a^8*e^(22*I*d*x + 10*I*c)*log(-I*
e^(I*d*x + I*c) - 1) - 468302026500*a^8*e^(20*I*d*x + 8*I*c)*log(-I*e^(I*d*
x + I*c) - 1) - 1561006755000*a^8*e^(18*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*
c) - 1) - 3512265198750*a^8*e^(16*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1
) - 5619624318000*a^8*e^(14*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 56
19624318000*a^8*e^(10*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 35122651
98750*a^8*e^(8*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1561006755000*a
^8*e^(6*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 468302026500*a^8*e^(4*
I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 85145823000*a^8*e^(2*I*d*x - 1
0*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6556228371000*a^8*e^(12*I*d*x)*log(-I*
e^(I*d*x + I*c) - 1) - 7095485250*a^8*e^(-12*I*c)*log(-I*e^(I*d*x + I*c) -
1) - 1231335*a^8*e^(24*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 147760
20*a^8*e^(22*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 81268110*a^8*e^(
20*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 270893700*a^8*e^(18*I*d*x +
 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 609510825*a^8*e^(16*I*d*x + 4*I*c)*lo
g(I*e^(I*d*x) + e^(-I*c)) - 975217320*a^8*e^(14*I*d*x + 2*I*c)*log(I*e^(I*d
*x) + e^(-I*c)) - 975217320*a^8*e^(10*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-
I*c)) - 609510825*a^8*e^(8*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 270
893700*a^8*e^(6*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 81268110*a^8*e
^(4*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 14776020*a^8*e^(2*I*d*x -
10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1137753540*a^8*e^(12*I*d*x)*log(I*e^(
I*d*x) + e^(-I*c)) - 1231335*a^8*e^(-12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) +
1231335*a^8*e^(24*I*d*x + 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 14776020*a
^8*e^(22*I*d*x + 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 81268110*a^8*e^(20*
I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 270893700*a^8*e^(18*I*d*x + 6
*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 609510825*a^8*e^(16*I*d*x + 4*I*c)*log
(-I*e^(I*d*x) + e^(-I*c)) + 975217320*a^8*e^(14*I*d*x + 2*I*c)*log(-I*e^(I*
d*x) + e^(-I*c)) + 975217320*a^8*e^(10*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^
(-I*c)) + 609510825*a^8*e^(8*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) +
270893700*a^8*e^(6*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 81268110*a
^8*e^(4*I*d*x - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 14776020*a^8*e^(2*I*d
*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1137753540*a^8*e^(12*I*d*x)*log
(-I*e^(I*d*x) + e^(-I*c)) + 1231335*a^8*e^(-12*I*c)*log(-I*e^(I*d*x) + e^(-
I*c)) - 18350080*I*a^8*e^(33*I*d*x + 21*I*c) - 243793920*I*a^8*e^(31*I*d*x
+ 19*I*c) - 1494220800*I*a^8*e^(29*I*d*x + 17*I*c) - 5594152960*I*a^8*e^(27
*I*d*x + 15*I*c) - 14273740800*I*a^8*e^(25*I*d*x + 13*I*c) - 26211778560*I*
a^8*e^(23*I*d*x + 11*I*c) - 35641098240*I*a^8*e^(21*I*d*x + 9*I*c) - 363331
58400*I*a^8*e^(19*I*d*x + 7*I*c) - 27768913920*I*a^8*e^(17*I*d*x + 5*I*c) -
 15715532800*I*a^8*e^(15*I*d*x + 3*I*c) - 6401556480*I*a^8*e^(13*I*d*x + I*
c) - 1777336320*I*a^8*e^(11*I*d*x - I*c) - 301465600*I*a^8*e^(9*I*d*x - 3*I
*c) - 23592960*I*a^8*e^(7*I*d*x - 5*I*c))/(d*e^(24*I*d*x + 12*I*c) + 12*d*e
^(22*I*d*x + 10*I*c) + 66*d*e^(20*I*d*x + 8*I*c) + 220*d*e^(18*I*d*x + 6*I*
c) + 495*d*e^(16*I*d*x + 4*I*c) + 792*d*e^(14*I*d*x + 2*I*c) + 792*d*e^(10*
I*d*x - 2*I*c) + 495*d*e^(8*I*d*x - 4*I*c) + 220*d*e^(6*I*d*x - 6*I*c) + 66
*d*e^(4*I*d*x - 8*I*c) + 12*d*e^(2*I*d*x - 10*I*c) + 924*d*e^(12*I*d*x) + d
*e^(-12*I*c))

3.96 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=136

$$-\frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d}-\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{1155d}-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}-$$

```
[Out] (((-2*I)/1155)*a^3*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/231
)*a^2*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^6)/d - ((I/33)*a*Cos[c + d*x]^9
*(a + I*a*Tan[c + d*x])^7)/d - ((I/11)*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x
])^8)/d
```

Rubi [A] time = 0.155986, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3497, 3488}

$$-\frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d}-\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{1155d}-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-2*I)/1155)*a^3*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/231
)*a^2*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^6)/d - ((I/33)*a*Cos[c + d*x]^9
*(a + I*a*Tan[c + d*x])^7)/d - ((I/11)*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x
])^8)/d
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx &= -\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}+\frac{1}{11}(3a)\int \cos^9(c+dx)(a+ia\tan(c+\\ &= -\frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d}-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}+\\ &= -\frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d}-\frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d}\\ &= -\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{1155d}-\frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx\end{aligned}$$

Mathematica [A] time = 1.27438, size = 73, normalized size = 0.54

$$\frac{a^8(-i(55\sin(c+dx)+63\sin(3(c+dx)))+440\cos(c+dx)+168\cos(3(c+dx)))(\sin(8(c+dx))-i\cos(8(c+dx)))}{4620d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(440*Cos[c + d*x] + 168*Cos[3*(c + d*x)] - I*(55*Sin[c + d*x] + 63*Sin
[3*(c + d*x)]))*((-I)*Cos[8*(c + d*x)] + Sin[8*(c + d*x)]))/(4620*d)
```

Maple [B] time = 0.094, size = 567, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/11*sin(d*x+c)^7*cos(d*x+c)^4-7/99*sin(d*x+c)^5*cos(d*x+c)^4-5/
99*sin(d*x+c)^3*cos(d*x+c)^4-1/33*sin(d*x+c)*cos(d*x+c)^4+1/99*(2+cos(d*x+c
)^2)*sin(d*x+c))-8*I*a^8*(-1/11*sin(d*x+c)^6*cos(d*x+c)^5-2/33*sin(d*x+c)^4
*cos(d*x+c)^5-8/231*sin(d*x+c)^2*cos(d*x+c)^5-16/1155*cos(d*x+c)^5)-28*a^8*
(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(d*x+c)^6-5/231*cos(d
*x+c)^6*sin(d*x+c)+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+56
*I*a^8*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/69
3*cos(d*x+c)^7)+70*a^8*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*sin(d*x+c)*cos
(d*x+c)^8+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d
*x+c))-56*I*a^8*(-1/11*sin(d*x+c)^2*cos(d*x+c)^9-2/99*cos(d*x+c)^9)-28*a^8*
(-1/11*sin(d*x+c)*cos(d*x+c)^10+1/99*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+
48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-8/11*I*a^8*cos(d*x+c)^11
+1/11*a^8*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c)^6+32/21*
cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 1.17487, size = 479, normalized size = 3.52

$$-\frac{2520i\,a^8\cos(dx+c)^{11}+24i\left(105\cos(dx+c)^{11}-385\cos(dx+c)^9+495\cos(dx+c)^7-231\cos(dx+c)^5\right)a^8+28}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/3465*(2520*I*a^8*cos(d*x + c)^11 + 24*I*(105*cos(d*x + c)^11 - 385*cos(d
*x + c)^9 + 495*cos(d*x + c)^7 - 231*cos(d*x + c)^5)*a^8 + 280*I*(63*cos(d*
x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)^7)*a^8 + 1960*I*(9*cos(d*x
 + c)^11 - 11*cos(d*x + c)^9)*a^8 + 28*(315*sin(d*x + c)^11 - 1540*sin(d*x
+ c)^9 + 2970*sin(d*x + c)^7 - 2772*sin(d*x + c)^5 + 1155*sin(d*x + c)^3)*a
^8 + 210*(105*sin(d*x + c)^11 - 385*sin(d*x + c)^9 + 495*sin(d*x + c)^7 - 2
31*sin(d*x + c)^5)*a^8 + 140*(63*sin(d*x + c)^11 - 154*sin(d*x + c)^9 + 99*
sin(d*x + c)^7)*a^8 + 5*(63*sin(d*x + c)^11 - 385*sin(d*x + c)^9 + 990*sin(
d*x + c)^7 - 1386*sin(d*x + c)^5 + 1155*sin(d*x + c)^3 - 693*sin(d*x + c))*
a^8 + 35*(9*sin(d*x + c)^11 - 11*sin(d*x + c)^9)*a^8)/d
```

Fricas [A] time = 2.09096, size = 190, normalized size = 1.4

$$\frac{-105i\,a^8e^{(11i\,dx+11i\,c)} - 385i\,a^8e^{(9i\,dx+9i\,c)} - 495i\,a^8e^{(7i\,dx+7i\,c)} - 231i\,a^8e^{(5i\,dx+5i\,c)}}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9240*(-105*I*a^8*e^(11*I*d*x + 11*I*c) - 385*I*a^8*e^(9*I*d*x + 9*I*c) -
495*I*a^8*e^(7*I*d*x + 7*I*c) - 231*I*a^8*e^(5*I*d*x + 5*I*c))/d
```

Sympy [A] time = 1.57391, size = 163, normalized size = 1.2

$$\begin{cases} \frac{-53760ia^8d^3e^{11ic}e^{11idx}-197120ia^8d^3e^{9ic}e^{9idx}-253440ia^8d^3e^{7ic}e^{7idx}-118272ia^8d^3e^{5ic}e^{5idx}}{4730880d^4} & \text{for } 4730880d^4 \neq 0 \\ x\left(\frac{a^8e^{11ic}}{8}+\frac{3a^8e^{9ic}}{8}+\frac{3a^8e^{7ic}}{8}+\frac{a^8e^{5ic}}{8}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-53760*I*a**8*d**3*exp(11*I*c)*exp(11*I*d*x) - 197120*I*a**8*d*
*3*exp(9*I*c)*exp(9*I*d*x) - 253440*I*a**8*d**3*exp(7*I*c)*exp(7*I*d*x) - 1
18272*I*a**8*d**3*exp(5*I*c)*exp(5*I*d*x))/(4730880*d**4), Ne(4730880*d**4,
 0)), (x*(a**8*exp(11*I*c)/8 + 3*a**8*exp(9*I*c)/8 + 3*a**8*exp(7*I*c)/8 +
a**8*exp(5*I*c)/8), True))
```

Giac [B] time = 3.81121, size = 3865, normalized size = 28.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/4844421120*(82027951005*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) +
 1) + 1148391314070*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) +
7464543541455*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 298581
74165820*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 821099789560
05*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 164219957912010*a^
8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 246329936868015*a^8*e^(
16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 246329936868015*a^8*e^(12*I*
d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 164219957912010*a^8*e^(10*I*d*x -
 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82109978956005*a^8*e^(8*I*d*x - 6*I*c)
*log(I*e^(I*d*x + I*c) + 1) + 29858174165820*a^8*e^(6*I*d*x - 8*I*c)*log(I*
e^(I*d*x + I*c) + 1) + 7464543541455*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*
x + I*c) + 1) + 1148391314070*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c
) + 1) + 281519927849160*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 8202
7951005*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82004266575*a^8*e^(28*
I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1148059732050*a^8*e^(26*I*d*x
+ 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7462388258325*a^8*e^(24*I*d*x + 10*I
```

```
*c)*log(I*e^(I*d*x + I*c) - 1) + 29849553033300*a^8*e^(22*I*d*x + 8*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) + 82086270841575*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^
(I*d*x + I*c) - 1) + 164172541683150*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 246258812524725*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I
*c) - 1) + 246258812524725*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 164172541683150*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 82086270841575*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 298495
53033300*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7462388258325
*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1148059732050*a^8*e^
(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 281438642885400*a^8*e^(14*I
*d*x)*log(I*e^(I*d*x + I*c) - 1) + 82004266575*a^8*e^(-14*I*c)*log(I*e^(I*d
*x + I*c) - 1) - 82027951005*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*
c) + 1) - 1148391314070*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) +
1) - 7464543541455*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) -
29858174165820*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 82109
978956005*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1642199579
12010*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 24632993686801
5*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 246329936868015*a^
8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 164219957912010*a^8*e^
(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 82109978956005*a^8*e^(8*I*
d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 29858174165820*a^8*e^(6*I*d*x -
8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 7464543541455*a^8*e^(4*I*d*x - 10*I*c)
*log(-I*e^(I*d*x + I*c) + 1) - 1148391314070*a^8*e^(2*I*d*x - 12*I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 281519927849160*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x +
 I*c) + 1) - 82027951005*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 8200
4266575*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11480597320
50*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7462388258325*a^
8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 29849553033300*a^8*e^
(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 82086270841575*a^8*e^(20*I
*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 164172541683150*a^8*e^(18*I*d*x
 + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 246258812524725*a^8*e^(16*I*d*x + 2
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 246258812524725*a^8*e^(12*I*d*x - 2*I*c
)*log(-I*e^(I*d*x + I*c) - 1) - 164172541683150*a^8*e^(10*I*d*x - 4*I*c)*lo
g(-I*e^(I*d*x + I*c) - 1) - 82086270841575*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 29849553033300*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*
x + I*c) - 1) - 7462388258325*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*
c) - 1) - 1148059732050*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1
) - 281438642885400*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 82004266
575*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 23684430*a^8*e^(28*I*d*x
+ 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 331582020*a^8*e^(26*I*d*x + 12*I*c)
*log(I*e^(I*d*x) + e^(-I*c)) - 2155283130*a^8*e^(24*I*d*x + 10*I*c)*log(I*e
^(I*d*x) + e^(-I*c)) - 8621132520*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x)
+ e^(-I*c)) - 23708114430*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*
c)) - 47416228860*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 71
124343290*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 7112434329
0*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 47416228860*a^8*e^
(10*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23708114430*a^8*e^(8*I*d*x
 - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 8621132520*a^8*e^(6*I*d*x - 8*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) - 2155283130*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(
I*d*x) + e^(-I*c)) - 331582020*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e
^(-I*c)) - 81284963760*a^8*e^(14*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 23684
430*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 23684430*a^8*e^(28*I*d*x
+ 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 331582020*a^8*e^(26*I*d*x + 12*I*c
)*log(-I*e^(I*d*x) + e^(-I*c)) + 2155283130*a^8*e^(24*I*d*x + 10*I*c)*log(-
I*e^(I*d*x) + e^(-I*c)) + 8621132520*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d
*x) + e^(-I*c)) + 23708114430*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e
^(-I*c)) + 47416228860*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)
) + 71124343290*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 711
24343290*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 4741622886
```

0*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 23708114430*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 8621132520*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2155283130*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 331582020*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 81284963760*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 23684430*a^8*e^(-14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 55050240*I*a^8*e^(39*I*d*x + 25*I*c) - 972554240*I*a^8*e^(37*I*d*x + 23*I*c) - 8095006720*I*a^8*e^(35*I*d*x + 21*I*c) - 42161143808*I*a^8*e^(33*I*d*x + 19*I*c) - 153891110912*I*a^8*e^(31*I*d*x + 17*I*c) - 417750581248*I*a^8*e^(29*I*d*x + 15*I*c) - 873287647232*I*a^8*e^(27*I*d*x + 13*I*c) - 1435886419968*I*a^8*e^(25*I*d*x + 11*I*c) - 1879877615616*I*a^8*e^(23*I*d*x + 9*I*c) - 1970745114624*I*a^8*e^(21*I*d*x + 7*I*c) - 1654208331776*I*a^8*e^(19*I*d*x + 5*I*c) - 1105350098944*I*a^8*e^(17*I*d*x + 3*I*c) - 580728651776*I*a^8*e^(15*I*d*x + I*c) - 234836983808*I*a^8*e^(13*I*d*x - I*c) - 70581747712*I*a^8*e^(11*I*d*x - 3*I*c) - 14856224768*I*a^8*e^(9*I*d*x - 5*I*c) - 1955069952*I*a^8*e^(7*I*d*x - 7*I*c) - 121110528*I*a^8*e^(5*I*d*x - 9*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))

3.97 $\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=211

$$-\frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d}-\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d}-\frac{8i\cos^5(c+dx)\left(a^2+ia^2\tan(c+\right.}{3003d}$$

```
[Out] (((-20*I)/3003)*a^3*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5)/d - (((20*I)/1
287)*a^2*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^6)/d - (((5*I)/143)*a*Cos[c
+ d*x]^11*(a + I*a*Tan[c + d*x])^7)/d - ((I/13)*Cos[c + d*x]^13*(a + I*a*Ta
n[c + d*x])^8)/d - (((8*I)/9009)*a^2*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*
x])^3)/d - (((8*I)/3003)*Cos[c + d*x]^5*(a^2 + I*a^2*Tan[c + d*x])^4)/d
```

Rubi [A] time = 0.253767, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3497, 3488}

$$-\frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d}-\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d}-\frac{8i\cos^5(c+dx)\left(a^2+ia^2\tan(c+\right.}{3003d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8,x]
```


```
[Out] (((-20*I)/3003)*a^3*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5)/d - (((20*I)/1
287)*a^2*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^6)/d - (((5*I)/143)*a*Cos[c
+ d*x]^11*(a + I*a*Tan[c + d*x])^7)/d - ((I/13)*Cos[c + d*x]^13*(a + I*a*Ta
n[c + d*x])^8)/d - (((8*I)/9009)*a^2*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*
x])^3)/d - (((8*I)/3003)*Cos[c + d*x]^5*(a^2 + I*a^2*Tan[c + d*x])^4)/d
```


Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d} + \frac{1}{13}(5a)\int \cos^{11}(c+dx)(a+ia\tan(c+$$
$$= -\frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d} - \frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}$$
$$= -\frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d} - \frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+d}{143d}$$
$$= -\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d} - \frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+}{1287d}$$
$$= -\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d} - \frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+}{1287d}$$
$$= -\frac{8ia^5\cos^3(c+dx)(a+ia\tan(c+dx))^3}{9009d} - \frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+d}{3003d}$$

Mathematica [A] time = 1.27266, size = 111, normalized size = 0.53

$$\frac{a^8(-1430i\sin(c+dx) - 2457i\sin(3(c+dx)) - 1155i\sin(5(c+dx)) + 11440\cos(c+dx) + 6552\cos(3(c+dx)) + 1848\,\mathrm{co}}{144144d(\cos(dx)+i\sin(dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(11440*Cos[c + d*x] + 6552*Cos[3*(c + d*x)] + 1848*Cos[5*(c + d*x)] -
(1430*I)*Sin[c + d*x] - (2457*I)*Sin[3*(c + d*x)] - (1155*I)*Sin[5*(c + d*x
)])*((-I)*Cos[8*(c + 2*d*x)] + Sin[8*(c + 2*d*x)]))/(144144*d*(Cos[d*x] + I
*Sin[d*x])^8)
```

Maple [B] time = 0.125, size = 617, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/13*sin(d*x+c)^7*cos(d*x+c)^6-7/143*sin(d*x+c)^5*cos(d*x+c)^6-3
5/1287*sin(d*x+c)^3*cos(d*x+c)^6-5/429*cos(d*x+c)^6*sin(d*x+c)+1/429*(8/3+c
os(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/13*sin(d*x+c)^6*cos(d
*x+c)^7-6/143*sin(d*x+c)^4*cos(d*x+c)^7-8/429*sin(d*x+c)^2*cos(d*x+c)^7-16/
3003*cos(d*x+c)^7)-28*a^8*(-1/13*sin(d*x+c)^5*cos(d*x+c)^8-5/143*sin(d*x+c)
^3*cos(d*x+c)^8-5/429*sin(d*x+c)*cos(d*x+c)^8+5/3003*(16/5+cos(d*x+c)^6+6/5
*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+56*I*a^8*(-1/13*sin(d*x+c)^4*co
s(d*x+c)^9-4/143*sin(d*x+c)^2*cos(d*x+c)^9-8/1287*cos(d*x+c)^9)+70*a^8*(-1/
13*sin(d*x+c)^3*cos(d*x+c)^10-3/143*sin(d*x+c)*cos(d*x+c)^10+1/429*(128/35+
cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*
x+c))-56*I*a^8*(-1/13*sin(d*x+c)^2*cos(d*x+c)^11-2/143*cos(d*x+c)^11)-28*a^
8*(-1/13*sin(d*x+c)*cos(d*x+c)^12+1/143*(256/63+cos(d*x+c)^10+10/9*cos(d*x+
c)^8+80/63*cos(d*x+c)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
-8/13*I*a^8*cos(d*x+c)^13+1/13*a^8*(1024/231+cos(d*x+c)^12+12/11*cos(d*x+c)
^10+40/33*cos(d*x+c)^8+320/231*cos(d*x+c)^6+128/77*cos(d*x+c)^4+512/231*cos
(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 1.17464, size = 547, normalized size = 2.59

$$-\frac{5544i\,a^8\cos(dx+c)^{13}+24i\left(231\,\cos(dx+c)^{13}-819\,\cos(dx+c)^{11}+1001\,\cos(dx+c)^9-429\,\cos(dx+c)^7\right)a^8+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/9009*(5544*I*a^8*cos(d*x + c)^13 + 24*I*(231*cos(d*x + c)^13 - 819*cos(d
*x + c)^11 + 1001*cos(d*x + c)^9 - 429*cos(d*x + c)^7)*a^8 + 392*I*(99*cos(
d*x + c)^13 - 234*cos(d*x + c)^11 + 143*cos(d*x + c)^9)*a^8 + 3528*I*(11*co
s(d*x + c)^13 - 13*cos(d*x + c)^11)*a^8 - 42*(1155*sin(d*x + c)^13 - 5460*s
in(d*x + c)^11 + 10010*sin(d*x + c)^9 - 8580*sin(d*x + c)^7 + 3003*sin(d*x
+ c)^5)*a^8 - 28*(693*sin(d*x + c)^13 - 4095*sin(d*x + c)^11 + 10010*sin(d*
x + c)^9 - 12870*sin(d*x + c)^7 + 9009*sin(d*x + c)^5 - 3003*sin(d*x + c)^3
)*a^8 - 84*(231*sin(d*x + c)^13 - 819*sin(d*x + c)^11 + 1001*sin(d*x + c)^9
 - 429*sin(d*x + c)^7)*a^8 - 3*(231*sin(d*x + c)^13 - 1638*sin(d*x + c)^11
+ 5005*sin(d*x + c)^9 - 8580*sin(d*x + c)^7 + 9009*sin(d*x + c)^5 - 6006*si
n(d*x + c)^3 + 3003*sin(d*x + c))*a^8 - 7*(99*sin(d*x + c)^13 - 234*sin(d*x
 + c)^11 + 143*sin(d*x + c)^9)*a^8)/d
```

Fricas [A] time = 2.24412, size = 292, normalized size = 1.38

$$\frac{-693i\,a^8e^{(13i\,dx+13i\,c)}-4095i\,a^8e^{(11i\,dx+11i\,c)}-10010i\,a^8e^{(9i\,dx+9i\,c)}-12870i\,a^8e^{(7i\,dx+7i\,c)}-9009i\,a^8e^{(5i\,dx+5i\,c)}-3003i\,a^8e}{288288\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/288288*(-693*I*a^8*e^(13*I*d*x + 13*I*c) - 4095*I*a^8*e^(11*I*d*x + 11*I*
c) - 10010*I*a^8*e^(9*I*d*x + 9*I*c) - 12870*I*a^8*e^(7*I*d*x + 7*I*c) - 90
09*I*a^8*e^(5*I*d*x + 5*I*c) - 3003*I*a^8*e^(3*I*d*x + 3*I*c))/d
```

Sympy [A] time = 2.03843, size = 241, normalized size = 1.14

$$\begin{cases}\frac{-17439916032ia^8d^5e^{13ic}e^{13idx}-103054049280ia^8d^5e^{11ic}e^{11idx}-251909898240ia^8d^5e^{9ic}e^{9idx}-323884154880ia^8d^5e^{7ic}e^{7idx}-226718908416ia^8d^5e^{5ic}e^{5idx}-7557}{7255005069312d^6} \\ x\left(\frac{a^8e^{13ic}}{32}+\frac{5a^8e^{11ic}}{32}+\frac{5a^8e^{9ic}}{16}+\frac{5a^8e^{7ic}}{16}+\frac{5a^8e^{5ic}}{32}+\frac{a^8e^{3ic}}{32}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**13*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-17439916032*I*a**8*d**5*exp(13*I*c)*exp(13*I*d*x) - 1030540492
80*I*a**8*d**5*exp(11*I*c)*exp(11*I*d*x) - 251909898240*I*a**8*d**5*exp(9*I
*c)*exp(9*I*d*x) - 323884154880*I*a**8*d**5*exp(7*I*c)*exp(7*I*d*x) - 22671
8908416*I*a**8*d**5*exp(5*I*c)*exp(5*I*d*x) - 75572969472*I*a**8*d**5*exp(3
*I*c)*exp(3*I*d*x))/(7255005069312*d**6), Ne(7255005069312*d**6, 0)), (x*(a
**8*exp(13*I*c)/32 + 5*a**8*exp(11*I*c)/32 + 5*a**8*exp(9*I*c)/16 + 5*a**8*
exp(7*I*c)/16 + 5*a**8*exp(5*I*c)/32 + a**8*exp(3*I*c)/32), True))
```

Giac [B] time = 4.3551, size = 3903, normalized size = 18.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/755729694720*(9725263833285*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*
c) + 1) + 136153693665990*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) +
 1) + 884999008828935*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1)
+ 3539996035315740*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 97
34989097118285*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 194699
78194236570*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 292049672
91354855*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 292049672913
54855*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 194699781942365
70*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9734989097118285*a
^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3539996035315740*a^8*e^
(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 884999008828935*a^8*e^(4*I*d
*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 136153693665990*a^8*e^(2*I*d*x -
12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 33377105475834120*a^8*e^(14*I*d*x)*log
(I*e^(I*d*x + I*c) + 1) + 9725263833285*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*
c) + 1) + 9720402036345*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1
) + 136085628508830*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) +
884556585307395*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 3538
226341229580*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 97301224
38381345*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 194602448767
62690*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 291903673151440
35*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 29190367315144035*
a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 19460244876762690*a^8
*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 9730122438381345*a^8*e^(
8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 3538226341229580*a^8*e^(6*I*d
*x - 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 884556585307395*a^8*e^(4*I*d*x - 1
0*I*c)*log(I*e^(I*d*x + I*c) - 1) + 136085628508830*a^8*e^(2*I*d*x - 12*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 33360419788736040*a^8*e^(14*I*d*x)*log(I*e^(
I*d*x + I*c) - 1) + 9720402036345*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1
) - 9725263833285*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1
36153693665990*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 8849
99008828935*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3539996
035315740*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 9734989097
118285*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1946997819423
6570*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 292049672913548
55*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 29204967291354855
*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 19469978194236570*a
^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 9734989097118285*a^8*
e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3539996035315740*a^8*e^(6
*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 884999008828935*a^8*e^(4*I*d*
x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 136153693665990*a^8*e^(2*I*d*x -
12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 33377105475834120*a^8*e^(14*I*d*x)*lo
g(-I*e^(I*d*x + I*c) + 1) - 9725263833285*a^8*e^(-14*I*c)*log(-I*e^(I*d*x +
 I*c) + 1) - 9720402036345*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c)
 - 1) - 136085628508830*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) -
1) - 884556585307395*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 3538226341229580*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9
730122438381345*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1946
0244876762690*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 291903
67315144035*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 29190367
315144035*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1946024487
```

```
6762690*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 973012243838
1345*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3538226341229580
*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 884556585307395*a^8*
e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 136085628508830*a^8*e^(2
*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 33360419788736040*a^8*e^(14*
I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 9720402036345*a^8*e^(-14*I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 4861796940*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x)
+ e^(-I*c)) - 68065157160*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I
*c)) - 442423521540*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) -
 1769694086160*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 48666
58736940*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 97333174738
80*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 14599976210820*a^
8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 14599976210820*a^8*e^(
12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 9733317473880*a^8*e^(10*I*d
*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 4866658736940*a^8*e^(8*I*d*x - 6*
I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1769694086160*a^8*e^(6*I*d*x - 8*I*c)*lo
g(I*e^(I*d*x) + e^(-I*c)) - 442423521540*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(
I*d*x) + e^(-I*c)) - 68065157160*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) +
 e^(-I*c)) - 16685687098080*a^8*e^(14*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) -
4861796940*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 4861796940*a^8*e^(
28*I*d*x + 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 68065157160*a^8*e^(26*I*d
*x + 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 442423521540*a^8*e^(24*I*d*x +
10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1769694086160*a^8*e^(22*I*d*x + 8*I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) + 4866658736940*a^8*e^(20*I*d*x + 6*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) + 9733317473880*a^8*e^(18*I*d*x + 4*I*c)*log(-I*
e^(I*d*x) + e^(-I*c)) + 14599976210820*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I
*d*x) + e^(-I*c)) + 14599976210820*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x
) + e^(-I*c)) + 9733317473880*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e
^(-I*c)) + 4866658736940*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c
)) + 1769694086160*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 4
42423521540*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 6806515
7160*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 16685687098080
*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 4861796940*a^8*e^(-14*I*c)
*log(-I*e^(I*d*x) + e^(-I*c)) - 1816657920*I*a^8*e^(41*I*d*x + 27*I*c) - 36
168007680*I*a^8*e^(39*I*d*x + 25*I*c) - 341843640320*I*a^8*e^(37*I*d*x + 23
*I*c) - 2039236526080*I*a^8*e^(35*I*d*x + 21*I*c) - 8609784135680*I*a^8*e^(
33*I*d*x + 19*I*c) - 27342720204800*I*a^8*e^(31*I*d*x + 17*I*c) - 677532663
80800*I*a^8*e^(29*I*d*x + 15*I*c) - 134089539584000*I*a^8*e^(27*I*d*x + 13*
I*c) - 215146797465600*I*a^8*e^(25*I*d*x + 11*I*c) - 282406740295680*I*a^8*
e^(23*I*d*x + 9*I*c) - 304579309731840*I*a^8*e^(21*I*d*x + 7*I*c) - 2699476
96578560*I*a^8*e^(19*I*d*x + 5*I*c) - 195820823511040*I*a^8*e^(17*I*d*x + 3
*I*c) - 115246062632960*I*a^8*e^(15*I*d*x + I*c) - 54220889784320*I*a^8*e^(
13*I*d*x - I*c) - 19924737064960*I*a^8*e^(11*I*d*x - 3*I*c) - 5513153085440
*I*a^8*e^(9*I*d*x - 5*I*c) - 1080738447360*I*a^8*e^(7*I*d*x - 7*I*c) - 1338
27133440*I*a^8*e^(5*I*d*x - 9*I*c) - 7872184320*I*a^8*e^(3*I*d*x - 11*I*c))
/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x +
 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*
d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x -
 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*
e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c)
 + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

3.98 $\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal. Leaf size=212

$$-\frac{a^8\sin^7(c+dx)}{1287d}+\frac{7a^8\sin^5(c+dx)}{2145d}-\frac{7a^8\sin^3(c+dx)}{1287d}+\frac{7a^8\sin(c+dx)}{1287d}-\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d}-\frac{2i}{}$$

```
[Out] (7*a^8*Sin[c + d*x])/(1287*d) - (7*a^8*Sin[c + d*x]^3)/(1287*d) + (7*a^8*Si
n[c + d*x]^5)/(2145*d) - (a^8*Sin[c + d*x]^7)/(1287*d) - (((2*I)/195)*a^3*C
os[c + d*x]^13*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/15)*a*Cos[c + d*x]^15*
(a + I*a*Tan[c + d*x])^7)/d - (((2*I)/715)*a^2*Cos[c + d*x]^11*(a^2 + I*a^2
*Tan[c + d*x])^3)/d - (((2*I)/1287)*Cos[c + d*x]^9*(a^8 + I*a^8*Tan[c + d*x
]))/d
```

Rubi [A] time = 0.19742, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3496, 2633}

$$-\frac{a^8\sin^7(c+dx)}{1287d}+\frac{7a^8\sin^5(c+dx)}{2145d}-\frac{7a^8\sin^3(c+dx)}{1287d}+\frac{7a^8\sin(c+dx)}{1287d}-\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d}-\frac{2i}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (7*a^8*Sin[c + d*x])/(1287*d) - (7*a^8*Sin[c + d*x]^3)/(1287*d) + (7*a^8*Si
n[c + d*x]^5)/(2145*d) - (a^8*Sin[c + d*x]^7)/(1287*d) - (((2*I)/195)*a^3*C
os[c + d*x]^13*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/15)*a*Cos[c + d*x]^15*
(a + I*a*Tan[c + d*x])^7)/d - (((2*I)/715)*a^2*Cos[c + d*x]^11*(a^2 + I*a^2
*Tan[c + d*x])^3)/d - (((2*I)/1287)*Cos[c + d*x]^9*(a^8 + I*a^8*Tan[c + d*x
]))/d
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d} + \frac{1}{15}a^2\int \cos^{13}(c+dx)(a+ia\tan$$
$$= -\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d} - \frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+$$
$$= -\frac{2ia^5\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{715d} - \frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c$$
$$= -\frac{2ia^5\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{715d} - \frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c$$
$$= -\frac{2ia^5\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{715d} - \frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c$$
$$= \frac{7a^8\sin(c+dx)}{1287d} - \frac{7a^8\sin^3(c+dx)}{1287d} + \frac{7a^8\sin^5(c+dx)}{2145d} - \frac{a^8\sin^7(c+dx)}{1287d} -$$

Mathematica [A] time = 3.10333, size = 133, normalized size = 0.63

$$\frac{a^8(-3575i\sin(c+dx) - 7371i\sin(3(c+dx)) - 5775i\sin(5(c+dx)) - 3003i\sin(7(c+dx)) + 28600\cos(c+dx) + 196}{411840d(\cos(dx) + i\sin}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(28600*Cos[c + d*x] + 19656*Cos[3*(c + d*x)] + 9240*Cos[5*(c + d*x)] +
 3432*Cos[7*(c + d*x)] - (3575*I)*Sin[c + d*x] - (7371*I)*Sin[3*(c + d*x)]
- (5775*I)*Sin[5*(c + d*x)] - (3003*I)*Sin[7*(c + d*x)])*((-I)*Cos[8*(c + 2
*d*x)] + Sin[8*(c + 2*d*x)]))/(411840*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] time = 0.131, size = 667, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/15*sin(d*x+c)^7*cos(d*x+c)^8-7/195*sin(d*x+c)^5*cos(d*x+c)^8-7
/429*sin(d*x+c)^3*cos(d*x+c)^8-7/1287*sin(d*x+c)*cos(d*x+c)^8+1/1287*(16/5+
cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/15*
sin(d*x+c)^6*cos(d*x+c)^9-2/65*sin(d*x+c)^4*cos(d*x+c)^9-8/715*sin(d*x+c)^2
*cos(d*x+c)^9-16/6435*cos(d*x+c)^9)-28*a^8*(-1/15*sin(d*x+c)^5*cos(d*x+c)^1
0-1/39*sin(d*x+c)^3*cos(d*x+c)^10-1/143*sin(d*x+c)*cos(d*x+c)^10+1/1287*(12
8/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*s
in(d*x+c))+56*I*a^8*(-1/15*sin(d*x+c)^4*cos(d*x+c)^11-4/195*sin(d*x+c)^2*co
s(d*x+c)^11-8/2145*cos(d*x+c)^11)+70*a^8*(-1/15*sin(d*x+c)^3*cos(d*x+c)^12-
1/65*sin(d*x+c)*cos(d*x+c)^12+1/715*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8
+80/63*cos(d*x+c)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))-56*
I*a^8*(-1/15*sin(d*x+c)^2*cos(d*x+c)^13-2/195*cos(d*x+c)^13)-28*a^8*(-1/15*
sin(d*x+c)*cos(d*x+c)^14+1/195*(1024/231+cos(d*x+c)^12+12/11*cos(d*x+c)^10+
40/33*cos(d*x+c)^8+320/231*cos(d*x+c)^6+128/77*cos(d*x+c)^4+512/231*cos(d*x
+c)^2)*sin(d*x+c))-8/15*I*a^8*cos(d*x+c)^15+1/15*a^8*(2048/429+cos(d*x+c)^1
4+14/13*cos(d*x+c)^12+168/143*cos(d*x+c)^10+560/429*cos(d*x+c)^8+640/429*co
s(d*x+c)^6+256/143*cos(d*x+c)^4+1024/429*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [B] time = 1.17846, size = 612, normalized size = 2.89

$$-\frac{3432i\,a^8\cos(dx+c)^{15}+8i\left(429\cos(dx+c)^{15}-1485\cos(dx+c)^{13}+1755\cos(dx+c)^{11}-715\cos(dx+c)^{9}\right)a^8+16\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/6435*(3432*I*a^8*cos(d*x + c)^15 + 8*I*(429*cos(d*x + c)^15 - 1485*cos(d
*x + c)^13 + 1755*cos(d*x + c)^11 - 715*cos(d*x + c)^9)*a^8 + 168*I*(143*co
s(d*x + c)^15 - 330*cos(d*x + c)^13 + 195*cos(d*x + c)^11)*a^8 + 1848*I*(13
*cos(d*x + c)^15 - 15*cos(d*x + c)^13)*a^8 + 4*(3003*sin(d*x + c)^15 - 1386
0*sin(d*x + c)^13 + 24570*sin(d*x + c)^11 - 20020*sin(d*x + c)^9 + 6435*sin
(d*x + c)^7)*a^8 + 10*(3003*sin(d*x + c)^15 - 17325*sin(d*x + c)^13 + 40950
*sin(d*x + c)^11 - 50050*sin(d*x + c)^9 + 32175*sin(d*x + c)^7 - 9009*sin(d
*x + c)^5)*a^8 + 4*(3003*sin(d*x + c)^15 - 20790*sin(d*x + c)^13 + 61425*si
n(d*x + c)^11 - 100100*sin(d*x + c)^9 + 96525*sin(d*x + c)^7 - 54054*sin(d*
x + c)^5 + 15015*sin(d*x + c)^3)*a^8 + (429*sin(d*x + c)^15 - 1485*sin(d*x
+ c)^13 + 1755*sin(d*x + c)^11 - 715*sin(d*x + c)^9)*a^8 + (429*sin(d*x + c
)^15 - 3465*sin(d*x + c)^13 + 12285*sin(d*x + c)^11 - 25025*sin(d*x + c)^9
+ 32175*sin(d*x + c)^7 - 27027*sin(d*x + c)^5 + 15015*sin(d*x + c)^3 - 6435
*sin(d*x + c))*a^8)/d
```

Fricas [A] time = 2.36828, size = 382, normalized size = 1.8

$$\frac{-429i\,a^8e^{(15i\,dx+15i\,c)}-3465i\,a^8e^{(13i\,dx+13i\,c)}-12285i\,a^8e^{(11i\,dx+11i\,c)}-25025i\,a^8e^{(9i\,dx+9i\,c)}-32175i\,a^8e^{(7i\,dx+7i\,c)}-27027i\,a^8\cdots}{823680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/823680*(-429*I*a^8*e^(15*I*d*x + 15*I*c) - 3465*I*a^8*e^(13*I*d*x + 13*I*
c) - 12285*I*a^8*e^(11*I*d*x + 11*I*c) - 25025*I*a^8*e^(9*I*d*x + 9*I*c) -
32175*I*a^8*e^(7*I*d*x + 7*I*c) - 27027*I*a^8*e^(5*I*d*x + 5*I*c) - 15015*I
*a^8*e^(3*I*d*x + 3*I*c) - 6435*I*a^8*e^(I*d*x + I*c))/d
```

Sympy [A] time = 2.45823, size = 314, normalized size = 1.48

$$\begin{cases}\frac{-1086774885079842816 0ia^8d^7e^{15ic}e^{15idx}-8777797148721807360 0ia^8d^7e^{13ic}e^{13idx}-311212808000136806400ia^8d^7e^{11ic}e^{11idx}-633952016296574976000ia^8d^7e^{9i\cdots}}{2086607779\cdots} \\ x\left(\frac{a^8e^{15ic}}{128}+\frac{7a^8e^{13ic}}{128}+\frac{21a^8e^{11ic}}{128}+\frac{35a^8e^{9ic}}{128}+\frac{35a^8e^{7ic}}{128}+\frac{21a^8e^{5ic}}{128}+\frac{7a^8e^{3ic}}{128}+\frac{a^8e^{ic}}{128}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**15*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-10867748850798428160*I*a**8*d**7*exp(15*I*c)*exp(15*I*d*x) - 8
7777971487218073600*I*a**8*d**7*exp(13*I*c)*exp(13*I*d*x) - 311212808000136
806400*I*a**8*d**7*exp(11*I*c)*exp(11*I*d*x) - 633952016296574976000*I*a**8
*d**7*exp(9*I*c)*exp(9*I*d*x) - 815081163809882112000*I*a**8*d**7*exp(7*I*c
```

```
)*exp(7*I*d*x) - 684668177600300974080*I*a**8*d**7*exp(5*I*c)*exp(5*I*d*x)
- 380371209777944985600*I*a**8*d**7*exp(3*I*c)*exp(3*I*d*x) - 163016232761976422400*I*a**8*d**7*exp(I*c)*exp(I*d*x))/(20866077793532982067200*d**8), Ne(20866077793532982067200*d**8, 0)), (x*(a**8*exp(15*I*c)/128 + 7*a**8*exp(13*I*c)/128 + 21*a**8*exp(11*I*c)/128 + 35*a**8*exp(9*I*c)/128 + 35*a**8*exp(7*I*c)/128 + 21*a**8*exp(5*I*c)/128 + 7*a**8*exp(3*I*c)/128 + a**8*exp(I*c)/128), True))
```

Giac [B] time = 4.30708, size = 3941, normalized size = 18.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/863691079680*(5682101344920*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 79549418828880*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 517071222387720*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2068284889550880*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5687783446264920*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11375566892529840*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 17063350338794760*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 17063350338794760*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11375566892529840*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5687783446264920*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2068284889550880*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 517071222387720*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 79549418828880*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 19500971815765440*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 5682101344920*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5674116082635*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 79437625156890*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 516344563519785*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2065378254079140*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5679790198717635*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 11359580397435270*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 17039370596152905*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 17039370596152905*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 11359580397435270*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5679790198717635*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2065378254079140*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 516344563519785*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 79437625156890*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 19473566395603320*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 5674116082635*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) - 5682101344920*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 79549418828880*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 517071222387720*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2068284889550880*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5687783446264920*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 11375566892529840*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 17063350338794760*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 17063350338794760*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 11375566892529840*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5687783446264920*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2068284889550880*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 517071222387720*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 79549418828880*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 19500971815765440*a^8*e^(14*I*d*x)*log(-I*e
```

^(I*d*x + I*c) + 1) - 5682101344920*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 5674116082635*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 79437625156890*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 51
6344563519785*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 20653
78254079140*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 56797901
98717635*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11359580397
435270*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1703937059615
2905*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 170393705961529
05*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11359580397435270
*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5679790198717635*a^
8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2065378254079140*a^8*e^
(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 516344563519785*a^8*e^(4*I*
d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 79437625156890*a^8*e^(2*I*d*x -
12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 19473566395603320*a^8*e^(14*I*d*x)*l
og(-I*e^(I*d*x + I*c) - 1) - 5674116082635*a^8*e^(-14*I*c)*log(-I*e^(I*d*x
+ I*c) - 1) - 7985262285*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*
c)) - 111793671990*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) -
726658867935*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 290663
5471740*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 799324754728
5*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 15986495094570*a^8
*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23979742641855*a^8*e^(1
6*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23979742641855*a^8*e^(12*I*d
*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 15986495094570*a^8*e^(10*I*d*x -
4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 7993247547285*a^8*e^(8*I*d*x - 6*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) - 2906635471740*a^8*e^(6*I*d*x - 8*I*c)*log(I*e
^(I*d*x) + e^(-I*c)) - 726658867935*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x
) + e^(-I*c)) - 111793671990*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e^(
-I*c)) - 27405420162120*a^8*e^(14*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 7985
262285*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 7985262285*a^8*e^(28*I
*d*x + 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 111793671990*a^8*e^(26*I*d*x
+ 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 726658867935*a^8*e^(24*I*d*x + 10*
I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2906635471740*a^8*e^(22*I*d*x + 8*I*c)*
log(-I*e^(I*d*x) + e^(-I*c)) + 7993247547285*a^8*e^(20*I*d*x + 6*I*c)*log(-
I*e^(I*d*x) + e^(-I*c)) + 15986495094570*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) + 23979742641855*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d
*x) + e^(-I*c)) + 23979742641855*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x)
+ e^(-I*c)) + 15986495094570*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^
(-I*c)) + 7993247547285*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)
) + 2906635471740*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 72
6658867935*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 11179367
1990*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 27405420162120
*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 7985262285*a^8*e^(-14*I*c)
*log(-I*e^(I*d*x) + e^(-I*c)) - 449839104*I*a^8*e^(43*I*d*x + 29*I*c) - 993
1063296*I*a^8*e^(41*I*d*x + 27*I*c) - 104683536384*I*a^8*e^(39*I*d*x + 25*I
*c) - 700958375936*I*a^8*e^(37*I*d*x + 23*I*c) - 3346162253824*I*a^8*e^(35*
I*d*x + 21*I*c) - 12115053117440*I*a^8*e^(33*I*d*x + 19*I*c) - 345536410419
20*I*a^8*e^(31*I*d*x + 17*I*c) - 79597529989120*I*a^8*e^(29*I*d*x + 15*I*c)
- 150652615393280*I*a^8*e^(27*I*d*x + 13*I*c) - 237078702981120*I*a^8*e^(2
5*I*d*x + 11*I*c) - 312733543170048*I*a^8*e^(23*I*d*x + 9*I*c) - 3475582872
45312*I*a^8*e^(21*I*d*x + 7*I*c) - 326158241497088*I*a^8*e^(19*I*d*x + 5*I*
c) - 258238371069952*I*a^8*e^(17*I*d*x + 3*I*c) - 171721273376768*I*a^8*e^(
15*I*d*x + I*c) - 95003913224192*I*a^8*e^(13*I*d*x - I*c) - 43034893877248*
I*a^8*e^(11*I*d*x - 3*I*c) - 15562783588352*I*a^8*e^(9*I*d*x - 5*I*c) - 431
9355076608*I*a^8*e^(7*I*d*x - 7*I*c) - 862791401472*I*a^8*e^(5*I*d*x - 9*I*
c) - 110210580480*I*a^8*e^(3*I*d*x - 11*I*c) - 6747586560*I*a^8*e^(I*d*x -
13*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24
*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c)
+ 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12
*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c)

```
+ 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x -
 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

3.99 $\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=107

$$-\frac{i(a-ia\tan(c+dx))^8}{8a^9d}+\frac{6i(a-ia\tan(c+dx))^7}{7a^8d}-\frac{2i(a-ia\tan(c+dx))^6}{a^7d}+\frac{8i(a-ia\tan(c+dx))^5}{5a^6d}$$

```
[Out] (((8*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^6*d) - ((2*I)*(a - I*a*Tan[c + d*x]
)^6)/(a^7*d) + (((6*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^8*d) - ((I/8)*(a - I
*a*Tan[c + d*x])^8)/(a^9*d)
```

Rubi [A] time = 0.0695333, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{i(a-ia\tan(c+dx))^8}{8a^9d}+\frac{6i(a-ia\tan(c+dx))^7}{7a^8d}-\frac{2i(a-ia\tan(c+dx))^6}{a^7d}+\frac{8i(a-ia\tan(c+dx))^5}{5a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((8*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^6*d) - ((2*I)*(a - I*a*Tan[c + d*x]
)^6)/(a^7*d) + (((6*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^8*d) - ((I/8)*(a - I
*a*Tan[c + d*x])^8)/(a^9*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^4(a+x)^3\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(8a^3(a-x)^4-12a^2(a-x)^5+6a(a-x)^6-(a-x)^7\right)dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= \frac{8i(a-ia\tan(c+dx))^5}{5a^6d}-\frac{2i(a-ia\tan(c+dx))^6}{a^7d}+\frac{6i(a-ia\tan(c+dx))^7}{7a^8d}-\frac{i(a-ia\tan(c+dx))^8}{8a^9d}\end{aligned}$$

Mathematica [A] time = 0.330652, size = 71, normalized size = 0.66

$$\frac{\sec(c)\sec^8(c+dx)(56\sin(c+2dx)+28\sin(3c+4dx)+8\sin(5c+6dx)+\sin(7c+8dx)-35\sin(c)-35i\cos(c))}{280ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^8*((-35*I)*Cos[c] - 35*Sin[c] + 56*Sin[c + 2*d*x] + 28
*Sin[3*c + 4*d*x] + 8*Sin[5*c + 6*d*x] + Sin[7*c + 8*d*x]))/(280*a*d)
```

Maple [A] time = 0.063, size = 87, normalized size = 0.8

$$\frac{1}{ad}\left(\tan(dx+c) - \frac{i}{8}(\tan(dx+c))^8 + \frac{(\tan(dx+c))^7}{7} - \frac{i}{2}(\tan(dx+c))^6 + \frac{3(\tan(dx+c))^5}{5} - \frac{3i}{4}(\tan(dx+c))^4 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/8*I*tan(d*x+c)^8+1/7*tan(d*x+c)^7-1/2*I*tan(d*x+c)^6+3/
5*tan(d*x+c)^5-3/4*I*tan(d*x+c)^4+tan(d*x+c)^3-1/2*I*tan(d*x+c)^2)
```

Maxima [A] time = 1.0558, size = 117, normalized size = 1.09

$$\frac{-105i\tan(dx+c)^8 + 120\tan(dx+c)^7 - 420i\tan(dx+c)^6 + 504\tan(dx+c)^5 - 630i\tan(dx+c)^4 + 840\tan(dx}{840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/840*(-105*I*tan(d*x + c)^8 + 120*tan(d*x + c)^7 - 420*I*tan(d*x + c)^6 +
504*tan(d*x + c)^5 - 630*I*tan(d*x + c)^4 + 840*tan(d*x + c)^3 - 420*I*tan(
d*x + c)^2 + 840*tan(d*x + c))/(a*d)
```

Fricas [A] time = 1.69649, size = 458, normalized size = 4.28

$$\frac{1792i\,e^{(6i\,dx+6i\,c)} + 896i\,e^{(4i\,dx+4i\,c)} + 256i\,e^{(2i\,dx+2i\,c)} + 32i}{35\left(ade^{(16i\,dx+16i\,c)} + 8\,ade^{(14i\,dx+14i\,c)} + 28\,ade^{(12i\,dx+12i\,c)} + 56\,ade^{(10i\,dx+10i\,c)} + 70\,ade^{(8i\,dx+8i\,c)} + 56\,ade^{(6i\,dx+6i\,c)} + 28\,ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/35*(1792*I*e^(6*I*d*x + 6*I*c) + 896*I*e^(4*I*d*x + 4*I*c) + 256*I*e^(2*I
*d*x + 2*I*c) + 32*I)/(a*d*e^(16*I*d*x + 16*I*c) + 8*a*d*e^(14*I*d*x + 14*I
*c) + 28*a*d*e^(12*I*d*x + 12*I*c) + 56*a*d*e^(10*I*d*x + 10*I*c) + 70*a*d*
e^(8*I*d*x + 8*I*c) + 56*a*d*e^(6*I*d*x + 6*I*c) + 28*a*d*e^(4*I*d*x + 4*I*
c) + 8*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.16582, size = 117, normalized size = 1.09

$$-\frac{35i\,\tan\left(dx+c\right)^{8}-40\,\tan\left(dx+c\right)^{7}+140i\,\tan\left(dx+c\right)^{6}-168\,\tan\left(dx+c\right)^{5}+210i\,\tan\left(dx+c\right)^{4}-280\,\tan\left(dx+c\right)^{3}}{280\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/280*(35*I*tan(d*x + c)^8 - 40*tan(d*x + c)^7 + 140*I*tan(d*x + c)^6 - 16
8*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 - 280*tan(d*x + c)^3 + 140*I*tan(d*
x + c)^2 - 280*tan(d*x + c))/(a*d)
```

3.100 $\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{i(a-ia\tan(c+dx))^6}{6a^7d}-\frac{4i(a-ia\tan(c+dx))^5}{5a^6d}+\frac{i(a-ia\tan(c+dx))^4}{a^5d}$$

```
[Out] (I*(a - I*a*Tan[c + d*x])^4)/(a^5*d) - (((4*I)/5)*(a - I*a*Tan[c + d*x])^5)
/(a^6*d) + ((I/6)*(a - I*a*Tan[c + d*x])^6)/(a^7*d)
```

Rubi [A] time = 0.0620516, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a-ia\tan(c+dx))^6}{6a^7d}-\frac{4i(a-ia\tan(c+dx))^5}{5a^6d}+\frac{i(a-ia\tan(c+dx))^4}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I*(a - I*a*Tan[c + d*x])^4)/(a^5*d) - (((4*I)/5)*(a - I*a*Tan[c + d*x])^5)
/(a^6*d) + ((I/6)*(a - I*a*Tan[c + d*x])^6)/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^3(a+x)^2\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a-x)^3-4a(a-x)^4+(a-x)^5\right)dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= \frac{i(a-ia\tan(c+dx))^4}{a^5d}-\frac{4i(a-ia\tan(c+dx))^5}{5a^6d}+\frac{i(a-ia\tan(c+dx))^6}{6a^7d}\end{aligned}$$

Mathematica [A] time = 0.258387, size = 60, normalized size = 0.75

$$\frac{\sec(c)\sec^6(c+dx)(15\sin(c+2dx)+6\sin(3c+4dx)+\sin(5c+6dx)-10\sin(c)-10i\cos(c))}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^6*((-10*I)*Cos[c] - 10*Sin[c] + 15*Sin[c + 2*d*x] + 6*
Sin[3*c + 4*d*x] + Sin[5*c + 6*d*x]))/(60*a*d)
```

Maple [A] time = 0.061, size = 68, normalized size = 0.9

$$\frac{1}{ad}\left(\tan(dx+c)-\frac{i}{6}(\tan(dx+c))^{6}+\frac{(\tan(dx+c))^{5}}{5}-\frac{i}{2}(\tan(dx+c))^{4}+\frac{2(\tan(dx+c))^{3}}{3}-\frac{i}{2}(\tan(dx+c))^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/6*I*tan(d*x+c)^6+1/5*tan(d*x+c)^5-1/2*I*tan(d*x+c)^4+2/
3*tan(d*x+c)^3-1/2*I*tan(d*x+c)^2)
```

Maxima [A] time = 1.13411, size = 90, normalized size = 1.12

$$-\frac{10i\tan(dx+c)^{6}-12\tan(dx+c)^{5}+30i\tan(dx+c)^{4}-40\tan(dx+c)^{3}+30i\tan(dx+c)^{2}-60\tan(dx+c)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(10*I*tan(d*x + c)^6 - 12*tan(d*x + c)^5 + 30*I*tan(d*x + c)^4 - 40*t
an(d*x + c)^3 + 30*I*tan(d*x + c)^2 - 60*tan(d*x + c))/(a*d)
```

Fricas [A] time = 1.86957, size = 333, normalized size = 4.16

$$\frac{240i\,e^{(4i\,dx+4i\,c)}+96i\,e^{(2i\,dx+2i\,c)}+16i}{15\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(240*I*e^(4*I*d*x + 4*I*c) + 96*I*e^(2*I*d*x + 2*I*c) + 16*I)/(a*d*e^(
12*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c) + 15*a*d*e^(8*I*d*x + 8*I*
c) + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*I*d*x + 4*I*c) + 6*a*d*e^(2*I
*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.15693, size = 90, normalized size = 1.12

$$-\frac{5i\tan(dx+c)^6-6\tan(dx+c)^5+15i\tan(dx+c)^4-20\tan(dx+c)^3+15i\tan(dx+c)^2-30\tan(dx+c)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(5*I*tan(d*x + c)^6 - 6*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 - 20*tan
(d*x + c)^3 + 15*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a*d)
```

3.101 $\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{2i(a-ia\tan(c+dx))^3}{3a^4d}-\frac{i(a-ia\tan(c+dx))^4}{4a^5d}$$

```
[Out] (((2*I)/3)*(a - I*a*Tan[c + d*x])^3)/(a^4*d) - ((I/4)*(a - I*a*Tan[c + d*x]
)^4)/(a^5*d)
```

Rubi [A] time = 0.0511579, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{2i(a-ia\tan(c+dx))^3}{3a^4d}-\frac{i(a-ia\tan(c+dx))^4}{4a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((2*I)/3)*(a - I*a*Tan[c + d*x])^3)/(a^4*d) - ((I/4)*(a - I*a*Tan[c + d*x]
)^4)/(a^5*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^2(a+x)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(2a(a-x)^2-(a-x)^3\right)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= \frac{2i(a-ia\tan(c+dx))^3}{3a^4d}-\frac{i(a-ia\tan(c+dx))^4}{4a^5d}\end{aligned}$$

Mathematica [A] time = 0.166073, size = 49, normalized size = 0.89

$$\frac{\sec(c)\sec^4(c+dx)(4\sin(c+2dx)+\sin(3c+4dx)-3\sin(c)-3i\cos(c))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^4*((-3*I)*Cos[c] - 3*Sin[c] + 4*Sin[c + 2*d*x] + Sin[3
*c + 4*d*x]))/(12*a*d)
```

Maple [A] time = 0.06, size = 47, normalized size = 0.9

$$\frac{1}{ad}\left(\tan(dx+c)-\frac{i}{4}(\tan(dx+c))^4+\frac{(\tan(dx+c))^3}{3}-\frac{i}{2}(\tan(dx+c))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/4*I*tan(d*x+c)^4+1/3*tan(d*x+c)^3-1/2*I*tan(d*x+c)^2)
```

Maxima [A] time = 1.39724, size = 63, normalized size = 1.15

$$\frac{-3i\tan(dx+c)^4+4\tan(dx+c)^3-6i\tan(dx+c)^2+12\tan(dx+c)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(-3*I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 - 6*I*tan(d*x + c)^2 + 12*tan(
d*x + c))/(a*d)
```

Fricas [A] time = 1.95761, size = 208, normalized size = 3.78

$$\frac{16i\,e^{(2i\,dx+2i\,c)}+4i}{3\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(16*I*e^(2*I*d*x + 2*I*c) + 4*I)/(a*d*e^(8*I*d*x + 8*I*c) + 4*a*d*e^(6*
I*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(2*I*d*x + 2*I*c) + a*
d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.19848, size = 63, normalized size = 1.15

$$-\frac{3i\tan\left(dx+c\right)^{4}-4\tan\left(dx+c\right)^{3}+6i\tan\left(dx+c\right)^{2}-12\tan\left(dx+c\right)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*I*tan(d*x + c)^4 - 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 12*tan(
d*x + c))/(a*d)
```

3.102 $\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\tan(c+dx)}{ad}-\frac{i\tan^2(c+dx)}{2ad}$$

```
[Out] Tan[c + d*x]/(a*d) - ((I/2)*Tan[c + d*x]^2)/(a*d)
```

Rubi [A] time = 0.0431182, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3487}

$$\frac{\tan(c+dx)}{ad}-\frac{i\tan^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] Tan[c + d*x]/(a*d) - ((I/2)*Tan[c + d*x]^2)/(a*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= \frac{\tan(c+dx)}{ad}-\frac{i\tan^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.163767, size = 35, normalized size = 1.03

$$\frac{\sec(c+dx)(2\sec(c)\sin(dx)-i\sec(c+dx))}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*((-I)*Sec[c + d*x] + 2*Sec[c]*Sin[d*x]))/(2*a*d)
```

Maple [A] time = 0.054, size = 26, normalized size = 0.8

$$\frac{\tan(dx+c)-\frac{i}{2}(\tan(dx+c))^2}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/2*I*tan(d*x+c)^2)
```

Maxima [A] time = 1.13882, size = 36, normalized size = 1.06

$$-\frac{i\tan\left(dx+c\right)^2-2\tan\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

Fricas [A] time = 1.78664, size = 88, normalized size = 2.59

$$\frac{2i}{ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2*I/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.15863, size = 36, normalized size = 1.06

$$-\frac{i\tan\left(dx+c\right)^2-2\tan\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

3.103 $\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

```
[Out] x/a + (I*Log[Cos[c + d*x]])/(a*d)
```

Rubi [A] time = 0.0408845, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 31}

$$\frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] x/a + (I*Log[Cos[c + d*x]])/(a*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,ia\tan(c+dx)\right)}{ad}\\ &= \frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}\end{aligned}$$

Mathematica [A] time = 0.0893654, size = 31, normalized size = 1.35

$$\frac{2\tan^{-1}(\tan(dx))+i\log\left(\cos^2(c+dx)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*ArcTan[Tan[d*x]] + I*Log[Cos[c + d*x]^2])/(2*a*d)
```

Maple [A] time = 0.03, size = 23, normalized size = 1.

$$\frac{-i\ln\left(a+ia\tan\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I/a/d*ln(a+I*a*tan(d*x+c))
```

Maxima [A] time = 1.16437, size = 27, normalized size = 1.17

$$-\frac{i\log\left(i\,a\tan\left(dx+c\right)+a\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -I*log(I*a*tan(d*x + c) + a)/(a*d)
```

Fricas [A] time = 2.21461, size = 65, normalized size = 2.83

$$\frac{2\,dx+i\,\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*d*x + I*log(e^(2*I*d*x + 2*I*c) + 1))/(a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.18413, size = 80, normalized size = 3.48

$$-\frac{\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a}-\frac{i\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{i\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*I*log(tan(1/2*d*x + 1/2*c) - I)/a - I*log(abs(tan(1/2*d*x + 1/2*c) + 1)
)/a - I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a)/d
```

3.104 $\int \frac{1}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

```
[Out] x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0120401, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3479, 8}

$$\frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-1),x]
```

```
[Out] x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+ia\tan(c+dx)}\,dx &= \frac{i}{2d(a+ia\tan(c+dx))}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.102145, size = 45, normalized size = 1.36

$$\frac{(2dx-i)\tan(c+dx)-2idx+1}{4ad(\tan(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-1),x]
```

```
[Out] (1 - (2*I)*d*x + (-I + 2*d*x)*Tan[c + d*x])/(4*a*d*(-I + Tan[c + d*x]))
```

Maple [B] time = 0.026, size = 59, normalized size = 1.8

$$\frac{-\frac{i}{4}\ln\left(\tan\left(dx+c\right)-i\right)}{ad}+\frac{1}{2\,ad\left(\tan\left(dx+c\right)-i\right)}+\frac{\frac{i}{4}\ln\left(\tan\left(dx+c\right)+i\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -1/4*I/a/d*ln(tan(d*x+c)-I)+1/2/a/d/(tan(d*x+c)-I)+1/4*I/a/d*ln(tan(d*x+c)+
I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.91847, size = 86, normalized size = 2.61

$$\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*x*e^(2*I*d*x + 2*I*c) + I)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A] time = 0.215058, size = 61, normalized size = 1.85

$$\begin{cases}\frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for } 4ade^{2ic}\neq 0\\ x\left(\frac{\left(e^{2ic}+1\right)e^{-2ic}}{2a}-\frac{1}{2a}\right) & \text{otherwise}\end{cases}+\frac{x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(4*a*d*exp(2*I*c), 0)), (
x*((exp(2*I*c) + 1)*exp(-2*I*c)/(2*a) - 1/(2*a)), True)) + x/(2*a)
```

Giac [B] time = 1.13752, size = 81, normalized size = 2.45

$$-\frac{\frac{i\log(\tan(dx+c)-i)}{a}-\frac{i\log(-i\tan(dx+c)+1)}{a}+\frac{-i\tan(dx+c)-3}{a(\tan(dx+c)-i)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(I*log(tan(d*x + c) - I)/a - I*log(-I*tan(d*x + c) + 1)/a + (-I*tan(d*
x + c) - 3)/(a*(tan(d*x + c) - I)))/d
```

3.105 $\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{ia}{8d(a+ia\tan(c+dx))^2}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{i}{4d(a+ia\tan(c+dx))}+\frac{3x}{8a}$$

```
[Out] (3*x)/(8*a) - (I/8)/(d*(a - I*a*Tan[c + d*x])) + ((I/8)*a)/(d*(a + I*a*Tan[
c + d*x])^2) + (I/4)/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0682929, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$\frac{ia}{8d(a+ia\tan(c+dx))^2}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{i}{4d(a+ia\tan(c+dx))}+\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) - (I/8)/(d*(a - I*a*Tan[c + d*x])) + ((I/8)*a)/(d*(a + I*a*Tan[
c + d*x])^2) + (I/4)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{(ia^3)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(ia^3)\,\mathrm{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3(a^2-x^2)}\right)dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{i}{8d(a-ia\tan(c+dx))}+\frac{ia}{8d(a+ia\tan(c+dx))^2}+\frac{i}{4d(a+ia\tan(c+dx))}-\frac{(3i)\,\mathrm{Subst}\left(\int \right.}{}$$
$$= \frac{3x}{8a}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{ia}{8d(a+ia\tan(c+dx))^2}+\frac{i}{4d(a+ia\tan(c+dx))}$$

Mathematica [A] time = 0.249609, size = 78, normalized size = 0.95

$$-\frac{2\cos(2(c+dx))-12dx\tan(c+dx)+6i\tan(c+dx)+3i\sin(3(c+dx))\sec(c+dx)+12idx-7}{32ad(\tan(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -(-7 + (12*I)*d*x + 2*Cos[2*(c + d*x)] + (3*I)*Sec[c + d*x]*Sin[3*(c + d*x)
] + (6*I)*Tan[c + d*x] - 12*d*x*Tan[c + d*x])/(32*a*d*(-I + Tan[c + d*x]))
```

Maple [A] time = 0.082, size = 98, normalized size = 1.2

$$\frac{-\frac{3i}{16}\ln(\tan(dx+c)-i)}{ad}-\frac{\frac{i}{8}}{ad(\tan(dx+c)-i)^2}+\frac{1}{4ad(\tan(dx+c)-i)}+\frac{\frac{3i}{16}\ln(\tan(dx+c)+i)}{ad}+\frac{1}{8ad(\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -3/16*I/a/d*ln(tan(d*x+c)-I)-1/8*I/a/d/(tan(d*x+c)-I)^2+1/4/a/d/(tan(d*x+c)
-I)+3/16*I/a/d*ln(tan(d*x+c)+I)+1/8/a/d/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.91309, size = 159, normalized size = 1.94

$$\frac{\left(12\,dxe^{(4i\,dx+4i\,c)}-2i\,e^{(6i\,dx+6i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{32\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(12*d*x*e^(4*I*d*x + 4*I*c) - 2*I*e^(6*I*d*x + 6*I*c) + 6*I*e^(2*I*d*x
 + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a*d)
```

Sympy [A] time = 0.494572, size = 153, normalized size = 1.87

$$\begin{cases} \frac{\left(-512ia^2d^2e^{8ic}e^{2idx}+1536ia^2d^2e^{4ic}e^{-2idx}+256ia^2d^2e^{2ic}e^{-4idx}\right)e^{-6ic}}{8192a^3d^3} & \text{for } 8192a^3d^3e^{6ic} \neq 0 \\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-4ic}}{8a}-\frac{3}{8a}\right) & \text{otherwise} \end{cases} + \frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-512*I*a**2*d**2*exp(8*I*c)*exp(2*I*d*x) + 1536*I*a**2*d**2*exp
(4*I*c)*exp(-2*I*d*x) + 256*I*a**2*d**2*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*
c)/(8192*a**3*d**3), Ne(8192*a**3*d**3*exp(6*I*c), 0)), (x*((exp(6*I*c) + 3
*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-4*I*c)/(8*a) - 3/(8*a)), True)) + 3*x/
(8*a)
```

Giac [A] time = 1.13896, size = 134, normalized size = 1.63

$$-\frac{\frac{6i\log(i\tan(dx+c)+1)}{a}-\frac{6i\log(i\tan(dx+c)-1)}{a}+\frac{2(3\tan(dx+c)+5i)}{a(-i\tan(dx+c)+1)}+\frac{-9i\tan(dx+c)^2-26\tan(dx+c)+21i}{a(\tan(dx+c)-i)^2}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(6*I*log(I*tan(d*x + c) + 1)/a - 6*I*log(I*tan(d*x + c) - 1)/a + 2*(3
*tan(d*x + c) + 5*I)/(a*(-I*tan(d*x + c) + 1)) + (-9*I*tan(d*x + c)^2 - 26*
tan(d*x + c) + 21*I)/(a*(tan(d*x + c) - I)^2))/d
```

3.106 $\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{ia^2}{24d(a+ia\tan(c+dx))^3}-\frac{ia}{32d(a-ia\tan(c+dx))^2}+\frac{3ia}{32d(a+ia\tan(c+dx))^2}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{3i}{16d(a+ia\,\mathrm{ta}}$$

```
[Out] (5*x)/(16*a) - ((I/32)*a)/(d*(a - I*a*Tan[c + d*x])^2) - (I/8)/(d*(a - I*a*
Tan[c + d*x])) + ((I/24)*a^2)/(d*(a + I*a*Tan[c + d*x])^3) + (((3*I)/32)*a)
/(d*(a + I*a*Tan[c + d*x])^2) + ((3*I)/16)/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0872302, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$\frac{ia^2}{24d(a+ia\tan(c+dx))^3}-\frac{ia}{32d(a-ia\tan(c+dx))^2}+\frac{3ia}{32d(a+ia\tan(c+dx))^2}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{3i}{16d(a+ia\,\mathrm{ta}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```


```
[Out] (5*x)/(16*a) - ((I/32)*a)/(d*(a - I*a*Tan[c + d*x])^2) - (I/8)/(d*(a - I*a*
Tan[c + d*x])) + ((I/24)*a^2)/(d*(a + I*a*Tan[c + d*x])^3) + (((3*I)/32)*a)
/(d*(a + I*a*Tan[c + d*x])^2) + ((3*I)/16)/(d*(a + I*a*Tan[c + d*x]))
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{(ia^5)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^4}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\mathrm{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^3}+\frac{1}{8a^5(a-x)^2}+\frac{1}{8a^3(a+x)^4}+\frac{3}{16a^4(a+x)^3}+\frac{3}{16a^5(a+x)^2}+\frac{5}{16a^5(a^2-x^2)}\right)dx, \ldots}{d}$$

$$= -\frac{ia}{32d(a-ia\tan(c+dx))^2}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{ia^2}{24d(a+ia\tan(c+dx))^3}+\frac{\ldots}{32d(a+\ldots}$$

$$= \frac{5x}{16a}-\frac{ia}{32d(a-ia\tan(c+dx))^2}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{ia^2}{24d(a+ia\tan(c+dx))^3}+\frac{\ldots}{32\ldots}$$

Mathematica [A] time = 0.20648, size = 109, normalized size = 0.81

$$-\frac{\sec(c+dx)(-120dx\sin(c+dx)+60i\sin(c+dx)+45i\sin(3(c+dx))+5i\sin(5(c+dx))+60i(2dx+i)\cos(c+dx)\ldots}{384ad(\tan(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]*((60*I)*(I + 2*d*x)*Cos[c + d*x] + 15*Cos[3*(c + d*x)] + Cos
[5*(c + d*x)] + (60*I)*Sin[c + d*x] - 120*d*x*Sin[c + d*x] + (45*I)*Sin[3*(
c + d*x)] + (5*I)*Sin[5*(c + d*x)]))/(384*a*d*(-I + Tan[c + d*x]))
```

Maple [A] time = 0.083, size = 137, normalized size = 1.

$$\frac{-\frac{5i}{32}\ln(\tan(dx+c)-i)}{ad}-\frac{\frac{3i}{32}}{ad(\tan(dx+c)-i)^2}-\frac{1}{24\,ad(\tan(dx+c)-i)^3}+\frac{3}{16\,ad(\tan(dx+c)-i)}+\frac{\frac{i}{32}}{ad(\tan(dx+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -5/32*I/a/d*ln(tan(d*x+c)-I)-3/32*I/a/d/(tan(d*x+c)-I)^2-1/24/a/d/(tan(d*x+
c)-I)^3+3/16/a/d/(tan(d*x+c)-I)+1/32*I/a/d/(tan(d*x+c)+I)^2+5/32*I/a/d*ln(t
an(d*x+c)+I)+1/8/a/d/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.94405, size = 242, normalized size = 1.81

$$\frac{\left(120\, dxe^{(6i\, dx+6i\, c)} - 3i\, e^{(10i\, dx+10i\, c)} - 30i\, e^{(8i\, dx+8i\, c)} + 60i\, e^{(4i\, dx+4i\, c)} + 15i\, e^{(2i\, dx+2i\, c)} + 2i\right)e^{(-6i\, dx-6i\, c)}}{384\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/384*(120*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(10*I*d*x + 10*I*c) - 30*I*e^(8*
I*d*x + 8*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 2*I)
*e^(-6*I*d*x - 6*I*c)/(a*d)
```

Sympy [A] time = 0.790826, size = 221, normalized size = 1.65

$$\begin{cases} \frac{\left(-50331648ia^4d^4e^{16ic}e^{4idx}-503316480ia^4d^4e^{14ic}e^{2idx}+1006632960ia^4d^4e^{10ic}e^{-2idx}+251658240ia^4d^4e^{8ic}e^{-4idx}+33554432ia^4d^4e^{6ic}e^{-6idx}\right)e^{-12ic}}{6442450944a^5d^5} & \text{for } 64424 \\ x\left(\frac{\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-6ic}}{32a} - \frac{5}{16a}\right) & \text{otherwis} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-50331648*I*a**4*d**4*exp(16*I*c)*exp(4*I*d*x) - 503316480*I*a*
*4*d**4*exp(14*I*c)*exp(2*I*d*x) + 1006632960*I*a**4*d**4*exp(10*I*c)*exp(-
2*I*d*x) + 251658240*I*a**4*d**4*exp(8*I*c)*exp(-4*I*d*x) + 33554432*I*a**4
*d**4*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(6442450944*a**5*d**5), Ne(644
2450944*a**5*d**5*exp(12*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*ex
p(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-6*I*c)/(32*a) - 5/(16*a))
, True)) + 5*x/(16*a)
```

Giac [A] time = 1.16066, size = 157, normalized size = 1.17

$$-\frac{-\frac{30i\log(\tan(dx+c)+i)}{a} + \frac{30i\log(\tan(dx+c)-i)}{a} + \frac{3\left(-15i\tan(dx+c)^2+38\tan(dx+c)+25i\right)}{a(-i\tan(dx+c)+1)^2} - \frac{55i\tan(dx+c)^3+201\tan(dx+c)^2-255i\tan(dx+c)-117}{a(\tan(dx+c)-i)^3}}{192\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(-30*I*log(tan(d*x + c) + I)/a + 30*I*log(tan(d*x + c) - I)/a + 3*(-
15*I*tan(d*x + c)^2 + 38*tan(d*x + c) + 25*I)/(a*(-I*tan(d*x + c) + 1)^2) -
 (55*I*tan(d*x + c)^3 + 201*tan(d*x + c)^2 - 255*I*tan(d*x + c) - 117)/(a*(
tan(d*x + c) - I)^3))/d
```

3.107 $\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{\tan(c+dx)\sec^3(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((I/5)*Sec[c + d*x]^5)/(a*d) + (3*Sec[c
 + d*x]*Tan[c + d*x])/(8*a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.0767625, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3501, 3768, 3770}

$$-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{\tan(c+dx)\sec^3(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((I/5)*Sec[c + d*x]^5)/(a*d) + (3*Sec[c
 + d*x]*Tan[c + d*x])/(8*a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a*d)
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\sec^5(c+dx)}{5ad}+\frac{\int \sec^5(c+dx)\,dx}{a}\\
&= -\frac{i\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)\tan(c+dx)}{4ad}+\frac{3\int \sec^3(c+dx)\,dx}{4a}\\
&= -\frac{i\sec^5(c+dx)}{5ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}+\frac{\sec^3(c+dx)\tan(c+dx)}{4ad}+\frac{3\int \sec(c+dx)}{8a}\\
&= \frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}+\frac{\sec^3(c+dx)\tan(c+}{4ad}
\end{aligned}$$

Mathematica [A] time = 0.331157, size = 60, normalized size = 0.71

$$\frac{240\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)+(70\sin(2(c+dx))+15\sin(4(c+dx))-64i)\sec^5(c+dx)}{320ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (240*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^5*(-64*I + 70*Sin
[2*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(320*a*d)
```

Maple [B] time = 0.08, size = 430, normalized size = 5.1

$$\frac{\frac{i}{5}}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-5}+\frac{5}{8\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{\frac{3\,i}{4}}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}+\frac{1}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{\frac{i}{2}}{ad}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 1/5*I/a/d/(tan(1/2*d*x+1/2*c)-1)^5+5/8/a/d/(tan(1/2*d*x+1/2*c)+1)+3/4*I/a/d
/(tan(1/2*d*x+1/2*c)-1)^3+1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^3+1/2*I/a/d/(tan(1
/2*d*x+1/2*c)-1)^4-1/4/a/d/(tan(1/2*d*x+1/2*c)+1)^4+1/2*I/a/d/(tan(1/2*d*x+
1/2*c)+1)^4-7/8/a/d/(tan(1/2*d*x+1/2*c)+1)^2+5/8*I/a/d/(tan(1/2*d*x+1/2*c)-
1)^2+3/8/a/d*ln(tan(1/2*d*x+1/2*c)+1)+3/8*I/a/d/(tan(1/2*d*x+1/2*c)-1)+7/8/
a/d/(tan(1/2*d*x+1/2*c)-1)^2-3/8*I/a/d/(tan(1/2*d*x+1/2*c)+1)+5/8/a/d/(tan(
1/2*d*x+1/2*c)-1)-3/4*I/a/d/(tan(1/2*d*x+1/2*c)+1)^3+1/2/a/d/(tan(1/2*d*x+1
/2*c)-1)^3+5/8*I/a/d/(tan(1/2*d*x+1/2*c)+1)^2+1/4/a/d/(tan(1/2*d*x+1/2*c)-1
)^4-1/5*I/a/d/(tan(1/2*d*x+1/2*c)+1)^5-3/8/a/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.27863, size = 390, normalized size = 4.64

$$\frac{\frac{16\left(-\frac{75i\sin(dx+c)}{\cos(dx+c)+1}+\frac{30i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{240\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{30i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{120\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{75i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-24\right)}{-120i\,a+\frac{600i\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200i\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200i\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600i\,a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120i\,a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(16*(-75*I*sin(d*x + c)/(cos(d*x + c) + 1) + 30*I*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 240*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 30*I*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 - 120*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 75*I*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 - 24)/(-120*I*a + 600*I*a*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 - 1200*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1200
*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a*sin(d*x + c)^8/(cos(d*x
+ c) + 1)^8 + 120*I*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 3*log(sin(d*
x + c)/(cos(d*x + c) + 1) + 1)/a - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a)/d
```

Fricas [B] time = 2.24375, size = 810, normalized size = 9.64

$$\frac{15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(10i\,dx+10i\,c)}+5\right.}{40\left(ade^{(10i\,dx+10i\,c)}+5\,ade^{(8\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6
*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
I*c) + I) - 15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d
*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*
d*x + I*c) - I) - 30*I*e^(9*I*d*x + 9*I*c) - 140*I*e^(7*I*d*x + 7*I*c) - 25
6*I*e^(5*I*d*x + 5*I*c) + 140*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))
/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^(8*I*d*x + 8*I*c) + 10*a*d*e^(6*I*d*x
 + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c) + 5*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.24486, size = 189, normalized size = 2.25

$$\frac{\frac{15\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{15\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(25\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+40i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+80i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+10\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}a}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 15*log(abs(tan(1/2*d*x + 1/
2*c) - 1))/a + 2*(25*tan(1/2*d*x + 1/2*c)^9 + 40*I*tan(1/2*d*x + 1/2*c)^8 -
 10*tan(1/2*d*x + 1/2*c)^7 + 80*I*tan(1/2*d*x + 1/2*c)^4 + 10*tan(1/2*d*x +
 1/2*c)^3 - 25*tan(1/2*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*
a))/d
```

3.108 $\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sec^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - ((I/3)*Sec[c + d*x]^3)/(a*d) + (Sec[c + d*x
]*Tan[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.0535457, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3501, 3768, 3770}

$$-\frac{i\sec^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - ((I/3)*Sec[c + d*x]^3)/(a*d) + (Sec[c + d*x
]*Tan[c + d*x])/(2*a*d)
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\sec^3(c+dx)}{3ad}+\frac{\int \sec^3(c+dx)\,dx}{a}\\
&= -\frac{i\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)\tan(c+dx)}{2ad}+\frac{\int \sec(c+dx)\,dx}{2a}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{i\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)\tan(c+dx)}{2ad}
\end{aligned}$$

Mathematica [A] time = 0.219742, size = 50, normalized size = 0.83

$$\frac{12\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)+(3\sin(2(c+dx))-4i)\sec^3(c+dx)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (12*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(-4*I + 3*Sin[2*
(c + d*x)]))/(12*a*d)
```

Maple [B] time = 0.072, size = 258, normalized size = 4.3

$$\frac{-\frac{i}{3}}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{1}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{\frac{i}{2}}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{1}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -1/3*I/a/d/(tan(1/2*d*x+1/2*c)+1)^3+1/2/a/d/(tan(1/2*d*x+1/2*c)+1)-1/2*I/a/
d/(tan(1/2*d*x+1/2*c)+1)-1/2/a/d/(tan(1/2*d*x+1/2*c)+1)^2+1/2*I/a/d/(tan(1/
2*d*x+1/2*c)+1)^2+1/2/a/d*ln(tan(1/2*d*x+1/2*c)+1)+1/3*I/a/d/(tan(1/2*d*x+1
/2*c)-1)^3+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)^2+1/2*I/a/d/(tan(1/2*d*x+1/2*c)-1
)^2+1/2/a/d/(tan(1/2*d*x+1/2*c)-1)+1/2*I/a/d/(tan(1/2*d*x+1/2*c)-1)-1/2/a/d
*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.03631, size = 251, normalized size = 4.18

$$\frac{\frac{4\left(\frac{3i\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2\right)}{6ia-\frac{18ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18ia\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6ia\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(3*I*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 3*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(6*I*a - 18*I*a*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 - 6*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [B] time = 2.17509, size = 505, normalized size = 8.42

$$\frac{3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\right.}\right.}{6\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c)
 + 1)*log(e^(I*d*x + I*c) + I) - 3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*
I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 6*I*e^(5*I*d*x
 + 5*I*c) - 16*I*e^(3*I*d*x + 3*I*c) + 6*I*e^(I*d*x + I*c))/(a*d*e^(6*I*d*x
 + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.19205, size = 136, normalized size = 2.27

$$\frac{\frac{3 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a} - \frac{3 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right|\right)}{a} + \frac{2\left(3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+6i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}-3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+2i\right)}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-1\right)^{3} a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*log(abs(tan(1/2*d*x + 1/2*c
) - 1))/a + 2*(3*tan(1/2*d*x + 1/2*c)^5 + 6*I*tan(1/2*d*x + 1/2*c)^4 - 3*ta
n(1/2*d*x + 1/2*c) + 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

3.109 $\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)
```

Rubi [A] time = 0.0435518, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3501, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx &= -\frac{i\sec(c+dx)}{ad}+\frac{\int \sec(c+dx)\,dx}{a}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A] time = 0.129121, size = 34, normalized size = 1.1

$$\frac{2\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)-i\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] - I*Sec[c + d*x])/(a*d)
```

Maple [B] time = 0.063, size = 85, normalized size = 2.7

$$\frac{-i}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{1}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{i}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{1}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I/a/d/(tan(1/2*d*x+1/2*c)+1)+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)+I/a/d/(tan(1/2
*d*x+1/2*c)-1)-1/a/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.05449, size = 112, normalized size = 3.61

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2}{-ia+\frac{ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c
) + 1) - 1)/a - 2/(-I*a + I*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Fricas [B] time = 2.10465, size = 217, normalized size = 7.

$$\frac{\left(e^{(2idx+2ic)}+1\right)\log\left(e^{(idx+ic)}+i\right)-\left(e^{(2idx+2ic)}+1\right)\log\left(e^{(idx+ic)}-i\right)-2ie^{(idx+ic)}}{ade^{(2idx+2ic)}+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - (e^(2*I*d*x + 2*I*c)
+ 1)*log(e^(I*d*x + I*c) - I) - 2*I*e^(I*d*x + I*c))/(a*d*e^(2*I*d*x + 2*I*
c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.20004, size = 81, normalized size = 2.61

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))/
a + 2*I/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

3.110 $\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

```
[Out] (I*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0228633, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3488}

$$\frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

Mathematica [A] time = 0.0267033, size = 25, normalized size = 0.89

$$\frac{\sec(c+dx)}{ad(\tan(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] Sec[c + d*x]/(a*d*(-I + Tan[c + d*x]))
```

Maple [A] time = 0.034, size = 23, normalized size = 0.8

$$2\,\frac{1}{ad\left(\tan\left(1/2\,dx+c/2\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c)),x)

[Out] 2/d/a/(tan(1/2*d*x+1/2*c)-I)
```

Maxima [A] time = 0.985451, size = 39, normalized size = 1.39

$$\frac{2}{\left(-i\,a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")

[Out] 2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

Fricas [A] time = 2.16242, size = 35, normalized size = 1.25

$$\frac{i\,e^{(-i\,dx-i\,c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")

[Out] I*e^(-I*d*x - I*c)/(a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x)

[Out] Exception raised: AttributeError
```

Giac [A] time = 1.1525, size = 28, normalized size = 1.

$$\frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")

[Out] 2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

3.111 $\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=47

$$\frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

```
[Out] (2*Sin[c + d*x])/(3*a*d) + ((I/3)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0370954, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3502, 2637}

$$\frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*Sin[c + d*x])/(3*a*d) + ((I/3)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx &= \frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}+\frac{2\int \cos(c+dx)\,dx}{3a}\\ &= \frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.117212, size = 50, normalized size = 1.06

$$-\frac{\sec(c+dx)(2i\sin(2(c+dx))+\cos(2(c+dx))-3)}{6ad(\tan(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]*(-3 + Cos[2*(c + d*x)] + (2*I)*Sin[2*(c + d*x)]))/(6*a*d*(-I
 + Tan[c + d*x]))
```

Maple [A] time = 0.078, size = 75, normalized size = 1.6

$$2\,\frac{1}{ad}\left(-1/3\,\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-3}+\frac{i/2}{\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{2}}+3/4\,\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-1}+1/4\,(\tan\left(1/2\,dx+c/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/d/a*(-1/3/(tan(1/2*d*x+1/2*c)-I)^3+1/2*I/(tan(1/2*d*x+1/2*c)-I)^2+3/4/(ta
n(1/2*d*x+1/2*c)-I)+1/4/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.96835, size = 122, normalized size = 2.6

$$\frac{\left(-3i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(-3*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x -
 3*I*c)/(a*d)
```

Sympy [A] time = 0.486814, size = 128, normalized size = 2.72

$$\begin{cases}\frac{\left(-24ia^{2}d^{2}e^{5ic}e^{idx}+48ia^{2}d^{2}e^{3ic}e^{-idx}+8ia^{2}d^{2}e^{ic}e^{-3idx}\right)e^{-4ic}}{96a^{3}d^{3}} & \text{for}\ 96a^{3}d^{3}e^{4ic}\neq 0\\ \frac{x\left(e^{4ic}+2e^{2ic}+1\right)e^{-3ic}}{4a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-24*I*a**2*d**2*exp(5*I*c)*exp(I*d*x) + 48*I*a**2*d**2*exp(3*I*
c)*exp(-I*d*x) + 8*I*a**2*d**2*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*c)/(96*a**3
```

```
*d**3), Ne(96*a**3*d**3*exp(4*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I*c) + 1)
*exp(-3*I*c)/(4*a), True))
```

Giac [A] time = 1.1344, size = 90, normalized size = 1.91

$$\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3/(a*(tan(1/2*d*x + 1/2*c) + I)) + (9*tan(1/2*d*x + 1/2*c)^2 - 12*I*ta
n(1/2*d*x + 1/2*c) - 7)/(a*(tan(1/2*d*x + 1/2*c) - I)^3))/d
```

3.112 $\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=67

$$-\frac{4\sin^3(c+dx)}{15ad}+\frac{4\sin(c+dx)}{5ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

```
[Out] (4*Sin[c + d*x])/(5*a*d) - (4*Sin[c + d*x]^3)/(15*a*d) + ((I/5)*Cos[c + d*x
]^3)/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0519919, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3502, 2633}

$$-\frac{4\sin^3(c+dx)}{15ad}+\frac{4\sin(c+dx)}{5ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (4*Sin[c + d*x])/(5*a*d) - (4*Sin[c + d*x]^3)/(15*a*d) + ((I/5)*Cos[c + d*x
]^3)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx&=\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}+\frac{4\int \cos^3(c+dx)\,dx}{5a}\\&=\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}-\frac{4\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{5ad}\\&=\frac{4\sin(c+dx)}{5ad}-\frac{4\sin^3(c+dx)}{15ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.151753, size = 72, normalized size = 1.07

$$-\frac{\sec(c+dx)(40i\sin(2(c+dx))+4i\sin(4(c+dx))+20\cos(2(c+dx))+\cos(4(c+dx))-45)}{120ad(\tan(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]*(-45 + 20*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + (40*I)*Sin[2
*(c + d*x)] + (4*I)*Sin[4*(c + d*x)]))/(120*a*d*(-I + Tan[c + d*x]))
```

Maple [B] time = 0.084, size = 141, normalized size = 2.1

$$2\,\frac{1}{ad}\left(\frac{-i/2}{\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{4}}+\frac{3/4\,i}{\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{2}}+1/5\,\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-5}-5/6\,\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/d/a*(-1/2*I/(tan(1/2*d*x+1/2*c)-I)^4+3/4*I/(tan(1/2*d*x+1/2*c)-I)^2+1/5/(
tan(1/2*d*x+1/2*c)-I)^5-5/6/(tan(1/2*d*x+1/2*c)-I)^3+11/16/(tan(1/2*d*x+1/2
*c)-I)-1/8*I/(tan(1/2*d*x+1/2*c)+I)^2-1/12/(tan(1/2*d*x+1/2*c)+I)^3+5/16/(t
an(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.03251, size = 200, normalized size = 2.99

$$\frac{\left(-5 i\, e^{(8 i\, dx+8 i\, c)}-60 i\, e^{(6 i\, dx+6 i\, c)}+90 i\, e^{(4 i\, dx+4 i\, c)}+20 i\, e^{(2 i\, dx+2 i\, c)}+3 i\right) e^{(-5 i\, dx-5 i\, c)}}{240\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(-5*I*e^(8*I*d*x + 8*I*c) - 60*I*e^(6*I*d*x + 6*I*c) + 90*I*e^(4*I*d*
x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a*d)
```

Sympy [A] time = 0.855722, size = 197, normalized size = 2.94

$$\begin{cases} \frac{\left(-30720 i a^{4} d^{4} e^{12 i c} e^{3 i d x}-368640 i a^{4} d^{4} e^{10 i c} e^{i d x}+552960 i a^{4} d^{4} e^{8 i c} e^{-i d x}+122880 i a^{4} d^{4} e^{6 i c} e^{-3 i d x}+18432 i a^{4} d^{4} e^{4 i c} e^{-5 i d x}\right) e^{-9 i c}}{1474560 a^{5} d^{5}} & \text{for}\: 1474560 a^{5} d^{5} e^{9 i c} \neq 0 \\ \frac{x \left(e^{8 i c}+4 e^{6 i c}+6 e^{4 i c}+4 e^{2 i c}+1\right) e^{-5 i c}}{16 a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-30720*I*a**4*d**4*exp(12*I*c)*exp(3*I*d*x) - 368640*I*a**4*d**
4*exp(10*I*c)*exp(I*d*x) + 552960*I*a**4*d**4*exp(8*I*c)*exp(-I*d*x) + 1228
80*I*a**4*d**4*exp(6*I*c)*exp(-3*I*d*x) + 18432*I*a**4*d**4*exp(4*I*c)*exp(
-5*I*d*x))*exp(-9*I*c)/(1474560*a**5*d**5), Ne(1474560*a**5*d**5*exp(9*I*c)
, 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp
(-5*I*c)/(16*a), True))
```

Giac [B] time = 1.15397, size = 161, normalized size = 2.4

$$\frac{\frac{5\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-13\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+400i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+113}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 + 24*I*tan(1/2*d*x + 1/2*c) - 13)/(a*(t
an(1/2*d*x + 1/2*c) + I)^3) + (165*tan(1/2*d*x + 1/2*c)^4 - 480*I*tan(1/2*d
*x + 1/2*c)^3 - 650*tan(1/2*d*x + 1/2*c)^2 + 400*I*tan(1/2*d*x + 1/2*c) + 1
13)/(a*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

3.113 $\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{6\sin^5(c+dx)}{35ad}-\frac{4\sin^3(c+dx)}{7ad}+\frac{6\sin(c+dx)}{7ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

```
[Out] (6*Sin[c + d*x])/(7*a*d) - (4*Sin[c + d*x]^3)/(7*a*d) + (6*Sin[c + d*x]^5)/
(35*a*d) + ((I/7)*Cos[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0553903, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3502, 2633}

$$\frac{6\sin^5(c+dx)}{35ad}-\frac{4\sin^3(c+dx)}{7ad}+\frac{6\sin(c+dx)}{7ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (6*Sin[c + d*x])/(7*a*d) - (4*Sin[c + d*x]^3)/(7*a*d) + (6*Sin[c + d*x]^5)/
(35*a*d) + ((I/7)*Cos[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx &= \frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}+\frac{6\int \cos^5(c+dx)\,dx}{7a}\\ &= \frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}-\frac{6\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, -\sin(c+dx)\right)}{7ad}\\ &= \frac{6\sin(c+dx)}{7ad}-\frac{4\sin^3(c+dx)}{7ad}+\frac{6\sin^5(c+dx)}{35ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.172666, size = 94, normalized size = 1.11

$$-\frac{\sec(c+dx)(350i\sin(2(c+dx))+56i\sin(4(c+dx))+6i\sin(6(c+dx))+175\cos(2(c+dx))+14\cos(4(c+dx))+\cos(}{1120ad(\tan(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]*(-350 + 175*Cos[2*(c + d*x)] + 14*Cos[4*(c + d*x)] + Cos[6*(
c + d*x)] + (350*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)] + (6*I)*Sin[
6*(c + d*x)]))/(1120*a*d*(-I + Tan[c + d*x]))
```

Maple [B] time = 0.085, size = 207, normalized size = 2.4

$$2\frac{1}{ad}\left(-1/7\,(\tan(1/2\,dx+c/2)-i)^{-7}+\frac{i/2}{(\tan(1/2\,dx+c/2)-i)^{6}}+\frac{\frac{15\,i}{16}}{(\tan(1/2\,dx+c/2)-i)^{2}}-\frac{\frac{11\,i}{8}}{(\tan(1/2\,dx+c/2)-i)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/d/a*(-1/7/(tan(1/2*d*x+1/2*c)-I)^7+1/2*I/(tan(1/2*d*x+1/2*c)-I)^6+15/16*I
/(tan(1/2*d*x+1/2*c)-I)^2-11/8*I/(tan(1/2*d*x+1/2*c)-I)^4+21/20/(tan(1/2*d*
x+1/2*c)-I)^5-11/8/(tan(1/2*d*x+1/2*c)-I)^3+21/32/(tan(1/2*d*x+1/2*c)-I)+1/
8*I/(tan(1/2*d*x+1/2*c)+I)^4-1/4*I/(tan(1/2*d*x+1/2*c)+I)^2+1/20/(tan(1/2*d
*x+1/2*c)+I)^5-1/4/(tan(1/2*d*x+1/2*c)+I)^3+11/32/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.03141, size = 284, normalized size = 3.34

$$\frac{\left(-7i\,e^{(12i\,dx+12i\,c)}-70i\,e^{(10i\,dx+10i\,c)}-525i\,e^{(8i\,dx+8i\,c)}+700i\,e^{(6i\,dx+6i\,c)}+175i\,e^{(4i\,dx+4i\,c)}+42i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{2240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2240*(-7*I*e^(12*I*d*x + 12*I*c) - 70*I*e^(10*I*d*x + 10*I*c) - 525*I*e^(
8*I*d*x + 8*I*c) + 700*I*e^(6*I*d*x + 6*I*c) + 175*I*e^(4*I*d*x + 4*I*c) +
42*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a*d)
```

Sympy [A] time = 1.32842, size = 265, normalized size = 3.12

$$\begin{cases}\frac{\left(-150323855360ia^{6}d^{6}e^{21ic}e^{5idx}-1503238553600ia^{6}d^{6}e^{19ic}e^{3idx}-11274289152000ia^{6}d^{6}e^{17ic}e^{idx}+15032385536000ia^{6}d^{6}e^{15ic}e^{-idx}+3758096384000ia^{6}d^{6}e^{13ic}\right.}{48103633715200a^{7}d^{7}}\\ \frac{x\left(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1\right)e^{-7ic}}{64a}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-150323855360*I*a**6*d**6*exp(21*I*c)*exp(5*I*d*x) - 1503238553
600*I*a**6*d**6*exp(19*I*c)*exp(3*I*d*x) - 11274289152000*I*a**6*d**6*exp(1
7*I*c)*exp(I*d*x) + 15032385536000*I*a**6*d**6*exp(15*I*c)*exp(-I*d*x) + 37
58096384000*I*a**6*d**6*exp(13*I*c)*exp(-3*I*d*x) + 901943132160*I*a**6*d**
6*exp(11*I*c)*exp(-5*I*d*x) + 107374182400*I*a**6*d**6*exp(9*I*c)*exp(-7*I*
d*x))*exp(-16*I*c)/(48103633715200*a**7*d**7), Ne(48103633715200*a**7*d**7*
exp(16*I*c), 0)), (x*(exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8*I*c) + 20*exp(
6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-7*I*c)/(64*a), True))
```

Giac [B] time = 1.15593, size = 231, normalized size = 2.72

$$\frac{\frac{7\left(55\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+180i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-250\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-160i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+43\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5}+\frac{735\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-3360i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-7315\tan\left(\frac{1}{2}dx\right.}{}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/560*(7*(55*tan(1/2*d*x + 1/2*c)^4 + 180*I*tan(1/2*d*x + 1/2*c)^3 - 250*ta
n(1/2*d*x + 1/2*c)^2 - 160*I*tan(1/2*d*x + 1/2*c) + 43)/(a*(tan(1/2*d*x + 1
/2*c) + I)^5) + (735*tan(1/2*d*x + 1/2*c)^6 - 3360*I*tan(1/2*d*x + 1/2*c)^5
 - 7315*tan(1/2*d*x + 1/2*c)^4 + 8820*I*tan(1/2*d*x + 1/2*c)^3 + 6321*tan(1
/2*d*x + 1/2*c)^2 - 2492*I*tan(1/2*d*x + 1/2*c) - 461)/(a*(tan(1/2*d*x + 1/
2*c) - I)^7))/d
```

3.114 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$\frac{i(a-ia\tan(c+dx))^7}{7a^9d}-\frac{2i(a-ia\tan(c+dx))^6}{3a^8d}+\frac{4i(a-ia\tan(c+dx))^5}{5a^7d}$$

```
[Out] (((4*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^7*d) - (((2*I)/3)*(a - I*a*Tan[c +
d*x])^6)/(a^8*d) + ((I/7)*(a - I*a*Tan[c + d*x])^7)/(a^9*d)
```

Rubi [A] time = 0.0625975, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a-ia\tan(c+dx))^7}{7a^9d}-\frac{2i(a-ia\tan(c+dx))^6}{3a^8d}+\frac{4i(a-ia\tan(c+dx))^5}{5a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((4*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^7*d) - (((2*I)/3)*(a - I*a*Tan[c +
d*x])^6)/(a^8*d) + ((I/7)*(a - I*a*Tan[c + d*x])^7)/(a^9*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^4(a+x)^2\,dx,x,ia\tan(c+dx)\right)}{a^9d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a-x)^4-4a(a-x)^5+(a-x)^6\right)dx,x,ia\tan(c+dx)\right)}{a^9d}\\ &= \frac{4i(a-ia\tan(c+dx))^5}{5a^7d}-\frac{2i(a-ia\tan(c+dx))^6}{3a^8d}+\frac{i(a-ia\tan(c+dx))^7}{7a^9d}\end{aligned}$$

Mathematica [A] time = 0.418424, size = 90, normalized size = 1.1

$$\frac{\sec(c)\sec^7(c+dx)(-35\sin(2c+dx)+42\sin(2c+3dx)+14\sin(4c+5dx)+2\sin(6c+7dx)-35i\cos(2c+dx)+35}{210a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^7*((-35*I)*Cos[d*x] - (35*I)*Cos[2*c + d*x] + 35*Sin[d
*x] - 35*Sin[2*c + d*x] + 42*Sin[2*c + 3*d*x] + 14*Sin[4*c + 5*d*x] + 2*Sin
[6*c + 7*d*x]))/(210*a^2*d)
```

Maple [A] time = 0.075, size = 78, normalized size = 1.

$$\frac{1}{a^2d}\left(\tan(dx+c) - \frac{(\tan(dx+c))^7}{7} - \frac{i}{3}(\tan(dx+c))^6 - \frac{(\tan(dx+c))^5}{5} - i(\tan(dx+c))^4 + \frac{(\tan(dx+c))^3}{3} - i(\tan(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(tan(d*x+c)-1/7*tan(d*x+c)^7-1/3*I*tan(d*x+c)^6-1/5*tan(d*x+c)^5-I*
tan(d*x+c)^4+1/3*tan(d*x+c)^3-I*tan(d*x+c)^2)
```

Maxima [A] time = 1.13657, size = 104, normalized size = 1.27

$$-\frac{15\,\tan(dx+c)^7 + 35i\,\tan(dx+c)^6 + 21\,\tan(dx+c)^5 + 105i\,\tan(dx+c)^4 - 35\,\tan(dx+c)^3 + 105i\,\tan(dx+c)^2 -}{105\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(15*tan(d*x + c)^7 + 35*I*tan(d*x + c)^6 + 21*tan(d*x + c)^5 + 105*I
*tan(d*x + c)^4 - 35*tan(d*x + c)^3 + 105*I*tan(d*x + c)^2 - 105*tan(d*x +
c))/(a^2*d)
```

Fricas [B] time = 2.11047, size = 402, normalized size = 4.9

$$\frac{2688i\,e^{(4i\,dx+4i\,c)} + 896i\,e^{(2i\,dx+2i\,c)} + 128i}{105\left(a^2de^{(14i\,dx+14i\,c)} + 7\,a^2de^{(12i\,dx+12i\,c)} + 21\,a^2de^{(10i\,dx+10i\,c)} + 35\,a^2de^{(8i\,dx+8i\,c)} + 35\,a^2de^{(6i\,dx+6i\,c)} + 21\,a^2de^{(4i\,dx+4i\,c)} + 7\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2688*I*e^(4*I*d*x + 4*I*c) + 896*I*e^(2*I*d*x + 2*I*c) + 128*I)/(a^2
*d*e^(14*I*d*x + 14*I*c) + 7*a^2*d*e^(12*I*d*x + 12*I*c) + 21*a^2*d*e^(10*I
*d*x + 10*I*c) + 35*a^2*d*e^(8*I*d*x + 8*I*c) + 35*a^2*d*e^(6*I*d*x + 6*I*c
) + 21*a^2*d*e^(4*I*d*x + 4*I*c) + 7*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.16977, size = 104, normalized size = 1.27

$$-\frac{15\tan\left(dx+c\right)^{7}+35i\tan\left(dx+c\right)^{6}+21\tan\left(dx+c\right)^{5}+105i\tan\left(dx+c\right)^{4}-35\tan\left(dx+c\right)^{3}+105i\tan\left(dx+c\right)^{2}}{105\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/105*(15*tan(d*x + c)^7 + 35*I*tan(d*x + c)^6 + 21*tan(d*x + c)^5 + 105*I
*tan(d*x + c)^4 - 35*tan(d*x + c)^3 + 105*I*tan(d*x + c)^2 - 105*tan(d*x +
c))/(a^2*d)
```

3.115 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2} dx$

Optimal. Leaf size=55

$$\frac{i(a-ia\tan(c+dx))^4}{2a^6d}-\frac{i(a-ia\tan(c+dx))^5}{5a^7d}$$

```
[Out] ((I/2)*(a - I*a*Tan[c + d*x])^4)/(a^6*d) - ((I/5)*(a - I*a*Tan[c + d*x])^5)
/(a^7*d)
```

Rubi [A] time = 0.0505928, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a-ia\tan(c+dx))^4}{2a^6d}-\frac{i(a-ia\tan(c+dx))^5}{5a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/2)*(a - I*a*Tan[c + d*x])^4)/(a^6*d) - ((I/5)*(a - I*a*Tan[c + d*x])^5)
/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^3(a+x)\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\
&= -\frac{i\,\text{Subst}\left(\int \left(2a(a-x)^3-(a-x)^4\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\
&= \frac{i(a-ia\tan(c+dx))^4}{2a^6d}-\frac{i(a-ia\tan(c+dx))^5}{5a^7d}
\end{aligned}$$

Mathematica [A] time = 0.328308, size = 77, normalized size = 1.4

$$\frac{\sec(c)\sec^5(c+dx)(-5\sin(2c+dx)+5\sin(2c+3dx)+\sin(4c+5dx)-5i\cos(2c+dx)+5\sin(dx)-5i\cos(dx))}{20a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^5*((-5*I)*Cos[d*x] - (5*I)*Cos[2*c + d*x] + 5*Sin[d*x]
 - 5*Sin[2*c + d*x] + 5*Sin[2*c + 3*d*x] + Sin[4*c + 5*d*x]))/(20*a^2*d)
```

Maple [A] time = 0.072, size = 47, normalized size = 0.9

$$\frac{1}{a^2d}\left(\tan(dx+c)-\frac{(\tan(dx+c))^5}{5}-\frac{i}{2}(\tan(dx+c))^4-i(\tan(dx+c))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(tan(d*x+c)-1/5*tan(d*x+c)^5-1/2*I*tan(d*x+c)^4-I*tan(d*x+c)^2)
```

Maxima [A] time = 1.15832, size = 63, normalized size = 1.15

$$-\frac{2\tan(dx+c)^5+5i\tan(dx+c)^4+10i\tan(dx+c)^2-10\tan(dx+c)}{10\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*tan
(d*x + c))/(a^2*d)
```

Fricas [B] time = 2.12755, size = 267, normalized size = 4.85

$$\frac{40i\,e^{(2i\,dx+2i\,c)}+8i}{5\left(a^2de^{(10i\,dx+10i\,c)}+5\,a^2de^{(8i\,dx+8i\,c)}+10\,a^2de^{(6i\,dx+6i\,c)}+10\,a^2de^{(4i\,dx+4i\,c)}+5\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5*(40*I*e^(2*I*d*x + 2*I*c) + 8*I)/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d
*e^(8*I*d*x + 8*I*c) + 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x +
 4*I*c) + 5*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.15277, size = 63, normalized size = 1.15

$$-\frac{2\tan\left(dx+c\right)^{5}+5i\tan\left(dx+c\right)^{4}+10i\tan\left(dx+c\right)^{2}-10\tan\left(dx+c\right)}{10\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*tan
(d*x + c))/(a^2*d)
```

3.116 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=27

$$\frac{i(a-ia\tan(c+dx))^3}{3a^5d}$$

```
[Out] ((I/3)*(a - I*a*Tan[c + d*x])^3)/(a^5*d)
```

Rubi [A] time = 0.0444558, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$\frac{i(a-ia\tan(c+dx))^3}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/3)*(a - I*a*Tan[c + d*x])^3)/(a^5*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{i\,\mathrm{Subst}\left(\int (a-x)^2\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= \frac{i(a-ia\tan(c+dx))^3}{3a^5d}\end{aligned}$$

Mathematica [B] time = 0.209109, size = 68, normalized size = 2.52

$$\frac{\sec(c)\sec^3(c+dx)(-3\sin(2c+dx)+2\sin(2c+3dx)-3i\cos(2c+dx)+3\sin(dx)-3i\cos(dx))}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^3*((-3*I)*Cos[d*x] - (3*I)*Cos[2*c + d*x] + 3*Sin[d*x]
 - 3*Sin[2*c + d*x] + 2*Sin[2*c + 3*d*x]))/(6*a^2*d)
```

Maple [A] time = 0.068, size = 36, normalized size = 1.3

$$\frac{1}{a^2d}\left(\tan\left(dx+c\right)-\frac{\left(\tan\left(dx+c\right)\right)^3}{3}-i\left(\tan\left(dx+c\right)\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(tan(d*x+c)-1/3*tan(d*x+c)^3-I*tan(d*x+c)^2)
```

Maxima [A] time = 1.09483, size = 47, normalized size = 1.74

$$-\frac{\tan\left(dx+c\right)^3+3i\tan\left(dx+c\right)^2-3\tan\left(dx+c\right)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

Fricas [B] time = 1.9394, size = 139, normalized size = 5.15

$$\frac{8i}{3\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 8/3*I/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.18927, size = 47, normalized size = 1.74

$$-\frac{\tan\left(dx+c\right)^3+3i\tan\left(dx+c\right)^2-3\tan\left(dx+c\right)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

3.117 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\tan(c+dx)}{a^2d}+\frac{2i\log(\cos(c+dx))}{a^2d}+\frac{2x}{a^2}$$

```
[Out] (2*x)/a^2 + ((2*I)*Log[Cos[c + d*x]])/(a^2*d) - Tan[c + d*x]/(a^2*d)
```

Rubi [A] time = 0.0487313, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{\tan(c+dx)}{a^2d}+\frac{2i\log(\cos(c+dx))}{a^2d}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 + ((2*I)*Log[Cos[c + d*x]])/(a^2*d) - Tan[c + d*x]/(a^2*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{a-x}{a+x}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(-1+\frac{2a}{a+x}\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= \frac{2x}{a^2}+\frac{2i\log(\cos(c+dx))}{a^2d}-\frac{\tan(c+dx)}{a^2d}\end{aligned}$$

Mathematica [A] time = 0.365947, size = 71, normalized size = 1.87

$$\frac{4\tan^{-1}(\tan(dx))+i\sec(c)\sec(c+dx)\left(\cos(dx)\log\left(\cos^2(c+dx)\right)+\cos(2c+dx)\log\left(\cos^2(c+dx)\right)+2i\sin(dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (4*ArcTan[Tan[d*x]] + I*Sec[c]*Sec[c + d*x]*(Cos[d*x]*Log[Cos[c + d*x]^2] +
 Cos[2*c + d*x]*Log[Cos[c + d*x]^2] + (2*I)*Sin[d*x]))/(2*a^2*d)
```

Maple [A] time = 0.063, size = 35, normalized size = 0.9

$$\frac{-2\,i\ln\left(\tan\left(dx+c\right)-i\right)}{a^{2}d}-\frac{\tan\left(dx+c\right)}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -2*I/a^2/d*ln(tan(d*x+c)-I)-tan(d*x+c)/a^2/d
```

Maxima [A] time = 0.962078, size = 43, normalized size = 1.13

$$\frac{-\frac{2i\,\log(i\,\tan(dx+c)+1)}{a^2}-\frac{\tan(dx+c)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (-2*I*log(I*tan(d*x + c) + 1)/a^2 - tan(d*x + c)/a^2)/d
```

Fricas [A] time = 2.36921, size = 192, normalized size = 5.05

$$\frac{4\,dxe^{(2i\,dx+2i\,c)}+4\,dx+\left(2i\,e^{(2i\,dx+2i\,c)}+2i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-2i}{a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (4*d*x*e^(2*I*d*x + 2*I*c) + 4*d*x + (2*I*e^(2*I*d*x + 2*I*c) + 2*I)*log(e^
(2*I*d*x + 2*I*c) + 1) - 2*I)/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.2018, size = 138, normalized size = 3.63

$$\frac{2\left(-\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^2}+\frac{i\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}+\frac{i\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{-i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(-2*I*log(tan(1/2*d*x + 1/2*c) - I)/a^2 + I*log(abs(tan(1/2*d*x + 1/2*c)
+ 1))/a^2 + I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + (-I*tan(1/2*d*x + 1/
2*c)^2 + tan(1/2*d*x + 1/2*c) + I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^2))/d
```

3.118 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=26

$$\frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] I/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.0432143, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$\frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] I/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.0401591, size = 32, normalized size = 1.23

$$\frac{i\sec^2(c+dx)}{2d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/2)*Sec[c + d*x]^2)/(d*(a + I*a*Tan[c + d*x])^2)
```

Maple [A] time = 0.032, size = 24, normalized size = 0.9

$$\frac{i}{ad\left(a+ia\tan\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] I/a/d/(a+I*a*tan(d*x+c))
```

Maxima [A] time = 0.955199, size = 28, normalized size = 1.08

$$\frac{i}{\left(i\,a\tan\left(dx+c\right)+a\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] I/((I*a*tan(d*x + c) + a)*a*d)
```

Fricas [A] time = 2.26377, size = 49, normalized size = 1.88

$$\frac{i\,e^{\left(-2i\,dx-2i\,c\right)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.19236, size = 41, normalized size = 1.58

$$-\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

3.119 $\int \frac{1}{(a+ia\tan(c+dx))^2} dx$

Optimal. Leaf size=61

$$\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$

```
[Out] x/(4*a^2) + (I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (I/4)/(d*(a^2 + I*a^2*Tan[
c + d*x]))
```

Rubi [A] time = 0.0285386, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3479, 8}

$$\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-2),x]
```

```
[Out] x/(4*a^2) + (I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (I/4)/(d*(a^2 + I*a^2*Tan[
c + d*x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx &= \frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{\int \frac{1}{a+ia\tan(c+dx)}\,dx}{2a}\\
&= \frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\int 1\,dx}{4a^2}\\
&= \frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\left(a^2+ia^2\tan(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.164979, size = 68, normalized size = 1.11

$$-\frac{\sec^2(c+dx)((1+4idx)\sin(2(c+dx))+(4dx+i)\cos(2(c+dx))+4i)}{16a^2d(\tan(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-2),x]
```

```
[Out] -(Sec[c + d*x]^2*(4*I + (I + 4*d*x)*Cos[2*(c + d*x)] + (1 + (4*I)*d*x)*Sin[
2*(c + d*x)]))/(16*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A] time = 0.023, size = 79, normalized size = 1.3

$$\frac{-\frac{i}{8}\ln\left(\tan\left(dx+c\right)-i\right)}{a^2d}-\frac{\frac{i}{4}}{a^2d\left(\tan\left(dx+c\right)-i\right)^2}+\frac{1}{4\,a^2d\left(\tan\left(dx+c\right)-i\right)}+\frac{\frac{i}{8}\ln\left(\tan\left(dx+c\right)+i\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -1/8*I/a^2/d*ln(tan(d*x+c)-I)-1/4*I/a^2/d/(tan(d*x+c)-I)^2+1/4/a^2/d/(tan(d
*x+c)-I)+1/8*I/a^2/d*ln(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.19297, size = 126, normalized size = 2.07

$$\frac{\left(4\,dxe^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x
- 4*I*c)/(a^2*d)
```

Sympy [A] time = 0.339543, size = 119, normalized size = 1.95

$$\begin{cases}\frac{\left(16ia^2de^{4ic}e^{-2idx}+4ia^2de^{2ic}e^{-4idx}\right)e^{-6ic}}{64a^4d^2} & \text{for } 64a^4d^2e^{6ic}\neq 0\\ x\left(\frac{\left(e^{4ic}+2e^{2ic}+1\right)e^{-4ic}}{4a^2}-\frac{1}{4a^2}\right) & \text{otherwise}\end{cases}+\frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((16*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**2*d*exp(2*I*c)*ex
p(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(64*a**4*d**2*exp(6*I*c), 0)), (
```

```
x*((exp(4*I*c) + 2*exp(2*I*c) + 1)*exp(-4*I*c)/(4*a**2) - 1/(4*a**2)), True
)) + x/(4*a**2)
```

Giac [A] time = 1.11783, size = 97, normalized size = 1.59

$$-\frac{\frac{2i\log(i\tan(dx+c)+1)}{a^2}-\frac{2i\log(i\tan(dx+c)-1)}{a^2}+\frac{-3i\tan(dx+c)^2-10\tan(dx+c)+11i}{a^2(\tan(dx+c)-i)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*I*log(I*tan(d*x + c) + 1)/a^2 - 2*I*log(I*tan(d*x + c) - 1)/a^2 +
(-3*I*tan(d*x + c)^2 - 10*tan(d*x + c) + 11*I)/(a^2*(tan(d*x + c) - I)^2))/
d
```

3.120 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$-\frac{i}{16d\left(a^2-ia^2\tan(c+dx)\right)}+\frac{3i}{16d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{x}{4a^2}+\frac{ia}{12d(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^2}$$

```
[Out] x/(4*a^2) + ((I/12)*a)/(d*(a + I*a*Tan[c + d*x])^3) + (I/8)/(d*(a + I*a*Tan
[c + d*x])^2) - (I/16)/(d*(a^2 - I*a^2*Tan[c + d*x])) + ((3*I)/16)/(d*(a^2
+ I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.0814118, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{i}{16d\left(a^2-ia^2\tan(c+dx)\right)}+\frac{3i}{16d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{x}{4a^2}+\frac{ia}{12d(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] x/(4*a^2) + ((I/12)*a)/(d*(a + I*a*Tan[c + d*x])^3) + (I/8)/(d*(a + I*a*Tan
[c + d*x])^2) - (I/16)/(d*(a^2 - I*a^2*Tan[c + d*x])) + ((3*I)/16)/(d*(a^2
+ I*a^2*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^4}dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\,\text{Subst}\left(\int\left(\frac{1}{16a^4(a-x)^2}+\frac{1}{4a^2(a+x)^4}+\frac{1}{4a^3(a+x)^3}+\frac{3}{16a^4(a+x)^2}+\frac{1}{4a^4(a^2-x^2)}\right)dx, x, ia\tan(c+\right.}{d}$$

$$= \frac{ia}{12d(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^2}-\frac{i}{16d\left(a^2-ia^2\tan(c+dx)\right)}+\frac{}{16d\left(a^2\right.}$$

$$= \frac{x}{4a^2}+\frac{ia}{12d(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^2}-\frac{i}{16d\left(a^2-ia^2\tan(c+dx)\right)}+\frac{}{1}$$

Mathematica [A] time = 0.25752, size = 95, normalized size = 0.83

$$\frac{i\sec^2(c+dx)(-12dx\sin(2(c+dx))+3i\sin(2(c+dx))+2i\sin(4(c+dx))+(-3+12idx)\cos(2(c+dx))+\cos(4(c+dx))}{48a^2d(\tan(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/48)*Sec[c + d*x]^2*(-9 + (-3 + (12*I)*d*x)*Cos[2*(c + d*x)] + Cos[4*(c
+ d*x)] + (3*I)*Sin[2*(c + d*x)] - 12*d*x*Sin[2*(c + d*x)] + (2*I)*Sin[4*(c
 + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A] time = 0.093, size = 117, normalized size = 1.

$$\frac{-\frac{i}{8}\ln(\tan(dx+c)-i)}{a^2d}-\frac{\frac{i}{8}}{a^2d(\tan(dx+c)-i)^2}-\frac{1}{12\,a^2d(\tan(dx+c)-i)^3}+\frac{3}{16\,a^2d(\tan(dx+c)-i)}+\frac{\frac{i}{8}\ln(\tan(dx-}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -1/8*I/a^2/d*ln(tan(d*x+c)-I)-1/8*I/a^2/d/(tan(d*x+c)-I)^2-1/12/a^2/d/(tan(
d*x+c)-I)^3+3/16/a^2/d/(tan(d*x+c)-I)+1/8*I/a^2/d*ln(tan(d*x+c)+I)+1/16/a^2
/d/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.24551, size = 198, normalized size = 1.74

$$\frac{\left(24\,dxe^{(6i\,dx+6i\,c)} - 3i\,e^{(8i\,dx+8i\,c)} + 18i\,e^{(4i\,dx+4i\,c)} + 6i\,e^{(2i\,dx+2i\,c)} + i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/96*(24*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(8*I*d*x + 8*I*c) + 18*I*e^(4*I*d*
x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [A] time = 0.680492, size = 190, normalized size = 1.67

$$\begin{cases} \frac{\left(-24576ia^6d^3e^{14ic}e^{2idx}+147456ia^6d^3e^{10ic}e^{-2idx}+49152ia^6d^3e^{8ic}e^{-4idx}+8192ia^6d^3e^{6ic}e^{-6idx}\right)e^{-12ic}}{786432a^8d^4} & \text{for } 786432a^8d^4e^{12ic} \neq 0 \\ x\left(\frac{\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-6ic}}{16a^2} - \frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-24576*I*a**6*d**3*exp(14*I*c)*exp(2*I*d*x) + 147456*I*a**6*d**
3*exp(10*I*c)*exp(-2*I*d*x) + 49152*I*a**6*d**3*exp(8*I*c)*exp(-4*I*d*x) +
8192*I*a**6*d**3*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(786432*a**8*d**4),
 Ne(786432*a**8*d**4*exp(12*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*e
xp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-6*I*c)/(16*a**2) - 1/(4*a**2)), True)) +
 x/(4*a**2)
```

Giac [A] time = 1.16101, size = 139, normalized size = 1.22

$$-\frac{-\frac{6i\,\log(\tan(dx+c)+i)}{a^2} + \frac{6i\,\log(\tan(dx+c)-i)}{a^2} + \frac{3\,(2i\,\tan(dx+c)-3)}{a^2(\tan(dx+c)+i)} + \frac{-11i\,\tan(dx+c)^3-42\,\tan(dx+c)^2+57i\,\tan(dx+c)+30}{a^2(\tan(dx+c)-i)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/48*(-6*I*log(tan(d*x + c) + I)/a^2 + 6*I*log(tan(d*x + c) - I)/a^2 + 3*(
2*I*tan(d*x + c) - 3)/(a^2*(tan(d*x + c) + I)) + (-11*I*tan(d*x + c)^3 - 42
*tan(d*x + c)^2 + 57*I*tan(d*x + c) + 30)/(a^2*(tan(d*x + c) - I)^3))/d
```

3.121 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=165

$$\frac{ia^2}{32d(a+ia\tan(c+dx))^4}-\frac{5i}{64d\left(a^2-ia^2\tan(c+dx)\right)}+\frac{5i}{32d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{15x}{64a^2}+\frac{ia}{16d(a+ia\tan(c+dx))^3}-$$

```
[Out] (15*x)/(64*a^2) - (I/64)/(d*(a - I*a*Tan[c + d*x])^2) + ((I/32)*a^2)/(d*(a
+ I*a*Tan[c + d*x])^4) + ((I/16)*a)/(d*(a + I*a*Tan[c + d*x])^3) + ((3*I)/3
2)/(d*(a + I*a*Tan[c + d*x])^2) - ((5*I)/64)/(d*(a^2 - I*a^2*Tan[c + d*x]))
 + ((5*I)/32)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.103233, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$\frac{ia^2}{32d(a+ia\tan(c+dx))^4}-\frac{5i}{64d\left(a^2-ia^2\tan(c+dx)\right)}+\frac{5i}{32d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{15x}{64a^2}+\frac{ia}{16d(a+ia\tan(c+dx))^3}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (15*x)/(64*a^2) - (I/64)/(d*(a - I*a*Tan[c + d*x])^2) + ((I/32)*a^2)/(d*(a
+ I*a*Tan[c + d*x])^4) + ((I/16)*a)/(d*(a + I*a*Tan[c + d*x])^3) + ((3*I)/3
2)/(d*(a + I*a*Tan[c + d*x])^2) - ((5*I)/64)/(d*(a^2 - I*a^2*Tan[c + d*x]))
 + ((5*I)/32)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^5}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^3} + \frac{5}{64a^6(a-x)^2} + \frac{1}{8a^3(a+x)^5} + \frac{3}{16a^4(a+x)^4} + \frac{3}{16a^5(a+x)^3} + \frac{5}{32a^6(a+x)^2} + \right.\right.}{d}$$
$$= -\frac{i}{64d(a-ia\tan(c+dx))^2} + \frac{ia^2}{32d(a+ia\tan(c+dx))^4} + \frac{ia}{16d(a+ia\tan(c+dx))^3} + \frac{}{32}$$
$$= \frac{15x}{64a^2} - \frac{i}{64d(a-ia\tan(c+dx))^2} + \frac{ia^2}{32d(a+ia\tan(c+dx))^4} + \frac{ia}{16d(a+ia\tan(c+dx))^3}$$

Mathematica [A] time = 0.252469, size = 120, normalized size = 0.73

$$\frac{i\sec^2(c+dx)(-120dx\sin(2(c+dx)) + 30i\sin(2(c+dx)) + 32i\sin(4(c+dx)) + 3i\sin(6(c+dx)) + 30i(4dx+i)\cos(2}{512a^2d(\tan(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/512)*Sec[c + d*x]^2*(-80 + (30*I)*(I + 4*d*x)*Cos[2*(c + d*x)] + 16*Cos
[4*(c + d*x)] + Cos[6*(c + d*x)] + (30*I)*Sin[2*(c + d*x)] - 120*d*x*Sin[2*
(c + d*x)] + (32*I)*Sin[4*(c + d*x)] + (3*I)*Sin[6*(c + d*x)]))/(a^2*d*(-I
+ Tan[c + d*x])^2)
```

Maple [A] time = 0.094, size = 157, normalized size = 1.

$$\frac{-\frac{15i}{128}\ln(\tan(dx+c)-i)}{a^2d} + \frac{\frac{i}{32}}{a^2d(\tan(dx+c)-i)^4} - \frac{\frac{3i}{32}}{a^2d(\tan(dx+c)-i)^2} - \frac{1}{16a^2d(\tan(dx+c)-i)^3} + \frac{}{32a^2d(\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -15/128*I/a^2/d*ln(tan(d*x+c)-I)+1/32*I/a^2/d/(tan(d*x+c)-I)^4-3/32*I/a^2/d
/(tan(d*x+c)-I)^2-1/16/a^2/d/(tan(d*x+c)-I)^3+5/32/a^2/d/(tan(d*x+c)-I)+1/6
4*I/a^2/d/(tan(d*x+c)+I)^2+15/128*I/a^2/d*ln(tan(d*x+c)+I)+5/64/a^2/d/(tan(
d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.29797, size = 279, normalized size = 1.69

$$\frac{\left(120\,dxe^{(8i\,dx+8i\,c)} - 2i\,e^{(12i\,dx+12i\,c)} - 24i\,e^{(10i\,dx+10i\,c)} + 80i\,e^{(6i\,dx+6i\,c)} + 30i\,e^{(4i\,dx+4i\,c)} + 8i\,e^{(2i\,dx+2i\,c)} + i\right)e^{(-8i\,dx-8i\,c)}}{512\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/512*(120*d*x*e^(8*I*d*x + 8*I*c) - 2*I*e^(12*I*d*x + 12*I*c) - 24*I*e^(10
*I*d*x + 10*I*c) + 80*I*e^(6*I*d*x + 6*I*c) + 30*I*e^(4*I*d*x + 4*I*c) + 8*
I*e^(2*I*d*x + 2*I*c) + I)*e^(-8*I*d*x - 8*I*c)/(a^2*d)
```

Sympy [A] time = 1.17119, size = 260, normalized size = 1.58

$$\begin{cases} \frac{\left(-17179869184ia^{10}d^5e^{24ic}e^{4idx} - 206158430208ia^{10}d^5e^{22ic}e^{2idx} + 687194767360ia^{10}d^5e^{18ic}e^{-2idx} + 257698037760ia^{10}d^5e^{16ic}e^{-4idx} + 68719476736ia^{10}d^5e^{14ic}e^{-6idx}\right.}{4398046511104a^{12}d^6} \\ x\left(\frac{\left(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1\right)e^{-8ic}}{64a^2} - \frac{15}{64a^2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-17179869184*I*a**10*d**5*exp(24*I*c)*exp(4*I*d*x) - 2061584302
08*I*a**10*d**5*exp(22*I*c)*exp(2*I*d*x) + 687194767360*I*a**10*d**5*exp(18
*I*c)*exp(-2*I*d*x) + 257698037760*I*a**10*d**5*exp(16*I*c)*exp(-4*I*d*x) +
 68719476736*I*a**10*d**5*exp(14*I*c)*exp(-6*I*d*x) + 8589934592*I*a**10*d*
*5*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(4398046511104*a**12*d**6), Ne(4
398046511104*a**12*d**6*exp(20*I*c), 0)), (x*((exp(12*I*c) + 6*exp(10*I*c)
+ 15*exp(8*I*c) + 20*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-8*
I*c)/(64*a**2) - 15/(64*a**2)), True)) + 15*x/(64*a**2)
```

Giac [A] time = 1.14435, size = 171, normalized size = 1.04

$$-\frac{\frac{60i\log(i\tan(dx+c)+1)}{a^2} - \frac{60i\log(i\tan(dx+c)-1)}{a^2} + \frac{2\left(45i\tan(dx+c)^2-110\tan(dx+c)-69i\right)}{a^2(\tan(dx+c)+i)^2} + \frac{-125i\tan(dx+c)^4-580\tan(dx+c)^3+1038i\tan(dx+c)^2+8}{a^2(\tan(dx+c)-i)^4}}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/512*(60*I*log(I*tan(d*x + c) + 1)/a^2 - 60*I*log(I*tan(d*x + c) - 1)/a^2
 + 2*(45*I*tan(d*x + c)^2 - 110*tan(d*x + c) - 69*I)/(a^2*(tan(d*x + c) + I
)^2) + (-125*I*tan(d*x + c)^4 - 580*tan(d*x + c)^3 + 1038*I*tan(d*x + c)^2
+ 868*tan(d*x + c) - 301*I)/(a^2*(tan(d*x + c) - I)^4))/d
```

3.122 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2} dx$

Optimal. Leaf size=124

$$\frac{7\tanh^{-1}(\sin(c+dx))}{16a^2d} - \frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{7\tan(c+dx)\sec^5(c+dx)}{30a^2d} + \frac{7\tan(c+dx)\sec^3(c+dx)}{24a^2d} + \frac{7\tan$$

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(16*a^2*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(16*a^
2*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(24*a^2*d) + (7*Sec[c + d*x]^5*Tan[c
 + d*x])/(30*a^2*d) - (((2*I)/5)*Sec[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rubi [A] time = 0.0814793, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3500, 3768, 3770}

$$\frac{7\tanh^{-1}(\sin(c+dx))}{16a^2d} - \frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{7\tan(c+dx)\sec^5(c+dx)}{30a^2d} + \frac{7\tan(c+dx)\sec^3(c+dx)}{24a^2d} + \frac{7\tan$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(16*a^2*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(16*a^
2*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(24*a^2*d) + (7*Sec[c + d*x]^5*Tan[c
 + d*x])/(30*a^2*d) - (((2*I)/5)*Sec[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{7\int \sec^7(c+dx)\,dx}{5a^2}$$
$$= \frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d} - \frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{7\int \sec^5(c+dx)\,dx}{6a^2}$$
$$= \frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d} + \frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d} - \frac{2i\sec^7(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{7\int}{}$$
$$= \frac{7\sec(c+dx)\tan(c+dx)}{16a^2d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d} + \frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d} - \frac{}{5d(}$$
$$= \frac{7\tanh^{-1}(\sin(c+dx))}{16a^2d} + \frac{7\sec(c+dx)\tan(c+dx)}{16a^2d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d} + \frac{7\sec^5(c}{}$$

Mathematica [B] time = 1.79663, size = 294, normalized size = 2.37

$$-\frac{\sec^6(c+dx)\left(5\left(60\sin(c+dx)-238\sin(3(c+dx))-42\sin(5(c+dx))+21\cos(6(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]^6*((3072*I)*Cos[c + d*x] + 5*(210*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + 21*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + 315*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 126*Cos[4*(c + d*x)]*(Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 21
0*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 21*Cos[6*(c + d*x)]*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] + 60*Sin[c + d*x] - 238*Sin[3*(c + d*x)] - 4
2*Sin[5*(c + d*x)])))/(7680*a^2*d)
```

Maple [B] time = 0.091, size = 514, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/2/a^2/d/(tan(1/2*d*x+1/2*c)+1)^4+3/2*I/a^2/d/(tan(1/2*d*x+1/2*c)-1)^3+9/1
6/a^2/d/(tan(1/2*d*x+1/2*c)+1)+3/4*I/a^2/d/(tan(1/2*d*x+1/2*c)-1)-1/6/a^2/d
/(tan(1/2*d*x+1/2*c)+1)^3+2/5*I/a^2/d/(tan(1/2*d*x+1/2*c)-1)^5-1/2/a^2/d/(t
an(1/2*d*x+1/2*c)+1)^5+5/4*I/a^2/d/(tan(1/2*d*x+1/2*c)-1)^2-9/16/a^2/d/(tan
(1/2*d*x+1/2*c)+1)^2-3/4*I/a^2/d/(tan(1/2*d*x+1/2*c)+1)+1/6/a^2/d/(tan(1/2*
d*x+1/2*c)+1)^6+7/16/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)-1/2/a^2/d/(tan(1/2*d*x+
1/2*c)-1)^4+I/a^2/d/(tan(1/2*d*x+1/2*c)+1)^4+9/16/a^2/d/(tan(1/2*d*x+1/2*c)
-1)^2-2/5*I/a^2/d/(tan(1/2*d*x+1/2*c)+1)^5+9/16/a^2/d/(tan(1/2*d*x+1/2*c)-1
)-3/2*I/a^2/d/(tan(1/2*d*x+1/2*c)+1)^3-1/6/a^2/d/(tan(1/2*d*x+1/2*c)-1)^3+5
/4*I/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2-1/2/a^2/d/(tan(1/2*d*x+1/2*c)-1)^5+I/a^
2/d/(tan(1/2*d*x+1/2*c)-1)^4-1/6/a^2/d/(tan(1/2*d*x+1/2*c)-1)^6-7/16/a^2/d*
ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.04093, size = 568, normalized size = 4.58

$$\frac{2\left(\frac{135 \sin(dx+c)}{\cos(dx+c)+1} + \frac{96i \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{445 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{960i \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{330 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{960i \sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{330 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{480i \sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{445 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} + \frac{480i \sin(dx+c}{(\cos(dx+c)+1}}{a^2 - \frac{6a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{15a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{20a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{15a^2 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{6a^2 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{a^2 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}$$
$$240\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(2*(135*sin(d*x + c)/(cos(d*x + c) + 1) + 96*I*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 445*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 960*I*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 330*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 960*I*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 330*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 - 480*I*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 445*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 + 480*I*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 135*sin(d*x +
 c)^11/(cos(d*x + c) + 1)^11 - 96*I)/(a^2 - 6*a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^2*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*
a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12) + 105*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 - 105*log(si
n(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [B] time = 2.49574, size = 996, normalized size = 8.03

$$\frac{105\left(e^{(12i\,dx+12i\,c)} + 6\,e^{(10i\,dx+10i\,c)} + 15\,e^{(8i\,dx+8i\,c)} + 20\,e^{(6i\,dx+6i\,c)} + 15\,e^{(4i\,dx+4i\,c)} + 6\,e^{(2i\,dx+2i\,c)} + 1\right)\log\left(e^{(i\,dx+i\,c)} + i\right) - 1}{240\left(a^2de\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(105*(e^(12*I*d*x + 12*I*c) + 6*e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*x
 + 8*I*c) + 20*e^(6*I*d*x + 6*I*c) + 15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x
+ 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 105*(e^(12*I*d*x + 12*I*c) + 6*e^(
10*I*d*x + 10*I*c) + 15*e^(8*I*d*x + 8*I*c) + 20*e^(6*I*d*x + 6*I*c) + 15*e
^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) -
210*I*e^(11*I*d*x + 11*I*c) - 1190*I*e^(9*I*d*x + 9*I*c) - 2772*I*e^(7*I*d*
x + 7*I*c) - 3372*I*e^(5*I*d*x + 5*I*c) + 1190*I*e^(3*I*d*x + 3*I*c) + 210*
I*e^(I*d*x + I*c))/(a^2*d*e^(12*I*d*x + 12*I*c) + 6*a^2*d*e^(10*I*d*x + 10*
I*c) + 15*a^2*d*e^(8*I*d*x + 8*I*c) + 20*a^2*d*e^(6*I*d*x + 6*I*c) + 15*a^2
*d*e^(4*I*d*x + 4*I*c) + 6*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.21702, size = 277, normalized size = 2.23

$$\frac{\frac{105 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{105 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{2\left(135 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+480i \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-445 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-480i \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}}{}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(105*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 105*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^2 + 2*(135*tan(1/2*d*x + 1/2*c)^11 + 480*I*tan(1/2*d*x +
1/2*c)^10 - 445*tan(1/2*d*x + 1/2*c)^9 - 480*I*tan(1/2*d*x + 1/2*c)^8 - 330
*tan(1/2*d*x + 1/2*c)^7 + 960*I*tan(1/2*d*x + 1/2*c)^6 - 330*tan(1/2*d*x +
1/2*c)^5 - 960*I*tan(1/2*d*x + 1/2*c)^4 - 445*tan(1/2*d*x + 1/2*c)^3 + 96*I
*tan(1/2*d*x + 1/2*c)^2 + 135*tan(1/2*d*x + 1/2*c) - 96*I)/((tan(1/2*d*x +
1/2*c)^2 - 1)^6*a^2))/d
```

3.123 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{5\tan(c+dx)\sec^3(c+dx)}{12a^2d}+\frac{5\tan(c+dx)\sec(c+dx)}{8a^2d}$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(8*a^2*d) + (5*Sec[c + d*x]*Tan[c + d*x])/(8*a^2*
d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(12*a^2*d) - (((2*I)/3)*Sec[c + d*x]^5
)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.0676607, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3500, 3768, 3770}

$$\frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{5\tan(c+dx)\sec^3(c+dx)}{12a^2d}+\frac{5\tan(c+dx)\sec(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(8*a^2*d) + (5*Sec[c + d*x]*Tan[c + d*x])/(8*a^2*
d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(12*a^2*d) - (((2*I)/3)*Sec[c + d*x]^5
)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{5\int \sec^5(c+dx)\,dx}{3a^2}$$
$$= \frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d} - \frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{5\int \sec^3(c+dx)\,dx}{4a^2}$$
$$= \frac{5\sec(c+dx)\tan(c+dx)}{8a^2d} + \frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d} - \frac{2i\sec^5(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{5\int \mathrm{s}$$
$$= \frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d} + \frac{5\sec(c+dx)\tan(c+dx)}{8a^2d} + \frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d} - \frac{2i}{3d\left(a^2+\right.}$$

Mathematica [B] time = 0.707641, size = 215, normalized size = 2.15

$$-\frac{\sec^4(c+dx)\left(18\sin(c+dx) - 30\sin(3(c+dx)) + 128i\cos(c+dx) + 45\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 60\,\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]^4*((128*I)*Cos[c + d*x] + 45*Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] + 60*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 15*Cos[4*(c + d*x)]*(Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
 - 45*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 18*Sin[c + d*x] - 30*Sin[3
*(c + d*x)]))/(192*a^2*d)
```

Maple [B] time = 0.086, size = 342, normalized size = 3.4

$$\frac{3}{8a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} + \frac{\frac{2i}{3}}{a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3} - \frac{1}{8a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} + \frac{i}{a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} - $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 3/8/a^2/d/(tan(1/2*d*x+1/2*c)+1)+2/3*I/a^2/d/(tan(1/2*d*x+1/2*c)-1)^3-1/8/a
^2/d/(tan(1/2*d*x+1/2*c)+1)^2+I/a^2/d/(tan(1/2*d*x+1/2*c)-1)-1/2/a^2/d/(tan
(1/2*d*x+1/2*c)+1)^3-I/a^2/d/(tan(1/2*d*x+1/2*c)+1)+1/4/a^2/d/(tan(1/2*d*x+
1/2*c)+1)^4+5/8/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)+3/8/a^2/d/(tan(1/2*d*x+1/2*c
)-1)-2/3*I/a^2/d/(tan(1/2*d*x+1/2*c)+1)^3+1/8/a^2/d/(tan(1/2*d*x+1/2*c)-1)^
2+I/a^2/d/(tan(1/2*d*x+1/2*c)+1)^2-1/2/a^2/d/(tan(1/2*d*x+1/2*c)-1)^3+I/a^2
/d/(tan(1/2*d*x+1/2*c)-1)^2-1/4/a^2/d/(tan(1/2*d*x+1/2*c)-1)^4-5/8/a^2/d*ln
(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.09258, size = 398, normalized size = 3.98

$$\frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{48i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{48i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-16i\right)}{a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}} + \frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} - \frac{15\log\left(\frac{\sin(d}{\cos(dx}\right.}{a^2}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*I*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*I*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*I*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 16*I
)/(a^2 - 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^2 - 15*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [B] time = 2.37643, size = 667, normalized size = 6.67

$$\frac{15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+}\right.}{24\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,dx+6i\,c)}+6\,a^2de^{(4i}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(15*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*
c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(8*I*d*x +
 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x + 2*
I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(7*I*d*x + 7*I*c) - 110*I*e^(5*
I*d*x + 5*I*c) - 146*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))/(a^2*d*e
^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^(4*I*d*x + 4*I
*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.24592, size = 207, normalized size = 2.07

$$\frac{\frac{15\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{15\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+48i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-48i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-33\,\mathrm{t}\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 15*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 48*I*tan(1/2*d*x + 1/2*c)^
6 - 33*tan(1/2*d*x + 1/2*c)^5 - 48*I*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*
x + 1/2*c)^3 + 16*I*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 16*I)
/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^2))/d
```

3.124 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=74

$$\frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{3\tan(c+dx)\sec(c+dx)}{2a^2d}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*
d) - ((2*I)*Sec[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.0580909, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3500, 3768, 3770}

$$\frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{3\tan(c+dx)\sec(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*
d) - ((2*I)*Sec[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= -\frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{3\int \sec^3(c+dx)\,dx}{a^2} \\
&= \frac{3\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{3\int \sec(c+dx)\,dx}{2a^2} \\
&= \frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d} + \frac{3\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{2i\sec^3(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] time = 0.36504, size = 146, normalized size = 1.97

$$
-\frac{\sec^2(c+dx)\left(2\sin(c+dx)+8i\cos(c+dx)+3\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\left(\log\left(\cos\left(\tfrac{1}{2}(\right.\right.\right.}{4a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]^2*((8*I)*Cos[c + d*x] + 3*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 3*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 2*Sin[c + d*x]))/(4*a^2*d)
```

Maple [B] time = 0.078, size = 170, normalized size = 2.3

$$
-\frac{1}{2a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{2i}{a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} + \frac{1}{2a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} + \frac{3}{2a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -1/2/a^2/d/(tan(1/2*d*x+1/2*c)+1)-2*I/a^2/d/(tan(1/2*d*x+1/2*c)+1)+1/2/a^2/
d/(tan(1/2*d*x+1/2*c)+1)^2+3/2/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)-1/2/a^2/d/(ta
n(1/2*d*x+1/2*c)-1)+2*I/a^2/d/(tan(1/2*d*x+1/2*c)-1)-1/2/a^2/d/(tan(1/2*d*x
+1/2*c)-1)^2-3/2/a^2/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.0168, size = 225, normalized size = 3.04

$$
-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+4i\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} - \frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{2d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*I)/(a^2 - 2*a^2*sin(d*x +
```

```
 c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*l
og(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^2)/d
```

Fricas [B] time = 2.42679, size = 375, normalized size = 5.07

$$\frac{3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(3i\,dx+3i\,c)}-1}{2\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c
) + I) - 3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
 I*c) - I) - 6*I*e^(3*I*d*x + 3*I*c) - 10*I*e^(I*d*x + I*c))/(a^2*d*e^(4*I*
d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.19146, size = 131, normalized size = 1.77

$$\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a^2 - 2*(tan(1/2*d*x + 1/2*c)^3 - 4*I*tan(1/2*d*x + 1/2*c)^2 + ta
n(1/2*d*x + 1/2*c) + 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

3.125 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=48

$$-\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*I)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan
[c + d*x]))
```

Rubi [A] time = 0.0471635, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3500, 3770}

$$-\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*I)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan
[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{\int \sec(c+dx)\,dx}{a^2}\\ &= -\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [B] time = 0.177561, size = 184, normalized size = 3.83

$$-\frac{\sec^2(c+dx)\left(\cos\left(\frac{3}{2}(c+dx)\right)+i\sin\left(\frac{3}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -((Sec[c + d*x]^2*(Cos[(c + d*x)/2]*(2*I + Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 + I*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] - I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*S
in[(c + d*x)/2])*(Cos[(3*(c + d*x))/2] + I*Sin[(3*(c + d*x))/2]))/(a^2*d*(-
I + Tan[c + d*x])^2))
```

Maple [A] time = 0.073, size = 63, normalized size = 1.3

$$-\frac{1}{a^2 d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{1}{a^2 d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\,\frac{1}{a^2 d\left(\tan\left(1/2\,dx+c/2\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -1/a^2/d*ln(tan(1/2*d*x+1/2*c)+1)+1/a^2/d*ln(tan(1/2*d*x+1/2*c)-1)+4/a^2/d/
(tan(1/2*d*x+1/2*c)-I)
```

Maxima [B] time = 1.49095, size = 158, normalized size = 3.29

$$-\frac{-2i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-2i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)-4i\cos\left(dx+c\right)+\log\left(\cos\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(-2*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*I*arctan2(cos(d*x +
c), -sin(d*x + c) + 1) - 4*I*cos(d*x + c) + log(cos(d*x + c)^2 + sin(d*x +
c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*
x + c) + 1) - 4*sin(d*x + c))/(a^2*d)
```

Fricas [A] time = 2.30481, size = 161, normalized size = 3.35

$$-\frac{\left(e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\right)e^{(-i\,dx-i\,c)}}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(e^(I*d*x + I*c)*log(e^(I*d*x + I*c) + I) - e^(I*d*x + I*c)*log(e^(I*d*x +
 I*c) - I) - 2*I)*e^(-I*d*x - I*c)/(a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.26314, size = 80, normalized size = 1.67

$$-\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1
))/a^2 - 4/(a^2*(tan(1/2*d*x + 1/2*c) - I)))/d
```

3.126 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{i\sec(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}$$

```
[Out] ((I/3)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(d
*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.0539423, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3502, 3488}

$$\frac{i\sec(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/3)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(d
*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}+\frac{\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx}{3a}\\ &= \frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}+\frac{i\sec(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.0738067, size = 38, normalized size = 0.58

$$\frac{(\tan(c+dx)-2i)\sec(c+dx)}{3a^2d(\tan(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(-2*I + Tan[c + d*x]))/(3*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A] time = 0.044, size = 57, normalized size = 0.9

$$2\frac{1}{a^2d}\left(\frac{i}{\left(\tan\left(1/2\,dx+c/2\right)-i\right)^2}+\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-1}-2/3\,\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(I/(tan(1/2*d*x+1/2*c)-I)^2+1/(tan(1/2*d*x+1/2*c)-I)-2/3/(tan(1/2*d
*x+1/2*c)-I)^3)
```

Maxima [A] time = 0.981063, size = 61, normalized size = 0.94

$$\frac{i\,\cos\left(3\,dx+3\,c\right)+3i\,\cos\left(dx+c\right)+\sin\left(3\,dx+3\,c\right)+3\,\sin\left(dx+c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) + sin(3*d*x + 3*c) + 3*sin(d*x +
 c))/(a^2*d)
```

Fricas [A] time = 2.21306, size = 86, normalized size = 1.32

$$\frac{\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.15592, size = 63, normalized size = 0.97

$$\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\right)}{3a^2d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 3*I*tan(1/2*d*x + 1/2*c) - 2)/(a^2*d*(tan(1
/2*d*x + 1/2*c) - I)^3)
```

3.127 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$-\frac{\sin^3(c+dx)}{5a^2d}+\frac{3\sin(c+dx)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] (3*Sin[c + d*x])/(5*a^2*d) - Sin[c + d*x]^3/(5*a^2*d) + (((2*I)/5)*Cos[c +
d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.048421, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3500, 2633}

$$-\frac{\sin^3(c+dx)}{5a^2d}+\frac{3\sin(c+dx)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*Sin[c + d*x])/(5*a^2*d) - Sin[c + d*x]^3/(5*a^2*d) + (((2*I)/5)*Cos[c +
d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{3\int \cos^3(c+dx)\,dx}{5a^2}\\ &= \frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{3\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{5a^2d}\\ &= \frac{3\sin(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.2722, size = 68, normalized size = 0.96

$$\frac{\sec(c+dx)(4i\cos(2(c+dx))+5\tan(c+dx)-3\sin(3(c+dx))\sec(c+dx)-12i)}{20a^2d(\tan(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(-12*I + (4*I)*Cos[2*(c + d*x)] - 3*Sec[c + d*x]*Sin[3*(c + d
*x)] + 5*Tan[c + d*x]))/(20*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A] time = 0.087, size = 108, normalized size = 1.5

$$2\frac{1}{a^2d}\left(\frac{-i}{(\tan(1/2\,dx+c/2)-i)^4}+\frac{5/4\,i}{(\tan(1/2\,dx+c/2)-i)^2}+2/5\,(\tan(1/2\,dx+c/2)-i)^{-5}-3/2\,(\tan(1/2\,dx+c/2)-i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(-I/(tan(1/2*d*x+1/2*c)-I)^4+5/4*I/(tan(1/2*d*x+1/2*c)-I)^2+2/5/(ta
n(1/2*d*x+1/2*c)-I)^5-3/2/(tan(1/2*d*x+1/2*c)-I)^3+7/8/(tan(1/2*d*x+1/2*c)-
I)+1/8/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.24008, size = 161, normalized size = 2.27

$$\frac{\left(-5i\,e^{(6i\,dx+6i\,c)}+15i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i\,c)}}{40\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/40*(-5*I*e^(6*I*d*x + 6*I*c) + 15*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x
+ 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [A] time = 0.722779, size = 165, normalized size = 2.32

$$\begin{cases}\frac{\left(-2560ia^6d^3e^{10ic}e^{idx}+7680ia^6d^3e^{8ic}e^{-idx}+2560ia^6d^3e^{6ic}e^{-3idx}+512ia^6d^3e^{4ic}e^{-5idx}\right)e^{-9ic}}{20480a^8d^4} & \text{for } 20480a^8d^4e^{9ic}\neq 0\\ \frac{x\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-5ic}}{8a^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-2560*I*a**6*d**3*exp(10*I*c)*exp(I*d*x) + 7680*I*a**6*d**3*exp
(8*I*c)*exp(-I*d*x) + 2560*I*a**6*d**3*exp(6*I*c)*exp(-3*I*d*x) + 512*I*a**
6*d**3*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(20480*a**8*d**4), Ne(20480*a*
*8*d**4*exp(9*I*c), 0)), (x*(exp(6*I*c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*
exp(-5*I*c)/(8*a**2), True))
```

Giac [A] time = 1.15062, size = 126, normalized size = 1.77

$$\frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)}+\frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-90i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+70i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+21}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^5}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) + I)) + (35*tan(1/2*d*x + 1/2*c)^4 - 90*
I*tan(1/2*d*x + 1/2*c)^3 - 120*tan(1/2*d*x + 1/2*c)^2 + 70*I*tan(1/2*d*x +
1/2*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

3.128 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{\sin^5(c+dx)}{7a^2d}-\frac{10\sin^3(c+dx)}{21a^2d}+\frac{5\sin(c+dx)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] (5*Sin[c + d*x])/(7*a^2*d) - (10*Sin[c + d*x]^3)/(21*a^2*d) + Sin[c + d*x]^
5/(7*a^2*d) + (((2*I)/7)*Cos[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.0585017, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3500, 2633}

$$\frac{\sin^5(c+dx)}{7a^2d}-\frac{10\sin^3(c+dx)}{21a^2d}+\frac{5\sin(c+dx)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*Sin[c + d*x])/(7*a^2*d) - (10*Sin[c + d*x]^3)/(21*a^2*d) + Sin[c + d*x]^
5/(7*a^2*d) + (((2*I)/7)*Cos[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{5\int \cos^5(c+dx)\,dx}{7a^2}\\ &= \frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{5\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{7a^2d}\\ &= \frac{5\sin(c+dx)}{7a^2d}-\frac{10\sin^3(c+dx)}{21a^2d}+\frac{\sin^5(c+dx)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.19092, size = 95, normalized size = 1.07

$$\frac{i\sec^2(c+dx)(-70i\sin(c+dx)+63i\sin(3(c+dx))+5i\sin(5(c+dx))-140\cos(c+dx)+42\cos(3(c+dx))+2\cos(5}{336a^2d(\tan(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/336)*Sec[c + d*x]^2*(-140*Cos[c + d*x] + 42*Cos[3*(c + d*x)] + 2*Cos[5*
(c + d*x)] - (70*I)*Sin[c + d*x] + (63*I)*Sin[3*(c + d*x)] + (5*I)*Sin[5*(c
 + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [B] time = 0.09, size = 174, normalized size = 2.

$$2\frac{1}{a^2d}\left(\frac{i}{(\tan(1/2\,dx+c/2)-i)^6}-\frac{5/2\,i}{(\tan(1/2\,dx+c/2)-i)^4}+\frac{\frac{23\,i}{16}}{(\tan(1/2\,dx+c/2)-i)^2}-2/7\,(\tan(1/2\,dx+c/2)-i)^{-7}+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(I/(tan(1/2*d*x+1/2*c)-I)^6-5/2*I/(tan(1/2*d*x+1/2*c)-I)^4+23/16*I/
(tan(1/2*d*x+1/2*c)-I)^2-2/7/(tan(1/2*d*x+1/2*c)-I)^7+2/(tan(1/2*d*x+1/2*c)
-I)^5-55/24/(tan(1/2*d*x+1/2*c)-I)^3+13/16/(tan(1/2*d*x+1/2*c)-I)-1/16*I/(t
an(1/2*d*x+1/2*c)+I)^2-1/24/(tan(1/2*d*x+1/2*c)+I)^3+3/16/(tan(1/2*d*x+1/2*
c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.32369, size = 244, normalized size = 2.74

$$\frac{\left(-7i\,e^{(10i\,dx+10i\,c)}-105i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-7i\,dx-7i\,c)}}{672\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/672*(-7*I*e^(10*I*d*x + 10*I*c) - 105*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(6*
I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*x + 2*I*c) + 3*I)
*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

Sympy [A] time = 1.22719, size = 233, normalized size = 2.62

$$\begin{cases}\frac{\left(-176160768ia^{10}d^5e^{19ic}e^{3idx}-2642411520ia^{10}d^5e^{17ic}e^{idx}+5284823040ia^{10}d^5e^{15ic}e^{-idx}+1761607680ia^{10}d^5e^{13ic}e^{-3idx}+528482304ia^{10}d^5e^{11ic}e^{-5idx}+75497472ia^{1}\right.}{16911433728a^{12}d^6} \\ \frac{x\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-7ic}}{32a^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-176160768*I*a**10*d**5*exp(19*I*c)*exp(3*I*d*x) - 2642411520*I
*a**10*d**5*exp(17*I*c)*exp(I*d*x) + 5284823040*I*a**10*d**5*exp(15*I*c)*ex
p(-I*d*x) + 1761607680*I*a**10*d**5*exp(13*I*c)*exp(-3*I*d*x) + 528482304*I
*a**10*d**5*exp(11*I*c)*exp(-5*I*d*x) + 75497472*I*a**10*d**5*exp(9*I*c)*ex
p(-7*I*d*x))*exp(-16*I*c)/(16911433728*a**12*d**6), Ne(16911433728*a**12*d*
*6*exp(16*I*c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*ex
p(4*I*c) + 5*exp(2*I*c) + 1)*exp(-7*I*c)/(32*a**2), True))
```

Giac [A] time = 1.1495, size = 196, normalized size = 2.2

$$\frac{\frac{7\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{273\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1155i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2870i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2037\tan\left(\frac{1}{2}\cdots\right.}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^7}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 + 15*I*tan(1/2*d*x + 1/2*c) - 8)/(a^2*(t
an(1/2*d*x + 1/2*c) + I)^3) + (273*tan(1/2*d*x + 1/2*c)^6 - 1155*I*tan(1/2*
d*x + 1/2*c)^5 - 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*I*tan(1/2*d*x + 1/2*c)^
3 + 2037*tan(1/2*d*x + 1/2*c)^2 - 791*I*tan(1/2*d*x + 1/2*c) - 152)/(a^2*(t
an(1/2*d*x + 1/2*c) - I)^7))/d
```

3.129 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=107

$$-\frac{\sin^7(c+dx)}{9a^2d}+\frac{7\sin^5(c+dx)}{15a^2d}-\frac{7\sin^3(c+dx)}{9a^2d}+\frac{7\sin(c+dx)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] (7*Sin[c + d*x])/(9*a^2*d) - (7*Sin[c + d*x]^3)/(9*a^2*d) + (7*Sin[c + d*x]
^5)/(15*a^2*d) - Sin[c + d*x]^7/(9*a^2*d) + (((2*I)/9)*Cos[c + d*x]^7)/(d*(
a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.0615628, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3500, 2633}

$$-\frac{\sin^7(c+dx)}{9a^2d}+\frac{7\sin^5(c+dx)}{15a^2d}-\frac{7\sin^3(c+dx)}{9a^2d}+\frac{7\sin(c+dx)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*Sin[c + d*x])/(9*a^2*d) - (7*Sin[c + d*x]^3)/(9*a^2*d) + (7*Sin[c + d*x]
^5)/(15*a^2*d) - Sin[c + d*x]^7/(9*a^2*d) + (((2*I)/9)*Cos[c + d*x]^7)/(d*(
a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx &= \frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{7\int \cos^7(c+dx)\,dx}{9a^2}\\ &= \frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{7\,\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{9a^2d}\\ &= \frac{7\sin(c+dx)}{9a^2d}-\frac{7\sin^3(c+dx)}{9a^2d}+\frac{7\sin^5(c+dx)}{15a^2d}-\frac{\sin^7(c+dx)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.348066, size = 117, normalized size = 1.09

$$\frac{i\sec^2(c+dx)(-525i\sin(c+dx)+567i\sin(3(c+dx))+75i\sin(5(c+dx))+7i\sin(7(c+dx))-1050\cos(c+dx)+37}{2880a^2d(\tan(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/2880)*Sec[c + d*x]^2*(-1050*Cos[c + d*x] + 378*Cos[3*(c + d*x)] + 30*Co
s[5*(c + d*x)] + 2*Cos[7*(c + d*x)] - (525*I)*Sin[c + d*x] + (567*I)*Sin[3*
(c + d*x)] + (75*I)*Sin[5*(c + d*x)] + (7*I)*Sin[7*(c + d*x)]))/(a^2*d*(-I
+ Tan[c + d*x])^2)
```

Maple [B] time = 0.092, size = 240, normalized size = 2.2

$$2\frac{1}{a^2d}\left(\frac{-i}{(\tan(1/2\,dx+c/2)-i)^8}+\frac{\frac{51\,i}{32}}{(\tan(1/2\,dx+c/2)-i)^2}+\frac{\frac{49\,i}{12}}{(\tan(1/2\,dx+c/2)-i)^6}-\frac{\frac{35\,i}{8}}{(\tan(1/2\,dx+c/2)-i)^4}+2/9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(-I/(tan(1/2*d*x+1/2*c)-I)^8+51/32*I/(tan(1/2*d*x+1/2*c)-I)^2+49/12
*I/(tan(1/2*d*x+1/2*c)-I)^6-35/8*I/(tan(1/2*d*x+1/2*c)-I)^4+2/9/(tan(1/2*d*
x+1/2*c)-I)^9-5/2/(tan(1/2*d*x+1/2*c)-I)^7+49/10/(tan(1/2*d*x+1/2*c)-I)^5-4
9/16/(tan(1/2*d*x+1/2*c)-I)^3+99/128/(tan(1/2*d*x+1/2*c)-I)+1/16*I/(tan(1/2
*d*x+1/2*c)+I)^4-9/64*I/(tan(1/2*d*x+1/2*c)+I)^2+1/40/(tan(1/2*d*x+1/2*c)+I
)^5-13/96/(tan(1/2*d*x+1/2*c)+I)^3+29/128/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.3389, size = 329, normalized size = 3.07

$$\frac{\left(-9i\,e^{(14i\,dx+14i\,c)}-105i\,e^{(12i\,dx+12i\,c)}-945i\,e^{(10i\,dx+10i\,c)}+1575i\,e^{(8i\,dx+8i\,c)}+525i\,e^{(6i\,dx+6i\,c)}+189i\,e^{(4i\,dx+4i\,c)}+45i\,e^{(2i\,dx}}{5760\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5760*(-9*I*e^(14*I*d*x + 14*I*c) - 105*I*e^(12*I*d*x + 12*I*c) - 945*I*e^
(10*I*d*x + 10*I*c) + 1575*I*e^(8*I*d*x + 8*I*c) + 525*I*e^(6*I*d*x + 6*I*c
```

```
) + 189*I*e^(4*I*d*x + 4*I*c) + 45*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-9*I*d*x
 - 9*I*c)/(a^2*d)
```

Sympy [A] time = 1.72632, size = 301, normalized size = 2.81

$$\begin{cases} \frac{(-227994731135631360ia^{14}d^7e^{30ic}e^{5idx}-2659938529915699200ia^{14}d^7e^{28ic}e^{3idx}-23939446769241292800ia^{14}d^7e^{26ic}e^{idx}+39899077948735488000ia^{14}d^7e^{24ic}e^{-ic}}{1459166} \\ \frac{x(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1)e^{-9ic}}{128a^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-227994731135631360*I*a**14*d**7*exp(30*I*c)*exp(5*I*d*x) - 265
9938529915699200*I*a**14*d**7*exp(28*I*c)*exp(3*I*d*x) - 239394467692412928
00*I*a**14*d**7*exp(26*I*c)*exp(I*d*x) + 39899077948735488000*I*a**14*d**7*
exp(24*I*c)*exp(-I*d*x) + 13299692649578496000*I*a**14*d**7*exp(22*I*c)*exp
(-3*I*d*x) + 4787889353848258560*I*a**14*d**7*exp(20*I*c)*exp(-5*I*d*x) + 1
139973655678156800*I*a**14*d**7*exp(18*I*c)*exp(-7*I*d*x) + 126663739519795
200*I*a**14*d**7*exp(16*I*c)*exp(-9*I*d*x))*exp(-25*I*c)/(14591662792680407
0400*a**16*d**8), Ne(145916627926804070400*a**16*d**8*exp(25*I*c), 0)), (x*
(exp(14*I*c) + 7*exp(12*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*
c) + 21*exp(4*I*c) + 7*exp(2*I*c) + 1)*exp(-9*I*c)/(128*a**2), True))
```

Giac [B] time = 1.16242, size = 266, normalized size = 2.49

$$\frac{\frac{3\left(435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1470i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2060\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1330i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+353\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5}+\frac{4455\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-26460i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-78120}{}}{2880d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2880*(3*(435*tan(1/2*d*x + 1/2*c)^4 + 1470*I*tan(1/2*d*x + 1/2*c)^3 - 206
0*tan(1/2*d*x + 1/2*c)^2 - 1330*I*tan(1/2*d*x + 1/2*c) + 353)/(a^2*(tan(1/2
*d*x + 1/2*c) + I)^5) + (4455*tan(1/2*d*x + 1/2*c)^8 - 26460*I*tan(1/2*d*x
+ 1/2*c)^7 - 78120*tan(1/2*d*x + 1/2*c)^6 + 137340*I*tan(1/2*d*x + 1/2*c)^5
 + 157374*tan(1/2*d*x + 1/2*c)^4 - 118356*I*tan(1/2*d*x + 1/2*c)^3 - 57744*
tan(1/2*d*x + 1/2*c)^2 + 16596*I*tan(1/2*d*x + 1/2*c) + 2339)/(a^2*(tan(1/2
*d*x + 1/2*c) - I)^9))/d
```

3.130 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$-\frac{i(a-ia\tan(c+dx))^{10}}{10a^{13}d}+\frac{2i(a-ia\tan(c+dx))^9}{3a^{12}d}-\frac{3i(a-ia\tan(c+dx))^8}{2a^{11}d}+\frac{8i(a-ia\tan(c+dx))^7}{7a^{10}d}$$

```
[Out] (((8*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^10*d) - (((3*I)/2)*(a - I*a*Tan[c +
 d*x])^8)/(a^11*d) + (((2*I)/3)*(a - I*a*Tan[c + d*x])^9)/(a^12*d) - ((I/10
)*(a - I*a*Tan[c + d*x])^10)/(a^13*d)
```

Rubi [A] time = 0.0688227, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{i(a-ia\tan(c+dx))^{10}}{10a^{13}d}+\frac{2i(a-ia\tan(c+dx))^9}{3a^{12}d}-\frac{3i(a-ia\tan(c+dx))^8}{2a^{11}d}+\frac{8i(a-ia\tan(c+dx))^7}{7a^{10}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((8*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^10*d) - (((3*I)/2)*(a - I*a*Tan[c +
 d*x])^8)/(a^11*d) + (((2*I)/3)*(a - I*a*Tan[c + d*x])^9)/(a^12*d) - ((I/10
)*(a - I*a*Tan[c + d*x])^10)/(a^13*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^6(a+x)^3\,dx,x,ia\tan(c+dx)\right)}{a^{13}d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(8a^3(a-x)^6-12a^2(a-x)^7+6a(a-x)^8-(a-x)^9\right)dx,x,ia\tan(c+dx)\right)}{a^{13}d}\\ &= \frac{8i(a-ia\tan(c+dx))^7}{7a^{10}d}-\frac{3i(a-ia\tan(c+dx))^8}{2a^{11}d}+\frac{2i(a-ia\tan(c+dx))^9}{3a^{12}d}-\frac{i(a-ia\tan}{10}\end{aligned}$$

Mathematica [A] time = 0.682843, size = 117, normalized size = 1.07

$$\frac{\sec(c)\sec^{10}(c+dx)(105\sin(c+2dx)-105\sin(3c+2dx)+120\sin(3c+4dx)+45\sin(5c+6dx)+10\sin(7c+8dx)-}{840a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^10*((-126*I)*Cos[c] - (105*I)*Cos[c + 2*d*x] - (105*I)
*Cos[3*c + 2*d*x] - 126*Sin[c] + 105*Sin[c + 2*d*x] - 105*Sin[3*c + 2*d*x]
+ 120*Sin[3*c + 4*d*x] + 45*Sin[5*c + 6*d*x] + 10*Sin[7*c + 8*d*x] + Sin[9*
c + 10*d*x]))/(840*a^3*d)
```

Maple [A] time = 0.084, size = 89, normalized size = 0.8

$$\frac{1}{da^3}\left(\tan(dx+c) + \frac{i}{10}(\tan(dx+c))^{10} - \frac{(\tan(dx+c))^9}{3} - \frac{8\,(\tan(dx+c))^7}{7} - i\,(\tan(dx+c))^6 - \frac{6\,(\tan(dx+c))^5}{5} - 2\,i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(tan(d*x+c)+1/10*I*tan(d*x+c)^10-1/3*tan(d*x+c)^9-8/7*tan(d*x+c)^7-
I*tan(d*x+c)^6-6/5*tan(d*x+c)^5-2*I*tan(d*x+c)^4-3/2*I*tan(d*x+c)^2)
```

Maxima [A] time = 1.0175, size = 117, normalized size = 1.07

$$\frac{42i\,\tan(dx+c)^{10} - 140\,\tan(dx+c)^9 - 480\,\tan(dx+c)^7 - 420i\,\tan(dx+c)^6 - 504\,\tan(dx+c)^5 - 840i\,\tan(dx+c)^4}{420\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/420*(42*I*tan(d*x + c)^10 - 140*tan(d*x + c)^9 - 480*tan(d*x + c)^7 - 420
*I*tan(d*x + c)^6 - 504*tan(d*x + c)^5 - 840*I*tan(d*x + c)^4 - 630*I*tan(d
*x + c)^2 + 420*tan(d*x + c))/(a^3*d)
```

Fricas [B] time = 2.61, size = 587, normalized size = 5.39

$$\frac{15360i\,e^{(6i\,dx+6i\,c)} + 5760i\,e^{(4i\,dx+4i\,c)} + 1280i\,e^{(2}}{105\left(a^3de^{(20i\,dx+20i\,c)} + 10\,a^3de^{(18i\,dx+18i\,c)} + 45\,a^3de^{(16i\,dx+16i\,c)} + 120\,a^3de^{(14i\,dx+14i\,c)} + 210\,a^3de^{(12i\,dx+12i\,c)} + 252\,a^3de^{(10i\,dx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(15360*I*e^(6*I*d*x + 6*I*c) + 5760*I*e^(4*I*d*x + 4*I*c) + 1280*I*e^
(2*I*d*x + 2*I*c) + 128*I)/(a^3*d*e^(20*I*d*x + 20*I*c) + 10*a^3*d*e^(18*I*
d*x + 18*I*c) + 45*a^3*d*e^(16*I*d*x + 16*I*c) + 120*a^3*d*e^(14*I*d*x + 14
*I*c) + 210*a^3*d*e^(12*I*d*x + 12*I*c) + 252*a^3*d*e^(10*I*d*x + 10*I*c) +
 210*a^3*d*e^(8*I*d*x + 8*I*c) + 120*a^3*d*e^(6*I*d*x + 6*I*c) + 45*a^3*d*e
^(4*I*d*x + 4*I*c) + 10*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.19416, size = 117, normalized size = 1.07

$$-\frac{-21i\tan(dx+c)^{10}+70\tan(dx+c)^{9}+240\tan(dx+c)^{7}+210i\tan(dx+c)^{6}+252\tan(dx+c)^{5}+420i\tan(dx}{210\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/210*(-21*I*tan(d*x + c)^10 + 70*tan(d*x + c)^9 + 240*tan(d*x + c)^7 + 21
0*I*tan(d*x + c)^6 + 252*tan(d*x + c)^5 + 420*I*tan(d*x + c)^4 + 315*I*tan(
d*x + c)^2 - 210*tan(d*x + c))/(a^3*d)
```

3.131 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$\frac{i(a-ia\tan(c+dx))^8}{8a^{11}d}-\frac{4i(a-ia\tan(c+dx))^7}{7a^{10}d}+\frac{2i(a-ia\tan(c+dx))^6}{3a^9d}$$

```
[Out] (((2*I)/3)*(a - I*a*Tan[c + d*x])^6)/(a^9*d) - (((4*I)/7)*(a - I*a*Tan[c +
d*x])^7)/(a^10*d) + ((I/8)*(a - I*a*Tan[c + d*x])^8)/(a^11*d)
```

Rubi [A] time = 0.0613849, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a-ia\tan(c+dx))^8}{8a^{11}d}-\frac{4i(a-ia\tan(c+dx))^7}{7a^{10}d}+\frac{2i(a-ia\tan(c+dx))^6}{3a^9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((2*I)/3)*(a - I*a*Tan[c + d*x])^6)/(a^9*d) - (((4*I)/7)*(a - I*a*Tan[c +
d*x])^7)/(a^10*d) + ((I/8)*(a - I*a*Tan[c + d*x])^8)/(a^11*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^5(a+x)^2\,dx, x, ia\tan(c+dx)\right)}{a^{11}d}\\
&= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a-x)^5-4a(a-x)^6+(a-x)^7\right)dx, x, ia\tan(c+dx)\right)}{a^{11}d}\\
&= \frac{2i(a-ia\tan(c+dx))^6}{3a^9d}-\frac{4i(a-ia\tan(c+dx))^7}{7a^{10}d}+\frac{i(a-ia\tan(c+dx))^8}{8a^{11}d}
\end{aligned}$$

Mathematica [A] time = 0.468781, size = 106, normalized size = 1.29

$$\frac{\sec(c)\sec^8(c+dx)(28\sin(c+2dx)-28\sin(3c+2dx)+28\sin(3c+4dx)+8\sin(5c+6dx)+\sin(7c+8dx)-28i\cos(c}{168a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^8*((-35*I)*Cos[c] - (28*I)*Cos[c + 2*d*x] - (28*I)*Cos
[3*c + 2*d*x] - 35*Sin[c] + 28*Sin[c + 2*d*x] - 28*Sin[3*c + 2*d*x] + 28*Si
n[3*c + 4*d*x] + 8*Sin[5*c + 6*d*x] + Sin[7*c + 8*d*x]))/(168*a^3*d)
```

Maple [A] time = 0.079, size = 89, normalized size = 1.1

$$\frac{1}{da^3}\left(\tan(dx+c)+\frac{i}{8}(\tan(dx+c))^8-\frac{3(\tan(dx+c))^7}{7}-\frac{i}{6}(\tan(dx+c))^6-(\tan(dx+c))^5-\frac{5i}{4}(\tan(dx+c))^4-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(tan(d*x+c)+1/8*I*tan(d*x+c)^8-3/7*tan(d*x+c)^7-1/6*I*tan(d*x+c)^6-
tan(d*x+c)^5-5/4*I*tan(d*x+c)^4-1/3*tan(d*x+c)^3-3/2*I*tan(d*x+c)^2)
```

Maxima [A] time = 0.986612, size = 117, normalized size = 1.43

$$-\frac{-21i\tan(dx+c)^8+72\tan(dx+c)^7+28i\tan(dx+c)^6+168\tan(dx+c)^5+210i\tan(dx+c)^4+56\tan(dx+c)}{168a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/168*(-21*I*tan(d*x + c)^8 + 72*tan(d*x + c)^7 + 28*I*tan(d*x + c)^6 + 16
8*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 + 56*tan(d*x + c)^3 + 252*I*tan(d*x
 + c)^2 - 168*tan(d*x + c))/(a^3*d)
```

Fricas [B] time = 2.34707, size = 443, normalized size = 5.4

$$\frac{896i\,e^{(4i\,dx+4i\,c)}+256i\,e^{(2i\,dx+2i\,c)}+32i}{21\left(a^3de^{(16i\,dx+16i\,c)}+8\,a^3de^{(14i\,dx+14i\,c)}+28\,a^3de^{(12i\,dx+12i\,c)}+56\,a^3de^{(10i\,dx+10i\,c)}+70\,a^3de^{(8i\,dx+8i\,c)}+56\,a^3de^{(6i\,dx+6i\,c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/21*(896*I*e^(4*I*d*x + 4*I*c) + 256*I*e^(2*I*d*x + 2*I*c) + 32*I)/(a^3*d*
e^(16*I*d*x + 16*I*c) + 8*a^3*d*e^(14*I*d*x + 14*I*c) + 28*a^3*d*e^(12*I*d*
x + 12*I*c) + 56*a^3*d*e^(10*I*d*x + 10*I*c) + 70*a^3*d*e^(8*I*d*x + 8*I*c)
 + 56*a^3*d*e^(6*I*d*x + 6*I*c) + 28*a^3*d*e^(4*I*d*x + 4*I*c) + 8*a^3*d*e^
(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.20249, size = 117, normalized size = 1.43

$$-\frac{-21i\tan\left(dx+c\right)^{8}+72\tan\left(dx+c\right)^{7}+28i\tan\left(dx+c\right)^{6}+168\tan\left(dx+c\right)^{5}+210i\tan\left(dx+c\right)^{4}+56\tan\left(dx+c\right)^{3}}{168\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/168*(-21*I*tan(d*x + c)^8 + 72*tan(d*x + c)^7 + 28*I*tan(d*x + c)^6 + 16
8*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 + 56*tan(d*x + c)^3 + 252*I*tan(d*x
 + c)^2 - 168*tan(d*x + c))/(a^3*d)
```

3.132 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=55

$$\frac{2i(a-ia\tan(c+dx))^5}{5a^8d}-\frac{i(a-ia\tan(c+dx))^6}{6a^9d}$$

```
[Out] (((2*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^8*d) - ((I/6)*(a - I*a*Tan[c + d*x]
)^6)/(a^9*d)
```

Rubi [A] time = 0.0473699, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{2i(a-ia\tan(c+dx))^5}{5a^8d}-\frac{i(a-ia\tan(c+dx))^6}{6a^9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((2*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^8*d) - ((I/6)*(a - I*a*Tan[c + d*x]
)^6)/(a^9*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^4(a+x)\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(2a(a-x)^4-(a-x)^5\right)\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= \frac{2i(a-ia\tan(c+dx))^5}{5a^8d}-\frac{i(a-ia\tan(c+dx))^6}{6a^9d}\end{aligned}$$

Mathematica [A] time = 0.435988, size = 97, normalized size = 1.76

$$\frac{\sec(c)\sec^6(c+dx)(15\sin(c+2dx)-15\sin(3c+2dx)+12\sin(3c+4dx)+2\sin(5c+6dx)-15i\cos(c+2dx)-15i\,\text{c}}{60a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^6*((-20*I)*Cos[c] - (15*I)*Cos[c + 2*d*x] - (15*I)*Cos
[3*c + 2*d*x] - 20*Sin[c] + 15*Sin[c + 2*d*x] - 15*Sin[3*c + 2*d*x] + 12*Si
n[3*c + 4*d*x] + 2*Sin[5*c + 6*d*x]))/(60*a^3*d)
```

Maple [A] time = 0.078, size = 68, normalized size = 1.2

$$\frac{1}{da^3}\left(\tan(dx+c)+\frac{i}{6}(\tan(dx+c))^6-\frac{3(\tan(dx+c))^5}{5}-\frac{i}{2}(\tan(dx+c))^4-\frac{2(\tan(dx+c))^3}{3}-\frac{3i}{2}(\tan(dx+c))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(tan(d*x+c)+1/6*I*tan(d*x+c)^6-3/5*tan(d*x+c)^5-1/2*I*tan(d*x+c)^4-
2/3*tan(d*x+c)^3-3/2*I*tan(d*x+c)^2)
```

Maxima [A] time = 1.05913, size = 90, normalized size = 1.64

$$\frac{5i\tan(dx+c)^6-18\tan(dx+c)^5-15i\tan(dx+c)^4-20\tan(dx+c)^3-45i\tan(dx+c)^2+30\tan(dx+c)}{30a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/30*(5*I*tan(d*x + c)^6 - 18*tan(d*x + c)^5 - 15*I*tan(d*x + c)^4 - 20*tan
(d*x + c)^3 - 45*I*tan(d*x + c)^2 + 30*tan(d*x + c))/(a^3*d)
```

Fricas [B] time = 2.35389, size = 316, normalized size = 5.75

$$\frac{192i\,e^{(2i\,dx+2i\,c)}+32i}{15\left(a^3de^{(12i\,dx+12i\,c)}+6\,a^3de^{(10i\,dx+10i\,c)}+15\,a^3de^{(8i\,dx+8i\,c)}+20\,a^3de^{(6i\,dx+6i\,c)}+15\,a^3de^{(4i\,dx+4i\,c)}+6\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(192*I*e^(2*I*d*x + 2*I*c) + 32*I)/(a^3*d*e^(12*I*d*x + 12*I*c) + 6*a^
3*d*e^(10*I*d*x + 10*I*c) + 15*a^3*d*e^(8*I*d*x + 8*I*c) + 20*a^3*d*e^(6*I*
d*x + 6*I*c) + 15*a^3*d*e^(4*I*d*x + 4*I*c) + 6*a^3*d*e^(2*I*d*x + 2*I*c) +
 a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.18569, size = 90, normalized size = 1.64

$$-\frac{-5i\tan(dx+c)^6 + 18\tan(dx+c)^5 + 15i\tan(dx+c)^4 + 20\tan(dx+c)^3 + 45i\tan(dx+c)^2 - 30\tan(dx+c)}{30a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/30*(-5*I*tan(d*x + c)^6 + 18*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 + 20*t
an(d*x + c)^3 + 45*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a^3*d)
```

3.133 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=27

$$\frac{i(a-ia\tan(c+dx))^4}{4a^7d}$$

```
[Out] ((I/4)*(a - I*a*Tan[c + d*x])^4)/(a^7*d)
```

Rubi [A] time = 0.039479, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$\frac{i(a-ia\tan(c+dx))^4}{4a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/4)*(a - I*a*Tan[c + d*x])^4)/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^3\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= \frac{i(a-ia\tan(c+dx))^4}{4a^7d}\end{aligned}$$

Mathematica [B] time = 0.378706, size = 84, normalized size = 3.11

$$\frac{\sec(c)\sec^4(c+dx)(2\sin(c+2dx)-2\sin(3c+2dx)+\sin(3c+4dx)-2i\cos(c+2dx)-2i\cos(3c+2dx)-3\sin(c)-3i\cos}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^4*((-3*I)*Cos[c] - (2*I)*Cos[c + 2*d*x] - (2*I)*Cos[3*
c + 2*d*x] - 3*Sin[c] + 2*Sin[c + 2*d*x] - 2*Sin[3*c + 2*d*x] + Sin[3*c + 4
```

```
*d*x]))/(4*a^3*d)
```

Maple [A] time = 0.073, size = 47, normalized size = 1.7

$$\frac{\tan\left(dx+c\right)+\frac{i}{4}\left(\tan\left(dx+c\right)\right)^{4}-\left(\tan\left(dx+c\right)\right)^{3}-\frac{3i}{2}\left(\tan\left(dx+c\right)\right)^{2}}{da^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(tan(d*x+c)+1/4*I*tan(d*x+c)^4-tan(d*x+c)^3-3/2*I*tan(d*x+c)^2)
```

Maxima [B] time = 0.970975, size = 63, normalized size = 2.33

$$-\frac{-i\tan\left(dx+c\right)^{4}+4\tan\left(dx+c\right)^{3}+6i\tan\left(dx+c\right)^{2}-4\tan\left(dx+c\right)}{4\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*x
 + c))/(a^3*d)
```

Fricas [B] time = 2.30315, size = 177, normalized size = 6.56

$$\frac{4i}{a^{3}de^{\left(8i\,dx+8i\,c\right)}+4\,a^{3}de^{\left(6i\,dx+6i\,c\right)}+6\,a^{3}de^{\left(4i\,dx+4i\,c\right)}+4\,a^{3}de^{\left(2i\,dx+2i\,c\right)}+a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 4*I/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a^3*d*e^(4
*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

<u>Giac</u> [B] time = 1.20958, size = 63, normalized size = 2.33

$$-\frac{-i\tan\left(dx+c\right)^{4}+4\tan\left(dx+c\right)^{3}+6i\tan\left(dx+c\right)^{2}-4\tan\left(dx+c\right)}{4a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*x
 + c))/(a^3*d)
```

3.134 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=58

$$\frac{i\tan^2(c+dx)}{2a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4i\log(\cos(c+dx))}{a^3d}+\frac{4x}{a^3}$$

```
[Out] (4*x)/a^3 + ((4*I)*Log[Cos[c + d*x]])/(a^3*d) - (3*Tan[c + d*x])/(a^3*d) +
((I/2)*Tan[c + d*x]^2)/(a^3*d)
```

Rubi [A] time = 0.0472475, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i\tan^2(c+dx)}{2a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4i\log(\cos(c+dx))}{a^3d}+\frac{4x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (4*x)/a^3 + ((4*I)*Log[Cos[c + d*x]])/(a^3*d) - (3*Tan[c + d*x])/(a^3*d) +
((I/2)*Tan[c + d*x]^2)/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{(a-x)^2}{a+x}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(-3a+x+\frac{4a^2}{a+x}\right)dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= \frac{4x}{a^3}+\frac{4i\log(\cos(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{i\tan^2(c+dx)}{2a^3d}\end{aligned}$$

Mathematica [A] time = 0.37478, size = 113, normalized size = 1.95

$$\frac{\sec(c)\sec^2(c+dx)(-3\sin(c+2dx)+2dx\cos(3c+2dx)+2i\cos(3c+2dx)\log(\cos(c+dx))+2\cos(c+2dx)(dx+i\,\text{l}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^2*(2*d*x*Cos[3*c + 2*d*x] + 2*Cos[c + 2*d*x]*(d*x + I*
Log[Cos[c + d*x]]) + Cos[c]*(I + 4*d*x + (4*I)*Log[Cos[c + d*x]]) + (2*I)*C
os[3*c + 2*d*x]*Log[Cos[c + d*x]] + 3*Sin[c] - 3*Sin[c + 2*d*x]))/(2*a^3*d)
```

Maple [A] time = 0.079, size = 52, normalized size = 0.9

$$-3\,{\frac{\tan \left(dx+c \right) }{d{a}^{3}}}+{\frac{{\frac{i}{2}} \left(\tan \left(dx+c \right) \right) ^{2}}{d{a}^{3}}}-{\frac{4\,i\ln \left(\tan \left(dx+c \right) -i \right) }{d{a}^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -3*tan(d*x+c)/a^3/d+1/2*I*tan(d*x+c)^2/a^3/d-4*I/a^3/d*ln(tan(d*x+c)-I)
```

Maxima [A] time = 0.968436, size = 61, normalized size = 1.05

$$\frac{\frac{i\,\tan(dx+c)^{2}-6\,\tan(dx+c)}{a^{3}}-\frac{8i\,\log(i\,\tan(dx+c)+1)}{a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((I*tan(d*x + c)^2 - 6*tan(d*x + c))/a^3 - 8*I*log(I*tan(d*x + c) + 1)/
a^3)/d
```

Fricas [B] time = 2.33946, size = 317, normalized size = 5.47

$$\frac{8\,dxe^{(4i\,dx+4i\,c)}+8\,dx+(16\,dx-4i)e^{(2i\,dx+2i\,c)}+\left(4i\,e^{(4i\,dx+4i\,c)}+8i\,e^{(2i\,dx+2i\,c)}+4i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-6i}{a^{3}de^{(4i\,dx+4i\,c)}+2\,a^{3}de^{(2i\,dx+2i\,c)}+a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (8*d*x*e^(4*I*d*x + 4*I*c) + 8*d*x + (16*d*x - 4*I)*e^(2*I*d*x + 2*I*c) + (
4*I*e^(4*I*d*x + 4*I*c) + 8*I*e^(2*I*d*x + 2*I*c) + 4*I)*log(e^(2*I*d*x + 2
*I*c) + 1) - 6*I)/(a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c)
+ a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.23457, size = 176, normalized size = 3.03

$$\frac{2\left(-\frac{4i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^3}+\frac{2i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{2i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{-3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+7i\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(-4*I*log(tan(1/2*d*x + 1/2*c) - I)/a^3 + 2*I*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/a^3 + 2*I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + (-3*I*tan(1/2*d*
x + 1/2*c)^4 + 3*tan(1/2*d*x + 1/2*c)^3 + 7*I*tan(1/2*d*x + 1/2*c)^2 - 3*ta
n(1/2*d*x + 1/2*c) - 3*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

3.135 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=50

$$\frac{2i}{d\left(a^3+ia^3\tan(c+dx)\right)}-\frac{i\log(\cos(c+dx))}{a^3d}-\frac{x}{a^3}$$

```
[Out] -(x/a^3) - (I*Log[Cos[c + d*x]])/(a^3*d) + (2*I)/(d*(a^3 + I*a^3*Tan[c + d*
x]))
```

Rubi [A] time = 0.0486496, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{2i}{d\left(a^3+ia^3\tan(c+dx)\right)}-\frac{i\log(\cos(c+dx))}{a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (I*Log[Cos[c + d*x]])/(a^3*d) + (2*I)/(d*(a^3 + I*a^3*Tan[c + d*
x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{a-x}{(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{2a}{(a+x)^2}\right)dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{x}{a^3}-\frac{i\log(\cos(c+dx))}{a^3d}+\frac{2i}{d\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.214589, size = 88, normalized size = 1.76

$$\frac{\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))(\log(\cos(c+dx))+\tan(c+dx)(i\log(\cos(c+dx))+dx+i)-idx-1)}{a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*(-1 - I*d*x + Log[C
os[c + d*x]] + (I + d*x + I*Log[Cos[c + d*x]])*Tan[c + d*x]))/(a^3*d*(-I +
Tan[c + d*x])^3)
```

Maple [A] time = 0.08, size = 40, normalized size = 0.8

$$\frac{i\ln\left(\tan\left(dx+c\right)-i\right)}{da^3}+2\,\frac{1}{da^3\left(\tan\left(dx+c\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] I/a^3/d*ln(tan(d*x+c)-I)+2/a^3/d/(tan(d*x+c)-I)
```

Maxima [A] time = 0.98214, size = 89, normalized size = 1.78

$$-\frac{\frac{4\,(-i\,\tan(dx+c)-1)}{2i\,a^3\,\tan(dx+c)^2+4\,a^3\,\tan(dx+c)-2i\,a^3}-\frac{i\,\log(i\,\tan(dx+c)+1)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(4*(-I*tan(d*x + c) - 1)/(2*I*a^3*tan(d*x + c)^2 + 4*a^3*tan(d*x + c) - 2*
I*a^3) - I*log(I*tan(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 2.26739, size = 157, normalized size = 3.14

$$-\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)e^{(-2i\,dx-2i\,c)}}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(2*d*x*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c)
 + 1) - I)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.20099, size = 138, normalized size = 2.76

$$-\frac{-\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^3}+\frac{i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3i}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(-2*I*log(tan(1/2*d*x + 1/2*c) - I)/a^3 + I*log(abs(tan(1/2*d*x + 1/2*c) +
 1))/a^3 + I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + (3*I*tan(1/2*d*x + 1/
2*c)^2 + 10*tan(1/2*d*x + 1/2*c) - 3*I)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^2))
/d
```

3.136 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=27

$$\frac{i}{2ad(a+ia\tan(c+dx))^2}$$

```
[Out] (I/2)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rubi [A] time = 0.0531279, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$\frac{i}{2ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (I/2)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{i}{2ad(a+ia\tan(c+dx))^2}\end{aligned}$$

Mathematica [A] time = 0.0702105, size = 42, normalized size = 1.56

$$-\frac{i(\tan(c+dx)-3i)\sec^2(c+dx)}{8a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-I/8)*Sec[c + d*x]^2*(-3*I + Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] time = 0.038, size = 24, normalized size = 0.9

$$\frac{\frac{i}{2}}{ad\left(a+ia\tan\left(dx+c\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/2*I/a/d/(a+I*a*tan(d*x+c))^2
```

Maxima [A] time = 0.946087, size = 28, normalized size = 1.04

$$\frac{i}{2\left(i\,a\tan\left(dx+c\right)+a\right)^{2}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*I/((I*a*tan(d*x + c) + a)^2*a*d)
```

Fricas [A] time = 2.22497, size = 86, normalized size = 3.19

$$\frac{\left(2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{8\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.16061, size = 77, normalized size = 2.85

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{3}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c
))/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^4)
```

3.137 $\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{x}{8a^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

```
[Out] x/(8*a^3) + (I/6)/(d*(a + I*a*Tan[c + d*x])^3) + (I/8)/(a*d*(a + I*a*Tan[c
+ d*x])^2) + (I/8)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rubi [A] time = 0.0470442, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3479, 8}

$$\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{x}{8a^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{6d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-3),x]
```

```
[Out] x/(8*a^3) + (I/6)/(d*(a + I*a*Tan[c + d*x])^3) + (I/8)/(a*d*(a + I*a*Tan[c
+ d*x])^2) + (I/8)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx &= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{2a}\\ &= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{\int \frac{1}{a+ia\tan(c+dx)}\,dx}{4a^2}\\ &= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{\int 1\,dx}{8a^3}\\ &= \frac{x}{8a^3}+\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.195165, size = 93, normalized size = 1.06

$$\frac{i\sec^3(c+dx)(-9\sin(c+dx)+12idx\sin(3(c+dx))+2\sin(3(c+dx))+27i\cos(c+dx)+2(6dx+i)\cos(3(c+dx)))}{96a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-3),x]
```

```
[Out] ((I/96)*Sec[c + d*x]^3*((27*I)*Cos[c + d*x] + 2*(I + 6*d*x)*Cos[3*(c + d*x)
] - 9*Sin[c + d*x] + 2*Sin[3*(c + d*x)] + (12*I)*d*x*Sin[3*(c + d*x)]))/(a^
3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] time = 0.026, size = 98, normalized size = 1.1

$$\frac{-\frac{i}{16}\ln\left(\tan\left(dx+c\right)-i\right)}{a^{3}d}-\frac{\frac{i}{8}}{a^{3}d\left(\tan\left(dx+c\right)-i\right)^{2}}-\frac{1}{6\,a^{3}d\left(\tan\left(dx+c\right)-i\right)^{3}}+\frac{1}{8\,a^{3}d\left(\tan\left(dx+c\right)-i\right)}+\frac{\frac{i}{16}\ln\left(\tan\left(d\right.\right.}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -1/16*I/a^3/d*ln(tan(d*x+c)-I)-1/8*I/a^3/d/(tan(d*x+c)-I)^2-1/6/a^3/d/(tan(
d*x+c)-I)^3+1/8/a^3/d/(tan(d*x+c)-I)+1/16*I/a^3/d*ln(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.2275, size = 166, normalized size = 1.89

$$\frac{\left(12\,dxe^{\left(6i\,dx+6i\,c\right)}+18i\,e^{\left(4i\,dx+4i\,c\right)}+9i\,e^{\left(2i\,dx+2i\,c\right)}+2i\right)e^{\left(-6i\,dx-6i\,c\right)}}{96\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(12*d*x*e^(6*I*d*x + 6*I*c) + 18*I*e^(4*I*d*x + 4*I*c) + 9*I*e^(2*I*d*
x + 2*I*c) + 2*I)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A] time = 0.592316, size = 156, normalized size = 1.77

$$\begin{cases}\frac{\left(4608ia^{6}d^{2}e^{10ic}e^{-2idx}+2304ia^{6}d^{2}e^{8ic}e^{-4idx}+512ia^{6}d^{2}e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^{9}d^{3}} & \text{for}\: 24576a^{9}d^{3}e^{12ic}\neq 0\\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-6ic}}{8a^{3}}-\frac{1}{8a^{3}}\right) & \text{otherwise}\end{cases}+\frac{x}{8a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((4608*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 2304*I*a**6*d**2*e
xp(8*I*c)*exp(-4*I*d*x) + 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12
*I*c)/(24576*a**9*d**3), Ne(24576*a**9*d**3*exp(12*I*c), 0)), (x*((exp(6*I*
c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-6*I*c)/(8*a**3) - 1/(8*a**3)), T
rue)) + x/(8*a**3)
```

Giac [A] time = 1.11168, size = 108, normalized size = 1.23

$$-\frac{\frac{6i\log(\tan(dx+c)-i)}{a^3}-\frac{6i\log(i\tan(dx+c)-1)}{a^3}+\frac{-11i\tan(dx+c)^3-45\tan(dx+c)^2+69i\tan(dx+c)+51}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(6*I*log(tan(d*x + c) - I)/a^3 - 6*I*log(I*tan(d*x + c) - 1)/a^3 + (-
11*I*tan(d*x + c)^3 - 45*tan(d*x + c)^2 + 69*I*tan(d*x + c) + 51)/(a^3*(tan
(d*x + c) - I)^3))/d
```

3.138 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=141

$$-\frac{i}{32d\left(a^3-ia^3\tan(c+dx)\right)}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{5x}{32a^3}+\frac{ia}{16d(a+ia\tan(c+dx))^4}+\frac{i}{12d(a+ia\tan(c+dx)}$$

```
[Out] (5*x)/(32*a^3) + ((I/16)*a)/(d*(a + I*a*Tan[c + d*x])^4) + (I/12)/(d*(a + I
*a*Tan[c + d*x])^3) + ((3*I)/32)/(a*d*(a + I*a*Tan[c + d*x])^2) - (I/32)/(d
*(a^3 - I*a^3*Tan[c + d*x])) + (I/8)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rubi [A] time = 0.0861982, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{i}{32d\left(a^3-ia^3\tan(c+dx)\right)}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{5x}{32a^3}+\frac{ia}{16d(a+ia\tan(c+dx))^4}+\frac{i}{12d(a+ia\tan(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (5*x)/(32*a^3) + ((I/16)*a)/(d*(a + I*a*Tan[c + d*x])^4) + (I/12)/(d*(a + I
*a*Tan[c + d*x])^3) + ((3*I)/32)/(a*d*(a + I*a*Tan[c + d*x])^2) - (I/32)/(d
*(a^3 - I*a^3*Tan[c + d*x])) + (I/8)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^5}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^2}+\frac{1}{4a^2(a+x)^5}+\frac{1}{4a^3(a+x)^4}+\frac{3}{16a^4(a+x)^3}+\frac{1}{8a^5(a+x)^2}+\frac{5}{32a^5(a^2-x^2)}\right)dx, x$$

$$= \frac{ia}{16d(a+ia\tan(c+dx))^4}+\frac{i}{12d(a+ia\tan(c+dx))^3}+\frac{3i}{32ad(a+ia\tan(c+dx))^2}-\frac{}{32d\,(a}$$

$$= \frac{5x}{32a^3}+\frac{ia}{16d(a+ia\tan(c+dx))^4}+\frac{i}{12d(a+ia\tan(c+dx))^3}+\frac{3i}{32ad(a+ia\tan(c+dx))^2}-$$

Mathematica [A] time = 0.223901, size = 115, normalized size = 0.82

$$\frac{\sec^3(c+dx)(-60i\sin(c+dx)-120dx\sin(3(c+dx))+20i\sin(3(c+dx))+15i\sin(5(c+dx))-180\cos(c+dx)+20i(6d}{768a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-180*Cos[c + d*x] + (20*I)*(I + 6*d*x)*Cos[3*(c + d*x)] +
9*Cos[5*(c + d*x)] - (60*I)*Sin[c + d*x] + (20*I)*Sin[3*(c + d*x)] - 120*d*
x*Sin[3*(c + d*x)] + (15*I)*Sin[5*(c + d*x)]))/(768*a^3*d*(-I + Tan[c + d*x
])^3)
```

Maple [A] time = 0.097, size = 137, normalized size = 1.

$$\frac{-\frac{5i}{64}\ln(\tan(dx+c)-i)}{a^3d}+\frac{\frac{i}{16}}{a^3d(\tan(dx+c)-i)^4}-\frac{\frac{3i}{32}}{a^3d(\tan(dx+c)-i)^2}-\frac{1}{12a^3d(\tan(dx+c)-i)^3}+\frac{1}{8a^3d(\tan(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -5/64*I/a^3/d*ln(tan(d*x+c)-I)+1/16*I/a^3/d/(tan(d*x+c)-I)^4-3/32*I/a^3/d/(
tan(d*x+c)-I)^2-1/12/a^3/d/(tan(d*x+c)-I)^3+1/8/a^3/d/(tan(d*x+c)-I)+5/64*I
/a^3/d*ln(tan(d*x+c)+I)+1/32/a^3/d/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.25761, size = 247, normalized size = 1.75

$$\frac{\left(120\,dxe^{(8i\,dx+8i\,c)}-12i\,e^{(10i\,dx+10i\,c)}+120i\,e^{(6i\,dx+6i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{768\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/768*(120*d*x*e^(8*I*d*x + 8*I*c) - 12*I*e^(10*I*d*x + 10*I*c) + 120*I*e^(
6*I*d*x + 6*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*
I)*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

Sympy [A] time = 1.04198, size = 226, normalized size = 1.6

$$\begin{cases}\frac{\left(-100663296ia^{12}d^4e^{22ic}e^{2idx}+1006632960ia^{12}d^4e^{18ic}e^{-2idx}+503316480ia^{12}d^4e^{16ic}e^{-4idx}+167772160ia^{12}d^4e^{14ic}e^{-6idx}+25165824ia^{12}d^4e^{12ic}e^{-8idx}\right)e^{-20ic}}{6442450944a^{15}d^5} \\ x\left(\frac{\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-8ic}}{32a^3}-\frac{5}{32a^3}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-100663296*I*a**12*d**4*exp(22*I*c)*exp(2*I*d*x) + 1006632960*I
*a**12*d**4*exp(18*I*c)*exp(-2*I*d*x) + 503316480*I*a**12*d**4*exp(16*I*c)*
exp(-4*I*d*x) + 167772160*I*a**12*d**4*exp(14*I*c)*exp(-6*I*d*x) + 25165824
*I*a**12*d**4*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(6442450944*a**15*d**
5), Ne(6442450944*a**15*d**5*exp(20*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I
*c) + 10*exp(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-8*I*c)/(32*a**
3) - 5/(32*a**3)), True)) + 5*x/(32*a**3)
```

Giac [A] time = 1.17716, size = 161, normalized size = 1.14

$$-\frac{-\frac{60i\log(-i\tan(dx+c)+1)}{a^3}+\frac{60i\log(-i\tan(dx+c)-1)}{a^3}-\frac{12(5\tan(dx+c)+7i)}{a^3(i\tan(dx+c)-1)}+\frac{-125i\tan(dx+c)^4-596\tan(dx+c)^3+1110i\tan(dx+c)^2+996\tan(dx}{a^3(\tan(dx+c)-i)^4}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/768*(-60*I*log(-I*tan(d*x + c) + 1)/a^3 + 60*I*log(-I*tan(d*x + c) - 1)/
a^3 - 12*(5*tan(d*x + c) + 7*I)/(a^3*(I*tan(d*x + c) - 1)) + (-125*I*tan(d*
x + c)^4 - 596*tan(d*x + c)^3 + 1110*I*tan(d*x + c)^2 + 996*tan(d*x + c) -
405*I)/(a^3*(tan(d*x + c) - I)^4))/d
```

3.139 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$\frac{ia^2}{40d(a+ia\tan(c+dx))^5}-\frac{3i}{64d\left(a^3-ia^3\tan(c+dx)\right)}+\frac{15i}{128d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{21x}{128a^3}+\frac{3ia}{64d(a+ia\tan(c+dx))^4}$$

```
[Out] (21*x)/(128*a^3) - (I/128)/(a*d*(a - I*a*Tan[c + d*x])^2) + ((I/40)*a^2)/(d
*(a + I*a*Tan[c + d*x])^5) + (((3*I)/64)*a)/(d*(a + I*a*Tan[c + d*x])^4) +
(I/16)/(d*(a + I*a*Tan[c + d*x])^3) + ((5*I)/64)/(a*d*(a + I*a*Tan[c + d*x]
)^2) - ((3*I)/64)/(d*(a^3 - I*a^3*Tan[c + d*x])) + ((15*I)/128)/(d*(a^3 + I
*a^3*Tan[c + d*x]))
```

Rubi [A] time = 0.116134, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$\frac{ia^2}{40d(a+ia\tan(c+dx))^5}-\frac{3i}{64d\left(a^3-ia^3\tan(c+dx)\right)}+\frac{15i}{128d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{21x}{128a^3}+\frac{3ia}{64d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (21*x)/(128*a^3) - (I/128)/(a*d*(a - I*a*Tan[c + d*x])^2) + ((I/40)*a^2)/(d
*(a + I*a*Tan[c + d*x])^5) + (((3*I)/64)*a)/(d*(a + I*a*Tan[c + d*x])^4) +
(I/16)/(d*(a + I*a*Tan[c + d*x])^3) + ((5*I)/64)/(a*d*(a + I*a*Tan[c + d*x]
)^2) - ((3*I)/64)/(d*(a^3 - I*a^3*Tan[c + d*x])) + ((15*I)/128)/(d*(a^3 + I
*a^3*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^6}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^3} + \frac{3}{64a^7(a-x)^2} + \frac{1}{8a^3(a+x)^6} + \frac{3}{16a^4(a+x)^5} + \frac{3}{16a^5(a+x)^4} + \frac{5}{32a^6(a+x)^3} + \right.\right.}{d}$$

$$= -\frac{i}{128ad(a-ia\tan(c+dx))^2} + \frac{ia^2}{40d(a+ia\tan(c+dx))^5} + \frac{3ia}{64d(a+ia\tan(c+dx))^4} +$$

$$= \frac{21x}{128a^3} - \frac{i}{128ad(a-ia\tan(c+dx))^2} + \frac{ia^2}{40d(a+ia\tan(c+dx))^5} + \frac{3ia}{64d(a+ia\tan(c+d}$$

Mathematica [A] time = 0.373805, size = 137, normalized size = 0.7

$$\frac{\sec^3(c+dx)(-350i\sin(c+dx) - 840dx\sin(3(c+dx)) + 140i\sin(3(c+dx)) + 175i\sin(5(c+dx)) + 14i\sin(7(c+dx))}{5120a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-1050*Cos[c + d*x] + (140*I)*(I + 6*d*x)*Cos[3*(c + d*x)]
+ 105*Cos[5*(c + d*x)] + 6*Cos[7*(c + d*x)] - (350*I)*Sin[c + d*x] + (140*I
)*Sin[3*(c + d*x)] - 840*d*x*Sin[3*(c + d*x)] + (175*I)*Sin[5*(c + d*x)] +
(14*I)*Sin[7*(c + d*x)]))/(5120*a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] time = 0.102, size = 176, normalized size = 0.9

$$\frac{-\frac{21i}{256}\ln(\tan(dx+c)-i)}{a^3d} + \frac{\frac{3i}{64}}{a^3d(\tan(dx+c)-i)^4} - \frac{\frac{5i}{64}}{a^3d(\tan(dx+c)-i)^2} + \frac{1}{40a^3d(\tan(dx+c)-i)^5} - \frac{}{16a^3d(\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -21/256*I/a^3/d*ln(tan(d*x+c)-I)+3/64*I/a^3/d/(tan(d*x+c)-I)^4-5/64*I/a^3/d
/(tan(d*x+c)-I)^2+1/40/a^3/d/(tan(d*x+c)-I)^5-1/16/a^3/d/(tan(d*x+c)-I)^3+1
5/128/a^3/d/(tan(d*x+c)-I)+1/128*I/a^3/d/(tan(d*x+c)+I)^2+21/256*I/a^3/d*ln
(tan(d*x+c)+I)+3/64/a^3/d/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.41382, size = 333, normalized size = 1.71

$$\frac{\left(840\,dxe^{(10i\,dx+10i\,c)} - 10i\,e^{(14i\,dx+14i\,c)} - 140i\,e^{(12i\,dx+12i\,c)} + 700i\,e^{(8i\,dx+8i\,c)} + 350i\,e^{(6i\,dx+6i\,c)} + 140i\,e^{(4i\,dx+4i\,c)} + 35i\,e^{(2i\,dx+2i}}{5120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5120*(840*d*x*e^(10*I*d*x + 10*I*c) - 10*I*e^(14*I*d*x + 14*I*c) - 140*I*
e^(12*I*d*x + 12*I*c) + 700*I*e^(8*I*d*x + 8*I*c) + 350*I*e^(6*I*d*x + 6*I*
c) + 140*I*e^(4*I*d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 4*I)*e^(-10*I*d
*x - 10*I*c)/(a^3*d)
```

Sympy [A] time = 1.33778, size = 294, normalized size = 1.51

$$\begin{cases} \frac{\left(-11258999068426240ia^{18}d^6e^{34ic}e^{4idx} - 157625986957967360ia^{18}d^6e^{32ic}e^{2idx} + 788129934789836800ia^{18}d^6e^{28ic}e^{-2idx} + 394064967394918400ia^{18}d^6e^{26ic}e^{-4idx} + 1}{5764607523034234880a^{21}d^7} \\ x\left(\frac{\left(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1\right)e^{-10ic}}{128a^3} - \frac{21}{128a^3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-11258999068426240*I*a**18*d**6*exp(34*I*c)*exp(4*I*d*x) - 1576
25986957967360*I*a**18*d**6*exp(32*I*c)*exp(2*I*d*x) + 788129934789836800*I
*a**18*d**6*exp(28*I*c)*exp(-2*I*d*x) + 394064967394918400*I*a**18*d**6*exp
(26*I*c)*exp(-4*I*d*x) + 157625986957967360*I*a**18*d**6*exp(24*I*c)*exp(-6
*I*d*x) + 39406496739491840*I*a**18*d**6*exp(22*I*c)*exp(-8*I*d*x) + 450359
9627370496*I*a**18*d**6*exp(20*I*c)*exp(-10*I*d*x))*exp(-30*I*c)/(576460752
3034234880*a**21*d**7), Ne(5764607523034234880*a**21*d**7*exp(30*I*c), 0)),
 (x*((exp(14*I*c) + 7*exp(12*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp
(6*I*c) + 21*exp(4*I*c) + 7*exp(2*I*c) + 1)*exp(-10*I*c)/(128*a**3) - 21/(1
28*a**3)), True)) + 21*x/(128*a**3)
```

Giac [A] time = 1.18832, size = 184, normalized size = 0.94

$$-\frac{-\frac{420i\log(\tan(dx+c)+i)}{a^3} + \frac{420i\log(\tan(dx+c)-i)}{a^3} + \frac{10\left(-63i\tan(dx+c)^2+150\tan(dx+c)+91i\right)}{a^3(i\tan(dx+c)-1)^2} - \frac{959i\tan(dx+c)^5+5395\tan(dx+c)^4-12390i\tan(dx+c}{a^3(\tan(dx+c}}{5120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/5120*(-420*I*log(tan(d*x + c) + I)/a^3 + 420*I*log(tan(d*x + c) - I)/a^3
 + 10*(-63*I*tan(d*x + c)^2 + 150*tan(d*x + c) + 91*I)/(a^3*(I*tan(d*x + c)
 - 1)^2) - (959*I*tan(d*x + c)^5 + 5395*tan(d*x + c)^4 - 12390*I*tan(d*x +
c)^3 - 14710*tan(d*x + c)^2 + 9275*I*tan(d*x + c) + 2647)/(a^3*(tan(d*x + c
) - I)^5))/d
```

3.140 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=119

$$-\frac{7i\sec^5(c+dx)}{15a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{7\tan(c+dx)\sec^3(c+dx)}{12a^3d}+\frac{7\tan(c+dx)\sec(c+dx)}{8a^3d}-\frac{2i\sec^7(c+}{3ad(a+ia\tan(}$$

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - (((7*I)/15)*Sec[c + d*x]^5)/(a^3*d) +
 (7*Sec[c + d*x]*Tan[c + d*x])/(8*a^3*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/
(12*a^3*d) - (((2*I)/3)*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rubi [A] time = 0.114983, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3500, 3501, 3768, 3770}

$$-\frac{7i\sec^5(c+dx)}{15a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{7\tan(c+dx)\sec^3(c+dx)}{12a^3d}+\frac{7\tan(c+dx)\sec(c+dx)}{8a^3d}-\frac{2i\sec^7(c+}{3ad(a+ia\tan(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^3,x]
```


```
[Out] (7*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - (((7*I)/15)*Sec[c + d*x]^5)/(a^3*d) +
 (7*Sec[c + d*x]*Tan[c + d*x])/(8*a^3*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/
(12*a^3*d) - (((2*I)/3)*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^2)
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2} + \frac{7\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx}{3a^2}$$
$$= -\frac{7i\sec^5(c+dx)}{15a^3d} - \frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2} + \frac{7\int \sec^5(c+dx)\,dx}{3a^3}$$
$$= -\frac{7i\sec^5(c+dx)}{15a^3d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{12a^3d} - \frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2} + \frac{7\int \sec^3(c+d}{4a^3}$$
$$= -\frac{7i\sec^5(c+dx)}{15a^3d} + \frac{7\sec(c+dx)\tan(c+dx)}{8a^3d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{12a^3d} - \frac{2i\sec^7(c}{3ad(a+ia\,\mathrm{ta}}$$
$$= \frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d} - \frac{7i\sec^5(c+dx)}{15a^3d} + \frac{7\sec(c+dx)\tan(c+dx)}{8a^3d} + \frac{7\sec^3(c+dx)\tan(}{12a^3d}$$

Mathematica [A] time = 0.37638, size = 113, normalized size = 0.95

$$\frac{\sec^8(c+dx)(\cos(3(c+dx))+i\sin(3(c+dx)))\left(-150i\sin(2(c+dx))+105i\sin(4(c+dx))+640\cos(2(c+dx))+1680i\,\mathrm{c}\right.}{960a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^8*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])*(448 + (1680*I)*Arc
Tanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]^5 + 640*Cos[2*(c + d*x)] -
(150*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(960*a^3*d*(-I + Tan[
c + d*x])^3)
```

Maple [B] time = 0.102, size = 430, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 11/8*I/a^3/d/(tan(1/2*d*x+1/2*c)-1)^2+5/8/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2-1/
2*I/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4+3/4/a^3/d/(tan(1/2*d*x+1/2*c)+1)^4+11/8*
I/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2+1/8/a^3/d/(tan(1/2*d*x+1/2*c)+1)-7/12*I/a^
3/d/(tan(1/2*d*x+1/2*c)+1)^3-3/2/a^3/d/(tan(1/2*d*x+1/2*c)+1)^3+1/5*I/a^3/d
/(tan(1/2*d*x+1/2*c)+1)^5+7/8/a^3/d*ln(tan(1/2*d*x+1/2*c)+1)-13/8*I/a^3/d/(
tan(1/2*d*x+1/2*c)+1)+1/8/a^3/d/(tan(1/2*d*x+1/2*c)-1)+13/8*I/a^3/d/(tan(1/
2*d*x+1/2*c)-1)-3/4/a^3/d/(tan(1/2*d*x+1/2*c)-1)^4-1/2*I/a^3/d/(tan(1/2*d*x
+1/2*c)-1)^4-5/8/a^3/d/(tan(1/2*d*x+1/2*c)-1)^2+7/12*I/a^3/d/(tan(1/2*d*x+1
/2*c)-1)^3-3/2/a^3/d/(tan(1/2*d*x+1/2*c)-1)^3-1/5*I/a^3/d/(tan(1/2*d*x+1/2*
c)-1)^5-7/8/a^3/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.08053, size = 460, normalized size = 3.87

$$\frac{\frac{16\left(-\frac{15i\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{390i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{400\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{360\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{15i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-136\right)}{-120i\,a^3+\frac{600i\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200i\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200i\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600i\,a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120i\,a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right.}{a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(16*(-15*I*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 390*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 400*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390*I*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 360*sin(d*x + c)^8/(cos(d*x + c) + 1)
^8 + 15*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 136)/(-120*I*a^3 + 600*I*a^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1200*I*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 1200*I*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a^3*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120*I*a^3*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10) + 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 - 7*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [B] time = 2.59923, size = 836, normalized size = 7.03

$$\frac{105\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(e^{(10i\,dx+10i\,c)}+\cdots\right.}{120\left(a^3de^{(10i\,dx+10i\,c)}+5\,a^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
+ I*c) + I) - 105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*
I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^
(I*d*x + I*c) - I) - 210*I*e^(9*I*d*x + 9*I*c) - 980*I*e^(7*I*d*x + 7*I*c)
- 1792*I*e^(5*I*d*x + 5*I*c) - 1580*I*e^(3*I*d*x + 3*I*c) + 210*I*e^(I*d*x
+ I*c))/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) + 10*a^3
*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(2*I*d*x
+ 2*I*c) + a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.23278, size = 224, normalized size = 1.88

$$\frac{\frac{105\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{105\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+360i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-390\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-960i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots\right.}{\left(\tan\left(\frac{1}{2}\cdots\right.\right.}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(105*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 105*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 360*I*tan(1/2*d*x + 1/
2*c)^8 - 390*tan(1/2*d*x + 1/2*c)^7 - 960*I*tan(1/2*d*x + 1/2*c)^6 + 400*I*
tan(1/2*d*x + 1/2*c)^4 + 390*tan(1/2*d*x + 1/2*c)^3 - 320*I*tan(1/2*d*x + 1
/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 136*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*
a^3))/d
```

3.141 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=93

$$-\frac{5i\sec^3(c+dx)}{3a^3d}+\frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{5\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (((5*I)/3)*Sec[c + d*x]^3)/(a^3*d) +
(5*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((2*I)*Sec[c + d*x]^5)/(a*d*(a +
I*a*Tan[c + d*x])^2)
```

Rubi [A] time = 0.0981361, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3500, 3501, 3768, 3770}

$$-\frac{5i\sec^3(c+dx)}{3a^3d}+\frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{5\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (((5*I)/3)*Sec[c + d*x]^3)/(a^3*d) +
(5*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((2*I)*Sec[c + d*x]^5)/(a*d*(a +
I*a*Tan[c + d*x])^2)
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= -\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}+\frac{5\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx}{a^2}\\
&= -\frac{5i\sec^3(c+dx)}{3a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}+\frac{5\int \sec^3(c+dx)\,dx}{a^3}\\
&= -\frac{5i\sec^3(c+dx)}{3a^3d}+\frac{5\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}+\frac{5\int \sec(c+dx)\,d}{2a^3}\\
&= \frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{5i\sec^3(c+dx)}{3a^3d}+\frac{5\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+d}
\end{aligned}
$$

Mathematica [A] time = 0.379595, size = 63, normalized size = 0.68

$$
\frac{60\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)-i\sec^3(c+dx)(-9i\sin(2(c+dx))+24\cos(2(c+dx))+20)}{12a^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (60*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] - I*Sec[c + d*x]^3*(20 + 24*Cos[2
*(c + d*x)] - (9*I)*Sin[2*(c + d*x)]))/(12*a^3*d)
```

Maple [B] time = 0.098, size = 258, normalized size = 2.8

$$
\frac{\frac{i}{3}}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{3}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{\frac{i}{2}}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{3}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/3*I/a^3/d/(tan(1/2*d*x+1/2*c)+1)^3+3/2/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2-1/2
*I/a^3/d/(tan(1/2*d*x+1/2*c)+1)^2-3/2/a^3/d/(tan(1/2*d*x+1/2*c)+1)-7/2*I/a^
3/d/(tan(1/2*d*x+1/2*c)+1)+5/2/a^3/d*ln(tan(1/2*d*x+1/2*c)+1)-1/3*I/a^3/d/(
tan(1/2*d*x+1/2*c)-1)^3-3/2/a^3/d/(tan(1/2*d*x+1/2*c)-1)^2-1/2*I/a^3/d/(tan
(1/2*d*x+1/2*c)-1)^2-3/2/a^3/d/(tan(1/2*d*x+1/2*c)-1)+7/2*I/a^3/d/(tan(1/2*
d*x+1/2*c)-1)-5/2/a^3/d*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.00669, size = 290, normalized size = 3.12

$$
\frac{\frac{4\left(-\frac{9i\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+22\right)}{6ia^3-\frac{18ia^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18ia^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6ia^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{2d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(-9*I*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9*I*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 + 22)/(6*I*a^3 - 18*I*a^3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 18*I*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*I*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6) + 5*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 -
 5*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [B] time = 2.49059, size = 521, normalized size = 5.6

$$\frac{15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(\right.}{6\left(a^3de^{(6i\,dx+6i\,c)}+3\,a^3de^{(4i\,dx+4i\,c)}+3\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c
) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x +
4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(5*I*
d*x + 5*I*c) - 80*I*e^(3*I*d*x + 3*I*c) - 66*I*e^(I*d*x + I*c))/(a^3*d*e^(6
*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c)
 + a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.2489, size = 154, normalized size = 1.66

$$\frac{\frac{15\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{15\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{2\left(9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-18i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-22i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 15*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/a^3 - 2*(9*tan(1/2*d*x + 1/2*c)^5 - 18*I*tan(1/2*d*x + 1/2*c)^4
 + 48*I*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 22*I)/((tan(1/2*d
*x + 1/2*c)^2 - 1)^3*a^3))/d
```

3.142 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=65

$$\frac{3i\sec(c+dx)}{a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((3*I)*Sec[c + d*x])/(a^3*d) + ((2*I)*
Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rubi [A] time = 0.0850484, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3500, 3501, 3770}

$$\frac{3i\sec(c+dx)}{a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((3*I)*Sec[c + d*x])/(a^3*d) + ((2*I)*
Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2} - \frac{3\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx}{a^2}$$
$$= \frac{3i\sec(c+dx)}{a^3d} + \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2} - \frac{3\int \sec(c+dx)\,dx}{a^3}$$
$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{3i\sec(c+dx)}{a^3d} + \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

Mathematica [A] time = 0.272839, size = 108, normalized size = 1.66

$$\frac{\sec^3(c+dx)(-\sin(dx)+i\cos(dx))^3\left((\tan(c+dx)-5i)(\cos(2c-dx)+i\sin(2c-dx))+6(\cos(3c)+i\sin(3c))\tanh^{-1}\right.}{a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(I*Cos[d*x] - Sin[d*x])^3*(6*ArcTanh[Sin[c] + Cos[c]*Tan[(d
*x)/2]]*(Cos[3*c] + I*Sin[3*c]) + (Cos[2*c - d*x] + I*Sin[2*c - d*x])*(-5*I
 + Tan[c + d*x])))/(a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] time = 0.079, size = 108, normalized size = 1.7

$$\frac{i}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - 3\,\frac{\ln(\tan(1/2\,dx+c/2)+1)}{da^3} + 8\,\frac{1}{da^3(\tan(1/2\,dx+c/2)-i)} - \frac{i}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] I/a^3/d/(tan(1/2*d*x+1/2*c)+1)-3/a^3/d*ln(tan(1/2*d*x+1/2*c)+1)+8/a^3/d/(ta
n(1/2*d*x+1/2*c)-I)-I/a^3/d/(tan(1/2*d*x+1/2*c)-1)+3/a^3/d*ln(tan(1/2*d*x+1
/2*c)-1)
```

Maxima [B] time = 1.54997, size = 444, normalized size = 6.83

$$\frac{(6\cos(3\,dx+3\,c)+6\cos(dx+c)+6i\sin(3\,dx+3\,c)+6i\sin(dx+c))\arctan(\cos(dx+c),\sin(dx+c)+1)+(6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] ((6*cos(3*d*x + 3*c) + 6*cos(d*x + c) + 6*I*sin(3*d*x + 3*c) + 6*I*sin(d*x
+ c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) + (6*cos(3*d*x + 3*c) + 6*cos
(d*x + c) + 6*I*sin(3*d*x + 3*c) + 6*I*sin(d*x + c))*arctan2(cos(d*x + c),
-sin(d*x + c) + 1) - (-3*I*cos(3*d*x + 3*c) - 3*I*cos(d*x + c) + 3*sin(3*d*
x + 3*c) + 3*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x
+ c) + 1) - (3*I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) - 3*sin(3*d*x + 3*c) -
```

```
 3*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)
+ 12*cos(2*d*x + 2*c) + 12*I*sin(2*d*x + 2*c) + 8)/((-2*I*a^3*cos(3*d*x + 3
*c) - 2*I*a^3*cos(d*x + c) + 2*a^3*sin(3*d*x + 3*c) + 2*a^3*sin(d*x + c))*d
)
```

Fricas [A] time = 2.35974, size = 302, normalized size = 4.65

$$-\frac{3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c)}-4i}{a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) + I) - 3*(e
^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 6*I*e^(2*I
*d*x + 2*I*c) - 4*I)/(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.22397, size = 151, normalized size = 2.32

$$-\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{2\left(4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 3*log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/a^3 - 2*(4*tan(1/2*d*x + 1/2*c)^2 - I*tan(1/2*d*x + 1/2*c) - 5)/((ta
n(1/2*d*x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + I)
*a^3))/d
```

3.143 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=32

$$\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

```
[Out] ((I/3)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3)
```

Rubi [A] time = 0.0372742, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3488}

$$\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/3)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3)
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

Mathematica [A] time = 0.046857, size = 32, normalized size = 1.

$$\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/3)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3)
```

Maple [A] time = 0.081, size = 57, normalized size = 1.8

$$2\,\frac{1}{da^3}\left(\frac{2\,i}{(\tan(1/2\,dx+c/2)-i)^2}+(\tan(1/2\,dx+c/2)-i)^{-1}-4/3\,(\tan(1/2\,dx+c/2)-i)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(2*I/(tan(1/2*d*x+1/2*c)-I)^2+1/(tan(1/2*d*x+1/2*c)-I)-4/3/(tan(1/2
*d*x+1/2*c)-I)^3)
```

Maxima [A] time = 0.989576, size = 39, normalized size = 1.22

$$\frac{i\cos\left(3\,dx+3\,c\right)+\sin\left(3\,dx+3\,c\right)}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

Fricas [A] time = 2.22272, size = 49, normalized size = 1.53

$$\frac{i\,e^{\left(-3i\,dx-3i\,c\right)}}{3\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.201, size = 49, normalized size = 1.53

$$\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{3\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

3.144 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=98

$$\frac{2i\sec(c+dx)}{15d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$

```
[Out] ((I/5)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (((2*I)/15)*Sec[c + d*x
])/(a*d*(a + I*a*Tan[c + d*x])^2) + (((2*I)/15)*Sec[c + d*x])/(d*(a^3 + I*a
^3*Tan[c + d*x]))
```

Rubi [A] time = 0.0766288, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3502, 3488}

$$\frac{2i\sec(c+dx)}{15d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2}+\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/5)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (((2*I)/15)*Sec[c + d*x
])/(a*d*(a + I*a*Tan[c + d*x])^2) + (((2*I)/15)*Sec[c + d*x])/(d*(a^3 + I*a
^3*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx&=\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{5a}\\&=\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2}+\frac{2\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx}{15a^2}\\&=\frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2}+\frac{2i\sec(c+dx)}{15d\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.117521, size = 54, normalized size = 0.55

$$-\frac{\sec^3(c+dx)(6i\sin(2(c+dx))+9\cos(2(c+dx))+5)}{30a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -(Sec[c + d*x]^3*(5 + 9*Cos[2*(c + d*x)] + (6*I)*Sin[2*(c + d*x)]))/(30*a^3
*d*(-I + Tan[c + d*x])^3)
```

Maple [A] time = 0.049, size = 90, normalized size = 0.9

$$2\frac{1}{da^3}\left(-8/3\,(\tan(1/2\,dx+c/2)-i)^{-3}-\frac{2\,i}{(\tan(1/2\,dx+c/2)-i)^4}+4/5\,(\tan(1/2\,dx+c/2)-i)^{-5}+\frac{2\,i}{(\tan(1/2\,dx+c/2)-i)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(-8/3/(tan(1/2*d*x+1/2*c)-I)^3-2*I/(tan(1/2*d*x+1/2*c)-I)^4+4/5/(ta
n(1/2*d*x+1/2*c)-I)^5+2*I/(tan(1/2*d*x+1/2*c)-I)^2+1/(tan(1/2*d*x+1/2*c)-I)
)
```

Maxima [A] time = 0.991177, size = 93, normalized size = 0.95

$$\frac{3i\cos(5\,dx+5\,c)+10i\cos(3\,dx+3\,c)+15i\cos(dx+c)+3\sin(5\,dx+5\,c)+10\sin(3\,dx+3\,c)+15\sin(dx+c)}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(3*I*cos(5*d*x + 5*c) + 10*I*cos(3*d*x + 3*c) + 15*I*cos(d*x + c) + 3*
sin(5*d*x + 5*c) + 10*sin(3*d*x + 3*c) + 15*sin(d*x + c))/(a^3*d)
```

Fricas [A] time = 2.02872, size = 128, normalized size = 1.31

$$\frac{\left(15i\,e^{(4i\,dx+4i\,c)}+10i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(15*I*e^(4*I*d*x + 4*I*c) + 10*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*
x - 5*I*c)/(a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.1774, size = 99, normalized size = 1.01

$$\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-30i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+20i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\right)}{15\,a^{3}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 30*I*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2*
d*x + 1/2*c)^2 + 20*I*tan(1/2*d*x + 1/2*c) + 7)/(a^3*d*(tan(1/2*d*x + 1/2*c
) - I)^5)
```

3.145 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=101

$$-\frac{4\sin^3(c+dx)}{35a^3d}+\frac{12\sin(c+dx)}{35a^3d}+\frac{8i\cos^3(c+dx)}{35d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

```
[Out] (12*Sin[c + d*x])/(35*a^3*d) - (4*Sin[c + d*x]^3)/(35*a^3*d) + ((I/7)*Cos[c
 + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (((8*I)/35)*Cos[c + d*x]^3)/(d*(a^3
 + I*a^3*Tan[c + d*x]))
```

Rubi [A] time = 0.0810969, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3502, 3500, 2633}

$$-\frac{4\sin^3(c+dx)}{35a^3d}+\frac{12\sin(c+dx)}{35a^3d}+\frac{8i\cos^3(c+dx)}{35d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (12*Sin[c + d*x])/(35*a^3*d) - (4*Sin[c + d*x]^3)/(35*a^3*d) + ((I/7)*Cos[c
 + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (((8*I)/35)*Cos[c + d*x]^3)/(d*(a^3
 + I*a^3*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3} + \frac{4\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{7a}$$
$$= \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3} + \frac{8i\cos^3(c+dx)}{35d\left(a^3+ia^3\tan(c+dx)\right)} + \frac{12\int \cos^3(c+dx)\,dx}{35a^3}$$
$$= \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3} + \frac{8i\cos^3(c+dx)}{35d\left(a^3+ia^3\tan(c+dx)\right)} - \frac{12\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin\right.}{35a^3d}$$
$$= \frac{12\sin(c+dx)}{35a^3d} - \frac{4\sin^3(c+dx)}{35a^3d} + \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3} + \frac{8i\cos^3(c+dx)}{35d\left(a^3+ia^3\tan(c+dx)\right)}$$

Mathematica [A] time = 0.177119, size = 76, normalized size = 0.75

$$-\frac{\sec^3(c+dx)(56i\sin(2(c+dx)) - 20i\sin(4(c+dx)) + 84\cos(2(c+dx)) - 15\cos(4(c+dx)) + 35)}{280a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -(Sec[c + d*x]^3*(35 + 84*Cos[2*(c + d*x)] - 15*Cos[4*(c + d*x)] + (56*I)*S
in[2*(c + d*x)] - (20*I)*Sin[4*(c + d*x)]))/(280*a^3*d*(-I + Tan[c + d*x])^
3)
```

Maple [A] time = 0.094, size = 141, normalized size = 1.4

$$2\,\frac{1}{da^3}\left(\frac{2\,i}{(\tan(1/2\,dx+c/2)-i)^6} - \frac{9/2\,i}{(\tan(1/2\,dx+c/2)-i)^4} + \frac{\frac{17\,i}{8}}{(\tan(1/2\,dx+c/2)-i)^2} - 4/7\,(\tan(1/2\,dx+c/2)-i)^{-7} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(2*I/(tan(1/2*d*x+1/2*c)-I)^6-9/2*I/(tan(1/2*d*x+1/2*c)-I)^4+17/8*I
/(tan(1/2*d*x+1/2*c)-I)^2-4/7/(tan(1/2*d*x+1/2*c)-I)^7+19/5/(tan(1/2*d*x+1/
2*c)-I)^5-15/4/(tan(1/2*d*x+1/2*c)-I)^3+15/16/(tan(1/2*d*x+1/2*c)-I)+1/16/(
tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.23384, size = 205, normalized size = 2.03

$$\frac{\left(-35i\,e^{(8i\,dx+8i\,c)}+140i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+28i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{560\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(-35*I*e^(8*I*d*x + 8*I*c) + 140*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*
d*x + 4*I*c) + 28*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^3*d)
```

Sympy [A] time = 1.05874, size = 199, normalized size = 1.97

$$\begin{cases} \frac{\left(-71680ia^{12}d^4e^{17ic}e^{idx}+286720ia^{12}d^4e^{15ic}e^{-idx}+143360ia^{12}d^4e^{13ic}e^{-3idx}+57344ia^{12}d^4e^{11ic}e^{-5idx}+10240ia^{12}d^4e^{9ic}e^{-7idx}\right)e^{-16ic}}{1146880a^{15}d^5} & \text{for } 1146880a^{15}d^5e^{16ic} \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-7ic}}{16a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-71680*I*a**12*d**4*exp(17*I*c)*exp(I*d*x) + 286720*I*a**12*d**
4*exp(15*I*c)*exp(-I*d*x) + 143360*I*a**12*d**4*exp(13*I*c)*exp(-3*I*d*x) +
 57344*I*a**12*d**4*exp(11*I*c)*exp(-5*I*d*x) + 10240*I*a**12*d**4*exp(9*I*
c)*exp(-7*I*d*x))*exp(-16*I*c)/(1146880*a**15*d**5), Ne(1146880*a**15*d**5*
exp(16*I*c), 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*
c) + 1)*exp(-7*I*c)/(16*a**3), True))
```

Giac [A] time = 1.1653, size = 161, normalized size = 1.59

$$\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1960i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4025\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+4480i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3143\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1176i\,\tan\left(\frac{1}{2}\,dx+\right.}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) + I)) + (525*tan(1/2*d*x + 1/2*c)^6 -
1960*I*tan(1/2*d*x + 1/2*c)^5 - 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*I*tan(1/
2*d*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 - 1176*I*tan(1/2*d*x + 1/2*c
) - 243)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

3.146 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=121

$$\frac{2\sin^5(c+dx)}{21a^3d}-\frac{20\sin^3(c+dx)}{63a^3d}+\frac{10\sin(c+dx)}{21a^3d}+\frac{4i\cos^5(c+dx)}{21d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

```
[Out] (10*Sin[c + d*x])/(21*a^3*d) - (20*Sin[c + d*x]^3)/(63*a^3*d) + (2*Sin[c +
d*x]^5)/(21*a^3*d) + ((I/9)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3) +
(((4*I)/21)*Cos[c + d*x]^5)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rubi [A] time = 0.11338, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3502, 3500, 2633}

$$\frac{2\sin^5(c+dx)}{21a^3d}-\frac{20\sin^3(c+dx)}{63a^3d}+\frac{10\sin(c+dx)}{21a^3d}+\frac{4i\cos^5(c+dx)}{21d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (10*Sin[c + d*x])/(21*a^3*d) - (20*Sin[c + d*x]^3)/(63*a^3*d) + (2*Sin[c +
d*x]^5)/(21*a^3*d) + ((I/9)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3) +
(((4*I)/21)*Cos[c + d*x]^5)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3} + \frac{2\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{3a}$$
$$= \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3} + \frac{4i\cos^5(c+dx)}{21d\left(a^3+ia^3\tan(c+dx)\right)} + \frac{10\int \cos^5(c+dx)\,dx}{21a^3}$$
$$= \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3} + \frac{4i\cos^5(c+dx)}{21d\left(a^3+ia^3\tan(c+dx)\right)} - \frac{10\,\mathrm{Subst}\left(\int\left(1-2x^2+x^4\right)dx,x,-\right.}{21a^3d}$$
$$= \frac{10\sin(c+dx)}{21a^3d} - \frac{20\sin^3(c+dx)}{63a^3d} + \frac{2\sin^5(c+dx)}{21a^3d} + \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3} + \frac{4i\cos}{21d\left(a^3+ia\right.}$$

Mathematica [A] time = 0.225129, size = 98, normalized size = 0.81

$$\frac{\sec^3(c+dx)(-378i\sin(2(c+dx))+216i\sin(4(c+dx))+14i\sin(6(c+dx))-567\cos(2(c+dx))+162\cos(4(c+dx))+}{2016a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-210 - 567*Cos[2*(c + d*x)] + 162*Cos[4*(c + d*x)] + 7*Cos
[6*(c + d*x)] - (378*I)*Sin[2*(c + d*x)] + (216*I)*Sin[4*(c + d*x)] + (14*I
)*Sin[6*(c + d*x)]))/(2016*a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] time = 0.103, size = 207, normalized size = 1.7

$$2\frac{1}{da^3}\left(\frac{\frac{23\,i}{3}}{(\tan(1/2\,dx+c/2)-i)^6} - \frac{2\,i}{(\tan(1/2\,dx+c/2)-i)^8} + \frac{9/4\,i}{(\tan(1/2\,dx+c/2)-i)^2} - \frac{\frac{59\,i}{8}}{(\tan(1/2\,dx+c/2)-i)^4} + 4/9\,(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(23/3*I/(tan(1/2*d*x+1/2*c)-I)^6-2*I/(tan(1/2*d*x+1/2*c)-I)^8+9/4*I
/(tan(1/2*d*x+1/2*c)-I)^2-59/8*I/(tan(1/2*d*x+1/2*c)-I)^4+4/9/(tan(1/2*d*x+
1/2*c)-I)^9-34/7/(tan(1/2*d*x+1/2*c)-I)^7+35/4/(tan(1/2*d*x+1/2*c)-I)^5-19/
4/(tan(1/2*d*x+1/2*c)-I)^3+57/64/(tan(1/2*d*x+1/2*c)-I)-1/32*I/(tan(1/2*d*x
+1/2*c)+I)^2-1/48/(tan(1/2*d*x+1/2*c)+I)^3+7/64/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.3048, size = 289, normalized size = 2.39

$$\frac{\left(-21 i e^{(12 i dx+12 i c)} - 378 i e^{(10 i dx+10 i c)} + 945 i e^{(8 i dx+8 i c)} + 420 i e^{(6 i dx+6 i c)} + 189 i e^{(4 i dx+4 i c)} + 54 i e^{(2 i dx+2 i c)} + 7 i\right) e^{(-9 i dx-9 i c)}}{4032 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4032*(-21*I*e^(12*I*d*x + 12*I*c) - 378*I*e^(10*I*d*x + 10*I*c) + 945*I*e
^(8*I*d*x + 8*I*c) + 420*I*e^(6*I*d*x + 6*I*c) + 189*I*e^(4*I*d*x + 4*I*c)
+ 54*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-9*I*d*x - 9*I*c)/(a^3*d)
```

Sympy [A] time = 1.59516, size = 267, normalized size = 2.21

$$\begin{cases} \frac{\left(-811748818944 i a^{18} d^6 e^{28 i c} e^{3 i d x} - 14611478740992 i a^{18} d^6 e^{26 i c} e^{i d x} + 36528696852480 i a^{18} d^6 e^{24 i c} e^{-i d x} + 16234976378880 i a^{18} d^6 e^{22 i c} e^{-3 i d x} + 7305739370496 i a^{18} \ldots\right.}{155855773237248 a^{21} d^7} \\ \frac{x\left(e^{12 i c} + 6 e^{10 i c} + 15 e^{8 i c} + 20 e^{6 i c} + 15 e^{4 i c} + 6 e^{2 i c} + 1\right) e^{-9 i c}}{64 a^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-811748818944*I*a**18*d**6*exp(28*I*c)*exp(3*I*d*x) - 146114787
40992*I*a**18*d**6*exp(26*I*c)*exp(I*d*x) + 36528696852480*I*a**18*d**6*exp
(24*I*c)*exp(-I*d*x) + 16234976378880*I*a**18*d**6*exp(22*I*c)*exp(-3*I*d*x
) + 7305739370496*I*a**18*d**6*exp(20*I*c)*exp(-5*I*d*x) + 2087354105856*I*
a**18*d**6*exp(18*I*c)*exp(-7*I*d*x) + 270582939648*I*a**18*d**6*exp(16*I*c
)*exp(-9*I*d*x))*exp(-25*I*c)/(155855773237248*a**21*d**7), Ne(155855773237
248*a**21*d**7*exp(25*I*c), 0)), (x*(exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8
*I*c) + 20*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-9*I*c)/(64*a
**3), True))
```

Giac [A] time = 1.17019, size = 231, normalized size = 1.91

$$\frac{\frac{21\left(21 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 + 36 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) - 19\right)}{a^3\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+i\right)^3} + \frac{3591 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8 - 19656 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7 - 56196 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6 + 95760 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5 + 107 \ldots}{a^3\left(\tan\left(\frac{1}{2} d \ldots\right.\right.}}{2016 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2016*(21*(21*tan(1/2*d*x + 1/2*c)^2 + 36*I*tan(1/2*d*x + 1/2*c) - 19)/(a^
3*(tan(1/2*d*x + 1/2*c) + I)^3) + (3591*tan(1/2*d*x + 1/2*c)^8 - 19656*I*ta
n(1/2*d*x + 1/2*c)^7 - 56196*tan(1/2*d*x + 1/2*c)^6 + 95760*I*tan(1/2*d*x +
 1/2*c)^5 + 107730*tan(1/2*d*x + 1/2*c)^4 - 79464*I*tan(1/2*d*x + 1/2*c)^3
- 38484*tan(1/2*d*x + 1/2*c)^2 + 10944*I*tan(1/2*d*x + 1/2*c) + 1615)/(a^3*
(tan(1/2*d*x + 1/2*c) - I)^9))/d
```

3.147 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=139

$$-\frac{8\sin^7(c+dx)}{99a^3d}+\frac{56\sin^5(c+dx)}{165a^3d}-\frac{56\sin^3(c+dx)}{99a^3d}+\frac{56\sin(c+dx)}{99a^3d}+\frac{16i\cos^7(c+dx)}{99d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+$$

```
[Out] (56*Sin[c + d*x])/(99*a^3*d) - (56*Sin[c + d*x]^3)/(99*a^3*d) + (56*Sin[c +
 d*x]^5)/(165*a^3*d) - (8*Sin[c + d*x]^7)/(99*a^3*d) + ((I/11)*Cos[c + d*x]
^5)/(d*(a + I*a*Tan[c + d*x])^3) + (((16*I)/99)*Cos[c + d*x]^7)/(d*(a^3 + I
*a^3*Tan[c + d*x]))
```

Rubi [A] time = 0.116676, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3502, 3500, 2633}

$$-\frac{8\sin^7(c+dx)}{99a^3d}+\frac{56\sin^5(c+dx)}{165a^3d}-\frac{56\sin^3(c+dx)}{99a^3d}+\frac{56\sin(c+dx)}{99a^3d}+\frac{16i\cos^7(c+dx)}{99d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (56*Sin[c + d*x])/(99*a^3*d) - (56*Sin[c + d*x]^3)/(99*a^3*d) + (56*Sin[c +
 d*x]^5)/(165*a^3*d) - (8*Sin[c + d*x]^7)/(99*a^3*d) + ((I/11)*Cos[c + d*x]
^5)/(d*(a + I*a*Tan[c + d*x])^3) + (((16*I)/99)*Cos[c + d*x]^7)/(d*(a^3 + I
*a^3*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3} + \frac{8\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{11a}$$
$$= \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3} + \frac{16i\cos^7(c+dx)}{99d\left(a^3+ia^3\tan(c+dx)\right)} + \frac{56\int\cos^7(c+dx)\,dx}{99a^3}$$
$$= \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3} + \frac{16i\cos^7(c+dx)}{99d\left(a^3+ia^3\tan(c+dx)\right)} - \frac{56\,\mathrm{Subst}\left(\int\left(1-3x^2+3x^4-x\right.\right.}{99a^3d}$$
$$= \frac{56\sin(c+dx)}{99a^3d} - \frac{56\sin^3(c+dx)}{99a^3d} + \frac{56\sin^5(c+dx)}{165a^3d} - \frac{8\sin^7(c+dx)}{99a^3d} + \frac{i\cos^5(c+d}{11d(a+ia\tan(c}$$

Mathematica [A] time = 0.547016, size = 120, normalized size = 0.86

$$\frac{\sec^3(c+dx)(-11088i\sin(2(c+dx))+7920i\sin(4(c+dx))+880i\sin(6(c+dx))+72i\sin(8(c+dx))-16632\cos(2(c}{63360a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-5775 - 16632*Cos[2*(c + d*x)] + 5940*Cos[4*(c + d*x)] + 4
40*Cos[6*(c + d*x)] + 27*Cos[8*(c + d*x)] - (11088*I)*Sin[2*(c + d*x)] + (7
920*I)*Sin[4*(c + d*x)] + (880*I)*Sin[6*(c + d*x)] + (72*I)*Sin[8*(c + d*x)
]))/(63360*a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [B] time = 0.103, size = 273, normalized size = 2.

$$2\,\frac{1}{da^3}\left(\frac{\frac{303\,i}{128}}{(\tan(1/2\,dx+c/2)-i)^2} - \frac{\frac{5\,i}{64}}{(\tan(1/2\,dx+c/2)+i)^2} + \frac{2\,i}{(\tan(1/2\,dx+c/2)-i)^{10}} - \frac{23/2\,i}{(\tan(1/2\,dx+c/2)-i)^8} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(303/128*I/(tan(1/2*d*x+1/2*c)-I)^2-5/64*I/(tan(1/2*d*x+1/2*c)+I)^2
+2*I/(tan(1/2*d*x+1/2*c)-I)^10-23/2*I/(tan(1/2*d*x+1/2*c)-I)^8+1/32*I/(tan(
1/2*d*x+1/2*c)+I)^4-4/11/(tan(1/2*d*x+1/2*c)-I)^11+53/9/(tan(1/2*d*x+1/2*c)
-I)^9-33/2/(tan(1/2*d*x+1/2*c)-I)^7+623/40/(tan(1/2*d*x+1/2*c)-I)^5-365/64/
(tan(1/2*d*x+1/2*c)-I)^3+219/256/(tan(1/2*d*x+1/2*c)-I)+217/12*I/(tan(1/2*d
*x+1/2*c)-I)^6-169/16*I/(tan(1/2*d*x+1/2*c)-I)^4+1/80/(tan(1/2*d*x+1/2*c)+I
)^5-7/96/(tan(1/2*d*x+1/2*c)+I)^3+37/256/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.40705, size = 390, normalized size = 2.81

$$\frac{\left(-99i\,e^{(16i\,dx+16i\,c)}-1320i\,e^{(14i\,dx+14i\,c)}-13860i\,e^{(12i\,dx+12i\,c)}+27720i\,e^{(10i\,dx+10i\,c)}+11550i\,e^{(8i\,dx+8i\,c)}+5544i\,e^{(6i\,dx+6i\,c)}+\cdots}{126720\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/126720*(-99*I*e^(16*I*d*x + 16*I*c) - 1320*I*e^(14*I*d*x + 14*I*c) - 1386
0*I*e^(12*I*d*x + 12*I*c) + 27720*I*e^(10*I*d*x + 10*I*c) + 11550*I*e^(8*I*
d*x + 8*I*c) + 5544*I*e^(6*I*d*x + 6*I*c) + 1980*I*e^(4*I*d*x + 4*I*c) + 44
0*I*e^(2*I*d*x + 2*I*c) + 45*I)*e^(-11*I*d*x - 11*I*c)/(a^3*d)
```

Sympy [A] time = 2.18172, size = 335, normalized size = 2.41

$$\begin{cases} \frac{\left(-626985510622986240ia^{24}d^8e^{41ic}e^{5idx}-835980680830648320 0ia^{24}d^8e^{39ic}e^{3idx}-87777971487218073600ia^{24}d^8e^{37ic}e^{idx}+175555942974436147200ia^{24}d^8e^{35ic}e^{-\cdots}\right.}{\cdots} \\ \frac{x\left(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1\right)e^{-11ic}}{256a^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-626985510622986240*I*a**24*d**8*exp(41*I*c)*exp(5*I*d*x) - 835
9806808306483200*I*a**24*d**8*exp(39*I*c)*exp(3*I*d*x) - 877779714872180736
00*I*a**24*d**8*exp(37*I*c)*exp(I*d*x) + 175555942974436147200*I*a**24*d**8
*exp(35*I*c)*exp(-I*d*x) + 73148309572681728000*I*a**24*d**8*exp(33*I*c)*ex
p(-3*I*d*x) + 35111188594887229440*I*a**24*d**8*exp(31*I*c)*exp(-5*I*d*x) +
 12539710212459724800*I*a**24*d**8*exp(29*I*c)*exp(-7*I*d*x) + 278660226943
5494400*I*a**24*d**8*exp(27*I*c)*exp(-9*I*d*x) + 284993413919539200*I*a**24
*d**8*exp(25*I*c)*exp(-11*I*d*x))*exp(-36*I*c)/(802541453597422387200*a**27
*d**9), Ne(802541453597422387200*a**27*d**9*exp(36*I*c), 0)), (x*(exp(16*I*
c) + 8*exp(14*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(8*I*c) + 56*e
xp(6*I*c) + 28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-11*I*c)/(256*a**3), True
))
```

Giac [A] time = 1.18331, size = 301, normalized size = 2.17

$$\frac{\frac{33\left(555\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1920i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2710\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1760i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+463\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^5}+\frac{108405\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-784080i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-2\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/63360*(33*(555*tan(1/2*d*x + 1/2*c)^4 + 1920*I*tan(1/2*d*x + 1/2*c)^3 - 2
710*tan(1/2*d*x + 1/2*c)^2 - 1760*I*tan(1/2*d*x + 1/2*c) + 463)/(a^3*(tan(1
```

```
/2*d*x + 1/2*c) + I)^5) + (108405*tan(1/2*d*x + 1/2*c)^10 - 784080*I*tan(1/
2*d*x + 1/2*c)^9 - 2901195*tan(1/2*d*x + 1/2*c)^8 + 6652800*I*tan(1/2*d*x +
 1/2*c)^7 + 10407474*tan(1/2*d*x + 1/2*c)^6 - 11435424*I*tan(1/2*d*x + 1/2*
c)^5 - 8949270*tan(1/2*d*x + 1/2*c)^4 + 4899840*I*tan(1/2*d*x + 1/2*c)^3 +
1816265*tan(1/2*d*x + 1/2*c)^2 - 411664*I*tan(1/2*d*x + 1/2*c) - 47279)/(a^
3*(tan(1/2*d*x + 1/2*c) - I)^11))/d
```

3.148 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=82

$$\frac{i(a-ia\tan(c+dx))^9}{9a^{13}d}-\frac{i(a-ia\tan(c+dx))^8}{2a^{12}d}+\frac{4i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

```
[Out] (((4*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^11*d) - ((I/2)*(a - I*a*Tan[c + d*x
])^8)/(a^12*d) + ((I/9)*(a - I*a*Tan[c + d*x])^9)/(a^13*d)
```

Rubi [A] time = 0.0636081, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a-ia\tan(c+dx))^9}{9a^{13}d}-\frac{i(a-ia\tan(c+dx))^8}{2a^{12}d}+\frac{4i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (((4*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^11*d) - ((I/2)*(a - I*a*Tan[c + d*x
])^8)/(a^12*d) + ((I/9)*(a - I*a*Tan[c + d*x])^9)/(a^13*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^6(a+x)^2\,dx,x,ia\tan(c+dx)\right)}{a^{13}d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a-x)^6-4a(a-x)^7+(a-x)^8\right)dx,x,ia\tan(c+dx)\right)}{a^{13}d}\\ &= \frac{4i(a-ia\tan(c+dx))^7}{7a^{11}d}-\frac{i(a-ia\tan(c+dx))^8}{2a^{12}d}+\frac{i(a-ia\tan(c+dx))^9}{9a^{13}d}\end{aligned}$$

Mathematica [A] time = 0.559942, size = 136, normalized size = 1.66

$$\frac{\sec(c)\sec^9(c+dx)(-63\sin(2c+dx)+42\sin(2c+3dx)-42\sin(4c+3dx)+36\sin(4c+5dx)+9\sin(6c+7dx)+\sin(8}{252a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^9*((-63*I)*Cos[d*x] - (63*I)*Cos[2*c + d*x] - (42*I)*C
os[2*c + 3*d*x] - (42*I)*Cos[4*c + 3*d*x] + 63*Sin[d*x] - 63*Sin[2*c + d*x]
 + 42*Sin[2*c + 3*d*x] - 42*Sin[4*c + 3*d*x] + 36*Sin[4*c + 5*d*x] + 9*Sin[
6*c + 7*d*x] + Sin[8*c + 9*d*x]))/(252*a^4*d)
```

Maple [A] time = 0.087, size = 99, normalized size = 1.2

$$\frac{1}{a^4 d}\left(\tan(dx+c) + \frac{(\tan(dx+c))^9}{9} + \frac{i}{2}(\tan(dx+c))^8 - \frac{4(\tan(dx+c))^7}{7} + \frac{2i}{3}(\tan(dx+c))^6 - 2(\tan(dx+c))^5 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*(tan(d*x+c)+1/9*tan(d*x+c)^9+1/2*I*tan(d*x+c)^8-4/7*tan(d*x+c)^7+2/
3*I*tan(d*x+c)^6-2*tan(d*x+c)^5-I*tan(d*x+c)^4-4/3*tan(d*x+c)^3-2*I*tan(d*x
+c)^2)
```

Maxima [A] time = 0.984934, size = 131, normalized size = 1.6

$$\frac{14\tan(dx+c)^9 + 63i\tan(dx+c)^8 - 72\tan(dx+c)^7 + 84i\tan(dx+c)^6 - 252\tan(dx+c)^5 - 126i\tan(dx+c)^4}{126a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/126*(14*tan(d*x + c)^9 + 63*I*tan(d*x + c)^8 - 72*tan(d*x + c)^7 + 84*I*t
an(d*x + c)^6 - 252*tan(d*x + c)^5 - 126*I*tan(d*x + c)^4 - 168*tan(d*x + c
)^3 - 252*I*tan(d*x + c)^2 + 126*tan(d*x + c))/(a^4*d)
```

Fricas [B] time = 2.8042, size = 494, normalized size = 6.02

$$\frac{4608i\,e^{(4i\,dx+4i\,c)} + 1152i\,e^{(2i\,dx+2i\,c)} + 128i}{63\left(a^4 de^{(18i\,dx+18i\,c)} + 9\,a^4 de^{(16i\,dx+16i\,c)} + 36\,a^4 de^{(14i\,dx+14i\,c)} + 84\,a^4 de^{(12i\,dx+12i\,c)} + 126\,a^4 de^{(10i\,dx+10i\,c)} + 126\,a^4 de^{(8i\,dx+8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/63*(4608*I*e^(4*I*d*x + 4*I*c) + 1152*I*e^(2*I*d*x + 2*I*c) + 128*I)/(a^4
*d*e^(18*I*d*x + 18*I*c) + 9*a^4*d*e^(16*I*d*x + 16*I*c) + 36*a^4*d*e^(14*I
*d*x + 14*I*c) + 84*a^4*d*e^(12*I*d*x + 12*I*c) + 126*a^4*d*e^(10*I*d*x + 1
0*I*c) + 126*a^4*d*e^(8*I*d*x + 8*I*c) + 84*a^4*d*e^(6*I*d*x + 6*I*c) + 36*
a^4*d*e^(4*I*d*x + 4*I*c) + 9*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.19941, size = 131, normalized size = 1.6

$$\frac{14\,\tan\left(dx+c\right)^{9}+63i\,\tan\left(dx+c\right)^{8}-72\,\tan\left(dx+c\right)^{7}+84i\,\tan\left(dx+c\right)^{6}-252\,\tan\left(dx+c\right)^{5}-126i\,\tan\left(dx+c\right)^{4}-1}{126\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/126*(14*tan(d*x + c)^9 + 63*I*tan(d*x + c)^8 - 72*tan(d*x + c)^7 + 84*I*t
an(d*x + c)^6 - 252*tan(d*x + c)^5 - 126*I*tan(d*x + c)^4 - 168*tan(d*x + c
)^3 - 252*I*tan(d*x + c)^2 + 126*tan(d*x + c))/(a^4*d)
```

3.149 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=55

$$\frac{i(a-ia\tan(c+dx))^6}{3a^{10}d}-\frac{i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

```
[Out] ((I/3)*(a - I*a*Tan[c + d*x])^6)/(a^10*d) - ((I/7)*(a - I*a*Tan[c + d*x])^7
)/(a^11*d)
```

Rubi [A] time = 0.0472249, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a-ia\tan(c+dx))^6}{3a^{10}d}-\frac{i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/3)*(a - I*a*Tan[c + d*x])^6)/(a^10*d) - ((I/7)*(a - I*a*Tan[c + d*x])^7
)/(a^11*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^5(a+x)\,dx, x, ia\tan(c+dx)\right)}{a^{11}d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(2a(a-x)^5-(a-x)^6\right)\,dx, x, ia\tan(c+dx)\right)}{a^{11}d}\\ &= \frac{i(a-ia\tan(c+dx))^6}{3a^{10}d}-\frac{i(a-ia\tan(c+dx))^7}{7a^{11}d}\end{aligned}$$

Mathematica [B] time = 0.347308, size = 127, normalized size = 2.31

$$\frac{\sec(c)\sec^7(c+dx)(-35\sin(2c+dx)+21\sin(2c+3dx)-21\sin(4c+3dx)+14\sin(4c+5dx)+2\sin(6c+7dx)-35\ldots}{84a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^7*((-35*I)*Cos[d*x] - (35*I)*Cos[2*c + d*x] - (21*I)*C
os[2*c + 3*d*x] - (21*I)*Cos[4*c + 3*d*x] + 35*Sin[d*x] - 35*Sin[2*c + d*x]
 + 21*Sin[2*c + 3*d*x] - 21*Sin[4*c + 3*d*x] + 14*Sin[4*c + 5*d*x] + 2*Sin[
6*c + 7*d*x]))/(84*a^4*d)
```

Maple [A] time = 0.085, size = 67, normalized size = 1.2

$$\frac{1}{a^4d}\left(\tan(dx+c)+\frac{(\tan(dx+c))^7}{7}+\frac{2i}{3}(\tan(dx+c))^6-(\tan(dx+c))^5-\frac{5(\tan(dx+c))^3}{3}-2i(\tan(dx+c))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*(tan(d*x+c)+1/7*tan(d*x+c)^7+2/3*I*tan(d*x+c)^6-tan(d*x+c)^5-5/3*ta
n(d*x+c)^3-2*I*tan(d*x+c)^2)
```

Maxima [A] time = 0.98677, size = 90, normalized size = 1.64

$$\frac{3\tan(dx+c)^7+14i\tan(dx+c)^6-21\tan(dx+c)^5-35\tan(dx+c)^3-42i\tan(dx+c)^2+21\tan(dx+c)}{21a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/21*(3*tan(d*x + c)^7 + 14*I*tan(d*x + c)^6 - 21*tan(d*x + c)^5 - 35*tan(d
*x + c)^3 - 42*I*tan(d*x + c)^2 + 21*tan(d*x + c))/(a^4*d)
```

Fricas [B] time = 2.50114, size = 360, normalized size = 6.55

$$\frac{448i\,e^{(2i\,dx+2i\,c)}+64i}{21\left(a^4de^{(14i\,dx+14i\,c)}+7\,a^4de^{(12i\,dx+12i\,c)}+21\,a^4de^{(10i\,dx+10i\,c)}+35\,a^4de^{(8i\,dx+8i\,c)}+35\,a^4de^{(6i\,dx+6i\,c)}+21\,a^4de^{(4i\,dx+4i\,c)}+7\,a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/21*(448*I*e^(2*I*d*x + 2*I*c) + 64*I)/(a^4*d*e^(14*I*d*x + 14*I*c) + 7*a^
4*d*e^(12*I*d*x + 12*I*c) + 21*a^4*d*e^(10*I*d*x + 10*I*c) + 35*a^4*d*e^(8*
I*d*x + 8*I*c) + 35*a^4*d*e^(6*I*d*x + 6*I*c) + 21*a^4*d*e^(4*I*d*x + 4*I*c
) + 7*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.16022, size = 90, normalized size = 1.64

$$\frac{3 \tan (dx+c)^7 + 14i \tan (dx+c)^6 - 21 \tan (dx+c)^5 - 35 \tan (dx+c)^3 - 42i \tan (dx+c)^2 + 21 \tan (dx+c)}{21 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/21*(3*tan(d*x + c)^7 + 14*I*tan(d*x + c)^6 - 21*tan(d*x + c)^5 - 35*tan(d
*x + c)^3 - 42*I*tan(d*x + c)^2 + 21*tan(d*x + c))/(a^4*d)
```

3.150 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=27

$$\frac{i(a-ia\tan(c+dx))^5}{5a^9d}$$

```
[Out] ((I/5)*(a - I*a*Tan[c + d*x])^5)/(a^9*d)
```

Rubi [A] time = 0.0400154, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$\frac{i(a-ia\tan(c+dx))^5}{5a^9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/5)*(a - I*a*Tan[c + d*x])^5)/(a^9*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^4\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= \frac{i(a-ia\tan(c+dx))^5}{5a^9d}\end{aligned}$$

Mathematica [B] time = 0.367759, size = 116, normalized size = 4.3

$$\frac{\sec(c)\sec^5(c+dx)(-10\sin(2c+dx)+5\sin(2c+3dx)-5\sin(4c+3dx)+2\sin(4c+5dx)-10i\cos(2c+dx)-5i\cos(2c}{10a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^5*((-10*I)*Cos[d*x] - (10*I)*Cos[2*c + d*x] - (5*I)*Co
s[2*c + 3*d*x] - (5*I)*Cos[4*c + 3*d*x] + 10*Sin[d*x] - 10*Sin[2*c + d*x] +
```

```
5*Sin[2*c + 3*d*x] - 5*Sin[4*c + 3*d*x] + 2*Sin[4*c + 5*d*x]))/(10*a^4*d)
```

Maple [B] time = 0.084, size = 57, normalized size = 2.1

$$\frac{1}{a^4 d}\left(\tan(dx+c)+\frac{(\tan(dx+c))^5}{5}+i(\tan(dx+c))^4-2(\tan(dx+c))^3-2i(\tan(dx+c))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*(tan(d*x+c)+1/5*tan(d*x+c)^5+I*tan(d*x+c)^4-2*tan(d*x+c)^3-2*I*tan(
d*x+c)^2)
```

Maxima [B] time = 0.977301, size = 77, normalized size = 2.85

$$\frac{3\tan(dx+c)^5+15i\tan(dx+c)^4-30\tan(dx+c)^3-30i\tan(dx+c)^2+15\tan(dx+c)}{15a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 - 30*tan(d*x + c)^3 - 30*I*tan
(d*x + c)^2 + 15*tan(d*x + c))/(a^4*d)
```

Fricas [B] time = 2.41311, size = 227, normalized size = 8.41

$$\frac{32i}{5\left(a^4de^{(10i\,dx+10i\,c)}+5\,a^4de^{(8i\,dx+8i\,c)}+10\,a^4de^{(6i\,dx+6i\,c)}+10\,a^4de^{(4i\,dx+4i\,c)}+5\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 32/5*I/(a^4*d*e^(10*I*d*x + 10*I*c) + 5*a^4*d*e^(8*I*d*x + 8*I*c) + 10*a^4*
d*e^(6*I*d*x + 6*I*c) + 10*a^4*d*e^(4*I*d*x + 4*I*c) + 5*a^4*d*e^(2*I*d*x +
 2*I*c) + a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.20862, size = 74, normalized size = 2.74

$$\frac{\tan(dx+c)^5 + 5i\tan(dx+c)^4 - 10\tan(dx+c)^3 - 10i\tan(dx+c)^2 + 5\tan(dx+c)}{5a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/5*(tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 - 10*tan(d*x + c)^3 - 10*I*tan(d*x
 + c)^2 + 5*tan(d*x + c))/(a^4*d)
```

3.151 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=90

$$-\frac{i(a-ia\tan(c+dx))^3}{3a^7d}-\frac{i(a-ia\tan(c+dx))^2}{a^6d}-\frac{4\tan(c+dx)}{a^4d}+\frac{8i\log(\cos(c+dx))}{a^4d}+\frac{8x}{a^4}$$

```
[Out] (8*x)/a^4 + ((8*I)*Log[Cos[c + d*x]])/(a^4*d) - (4*Tan[c + d*x])/(a^4*d) -
(I*(a - I*a*Tan[c + d*x])^2)/(a^6*d) - ((I/3)*(a - I*a*Tan[c + d*x])^3)/(a^
7*d)
```

Rubi [A] time = 0.0560963, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{i(a-ia\tan(c+dx))^3}{3a^7d}-\frac{i(a-ia\tan(c+dx))^2}{a^6d}-\frac{4\tan(c+dx)}{a^4d}+\frac{8i\log(\cos(c+dx))}{a^4d}+\frac{8x}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (8*x)/a^4 + ((8*I)*Log[Cos[c + d*x]])/(a^4*d) - (4*Tan[c + d*x])/(a^4*d) -
(I*(a - I*a*Tan[c + d*x])^2)/(a^6*d) - ((I/3)*(a - I*a*Tan[c + d*x])^3)/(a^
7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{(a-x)^3}{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(-4a^2-2a(a-x)-(a-x)^2+\frac{8a^3}{a+x}\right)dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= \frac{8x}{a^4}+\frac{8i\log(\cos(c+dx))}{a^4d}-\frac{4\tan(c+dx)}{a^4d}-\frac{i(a-ia\tan(c+dx))^2}{a^6d}-\frac{i(a-ia\tan(c+dx}{3a^7d}\end{aligned}$$

Mathematica [A] time = 0.669694, size = 168, normalized size = 1.87

$\sec(c)\sec^3(c+dx)(12\sin(2c+dx)-11\sin(2c+3dx)+6dx\cos(2c+3dx)+6dx\cos(4c+3dx)+6i\cos(2c+3dx)\log$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^3*(6*d*x*Cos[2*c + 3*d*x] + 6*d*x*Cos[4*c + 3*d*x] + 6
*Cos[d*x]*(I + 3*d*x + (3*I)*Log[Cos[c + d*x]]) + 6*Cos[2*c + d*x]*(I + 3*d
*x + (3*I)*Log[Cos[c + d*x]]) + (6*I)*Cos[2*c + 3*d*x]*Log[Cos[c + d*x]] +
(6*I)*Cos[4*c + 3*d*x]*Log[Cos[c + d*x]] - 21*Sin[d*x] + 12*Sin[2*c + d*x]
- 11*Sin[2*c + 3*d*x]))/(6*a^4*d)
```

Maple [A] time = 0.085, size = 68, normalized size = 0.8

$$-7\frac{\tan(dx+c)}{a^4d}+\frac{(\tan(dx+c))^3}{3a^4d}+\frac{2i(\tan(dx+c))^2}{a^4d}-\frac{8i\ln(\tan(dx+c)-i)}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] -7*tan(d*x+c)/a^4/d+1/3/d/a^4*tan(d*x+c)^3+2*I/d/a^4*tan(d*x+c)^2-8*I/d/a^4
*ln(tan(d*x+c)-I)
```

Maxima [A] time = 0.995178, size = 72, normalized size = 0.8

$$\frac{\frac{\tan(dx+c)^3+6i\tan(dx+c)^2-21\tan(dx+c)}{a^4}-\frac{24i\log(i\tan(dx+c)+1)}{a^4}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 21*tan(d*x + c))/a^4 - 24*I*log
(I*tan(d*x + c) + 1)/a^4)/d
```

Fricas [A] time = 2.48496, size = 463, normalized size = 5.14

$$\frac{48\,dxe^{(6i\,dx+6i\,c)}+48\,dx+(144\,dx-24i)e^{(4i\,dx+4i\,c)}+(144\,dx-60i)e^{(2i\,dx+2i\,c)}+\left(24i\,e^{(6i\,dx+6i\,c)}+72i\,e^{(4i\,dx+4i\,c)}+72i\,e^{(2i\,dx+}\right.}{3\left(a^4de^{(6i\,dx+6i\,c)}+3\,a^4de^{(4i\,dx+4i\,c)}+3\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(48*d*x*e^(6*I*d*x + 6*I*c) + 48*d*x + (144*d*x - 24*I)*e^(4*I*d*x + 4*
I*c) + (144*d*x - 60*I)*e^(2*I*d*x + 2*I*c) + (24*I*e^(6*I*d*x + 6*I*c) + 7
2*I*e^(4*I*d*x + 4*I*c) + 72*I*e^(2*I*d*x + 2*I*c) + 24*I)*log(e^(2*I*d*x +
 2*I*c) + 1) - 44*I)/(a^4*d*e^(6*I*d*x + 6*I*c) + 3*a^4*d*e^(4*I*d*x + 4*I*
c) + 3*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.20122, size = 211, normalized size = 2.34

$$-\frac{2\left(\frac{24i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^4}-\frac{12i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{12i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}+\frac{22i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-78i\tan\left(\frac{1}{2}\right.}{\left(\right.}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -2/3*(24*I*log(tan(1/2*d*x + 1/2*c) - I)/a^4 - 12*I*log(abs(tan(1/2*d*x + 1
/2*c) + 1))/a^4 - 12*I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + (22*I*tan(1
/2*d*x + 1/2*c)^6 - 21*tan(1/2*d*x + 1/2*c)^5 - 78*I*tan(1/2*d*x + 1/2*c)^4
 + 46*tan(1/2*d*x + 1/2*c)^3 + 78*I*tan(1/2*d*x + 1/2*c)^2 - 21*tan(1/2*d*x
 + 1/2*c) - 22*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^4))/d
```

3.152 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4} dx$

Optimal. Leaf size=63

$$\frac{\tan(c+dx)}{a^4d}+\frac{4i}{d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{4i\log(\cos(c+dx))}{a^4d}-\frac{4x}{a^4}$$

```
[Out] (-4*x)/a^4 - ((4*I)*Log[Cos[c + d*x]])/(a^4*d) + Tan[c + d*x]/(a^4*d) + (4*
I)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.053086, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{\tan(c+dx)}{a^4d}+\frac{4i}{d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{4i\log(\cos(c+dx))}{a^4d}-\frac{4x}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-4*x)/a^4 - ((4*I)*Log[Cos[c + d*x]])/(a^4*d) + Tan[c + d*x]/(a^4*d) + (4*
I)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^2}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{i\,\text{Subst}\left(\int \left(1+\frac{4a^2}{(a+x)^2}-\frac{4a}{a+x}\right)dx,x,ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{4x}{a^4}-\frac{4i\log(\cos(c+dx))}{a^4d}+\frac{\tan(c+dx)}{a^4d}+\frac{4i}{d\left(a^4+ia^4\tan(c+dx)\right)}
\end{aligned}$$

Mathematica [B] time = 0.571658, size = 214, normalized size = 3.4

$\sec(c)\sec(c+dx)(-\cos(c+dx)+i\sin(c+dx))(2idx\sin(c+2dx)-2\sin(c+2dx)+2idx\sin(3c+2dx)-\sin(3c+2dx)$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]*(-Cos[c + d*x] + I*Sin[c + d*x])*((-I)*Cos[3*c + 2*d*x
] + 2*d*x*Cos[3*c + 2*d*x] + 2*Cos[c + 2*d*x]*(d*x + I*Log[Cos[c + d*x]]) +
 Cos[c]*(-3*I + 4*d*x + (4*I)*Log[Cos[c + d*x]]) + (2*I)*Cos[3*c + 2*d*x]*L
og[Cos[c + d*x]] + Sin[c] - 2*Sin[c + 2*d*x] + (2*I)*d*x*Sin[c + 2*d*x] - 2
*Log[Cos[c + d*x]]*Sin[c + 2*d*x] - Sin[3*c + 2*d*x] + (2*I)*d*x*Sin[3*c +
2*d*x] - 2*Log[Cos[c + d*x]]*Sin[3*c + 2*d*x]))/(2*a^4*d)
```

Maple [A] time = 0.073, size = 53, normalized size = 0.8

$$\frac{\tan(dx+c)}{a^4d}+4\frac{1}{a^4d(\tan(dx+c)-i)}+\frac{4i\ln(\tan(dx+c)-i)}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] tan(d*x+c)/a^4/d+4/d/a^4/(tan(d*x+c)-I)+4*I/d/a^4*ln(tan(d*x+c)-I)
```

Maxima [A] time = 0.997645, size = 130, normalized size = 2.06

$$\frac{\frac{12\left(\tan(dx+c)^2-2i\tan(dx+c)-1\right)}{3a^4\tan(dx+c)^3-9ia^4\tan(dx+c)^2-9a^4\tan(dx+c)+3ia^4}+\frac{4i\log(i\tan(dx+c)+1)}{a^4}+\frac{\tan(dx+c)}{a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] (12*(tan(d*x + c)^2 - 2*I*tan(d*x + c) - 1)/(3*a^4*tan(d*x + c)^3 - 9*I*a^4
*tan(d*x + c)^2 - 9*a^4*tan(d*x + c) + 3*I*a^4) + 4*I*log(I*tan(d*x + c) +
1)/a^4 + tan(d*x + c)/a^4)/d
```

Fricas [A] time = 2.42734, size = 286, normalized size = 4.54

$$-\frac{8\,dxe^{(4i\,dx+4i\,c)}+(8\,dx-4i)e^{(2i\,dx+2i\,c)}-\left(-4i\,e^{(4i\,dx+4i\,c)}-4i\,e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-2i}{a^4de^{(4i\,dx+4i\,c)}+a^4de^{(2i\,dx+2i\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -(8*d*x*e^(4*I*d*x + 4*I*c) + (8*d*x - 4*I)*e^(2*I*d*x + 2*I*c) - (-4*I*e^(
4*I*d*x + 4*I*c) - 4*I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) -
2*I)/(a^4*d*e^(4*I*d*x + 4*I*c) + a^4*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.19539, size = 198, normalized size = 3.14

$$\frac{2\left(\frac{4i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^4}-\frac{2i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{2i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}+\frac{2i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2i}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^4}+\frac{-6i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 \cdots}{a^4\left(\tan\left(\frac{1}{2} \cdots\right.\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 2*(4*I*log(tan(1/2*d*x + 1/2*c) - I)/a^4 - 2*I*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/a^4 - 2*I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + (2*I*tan(1/2*d*x
+ 1/2*c)^2 - tan(1/2*d*x + 1/2*c) - 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^4)
 + (-6*I*tan(1/2*d*x + 1/2*c)^2 - 16*tan(1/2*d*x + 1/2*c) + 6*I)/(a^4*(tan(
1/2*d*x + 1/2*c) - I)^2))/d
```

3.153 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=29

$$\frac{\tan(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)^2}$$

```
[Out] Tan[c + d*x]/(d*(a^2 + I*a^2*Tan[c + d*x])^2)
```

Rubi [A] time = 0.0412489, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 34}

$$\frac{\tan(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] Tan[c + d*x]/(d*(a^2 + I*a^2*Tan[c + d*x])^2)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{a-x}{(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{a^3 d}\\ &= \frac{\tan(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)^2}\end{aligned}$$

Mathematica [A] time = 0.0511495, size = 32, normalized size = 1.1

$$\frac{i\sec^4(c+dx)}{4d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/4)*Sec[c + d*x]^4)/(d*(a + I*a*Tan[c + d*x])^4)
```

Maple [A] time = 0.084, size = 36, normalized size = 1.2

$$\frac{1}{a^4 d}\left(\frac{-i}{\left(\tan\left(dx+c\right)-i\right)^2}-\left(\tan\left(dx+c\right)-i\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*(-I/(tan(d*x+c)-I)^2-1/(tan(d*x+c)-I))
```

Maxima [B] time = 1.00374, size = 90, normalized size = 3.1

$$-\frac{3\left(\tan\left(dx+c\right)^2-i\tan\left(dx+c\right)\right)}{\left(3a^4\tan\left(dx+c\right)^3-9ia^4\tan\left(dx+c\right)^2-9a^4\tan\left(dx+c\right)+3ia^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -3*(tan(d*x + c)^2 - I*tan(d*x + c))/((3*a^4*tan(d*x + c)^3 - 9*I*a^4*tan(d
*x + c)^2 - 9*a^4*tan(d*x + c) + 3*I*a^4)*d)
```

Fricas [A] time = 2.37237, size = 49, normalized size = 1.69

$$\frac{ie^{\left(-4i\,dx-4i\,c\right)}}{4a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*I*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.17843, size = 59, normalized size = 2.03

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{4}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))/(a^4*d*(tan(1/2*d*x + 1/
2*c) - I)^4)
```

3.154 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=27

$$\frac{i}{3ad(a+ia\tan(c+dx))^3}$$

```
[Out] (I/3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

Rubi [A] time = 0.039359, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$\frac{i}{3ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (I/3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1}{(a+x)^4}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{i}{3ad(a+ia\tan(c+dx))^3}\end{aligned}$$

Mathematica [B] time = 0.143224, size = 56, normalized size = 2.07

$$\frac{i\sec^4(c+dx)(2i\sin(2(c+dx))+4\cos(2(c+dx))+3)}{24a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/24)*Sec[c + d*x]^4*(3 + 4*Cos[2*(c + d*x)] + (2*I)*Sin[2*(c + d*x)]))/(
a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.039, size = 24, normalized size = 0.9

$$\frac{\frac{i}{3}}{ad\left(a+ia\tan\left(dx+c\right)\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/3*I/a/d/(a+I*a*tan(d*x+c))^3
```

Maxima [A] time = 0.963541, size = 28, normalized size = 1.04

$$\frac{i}{3\left(i\,a\tan\left(dx+c\right)+a\right)^{3}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*I/((I*a*tan(d*x + c) + a)^3*a*d)
```

Fricas [A] time = 2.38951, size = 123, normalized size = 4.56

$$\frac{\left(3i\,e^{(4i\,dx+4i\,c)}+3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{24\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x -
6*I*c)/(a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.19635, size = 115, normalized size = 4.26

$$-\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{3\,a^{4}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*d*x + 1/2*c)^5 - 6*I*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d*
x + 1/2*c)^3 + 6*I*tan(1/2*d*x + 1/2*c)^2 + 3*tan(1/2*d*x + 1/2*c))/(a^4*d*
(tan(1/2*d*x + 1/2*c) - I)^6)
```

3.155 $\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=116

$$\frac{i}{16d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{x}{16a^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx)}$$

```
[Out] x/(16*a^4) + (I/8)/(d*(a + I*a*Tan[c + d*x])^4) + (I/12)/(a*d*(a + I*a*Tan[
c + d*x])^3) + (I/16)/(d*(a^2 + I*a^2*Tan[c + d*x])^2) + (I/16)/(d*(a^4 + I
*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.0620533, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3479, 8}

$$\frac{i}{16d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{x}{16a^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-4),x]
```

```
[Out] x/(16*a^4) + (I/8)/(d*(a + I*a*Tan[c + d*x])^4) + (I/12)/(a*d*(a + I*a*Tan[
c + d*x])^3) + (I/16)/(d*(a^2 + I*a^2*Tan[c + d*x])^2) + (I/16)/(d*(a^4 + I
*a^4*Tan[c + d*x]))
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx &= \frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx}{2a}\\
&= \frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{4a^2}\\
&= \frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\int\\
&= \frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{}{16}\\
&= \frac{x}{16a^4}+\frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx\right.}
\end{aligned}$$

Mathematica [A] time = 0.183339, size = 98, normalized size = 0.84

$$\frac{\sec^4(c+dx)(-32\sin(2(c+dx))+24idx\sin(4(c+dx))+3\sin(4(c+dx))+64i\cos(2(c+dx))+3(8dx+i)\cos(4(c+dx)}{384a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-4),x]
```

```
[Out] (Sec[c + d*x]^4*(36*I + (64*I)*Cos[2*(c + d*x)] + 3*(I + 8*d*x)*Cos[4*(c +
d*x)] - 32*Sin[2*(c + d*x)] + 3*Sin[4*(c + d*x)] + (24*I)*d*x*Sin[4*(c + d*
x)]))/(384*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.027, size = 118, normalized size = 1.

$$\frac{\frac{i}{8}}{a^4d(\tan(dx+c)-i)^4}-\frac{\frac{i}{32}\ln(\tan(dx+c)-i)}{a^4d}-\frac{\frac{i}{16}}{a^4d(\tan(dx+c)-i)^2}-\frac{1}{12a^4d(\tan(dx+c)-i)^3}+\frac{1}{16a^4d(\tan(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/8*I/d/a^4/(tan(d*x+c)-I)^4-1/32*I/d/a^4*ln(tan(d*x+c)-I)-1/16*I/d/a^4/(ta
n(d*x+c)-I)^2-1/12/d/a^4/(tan(d*x+c)-I)^3+1/16/d/a^4/(tan(d*x+c)-I)+1/32*I/
d/a^4*ln(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.38736, size = 205, normalized size = 1.77

$$\frac{\left(24\,dxe^{(8i\,dx+8i\,c)}+48i\,e^{(6i\,dx+6i\,c)}+36i\,e^{(4i\,dx+4i\,c)}+16i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{384\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(24*d*x*e^(8*I*d*x + 8*I*c) + 48*I*e^(6*I*d*x + 6*I*c) + 36*I*e^(4*I*
d*x + 4*I*c) + 16*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A] time = 0.818238, size = 190, normalized size = 1.64

$$\begin{cases} \frac{\left(98304ia^{12}d^3e^{18ic}e^{-2idx}+73728ia^{12}d^3e^{16ic}e^{-4idx}+32768ia^{12}d^3e^{14ic}e^{-6idx}+6144ia^{12}d^3e^{12ic}e^{-8idx}\right)e^{-20ic}}{786432a^{16}d^4} & \text{for } 786432a^{16}d^4e^{20ic} \neq 0 \\ x\left(\frac{\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-8ic}}{16a^4}-\frac{1}{16a^4}\right) & \text{otherwise} \end{cases} + \frac{x}{16a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((98304*I*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) + 73728*I*a**12*d*
*3*exp(16*I*c)*exp(-4*I*d*x) + 32768*I*a**12*d**3*exp(14*I*c)*exp(-6*I*d*x)
 + 6144*I*a**12*d**3*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(786432*a**16*
d**4), Ne(786432*a**16*d**4*exp(20*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c
) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-8*I*c)/(16*a**4) - 1/(16*a**4)),
True)) + x/(16*a**4)
```

Giac [A] time = 1.12048, size = 124, normalized size = 1.07

$$-\frac{-\frac{12i\log(-i\tan(dx+c)+1)}{a^4}+\frac{12i\log(-i\tan(dx+c)-1)}{a^4}+\frac{-25i\tan(dx+c)^4-124\tan(dx+c)^3+246i\tan(dx+c)^2+252\tan(dx+c)-153i}{a^4(\tan(dx+c)-i)^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/384*(-12*I*log(-I*tan(d*x + c) + 1)/a^4 + 12*I*log(-I*tan(d*x + c) - 1)/
a^4 + (-25*I*tan(d*x + c)^4 - 124*tan(d*x + c)^3 + 246*I*tan(d*x + c)^2 + 2
52*tan(d*x + c) - 153*I)/(a^4*(tan(d*x + c) - I)^4))/d
```

3.156 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=169

$$-\frac{i}{64d\left(a^4-ia^4\tan(c+dx)\right)}+\frac{5i}{64d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{3x}{32a^4}+\frac{ia}{20d(a+ia\tan(c+dx}$$

```
[Out] (3*x)/(32*a^4) + ((I/20)*a)/(d*(a + I*a*Tan[c + d*x])^5) + (I/16)/(d*(a + I
*a*Tan[c + d*x])^4) + (I/16)/(a*d*(a + I*a*Tan[c + d*x])^3) + (I/16)/(d*(a^
2 + I*a^2*Tan[c + d*x])^2) - (I/64)/(d*(a^4 - I*a^4*Tan[c + d*x])) + ((5*I)
/64)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.0964324, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{i}{64d\left(a^4-ia^4\tan(c+dx)\right)}+\frac{5i}{64d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{3x}{32a^4}+\frac{ia}{20d(a+ia\tan(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (3*x)/(32*a^4) + ((I/20)*a)/(d*(a + I*a*Tan[c + d*x])^5) + (I/16)/(d*(a + I
*a*Tan[c + d*x])^4) + (I/16)/(a*d*(a + I*a*Tan[c + d*x])^3) + (I/16)/(d*(a^
2 + I*a^2*Tan[c + d*x])^2) - (I/64)/(d*(a^4 - I*a^4*Tan[c + d*x])) + ((5*I)
/64)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^6}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^2} + \frac{1}{4a^2(a+x)^6} + \frac{1}{4a^3(a+x)^5} + \frac{3}{16a^4(a+x)^4} + \frac{1}{8a^5(a+x)^3} + \frac{5}{64a^6(a+x)^2} + \cdots\right.\right.}{d}$$

$$= \frac{ia}{20d(a+ia\tan(c+dx))^5} + \frac{i}{16d(a+ia\tan(c+dx))^4} + \frac{i}{16ad(a+ia\tan(c+dx))^3} + \cdots$$

$$= \frac{3x}{32a^4} + \frac{ia}{20d(a+ia\tan(c+dx))^5} + \frac{i}{16d(a+ia\tan(c+dx))^4} + \frac{i}{16ad(a+ia\tan(c+dx))} \cdots$$

Mathematica [A] time = 0.253351, size = 120, normalized size = 0.71

$$\frac{\sec^4(c+dx)(-100\sin(2(c+dx)) + 120idx\sin(4(c+dx)) + 15\sin(4(c+dx)) + 12\sin(6(c+dx)) + 200i\cos(2(c+dx)) \cdots}{1280a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(100*I + (200*I)*Cos[2*(c + d*x)] + 15*(I + 8*d*x)*Cos[4*(c
 + d*x)] - (8*I)*Cos[6*(c + d*x)] - 100*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*
x)] + (120*I)*d*x*Sin[4*(c + d*x)] + 12*Sin[6*(c + d*x)]))/(1280*a^4*d*(-I
+ Tan[c + d*x])^4)
```

Maple [A] time = 0.103, size = 156, normalized size = 0.9

$$\frac{-\frac{3i}{64}\ln(\tan(dx+c)-i)}{a^4d} + \frac{\frac{i}{16}}{a^4d(\tan(dx+c)-i)^4} - \frac{\frac{i}{16}}{a^4d(\tan(dx+c)-i)^2} + \frac{1}{20a^4d(\tan(dx+c)-i)^5} - \frac{\cdots}{16a^4d(\tan \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] -3/64*I/d/a^4*ln(tan(d*x+c)-I)+1/16*I/d/a^4/(tan(d*x+c)-I)^4-1/16*I/d/a^4/(
tan(d*x+c)-I)^2+1/20/d/a^4/(tan(d*x+c)-I)^5-1/16/d/a^4/(tan(d*x+c)-I)^3+5/6
4/d/a^4/(tan(d*x+c)-I)+3/64*I/d/a^4*ln(tan(d*x+c)+I)+1/64/d/a^4/(tan(d*x+c)
+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.35943, size = 292, normalized size = 1.73

$$\frac{\left(120\,dxe^{(10i\,dx+10i\,c)} - 10i\,e^{(12i\,dx+12i\,c)} + 150i\,e^{(8i\,dx+8i\,c)} + 100i\,e^{(6i\,dx+6i\,c)} + 50i\,e^{(4i\,dx+4i\,c)} + 15i\,e^{(2i\,dx+2i\,c)} + 2i\right)e^{(-10i\,dx-10i\,c)}}{1280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(120*d*x*e^(10*I*d*x + 10*I*c) - 10*I*e^(12*I*d*x + 12*I*c) + 150*I*
e^(8*I*d*x + 8*I*c) + 100*I*e^(6*I*d*x + 6*I*c) + 50*I*e^(4*I*d*x + 4*I*c)
+ 15*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(-10*I*d*x - 10*I*c)/(a^4*d)
```

Sympy [A] time = 1.23209, size = 260, normalized size = 1.54

$$\begin{cases} \frac{\left(-171798691840ia^{20}d^5e^{32ic}e^{2idx}+2576980377600ia^{20}d^5e^{28ic}e^{-2idx}+1717986918400ia^{20}d^5e^{26ic}e^{-4idx}+858993459200ia^{20}d^5e^{24ic}e^{-6idx}+257698037760ia^{20}d^5e^{22ic}\cdots\right.}{21990232555520a^{24}d^6} \\ x\left(\frac{\left(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1\right)e^{-10ic}}{64a^4} - \frac{3}{32a^4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-171798691840*I*a**20*d**5*exp(32*I*c)*exp(2*I*d*x) + 257698037
7600*I*a**20*d**5*exp(28*I*c)*exp(-2*I*d*x) + 1717986918400*I*a**20*d**5*ex
p(26*I*c)*exp(-4*I*d*x) + 858993459200*I*a**20*d**5*exp(24*I*c)*exp(-6*I*d*
x) + 257698037760*I*a**20*d**5*exp(22*I*c)*exp(-8*I*d*x) + 34359738368*I*a*
*20*d**5*exp(20*I*c)*exp(-10*I*d*x))*exp(-30*I*c)/(21990232555520*a**24*d**
6), Ne(21990232555520*a**24*d**6*exp(30*I*c), 0)), (x*((exp(12*I*c) + 6*exp
(10*I*c) + 15*exp(8*I*c) + 20*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1
)*exp(-10*I*c)/(64*a**4) - 3/(32*a**4)), True)) + 3*x/(32*a**4)
```

Giac [A] time = 1.14504, size = 166, normalized size = 0.98

$$-\frac{-\frac{60i\log(\tan(dx+c)+i)}{a^4} + \frac{60i\log(\tan(dx+c)-i)}{a^4} + \frac{20\,(3i\tan(dx+c)-4)}{a^4(\tan(dx+c)+i)} + \frac{-137i\tan(dx+c)^5-785\tan(dx+c)^4+1850i\tan(dx+c)^3+2290\tan(dx+c)^2-156\cdots}{a^4(\tan(dx+c)-i)^5}}{1280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/1280*(-60*I*log(tan(d*x + c) + I)/a^4 + 60*I*log(tan(d*x + c) - I)/a^4 +
 20*(3*I*tan(d*x + c) - 4)/(a^4*(tan(d*x + c) + I)) + (-137*I*tan(d*x + c)^
5 - 785*tan(d*x + c)^4 + 1850*I*tan(d*x + c)^3 + 2290*tan(d*x + c)^2 - 1565
*I*tan(d*x + c) - 541)/(a^4*(tan(d*x + c) - I)^5))/d
```

3.157 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=224

$$\frac{ia^2}{48d(a+ia\tan(c+dx))^6}-\frac{7i}{256d\left(a^4-ia^4\tan(c+dx)\right)}+\frac{21i}{256d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{i}{256d\left(a^2-ia^2\tan(c+dx)\right)^2}$$

```
[Out] (7*x)/(64*a^4) + ((I/48)*a^2)/(d*(a + I*a*Tan[c + d*x])^6) + (((3*I)/80)*a)
/(d*(a + I*a*Tan[c + d*x])^5) + ((3*I)/64)/(d*(a + I*a*Tan[c + d*x])^4) + (
(5*I)/96)/(a*d*(a + I*a*Tan[c + d*x])^3) - (I/256)/(d*(a^2 - I*a^2*Tan[c +
d*x])^2) + ((15*I)/256)/(d*(a^2 + I*a^2*Tan[c + d*x])^2) - ((7*I)/256)/(d*(
a^4 - I*a^4*Tan[c + d*x])) + ((21*I)/256)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.124897, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$\frac{ia^2}{48d(a+ia\tan(c+dx))^6}-\frac{7i}{256d\left(a^4-ia^4\tan(c+dx)\right)}+\frac{21i}{256d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{i}{256d\left(a^2-ia^2\tan(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```


```
[Out] (7*x)/(64*a^4) + ((I/48)*a^2)/(d*(a + I*a*Tan[c + d*x])^6) + (((3*I)/80)*a)
/(d*(a + I*a*Tan[c + d*x])^5) + ((3*I)/64)/(d*(a + I*a*Tan[c + d*x])^4) + (
(5*I)/96)/(a*d*(a + I*a*Tan[c + d*x])^3) - (I/256)/(d*(a^2 - I*a^2*Tan[c +
d*x])^2) + ((15*I)/256)/(d*(a^2 + I*a^2*Tan[c + d*x])^2) - ((7*I)/256)/(d*(
a^4 - I*a^4*Tan[c + d*x])) + ((21*I)/256)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^7}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{128a^7(a-x)^3} + \frac{7}{256a^8(a-x)^2} + \frac{1}{8a^3(a+x)^7} + \frac{3}{16a^4(a+x)^6} + \frac{3}{16a^5(a+x)^5} + \frac{5}{32a^6(a+x)^4} + \right.\right.}{d}$$
$$= \frac{ia^2}{48d(a+ia\tan(c+dx))^6} + \frac{3ia}{80d(a+ia\tan(c+dx))^5} + \frac{3i}{64d(a+ia\tan(c+dx))^4} + \frac{}{96ad(a}$$
$$= \frac{7x}{64a^4} + \frac{ia^2}{48d(a+ia\tan(c+dx))^6} + \frac{3ia}{80d(a+ia\tan(c+dx))^5} + \frac{3i}{64d(a+ia\tan(c+dx))^4} +$$

Mathematica [A] time = 0.495187, size = 142, normalized size = 0.63

$$\frac{\sec^4(c+dx)(-560\sin(2(c+dx)) + 840idx\sin(4(c+dx)) + 105\sin(4(c+dx)) + 144\sin(6(c+dx)) + 10\sin(8(c+dx)) +}{7680a^4d(\tan(c+dx) - i}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(525*I + (1120*I)*Cos[2*(c + d*x)] + 105*(I + 8*d*x)*Cos[4*
(c + d*x)] - (96*I)*Cos[6*(c + d*x)] - (5*I)*Cos[8*(c + d*x)] - 560*Sin[2*(
c + d*x)] + 105*Sin[4*(c + d*x)] + (840*I)*d*x*Sin[4*(c + d*x)] + 144*Sin[6
*(c + d*x)] + 10*Sin[8*(c + d*x)]))/(7680*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.1, size = 196, normalized size = 0.9

$$-\frac{\frac{7i}{128}\ln(\tan(dx+c)-i)}{a^4d} + \frac{\frac{3i}{64}}{a^4d(\tan(dx+c)-i)^4} - \frac{\frac{i}{48}}{a^4d(\tan(dx+c)-i)^6} - \frac{\frac{15i}{256}}{a^4d(\tan(dx+c)-i)^2} + \frac{3}{80a^4d(\tan(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] -7/128*I/d/a^4*ln(tan(d*x+c)-I)+3/64*I/d/a^4/(tan(d*x+c)-I)^4-1/48*I/d/a^4/
(tan(d*x+c)-I)^6-15/256*I/d/a^4/(tan(d*x+c)-I)^2+3/80/d/a^4/(tan(d*x+c)-I)^
5-5/96/d/a^4/(tan(d*x+c)-I)^3+21/256/d/a^4/(tan(d*x+c)-I)+1/256*I/d/a^4/(ta
n(d*x+c)+I)^2+7/128*I/d/a^4*ln(tan(d*x+c)+I)+7/256/d/a^4/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.3894, size = 379, normalized size = 1.69

$$\frac{\left(1680\,dxe^{(12i\,dx+12i\,c)} - 15i\,e^{(16i\,dx+16i\,c)} - 240i\,e^{(14i\,dx+14i\,c)} + 1680i\,e^{(10i\,dx+10i\,c)} + 1050i\,e^{(8i\,dx+8i\,c)} + 560i\,e^{(6i\,dx+6i\,c)} + 210\right.}{15360\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/15360*(1680*d*x*e^(12*I*d*x + 12*I*c) - 15*I*e^(16*I*d*x + 16*I*c) - 240*
I*e^(14*I*d*x + 14*I*c) + 1680*I*e^(10*I*d*x + 10*I*c) + 1050*I*e^(8*I*d*x
+ 8*I*c) + 560*I*e^(6*I*d*x + 6*I*c) + 210*I*e^(4*I*d*x + 4*I*c) + 48*I*e^(
2*I*d*x + 2*I*c) + 5*I)*e^(-12*I*d*x - 12*I*c)/(a^4*d)
```

Sympy [A] time = 1.52514, size = 328, normalized size = 1.46

$$\begin{cases} \frac{\left(-202661983231672320ia^{28}d^7e^{46ic}e^{4idx} - 324259173170675712 0ia^{28}d^7e^{44ic}e^{2idx} + 22698142121947299840ia^{28}d^7e^{40ic}e^{-2idx} + 14186338826217062400ia^{28}d^7e\right.}{207} \\ x\left(\frac{\left(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1\right)e^{-12ic}}{256a^4} - \frac{7}{64a^4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-202661983231672320*I*a**28*d**7*exp(46*I*c)*exp(4*I*d*x) - 324
2591731706757120*I*a**28*d**7*exp(44*I*c)*exp(2*I*d*x) + 226981421219472998
40*I*a**28*d**7*exp(40*I*c)*exp(-2*I*d*x) + 14186338826217062400*I*a**28*d*
*7*exp(38*I*c)*exp(-4*I*d*x) + 7566047373982433280*I*a**28*d**7*exp(36*I*c)
*exp(-6*I*d*x) + 2837267765243412480*I*a**28*d**7*exp(34*I*c)*exp(-8*I*d*x)
 + 648518346341351424*I*a**28*d**7*exp(32*I*c)*exp(-10*I*d*x) + 67553994410
557440*I*a**28*d**7*exp(30*I*c)*exp(-12*I*d*x))*exp(-42*I*c)/(2075258708292
32455680*a**32*d**8), Ne(207525870829232455680*a**32*d**8*exp(42*I*c), 0)),
 (x*((exp(16*I*c) + 8*exp(14*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*ex
p(8*I*c) + 56*exp(6*I*c) + 28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-12*I*c)/(
256*a**4) - 7/(64*a**4)), True)) + 7*x/(64*a**4)
```

Giac [A] time = 1.18837, size = 198, normalized size = 0.88

$$-\frac{-\frac{420i\log(-i\tan(dx+c)+1)}{a^4} + \frac{420i\log(-i\tan(dx+c)-1)}{a^4} + \frac{30\left(21i\tan(dx+c)^2-49\tan(dx+c)-29i\right)}{a^4(\tan(dx+c)+i)^2} + \frac{-1029i\tan(dx+c)^6-6804\tan(dx+c)^5+19035}{}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/7680*(-420*I*log(-I*tan(d*x + c) + 1)/a^4 + 420*I*log(-I*tan(d*x + c) -
1)/a^4 + 30*(21*I*tan(d*x + c)^2 - 49*tan(d*x + c) - 29*I)/(a^4*(tan(d*x +
c) + I)^2) + (-1029*I*tan(d*x + c)^6 - 6804*tan(d*x + c)^5 + 19035*I*tan(d*
x + c)^4 + 29080*tan(d*x + c)^3 - 25995*I*tan(d*x + c)^2 - 13332*tan(d*x +
c) + 3317*I)/(a^4*(tan(d*x + c) - I)^6))/d
```

3.158 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=133

$$\frac{35\tanh^{-1}(\sin(c+dx))}{8a^4d} - \frac{14i\sec^5(c+dx)}{3d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{35\tan(c+dx)\sec^3(c+dx)}{12a^4d} + \frac{35\tan(c+dx)\sec(c+dx)}{8a^4d} - \frac{2i\cdots}{ad(a+\cdots}$$

```
[Out] (35*ArcTanh[Sin[c + d*x]])/(8*a^4*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(8*a^
4*d) + (35*Sec[c + d*x]^3*Tan[c + d*x])/(12*a^4*d) - ((2*I)*Sec[c + d*x]^7)
/(a*d*(a + I*a*Tan[c + d*x])^3) - (((14*I)/3)*Sec[c + d*x]^5)/(d*(a^4 + I*a
^4*Tan[c + d*x]))
```

Rubi [A] time = 0.114347, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3500, 3768, 3770}

$$\frac{35\tanh^{-1}(\sin(c+dx))}{8a^4d} - \frac{14i\sec^5(c+dx)}{3d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{35\tan(c+dx)\sec^3(c+dx)}{12a^4d} + \frac{35\tan(c+dx)\sec(c+dx)}{8a^4d} - \frac{2i\cdots}{ad(a+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (35*ArcTanh[Sin[c + d*x]])/(8*a^4*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(8*a^
4*d) + (35*Sec[c + d*x]^3*Tan[c + d*x])/(12*a^4*d) - ((2*I)*Sec[c + d*x]^7)
/(a*d*(a + I*a*Tan[c + d*x])^3) - (((14*I)/3)*Sec[c + d*x]^5)/(d*(a^4 + I*a
^4*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3} + \frac{7\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^2}$$
$$= -\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3} - \frac{14i\sec^5(c+dx)}{3d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{35\int \sec^5(c+dx)\,dx}{3a^4}$$
$$= \frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d} - \frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3} - \frac{14i\sec^5(c+dx)}{3d\left(a^4+ia^4\tan(c+dx)\right)} + 35$$
$$= \frac{35\sec(c+dx)\tan(c+dx)}{8a^4d} + \frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d} - \frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3} -$$
$$= \frac{35\tanh^{-1}(\sin(c+dx))}{8a^4d} + \frac{35\sec(c+dx)\tan(c+dx)}{8a^4d} + \frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d} -$$

Mathematica [A] time = 0.961373, size = 237, normalized size = 1.78

$$-\frac{\sec^4(c+dx)\left(896i\cos(c+dx)+3\left(42\sin(c+dx)+58\sin(3(c+dx))+128i\cos(3(c+dx))+35\cos(4(c+dx))\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -(Sec[c + d*x]^4*((896*I)*Cos[c + d*x] + 3*((128*I)*Cos[3*(c + d*x)] + 105*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 35*Cos[4*(c + d*x)]*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] + 140*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 105*Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]] - 35*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + 42*Sin[c + d*x] + 58*Sin[3*(c + d*x)])))/(192*a^4*
d)
```

Maple [B] time = 0.099, size = 342, normalized size = 2.6

$$\frac{25}{8a^4d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} - \frac{2i}{a^4d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} + \frac{1}{2a^4d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3} + \frac{\frac{4i}{3}}{a^4d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 25/8/d/a^4/(tan(1/2*d*x+1/2*c)+1)^2-2*I/d/a^4/(tan(1/2*d*x+1/2*c)+1)^2+1/2/
d/a^4/(tan(1/2*d*x+1/2*c)+1)^3+4/3*I/d/a^4/(tan(1/2*d*x+1/2*c)+1)^3-27/8/d/
a^4/(tan(1/2*d*x+1/2*c)+1)-6*I/d/a^4/(tan(1/2*d*x+1/2*c)+1)-1/4/d/a^4/(tan(
1/2*d*x+1/2*c)+1)^4+35/8/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)+1/2/d/a^4/(tan(1/2*
d*x+1/2*c)-1)^3-4/3*I/d/a^4/(tan(1/2*d*x+1/2*c)-1)^3-25/8/d/a^4/(tan(1/2*d*
x+1/2*c)-1)^2-2*I/d/a^4/(tan(1/2*d*x+1/2*c)-1)^2-27/8/d/a^4/(tan(1/2*d*x+1/
2*c)-1)+6*I/d/a^4/(tan(1/2*d*x+1/2*c)-1)+1/4/d/a^4/(tan(1/2*d*x+1/2*c)-1)^4
-35/8/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.0538, size = 398, normalized size = 2.99

$$-\frac{\frac{2\left(\frac{81\sin(dx+c)}{\cos(dx+c)+1}-\frac{544i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{105\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{480i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{105\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{96i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{81\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+160i\right)}{a^4-\frac{4a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{105\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{105\log(\cdots}{\cdots}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/24*(2*(81*sin(d*x + c)/(cos(d*x + c) + 1) - 544*I*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 105*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 480*I*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 105*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 96*I*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 81*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 + 160*I)/(a^4 - 4*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^4*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 4*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a
^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 105*log(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)/a^4 + 105*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

Fricas [A] time = 2.51169, size = 674, normalized size = 5.07

$$\frac{105\left(e^{(8idx+8ic)}+4e^{(6idx+6ic)}+6e^{(4idx+4ic)}+4e^{(2idx+2ic)}+1\right)\log\left(e^{(idx+ic)}+i\right)-105\left(e^{(8idx+8ic)}+4e^{(6idx+6ic)}+6e^{(4idx+\cdots}\right.}{24\left(a^4de^{(8idx+8ic)}+4a^4de^{(6idx+6ic)}+6a^4de^{(4i\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(105*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I
*c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 105*(e^(8*I*d*x
 + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x +
2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 210*I*e^(7*I*d*x + 7*I*c) - 770*I*e^
(5*I*d*x + 5*I*c) - 1022*I*e^(3*I*d*x + 3*I*c) - 558*I*e^(I*d*x + I*c))/(a^
4*d*e^(8*I*d*x + 8*I*c) + 4*a^4*d*e^(6*I*d*x + 6*I*c) + 6*a^4*d*e^(4*I*d*x
+ 4*I*c) + 4*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.23231, size = 207, normalized size = 1.56

$$\frac{\frac{105\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{105\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}-\frac{2\left(81\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-96i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-105\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4a^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 105*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^4 - 2*(81*tan(1/2*d*x + 1/2*c)^7 - 96*I*tan(1/2*d*x + 1/2*
c)^6 - 105*tan(1/2*d*x + 1/2*c)^5 + 480*I*tan(1/2*d*x + 1/2*c)^4 - 105*tan(
1/2*d*x + 1/2*c)^3 - 544*I*tan(1/2*d*x + 1/2*c)^2 + 81*tan(1/2*d*x + 1/2*c)
 + 160*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^4))/d
```

3.159 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=107

$$-\frac{15\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{10i\sec^3(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{15\tan(c+dx)\sec(c+dx)}{2a^4d}+\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

```
[Out] (-15*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (15*Sec[c + d*x]*Tan[c + d*x])/(2*a
^4*d) + ((2*I)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^3) + ((10*I)*Sec
[c + d*x]^3)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.100898, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3500, 3768, 3770}

$$-\frac{15\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{10i\sec^3(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{15\tan(c+dx)\sec(c+dx)}{2a^4d}+\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-15*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (15*Sec[c + d*x]*Tan[c + d*x])/(2*a
^4*d) + ((2*I)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^3) + ((10*I)*Sec
[c + d*x]^3)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} - \frac{5\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^2}$$
$$= \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} + \frac{10i\sec^3(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{15\int \sec^3(c+dx)\,dx}{a^4}$$
$$= -\frac{15\sec(c+dx)\tan(c+dx)}{2a^4d} + \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} + \frac{10i\sec^3(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{15}{}$$
$$= -\frac{15\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{15\sec(c+dx)\tan(c+dx)}{2a^4d} + \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} + \frac{}{d\left(a\right.}$$

Mathematica [B] time = 6.12355, size = 988, normalized size = 9.23

$$\frac{15\cos(4c)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4}{2d(i\tan(c+dx)a+a)^4} - \frac{15\cos(4c)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\right.\right.}{2d(i\tan}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (15*Cos[4*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*(C
os[d*x] + I*Sin[d*x])^4)/(2*d*(a + I*a*Tan[c + d*x])^4) - (15*Cos[4*c]*Log[
Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d
*x])^4)/(2*d*(a + I*a*Tan[c + d*x])^4) + (Cos[d*x]*Sec[c + d*x]^4*((8*I)*Co
s[3*c] - 8*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(a + I*a*Tan[c + d*x])^4
) + (Sec[c]*Sec[c + d*x]^4*((4*I)*Cos[4*c] - 4*Sin[4*c])*(Cos[d*x] + I*Sin[
d*x])^4)/(d*(a + I*a*Tan[c + d*x])^4) + (((15*I)/2)*Log[Cos[c/2 + (d*x)/2]
- Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*Sin[4*c]*(Cos[d*x] + I*Sin[d*x])^4)/(d
*(a + I*a*Tan[c + d*x])^4) - (((15*I)/2)*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 +
 (d*x)/2]]*Sec[c + d*x]^4*Sin[4*c]*(Cos[d*x] + I*Sin[d*x])^4)/(d*(a + I*a*T
an[c + d*x])^4) + (Sec[c + d*x]^4*(8*Cos[3*c] + (8*I)*Sin[3*c])*(Cos[d*x] +
 I*Sin[d*x])^4*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (Sec[c + d*x]^4*(Co
s[4*c]/4 + (I/4)*Sin[4*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(Cos[c/2 + (d*x)/2
] - Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^4) + (Sec[c + d*x]^4*(-Cos
[4*c]/4 - (I/4)*Sin[4*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^4) + (4*Sec[c + d*x]^4*(Cos
[d*x] + I*Sin[d*x])^4*(Cos[4*c - (d*x)/2]/2 - Cos[4*c + (d*x)/2]/2 + (I/2)*
Sin[4*c - (d*x)/2] - (I/2)*Sin[4*c + (d*x)/2]))/(d*(Cos[c/2] + Sin[c/2])*(C
os[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*(a + I*a*Tan[c + d*x])^4) + (4*Sec[
c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(-Cos[4*c - (d*x)/2]/2 + Cos[4*c + (d*
x)/2]/2 - (I/2)*Sin[4*c - (d*x)/2] + (I/2)*Sin[4*c + (d*x)/2]))/(d*(Cos[c/2
] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])*(a + I*a*Tan[c + d*
x])^4)
```

Maple [A] time = 0.095, size = 192, normalized size = 1.8

$$\frac{1}{2a^4d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} + \frac{4i}{a^4d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{1}{2a^4d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} - \frac{15}{2a^4d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)+4*I/d/a^4/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^4
/(tan(1/2*d*x+1/2*c)+1)^2-15/2/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)+16/d/a^4/(tan
(1/2*d*x+1/2*c)-I)+1/2/d/a^4/(tan(1/2*d*x+1/2*c)-1)-4*I/d/a^4/(tan(1/2*d*x+
1/2*c)-1)+1/2/d/a^4/(tan(1/2*d*x+1/2*c)-1)^2+15/2/d/a^4*ln(tan(1/2*d*x+1/2*
c)-1)
```

Maxima [B] time = 1.61453, size = 630, normalized size = 5.89

$$\frac{(30\cos(5\,dx+5\,c)+60\cos(3\,dx+3\,c)+30\cos(dx+c)+30i\sin(5\,dx+5\,c)+60i\sin(3\,dx+3\,c)+30i\sin(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] ((30*cos(5*d*x + 5*c) + 60*cos(3*d*x + 3*c) + 30*cos(d*x + c) + 30*I*sin(5*
d*x + 5*c) + 60*I*sin(3*d*x + 3*c) + 30*I*sin(d*x + c))*arctan2(cos(d*x + c
), sin(d*x + c) + 1) + (30*cos(5*d*x + 5*c) + 60*cos(3*d*x + 3*c) + 30*cos(
d*x + c) + 30*I*sin(5*d*x + 5*c) + 60*I*sin(3*d*x + 3*c) + 30*I*sin(d*x + c
))*arctan2(cos(d*x + c), -sin(d*x + c) + 1) - (-15*I*cos(5*d*x + 5*c) - 30*
I*cos(3*d*x + 3*c) - 15*I*cos(d*x + c) + 15*sin(5*d*x + 5*c) + 30*sin(3*d*x
 + 3*c) + 15*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x
+ c) + 1) - (15*I*cos(5*d*x + 5*c) + 30*I*cos(3*d*x + 3*c) + 15*I*cos(d*x +
 c) - 15*sin(5*d*x + 5*c) - 30*sin(3*d*x + 3*c) - 15*sin(d*x + c))*log(cos(
d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 60*cos(4*d*x + 4*c) + 1
00*cos(2*d*x + 2*c) + 60*I*sin(4*d*x + 4*c) + 100*I*sin(2*d*x + 2*c) + 32)/
((-4*I*a^4*cos(5*d*x + 5*c) - 8*I*a^4*cos(3*d*x + 3*c) - 4*I*a^4*cos(d*x +
c) + 4*a^4*sin(5*d*x + 5*c) + 8*a^4*sin(3*d*x + 3*c) + 4*a^4*sin(d*x + c))*
d)
```

Fricas [A] time = 2.41002, size = 455, normalized size = 4.25

$$-\frac{15\left(e^{(5i\,dx+5i\,c)}+2\,e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(5i\,dx+5i\,c)}+2\,e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-30}{2\left(a^4de^{(5i\,dx+5i\,c)}+2\,a^4de^{(3i\,dx+3i\,c)}+a^4de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/2*(15*(e^(5*I*d*x + 5*I*c) + 2*e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*lo
g(e^(I*d*x + I*c) + I) - 15*(e^(5*I*d*x + 5*I*c) + 2*e^(3*I*d*x + 3*I*c) +
e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 30*I*e^(4*I*d*x + 4*I*c) - 50*I
*e^(2*I*d*x + 2*I*c) - 16*I)/(a^4*d*e^(5*I*d*x + 5*I*c) + 2*a^4*d*e^(3*I*d*
x + 3*I*c) + a^4*d*e^(I*d*x + I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.20373, size = 155, normalized size = 1.45

$$-\frac{\frac{15\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{15\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}-\frac{2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-8i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+8i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^4}-\frac{32}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/2*(15*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 15*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^4 - 2*(tan(1/2*d*x + 1/2*c)^3 - 8*I*tan(1/2*d*x + 1/2*c)^2 +
 tan(1/2*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - 32/(a^4
*(tan(1/2*d*x + 1/2*c) - I)))/d
```

3.160 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=82

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{2i\sec(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) + (((2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Ta
n[c + d*x])^3) - ((2*I)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.0866693, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3500, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{2i\sec(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) + (((2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Ta
n[c + d*x])^3) - ((2*I)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^2}\\ &= \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{2i\sec(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{\int \sec(c+dx)\,dx}{a^4}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}-\frac{2i\sec(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

Mathematica [B] time = 0.241428, size = 247, normalized size = 3.01

$$\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4\left(-6i\sin(3c)\sin(dx)+2i\sin(c)\sin(3dx)-2\sin(c)\cos(3dx)+6\sin(3c)\cos(dx)+\cos(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(-3*Cos[4*c]*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] + 3*Cos[4*c]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
- 2*Cos[3*d*x]*Sin[c] + 6*Cos[d*x]*Sin[3*c] - (3*I)*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]]*Sin[4*c] + (3*I)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
*Sin[4*c] + Cos[3*c]*((-6*I)*Cos[d*x] - 6*Sin[d*x]) - (6*I)*Sin[3*c]*Sin[d*
x] + (2*I)*Sin[c]*Sin[3*d*x] + 2*Cos[c]*(I*Cos[3*d*x] + Sin[3*d*x])))/(3*a^
4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.09, size = 86, normalized size = 1.1

$$\frac{1}{a^4 d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8\,i}{a^4 d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{-2}-\frac{16}{3\,a^4 d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{-3}-\frac{1}{a^4 d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)+8*I/d/a^4/(tan(1/2*d*x+1/2*c)-I)^2-16/3/d/
a^4/(tan(1/2*d*x+1/2*c)-I)^3-1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.51308, size = 190, normalized size = 2.32

$$\frac{-6i\,\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-6i\,\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)+4i\,\cos\left(3\,dx+3\,c\right)-12i\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(-6*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 6*I*arctan2(cos(d*x + c
), -sin(d*x + c) + 1) + 4*I*cos(3*d*x + 3*c) - 12*I*cos(d*x + c) + 3*log(co
s(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - 3*log(cos(d*x + c)^2
+ sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4*sin(3*d*x + 3*c) - 12*sin(d*x +
c))/(a^4*d)
```

Fricas [A] time = 2.51307, size = 221, normalized size = 2.7

$$\frac{\left(3\,e^{(3i\,dx+3i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-3\,e^{(3i\,dx+3i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-3i\,dx-3i\,c)}}{3\,a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*e^(3*I*d*x + 3*I*c)*log(e^(I*d*x + I*c) + I) - 3*e^(3*I*d*x + 3*I*c)
*log(e^(I*d*x + I*c) - I) - 6*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(-3*I*d*x - 3*
I*c)/(a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.20956, size = 99, normalized size = 1.21

$$\frac{\frac{3 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a^4}-\frac{3 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right|\right)}{a^4}+\frac{8\left(3i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}{a^4\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-i\right)^3}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a^4 + 8*(3*I*tan(1/2*d*x + 1/2*c) + 1)/(a^4*(tan(1/2*d*x + 1/2*c)
 - I)^3))/d
```

3.161 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=68

$$\frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}$$

```
[Out] ((I/5)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/15)*Sec[c + d*x]^
3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

Rubi [A] time = 0.0793596, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3502, 3488}

$$\frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}+\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/5)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/15)*Sec[c + d*x]^
3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx&=\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}+\frac{\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{5a}\\&=\frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}+\frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}\end{aligned}$$

Mathematica [A] time = 0.0796416, size = 40, normalized size = 0.59

$$-\frac{(\tan(c+dx)-4i)\sec^3(c+dx)}{15a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -(Sec[c + d*x]^3*(-4*I + Tan[c + d*x]))/(15*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.089, size = 90, normalized size = 1.3

$$2\,\frac{1}{a^4d}\left(8/5\,\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-5}+\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-1}-\frac{4\,i}{\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{4}}+\frac{3\,i}{\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{2}}-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/d/a^4*(8/5/(tan(1/2*d*x+1/2*c)-I)^5+1/(tan(1/2*d*x+1/2*c)-I)-4*I/(tan(1/2
*d*x+1/2*c)-I)^4+3*I/(tan(1/2*d*x+1/2*c)-I)^2-14/3/(tan(1/2*d*x+1/2*c)-I)^3
)
```

Maxima [A] time = 0.99218, size = 72, normalized size = 1.06

$$\frac{3i\,\cos\left(5\,dx+5\,c\right)+5i\,\cos\left(3\,dx+3\,c\right)+3\,\sin\left(5\,dx+5\,c\right)+5\,\sin\left(3\,dx+3\,c\right)}{30\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/30*(3*I*cos(5*d*x + 5*c) + 5*I*cos(3*d*x + 3*c) + 3*sin(5*d*x + 5*c) + 5*
sin(3*d*x + 3*c))/(a^4*d)
```

Fricas [A] time = 2.32098, size = 90, normalized size = 1.32

$$\frac{\left(5i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{30\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(5*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.17768, size = 99, normalized size = 1.46

$$\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-15i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+5i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{15\,a^{4}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 15*I*tan(1/2*d*x + 1/2*c)^3 - 25*tan(1/2*
d*x + 1/2*c)^2 + 5*I*tan(1/2*d*x + 1/2*c) + 4)/(a^4*d*(tan(1/2*d*x + 1/2*c)
 - I)^5)
```

3.162 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=132

$$\frac{2i\sec(c+dx)}{35d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{2i\sec(c+dx)}{35d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

```
[Out] ((I/7)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (((3*I)/35)*Sec[c + d*x
])/(a*d*(a + I*a*Tan[c + d*x])^3) + (((2*I)/35)*Sec[c + d*x])/(d*(a^2 + I*a
^2*Tan[c + d*x])^2) + (((2*I)/35)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x
]))
```

Rubi [A] time = 0.10815, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3502, 3488}

$$\frac{2i\sec(c+dx)}{35d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{2i\sec(c+dx)}{35d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3}+\frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/7)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (((3*I)/35)*Sec[c + d*x
])/(a*d*(a + I*a*Tan[c + d*x])^3) + (((2*I)/35)*Sec[c + d*x])/(d*(a^2 + I*a
^2*Tan[c + d*x])^2) + (((2*I)/35)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x
]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4} + \frac{3\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{7a}$$
$$= \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4} + \frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3} + \frac{6\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{35a^2}$$
$$= \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4} + \frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3} + \frac{2i\sec(c+dx)}{35d\left(a^2+ia^2\tan(c+dx)\right)^2} + \frac{2}{}$$
$$= \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4} + \frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3} + \frac{2i\sec(c+dx)}{35d\left(a^2+ia^2\tan(c+dx)\right)^2} + \frac{}{3}$$

Mathematica [A] time = 0.185143, size = 73, normalized size = 0.55

$$\frac{i\sec^4(c+dx)(7i\sin(c+dx)+15i\sin(3(c+dx))+28\cos(c+dx)+20\cos(3(c+dx)))}{140a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/140)*Sec[c + d*x]^4*(28*Cos[c + d*x] + 20*Cos[3*(c + d*x)] + (7*I)*Sin[
c + d*x] + (15*I)*Sin[3*(c + d*x)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.05, size = 123, normalized size = 0.9

$$2\,\frac{1}{a^4d}\left(\frac{3\,i}{(\tan(1/2\,dx+c/2)-i)^2}+\frac{36}{5\,(\tan(1/2\,dx+c/2)-i)^5}+\frac{4\,i}{(\tan(1/2\,dx+c/2)-i)^6}-\frac{8\,i}{(\tan(1/2\,dx+c/2)-i)^4}-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/d/a^4*(3*I/(tan(1/2*d*x+1/2*c)-I)^2+36/5/(tan(1/2*d*x+1/2*c)-I)^5+4*I/(ta
n(1/2*d*x+1/2*c)-I)^6-8*I/(tan(1/2*d*x+1/2*c)-I)^4-6/(tan(1/2*d*x+1/2*c)-I)
^3+1/(tan(1/2*d*x+1/2*c)-I)-8/7/(tan(1/2*d*x+1/2*c)-I)^7)
```

Maxima [A] time = 1.01088, size = 123, normalized size = 0.93

$$\frac{5i\cos(7\,dx+7\,c)+21i\cos(5\,dx+5\,c)+35i\cos(3\,dx+3\,c)+35i\cos(dx+c)+5\sin(7\,dx+7\,c)+21\sin(5\,dx+}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(5*I*cos(7*d*x + 7*c) + 21*I*cos(5*d*x + 5*c) + 35*I*cos(3*d*x + 3*c)
 + 35*I*cos(d*x + c) + 5*sin(7*d*x + 7*c) + 21*sin(5*d*x + 5*c) + 35*sin(3*
d*x + 3*c) + 35*sin(d*x + c))/(a^4*d)
```

Fricas [A] time = 2.24299, size = 166, normalized size = 1.26

$$\frac{\left(35i\,e^{(6i\,dx+6i\,c)}+35i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/280*(35*I*e^(6*I*d*x + 6*I*c) + 35*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*
x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.14042, size = 134, normalized size = 1.02

$$\frac{2\left(35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-105i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+210i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+147\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{35\,a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 2/35*(35*tan(1/2*d*x + 1/2*c)^6 - 105*I*tan(1/2*d*x + 1/2*c)^5 - 210*tan(1/
2*d*x + 1/2*c)^4 + 210*I*tan(1/2*d*x + 1/2*c)^3 + 147*tan(1/2*d*x + 1/2*c)^
2 - 49*I*tan(1/2*d*x + 1/2*c) - 12)/(a^4*d*(tan(1/2*d*x + 1/2*c) - I)^7)
```

3.163 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=134

$$-\frac{4\sin^3(c+dx)}{63a^4d}+\frac{4\sin(c+dx)}{21a^4d}+\frac{8i\cos^3(c+dx)}{63d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

```
[Out] (4*Sin[c + d*x])/(21*a^4*d) - (4*Sin[c + d*x]^3)/(63*a^4*d) + ((I/9)*Cos[c
+ d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (((5*I)/63)*Cos[c + d*x])/(a*d*(a +
I*a*Tan[c + d*x])^3) + (((8*I)/63)*Cos[c + d*x]^3)/(d*(a^4 + I*a^4*Tan[c +
d*x]))
```

Rubi [A] time = 0.120334, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3502, 3500, 2633}

$$-\frac{4\sin^3(c+dx)}{63a^4d}+\frac{4\sin(c+dx)}{21a^4d}+\frac{8i\cos^3(c+dx)}{63d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```


```
[Out] (4*Sin[c + d*x])/(21*a^4*d) - (4*Sin[c + d*x]^3)/(63*a^4*d) + ((I/9)*Cos[c
+ d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (((5*I)/63)*Cos[c + d*x])/(a*d*(a +
I*a*Tan[c + d*x])^3) + (((8*I)/63)*Cos[c + d*x]^3)/(d*(a^4 + I*a^4*Tan[c +
d*x]))
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4} + \frac{5\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{9a}$$

$$= \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4} + \frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3} + \frac{20\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{63a^2}$$

$$= \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4} + \frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3} + \frac{8i\cos^3(c+dx)}{63d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{4\int \mathrm{co}}{}$$

$$= \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4} + \frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3} + \frac{8i\cos^3(c+dx)}{63d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{4\,\mathrm{Sub}}{}$$

$$= \frac{4\sin(c+dx)}{21a^4d} - \frac{4\sin^3(c+dx)}{63a^4d} + \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4} + \frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3} + \frac{}{63}$$

Mathematica [A] time = 0.190985, size = 95, normalized size = 0.71

$$-\frac{i\sec^4(c+dx)(-42i\sin(c+dx) - 135i\sin(3(c+dx)) + 35i\sin(5(c+dx)) - 168\cos(c+dx) - 180\cos(3(c+dx)) + 28\,\mathrm{co}}{1008a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-I/1008)*Sec[c + d*x]^4*(-168*Cos[c + d*x] - 180*Cos[3*(c + d*x)] + 28*Co
s[5*(c + d*x)] - (42*I)*Sin[c + d*x] - (135*I)*Sin[3*(c + d*x)] + (35*I)*Si
n[5*(c + d*x)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.104, size = 174, normalized size = 1.3

$$2\,\frac{1}{a^4d}\left(\frac{\frac{43\,i}{3}}{(\tan(1/2\,dx+c/2)-i)^6} - \frac{4\,i}{(\tan(1/2\,dx+c/2)-i)^8} - \frac{\frac{49\,i}{4}}{(\tan(1/2\,dx+c/2)-i)^4} + \frac{\frac{49\,i}{16}}{(\tan(1/2\,dx+c/2)-i)^2} + \frac{}{9\,(\tan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/d/a^4*(43/3*I/(tan(1/2*d*x+1/2*c)-I)^6-4*I/(tan(1/2*d*x+1/2*c)-I)^8-49/4*
I/(tan(1/2*d*x+1/2*c)-I)^4+49/16*I/(tan(1/2*d*x+1/2*c)-I)^2+8/9/(tan(1/2*d*
x+1/2*c)-I)^9-66/7/(tan(1/2*d*x+1/2*c)-I)^7+31/2/(tan(1/2*d*x+1/2*c)-I)^5-1
73/24/(tan(1/2*d*x+1/2*c)-I)^3+31/32/(tan(1/2*d*x+1/2*c)-I)+1/32/(tan(1/2*d
*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.32711, size = 248, normalized size = 1.85

$$\frac{\left(-63i\,e^{(10i\,dx+10i\,c)}+315i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+126i\,e^{(4i\,dx+4i\,c)}+45i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{(-9i\,dx-9i\,c)}}{2016\,a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2016*(-63*I*e^(10*I*d*x + 10*I*c) + 315*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(
6*I*d*x + 6*I*c) + 126*I*e^(4*I*d*x + 4*I*c) + 45*I*e^(2*I*d*x + 2*I*c) + 7
*I)*e^(-9*I*d*x - 9*I*c)/(a^4*d)
```

Sympy [A] time = 1.6366, size = 233, normalized size = 1.74

$$\begin{cases} \frac{\left(-1585446912ia^{20}d^5e^{26ic}e^{idx}+7927234560ia^{20}d^5e^{24ic}e^{-idx}+5284823040ia^{20}d^5e^{22ic}e^{-3idx}+3170893824ia^{20}d^5e^{20ic}e^{-5idx}+1132462080ia^{20}d^5e^{18ic}e^{-7idx}+1761\right.}{50734301184a^{24}d^6} \\ \frac{x\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-9ic}}{32a^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-1585446912*I*a**20*d**5*exp(26*I*c)*exp(I*d*x) + 7927234560*I*
a**20*d**5*exp(24*I*c)*exp(-I*d*x) + 5284823040*I*a**20*d**5*exp(22*I*c)*ex
p(-3*I*d*x) + 3170893824*I*a**20*d**5*exp(20*I*c)*exp(-5*I*d*x) + 113246208
0*I*a**20*d**5*exp(18*I*c)*exp(-7*I*d*x) + 176160768*I*a**20*d**5*exp(16*I*
c)*exp(-9*I*d*x))*exp(-25*I*c)/(50734301184*a**24*d**6), Ne(50734301184*a**
24*d**6*exp(25*I*c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) +
10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-9*I*c)/(32*a**4), True))
```

Giac [A] time = 1.18235, size = 196, normalized size = 1.46

$$\frac{\frac{63}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)}+\frac{1953\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-9450i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-25998\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+42210i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+46368\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-33054i}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^9}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/1008*(63/(a^4*(tan(1/2*d*x + 1/2*c) + I)) + (1953*tan(1/2*d*x + 1/2*c)^8
- 9450*I*tan(1/2*d*x + 1/2*c)^7 - 25998*tan(1/2*d*x + 1/2*c)^6 + 42210*I*ta
n(1/2*d*x + 1/2*c)^5 + 46368*tan(1/2*d*x + 1/2*c)^4 - 33054*I*tan(1/2*d*x +
 1/2*c)^3 - 15858*tan(1/2*d*x + 1/2*c)^2 + 4374*I*tan(1/2*d*x + 1/2*c) + 70
3)/(a^4*(tan(1/2*d*x + 1/2*c) - I)^9))/d
```

3.164 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=156

$$\frac{2\sin^5(c+dx)}{33a^4d}-\frac{20\sin^3(c+dx)}{99a^4d}+\frac{10\sin(c+dx)}{33a^4d}+\frac{4i\cos^5(c+dx)}{33d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3}+\frac{i\cos}{11d(a+ia}$$

```
[Out] (10*Sin[c + d*x])/(33*a^4*d) - (20*Sin[c + d*x]^3)/(99*a^4*d) + (2*Sin[c +
d*x]^5)/(33*a^4*d) + ((I/11)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) +
 (((7*I)/99)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^3) + (((4*I)/33)*C
os[c + d*x]^5)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.147537, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3502, 3500, 2633}

$$\frac{2\sin^5(c+dx)}{33a^4d}-\frac{20\sin^3(c+dx)}{99a^4d}+\frac{10\sin(c+dx)}{33a^4d}+\frac{4i\cos^5(c+dx)}{33d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3}+\frac{i\cos}{11d(a+ia}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (10*Sin[c + d*x])/(33*a^4*d) - (20*Sin[c + d*x]^3)/(99*a^4*d) + (2*Sin[c +
d*x]^5)/(33*a^4*d) + ((I/11)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) +
 (((7*I)/99)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^3) + (((4*I)/33)*C
os[c + d*x]^5)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4} + \frac{7\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{11a}$$

$$= \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4} + \frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3} + \frac{14\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{33a^2}$$

$$= \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4} + \frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3} + \frac{4i\cos^5(c+dx)}{33d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{1\!\!\!}{}$$

$$= \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4} + \frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3} + \frac{4i\cos^5(c+dx)}{33d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{1\!\!\!}{}$$

$$= \frac{10\sin(c+dx)}{33a^4d} - \frac{20\sin^3(c+dx)}{99a^4d} + \frac{2\sin^5(c+dx)}{33a^4d} + \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4} + \frac{7i}{99ad(a}$$

Mathematica [A] time = 0.353103, size = 117, normalized size = 0.75

$$-\frac{i\sec^4(c+dx)(-231i\sin(c+dx)-891i\sin(3(c+dx))+385i\sin(5(c+dx))+21i\sin(7(c+dx))-924\cos(c+dx)-}{6336a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-I/6336)*Sec[c + d*x]^4*(-924*Cos[c + d*x] - 1188*Cos[3*(c + d*x)] + 308*
Cos[5*(c + d*x)] + 12*Cos[7*(c + d*x)] - (231*I)*Sin[c + d*x] - (891*I)*Sin
[3*(c + d*x)] + (385*I)*Sin[5*(c + d*x)] + (21*I)*Sin[7*(c + d*x)]))/(a^4*d
*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.107, size = 240, normalized size = 1.5

$$2\frac{1}{a^4d}\left(\frac{4i}{(\tan(1/2\,dx+c/2)-i)^{10}} - \frac{\frac{67i}{4}}{(\tan(1/2\,dx+c/2)-i)^4} - \frac{22i}{(\tan(1/2\,dx+c/2)-i)^8} + \frac{\frac{385i}{12}}{(\tan(1/2\,dx+c/2)-i)^6} + \frac{}{(\mathrm{t}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/d/a^4*(4*I/(tan(1/2*d*x+1/2*c)-I)^10-67/4*I/(tan(1/2*d*x+1/2*c)-I)^4-22*I
/(tan(1/2*d*x+1/2*c)-I)^8+385/12*I/(tan(1/2*d*x+1/2*c)-I)^6+201/64*I/(tan(1
/2*d*x+1/2*c)-I)^2-8/11/(tan(1/2*d*x+1/2*c)-I)^11+104/9/(tan(1/2*d*x+1/2*c)
-I)^9-61/2/(tan(1/2*d*x+1/2*c)-I)^7+105/4/(tan(1/2*d*x+1/2*c)-I)^5-267/32/(
tan(1/2*d*x+1/2*c)-I)^3+15/16/(tan(1/2*d*x+1/2*c)-I)-1/64*I/(tan(1/2*d*x+1/
2*c)+I)^2-1/96/(tan(1/2*d*x+1/2*c)+I)^3+1/16/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.32921, size = 336, normalized size = 2.15

$$\frac{\left(-33i\,e^{(14i\,dx+14i\,c)} - 693i\,e^{(12i\,dx+12i\,c)} + 2079i\,e^{(10i\,dx+10i\,c)} + 1155i\,e^{(8i\,dx+8i\,c)} + 693i\,e^{(6i\,dx+6i\,c)} + 297i\,e^{(4i\,dx+4i\,c)} + 77i\,e^{(2i\,dx\cdots}}{12672\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12672*(-33*I*e^(14*I*d*x + 14*I*c) - 693*I*e^(12*I*d*x + 12*I*c) + 2079*I
*e^(10*I*d*x + 10*I*c) + 1155*I*e^(8*I*d*x + 8*I*c) + 693*I*e^(6*I*d*x + 6*
I*c) + 297*I*e^(4*I*d*x + 4*I*c) + 77*I*e^(2*I*d*x + 2*I*c) + 9*I)*e^(-11*I
*d*x - 11*I*c)/(a^4*d)
```

Sympy [A] time = 2.02176, size = 301, normalized size = 1.93

$$\begin{cases} \frac{\left(-167196136166129664ia^{28}d^7e^{39ic}e^{3idx}-3511118859488722944ia^{28}d^7e^{37ic}e^{idx}+10533356578466168832ia^{28}d^7e^{35ic}e^{-idx}+5851864765814538240ia^{28}d^7e^{33ic}e^{-3i\cdots}}{64203316\cdots} \\ \frac{x\left(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1\right)e^{-11ic}}{128a^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-167196136166129664*I*a**28*d**7*exp(39*I*c)*exp(3*I*d*x) - 351
1118859488722944*I*a**28*d**7*exp(37*I*c)*exp(I*d*x) + 10533356578466168832
*I*a**28*d**7*exp(35*I*c)*exp(-I*d*x) + 5851864765814538240*I*a**28*d**7*ex
p(33*I*c)*exp(-3*I*d*x) + 3511118859488722944*I*a**28*d**7*exp(31*I*c)*exp(
-5*I*d*x) + 1504765225495166976*I*a**28*d**7*exp(29*I*c)*exp(-7*I*d*x) + 39
0124317720969216*I*a**28*d**7*exp(27*I*c)*exp(-9*I*d*x) + 45598946227126272
*I*a**28*d**7*exp(25*I*c)*exp(-11*I*d*x))*exp(-36*I*c)/(642033162877937909
6*a**32*d**8), Ne(6420331628779379097 6*a**32*d**8*exp(36*I*c), 0)), (x*(exp
(14*I*c) + 7*exp(12*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) +
 21*exp(4*I*c) + 7*exp(2*I*c) + 1)*exp(-11*I*c)/(128*a**4), True))
```

Giac [A] time = 1.15726, size = 266, normalized size = 1.71

$$\frac{\frac{33\left(12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+21i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-11\right)}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^3}+\frac{5940\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-39501i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-141075\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+313236i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+4795\cdots}{\cdots}}{3168\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3168*(33*(12*tan(1/2*d*x + 1/2*c)^2 + 21*I*tan(1/2*d*x + 1/2*c) - 11)/(a^
4*(tan(1/2*d*x + 1/2*c) + I)^3) + (5940*tan(1/2*d*x + 1/2*c)^10 - 39501*I*t
```

```
an(1/2*d*x + 1/2*c)^9 - 141075*tan(1/2*d*x + 1/2*c)^8 + 313236*I*tan(1/2*d*
x + 1/2*c)^7 + 479556*tan(1/2*d*x + 1/2*c)^6 - 516054*I*tan(1/2*d*x + 1/2*c
)^5 - 397914*tan(1/2*d*x + 1/2*c)^4 + 214500*I*tan(1/2*d*x + 1/2*c)^3 + 790
24*tan(1/2*d*x + 1/2*c)^2 - 17765*I*tan(1/2*d*x + 1/2*c) - 2155)/(a^4*(tan(
1/2*d*x + 1/2*c) - I)^11))/d
```

3.165 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=174

$$-\frac{8\sin^7(c+dx)}{143a^4d}+\frac{168\sin^5(c+dx)}{715a^4d}-\frac{56\sin^3(c+dx)}{143a^4d}+\frac{56\sin(c+dx)}{143a^4d}+\frac{16i\cos^7(c+dx)}{143d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{9i\cos^5(c+}{143ad(a+ia\tan}$$

```
[Out] (56*Sin[c + d*x])/(143*a^4*d) - (56*Sin[c + d*x]^3)/(143*a^4*d) + (168*Sin[
c + d*x]^5)/(715*a^4*d) - (8*Sin[c + d*x]^7)/(143*a^4*d) + ((I/13)*Cos[c +
d*x]^5)/(d*(a + I*a*Tan[c + d*x])^4) + (((9*I)/143)*Cos[c + d*x]^5)/(a*d*(a
 + I*a*Tan[c + d*x])^3) + (((16*I)/143)*Cos[c + d*x]^7)/(d*(a^4 + I*a^4*Tan
[c + d*x]))
```

Rubi [A] time = 0.158102, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3502, 3500, 2633}

$$-\frac{8\sin^7(c+dx)}{143a^4d}+\frac{168\sin^5(c+dx)}{715a^4d}-\frac{56\sin^3(c+dx)}{143a^4d}+\frac{56\sin(c+dx)}{143a^4d}+\frac{16i\cos^7(c+dx)}{143d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{9i\cos^5(c+}{143ad(a+ia\tan}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (56*Sin[c + d*x])/(143*a^4*d) - (56*Sin[c + d*x]^3)/(143*a^4*d) + (168*Sin[
c + d*x]^5)/(715*a^4*d) - (8*Sin[c + d*x]^7)/(143*a^4*d) + ((I/13)*Cos[c +
d*x]^5)/(d*(a + I*a*Tan[c + d*x])^4) + (((9*I)/143)*Cos[c + d*x]^5)/(a*d*(a
 + I*a*Tan[c + d*x])^3) + (((16*I)/143)*Cos[c + d*x]^7)/(d*(a^4 + I*a^4*Tan
[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4} + \frac{9\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{13a}$$
$$= \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4} + \frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3} + \frac{72\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{143a^2}$$
$$= \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4} + \frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3} + \frac{16i\cos^7(c+dx)}{143d\left(a^4+ia^4\tan(c+dx)\right)} +$$
$$= \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4} + \frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3} + \frac{16i\cos^7(c+dx)}{143d\left(a^4+ia^4\tan(c+dx)\right)} -$$
$$= \frac{56\sin(c+dx)}{143a^4d} - \frac{56\sin^3(c+dx)}{143a^4d} + \frac{168\sin^5(c+dx)}{715a^4d} - \frac{8\sin^7(c+dx)}{143a^4d} + \frac{i\cos^5(c+}{13d(a+ia\tan(}$$

Mathematica [A] time = 0.79145, size = 139, normalized size = 0.8

$$-\frac{i\sec^4(c+dx)(-6006i\sin(c+dx)-25740i\sin(3(c+dx))+14300i\sin(5(c+dx))+1365i\sin(7(c+dx))+99i\sin(9(}{183040a^4d(\mathrm{t}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-I/183040)*Sec[c + d*x]^4*(-24024*Cos[c + d*x] - 34320*Cos[3*(c + d*x)] +
 11440*Cos[5*(c + d*x)] + 780*Cos[7*(c + d*x)] + 44*Cos[9*(c + d*x)] - (600
6*I)*Sin[c + d*x] - (25740*I)*Sin[3*(c + d*x)] + (14300*I)*Sin[5*(c + d*x)]
 + (1365*I)*Sin[7*(c + d*x)] + (99*I)*Sin[9*(c + d*x)]))/(a^4*d*(-I + Tan[c
 + d*x])^4)
```

Maple [A] time = 0.11, size = 306, normalized size = 1.8

$$2\frac{1}{a^4d}\left(\frac{\frac{825i}{256}}{(\tan(1/2\,dx+c/2)-i)^2} - \frac{4i}{(\tan(1/2\,dx+c/2)-i)^{12}} - \frac{\frac{1375i}{64}}{(\tan(1/2\,dx+c/2)-i)^4} + \frac{31i}{(\tan(1/2\,dx+c/2)-i)^{10}} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/d/a^4*(825/256*I/(tan(1/2*d*x+1/2*c)-I)^2-4*I/(tan(1/2*d*x+1/2*c)-I)^12-1
375/64*I/(tan(1/2*d*x+1/2*c)-I)^4+31*I/(tan(1/2*d*x+1/2*c)-I)^10+1/64*I/(ta
n(1/2*d*x+1/2*c)+I)^4+465/8*I/(tan(1/2*d*x+1/2*c)-I)^6+8/13/(tan(1/2*d*x+1/
2*c)-I)^13-150/11/(tan(1/2*d*x+1/2*c)-I)^11+52/(tan(1/2*d*x+1/2*c)-I)^9-279
/4/(tan(1/2*d*x+1/2*c)-I)^7+6291/160/(tan(1/2*d*x+1/2*c)-I)^5-1207/128/(tan
(1/2*d*x+1/2*c)-I)^3+233/256/(tan(1/2*d*x+1/2*c)-I)-11/256*I/(tan(1/2*d*x+1
/2*c)+I)^2-135/2*I/(tan(1/2*d*x+1/2*c)-I)^8+1/160/(tan(1/2*d*x+1/2*c)+I)^5-
5/128/(tan(1/2*d*x+1/2*c)+I)^3+23/256/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.48283, size = 435, normalized size = 2.5

$$\frac{\left(-143 i e^{(18 i dx+18 i c)} - 2145 i e^{(16 i dx+16 i c)} - 25740 i e^{(14 i dx+14 i c)} + 60060 i e^{(12 i dx+12 i c)} + 30030 i e^{(10 i dx+10 i c)} + 18018 i e^{(8 i dx+8 i c)} \cdots\right.}{366080\, a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/366080*(-143*I*e^(18*I*d*x + 18*I*c) - 2145*I*e^(16*I*d*x + 16*I*c) - 257
40*I*e^(14*I*d*x + 14*I*c) + 60060*I*e^(12*I*d*x + 12*I*c) + 30030*I*e^(10*
I*d*x + 10*I*c) + 18018*I*e^(8*I*d*x + 8*I*c) + 8580*I*e^(6*I*d*x + 6*I*c)
+ 2860*I*e^(4*I*d*x + 4*I*c) + 585*I*e^(2*I*d*x + 2*I*c) + 55*I)*e^(-13*I*d
*x - 13*I*c)/(a^4*d)
```

Sympy [A] time = 2.60521, size = 369, normalized size = 2.12

$$\begin{cases} \frac{\left(-168824601762589816389632 0 i a^{36} d^{9} e^{54 i c} e^{5 i d x} - 2532369026438847245844480 0 i a^{36} d^{9} e^{52 i c} e^{3 i d x} - 3038842831726616695013376 00 i a^{36} d^{9} e^{50 i c} e^{i d x} + 70906332740 \cdots\right.}{\cdots} \\ \frac{x\left(e^{18 i c} + 9 e^{16 i c} + 36 e^{14 i c} + 84 e^{12 i c} + 126 e^{10 i c} + 126 e^{8 i c} + 84 e^{6 i c} + 36 e^{4 i c} + 9 e^{2 i c} + 1\right) e^{-13 i c}}{512 a^{4}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-1688246017625898163896320*I*a**36*d**9*exp(54*I*c)*exp(5*I*d*x
) - 25323690264388472458444800*I*a**36*d**9*exp(52*I*c)*exp(3*I*d*x) - 3038
84283172661669501337600*I*a**36*d**9*exp(50*I*c)*exp(I*d*x) + 7090633274028
77228836454400*I*a**36*d**9*exp(48*I*c)*exp(-I*d*x) + 354531663701438614418
227200*I*a**36*d**9*exp(46*I*c)*exp(-3*I*d*x) + 212718998220863168650936320
*I*a**36*d**9*exp(44*I*c)*exp(-5*I*d*x) + 101294761057553889833779200*I*a**
36*d**9*exp(42*I*c)*exp(-7*I*d*x) + 33764920352517963277926400*I*a**36*d**9
*exp(40*I*c)*exp(-9*I*d*x) + 6906460981196856125030400*I*a**36*d**9*exp(38*
I*c)*exp(-11*I*d*x) + 649325391394576216883200*I*a**36*d**9*exp(36*I*c)*exp
(-13*I*d*x))*exp(-49*I*c)/(4321909805122299299574579200*a**40*d**10), Ne(43
21909805122299299574579200*a**40*d**10*exp(49*I*c), 0)), (x*(exp(18*I*c) +
9*exp(16*I*c) + 36*exp(14*I*c) + 84*exp(12*I*c) + 126*exp(10*I*c) + 126*exp
(8*I*c) + 84*exp(6*I*c) + 36*exp(4*I*c) + 9*exp(2*I*c) + 1)*exp(-13*I*c)/(5
12*a**4), True))
```

Giac [A] time = 1.17725, size = 336, normalized size = 1.93

$$\frac{\frac{143\left(115 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}+405 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}-575 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-375 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+98\right)}{a^{4}\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+i\right)^{5}} + \frac{166595 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{12}-1409265 i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{11}-62 \cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/91520*(143*(115*tan(1/2*d*x + 1/2*c)^4 + 405*I*tan(1/2*d*x + 1/2*c)^3 - 5
75*tan(1/2*d*x + 1/2*c)^2 - 375*I*tan(1/2*d*x + 1/2*c) + 98)/(a^4*(tan(1/2*
d*x + 1/2*c) + I)^5) + (166595*tan(1/2*d*x + 1/2*c)^12 - 1409265*I*tan(1/2*
d*x + 1/2*c)^11 - 6232655*tan(1/2*d*x + 1/2*c)^10 + 17535375*I*tan(1/2*d*x
+ 1/2*c)^9 + 34610004*tan(1/2*d*x + 1/2*c)^8 - 49771722*I*tan(1/2*d*x + 1/2
*c)^7 - 53349582*tan(1/2*d*x + 1/2*c)^6 + 42730974*I*tan(1/2*d*x + 1/2*c)^5
 + 25431835*tan(1/2*d*x + 1/2*c)^4 - 10954229*I*tan(1/2*d*x + 1/2*c)^3 - 32
78067*tan(1/2*d*x + 1/2*c)^2 + 614627*I*tan(1/2*d*x + 1/2*c) + 60094)/(a^4*
(tan(1/2*d*x + 1/2*c) - I)^13))/d
```

3.166 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=134

$$\frac{\tan^5(c+dx)}{5a^8d}+\frac{2i\tan^4(c+dx)}{a^8d}-\frac{10\tan^3(c+dx)}{a^8d}-\frac{36i\tan^2(c+dx)}{a^8d}+\frac{129\tan(c+dx)}{a^8d}+\frac{64i}{d\left(a^8+ia^8\tan(c+dx)\right)}-19$$

```
[Out] (-192*x)/a^8 - ((192*I)*Log[Cos[c + d*x]])/(a^8*d) + (129*Tan[c + d*x])/(a^
8*d) - ((36*I)*Tan[c + d*x]^2)/(a^8*d) - (10*Tan[c + d*x]^3)/(a^8*d) + ((2*
I)*Tan[c + d*x]^4)/(a^8*d) + Tan[c + d*x]^5/(5*a^8*d) + (64*I)/(d*(a^8 + I*
a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.0792108, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{\tan^5(c+dx)}{5a^8d}+\frac{2i\tan^4(c+dx)}{a^8d}-\frac{10\tan^3(c+dx)}{a^8d}-\frac{36i\tan^2(c+dx)}{a^8d}+\frac{129\tan(c+dx)}{a^8d}+\frac{64i}{d\left(a^8+ia^8\tan(c+dx)\right)}-19$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-192*x)/a^8 - ((192*I)*Log[Cos[c + d*x]])/(a^8*d) + (129*Tan[c + d*x])/(a^
8*d) - ((36*I)*Tan[c + d*x]^2)/(a^8*d) - (10*Tan[c + d*x]^3)/(a^8*d) + ((2*
I)*Tan[c + d*x]^4)/(a^8*d) + Tan[c + d*x]^5/(5*a^8*d) + (64*I)/(d*(a^8 + I*
a^8*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{(a-x)^6}{(a+x)^2}\,dx,x,ia\tan(c+dx)\right)}{a^{13}d}\\
&= -\frac{i\,\text{Subst}\left(\int \left(129a^4-72a^3x+30a^2x^2-8ax^3+x^4+\frac{64a^6}{(a+x)^2}-\frac{192a^5}{a+x}\right)dx,x,ia\tan(c+dx)\right)}{a^{13}d}\\
&= -\frac{192x}{a^8}-\frac{192i\log(\cos(c+dx))}{a^8d}+\frac{129\tan(c+dx)}{a^8d}-\frac{36i\tan^2(c+dx)}{a^8d}-\frac{10\tan^3(c+dx)}{a^8d}+
\end{aligned}$$

Mathematica [B] time = 2.71195, size = 599, normalized size = 4.47

$$\sec(c)\sec^{13}(c+dx)(-\cos(7(c+dx))-i\sin(7(c+dx)))(300idx\sin(c+2dx)-985\sin(c+2dx)+300idx\sin(3c+2dx$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^13*(-Cos[7*(c + d*x)] - I*Sin[7*(c + d*x)])*((-220*I)*
Cos[3*c + 2*d*x] + 900*d*x*Cos[3*c + 2*d*x] + (238*I)*Cos[3*c + 4*d*x] + 36
0*d*x*Cos[3*c + 4*d*x] - (110*I)*Cos[5*c + 4*d*x] + 360*d*x*Cos[5*c + 4*d*x
] + (77*I)*Cos[5*c + 6*d*x] + 60*d*x*Cos[5*c + 6*d*x] - (10*I)*Cos[7*c + 6*
d*x] + 60*d*x*Cos[7*c + 6*d*x] + 10*Cos[c]*(-7*I + 120*d*x + (120*I)*Log[Co
s[c + d*x]]) + 5*Cos[c + 2*d*x]*(43*I + 180*d*x + (180*I)*Log[Cos[c + d*x]]
) + (900*I)*Cos[3*c + 2*d*x]*Log[Cos[c + d*x]] + (360*I)*Cos[3*c + 4*d*x]*L
og[Cos[c + d*x]] + (360*I)*Cos[5*c + 4*d*x]*Log[Cos[c + d*x]] + (60*I)*Cos[
5*c + 6*d*x]*Log[Cos[c + d*x]] + (60*I)*Cos[7*c + 6*d*x]*Log[Cos[c + d*x]]
+ 870*Sin[c] - 985*Sin[c + 2*d*x] + (300*I)*d*x*Sin[c + 2*d*x] - 300*Log[Co
s[c + d*x]]*Sin[c + 2*d*x] + 320*Sin[3*c + 2*d*x] + (300*I)*d*x*Sin[3*c + 2
*d*x] - 300*Log[Cos[c + d*x]]*Sin[3*c + 2*d*x] - 512*Sin[3*c + 4*d*x] + (24
0*I)*d*x*Sin[3*c + 4*d*x] - 240*Log[Cos[c + d*x]]*Sin[3*c + 4*d*x] + 10*Sin
[5*c + 4*d*x] + (240*I)*d*x*Sin[5*c + 4*d*x] - 240*Log[Cos[c + d*x]]*Sin[5*
c + 4*d*x] - 97*Sin[5*c + 6*d*x] + (60*I)*d*x*Sin[5*c + 6*d*x] - 60*Log[Cos
[c + d*x]]*Sin[5*c + 6*d*x] - 10*Sin[7*c + 6*d*x] + (60*I)*d*x*Sin[7*c + 6*
d*x] - 60*Log[Cos[c + d*x]]*Sin[7*c + 6*d*x]))/(20*a^8*d*(-I + Tan[c + d*x]
)^8)
```

Maple [A] time = 0.132, size = 120, normalized size = 0.9

$$129\,\frac{\tan(dx+c)}{da^8}+\frac{(\tan(dx+c))^5}{5\,da^8}+\frac{2\,i\,(\tan(dx+c))^4}{da^8}-10\,\frac{(\tan(dx+c))^3}{da^8}-\frac{36\,i\,(\tan(dx+c))^2}{da^8}+\frac{192\,i\ln(\tan(dx}{da^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 129*tan(d*x+c)/a^8/d+1/5*tan(d*x+c)^5/a^8/d+2*I*tan(d*x+c)^4/a^8/d-10*tan(d
*x+c)^3/a^8/d-36*I*tan(d*x+c)^2/a^8/d+192*I/d/a^8*ln(tan(d*x+c)-I)+64/d/a^8
/(tan(d*x+c)-I)
```

Maxima [A] time = 1.19722, size = 313, normalized size = 2.34

$$\frac{\frac{5\left(2240\tan(dx+c)^6-13440i\tan(dx+c)^5-33600\tan(dx+c)^4+44800i\tan(dx+c)^3+33600\tan(dx+c)^2-13440i\tan(dx+c)-2240\right)}{35a^8\tan(dx+c)^7-245ia^8\tan(dx+c)^6-735a^8\tan(dx+c)^5+1225ia^8\tan(dx+c)^4+1225a^8\tan(dx+c)^3-735ia^8\tan(dx+c)^2-245a^8\tan(dx+c)+35ia^8}+\frac{\tan(}{}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(5*(2240*tan(d*x + c)^6 - 13440*I*tan(d*x + c)^5 - 33600*tan(d*x + c)^4
 + 44800*I*tan(d*x + c)^3 + 33600*tan(d*x + c)^2 - 13440*I*tan(d*x + c) - 2
240)/(35*a^8*tan(d*x + c)^7 - 245*I*a^8*tan(d*x + c)^6 - 735*a^8*tan(d*x +
```

```
c)^5 + 1225*I*a^8*tan(d*x + c)^4 + 1225*a^8*tan(d*x + c)^3 - 735*I*a^8*tan(
d*x + c)^2 - 245*a^8*tan(d*x + c) + 35*I*a^8) + (tan(d*x + c)^5 + 10*I*tan(
d*x + c)^4 - 50*tan(d*x + c)^3 - 180*I*tan(d*x + c)^2 + 645*tan(d*x + c))/a
^8 + 960*I*log(I*tan(d*x + c) + 1)/a^8)/d
```

Fricas [B] time = 3.33474, size = 878, normalized size = 6.55

$$-\frac{1920\,dxe^{(12i\,dx+12i\,c)} + (9600\,dx - 960i)e^{(10i\,dx+10i\,c)} + (19200\,dx - 4320i)e^{(8i\,dx+8i\,c)} + (19200\,dx - 7520i)e^{(6i\,dx+6i\,c)} + (960}{5\left(a^8de^{(12i\,dx+12i\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/5*(1920*d*x*e^(12*I*d*x + 12*I*c) + (9600*d*x - 960*I)*e^(10*I*d*x + 10*
I*c) + (19200*d*x - 4320*I)*e^(8*I*d*x + 8*I*c) + (19200*d*x - 7520*I)*e^(6
*I*d*x + 6*I*c) + (9600*d*x - 6160*I)*e^(4*I*d*x + 4*I*c) + (1920*d*x - 219
2*I)*e^(2*I*d*x + 2*I*c) - (-960*I*e^(12*I*d*x + 12*I*c) - 4800*I*e^(10*I*d
*x + 10*I*c) - 9600*I*e^(8*I*d*x + 8*I*c) - 9600*I*e^(6*I*d*x + 6*I*c) - 48
00*I*e^(4*I*d*x + 4*I*c) - 960*I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*
c) + 1) - 160*I)/(a^8*d*e^(12*I*d*x + 12*I*c) + 5*a^8*d*e^(10*I*d*x + 10*I*
c) + 10*a^8*d*e^(8*I*d*x + 8*I*c) + 10*a^8*d*e^(6*I*d*x + 6*I*c) + 5*a^8*d*
e^(4*I*d*x + 4*I*c) + a^8*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.25056, size = 340, normalized size = 2.54

$$-\frac{2\left(-\frac{960i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^8} + \frac{480i\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^8} + \frac{480i\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^8} - \frac{5\left(-288i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-640\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+288}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/5*(-960*I*log(tan(1/2*d*x + 1/2*c) - I)/a^8 + 480*I*log(abs(tan(1/2*d*x
+ 1/2*c) + 1))/a^8 + 480*I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^8 - 5*(-288
*I*tan(1/2*d*x + 1/2*c)^2 - 640*tan(1/2*d*x + 1/2*c) + 288*I)/(a^8*(tan(1/2
*d*x + 1/2*c) - I)^2) + (-1096*I*tan(1/2*d*x + 1/2*c)^10 + 645*tan(1/2*d*x
+ 1/2*c)^9 + 5840*I*tan(1/2*d*x + 1/2*c)^8 - 2780*tan(1/2*d*x + 1/2*c)^7 -
12120*I*tan(1/2*d*x + 1/2*c)^6 + 4286*tan(1/2*d*x + 1/2*c)^5 + 12120*I*tan(
```

```
1/2*d*x + 1/2*c)^4 - 2780*tan(1/2*d*x + 1/2*c)^3 - 5840*I*tan(1/2*d*x + 1/2
*c)^2 + 645*tan(1/2*d*x + 1/2*c) + 1096*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*
a^8))/d
```

3.167 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=126

$$\frac{\tan^3(c+dx)}{3a^8d}+\frac{4i\tan^2(c+dx)}{a^8d}-\frac{31\tan(c+dx)}{a^8d}-\frac{80i}{d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{80i\log(\cos(c}{a^8d}$$

```
[Out] (80*x)/a^8 + ((80*I)*Log[Cos[c + d*x]])/(a^8*d) - (31*Tan[c + d*x])/(a^8*d)
 + ((4*I)*Tan[c + d*x]^2)/(a^8*d) + Tan[c + d*x]^3/(3*a^8*d) + (16*I)/(d*(a
^4 + I*a^4*Tan[c + d*x])^2) - (80*I)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.0768581, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{\tan^3(c+dx)}{3a^8d}+\frac{4i\tan^2(c+dx)}{a^8d}-\frac{31\tan(c+dx)}{a^8d}-\frac{80i}{d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{80i\log(\cos(c}{a^8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (80*x)/a^8 + ((80*I)*Log[Cos[c + d*x]])/(a^8*d) - (31*Tan[c + d*x])/(a^8*d)
 + ((4*I)*Tan[c + d*x]^2)/(a^8*d) + Tan[c + d*x]^3/(3*a^8*d) + (16*I)/(d*(a
^4 + I*a^4*Tan[c + d*x])^2) - (80*I)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{(a-x)^5}{(a+x)^3}\,dx,x,ia\tan(c+dx)\right)}{a^{11}d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(-31a^2+8ax-x^2+\frac{32a^5}{(a+x)^3}-\frac{80a^4}{(a+x)^2}+\frac{80a^3}{a+x}\right)dx,x,ia\tan(c+dx)\right)}{a^{11}d}\\ &= \frac{80x}{a^8}+\frac{80i\log(\cos(c+dx))}{a^8d}-\frac{31\tan(c+dx)}{a^8d}+\frac{4i\tan^2(c+dx)}{a^8d}+\frac{\tan^3(c+dx)}{3a^8d}+\frac{}{d\left(a^4+i\right.}\end{aligned}$$

Mathematica [B] time = 1.38642, size = 537, normalized size = 4.26

$$\sec(c)\sec^{11}(c+dx)(\cos(6(c+dx))+i\sin(6(c+dx)))(120idx\sin(2c+dx)+87\sin(2c+dx)+180idx\sin(2c+3dx)-$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^11*(Cos[6*(c + d*x)] + I*Sin[6*(c + d*x)])*((66*I)*Cos
[2*c + 3*d*x] + 180*d*x*Cos[2*c + 3*d*x] - (75*I)*Cos[4*c + 3*d*x] + 180*d*
x*Cos[4*c + 3*d*x] + (50*I)*Cos[4*c + 5*d*x] + 60*d*x*Cos[4*c + 5*d*x] + (3
*I)*Cos[6*c + 5*d*x] + 60*d*x*Cos[6*c + 5*d*x] + 3*Cos[2*c + d*x]*(-71*I +
80*d*x + (80*I)*Log[Cos[c + d*x]]) + Cos[d*x]*(-119*I + 240*d*x + (240*I)*L
og[Cos[c + d*x]]) + (180*I)*Cos[2*c + 3*d*x]*Log[Cos[c + d*x]] + (180*I)*Co
s[4*c + 3*d*x]*Log[Cos[c + d*x]] + (60*I)*Cos[4*c + 5*d*x]*Log[Cos[c + d*x]
] + (60*I)*Cos[6*c + 5*d*x]*Log[Cos[c + d*x]] - 101*Sin[d*x] + (120*I)*d*x*
Sin[d*x] - 120*Log[Cos[c + d*x]]*Sin[d*x] + 87*Sin[2*c + d*x] + (120*I)*d*x
*Sin[2*c + d*x] - 120*Log[Cos[c + d*x]]*Sin[2*c + d*x] - 96*Sin[2*c + 3*d*x
] + (180*I)*d*x*Sin[2*c + 3*d*x] - 180*Log[Cos[c + d*x]]*Sin[2*c + 3*d*x] +
 45*Sin[4*c + 3*d*x] + (180*I)*d*x*Sin[4*c + 3*d*x] - 180*Log[Cos[c + d*x]]
*Sin[4*c + 3*d*x] - 44*Sin[4*c + 5*d*x] + (60*I)*d*x*Sin[4*c + 5*d*x] - 60*
Log[Cos[c + d*x]]*Sin[4*c + 5*d*x] + 3*Sin[6*c + 5*d*x] + (60*I)*d*x*Sin[6*
c + 5*d*x] - 60*Log[Cos[c + d*x]]*Sin[6*c + 5*d*x]))/(12*a^8*d*(-I + Tan[c
+ d*x])^8)
```

Maple [A] time = 0.12, size = 107, normalized size = 0.9

$$-31\frac{\tan(dx+c)}{da^8}+\frac{(\tan(dx+c))^3}{3\,da^8}+\frac{4\,i\,(\tan(dx+c))^2}{da^8}-80\frac{1}{da^8(\tan(dx+c)-i)}-\frac{80\,i\ln(\tan(dx+c)-i)}{da^8}-\frac{}{da^8(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] -31*tan(d*x+c)/a^8/d+1/3*tan(d*x+c)^3/a^8/d+4*I*tan(d*x+c)^2/a^8/d-80/d/a^8
/(tan(d*x+c)-I)-80*I/d/a^8*ln(tan(d*x+c)-I)-16*I/d/a^8/(tan(d*x+c)-I)^2
```

Maxima [A] time = 1.24267, size = 288, normalized size = 2.29

$$-\frac{\frac{3\left(1680\tan(dx+c)^6-9744i\tan(dx+c)^5-23520\tan(dx+c)^4+30240i\tan(dx+c)^3+21840\tan(dx+c)^2-8400i\tan(dx+c)-1344\right)}{21a^8\tan(dx+c)^7-147ia^8\tan(dx+c)^6-441a^8\tan(dx+c)^5+735ia^8\tan(dx+c)^4+735a^8\tan(dx+c)^3-441ia^8\tan(dx+c)^2-147a^8\tan(dx+c)+21ia^8}-\frac{\tan(}{}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(1680*tan(d*x + c)^6 - 9744*I*tan(d*x + c)^5 - 23520*tan(d*x + c)^4
 + 30240*I*tan(d*x + c)^3 + 21840*tan(d*x + c)^2 - 8400*I*tan(d*x + c) - 13
44)/(21*a^8*tan(d*x + c)^7 - 147*I*a^8*tan(d*x + c)^6 - 441*a^8*tan(d*x + c
)^5 + 735*I*a^8*tan(d*x + c)^4 + 735*a^8*tan(d*x + c)^3 - 441*I*a^8*tan(d*x
 + c)^2 - 147*a^8*tan(d*x + c) + 21*I*a^8) - (tan(d*x + c)^3 + 12*I*tan(d*x
 + c)^2 - 93*tan(d*x + c))/a^8 + 240*I*log(I*tan(d*x + c) + 1)/a^8)/d
```

Fricas [A] time = 3.00845, size = 616, normalized size = 4.89

$$\frac{480\,dxe^{(10i\,dx+10i\,c)} + (1440\,dx - 240i)e^{(8i\,dx+8i\,c)} + (1440\,dx - 600i)e^{(6i\,dx+6i\,c)} + (480\,dx - 440i)e^{(4i\,dx+4i\,c)} + \left(240i\,e^{(10i\,dx+10}\right.}{3\left(a^8de^{(10i\,dx+10i\,c)} + 3\,a^8de^{(8i\,dx+8i\,c)} + 3\,a^8de^{(6i\,d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/3*(480*d*x*e^(10*I*d*x + 10*I*c) + (1440*d*x - 240*I)*e^(8*I*d*x + 8*I*c)
 + (1440*d*x - 600*I)*e^(6*I*d*x + 6*I*c) + (480*d*x - 440*I)*e^(4*I*d*x +
4*I*c) + (240*I*e^(10*I*d*x + 10*I*c) + 720*I*e^(8*I*d*x + 8*I*c) + 720*I*e
^(6*I*d*x + 6*I*c) + 240*I*e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c) + 1
) - 60*I*e^(2*I*d*x + 2*I*c) + 12*I)/(a^8*d*e^(10*I*d*x + 10*I*c) + 3*a^8*d
*e^(8*I*d*x + 8*I*c) + 3*a^8*d*e^(6*I*d*x + 6*I*c) + a^8*d*e^(4*I*d*x + 4*I
*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.21734, size = 304, normalized size = 2.41

$$-\frac{2\left(\frac{240i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^8} - \frac{120i\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^8} - \frac{120i\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^8} + \frac{220i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 93\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 684i\,\tan\left(\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/3*(240*I*log(tan(1/2*d*x + 1/2*c) - I)/a^8 - 120*I*log(abs(tan(1/2*d*x +
 1/2*c) + 1))/a^8 - 120*I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^8 + (220*I*t
an(1/2*d*x + 1/2*c)^6 - 93*tan(1/2*d*x + 1/2*c)^5 - 684*I*tan(1/2*d*x + 1/2
*c)^4 + 190*tan(1/2*d*x + 1/2*c)^3 + 684*I*tan(1/2*d*x + 1/2*c)^2 - 93*tan(
1/2*d*x + 1/2*c) - 220*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^8) + (-500*I*ta
n(1/2*d*x + 1/2*c)^4 - 2144*tan(1/2*d*x + 1/2*c)^3 + 3384*I*tan(1/2*d*x + 1
/2*c)^2 + 2144*tan(1/2*d*x + 1/2*c) - 500*I)/(a^8*(tan(1/2*d*x + 1/2*c) - I
)^4))/d
```

3.168 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8} dx$

Optimal. Leaf size=116

$$\frac{\tan(c+dx)}{a^8d}+\frac{24i}{d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{16i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{8i\log(\cos(c+dx))}{a^8d}$$

```
[Out] (-8*x)/a^8 - ((8*I)*Log[Cos[c + d*x]])/(a^8*d) + Tan[c + d*x]/(a^8*d) + ((1
6*I)/3)/(a^5*d*(a + I*a*Tan[c + d*x])^3) - (16*I)/(d*(a^4 + I*a^4*Tan[c + d
*x])^2) + (24*I)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.0682109, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{\tan(c+dx)}{a^8d}+\frac{24i}{d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{16i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{8i\log(\cos(c+dx))}{a^8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-8*x)/a^8 - ((8*I)*Log[Cos[c + d*x]])/(a^8*d) + Tan[c + d*x]/(a^8*d) + ((1
6*I)/3)/(a^5*d*(a + I*a*Tan[c + d*x])^3) - (16*I)/(d*(a^4 + I*a^4*Tan[c + d
*x])^2) + (24*I)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}dx &= -\frac{i\,\text{Subst}\left(\int \frac{(a-x)^4}{(a+x)^4}dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(1+\frac{16a^4}{(a+x)^4}-\frac{32a^3}{(a+x)^3}+\frac{24a^2}{(a+x)^2}-\frac{8a}{a+x}\right)dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= -\frac{8x}{a^8}-\frac{8i\log(\cos(c+dx))}{a^8d}+\frac{\tan(c+dx)}{a^8d}+\frac{16i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{16i}{d\left(a^4+ia^4\tan\right.}\end{aligned}$$

Mathematica [B] time = 0.885939, size = 397, normalized size = 3.42

$\sec(c)\sec^9(c+dx)(-\cos(5(c+dx))-i\sin(5(c+dx)))(12idx\sin(c+2dx)+11\sin(c+2dx)+12idx\sin(3c+2dx)+1$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^9*(-Cos[5*(c + d*x)] - I*Sin[5*(c + d*x)])*((-12*I)*Co
s[c] - (10*I)*Cos[3*c + 2*d*x] + 12*d*x*Cos[3*c + 2*d*x] + (2*I)*Cos[3*c +
4*d*x] + 12*d*x*Cos[3*c + 4*d*x] - I*Cos[5*c + 4*d*x] + 12*d*x*Cos[5*c + 4*
d*x] + Cos[c + 2*d*x]*(-7*I + 12*d*x + (12*I)*Log[Cos[c + d*x]]) + (12*I)*C
os[3*c + 2*d*x]*Log[Cos[c + d*x]] + (12*I)*Cos[3*c + 4*d*x]*Log[Cos[c + d*x
]] + (12*I)*Cos[5*c + 4*d*x]*Log[Cos[c + d*x]] + 11*Sin[c + 2*d*x] + (12*I)
*d*x*Sin[c + 2*d*x] - 12*Log[Cos[c + d*x]]*Sin[c + 2*d*x] + 14*Sin[3*c + 2*
d*x] + (12*I)*d*x*Sin[3*c + 2*d*x] - 12*Log[Cos[c + d*x]]*Sin[3*c + 2*d*x]
- 4*Sin[3*c + 4*d*x] + (12*I)*d*x*Sin[3*c + 4*d*x] - 12*Log[Cos[c + d*x]]*S
in[3*c + 4*d*x] - Sin[5*c + 4*d*x] + (12*I)*d*x*Sin[5*c + 4*d*x] - 12*Log[C
os[c + d*x]]*Sin[5*c + 4*d*x]))/(6*a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] time = 0.094, size = 92, normalized size = 0.8

$$\frac{\tan(dx+c)}{da^8}-\frac{16}{3\,da^8\,(\tan(dx+c)-i)^3}+\frac{8\,i\ln(\tan(dx+c)-i)}{da^8}+24\,\frac{1}{da^8\,(\tan(dx+c)-i)}+\frac{16\,i}{da^8\,(\tan(dx+c)-i)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] tan(d*x+c)/a^8/d-16/3/d/a^8/(tan(d*x+c)-I)^3+8*I/d/a^8*ln(tan(d*x+c)-I)+24/
d/a^8/(tan(d*x+c)-I)+16*I/d/a^8/(tan(d*x+c)-I)^2
```

Maxima [A] time = 1.25337, size = 255, normalized size = 2.2

$$\frac{\frac{2520\tan(dx+c)^6-13440i\tan(dx+c)^5-29960\tan(dx+c)^4+35840i\tan(dx+c)^3+24360\tan(dx+c)^2-8960i\tan(dx+c)-1400}{105a^8\tan(dx+c)^7-735ia^8\tan(dx+c)^6-2205a^8\tan(dx+c)^5+3675ia^8\tan(dx+c)^4+3675a^8\tan(dx+c)^3-2205ia^8\tan(dx+c)^2-735a^8\tan(dx+c)+105ia^8}+\frac{8i\,\mathrm{lc}\cdots}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] ((2520*tan(d*x + c)^6 - 13440*I*tan(d*x + c)^5 - 29960*tan(d*x + c)^4 + 358
40*I*tan(d*x + c)^3 + 24360*tan(d*x + c)^2 - 8960*I*tan(d*x + c) - 1400)/(1
05*a^8*tan(d*x + c)^7 - 735*I*a^8*tan(d*x + c)^6 - 2205*a^8*tan(d*x + c)^5
+ 3675*I*a^8*tan(d*x + c)^4 + 3675*a^8*tan(d*x + c)^3 - 2205*I*a^8*tan(d*x
+ c)^2 - 735*a^8*tan(d*x + c) + 105*I*a^8) + 8*I*log(I*tan(d*x + c) + 1)/a^
8 + tan(d*x + c)/a^8)/d
```

Fricas [A] time = 2.85278, size = 370, normalized size = 3.19

$$-\frac{48\,dxe^{(8i\,dx+8i\,c)}+(48\,dx-24i)e^{(6i\,dx+6i\,c)}-\left(-24i\,e^{(8i\,dx+8i\,c)}-24i\,e^{(6i\,dx+6i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(2i\cdots}}{3\left(a^8de^{(8i\,dx+8i\,c)}+a^8de^{(6i\,dx+6i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/3*(48*d*x*e^(8*I*d*x + 8*I*c) + (48*d*x - 24*I)*e^(6*I*d*x + 6*I*c) - (-
24*I*e^(8*I*d*x + 8*I*c) - 24*I*e^(6*I*d*x + 6*I*c))*log(e^(2*I*d*x + 2*I*c
) + 1) - 12*I*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) - 2*I)/(a^8*d*e
^(8*I*d*x + 8*I*c) + a^8*d*e^(6*I*d*x + 6*I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.21933, size = 270, normalized size = 2.33

$$-\frac{2\left(-\frac{120i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^8}+\frac{60i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^8}+\frac{60i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^8}-\frac{15\left(4i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^8}+\cdots\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/15*(-120*I*log(tan(1/2*d*x + 1/2*c) - I)/a^8 + 60*I*log(abs(tan(1/2*d*x
+ 1/2*c) + 1))/a^8 + 60*I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^8 - 15*(4*I*
tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) - 4*I)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a^8) + (294*I*tan(1/2*d*x + 1/2*c)^6 + 1884*tan(1/2*d*x + 1/2*c)^5
- 4890*I*tan(1/2*d*x + 1/2*c)^4 - 6920*tan(1/2*d*x + 1/2*c)^3 + 4890*I*tan(
1/2*d*x + 1/2*c)^2 + 1884*tan(1/2*d*x + 1/2*c) - 294*I)/(a^8*(tan(1/2*d*x +
 1/2*c) - I)^6))/d
```

3.169 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=43

$$\frac{i(a-ia\tan(c+dx))^4}{8d\left(a^3+ia^3\tan(c+dx)\right)^4}$$

```
[Out] ((I/8)*(a - I*a*Tan[c + d*x])^4)/(d*(a^3 + I*a^3*Tan[c + d*x])^4)
```

Rubi [A] time = 0.0431671, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 37}

$$\frac{i(a-ia\tan(c+dx))^4}{8d\left(a^3+ia^3\tan(c+dx)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/8)*(a - I*a*Tan[c + d*x])^4)/(d*(a^3 + I*a^3*Tan[c + d*x])^4)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{(a-x)^3}{(a+x)^5}\,dx, x, ia\tan(c+dx)\right)}{a^7 d}\\ &= \frac{i(a-ia\tan(c+dx))^4}{8d\left(a^3+ia^3\tan(c+dx)\right)^4}\end{aligned}$$

Mathematica [A] time = 0.0594251, size = 32, normalized size = 0.74

$$\frac{i\sec^8(c+dx)}{8d(a+ia\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/8)*Sec[c + d*x]^8)/(d*(a + I*a*Tan[c + d*x])^8)
```

Maple [A] time = 0.115, size = 63, normalized size = 1.5

$$\frac{1}{da^8}\left(-\left(\tan\left(dx+c\right)-i\right)^{-1}-\frac{3\,i}{\left(\tan\left(dx+c\right)-i\right)^{2}}+4\,\left(\tan\left(dx+c\right)-i\right)^{-3}+\frac{2\,i}{\left(\tan\left(dx+c\right)-i\right)^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d/a^8*(-1/(tan(d*x+c)-I)-3*I/(tan(d*x+c)-I)^2+4/(tan(d*x+c)-I)^3+2*I/(tan
(d*x+c)-I)^4)
```

Maima [B] time = 1.14773, size = 217, normalized size = 5.05

$$-\frac{35\,\tan\left(dx+c\right)^{6}-105i\,\tan\left(dx+c\right)^{5}-140\,\tan\left(dx+c\right)^{4}+140i\,\tan\left(dx+c\right)^{3}+105\,\tan}{\left(35\,a^{8}\tan\left(dx+c\right)^{7}-245i\,a^{8}\tan\left(dx+c\right)^{6}-735\,a^{8}\tan\left(dx+c\right)^{5}+1225i\,a^{8}\tan\left(dx+c\right)^{4}+1225\,a^{8}\tan\left(dx+c\right)^{3}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -(35*tan(d*x + c)^6 - 105*I*tan(d*x + c)^5 - 140*tan(d*x + c)^4 + 140*I*tan
(d*x + c)^3 + 105*tan(d*x + c)^2 - 35*I*tan(d*x + c))/((35*a^8*tan(d*x + c)
^7 - 245*I*a^8*tan(d*x + c)^6 - 735*a^8*tan(d*x + c)^5 + 1225*I*a^8*tan(d*x
 + c)^4 + 1225*a^8*tan(d*x + c)^3 - 735*I*a^8*tan(d*x + c)^2 - 245*a^8*tan(
d*x + c) + 35*I*a^8)*d)
```

Fricas [A] time = 2.52265, size = 49, normalized size = 1.14

$$\frac{i\,e^{\left(-8i\,dx-8i\,c\right)}}{8\,a^{8}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/8*I*e^(-8*I*d*x - 8*I*c)/(a^8*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.19459, size = 95, normalized size = 2.21

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{8}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^7 - 7*tan(1/2*d*x + 1/2*c)^5 + 7*tan(1/2*d*x + 1/2
*c)^3 - tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^8)
```

3.170 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=81

$$\frac{i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{4i}{5a^3d(a+ia\tan(c+dx))^5}$$

```
[Out] ((4*I)/5)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (I/3)/(a^5*d*(a + I*a*Tan[c + d*x])^3) - I/(d*(a^2 + I*a^2*Tan[c + d*x])^4)
```

Rubi [A] time = 0.0556106, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{4i}{5a^3d(a+ia\tan(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((4*I)/5)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (I/3)/(a^5*d*(a + I*a*Tan[c + d*x])^3) - I/(d*(a^2 + I*a^2*Tan[c + d*x])^4)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^6}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(\frac{4a^2}{(a+x)^6}-\frac{4a}{(a+x)^5}+\frac{1}{(a+x)^4}\right)dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= \frac{4i}{5a^3d(a+ia\tan(c+dx))^5}+\frac{i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)^4}\end{aligned}$$

Mathematica [A] time = 0.153923, size = 56, normalized size = 0.69

$$\frac{i\sec^8(c+dx)(4i\sin(2(c+dx))+16\cos(2(c+dx))+15)}{240a^8d(\tan(c+dx)-i)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/240)*Sec[c + d*x]^8*(15 + 16*Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)])
)/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] time = 0.102, size = 49, normalized size = 0.6

$$\frac{1}{da^{8}}\left(\frac{-i}{\left(\tan\left(dx+c\right)-i\right)^{4}}+\frac{4}{5\left(\tan\left(dx+c\right)-i\right)^{5}}-\frac{1}{3\left(\tan\left(dx+c\right)-i\right)^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d/a^8*(-I/(tan(d*x+c)-I)^4+4/5/(tan(d*x+c)-I)^5-1/3/(tan(d*x+c)-I)^3)
```

Maxima [B] time = 1.14954, size = 192, normalized size = 2.37

$$-\frac{35\tan\left(dx+c\right)^{4}-35i\tan\left(dx+c\right)^{3}+21\tan\left(dx+c\right)^{2}-7i\tan\left(dx+\right.}{\left(105a^{8}\tan\left(dx+c\right)^{7}-735ia^{8}\tan\left(dx+c\right)^{6}-2205a^{8}\tan\left(dx+c\right)^{5}+3675ia^{8}\tan\left(dx+c\right)^{4}+3675a^{8}\tan\left(dx+c\right)^{3}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -(35*tan(d*x + c)^4 - 35*I*tan(d*x + c)^3 + 21*tan(d*x + c)^2 - 7*I*tan(d*x
 + c) + 14)/((105*a^8*tan(d*x + c)^7 - 735*I*a^8*tan(d*x + c)^6 - 2205*a^8*
tan(d*x + c)^5 + 3675*I*a^8*tan(d*x + c)^4 + 3675*a^8*tan(d*x + c)^3 - 2205
*I*a^8*tan(d*x + c)^2 - 735*a^8*tan(d*x + c) + 105*I*a^8)*d)
```

Fricas [A] time = 2.46611, size = 132, normalized size = 1.63

$$\frac{\left(10i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}+6i\right)e^{(-10i\,dx-10i\,c)}}{240\,a^{8}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(10*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 6*I)*e^(-10*I*
d*x - 10*I*c)/(a^8*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.18484, size = 185, normalized size = 2.28

$$-\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-30i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+170i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+282\tan\left(\frac{1}{2}dx+\right.\right.}{15a^{8}d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*d*x + 1/2*c)^9 - 30*I*tan(1/2*d*x + 1/2*c)^8 - 140*tan(1/
2*d*x + 1/2*c)^7 + 170*I*tan(1/2*d*x + 1/2*c)^6 + 282*tan(1/2*d*x + 1/2*c)^
5 - 170*I*tan(1/2*d*x + 1/2*c)^4 - 140*tan(1/2*d*x + 1/2*c)^3 + 30*I*tan(1/
2*d*x + 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) -
I)^10)
```

3.171 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=55

$$\frac{i}{3a^2d(a+ia\tan(c+dx))^6}-\frac{i}{5a^3d(a+ia\tan(c+dx))^5}$$

```
[Out] (I/3)/(a^2*d*(a + I*a*Tan[c + d*x])^6) - (I/5)/(a^3*d*(a + I*a*Tan[c + d*x]
)^5)
```

Rubi [A] time = 0.0479161, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i}{3a^2d(a+ia\tan(c+dx))^6}-\frac{i}{5a^3d(a+ia\tan(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (I/3)/(a^2*d*(a + I*a*Tan[c + d*x])^6) - (I/5)/(a^3*d*(a + I*a*Tan[c + d*x]
)^5)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{a-x}{(a+x)^7}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(\frac{2a}{(a+x)^7}-\frac{1}{(a+x)^6}\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= \frac{i}{3a^2d(a+ia\tan(c+dx))^6}-\frac{i}{5a^3d(a+ia\tan(c+dx))^5}\end{aligned}$$

Mathematica [A] time = 0.20986, size = 78, normalized size = 1.42

$$\frac{i\sec^8(c+dx)(16i\sin(2(c+dx))+10i\sin(4(c+dx))+64\cos(2(c+dx))+20\cos(4(c+dx))+45)}{960a^8d(\tan(c+dx)-i)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/960)*Sec[c + d*x]^8*(45 + 64*Cos[2*(c + d*x)] + 20*Cos[4*(c + d*x)] + (
16*I)*Sin[2*(c + d*x)] + (10*I)*Sin[4*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x
])^8)
```

Maple [A] time = 0.12, size = 36, normalized size = 0.7

$$\frac{1}{d a^8}\left(-\frac{1}{5\left(\tan\left(dx+c\right)-i\right)^5}-\frac{\frac{i}{3}}{\left(\tan\left(dx+c\right)-i\right)^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d/a^8*(-1/5/(tan(d*x+c)-I)^5-1/3*I/(tan(d*x+c)-I)^6)
```

Maxima [B] time = 1.1587, size = 165, normalized size = 3.

$$-\frac{7\left(3\tan\left(dx+c\right)^2-i\tan\left(dx+c\right)+2\right)}{\left(105\,a^8\tan\left(dx+c\right)^7-735i\,a^8\tan\left(dx+c\right)^6-2205\,a^8\tan\left(dx+c\right)^5+3675i\,a^8\tan\left(dx+c\right)^4+3675\,a^8\tan\left(dx+c\right)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -7*(3*tan(d*x + c)^2 - I*tan(d*x + c) + 2)/((105*a^8*tan(d*x + c)^7 - 735*I
*a^8*tan(d*x + c)^6 - 2205*a^8*tan(d*x + c)^5 + 3675*I*a^8*tan(d*x + c)^4 +
 3675*a^8*tan(d*x + c)^3 - 2205*I*a^8*tan(d*x + c)^2 - 735*a^8*tan(d*x + c)
 + 105*I*a^8)*d)
```

Fricas [A] time = 2.32729, size = 205, normalized size = 3.73

$$\frac{\left(15i\,e^{(8i\,dx+8i\,c)}+40i\,e^{(6i\,dx+6i\,c)}+45i\,e^{(4i\,dx+4i\,c)}+24i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-12i\,dx-12i\,c)}}{960\,a^8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/960*(15*I*e^(8*I*d*x + 8*I*c) + 40*I*e^(6*I*d*x + 6*I*c) + 45*I*e^(4*I*d*
x + 4*I*c) + 24*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-12*I*d*x - 12*I*c)/(a^8*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.17525, size = 220, normalized size = 4.

$$-\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-60i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-235\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+822\tan\left(\frac{1}{2}dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*d*x + 1/2*c)^11 - 60*I*tan(1/2*d*x + 1/2*c)^10 - 235*tan(
1/2*d*x + 1/2*c)^9 + 480*I*tan(1/2*d*x + 1/2*c)^8 + 822*tan(1/2*d*x + 1/2*c
)^7 - 904*I*tan(1/2*d*x + 1/2*c)^6 - 822*tan(1/2*d*x + 1/2*c)^5 + 480*I*tan
(1/2*d*x + 1/2*c)^4 + 235*tan(1/2*d*x + 1/2*c)^3 - 60*I*tan(1/2*d*x + 1/2*c
)^2 - 15*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^12)
```

3.172 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=27

$$\frac{i}{7ad(a+ia\tan(c+dx))^7}$$

```
[Out] (I/7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

Rubi [A] time = 0.0388354, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$\frac{i}{7ad(a+ia\tan(c+dx))^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (I/7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1}{(a+x)^8}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{i}{7ad(a+ia\tan(c+dx))^7}\end{aligned}$$

Mathematica [B] time = 0.228745, size = 100, normalized size = 3.7

$$\frac{i\sec^8(c+dx)(14i\sin(2(c+dx))+14i\sin(4(c+dx))+6i\sin(6(c+dx))+56\cos(2(c+dx))+28\cos(4(c+dx))+8\,\text{co}}{896a^8d(\tan(c+dx)-i)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/896)*Sec[c + d*x]^8*(35 + 56*Cos[2*(c + d*x)] + 28*Cos[4*(c + d*x)] + 8
*Cos[6*(c + d*x)] + (14*I)*Sin[2*(c + d*x)] + (14*I)*Sin[4*(c + d*x)] + (6*
```

```
I)*Sin[6*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] time = 0.054, size = 24, normalized size = 0.9

$$\frac{\frac{i}{7}}{ad\left(a+ia\tan\left(dx+c\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/7*I/a/d/(a+I*a*tan(d*x+c))^7
```

Maxima [A] time = 1.19162, size = 28, normalized size = 1.04

$$\frac{i}{7\left(i\,a\tan\left(dx+c\right)+a\right)^{7}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/7*I/((I*a*tan(d*x + c) + a)^7*a*d)
```

Fricas [B] time = 2.35175, size = 278, normalized size = 10.3

$$\frac{\left(7i\,e^{(12i\,dx+12i\,c)}+21i\,e^{(10i\,dx+10i\,c)}+35i\,e^{(8i\,dx+8i\,c)}+35i\,e^{(6i\,dx+6i\,c)}+21i\,e^{(4i\,dx+4i\,c)}+7i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-14i\,dx-14i\,c)}}{896\,a^{8}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/896*(7*I*e^(12*I*d*x + 12*I*c) + 21*I*e^(10*I*d*x + 10*I*c) + 35*I*e^(8*I
*d*x + 8*I*c) + 35*I*e^(6*I*d*x + 6*I*c) + 21*I*e^(4*I*d*x + 4*I*c) + 7*I*e
^(2*I*d*x + 2*I*c) + I)*e^(-14*I*d*x - 14*I*c)/(a^8*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

<u>Giac</u> [B] time = 1.24341, size = 255, normalized size = 9.44

$$-\frac{2\left(7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-42i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}-182\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+490i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+1001\tan\left(\frac{1}{2}d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/7*(7*tan(1/2*d*x + 1/2*c)^13 - 42*I*tan(1/2*d*x + 1/2*c)^12 - 182*tan(1/
2*d*x + 1/2*c)^11 + 490*I*tan(1/2*d*x + 1/2*c)^10 + 1001*tan(1/2*d*x + 1/2*
c)^9 - 1484*I*tan(1/2*d*x + 1/2*c)^8 - 1716*tan(1/2*d*x + 1/2*c)^7 + 1484*I
*tan(1/2*d*x + 1/2*c)^6 + 1001*tan(1/2*d*x + 1/2*c)^5 - 490*I*tan(1/2*d*x +
 1/2*c)^4 - 182*tan(1/2*d*x + 1/2*c)^3 + 42*I*tan(1/2*d*x + 1/2*c)^2 + 7*ta
n(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^14)
```

3.173 $\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=229

$$\frac{i}{256d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{i}{256d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{i}{192a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{i}{128d\left(a^2+ia^2\tan(c+d\right.}$$

```
[Out] x/(256*a^8) + (I/16)/(d*(a + I*a*Tan[c + d*x])^8) + (I/28)/(a*d*(a + I*a*Ta
n[c + d*x])^7) + (I/48)/(a^2*d*(a + I*a*Tan[c + d*x])^6) + (I/80)/(a^3*d*(a
 + I*a*Tan[c + d*x])^5) + (I/128)/(d*(a^2 + I*a^2*Tan[c + d*x])^4) + (I/192
)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (I/256)/(d*(a^4 + I*a^4*Tan[c + d*
x])^2) + (I/256)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.153116, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3479, 8}

$$\frac{i}{256d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{i}{256d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{i}{192a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{i}{128d\left(a^2+ia^2\tan(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(-8),x]
```


```
[Out] x/(256*a^8) + (I/16)/(d*(a + I*a*Tan[c + d*x])^8) + (I/28)/(a*d*(a + I*a*Ta
n[c + d*x])^7) + (I/48)/(a^2*d*(a + I*a*Tan[c + d*x])^6) + (I/80)/(a^3*d*(a
 + I*a*Tan[c + d*x])^5) + (I/128)/(d*(a^2 + I*a^2*Tan[c + d*x])^4) + (I/192
)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (I/256)/(d*(a^4 + I*a^4*Tan[c + d*
x])^2) + (I/256)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```


Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^7}\,dx}{2a}$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^6}\,dx}{4a^2}$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \ldots$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \ldots$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \ldots$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \ldots$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \ldots$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \ldots$$

$$= \frac{x}{256a^8} + \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+\ldots}$$

Mathematica [A] time = 0.483862, size = 148, normalized size = 0.65

$$\frac{\sec^8(c+dx)(-6272\sin(2(c+dx)) - 7840\sin(4(c+dx)) - 5760\sin(6(c+dx)) + 1680idx\sin(8(c+dx)) + 105\sin(8(c\ldots}{4300\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-8),x]
```

```
[Out] (Sec[c + d*x]^8*(14700*I + (25088*I)*Cos[2*(c + d*x)] + (15680*I)*Cos[4*(c
+ d*x)] + (7680*I)*Cos[6*(c + d*x)] + (105*I)*Cos[8*(c + d*x)] + 1680*d*x*C
os[8*(c + d*x)] - 6272*Sin[2*(c + d*x)] - 7840*Sin[4*(c + d*x)] - 5760*Sin[
6*(c + d*x)] + 105*Sin[8*(c + d*x)] + (1680*I)*d*x*Sin[8*(c + d*x)]))/(4300
80*a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] time = 0.029, size = 196, normalized size = 0.9

$$\frac{\frac{i}{16}}{da^8(\tan(dx+c)-i)^8} + \frac{\frac{i}{128}}{da^8(\tan(dx+c)-i)^4} - \frac{\frac{i}{512}\ln(\tan(dx+c)-i)}{da^8} - \frac{\frac{i}{48}}{da^8(\tan(dx+c)-i)^6} - \frac{\frac{i}{256}}{da^8(\tan(dx+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/16*I/d/a^8/(tan(d*x+c)-I)^8+1/128*I/d/a^8/(tan(d*x+c)-I)^4-1/512*I/d/a^8*
ln(tan(d*x+c)-I)-1/48*I/d/a^8/(tan(d*x+c)-I)^6-1/256*I/d/a^8/(tan(d*x+c)-I)
```

```
^2-1/28/d/a^8/(tan(d*x+c)-I)^7+1/80/d/a^8/(tan(d*x+c)-I)^5-1/192/d/a^8/(tan
(d*x+c)-I)^3+1/256/d/a^8/(tan(d*x+c)-I)+1/512*I/d/a^8*ln(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.40363, size = 396, normalized size = 1.73

$$\frac{\left(1680\, dxe^{(16i\, dx+16i\, c)} + 6720i\, e^{(14i\, dx+14i\, c)} + 11760i\, e^{(12i\, dx+12i\, c)} + 15680i\, e^{(10i\, dx+10i\, c)} + 14700i\, e^{(8i\, dx+8i\, c)} + 9408i\, e^{(6i\, dx+6i\, c)}}{430080\, a^8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/430080*(1680*d*x*e^(16*I*d*x + 16*I*c) + 6720*I*e^(14*I*d*x + 14*I*c) + 1
1760*I*e^(12*I*d*x + 12*I*c) + 15680*I*e^(10*I*d*x + 10*I*c) + 14700*I*e^(8
*I*d*x + 8*I*c) + 9408*I*e^(6*I*d*x + 6*I*c) + 3920*I*e^(4*I*d*x + 4*I*c) +
 960*I*e^(2*I*d*x + 2*I*c) + 105*I)*e^(-16*I*d*x - 16*I*c)/(a^8*d)
```

Sympy [A] time = 2.11228, size = 326, normalized size = 1.42

$$\begin{cases} \frac{(226981421219472998 40 i a^{56} d^7 e^{70ic} e^{-2idx} + 397217487134077747 20 i a^{56} d^7 e^{68ic} e^{-4idx} + 529623316178770329 60 i a^{56} d^7 e^{66ic} e^{-6idx} + 496521858917597184 00 i a^{56} d^7}{14} \\ x \left(\frac{\left(e^{16ic} + 8e^{14ic} + 28e^{12ic} + 56e^{10ic} + 70e^{8ic} + 56e^{6ic} + 28e^{4ic} + 8e^{2ic} + 1\right) e^{-16ic}}{256a^8} - \frac{1}{256a^8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((22698142121947299840*I*a**56*d**7*exp(70*I*c)*exp(-2*I*d*x) + 3
9721748713407774720*I*a**56*d**7*exp(68*I*c)*exp(-4*I*d*x) + 52962331617877
032960*I*a**56*d**7*exp(66*I*c)*exp(-6*I*d*x) + 49652185891759718400*I*a**5
6*d**7*exp(64*I*c)*exp(-8*I*d*x) + 31777398970726219776*I*a**56*d**7*exp(62
*I*c)*exp(-10*I*d*x) + 13240582904469258240*I*a**56*d**7*exp(60*I*c)*exp(-1
2*I*d*x) + 3242591731706757120*I*a**56*d**7*exp(58*I*c)*exp(-14*I*d*x) + 35
4658470655426560*I*a**56*d**7*exp(56*I*c)*exp(-16*I*d*x))*exp(-72*I*c)/(145
26810958046271897 60*a**64*d**8), Ne(145268109580462718976 0*a**64*d**8*exp(7
2*I*c), 0)), (x*((exp(16*I*c) + 8*exp(14*I*c) + 28*exp(12*I*c) + 56*exp(10*
I*c) + 70*exp(8*I*c) + 56*exp(6*I*c) + 28*exp(4*I*c) + 8*exp(2*I*c) + 1)*ex
p(-16*I*c)/(256*a**8) - 1/(256*a**8)), True)) + x/(256*a**8)
```

Giac [A] time = 1.10688, size = 178, normalized size = 0.78

$$-\frac{-\frac{840i\log(-i\tan(dx+c)+1)}{a^8}+\frac{840i\log(-i\tan(dx+c)-1)}{a^8}+\frac{-2283i\tan(dx+c)^8-19944\tan(dx+c)^7+77364i\tan(dx+c)^6+175448\tan(dx+c)^5-258370i\cdots}{a^8(\tan(dx+\cdots}}{430080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/430080*(-840*I*log(-I*tan(d*x + c) + 1)/a^8 + 840*I*log(-I*tan(d*x + c)
- 1)/a^8 + (-2283*I*tan(d*x + c)^8 - 19944*tan(d*x + c)^7 + 77364*I*tan(d*x
 + c)^6 + 175448*tan(d*x + c)^5 - 258370*I*tan(d*x + c)^4 - 261464*tan(d*x
+ c)^3 + 192052*I*tan(d*x + c)^2 + 114152*tan(d*x + c) - 67819*I)/(a^8*(tan
(d*x + c) - I)^8))/d
```

3.174 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=278

$$-\frac{i}{1024d\left(a^8-ia^8\tan(c+dx)\right)}+\frac{9i}{1024d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{i}{128d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{3i}{256d\left(a^2+ia^2\tan(c+d\right.}$$

```
[Out] (5*x)/(512*a^8) + ((I/36)*a)/(d*(a + I*a*Tan[c + d*x])^9) + (I/32)/(d*(a +
I*a*Tan[c + d*x])^8) + ((3*I)/112)/(a*d*(a + I*a*Tan[c + d*x])^7) + (I/48)/
(a^2*d*(a + I*a*Tan[c + d*x])^6) + (I/64)/(a^3*d*(a + I*a*Tan[c + d*x])^5)
+ ((7*I)/768)/(a^5*d*(a + I*a*Tan[c + d*x])^3) + ((3*I)/256)/(d*(a^2 + I*a^
2*Tan[c + d*x])^4) + (I/128)/(d*(a^4 + I*a^4*Tan[c + d*x])^2) - (I/1024)/(d
*(a^8 - I*a^8*Tan[c + d*x])) + ((9*I)/1024)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.143577, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$-\frac{i}{1024d\left(a^8-ia^8\tan(c+dx)\right)}+\frac{9i}{1024d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{i}{128d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{3i}{256d\left(a^2+ia^2\tan(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (5*x)/(512*a^8) + ((I/36)*a)/(d*(a + I*a*Tan[c + d*x])^9) + (I/32)/(d*(a +
I*a*Tan[c + d*x])^8) + ((3*I)/112)/(a*d*(a + I*a*Tan[c + d*x])^7) + (I/48)/
(a^2*d*(a + I*a*Tan[c + d*x])^6) + (I/64)/(a^3*d*(a + I*a*Tan[c + d*x])^5)
+ ((7*I)/768)/(a^5*d*(a + I*a*Tan[c + d*x])^3) + ((3*I)/256)/(d*(a^2 + I*a^
2*Tan[c + d*x])^4) + (I/128)/(d*(a^4 + I*a^4*Tan[c + d*x])^2) - (I/1024)/(d
*(a^8 - I*a^8*Tan[c + d*x])) + ((9*I)/1024)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{10}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{1}{1024a^{10}(a-x)^2}+\frac{1}{4a^2(a+x)^{10}}+\frac{1}{4a^3(a+x)^9}+\frac{3}{16a^4(a+x)^8}+\frac{1}{8a^5(a+x)^7}+\frac{5}{64a^6(a+x)^6}\right.\right.}{}$$

$$= \frac{ia}{36d(a+ia\tan(c+dx))^9}+\frac{i}{32d(a+ia\tan(c+dx))^8}+\frac{3i}{112ad(a+ia\tan(c+dx))^7}+$$

$$= \frac{5x}{512a^8}+\frac{ia}{36d(a+ia\tan(c+dx))^9}+\frac{i}{32d(a+ia\tan(c+dx))^8}+\frac{3i}{112ad(a+ia\tan(c+d}$$

Mathematica [A] time = 0.966275, size = 170, normalized size = 0.61

$$\frac{\sec^8(c+dx)(-7056\sin(2(c+dx))-10080\sin(4(c+dx))-9720\sin(6(c+dx))+5040idx\sin(8(c+dx))+315\sin(8(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^8*(15876*I + (28224*I)*Cos[2*(c + d*x)] + (20160*I)*Cos[4*(c
+ d*x)] + (12960*I)*Cos[6*(c + d*x)] + (315*I)*Cos[8*(c + d*x)] + 5040*d*x*
Cos[8*(c + d*x)] - (224*I)*Cos[10*(c + d*x)] - 7056*Sin[2*(c + d*x)] - 1008
0*Sin[4*(c + d*x)] - 9720*Sin[6*(c + d*x)] + 315*Sin[8*(c + d*x)] + (5040*I
)*d*x*Sin[8*(c + d*x)] + 280*Sin[10*(c + d*x)]))/(516096*a^8*d*(-I + Tan[c
+ d*x])^8)
```

Maple [A] time = 0.119, size = 234, normalized size = 0.8

$$\frac{-\frac{5i}{1024}\ln(\tan(dx+c)-i)}{da^8}+\frac{\frac{3i}{256}}{da^8(\tan(dx+c)-i)^4}+\frac{\frac{i}{32}}{da^8(\tan(dx+c)-i)^8}-\frac{\frac{i}{48}}{da^8(\tan(dx+c)-i)^6}-\frac{\frac{i}{128}}{da^8(\tan(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] -5/1024*I/d/a^8*ln(tan(d*x+c)-I)+3/256*I/d/a^8/(tan(d*x+c)-I)^4+1/32*I/d/a^
8/(tan(d*x+c)-I)^8-1/48*I/d/a^8/(tan(d*x+c)-I)^6-1/128*I/d/a^8/(tan(d*x+c)-
I)^2+1/36/d/a^8/(tan(d*x+c)-I)^9-3/112/d/a^8/(tan(d*x+c)-I)^7+1/64/d/a^8/(t
an(d*x+c)-I)^5-7/768/d/a^8/(tan(d*x+c)-I)^3+9/1024/d/a^8/(tan(d*x+c)-I)+5/1
024*I/d/a^8*ln(tan(d*x+c)+I)+1/1024/d/a^8/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.44525, size = 479, normalized size = 1.72

$$\frac{\left(5040\,dxe^{(18i\,dx+18i\,c)}-252i\,e^{(20i\,dx+20i\,c)}+11340i\,e^{(16i\,dx+16i\,c)}+15120i\,e^{(14i\,dx+14i\,c)}+17640i\,e^{(12i\,dx+12i\,c)}+15876i\,e^{(10i\,dx+1}\right.}{516096\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/516096*(5040*d*x*e^(18*I*d*x + 18*I*c) - 252*I*e^(20*I*d*x + 20*I*c) + 11
340*I*e^(16*I*d*x + 16*I*c) + 15120*I*e^(14*I*d*x + 14*I*c) + 17640*I*e^(12
*I*d*x + 12*I*c) + 15876*I*e^(10*I*d*x + 10*I*c) + 10584*I*e^(8*I*d*x + 8*I
*c) + 5040*I*e^(6*I*d*x + 6*I*c) + 1620*I*e^(4*I*d*x + 4*I*c) + 315*I*e^(2*
I*d*x + 2*I*c) + 28*I)*e^(-18*I*d*x - 18*I*c)/(a^8*d)
```

Sympy [A] time = 2.735, size = 396, normalized size = 1.42

$$\begin{cases} \frac{\left(-2495687119199326634196634435584ia^{72}d^9e^{92ic}e^{2idx}+112305920363969698538848549601280ia^{72}d^9e^{88ic}e^{-2idx}+149741227151959598051798066135040ia^{72}\right.}{} \\ x\left(\frac{\left(e^{20ic}+10e^{18ic}+45e^{16ic}+120e^{14ic}+210e^{12ic}+252e^{10ic}+210e^{8ic}+120e^{6ic}+45e^{4ic}+10e^{2ic}+1\right)e^{-18ic}}{1024a^8}-\frac{5}{512a^8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-2495687119199326634196634435584*I*a**72*d**9*exp(92*I*c)*exp(2
*I*d*x) + 112305920363969698538848549601280*I*a**72*d**9*exp(88*I*c)*exp(-2
*I*d*x) + 149741227151959598051798066135040*I*a**72*d**9*exp(86*I*c)*exp(-4
*I*d*x) + 174698098343952864393764410490880*I*a**72*d**9*exp(84*I*c)*exp(-6
*I*d*x) + 157228288509557577954387969441792*I*a**72*d**9*exp(82*I*c)*exp(-8
*I*d*x) + 104818859006371718636258646294528*I*a**72*d**9*exp(80*I*c)*exp(-1
0*I*d*x) + 49913742383986532683932688711680*I*a**72*d**9*exp(78*I*c)*exp(-1
2*I*d*x) + 16043702909138528362692649943040*I*a**72*d**9*exp(76*I*c)*exp(-1
4*I*d*x) + 3119608898999158292745793044480*I*a**72*d**9*exp(74*I*c)*exp(-16
*I*d*x) + 277298568799925181577403826176*I*a**72*d**9*exp(72*I*c)*exp(-18*I
*d*x))*exp(-90*I*c)/(5111167220120220946834707324076032*a**80*d**10), Ne(51
11167220120220946834707324076032*a**80*d**10*exp(90*I*c), 0)), (x*((exp(20*
I*c) + 10*exp(18*I*c) + 45*exp(16*I*c) + 120*exp(14*I*c) + 210*exp(12*I*c)
+ 252*exp(10*I*c) + 210*exp(8*I*c) + 120*exp(6*I*c) + 45*exp(4*I*c) + 10*ex
p(2*I*c) + 1)*exp(-18*I*c)/(1024*a**8) - 5/(512*a**8)), True)) + 5*x/(512*a
**8)
```

Giac [A] time = 1.1867, size = 220, normalized size = 0.79

$$-\frac{-\frac{2520i\log(\tan(dx+c)+i)}{a^8}+\frac{2520i\log(\tan(dx+c)-i)}{a^8}+\frac{504\,(5i\tan(dx+c)-6)}{a^8(\tan(dx+c)+i)}+\frac{-7129i\tan(dx+c)^9-68697\tan(dx+c)^8+296964i\tan(dx+c)^7+758772\,\text{ta}}{}}{516096\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/516096*(-2520*I*log(tan(d*x + c) + I)/a^8 + 2520*I*log(tan(d*x + c) - I)
/a^8 + 504*(5*I*tan(d*x + c) - 6)/(a^8*(tan(d*x + c) + I)) + (-7129*I*tan(d
*x + c)^9 - 68697*tan(d*x + c)^8 + 296964*I*tan(d*x + c)^7 + 758772*tan(d*x
 + c)^6 - 1271214*I*tan(d*x + c)^5 - 1465758*tan(d*x + c)^4 + 1191540*I*tan
(d*x + c)^3 + 693828*tan(d*x + c)^2 - 295425*I*tan(d*x + c) - 89553)/(a^8*(
tan(d*x + c) - I)^9))/d
```

3.175 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=333

$$\frac{ia^2}{80d(a+ia\tan(c+dx))^{10}}-\frac{11i}{4096d\left(a^8-ia^8\tan(c+dx)\right)}+\frac{55i}{4096d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{i}{4096d\left(a^4-ia^4\tan(c+dx)\right)}$$

```
[Out] (33*x)/(2048*a^8) + ((I/80)*a^2)/(d*(a + I*a*Tan[c + d*x])^10) + ((I/48)*a)
/(d*(a + I*a*Tan[c + d*x])^9) + ((3*I)/128)/(d*(a + I*a*Tan[c + d*x])^8) +
((5*I)/224)/(a*d*(a + I*a*Tan[c + d*x])^7) + ((5*I)/256)/(a^2*d*(a + I*a*Ta
n[c + d*x])^6) + ((21*I)/1280)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + ((3*I)/25
6)/(a^5*d*(a + I*a*Tan[c + d*x])^3) + ((7*I)/512)/(d*(a^2 + I*a^2*Tan[c + d
*x])^4) - (I/4096)/(d*(a^4 - I*a^4*Tan[c + d*x])^2) + ((45*I)/4096)/(d*(a^4
 + I*a^4*Tan[c + d*x])^2) - ((11*I)/4096)/(d*(a^8 - I*a^8*Tan[c + d*x])) +
((55*I)/4096)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.180717, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3487, 44, 206}

$$\frac{ia^2}{80d(a+ia\tan(c+dx))^{10}}-\frac{11i}{4096d\left(a^8-ia^8\tan(c+dx)\right)}+\frac{55i}{4096d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{i}{4096d\left(a^4-ia^4\tan(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (33*x)/(2048*a^8) + ((I/80)*a^2)/(d*(a + I*a*Tan[c + d*x])^10) + ((I/48)*a)
/(d*(a + I*a*Tan[c + d*x])^9) + ((3*I)/128)/(d*(a + I*a*Tan[c + d*x])^8) +
((5*I)/224)/(a*d*(a + I*a*Tan[c + d*x])^7) + ((5*I)/256)/(a^2*d*(a + I*a*Ta
n[c + d*x])^6) + ((21*I)/1280)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + ((3*I)/25
6)/(a^5*d*(a + I*a*Tan[c + d*x])^3) + ((7*I)/512)/(d*(a^2 + I*a^2*Tan[c + d
*x])^4) - (I/4096)/(d*(a^4 - I*a^4*Tan[c + d*x])^2) + ((45*I)/4096)/(d*(a^4
 + I*a^4*Tan[c + d*x])^2) - ((11*I)/4096)/(d*(a^8 - I*a^8*Tan[c + d*x])) +
((55*I)/4096)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{\left(ia^5\right)\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{11}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{\left(ia^5\right)\text{Subst}\left(\int \left(\frac{1}{2048a^{11}(a-x)^3} + \frac{11}{4096a^{12}(a-x)^2} + \frac{1}{8a^3(a+x)^{11}} + \frac{3}{16a^4(a+x)^{10}} + \frac{3}{16a^5(a+x)^9} + \frac{5}{32a^6(\cdots}\right.\right.}{}$$

$$= \frac{ia^2}{80d(a+ia\tan(c+dx))^{10}} + \frac{ia}{48d(a+ia\tan(c+dx))^9} + \frac{3i}{128d(a+ia\tan(c+dx))^8} + \frac{\cdots}{22\cdots}$$

$$= \frac{33x}{2048a^8} + \frac{ia^2}{80d(a+ia\tan(c+dx))^{10}} + \frac{ia}{48d(a+ia\tan(c+dx))^9} + \frac{3i}{128d(a+ia\tan(c+\cdots}$$

Mathematica [A] time = 1.49975, size = 192, normalized size = 0.58

$$\frac{\sec^8(c+dx)(-44352\sin(2(c+dx)) - 69300\sin(4(c+dx)) - 79200\sin(6(c+dx)) + 55440idx\sin(8(c+dx)) + 3465\,\text{s}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^8*(97020*I + (177408*I)*Cos[2*(c + d*x)] + (138600*I)*Cos[4*(
c + d*x)] + (105600*I)*Cos[6*(c + d*x)] + (3465*I)*Cos[8*(c + d*x)] + 55440
*d*x*Cos[8*(c + d*x)] - (4480*I)*Cos[10*(c + d*x)] - (168*I)*Cos[12*(c + d*
x)] - 44352*Sin[2*(c + d*x)] - 69300*Sin[4*(c + d*x)] - 79200*Sin[6*(c + d*
x)] + 3465*Sin[8*(c + d*x)] + (55440*I)*d*x*Sin[8*(c + d*x)] + 5600*Sin[10*
(c + d*x)] + 252*Sin[12*(c + d*x)]))/(3440640*a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] time = 0.122, size = 274, normalized size = 0.8

$$-\frac{\frac{33i}{4096}\ln(\tan(dx+c)-i)}{da^8} + \frac{\frac{7i}{512}}{da^8(\tan(dx+c)-i)^4} + \frac{\frac{3i}{128}}{da^8(\tan(dx+c)-i)^8} - \frac{\frac{i}{80}}{da^8(\tan(dx+c)-i)^{10}} - \frac{\frac{5i}{256}}{da^8(\tan(dx\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] -33/4096*I/d/a^8*ln(tan(d*x+c)-I)+7/512*I/d/a^8/(tan(d*x+c)-I)^4+3/128*I/d/
a^8/(tan(d*x+c)-I)^8-1/80*I/d/a^8/(tan(d*x+c)-I)^10-5/256*I/d/a^8/(tan(d*x+
c)-I)^6-45/4096*I/d/a^8/(tan(d*x+c)-I)^2+1/48/d/a^8/(tan(d*x+c)-I)^9-5/224/
d/a^8/(tan(d*x+c)-I)^7+21/1280/d/a^8/(tan(d*x+c)-I)^5-3/256/d/a^8/(tan(d*x+
c)-I)^3+55/4096/d/a^8/(tan(d*x+c)-I)+1/4096*I/d/a^8/(tan(d*x+c)+I)^2+33/409
6*I/d/a^8*ln(tan(d*x+c)+I)+11/4096/d/a^8/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.55239, size = 571, normalized size = 1.71

$$\frac{\left(55440\, dxe^{(20i\, dx+20i\, c)} - 210i\, e^{(24i\, dx+24i\, c)} - 5040i\, e^{(22i\, dx+22i\, c)} + 92400i\, e^{(18i\, dx+18i\, c)} + 103950i\, e^{(16i\, dx+16i\, c)} + 110880i\, e^{(14i\, d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/3440640*(55440*d*x*e^(20*I*d*x + 20*I*c) - 210*I*e^(24*I*d*x + 24*I*c) -
5040*I*e^(22*I*d*x + 22*I*c) + 92400*I*e^(18*I*d*x + 18*I*c) + 103950*I*e^(
16*I*d*x + 16*I*c) + 110880*I*e^(14*I*d*x + 14*I*c) + 97020*I*e^(12*I*d*x +
 12*I*c) + 66528*I*e^(10*I*d*x + 10*I*c) + 34650*I*e^(8*I*d*x + 8*I*c) + 13
200*I*e^(6*I*d*x + 6*I*c) + 3465*I*e^(4*I*d*x + 4*I*c) + 560*I*e^(2*I*d*x +
 2*I*c) + 42*I)*e^(-20*I*d*x - 20*I*c)/(a^8*d)
```

Sympy [A] time = 2.0115, size = 464, normalized size = 1.39

$$\begin{cases} \frac{\left(-1143348752854353237236938680970741190492160 0ia^{88}d^{11}e^{114ic}e^{4idx} - 2744037006850447769368652834329778857181184 00ia^{88}d^{11}e^{112ic}e^{2idx} + 503073451\right.}{} \\ x\left(\frac{\left(e^{24ic}+12e^{22ic}+66e^{20ic}+220e^{18ic}+495e^{16ic}+792e^{14ic}+924e^{12ic}+792e^{10ic}+495e^{8ic}+220e^{6ic}+66e^{4ic}+12e^{2ic}+1\right)e^{-20ic}}{4096a^8} - \frac{33}{2048a^8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-11433487528543532372369386809707411904921600*I*a**88*d**11*exp
(114*I*c)*exp(4*I*d*x) - 27440370068504477693686528343297788571811840 0*I*a*
*88*d**11*exp(112*I*c)*exp(2*I*d*x) + 5030734512559154243842530196271261238
165504000*I*a**88*d**11*exp(108*I*c)*exp(-2*I*d*x) + 5659576326629048524322
846470805168892936192000*I*a**88*d**11*exp(106*I*c)*exp(-4*I*d*x) + 6036881
415070985092611036235525513485798604800*I*a**88*d**11*exp(104*I*c)*exp(-6*I
*d*x) + 5282271238187111956034656706084824300073779200*I*a**88*d**11*exp(10
2*I*c)*exp(-8*I*d*x) + 3622128849042591055566621741315308091479162880*I*a**
88*d**11*exp(100*I*c)*exp(-10*I*d*x) + 188652544220968284144094882360172296
4312064000*I*a**88*d**11*exp(98*I*c)*exp(-12*I*d*x) + 718676358937022034834
647170895894462595072000*I*a**88*d**11*exp(96*I*c)*exp(-14*I*d*x) + 1886525
44220968284144094882360172296431206400*I*a**88*d**11*exp(94*I*c)*exp(-16*I*
d*x) + 30489300076116086326318364825886431746457600*I*a**88*d**11*exp(92*I*
c)*exp(-18*I*d*x) + 2286697505708706474473877361941482380984320*I*a**88*d**
11*exp(90*I*c)*exp(-20*I*d*x))*exp(-110*I*c)/(187326259667657234388900033 49
0246236650235494400*a**96*d**12), Ne(1873262596676572343889000334902462366 5
0235494400*a**96*d**12*exp(110*I*c), 0)), (x*((exp(24*I*c) + 12*exp(22*I*c)
 + 66*exp(20*I*c) + 220*exp(18*I*c) + 495*exp(16*I*c) + 792*exp(14*I*c) + 9
24*exp(12*I*c) + 792*exp(10*I*c) + 495*exp(8*I*c) + 220*exp(6*I*c) + 66*exp
(4*I*c) + 12*exp(2*I*c) + 1)*exp(-20*I*c)/(4096*a**8) - 33/(2048*a**8)), Tr
ue)) + 33*x/(2048*a**8)
```

Giac [A] time = 1.17036, size = 254, normalized size = 0.76

$$-\frac{-\frac{27720i\log(-i\tan(dx+c)+1)}{a^8}+\frac{27720i\log(-i\tan(dx+c)-1)}{a^8}+\frac{420\left(99i\tan(dx+c)^2-220\tan(dx+c)-123i\right)}{a^8(\tan(dx+c)+i)^2}-\frac{81191i\tan(dx+c)^{10}+858110\tan(dx+\ldots}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/3440640*(-27720*I*log(-I*tan(d*x + c) + 1)/a^8 + 27720*I*log(-I*tan(d*x
+ c) - 1)/a^8 + 420*(99*I*tan(d*x + c)^2 - 220*tan(d*x + c) - 123*I)/(a^8*(
tan(d*x + c) + I)^2) - (81191*I*tan(d*x + c)^10 + 858110*tan(d*x + c)^9 - 4
107195*I*tan(d*x + c)^8 - 11748840*tan(d*x + c)^7 + 22318590*I*tan(d*x + c)
^6 + 29583540*tan(d*x + c)^5 - 27983550*I*tan(d*x + c)^4 - 19002600*tan(d*x
 + c)^3 + 9206235*I*tan(d*x + c)^2 + 3108990*tan(d*x + c) - 648327*I)/(a^8*
(tan(d*x + c) - I)^10))/d
```

3.176 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=205

$$\frac{1155\tanh^{-1}(\sin(c+dx))}{8a^8d}-\frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5}-\frac{66i\sec^7(c+dx)}{a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}-\frac{154i\sec^5(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{385\,\text{ta}}{}$$

```
[Out] (1155*ArcTanh[Sin[c + d*x]])/(8*a^8*d) + (1155*Sec[c + d*x]*Tan[c + d*x])/(
8*a^8*d) + (385*Sec[c + d*x]^3*Tan[c + d*x])/(4*a^8*d) + (((2*I)/3)*Sec[c +
 d*x]^11)/(a*d*(a + I*a*Tan[c + d*x])^7) - (((22*I)/3)*Sec[c + d*x]^9)/(a^3
*d*(a + I*a*Tan[c + d*x])^5) - ((66*I)*Sec[c + d*x]^7)/(a^2*d*(a^2 + I*a^2*
Tan[c + d*x])^3) - ((154*I)*Sec[c + d*x]^5)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.216992, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3500, 3768, 3770}

$$\frac{1155\tanh^{-1}(\sin(c+dx))}{8a^8d}-\frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5}-\frac{66i\sec^7(c+dx)}{a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}-\frac{154i\sec^5(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{385\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (1155*ArcTanh[Sin[c + d*x]])/(8*a^8*d) + (1155*Sec[c + d*x]*Tan[c + d*x])/(
8*a^8*d) + (385*Sec[c + d*x]^3*Tan[c + d*x])/(4*a^8*d) + (((2*I)/3)*Sec[c +
 d*x]^11)/(a*d*(a + I*a*Tan[c + d*x])^7) - (((22*I)/3)*Sec[c + d*x]^9)/(a^3
*d*(a + I*a*Tan[c + d*x])^5) - ((66*I)*Sec[c + d*x]^7)/(a^2*d*(a^2 + I*a^2*
Tan[c + d*x])^3) - ((154*I)*Sec[c + d*x]^5)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{11\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{3a^2}$$

$$= \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} + \frac{33\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx}{a^4}$$

$$= \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^7(c+dx)}{a^5d(a+ia\tan(c+dx))^3} + 231$$

$$= \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^7(c+dx)}{a^5d(a+ia\tan(c+dx))^3} - \frac{1}{d\left(a\right.}$$

$$= \frac{385\sec^3(c+dx)\tan(c+dx)}{4a^8d} + \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} -$$

$$= \frac{1155\sec(c+dx)\tan(c+dx)}{8a^8d} + \frac{385\sec^3(c+dx)\tan(c+dx)}{4a^8d} + \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}$$

$$= \frac{1155\tanh^{-1}(\sin(c+dx))}{8a^8d} + \frac{1155\sec(c+dx)\tan(c+dx)}{8a^8d} + \frac{385\sec^3(c+dx)\tan(c+d}{4a^8d}$$

<u>Mathematica</u> [B] time = 6.2131, size = 1704, normalized size = 8.31

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-1155*Cos[8*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8
*(Cos[d*x] + I*Sin[d*x])^8)/(8*d*(a + I*a*Tan[c + d*x])^8) + (1155*Cos[8*c]
*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(Cos[d*x] + I*
Sin[d*x])^8)/(8*d*(a + I*a*Tan[c + d*x])^8) + (Cos[3*d*x]*Sec[c + d*x]^8*((
(32*I)/3)*Cos[5*c] - (32*Sin[5*c])/3)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*
a*Tan[c + d*x])^8) + (Cos[d*x]*Sec[c + d*x]^8*((-160*I)*Cos[7*c] + 160*Sin[
7*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) - (((1155*I)/
8)*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Co
s[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (((1155*I)/8)*Log[Co
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Cos[d*x] +
I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c]*Sec[c + d*x]^8*(((-23
6*I)/3)*Cos[8*c] + (236*Sin[8*c])/3)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a
*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(-160*Cos[7*c] - (160*I)*Sin[7*c])*(Cos
[d*x] + I*Sin[d*x])^8*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x
]^8*((32*Cos[5*c])/3 + ((32*I)/3)*Sin[5*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[3
*d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(Cos[8*c]/16 + (I/16)
*Sin[8*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2])^4*(a + I*a*Tan[c + d*x])^8) - ((1/96 + I/96)*Sec[c + d*x]^8*((-407*
I)*Cos[(15*c)/2] + 343*Cos[(17*c)/2] + 407*Sin[(15*c)/2] + (343*I)*Sin[(17*
c)/2])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)
/2] - Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(-C
os[8*c]/16 - (I/16)*Sin[8*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2])^4*(a + I*a*Tan[c + d*x])^8) + ((1/96 + I/96)*Sec
[c + d*x]^8*(407*Cos[(15*c)/2] - (343*I)*Cos[(17*c)/2] + (407*I)*Sin[(15*c)
/2] + 343*Sin[(17*c)/2])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2] + Sin[c/2]
)*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^8) + (
236*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(Cos[8*c - (d*x)/2]/2 - Cos[8*
c + (d*x)/2]/2 + (I/2)*Sin[8*c - (d*x)/2] - (I/2)*Sin[8*c + (d*x)/2]))/(3*d
```

```
*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])*(a + I*a*T
an[c + d*x])^8) + (4*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(Cos[8*c - (d
*x)/2]/2 - Cos[8*c + (d*x)/2]/2 + (I/2)*Sin[8*c - (d*x)/2] - (I/2)*Sin[8*c
+ (d*x)/2]))/(3*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*
x)/2])^3*(a + I*a*Tan[c + d*x])^8) + (4*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*
x])^8*(-Cos[8*c - (d*x)/2]/2 + Cos[8*c + (d*x)/2]/2 - (I/2)*Sin[8*c - (d*x)
/2] + (I/2)*Sin[8*c + (d*x)/2]))/(3*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x
)/2] - Sin[c/2 + (d*x)/2])^3*(a + I*a*Tan[c + d*x])^8) + (236*Sec[c + d*x]^
8*(Cos[d*x] + I*Sin[d*x])^8*(-Cos[8*c - (d*x)/2]/2 + Cos[8*c + (d*x)/2]/2 -
 (I/2)*Sin[8*c - (d*x)/2] + (I/2)*Sin[8*c + (d*x)/2]))/(3*d*(Cos[c/2] + Sin
[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*(a + I*a*Tan[c + d*x])^8)
```

Maple [B] time = 0.146, size = 409, normalized size = 2.

$$\frac{121}{8\,d a^{8}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{76\,i}{d a^{8}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{1}{2\,d a^{8}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{\frac{8i}{3}}{d a^{8}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 121/8/d/a^8/(tan(1/2*d*x+1/2*c)+1)^2-76*I/d/a^8/(tan(1/2*d*x+1/2*c)+1)+1/2/
d/a^8/(tan(1/2*d*x+1/2*c)+1)^3+8/3*I/d/a^8/(tan(1/2*d*x+1/2*c)+1)^3-123/8/d
/a^8/(tan(1/2*d*x+1/2*c)+1)+128*I/d/a^8/(tan(1/2*d*x+1/2*c)-I)^2-1/4/d/a^8/
(tan(1/2*d*x+1/2*c)+1)^4+1155/8/d/a^8*ln(tan(1/2*d*x+1/2*c)+1)-4*I/d/a^8/(t
an(1/2*d*x+1/2*c)+1)^2-256/3/d/a^8/(tan(1/2*d*x+1/2*c)-I)^3-256/d/a^8/(tan(
1/2*d*x+1/2*c)-I)+1/2/d/a^8/(tan(1/2*d*x+1/2*c)-1)^3+76*I/d/a^8/(tan(1/2*d*
x+1/2*c)-1)-121/8/d/a^8/(tan(1/2*d*x+1/2*c)-1)^2-8/3*I/d/a^8/(tan(1/2*d*x+1
/2*c)-1)^3-123/8/d/a^8/(tan(1/2*d*x+1/2*c)-1)-4*I/d/a^8/(tan(1/2*d*x+1/2*c)
-1)^2+1/4/d/a^8/(tan(1/2*d*x+1/2*c)-1)^4-1155/8/d/a^8*ln(tan(1/2*d*x+1/2*c)
-1)
```

Maxima [B] time = 2.09246, size = 1075, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -((6930*cos(11*d*x + 11*c) + 27720*cos(9*d*x + 9*c) + 41580*cos(7*d*x + 7*c
) + 27720*cos(5*d*x + 5*c) + 6930*cos(3*d*x + 3*c) + 6930*I*sin(11*d*x + 11
*c) + 27720*I*sin(9*d*x + 9*c) + 41580*I*sin(7*d*x + 7*c) + 27720*I*sin(5*d
*x + 5*c) + 6930*I*sin(3*d*x + 3*c))*arctan2(cos(d*x + c), sin(d*x + c) + 1
) + (6930*cos(11*d*x + 11*c) + 27720*cos(9*d*x + 9*c) + 41580*cos(7*d*x + 7
*c) + 27720*cos(5*d*x + 5*c) + 6930*cos(3*d*x + 3*c) + 6930*I*sin(11*d*x +
11*c) + 27720*I*sin(9*d*x + 9*c) + 41580*I*sin(7*d*x + 7*c) + 27720*I*sin(5
*d*x + 5*c) + 6930*I*sin(3*d*x + 3*c))*arctan2(cos(d*x + c), -sin(d*x + c)
+ 1) - (-3465*I*cos(11*d*x + 11*c) - 13860*I*cos(9*d*x + 9*c) - 20790*I*cos
(7*d*x + 7*c) - 13860*I*cos(5*d*x + 5*c) - 3465*I*cos(3*d*x + 3*c) + 3465*s
in(11*d*x + 11*c) + 13860*sin(9*d*x + 9*c) + 20790*sin(7*d*x + 7*c) + 13860
*sin(5*d*x + 5*c) + 3465*sin(3*d*x + 3*c))*log(cos(d*x + c)^2 + sin(d*x + c
)^2 + 2*sin(d*x + c) + 1) - (3465*I*cos(11*d*x + 11*c) + 13860*I*cos(9*d*x
+ 9*c) + 20790*I*cos(7*d*x + 7*c) + 13860*I*cos(5*d*x + 5*c) + 3465*I*cos(3
```

```
*d*x + 3*c) - 3465*sin(11*d*x + 11*c) - 13860*sin(9*d*x + 9*c) - 20790*sin(
7*d*x + 7*c) - 13860*sin(5*d*x + 5*c) - 3465*sin(3*d*x + 3*c))*log(cos(d*x
+ c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 13860*cos(10*d*x + 10*c) +
50820*cos(8*d*x + 8*c) + 67452*cos(6*d*x + 6*c) + 36828*cos(4*d*x + 4*c) +
5632*cos(2*d*x + 2*c) + 13860*I*sin(10*d*x + 10*c) + 50820*I*sin(8*d*x + 8*
c) + 67452*I*sin(6*d*x + 6*c) + 36828*I*sin(4*d*x + 4*c) + 5632*I*sin(2*d*x
 + 2*c) - 512)/((-48*I*a^8*cos(11*d*x + 11*c) - 192*I*a^8*cos(9*d*x + 9*c)
- 288*I*a^8*cos(7*d*x + 7*c) - 192*I*a^8*cos(5*d*x + 5*c) - 48*I*a^8*cos(3*
d*x + 3*c) + 48*a^8*sin(11*d*x + 11*c) + 192*a^8*sin(9*d*x + 9*c) + 288*a^8
*sin(7*d*x + 7*c) + 192*a^8*sin(5*d*x + 5*c) + 48*a^8*sin(3*d*x + 3*c))*d)
```

Fricas [A] time = 2.95583, size = 826, normalized size = 4.03

$$\frac{3465\left(e^{(11idx+11ic)}+4e^{(9idx+9ic)}+6e^{(7idx+7ic)}+4e^{(5idx+5ic)}+e^{(3idx+3ic)}\right)\log\left(e^{(idx+ic)}+i\right)-3465\left(e^{(11idx+11ic)}+4e^{(9}\right.}{24\left(a^8de^{(11idx+11ic)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/24*(3465*(e^(11*I*d*x + 11*I*c) + 4*e^(9*I*d*x + 9*I*c) + 6*e^(7*I*d*x +
7*I*c) + 4*e^(5*I*d*x + 5*I*c) + e^(3*I*d*x + 3*I*c))*log(e^(I*d*x + I*c) +
 I) - 3465*(e^(11*I*d*x + 11*I*c) + 4*e^(9*I*d*x + 9*I*c) + 6*e^(7*I*d*x +
7*I*c) + 4*e^(5*I*d*x + 5*I*c) + e^(3*I*d*x + 3*I*c))*log(e^(I*d*x + I*c) -
 I) - 6930*I*e^(10*I*d*x + 10*I*c) - 25410*I*e^(8*I*d*x + 8*I*c) - 33726*I*
e^(6*I*d*x + 6*I*c) - 18414*I*e^(4*I*d*x + 4*I*c) - 2816*I*e^(2*I*d*x + 2*I
*c) + 256*I)/(a^8*d*e^(11*I*d*x + 11*I*c) + 4*a^8*d*e^(9*I*d*x + 9*I*c) + 6
*a^8*d*e^(7*I*d*x + 7*I*c) + 4*a^8*d*e^(5*I*d*x + 5*I*c) + a^8*d*e^(3*I*d*x
 + 3*I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.24793, size = 266, normalized size = 1.3

$$\frac{\frac{3465\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^8}-\frac{3465\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^8}-\frac{1024\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-7\right)}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^3}-\frac{2\left(369\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-1728i\tan\right.}{}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/24*(3465*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^8 - 3465*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^8 - 1024*(6*tan(1/2*d*x + 1/2*c)^2 - 15*I*tan(1/2*d*x +
1/2*c) - 7)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^3) - 2*(369*tan(1/2*d*x + 1/2*c
)^7 - 1728*I*tan(1/2*d*x + 1/2*c)^6 - 393*tan(1/2*d*x + 1/2*c)^5 + 5568*I*t
an(1/2*d*x + 1/2*c)^4 - 393*tan(1/2*d*x + 1/2*c)^3 - 5696*I*tan(1/2*d*x + 1
/2*c)^2 + 369*tan(1/2*d*x + 1/2*c) + 1856*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^
4*a^8))/d
```

3.177 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=183

$$-\frac{63\tanh^{-1}(\sin(c+dx))}{2a^8d}-\frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}+\frac{42i\sec^5(c+dx)}{5a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{42i\sec^3(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{63}{}$$

```
[Out] (-63*ArcTanh[Sin[c + d*x]])/(2*a^8*d) - (63*Sec[c + d*x]*Tan[c + d*x])/(2*a
^8*d) + (((2*I)/5)*Sec[c + d*x]^9)/(a*d*(a + I*a*Tan[c + d*x])^7) - (((6*I)
/5)*Sec[c + d*x]^7)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (((42*I)/5)*Sec[c +
d*x]^5)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + ((42*I)*Sec[c + d*x]^3)/(d*(
a^8 + I*a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.200439, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3500, 3768, 3770}

$$-\frac{63\tanh^{-1}(\sin(c+dx))}{2a^8d}-\frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}+\frac{42i\sec^5(c+dx)}{5a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{42i\sec^3(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{63}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-63*ArcTanh[Sin[c + d*x]])/(2*a^8*d) - (63*Sec[c + d*x]*Tan[c + d*x])/(2*a
^8*d) + (((2*I)/5)*Sec[c + d*x]^9)/(a*d*(a + I*a*Tan[c + d*x])^7) - (((6*I)
/5)*Sec[c + d*x]^7)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (((42*I)/5)*Sec[c +
d*x]^5)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + ((42*I)*Sec[c + d*x]^3)/(d*(
a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{9\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{5a^2}$$

$$= \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{21\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx}{5a^4}$$

$$= \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{42i\sec^5(c+dx)}{5a^5d(a+ia\tan(c+dx))^3} - \frac{21\int}{}$$

$$= \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{42i\sec^5(c+dx)}{5a^5d(a+ia\tan(c+dx))^3} + \frac{42}{d\left(a^8\right.}$$

$$= -\frac{63\sec(c+dx)\tan(c+dx)}{2a^8d} + \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{}{5a^5}$$

$$= -\frac{63\tanh^{-1}(\sin(c+dx))}{2a^8d} - \frac{63\sec(c+dx)\tan(c+dx)}{2a^8d} + \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{}{5a^3d(}$$

Mathematica [B] time = 6.18385, size = 1244, normalized size = 6.8

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (63*Cos[8*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(C
os[d*x] + I*Sin[d*x])^8)/(2*d*(a + I*a*Tan[c + d*x])^8) - (63*Cos[8*c]*Log[
Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d
*x])^8)/(2*d*(a + I*a*Tan[c + d*x])^8) + (Cos[5*d*x]*Sec[c + d*x]^8*(((8*I)
/5)*Cos[3*c] - (8*Sin[3*c])/5)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c
 + d*x])^8) + (Cos[3*d*x]*Sec[c + d*x]^8*((-8*I)*Cos[5*c] + 8*Sin[5*c])*(Co
s[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Cos[d*x]*Sec[c + d*
x]^8*((48*I)*Cos[7*c] - 48*Sin[7*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a
*Tan[c + d*x])^8) + (Sec[c]*Sec[c + d*x]^8*((8*I)*Cos[8*c] - 8*Sin[8*c])*(C
os[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (((63*I)/2)*Log[Cos
[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Cos[d*x] + I
*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) - (((63*I)/2)*Log[Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Cos[d*x] + I*Sin[d*x])^
8)/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(48*Cos[7*c] + (48*I)*Sin
[7*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (
Sec[c + d*x]^8*(-8*Cos[5*c] - (8*I)*Sin[5*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin
[3*d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*((8*Cos[3*c])/5 + (
(8*I)/5)*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[5*d*x])/(d*(a + I*a*Tan[c
+ d*x])^8) + (Sec[c + d*x]^8*(Cos[8*c]/4 + (I/4)*Sin[8*c])*(Cos[d*x] + I*Si
n[d*x])^8)/(d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c +
d*x])^8) + (Sec[c + d*x]^8*(-Cos[8*c]/4 - (I/4)*Sin[8*c])*(Cos[d*x] + I*Sin
[d*x])^8)/(d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d
*x])^8) + (8*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(Cos[8*c - (d*x)/2]/2
 - Cos[8*c + (d*x)/2]/2 + (I/2)*Sin[8*c - (d*x)/2] - (I/2)*Sin[8*c + (d*x)/
2]))/(d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*(a
+ I*a*Tan[c + d*x])^8) + (8*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(-Cos[
8*c - (d*x)/2]/2 + Cos[8*c + (d*x)/2]/2 - (I/2)*Sin[8*c - (d*x)/2] + (I/2)*
Sin[8*c + (d*x)/2]))/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2
 + (d*x)/2])*(a + I*a*Tan[c + d*x])^8)
```

Maple [A] time = 0.12, size = 282, normalized size = 1.5

$$\frac{1}{2\,d a^8}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{8\,i}{d a^8}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{1}{2\,d a^8}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{63}{2\,d a^8}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/2/d/a^8/(tan(1/2*d*x+1/2*c)+1)+8*I/d/a^8/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^8
/(tan(1/2*d*x+1/2*c)+1)^2-63/2/d/a^8*ln(tan(1/2*d*x+1/2*c)+1)-32*I/d/a^8/(t
an(1/2*d*x+1/2*c)-I)^2-128*I/d/a^8/(tan(1/2*d*x+1/2*c)-I)^4+256/5/d/a^8/(ta
n(1/2*d*x+1/2*c)-I)^5-64/d/a^8/(tan(1/2*d*x+1/2*c)-I)^3+64/d/a^8/(tan(1/2*d
*x+1/2*c)-I)+1/2/d/a^8/(tan(1/2*d*x+1/2*c)-1)-8*I/d/a^8/(tan(1/2*d*x+1/2*c)
-1)+1/2/d/a^8/(tan(1/2*d*x+1/2*c)-1)^2+63/2/d/a^8*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.90993, size = 730, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] ((630*cos(9*d*x + 9*c) + 1260*cos(7*d*x + 7*c) + 630*cos(5*d*x + 5*c) + 630
*I*sin(9*d*x + 9*c) + 1260*I*sin(7*d*x + 7*c) + 630*I*sin(5*d*x + 5*c))*arc
tan2(cos(d*x + c), sin(d*x + c) + 1) + (630*cos(9*d*x + 9*c) + 1260*cos(7*d
*x + 7*c) + 630*cos(5*d*x + 5*c) + 630*I*sin(9*d*x + 9*c) + 1260*I*sin(7*d*
x + 7*c) + 630*I*sin(5*d*x + 5*c))*arctan2(cos(d*x + c), -sin(d*x + c) + 1)
 - (-315*I*cos(9*d*x + 9*c) - 630*I*cos(7*d*x + 7*c) - 315*I*cos(5*d*x + 5*
c) + 315*sin(9*d*x + 9*c) + 630*sin(7*d*x + 7*c) + 315*sin(5*d*x + 5*c))*lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - (315*I*cos(9*d*x
+ 9*c) + 630*I*cos(7*d*x + 7*c) + 315*I*cos(5*d*x + 5*c) - 315*sin(9*d*x +
9*c) - 630*sin(7*d*x + 7*c) - 315*sin(5*d*x + 5*c))*log(cos(d*x + c)^2 + si
n(d*x + c)^2 - 2*sin(d*x + c) + 1) + 1260*cos(8*d*x + 8*c) + 2100*cos(6*d*x
 + 6*c) + 672*cos(4*d*x + 4*c) - 96*cos(2*d*x + 2*c) + 1260*I*sin(8*d*x + 8
*c) + 2100*I*sin(6*d*x + 6*c) + 672*I*sin(4*d*x + 4*c) - 96*I*sin(2*d*x + 2
*c) + 32)/((-20*I*a^8*cos(9*d*x + 9*c) - 40*I*a^8*cos(7*d*x + 7*c) - 20*I*a
^8*cos(5*d*x + 5*c) + 20*a^8*sin(9*d*x + 9*c) + 40*a^8*sin(7*d*x + 7*c) + 2
0*a^8*sin(5*d*x + 5*c))*d)
```

Fricas [A] time = 3.02538, size = 554, normalized size = 3.03

$$-\frac{315\left(e^{(9i\,dx+9i\,c)}+2\,e^{(7i\,dx+7i\,c)}+e^{(5i\,dx+5i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-315\left(e^{(9i\,dx+9i\,c)}+2\,e^{(7i\,dx+7i\,c)}+e^{(5i\,dx+5i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}\right.}{10\left(a^8de^{(9i\,dx+9i\,c)}+2\,a^8de^{(7i\,dx+7i\,c)}+a^8de^{(5i\,d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/10*(315*(e^(9*I*d*x + 9*I*c) + 2*e^(7*I*d*x + 7*I*c) + e^(5*I*d*x + 5*I*
c))*log(e^(I*d*x + I*c) + I) - 315*(e^(9*I*d*x + 9*I*c) + 2*e^(7*I*d*x + 7*
I*c) + e^(5*I*d*x + 5*I*c))*log(e^(I*d*x + I*c) - I) - 630*I*e^(8*I*d*x + 8
```

```
*I*c) - 1050*I*e^(6*I*d*x + 6*I*c) - 336*I*e^(4*I*d*x + 4*I*c) + 48*I*e^(2*
I*d*x + 2*I*c) - 16*I)/(a^8*d*e^(9*I*d*x + 9*I*c) + 2*a^8*d*e^(7*I*d*x + 7*
I*c) + a^8*d*e^(5*I*d*x + 5*I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.25661, size = 225, normalized size = 1.23

$$-\frac{\frac{315\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^8}-\frac{315\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^8}-\frac{10\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-16i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+16i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^8}-\frac{4\left(160\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4\cdots\right.}{\cdots}}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/10*(315*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^8 - 315*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^8 - 10*(tan(1/2*d*x + 1/2*c)^3 - 16*I*tan(1/2*d*x + 1/2*c
)^2 + tan(1/2*d*x + 1/2*c) + 16*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^8) - 4
*(160*tan(1/2*d*x + 1/2*c)^4 - 720*I*tan(1/2*d*x + 1/2*c)^3 - 1360*tan(1/2*
d*x + 1/2*c)^2 + 880*I*tan(1/2*d*x + 1/2*c) + 208)/(a^8*(tan(1/2*d*x + 1/2*
c) - I)^5))/d
```

3.178 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=156

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^8d}-\frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}+\frac{2i\sec^3(c+dx)}{3a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}-\frac{2i\sec(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{2i}{7ad(a+}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a^8*d) + (((2*I)/7)*Sec[c + d*x]^7)/(a*d*(a + I*a*Ta
n[c + d*x])^7) - (((2*I)/5)*Sec[c + d*x]^5)/(a^3*d*(a + I*a*Tan[c + d*x])^5
) + (((2*I)/3)*Sec[c + d*x]^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) - ((2*I
)*Sec[c + d*x])/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.216498, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3500, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^8d}-\frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}+\frac{2i\sec^3(c+dx)}{3a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}-\frac{2i\sec(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{2i}{7ad(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^8*d) + (((2*I)/7)*Sec[c + d*x]^7)/(a*d*(a + I*a*Ta
n[c + d*x])^7) - (((2*I)/5)*Sec[c + d*x]^5)/(a^3*d*(a + I*a*Tan[c + d*x])^5
) + (((2*I)/3)*Sec[c + d*x]^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) - ((2*I
)*Sec[c + d*x])/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{a^2}$$

$$= \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx}{a^4}$$

$$= \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{2i\sec^3(c+dx)}{3a^5d(a+ia\tan(c+dx))^3} - \int \frac{\text{s}}{(a+i}$$

$$= \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{2i\sec^3(c+dx)}{3a^5d(a+ia\tan(c+dx))^3} - \frac{2}{d\left(a^8\right.}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^8d} + \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{2i\sec}{3a^5d(a+ia}$$

Mathematica [A] time = 1.03443, size = 304, normalized size = 1.95

$$\frac{\sec^8(c+dx)\left(\cos\left(\frac{9}{2}(c+dx)\right)+i\sin\left(\frac{9}{2}(c+dx)\right)\right)\left(-70\sin\left(\frac{1}{2}(c+dx)\right)-42\sin\left(\frac{3}{2}(c+dx)\right)+210\sin\left(\frac{5}{2}(c+dx)\right)+30\,\text{si}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^8*((70*I)*Cos[(c + d*x)/2] - (42*I)*Cos[(3*(c + d*x))/2] - (2
10*I)*Cos[(5*(c + d*x))/2] + (30*I)*Cos[(7*(c + d*x))/2] - 105*Cos[(7*(c +
d*x))/2]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*Cos[(7*(c + d*x))/2
]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 70*Sin[(c + d*x)/2] - 42*Sin[(
3*(c + d*x))/2] + 210*Sin[(5*(c + d*x))/2] + 30*Sin[(7*(c + d*x))/2] - (105
*I)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] + (105*I)
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2])*(Cos[(9*(c
+ d*x))/2] + I*Sin[(9*(c + d*x))/2]))/(105*a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] time = 0.11, size = 176, normalized size = 1.1

$$\frac{1}{da^8}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{128\,i}{da^8}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{-6}+\frac{16\,i}{da^8}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{-2}-\frac{128\,i}{da^8}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{-4}-\frac{2!}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d/a^8*ln(tan(1/2*d*x+1/2*c)+1)+128*I/d/a^8/(tan(1/2*d*x+1/2*c)-I)^6+16*I/
d/a^8/(tan(1/2*d*x+1/2*c)-I)^2-128*I/d/a^8/(tan(1/2*d*x+1/2*c)-I)^4-256/7/d
/a^8/(tan(1/2*d*x+1/2*c)-I)^7+896/5/d/a^8/(tan(1/2*d*x+1/2*c)-I)^5-160/3/d/
a^8/(tan(1/2*d*x+1/2*c)-I)^3-1/d/a^8*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [A] time = 1.85278, size = 250, normalized size = 1.6

$$\frac{-210i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-210i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)+60i\cos\left(7\,dx+7\,c\right)-84i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/210*(-210*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 210*I*arctan2(cos(d
*x + c), -sin(d*x + c) + 1) + 60*I*cos(7*d*x + 7*c) - 84*I*cos(5*d*x + 5*c)
 + 140*I*cos(3*d*x + 3*c) - 420*I*cos(d*x + c) + 105*log(cos(d*x + c)^2 + s
in(d*x + c)^2 + 2*sin(d*x + c) + 1) - 105*log(cos(d*x + c)^2 + sin(d*x + c)
^2 - 2*sin(d*x + c) + 1) + 60*sin(7*d*x + 7*c) - 84*sin(5*d*x + 5*c) + 140*
sin(3*d*x + 3*c) - 420*sin(d*x + c))/(a^8*d)
```

Fricas [A] time = 2.65172, size = 306, normalized size = 1.96

$$\frac{\left(105\,e^{(7i\,dx+7i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-105\,e^{(7i\,dx+7i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}-42i\,e^{(2i\,dx+2i\,c)}+3}{105\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*e^(7*I*d*x + 7*I*c)*log(e^(I*d*x + I*c) + I) - 105*e^(7*I*d*x +
7*I*c)*log(e^(I*d*x + I*c) - I) - 210*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*d
*x + 4*I*c) - 42*I*e^(2*I*d*x + 2*I*c) + 30*I)*e^(-7*I*d*x - 7*I*c)/(a^8*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.23051, size = 169, normalized size = 1.08

$$\frac{\frac{105\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^8}-\frac{105\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^8}-\frac{2\left(-840i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1400\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3920i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2352\,\tan\left(\frac{1}{2}\,dx\right.}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/105*(105*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^8 - 105*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^8 - 2*(-840*I*tan(1/2*d*x + 1/2*c)^5 - 1400*tan(1/2*d*x +
 1/2*c)^4 + 3920*I*tan(1/2*d*x + 1/2*c)^3 + 2352*tan(1/2*d*x + 1/2*c)^2 - 1
064*I*tan(1/2*d*x + 1/2*c) - 152)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

3.179 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=68

$$\frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}+\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}$$

```
[Out] ((I/9)*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/63)*Sec[c + d*x]^
7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

Rubi [A] time = 0.0803466, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3502, 3488}

$$\frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}+\frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/9)*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/63)*Sec[c + d*x]^
7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}+\frac{\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{9a}\\ &= \frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}+\frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}\end{aligned}$$

Mathematica [A] time = 0.101362, size = 40, normalized size = 0.59

$$-\frac{(\tan(c+dx)-8i)\sec^7(c+dx)}{63a^8d(\tan(c+dx)-i)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] -(Sec[c + d*x]^7*(-8*I + Tan[c + d*x]))/(63*a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [B] time = 0.108, size = 156, normalized size = 2.3

$$2\,{\frac {1}{d{a}^{8}} \left(-{\frac {86}{3\, \left(\tan \left(1/2\,dx+c/2 \right) -i \right) ^{3}}}+136\, \left(\tan \left(1/2\,dx+c/2 \right) -i \right) ^{-5}+{\frac {7\,i}{ \left(\tan \left(1/2\,dx+c/2 \right) -i \right) ^{2}}}+{\frac {128}{9\, \left(\tan \left(1/2\,dx+c/2 \right) \right.}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(-86/3/(tan(1/2*d*x+1/2*c)-I)^3+136/(tan(1/2*d*x+1/2*c)-I)^5+7*I/(t
an(1/2*d*x+1/2*c)-I)^2+128/9/(tan(1/2*d*x+1/2*c)-I)^9+496/3*I/(tan(1/2*d*x+
1/2*c)-I)^6-928/7/(tan(1/2*d*x+1/2*c)-I)^7+1/(tan(1/2*d*x+1/2*c)-I)-64*I/(t
an(1/2*d*x+1/2*c)-I)^8-76*I/(tan(1/2*d*x+1/2*c)-I)^4)
```

Maxima [A] time = 1.21565, size = 72, normalized size = 1.06

$$\frac{7i \cos \left(9 \, d x + 9 \, c\right) + 9i \cos \left(7 \, d x + 7 \, c\right) + 7 \, \sin \left(9 \, d x + 9 \, c\right) + 9 \, \sin \left(7 \, d x + 7 \, c\right)}{126 \, a^{8} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/126*(7*I*cos(9*d*x + 9*c) + 9*I*cos(7*d*x + 7*c) + 7*sin(9*d*x + 9*c) + 9
*sin(7*d*x + 7*c))/(a^8*d)
```

Fricas [A] time = 2.44531, size = 92, normalized size = 1.35

$$\frac{{\left(9i \, e^{\left(2i \, d x + 2i \, c\right)} + 7i\right)} e^{\left(-9i \, d x - 9i \, c\right)}}{126 \, a^{8} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/126*(9*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-9*I*d*x - 9*I*c)/(a^8*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.22857, size = 169, normalized size = 2.49

$$\frac{2\left(63\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-63i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-483\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+315i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+693\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{63\,a^{8}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/63*(63*tan(1/2*d*x + 1/2*c)^8 - 63*I*tan(1/2*d*x + 1/2*c)^7 - 483*tan(1/2
*d*x + 1/2*c)^6 + 315*I*tan(1/2*d*x + 1/2*c)^5 + 693*tan(1/2*d*x + 1/2*c)^4
 - 189*I*tan(1/2*d*x + 1/2*c)^3 - 225*tan(1/2*d*x + 1/2*c)^2 + 9*I*tan(1/2*
d*x + 1/2*c) + 8)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^9)
```

3.180 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=138

$$\frac{2i\sec^5(c+dx)}{1155a^3d(a+ia\tan(c+dx))^5}+\frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6}+\frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

```
[Out] ((I/11)*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/33)*Sec[c + d*x]
^5)/(a*d*(a + I*a*Tan[c + d*x])^7) + (((2*I)/231)*Sec[c + d*x]^5)/(a^2*d*(a
 + I*a*Tan[c + d*x])^6) + (((2*I)/1155)*Sec[c + d*x]^5)/(a^3*d*(a + I*a*Tan
[c + d*x])^5)
```

Rubi [A] time = 0.175295, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3502, 3488}

$$\frac{2i\sec^5(c+dx)}{1155a^3d(a+ia\tan(c+dx))^5}+\frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6}+\frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7}+\frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/11)*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/33)*Sec[c + d*x]
^5)/(a*d*(a + I*a*Tan[c + d*x])^7) + (((2*I)/231)*Sec[c + d*x]^5)/(a^2*d*(a
 + I*a*Tan[c + d*x])^6) + (((2*I)/1155)*Sec[c + d*x]^5)/(a^3*d*(a + I*a*Tan
[c + d*x])^5)
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8} + \frac{3\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{11a}$$
$$= \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8} + \frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7} + \frac{2\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{33a^2}$$
$$= \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8} + \frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7} + \frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6} + \frac{2\int}{}$$
$$= \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8} + \frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7} + \frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6} + \frac{}{115}$$

Mathematica [A] time = 0.208569, size = 73, normalized size = 0.53

$$\frac{i\sec^8(c+dx)(55i\sin(c+dx)+63i\sin(3(c+dx))+440\cos(c+dx)+168\cos(3(c+dx)))}{4620a^8d(\tan(c+dx)-i)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/4620)*Sec[c + d*x]^8*(440*Cos[c + d*x] + 168*Cos[3*(c + d*x)] + (55*I)*
Sin[c + d*x] + (63*I)*Sin[3*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] time = 0.109, size = 189, normalized size = 1.4

$$2\,\frac{1}{da^8}\left((\tan(1/2\,dx+c/2)-i)^{-1}+\frac{512}{3\,(\tan(1/2\,dx+c/2)-i)^9}-\frac{88\,i}{(\tan(1/2\,dx+c/2)-i)^4}+\frac{932}{5\,(\tan(1/2\,dx+c/2)-i)^5}-\frac{}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(1/(tan(1/2*d*x+1/2*c)-I)+512/3/(tan(1/2*d*x+1/2*c)-I)^9-88*I/(tan(
1/2*d*x+1/2*c)-I)^4+932/5/(tan(1/2*d*x+1/2*c)-I)^5-128/11/(tan(1/2*d*x+1/2*
c)-I)^11-2376/7/(tan(1/2*d*x+1/2*c)-I)^7-288*I/(tan(1/2*d*x+1/2*c)-I)^8+7*I
/(tan(1/2*d*x+1/2*c)-I)^2+64*I/(tan(1/2*d*x+1/2*c)-I)^10+292*I/(tan(1/2*d*x
+1/2*c)-I)^6-30/(tan(1/2*d*x+1/2*c)-I)^3)
```

Maxima [A] time = 1.23661, size = 131, normalized size = 0.95

$$\frac{105i\,\cos(11\,dx+11\,c)+385i\,\cos(9\,dx+9\,c)+495i\,\cos(7\,dx+7\,c)+231i\,\cos(5\,dx+5\,c)+105\,\sin(11\,dx+11\,c)+38}{9240\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/9240*(105*I*cos(11*d*x + 11*c) + 385*I*cos(9*d*x + 9*c) + 495*I*cos(7*d*x
 + 7*c) + 231*I*cos(5*d*x + 5*c) + 105*sin(11*d*x + 11*c) + 385*sin(9*d*x +
 9*c) + 495*sin(7*d*x + 7*c) + 231*sin(5*d*x + 5*c))/(a^8*d)
```

Fricas [A] time = 2.4299, size = 177, normalized size = 1.28

$$\frac{\left(231i\,e^{(6i\,dx+6i\,c)} + 495i\,e^{(4i\,dx+4i\,c)} + 385i\,e^{(2i\,dx+2i\,c)} + 105i\right)e^{(-11i\,dx-11i\,c)}}{9240\,a^8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9240*(231*I*e^(6*I*d*x + 6*I*c) + 495*I*e^(4*I*d*x + 4*I*c) + 385*I*e^(2*
I*d*x + 2*I*c) + 105*I)*e^(-11*I*d*x - 11*I*c)/(a^8*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.25228, size = 204, normalized size = 1.48

$$\frac{2\left(1155\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10} - 3465i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9} - 13860\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8} + 23100i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7} + 37422\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/1155*(1155*tan(1/2*d*x + 1/2*c)^10 - 3465*I*tan(1/2*d*x + 1/2*c)^9 - 1386
0*tan(1/2*d*x + 1/2*c)^8 + 23100*I*tan(1/2*d*x + 1/2*c)^7 + 37422*tan(1/2*d
*x + 1/2*c)^6 - 32802*I*tan(1/2*d*x + 1/2*c)^5 - 27060*tan(1/2*d*x + 1/2*c)
^4 + 11220*I*tan(1/2*d*x + 1/2*c)^3 + 4895*tan(1/2*d*x + 1/2*c)^2 - 517*I*t
an(1/2*d*x + 1/2*c) - 152)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^11)
```

3.181 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=213

$$\frac{8i\sec^3(c+dx)}{9009a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{8i\sec^3(c+dx)}{3003d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{20i\sec^3(c+dx)}{3003a^3d(a+ia\tan(c+dx))^5}+\frac{20i\sec^3(c+}{1287a^2d(a+ia\tan(}$$

```
[Out] ((I/13)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^8) + (((5*I)/143)*Sec[c +
 d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^7) + (((20*I)/1287)*Sec[c + d*x]^3)/(a
^2*d*(a + I*a*Tan[c + d*x])^6) + (((20*I)/3003)*Sec[c + d*x]^3)/(a^3*d*(a +
 I*a*Tan[c + d*x])^5) + (((8*I)/3003)*Sec[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c
 + d*x])^4) + (((8*I)/9009)*Sec[c + d*x]^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x
])^3)
```

Rubi [A] time = 0.275823, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3502, 3488}

$$\frac{8i\sec^3(c+dx)}{9009a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{8i\sec^3(c+dx)}{3003d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{20i\sec^3(c+dx)}{3003a^3d(a+ia\tan(c+dx))^5}+\frac{20i\sec^3(c+}{1287a^2d(a+ia\tan(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```


```
[Out] ((I/13)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^8) + (((5*I)/143)*Sec[c +
 d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^7) + (((20*I)/1287)*Sec[c + d*x]^3)/(a
^2*d*(a + I*a*Tan[c + d*x])^6) + (((20*I)/3003)*Sec[c + d*x]^3)/(a^3*d*(a +
 I*a*Tan[c + d*x])^5) + (((8*I)/3003)*Sec[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c
 + d*x])^4) + (((8*I)/9009)*Sec[c + d*x]^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x
])^3)
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{13a}$$
$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{143a^2}$$
$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6}$$
$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6}$$
$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6}$$
$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6}$$

Mathematica [A] time = 0.249824, size = 95, normalized size = 0.45

$$\frac{i\sec^8(c+dx)(1430i\sin(c+dx)+2457i\sin(3(c+dx))+1155i\sin(5(c+dx))+11440\cos(c+dx)+6552\cos(3(c+d}{144144a^8d(\tan(c+dx)-i)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/144144)*Sec[c + d*x]^8*(11440*Cos[c + d*x] + 6552*Cos[3*(c + d*x)] + 18
48*Cos[5*(c + d*x)] + (1430*I)*Sin[c + d*x] + (2457*I)*Sin[3*(c + d*x)] + (
1155*I)*Sin[5*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] time = 0.125, size = 222, normalized size = 1.

$$2\frac{1}{da^8}\left(\frac{5840}{9\left(\tan(1/2\,dx+c/2)-i\right)^9} - \frac{4528}{7\left(\tan(1/2\,dx+c/2)-i\right)^7} + \frac{128}{13\left(\tan(1/2\,dx+c/2)-i\right)^{13}} + \frac{432\,i}{(\tan(1/2\,dx+c/2)-i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(5840/9/(tan(1/2*d*x+1/2*c)-I)^9-4528/7/(tan(1/2*d*x+1/2*c)-I)^7+12
8/13/(tan(1/2*d*x+1/2*c)-I)^13+432*I/(tan(1/2*d*x+1/2*c)-I)^10-2272/11/(tan
(1/2*d*x+1/2*c)-I)^11-736*I/(tan(1/2*d*x+1/2*c)-I)^8-94/3/(tan(1/2*d*x+1/2*
c)-I)^3-64*I/(tan(1/2*d*x+1/2*c)-I)^12+240/(tan(1/2*d*x+1/2*c)-I)^5+1/(tan(
1/2*d*x+1/2*c)-I)-100*I/(tan(1/2*d*x+1/2*c)-I)^4+1336/3*I/(tan(1/2*d*x+1/2*
c)-I)^6+7*I/(tan(1/2*d*x+1/2*c)-I)^2)
```

Maxima [A] time = 1.24318, size = 190, normalized size = 0.89

$$\frac{693i\cos(13\,dx+13\,c)+4095i\cos(11\,dx+11\,c)+10010i\cos(9\,dx+9\,c)+12870i\cos(7\,dx+7\,c)+9009i\cos(5\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/288288*(693*I*cos(13*d*x + 13*c) + 4095*I*cos(11*d*x + 11*c) + 10010*I*co
s(9*d*x + 9*c) + 12870*I*cos(7*d*x + 7*c) + 9009*I*cos(5*d*x + 5*c) + 3003*
I*cos(3*d*x + 3*c) + 693*sin(13*d*x + 13*c) + 4095*sin(11*d*x + 11*c) + 100
10*sin(9*d*x + 9*c) + 12870*sin(7*d*x + 7*c) + 9009*sin(5*d*x + 5*c) + 3003
*sin(3*d*x + 3*c))/(a^8*d)
```

Fricas [A] time = 2.31983, size = 267, normalized size = 1.25

$$\frac{\left(3003i\,e^{(10i\,dx+10i\,c)}+9009i\,e^{(8i\,dx+8i\,c)}+12870i\,e^{(6i\,dx+6i\,c)}+10010i\,e^{(4i\,dx+4i\,c)}+4095i\,e^{(2i\,dx+2i\,c)}+693i\right)e^{(-13i\,dx-13i\,c)}}{288288\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/288288*(3003*I*e^(10*I*d*x + 10*I*c) + 9009*I*e^(8*I*d*x + 8*I*c) + 12870
*I*e^(6*I*d*x + 6*I*c) + 10010*I*e^(4*I*d*x + 4*I*c) + 4095*I*e^(2*I*d*x +
2*I*c) + 693*I)*e^(-13*I*d*x - 13*I*c)/(a^8*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.19599, size = 239, normalized size = 1.12

$$\frac{2\left(9009\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-45045i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-183183\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+435435i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+8108\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/9009*(9009*tan(1/2*d*x + 1/2*c)^12 - 45045*I*tan(1/2*d*x + 1/2*c)^11 - 18
3183*tan(1/2*d*x + 1/2*c)^10 + 435435*I*tan(1/2*d*x + 1/2*c)^9 + 810810*tan
(1/2*d*x + 1/2*c)^8 - 1051050*I*tan(1/2*d*x + 1/2*c)^7 - 1076790*tan(1/2*d*
x + 1/2*c)^6 + 785070*I*tan(1/2*d*x + 1/2*c)^5 + 451165*tan(1/2*d*x + 1/2*c
)^4 - 171457*I*tan(1/2*d*x + 1/2*c)^3 - 51675*tan(1/2*d*x + 1/2*c)^2 + 7111
*I*tan(1/2*d*x + 1/2*c) + 1240)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^13)
```

3.182 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=269

$$\frac{16i\sec(c+dx)}{6435d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{16i\sec(c+dx)}{6435d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{8i\sec(c+dx)}{2145a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{8i\sec(c+}{1287d\left(a^2+ia^2\,\text{ta}\right.}$$

```
[Out] ((I/15)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (((7*I)/195)*Sec[c + d
*x])/(a*d*(a + I*a*Tan[c + d*x])^7) + (((14*I)/715)*Sec[c + d*x])/(a^2*d*(a
 + I*a*Tan[c + d*x])^6) + (((14*I)/1287)*Sec[c + d*x])/(a^3*d*(a + I*a*Tan[
c + d*x])^5) + (((8*I)/1287)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])^4)
 + (((8*I)/2145)*Sec[c + d*x])/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((16
*I)/6435)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x])^2) + (((16*I)/6435)*S
ec[c + d*x])/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.259754, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3502, 3488}

$$\frac{16i\sec(c+dx)}{6435d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{16i\sec(c+dx)}{6435d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{8i\sec(c+dx)}{2145a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{8i\sec(c+}{1287d\left(a^2+ia^2\,\text{ta}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```


```
[Out] ((I/15)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (((7*I)/195)*Sec[c + d
*x])/(a*d*(a + I*a*Tan[c + d*x])^7) + (((14*I)/715)*Sec[c + d*x])/(a^2*d*(a
 + I*a*Tan[c + d*x])^6) + (((14*I)/1287)*Sec[c + d*x])/(a^3*d*(a + I*a*Tan[
c + d*x])^5) + (((8*I)/1287)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])^4)
 + (((8*I)/2145)*Sec[c + d*x])/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((16
*I)/6435)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x])^2) + (((16*I)/6435)*S
ec[c + d*x])/(d*(a^8 + I*a^8*Tan[c + d*x]))
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{15a}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{65a^2}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + 14$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{}{12}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{}{12}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{}{12}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{}{12}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{}{12}$$

Mathematica [A] time = 0.428746, size = 117, normalized size = 0.43

$$\frac{i\sec^8(c+dx)(3575i\sin(c+dx)+7371i\sin(3(c+dx))+5775i\sin(5(c+dx))+3003i\sin(7(c+dx))+28600\cos(c+dx)}{411840a^8d(\tan(c+dx)-i)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/411840)*Sec[c + d*x]^8*(28600*Cos[c + d*x] + 19656*Cos[3*(c + d*x)] + 9
240*Cos[5*(c + d*x)] + 3432*Cos[7*(c + d*x)] + (3575*I)*Sin[c + d*x] + (737
1*I)*Sin[3*(c + d*x)] + (5775*I)*Sin[5*(c + d*x)] + (3003*I)*Sin[7*(c + d*x
)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] time = 0.059, size = 255, normalized size = 1.

$$2\frac{1}{da^8}\left(\frac{-\frac{1792\,i}{3}}{(\tan(1/2\,dx+c/2)-i)^{12}}+\frac{\frac{1876\,i}{3}}{(\tan(1/2\,dx+c/2)-i)^{6}}-\frac{1472\,i}{(\tan(1/2\,dx+c/2)-i)^{8}}+(\tan(1/2\,dx+c/2)-i)^{-1}+\frac{}{9\,(\mathrm{ta}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(-1792/3*I/(tan(1/2*d*x+1/2*c)-I)^12+1876/3*I/(tan(1/2*d*x+1/2*c)-I
)^6-1472*I/(tan(1/2*d*x+1/2*c)-I)^8+1/(tan(1/2*d*x+1/2*c)-I)+14896/9/(tan(1
/2*d*x+1/2*c)-I)^9-112*I/(tan(1/2*d*x+1/2*c)-I)^4+64*I/(tan(1/2*d*x+1/2*c)-
I)^14-11872/11/(tan(1/2*d*x+1/2*c)-I)^11+7*I/(tan(1/2*d*x+1/2*c)-I)^2+7504/
5*I/(tan(1/2*d*x+1/2*c)-I)^10-1064/(tan(1/2*d*x+1/2*c)-I)^7-98/3/(tan(1/2*d
*x+1/2*c)-I)^3+3136/13/(tan(1/2*d*x+1/2*c)-I)^13+1484/5/(tan(1/2*d*x+1/2*c)
```

```
-I)^5-128/15/(tan(1/2*d*x+1/2*c)-I)^15)
```

Maxima [A] time = 1.25304, size = 242, normalized size = 0.9

$$\frac{429i\cos(15\,dx+15\,c)+3465i\cos(13\,dx+13\,c)+12285i\cos(11\,dx+11\,c)+25025i\cos(9\,dx+9\,c)+32175i\cos(\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/823680*(429*I*cos(15*d*x + 15*c) + 3465*I*cos(13*d*x + 13*c) + 12285*I*co
s(11*d*x + 11*c) + 25025*I*cos(9*d*x + 9*c) + 32175*I*cos(7*d*x + 7*c) + 27
027*I*cos(5*d*x + 5*c) + 15015*I*cos(3*d*x + 3*c) + 6435*I*cos(d*x + c) + 4
29*sin(15*d*x + 15*c) + 3465*sin(13*d*x + 13*c) + 12285*sin(11*d*x + 11*c)
+ 25025*sin(9*d*x + 9*c) + 32175*sin(7*d*x + 7*c) + 27027*sin(5*d*x + 5*c)
+ 15015*sin(3*d*x + 3*c) + 6435*sin(d*x + c))/(a^8*d)
```

Fricas [A] time = 2.53364, size = 355, normalized size = 1.32

$$\frac{\left(6435i\,e^{(14i\,dx+14i\,c)}+15015i\,e^{(12i\,dx+12i\,c)}+27027i\,e^{(10i\,dx+10i\,c)}+32175i\,e^{(8i\,dx+8i\,c)}+25025i\,e^{(6i\,dx+6i\,c)}+12285i\,e^{(4i\,dx+4\ldots}{823680\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/823680*(6435*I*e^(14*I*d*x + 14*I*c) + 15015*I*e^(12*I*d*x + 12*I*c) + 27
027*I*e^(10*I*d*x + 10*I*c) + 32175*I*e^(8*I*d*x + 8*I*c) + 25025*I*e^(6*I*
d*x + 6*I*c) + 12285*I*e^(4*I*d*x + 4*I*c) + 3465*I*e^(2*I*d*x + 2*I*c) + 4
29*I)*e^(-15*I*d*x - 15*I*c)/(a^8*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.16317, size = 274, normalized size = 1.02

$$\frac{2\left(6435\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}-45045i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-210210\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+630630i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/6435*(6435*tan(1/2*d*x + 1/2*c)^14 - 45045*I*tan(1/2*d*x + 1/2*c)^13 - 21
0210*tan(1/2*d*x + 1/2*c)^12 + 630630*I*tan(1/2*d*x + 1/2*c)^11 + 1414413*t
an(1/2*d*x + 1/2*c)^10 - 2357355*I*tan(1/2*d*x + 1/2*c)^9 - 3063060*tan(1/2
*d*x + 1/2*c)^8 + 3063060*I*tan(1/2*d*x + 1/2*c)^7 + 2407405*tan(1/2*d*x +
1/2*c)^6 - 1444443*I*tan(1/2*d*x + 1/2*c)^5 - 668850*tan(1/2*d*x + 1/2*c)^4
 + 222950*I*tan(1/2*d*x + 1/2*c)^3 + 54915*tan(1/2*d*x + 1/2*c)^2 - 7845*I*
tan(1/2*d*x + 1/2*c) - 952)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^15)
```

3.183 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=271

$$-\frac{64\sin^3(c+dx)}{12155a^8d}+\frac{192\sin(c+dx)}{12155a^8d}+\frac{128i\cos^3(c+dx)}{12155d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{16i\cos(c+dx)}{2431a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{112\ldots}{12155d\left(a^2\ldots\right.}$$

```
[Out] (192*Sin[c + d*x])/(12155*a^8*d) - (64*Sin[c + d*x]^3)/(12155*a^8*d) + ((I/
17)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (((3*I)/85)*Cos[c + d*x])/
(a*d*(a + I*a*Tan[c + d*x])^7) + (((24*I)/1105)*Cos[c + d*x])/(a^2*d*(a + I
*a*Tan[c + d*x])^6) + (((168*I)/12155)*Cos[c + d*x])/(a^3*d*(a + I*a*Tan[c
+ d*x])^5) + (((112*I)/12155)*Cos[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])^4
) + (((16*I)/2431)*Cos[c + d*x])/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((
128*I)/12155)*Cos[c + d*x]^3)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.312019, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3502, 3500, 2633}

$$-\frac{64\sin^3(c+dx)}{12155a^8d}+\frac{192\sin(c+dx)}{12155a^8d}+\frac{128i\cos^3(c+dx)}{12155d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{16i\cos(c+dx)}{2431a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{112\ldots}{12155d\left(a^2\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (192*Sin[c + d*x])/(12155*a^8*d) - (64*Sin[c + d*x]^3)/(12155*a^8*d) + ((I/
17)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (((3*I)/85)*Cos[c + d*x])/
(a*d*(a + I*a*Tan[c + d*x])^7) + (((24*I)/1105)*Cos[c + d*x])/(a^2*d*(a + I
*a*Tan[c + d*x])^6) + (((168*I)/12155)*Cos[c + d*x])/(a^3*d*(a + I*a*Tan[c
+ d*x])^5) + (((112*I)/12155)*Cos[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])^4
) + (((16*I)/2431)*Cos[c + d*x])/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((
128*I)/12155)*Cos[c + d*x]^3)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{9\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{17a}$$

$$= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{85a^2}$$

$$= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{16}{}$$

$$= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{}{12}$$

$$= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{}{12}$$

$$= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{}{12}$$

$$= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{}{12}$$

$$= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{}{12}$$

$$= \frac{192\sin(c+dx)}{12155a^8d} - \frac{64\sin^3(c+dx)}{12155a^8d} + \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7}$$

Mathematica [A] time = 0.900893, size = 139, normalized size = 0.51

$$-\frac{i\sec^8(c+dx)(-24310i\sin(c+dx) - 55692i\sin(3(c+dx)) - 56100i\sin(5(c+dx)) - 51051i\sin(7(c+dx)) + 6435i\sin(9}{3111680a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-I/3111680)*Sec[c + d*x]^8*(-194480*Cos[c + d*x] - 148512*Cos[3*(c + d*x)
] - 89760*Cos[5*(c + d*x)] - 58344*Cos[7*(c + d*x)] + 5720*Cos[9*(c + d*x)]
 - (24310*I)*Sin[c + d*x] - (55692*I)*Sin[3*(c + d*x)] - (56100*I)*Sin[5*(c
 + d*x)] - (51051*I)*Sin[7*(c + d*x)] + (6435*I)*Sin[9*(c + d*x)]))/(a^8*d*
(-I + Tan[c + d*x])^8)
```

Maple [A] time = 0.118, size = 306, normalized size = 1.1

$$2\frac{1}{da^8}\left(\frac{\frac{19109i}{5}}{(\tan(1/2dx+c/2)-i)^{10}} + \frac{784i}{(\tan(1/2dx+c/2)-i)^{14}} + \frac{\frac{1793i}{256}}{(\tan(1/2dx+c/2)-i)^{2}} - \frac{2692i}{(\tan(1/2dx+c/2)-i)^{12}} - \frac{}{(\tan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(19109/5*I/(tan(1/2*d*x+1/2*c)-I)^10+784*I/(tan(1/2*d*x+1/2*c)-I)^1
4+1793/256*I/(tan(1/2*d*x+1/2*c)-I)^2-2692*I/(tan(1/2*d*x+1/2*c)-I)^12-7937
/64*I/(tan(1/2*d*x+1/2*c)-I)^4-64*I/(tan(1/2*d*x+1/2*c)-I)^16-10241/4*I/(ta
n(1/2*d*x+1/2*c)-I)^8+13313/16*I/(tan(1/2*d*x+1/2*c)-I)^6+128/17/(tan(1/2*d
*x+1/2*c)-I)^17-1376/5/(tan(1/2*d*x+1/2*c)-I)^15+21400/13/(tan(1/2*d*x+1/2*
c)-I)^13-38954/11/(tan(1/2*d*x+1/2*c)-I)^11+6847/2/(tan(1/2*d*x+1/2*c)-I)^9
-12799/8/(tan(1/2*d*x+1/2*c)-I)^7+57083/160/(tan(1/2*d*x+1/2*c)-I)^5-4351/1
28/(tan(1/2*d*x+1/2*c)-I)^3+511/512/(tan(1/2*d*x+1/2*c)-I)+1/512/(tan(1/2*d
*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.66626, size = 452, normalized size = 1.67

$$\frac{\left(-12155i\,e^{(18i\,dx+18i\,c)} + 109395i\,e^{(16i\,dx+16i\,c)} + 145860i\,e^{(14i\,dx+14i\,c)} + 204204i\,e^{(12i\,dx+12i\,c)} + 218790i\,e^{(10i\,dx+10i\,c)} + 170\right.}{6223360\,a^8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/6223360*(-12155*I*e^(18*I*d*x + 18*I*c) + 109395*I*e^(16*I*d*x + 16*I*c)
+ 145860*I*e^(14*I*d*x + 14*I*c) + 204204*I*e^(12*I*d*x + 12*I*c) + 218790*
I*e^(10*I*d*x + 10*I*c) + 170170*I*e^(8*I*d*x + 8*I*c) + 92820*I*e^(6*I*d*x
 + 6*I*c) + 33660*I*e^(4*I*d*x + 4*I*c) + 7293*I*e^(2*I*d*x + 2*I*c) + 715*
I)*e^(-17*I*d*x - 17*I*c)/(a^8*d)
```

Sympy [A] time = 3.49903, size = 369, normalized size = 1.36

$$\begin{cases} \frac{\left(-14350091149820134393118720 0ia^{72}d^9e^{82ic}e^{idx}+129150820348381209538068480 0ia^{72}d^9e^{80ic}e^{-idx}+172201093797841612717424640 0ia^{72}d^9e^{78ic}e^{-3idx}+\right.}{} \\ \frac{x\left(e^{18ic}+9e^{16ic}+36e^{14ic}+84e^{12ic}+126e^{10ic}+126e^{8ic}+84e^{6ic}+36e^{4ic}+9e^{2ic}+1\right)e^{-17ic}}{512a^8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-143500911498201343931187200*I*a**72*d**9*exp(82*I*c)*exp(I*d*x
) + 1291508203483812095380684800*I*a**72*d**9*exp(80*I*c)*exp(-I*d*x) + 172
2010937978416127174246400*I*a**72*d**9*exp(78*I*c)*exp(-3*I*d*x) + 24108153
1316978257804394496O*I*a**72*d**9*exp(76*I*c)*exp(-5*I*d*x) + 2583016406967
```

```
624190761369600*I*a**72*d**9*exp(74*I*c)*exp(-7*I*d*x) + 200901276097481881
5036620800*I*a**72*d**9*exp(72*I*c)*exp(-9*I*d*x) + 109582514234990117183815
56800*I*a**72*d**9*exp(70*I*c)*exp(-11*I*d*x) + 397387139533480644732518400
*I*a**72*d**9*exp(68*I*c)*exp(-13*I*d*x) + 861005468989208063587123 20*I*a**
72*d**9*exp(66*I*c)*exp(-15*I*d*x) + 844123008812949081948 1600*I*a**72*d**9
*exp(64*I*c)*exp(-17*I*d*x))*exp(-81*I*c)/(734724666870790880927678464 00*a*
*80*d**10), Ne(734724666870790880927678464 00*a**80*d**10*exp(81*I*c), 0)),
(x*(exp(18*I*c) + 9*exp(16*I*c) + 36*exp(14*I*c) + 84*exp(12*I*c) + 126*exp
(10*I*c) + 126*exp(8*I*c) + 84*exp(6*I*c) + 36*exp(4*I*c) + 9*exp(2*I*c) +
1)*exp(-17*I*c)/(512*a**8), True))
```

Giac [A] time = 1.17698, size = 336, normalized size = 1.24

$$\frac{\frac{12155}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{6211205\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16}-55791450i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-303072770\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}+1091397450i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+2909561798}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/3111680*(12155/(a^8*(tan(1/2*d*x + 1/2*c) + I)) + (6211205*tan(1/2*d*x +
1/2*c)^16 - 55791450*I*tan(1/2*d*x + 1/2*c)^15 - 303072770*tan(1/2*d*x + 1/
2*c)^14 + 1091397450*I*tan(1/2*d*x + 1/2*c)^13 + 2909561798*tan(1/2*d*x + 1
/2*c)^12 - 5901218466*I*tan(1/2*d*x + 1/2*c)^11 - 9405145178*tan(1/2*d*x +
1/2*c)^10 + 11877161010*I*tan(1/2*d*x + 1/2*c)^9 + 12017308160*tan(1/2*d*x
+ 1/2*c)^8 - 9710430158*I*tan(1/2*d*x + 1/2*c)^7 - 6263238566*tan(1/2*d*x +
 1/2*c)^6 + 3172666718*I*tan(1/2*d*x + 1/2*c)^5 + 1247921210*tan(1/2*d*x +
1/2*c)^4 - 365303990*I*tan(1/2*d*x + 1/2*c)^3 - 77883902*tan(1/2*d*x + 1/2*
c)^2 + 10498214*I*tan(1/2*d*x + 1/2*c) + 982907)/(a^8*(tan(1/2*d*x + 1/2*c)
 - I)^17))/d
```

3.184 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal. Leaf size=301

$$\frac{32\sin^5(c+dx)}{4199a^8d}-\frac{320\sin^3(c+dx)}{12597a^8d}+\frac{160\sin(c+dx)}{4199a^8d}+\frac{64i\cos^5(c+dx)}{4199d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{112i\cos^3(c+dx)}{12597a^2d\left(a^2+ia^2\tan(c+d\right.}$$

```
[Out] (160*Sin[c + d*x])/(4199*a^8*d) - (320*Sin[c + d*x]^3)/(12597*a^8*d) + (32*
Sin[c + d*x]^5)/(4199*a^8*d) + ((I/19)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c +
d*x])^8) + (((11*I)/323)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^7) + (
((22*I)/969)*Cos[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^6) + (((66*I)/41
99)*Cos[c + d*x]^3)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (((48*I)/4199)*Cos[c
 + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x])^4) + (((112*I)/12597)*Cos[c + d*x]
^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((64*I)/4199)*Cos[c + d*x]^5)/(
d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rubi [A] time = 0.384034, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3502, 3500, 2633}

$$\frac{32\sin^5(c+dx)}{4199a^8d}-\frac{320\sin^3(c+dx)}{12597a^8d}+\frac{160\sin(c+dx)}{4199a^8d}+\frac{64i\cos^5(c+dx)}{4199d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{112i\cos^3(c+dx)}{12597a^2d\left(a^2+ia^2\tan(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```


```
[Out] (160*Sin[c + d*x])/(4199*a^8*d) - (320*Sin[c + d*x]^3)/(12597*a^8*d) + (32*
Sin[c + d*x]^5)/(4199*a^8*d) + ((I/19)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c +
d*x])^8) + (((11*I)/323)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^7) + (
((22*I)/969)*Cos[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^6) + (((66*I)/41
99)*Cos[c + d*x]^3)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (((48*I)/4199)*Cos[c
 + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x])^4) + (((112*I)/12597)*Cos[c + d*x]
^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((64*I)/4199)*Cos[c + d*x]^5)/(
d*(a^8 + I*a^8*Tan[c + d*x]))
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{19a}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{110\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{323a^2}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6} + \frac{66}{}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6} + \frac{}{4}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6} + \frac{}{4}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6} + \frac{}{4}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6} + \frac{}{4}$$

$$= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} + \frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6} + \frac{}{4}$$

$$= \frac{160\sin(c+dx)}{4199a^8d} - \frac{320\sin^3(c+dx)}{12597a^8d} + \frac{32\sin^5(c+dx)}{4199a^8d} + \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} + \frac{1}{323ad(}$$

Mathematica [A] time = 1.31914, size = 161, normalized size = 0.53

$$-\frac{i\sec^8(c+dx)(-92378i\sin(c+dx) - 226746i\sin(3(c+dx)) - 266475i\sin(5(c+dx)) - 323323i\sin(7(c+dx)) + 73359}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-I/12899328)*Sec[c + d*x]^8*(-739024*Cos[c + d*x] - 604656*Cos[3*(c + d*x
)] - 426360*Cos[5*(c + d*x)] - 369512*Cos[7*(c + d*x)] + 65208*Cos[9*(c + d
*x)] + 1768*Cos[11*(c + d*x)] - (92378*I)*Sin[c + d*x] - (226746*I)*Sin[3*(
c + d*x)] - (266475*I)*Sin[5*(c + d*x)] - (323323*I)*Sin[7*(c + d*x)] + (73
359*I)*Sin[9*(c + d*x)] + (2431*I)*Sin[11*(c + d*x)]))/(a^8*d*(-I + Tan[c +
 d*x])^8)
```

Maple [A] time = 0.125, size = 372, normalized size = 1.2

$$2\frac{1}{da^8}\left(\frac{-992i}{(\tan(1/2dx+c/2)-i)^{16}} - \frac{\frac{32525i}{8}}{(\tan(1/2dx+c/2)-i)^{8}} + \frac{\frac{7181i}{1024}}{(\tan(1/2dx+c/2)-i)^{2}} + \frac{\frac{32417i}{4}}{(\tan(1/2dx+c/2)-i)^{10}} - \frac{}{(\tan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 2/d/a^8*(-992*I/(tan(1/2*d*x+1/2*c)-I)^16-32525/8*I/(tan(1/2*d*x+1/2*c)-I)^
8+7181/1024*I/(tan(1/2*d*x+1/2*c)-I)^2+32417/4*I/(tan(1/2*d*x+1/2*c)-I)^10-
1/1024*I/(tan(1/2*d*x+1/2*c)+I)^2-25468/3*I/(tan(1/2*d*x+1/2*c)-I)^12+4428*
I/(tan(1/2*d*x+1/2*c)-I)^14-2177/16*I/(tan(1/2*d*x+1/2*c)-I)^4+64*I/(tan(1/
2*d*x+1/2*c)-I)^18-128/19/(tan(1/2*d*x+1/2*c)-I)^19+5248/17/(tan(1/2*d*x+1/
2*c)-I)^17-7096/3/(tan(1/2*d*x+1/2*c)-I)^15+87508/13/(tan(1/2*d*x+1/2*c)-I)
^13-18011/2/(tan(1/2*d*x+1/2*c)-I)^11+6215/(tan(1/2*d*x+1/2*c)-I)^9-72425/3
2/(tan(1/2*d*x+1/2*c)-I)^7+26871/64/(tan(1/2*d*x+1/2*c)-I)^5-54229/1536/(ta
n(1/2*d*x+1/2*c)-I)^3+509/512/(tan(1/2*d*x+1/2*c)-I)+204605/192*I/(tan(1/2*
d*x+1/2*c)-I)^6-1/1536/(tan(1/2*d*x+1/2*c)+I)^3+3/512/(tan(1/2*d*x+1/2*c)+I
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.54902, size = 543, normalized size = 1.8

$$\frac{\left(-4199i\,e^{(22i\,dx+22i\,c)} - 138567i\,e^{(20i\,dx+20i\,c)} + 692835i\,e^{(18i\,dx+18i\,c)} + 692835i\,e^{(16i\,dx+16i\,c)} + 831402i\,e^{(14i\,dx+14i\,c)} + 8314\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/25798656*(-4199*I*e^(22*I*d*x + 22*I*c) - 138567*I*e^(20*I*d*x + 20*I*c)
+ 692835*I*e^(18*I*d*x + 18*I*c) + 692835*I*e^(16*I*d*x + 16*I*c) + 831402*
I*e^(14*I*d*x + 14*I*c) + 831402*I*e^(12*I*d*x + 12*I*c) + 646646*I*e^(10*I
*d*x + 10*I*c) + 377910*I*e^(8*I*d*x + 8*I*c) + 159885*I*e^(6*I*d*x + 6*I*c
) + 46189*I*e^(4*I*d*x + 4*I*c) + 8151*I*e^(2*I*d*x + 2*I*c) + 663*I)*e^(-1
9*I*d*x - 19*I*c)/(a^8*d)
```

Sympy [A] time = 4.39548, size = 437, normalized size = 1.45

$$\begin{cases} \frac{\left(-627910689858846946911347157688181273395 2ia^{88}d^{11}e^{103ic}e^{3idx} - 207210527653419492480744562037099820220416ia^{88}d^{11}e^{101ic}e^{idx} + 1036052638267\cdots}{} \\ \frac{x\left(e^{22ic}+11e^{20ic}+55e^{18ic}+165e^{16ic}+330e^{14ic}+462e^{12ic}+462e^{10ic}+330e^{8ic}+165e^{6ic}+55e^{4ic}+11e^{2ic}+1\right)e^{-19ic}}{2048a^8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-6279106898588469469113471576881812733952*I*a**88*d**11*exp(103
*I*c)*exp(3*I*d*x) - 207210527653419492480744562037099820220416*I*a**88*d**
11*exp(101*I*c)*exp(I*d*x) + 1036052638267097462403722810185499101102080*I*
a**88*d**11*exp(99*I*c)*exp(-I*d*x) + 1036052638267097462403722810185499101
102080*I*a**88*d**11*exp(97*I*c)*exp(-3*I*d*x) + 12432631659205169548844673
72222598921322496*I*a**88*d**11*exp(95*I*c)*exp(-5*I*d*x) + 124326316592051
6954884467372222598921322496*I*a**88*d**11*exp(93*I*c)*exp(-7*I*d*x) + 9669
82462382624298243474622839799161028608*I*a**88*d**11*exp(91*I*c)*exp(-9*I*d
*x) + 565119620872962252220212441919363146055680*I*a**88*d**11*exp(89*I*c)*
exp(-11*I*d*x) + 239089070369330183631628340812038254100480*I*a**88*d**11*e
xp(87*I*c)*exp(-13*I*d*x) + 69070175884473164160248187345699940073472*I*a**
88*d**11*exp(85*I*c)*exp(-15*I*d*x) + 1218885456784820544004379776688822471
8848*I*a**88*d**11*exp(83*I*c)*exp(-17*I*d*x) + 991437931356074126702127091
086602010624*I*a**88*d**11*exp(81*I*c)*exp(-19*I*d*x))*exp(-100*I*c)/(38578
832784927556418233169368361857437401088*a**96*d**12), Ne(385788327849275564
18233169368361857437401088*a**96*d**12*exp(100*I*c), 0)), (x*(exp(22*I*c) +
 11*exp(20*I*c) + 55*exp(18*I*c) + 165*exp(16*I*c) + 330*exp(14*I*c) + 462*
exp(12*I*c) + 462*exp(10*I*c) + 330*exp(8*I*c) + 165*exp(6*I*c) + 55*exp(4*
I*c) + 11*exp(2*I*c) + 1)*exp(-19*I*c)/(2048*a**8), True))
```

Giac [A] time = 1.19953, size = 406, normalized size = 1.35

$$\frac{4199\left(18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+33i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-17\right)}{a^{8}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^{3}}+\frac{12823746\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{18}-140368371i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{17}-879644311\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{16}+3693272440i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/6449664*(4199*(18*tan(1/2*d*x + 1/2*c)^2 + 33*I*tan(1/2*d*x + 1/2*c) - 17
)/(a^8*(tan(1/2*d*x + 1/2*c) + I)^3) + (12823746*tan(1/2*d*x + 1/2*c)^18 -
140368371*I*tan(1/2*d*x + 1/2*c)^17 - 879644311*tan(1/2*d*x + 1/2*c)^16 + 3
693272440*I*tan(1/2*d*x + 1/2*c)^15 + 11467502592*tan(1/2*d*x + 1/2*c)^14 -
 27403194676*I*tan(1/2*d*x + 1/2*c)^13 - 51919375300*tan(1/2*d*x + 1/2*c)^1
2 + 79183835016*I*tan(1/2*d*x + 1/2*c)^11 + 98304418212*tan(1/2*d*x + 1/2*c
)^10 - 99750226290*I*tan(1/2*d*x + 1/2*c)^9 - 82860874122*tan(1/2*d*x + 1/2
*c)^8 + 56110430792*I*tan(1/2*d*x + 1/2*c)^7 + 30766700912*tan(1/2*d*x + 1/
2*c)^6 - 13462452660*I*tan(1/2*d*x + 1/2*c)^5 - 4616712644*tan(1/2*d*x + 1/
2*c)^4 + 1197851960*I*tan(1/2*d*x + 1/2*c)^3 + 226248618*tan(1/2*d*x + 1/2*
c)^2 - 27911475*I*tan(1/2*d*x + 1/2*c) - 2143959)/(a^8*(tan(1/2*d*x + 1/2*c
) - I)^19))/d
```

3.185 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=123

$$\frac{6ae^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d}-\frac{6ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}+\frac{2ae\sin(c+dx)(e\sec(c+d}{5d}$$

```
[Out] (-6*a*e^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((2*I)/7)*a*(e*Sec[c + d*x])^(7/2))/d + (6*a*e^3*Sqrt[e*Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*a*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*
d)
```

Rubi [A] time = 0.0905456, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3486, 3768, 3771, 2639}

$$\frac{6ae^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d}-\frac{6ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}+\frac{2ae\sin(c+dx)(e\sec(c+d}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-6*a*e^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((2*I)/7)*a*(e*Sec[c + d*x])^(7/2))/d + (6*a*e^3*Sqrt[e*Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*a*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*
d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + a\int (e\sec(c+dx))^{7/2}\,dx$$
$$= \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \frac{2ae(e\sec(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(3ae^2\right)\int (e\sec($$
$$= \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \frac{6ae^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2ae(e\sec(c+dx))^5}{5d}$$
$$= \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \frac{6ae^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2ae(e\sec(c+dx))^5}{5d}$$
$$= -\frac{6ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \frac{6ae^3\sqrt{e\sec(c+d}}{5d}$$

Mathematica [C] time = 2.13883, size = 156, normalized size = 1.27

$$\frac{aee^{-idx}(\cos(dx)-i\sin(dx))(e\sec(c+dx))^{5/2}(\cos(c+3dx)+i\sin(c+3dx))\left(7ie^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometri}\right.}{70d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*e*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] - I*Sin[d*x])*(Cos[c + 3*d*x] + I*Sin
[c + 3*d*x])*(-36*I - (28*I)*Cos[2*(c + d*x)] + ((7*I)*(1 + E^((2*I)*(c + d
*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I
)*(c + d*x)) + 7*Sec[c + d*x]*Sin[3*(c + d*x)] + 27*Tan[c + d*x]))/(70*d*E^
(I*d*x))
```

Maple [B] time = 0.365, size = 365, normalized size = 3.

$$\frac{2a(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2}{35d(\sin(dx+c))^5}\left(21i(\cos(dx+c))^4\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/35*a/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(21*I*cos(d*x+c)^4*sin(d*x+c)*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d
*x+c)-1)/sin(d*x+c),I)-21*I*cos(d*x+c)^4*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I
)+21*I*cos(d*x+c)^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-21*I*cos(d*x+c)^3*s
in(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipt
icF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-21*cos(d*x+c)^4+14*cos(d*x+c)^3+5*I*sin(
d*x+c)+7*cos(d*x+c))*(e/cos(d*x+c))^(7/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{7}{2}}(ia\tan(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-42i\,ae^{3}e^{(7i\,dx+7i\,c)} - 154i\,ae^{3}e^{(5i\,dx+5i\,c)} - 46i\,ae^{3}e^{(3i\,dx+3i\,c)} - 14i\,ae^{3}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + 35\left(de^{(6i\,dx+6i}\right.}{35\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4i\,dx+4i\,c)} + 3\,de^{(2i\,dx+2i\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/35*(sqrt(2)*(-42*I*a*e^3*e^(7*I*d*x + 7*I*c) - 154*I*a*e^3*e^(5*I*d*x + 5
*I*c) - 46*I*a*e^3*e^(3*I*d*x + 3*I*c) - 14*I*a*e^3*e^(I*d*x + I*c))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 35*(d*e^(6*I*d*x + 6*
I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*integral(3/5*
I*sqrt(2)*a*e^3*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/d
, x))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2
*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{7}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

3.186 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=94

$$\frac{2ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}+\frac{2ae\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}$$

```
[Out] (2*a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(3*d) + (((2*I)/5)*a*(e*Sec[c + d*x])^(5/2))/d + (2*a*e*(e*Sec[c + d*x])^(
3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.060312, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3486, 3768, 3771, 2641}

$$\frac{2ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{2ia(e\sec(c+dx))^{5/2}}{5d}+\frac{2ae\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(3*d) + (((2*I)/5)*a*(e*Sec[c + d*x])^(5/2))/d + (2*a*e*(e*Sec[c + d*x])^(
3/2)*Sin[c + d*x])/(3*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + a\int (e\sec(c+dx))^{5/2}\,dx$$
$$= \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + \frac{2ae(e\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\left(ae^2\right)\int \sqrt{e\,s$$
$$= \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + \frac{2ae(e\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\left(ae^2\sqrt{\cos(c$$
$$= \frac{2ae^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3d} + \frac{2ia(e\sec(c+dx))^{5/2}}{5d}$$

Mathematica [A] time = 0.504491, size = 57, normalized size = 0.61

$$\frac{a(e\sec(c+dx))^{5/2}\left(5\sin(2(c+dx))+10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+6i\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*(e*Sec[c + d*x])^(5/2)*(6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/
2, 2] + 5*Sin[2*(c + d*x)]))/(15*d)
```

Maple [A] time = 0.232, size = 192, normalized size = 2.

$$\frac{2\,a\left(\cos(dx+c)+1\right)^2\left(\cos(dx+c)-1\right)^2}{15\,d\left(\sin(dx+c)\right)^4}\left(5\,i\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(\cos(dx+c)\right)^3 \mathrm{EllipticF}\left(\frac{i\,(\cos}{\mathrm{si}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/15*a/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticF(I*(cos(d*x+c)-1)/sin
(d*x+c),I)+5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)^2*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+5*cos(d*x+c)*sin(d*x+c
)+3*I)*(e/cos(d*x+c))^(5/2)/sin(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{5}{2}}\left(i\,a\tan(dx+c)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-10i\,ae^2e^{(4i\,dx+4i\,c)}+24i\,ae^2e^{(2i\,dx+2i\,c)}+10i\,ae^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)\text{integral}}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(sqrt(2)*(-10*I*a*e^2*e^(4*I*d*x + 4*I*c) + 24*I*a*e^2*e^(2*I*d*x + 2*
I*c) + 10*I*a*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
) + 15*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*integral(-1/3*
I*sqrt(2)*a*e^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/
d, x))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{5}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

3.187 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=90

$$-\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}+\frac{2ae\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}$$

```
[Out] (-2*a*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d
*x]]) + (((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + (2*a*e*Sqrt[e*Sec[c + d*x]
]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0605921, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3486, 3768, 3771, 2639}

$$-\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(e\sec(c+dx))^{3/2}}{3d}+\frac{2ae\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-2*a*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d
*x]]) + (((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + (2*a*e*Sqrt[e*Sec[c + d*x]
]*Sin[c + d*x])/d
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + a\int (e\sec(c+dx))^{3/2}\,dx$$
$$= \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} - \left(ae^2\right)\int \frac{1}{\sqrt{e\sec(c+}}$$
$$= \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} - \frac{\left(ae^2\right)\int \sqrt{\cos(c+}}{\sqrt{\cos(c+dx)}\sqrt{e\sec}}$$
$$= -\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sqrt{e\sec(c+dx)}}{d}$$

Mathematica [C] time = 0.801333, size = 102, normalized size = 1.13

$$\frac{2aee^{-2idx}\sqrt{e\sec(c+dx)}(\cos(c+3dx)+i\sin(c+3dx))\left(i\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+\tan(c\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*e*Sqrt[e*Sec[c + d*x]]*(Cos[c + 3*d*x] + I*Sin[c + 3*d*x])*(-2*I + I*S
qrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
+ d*x))] + Tan[c + d*x]))/(3*d*E^((2*I)*d*x))
```

Maple [B] time = 0.21, size = 351, normalized size = 3.9

$$\frac{2\,a\left(\cos\left(dx+c\right)+1\right)^{2}\left(\cos\left(dx+c\right)-1\right)^{2}}{3\,d\left(\sin\left(dx+c\right)\right)^{5}}\left(3\,i\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{2}\sqrt{\left(\cos\left(dx+c\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\text{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/3*a/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(3*I*sin(d*x+c)*cos(d*x+c)^2*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x
+c)-1)/sin(d*x+c),I)-3*I*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+3
*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(
cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c)
,I)*cos(d*x+c)*sin(d*x+c)+I*sin(d*x+c)-3*cos(d*x+c)^2+3*cos(d*x+c))*(e/cos(
d*x+c))^(3/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-6i\,aee^{(3i\,dx+3i\,c)}-2i\,aee^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\left(de^{(2i\,dx+2i\,c)}+d\right)\mathrm{integral}\left(\frac{i\sqrt{2}ae\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{d},\right.}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-6*I*a*e*e^(3*I*d*x + 3*I*c) - 2*I*a*e*e^(I*d*x + I*c))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*(d*e^(2*I*d*x + 2*
I*c) + d)*integral(I*sqrt(2)*a*e*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I
*d*x + 1/2*I*c)/d, x))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int (e\sec(c+dx))^{\frac{3}{2}}\,dx+\int i\,(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*(Integral((e*sec(c + d*x))**(3/2), x) + Integral(I*(e*sec(c + d*x))**(3/2
)*tan(c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

3.188 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

```
[Out] ((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d
```

Rubi [A] time = 0.0434359, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3486, 3771, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx &= \frac{2ia\sqrt{e\sec(c+dx)}}{d}+a\int \sqrt{e\sec(c+dx)}\,dx\\
&= \frac{2ia\sqrt{e\sec(c+dx)}}{d}+\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\
&= \frac{2ia\sqrt{e\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.236105, size = 44, normalized size = 0.73

$$\frac{2a\sqrt{e\sec(c+dx)}\left(\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+i\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*(I + Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[e*Sec[c + d*x]
])/d
```

Maple [B] time = 0.238, size = 164, normalized size = 2.7

$$\frac{2\,ia\left(\cos\left(dx+c\right)+1\right)^{2}\left(\cos\left(dx+c\right)-1\right)^{2}}{d\left(\sin\left(dx+c\right)\right)^{4}}\sqrt{\frac{e}{\cos\left(dx+c\right)}}\left(\cos\left(dx+c\right)\mathrm{EllipticF}\left(\frac{i\left(\cos\left(dx+c\right)-1\right)}{\sin\left(dx+c\right)},i\right)\sqrt{(\cos\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2*I*a/d*(e/cos(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(cos(d*x+c)*
EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1)/sin(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2i\sqrt{2}a\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+d\mathrm{integral}\left(-\frac{i\sqrt{2}a\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{d},x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*I*sqrt(2)*a*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) +
d*integral(-I*sqrt(2)*a*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1
```

```
/2*I*c)/d, x))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sqrt{e\sec\left(c+dx\right)}\,dx+\int i\sqrt{e\sec\left(c+dx\right)}\tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*(Integral(sqrt(e*sec(c + d*x)), x) + Integral(I*sqrt(e*sec(c + d*x))*tan(
c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

3.189 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

```
[Out] ((-2*I)*a)/(d*Sqrt[e*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, 2])/(d*Sq
rt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.0467685, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3486, 3771, 2639}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*a)/(d*Sqrt[e*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, 2])/(d*Sq
rt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx&=-\frac{2ia}{d\sqrt{e\sec(c+dx)}}+a\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx\\&=-\frac{2ia}{d\sqrt{e\sec(c+dx)}}+\frac{a\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\&=-\frac{2ia}{d\sqrt{e\sec(c+dx)}}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.343589, size = 73, normalized size = 1.22

$$-\frac{4iae^{2i(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{3d\sqrt{1+e^{2i(c+dx)}}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (((-4*I)/3)*a*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I
)*(c + d*x))])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[e*Sec[c + d*x]])
```

Maple [B] time = 0.243, size = 910, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a/d*(cos(d*x+c)-1)*(4*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*sin(d*x+c
)*cos(d*x+c)^2*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-4*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+8*I*(-cos(d*x+c)/(co
s(d*x+c)+1)^2)^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d
*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-8*I*(-c
os(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticE(I*(cos(d*
x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)+4*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin
(d*x+c)-4*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-co
s(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*s
in(d*x+c)-4*I*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-
4*I*sin(d*x+c)*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+I*ln(-2*(2*(
-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*cos(d*x+c)*sin(d*x+
c)-I*cos(d*x+c)*ln(-(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-co
s(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c
)^2)*sin(d*x+c)-4*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+4*cos(d
*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2))/(-cos(d*x+c)/(cos(d*x+c)+1)^2)^
(1/2)/sin(d*x+c)^3/cos(d*x+c)/(e/cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,a\tan\left(dx+c\right)+a}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/sqrt(e*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-2i\,ae^{(2i\,dx+2i\,c)}-2i\,a\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\left(dee^{(i\,dx+i\,c)}-de\right)\mathrm{integral}\left(\frac{\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)}-2i\,ae^{(i\,dx+i\,c)}-i\,a\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}}{dee^{(3i\,dx+3i\,c)}-2\,dee^{(2i\,dx+2i\,c)}+dee^{(i\,dx+i\,c)}}\right.}{dee^{(i\,dx+i\,c)}-de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(-2*I*a*e^(2*I*d*x + 2*I*c) - 2*I*a)*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(1/2*I*d*x + 1/2*I*c) + (d*e*e^(I*d*x + I*c) - d*e)*integral(sqrt(2)
*(-I*a*e^(2*I*d*x + 2*I*c) - 2*I*a*e^(I*d*x + I*c) - I*a)*sqrt(e/(e^(2*I*d*
x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e*e^(3*I*d*x + 3*I*c) - 2*d*e*e
^(2*I*d*x + 2*I*c) + d*e*e^(I*d*x + I*c)), x))/(d*e*e^(I*d*x + I*c) - d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{1}{\sqrt{e\sec\left(c+dx\right)}}\,dx+\int\frac{i\tan\left(c+dx\right)}{\sqrt{e\sec\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*sec(c + d*x)), x) + Integral(I*tan(c + d*x)/sqrt(e*sec
(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{i\,a\tan\left(dx+c\right)+a}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/sqrt(e*sec(d*x + c)), x)
```

3.190 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}$$

```
[Out] (((-2*I)/3)*a)/(d*(e*Sec[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*a*Sin[c + d*x])/(3*
d*e*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.0680948, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3486, 3769, 3771, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*a)/(d*(e*Sec[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*a*Sin[c + d*x])/(3*
d*e*Sqrt[e*Sec[c + d*x]])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx &= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}} + a\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx \\ &= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}} + \frac{a\int \sqrt{e\sec(c+dx)}\,dx}{3e^2} \\ &= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}} + \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,d}{3e^2} \\ &= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2} + \frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+d}}\end{aligned}$$

Mathematica [A] time = 0.394509, size = 62, normalized size = 0.65

$$\frac{2a\left(\sin(c+dx)-i\cos(c+dx)+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{\sqrt{\cos(c+dx)}}\right)}{3de\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*((-I)*Cos[c + d*x] + EllipticF[(c + d*x)/2, 2]/Sqrt[Cos[c + d*x]] + Si
n[c + d*x]))/(3*d*e*Sqrt[e*Sec[c + d*x]])
```

Maple [A] time = 0.199, size = 170, normalized size = 1.8

$$\frac{2\,a}{3\,d\left(\cos\left(dx+c\right)\right)^{2}}\left(i\cos\left(dx+c\right)\sqrt{\left(\cos\left(dx+c\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\mathrm{EllipticF}\left(\frac{i\left(\cos\left(dx+c\right)-1\right)}{\sin\left(dx+c\right)},i\right)+i\sqrt{(\cos}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/3*a/d*(I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+I*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-I
*cos(d*x+c)^2+cos(d*x+c)*sin(d*x+c))/cos(d*x+c)^2/(e/cos(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,de^2\text{integral}\left(-\frac{i\sqrt{2}a\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,de^2},x\right)+\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{3\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*e^2*integral(-1/3*I*sqrt(2)*a*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^
(-1/2*I*d*x - 1/2*I*c)/(d*e^2), x) + sqrt(2)*(-I*a*e^(2*I*d*x + 2*I*c) - I*
a)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{1}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\frac{i\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*sec(c + d*x))**(-3/2), x) + Integral(I*tan(c + d*x)/(e*sec(c
 + d*x))**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{i\,a\tan(dx+c)+a}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(3/2), x)
```

3.191 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=96

$$\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}$$

```
[Out] (((-2*I)/5)*a)/(d*(e*Sec[c + d*x])^(5/2)) + (6*a*EllipticE[(c + d*x)/2, 2])
/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*a*Sin[c + d*x])/(5*
d*e*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0652379, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3486, 3769, 3771, 2639}

$$\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*a)/(d*(e*Sec[c + d*x])^(5/2)) + (6*a*EllipticE[(c + d*x)/2, 2])
/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*a*Sin[c + d*x])/(5*
d*e*(e*Sec[c + d*x])^(3/2))
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2ia}{5d(e\sec(c+dx))^{5/2}} + a\int \frac{1}{(e\sec(c+dx))^{5/2}}\,dx$$
$$= -\frac{2ia}{5d(e\sec(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}} + \frac{(3a)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}$$
$$= -\frac{2ia}{5d(e\sec(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}} + \frac{(3a)\int \sqrt{\cos(c+dx)}\,dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= -\frac{2ia}{5d(e\sec(c+dx))^{5/2}} + \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}$$

Mathematica [C] time = 0.75465, size = 99, normalized size = 1.03

$$-\frac{a(\tan(c+dx)-i)\left(-2\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3i\sin(2(c+dx))+2\cos(2(c+dx))+2\right)}{5de^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] -(a*(2 + 2*Cos[2*(c + d*x)] - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] - (3*I)*Sin[2*(c + d*x)])*(-I + T
an[c + d*x]))/(5*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [B] time = 0.191, size = 339, normalized size = 3.5

$$-\frac{2a}{5d\sin(dx+c)(\cos(dx+c))^3}\left(3i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\cos(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/5*a/d*(3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-3*I*(1/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)
/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+I*sin(d*x+c)*cos(d*x+c)^3+3*I*sin(d*x+
c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d
*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*x
+c)^4+2*cos(d*x+c)^2-3*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^3/(e/cos(d*x+c))^(
5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,a\tan(dx+c)+a}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-i\,ae^{(5i\,dx+5i\,c)}+i\,ae^{(4i\,dx+4i\,c)}-8i\,ae^{(3i\,dx+3i\,c)}-4i\,ae^{(2i\,dx+2i\,c)}-7i\,ae^{(i\,dx+i\,c)}-5i\,a\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+10\left(de^{3}\right.}{10\left(de^{3}e^{(2i\,dx+2i\,c)}-de^{3}e^{(i\,dx+i\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(2)*(-I*a*e^(5*I*d*x + 5*I*c) + I*a*e^(4*I*d*x + 4*I*c) - 8*I*a*e
^(3*I*d*x + 3*I*c) - 4*I*a*e^(2*I*d*x + 2*I*c) - 7*I*a*e^(I*d*x + I*c) - 5*
I*a)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 10*(d*e^3*
e^(2*I*d*x + 2*I*c) - d*e^3*e^(I*d*x + I*c))*integral(1/5*sqrt(2)*(-3*I*a*e
^(2*I*d*x + 2*I*c) - 6*I*a*e^(I*d*x + I*c) - 3*I*a)*sqrt(e/(e^(2*I*d*x + 2*
I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^3*e^(3*I*d*x + 3*I*c) - 2*d*e^3*e^(
2*I*d*x + 2*I*c) + d*e^3*e^(I*d*x + I*c)), x))/(d*e^3*e^(2*I*d*x + 2*I*c) -
 d*e^3*e^(I*d*x + I*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{1}{\left(e\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx+\int\frac{i\tan\left(c+dx\right)}{\left(e\sec\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] a*(Integral((e*sec(c + d*x))**(-5/2), x) + Integral(I*tan(c + d*x)/(e*sec(c
 + d*x))**(5/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{i\,a\tan\left(dx+c\right)+a}{\left(e\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(5/2), x)
```

3.192 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=125

$$\frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}+\frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}$$

```
[Out] (((-2*I)/7)*a)/(d*(e*Sec[c + d*x])^(7/2)) + (10*a*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(21*d*e^4) + (2*a*Sin[c + d*x])/(
7*d*e*(e*Sec[c + d*x])^(5/2)) + (10*a*Sin[c + d*x])/(21*d*e^3*Sqrt[e*Sec[c
+ d*x]])
```

Rubi [A] time = 0.0823461, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3486, 3769, 3771, 2641}

$$\frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}-\frac{2ia}{7d(e\sec(c+dx))^{7/2}}+\frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/7)*a)/(d*(e*Sec[c + d*x])^(7/2)) + (10*a*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(21*d*e^4) + (2*a*Sin[c + d*x])/(
7*d*e*(e*Sec[c + d*x])^(5/2)) + (10*a*Sin[c + d*x])/(21*d*e^3*Sqrt[e*Sec[c
+ d*x]])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}dx = -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + a\int \frac{1}{(e\sec(c+dx))^{7/2}}dx$$
$$= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{(5a)\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7e^2}$$
$$= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}} + \frac{(5a)\int \sqrt{e\sec(c+a}}{21e^4}$$
$$= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}} + \frac{\left(5a\sqrt{\cos(c+dx)}\sqrt{\right.}}{}$$
$$= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{10a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4} + \frac{2a\sin(c+d}{7de(e\sec(c+d}$$

Mathematica [A] time = 0.67352, size = 121, normalized size = 0.97

$$\frac{a\sqrt{e\sec(c+dx)}(\cos(c+dx)+i\sin(c+dx))\left(5\sin(c+dx)+5\sin(3(c+dx))-14i\cos(c+dx)+2i\cos(3(c+dx))+2\right.}{42de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (a*Sqrt[e*Sec[c + d*x]]*(Cos[c + d*x] + I*Sin[c + d*x])*((-14*I)*Cos[c + d*
x] + (2*I)*Cos[3*(c + d*x)] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*(Cos[c + d*x] - I*Sin[c + d*x]) + 5*Sin[c + d*x] + 5*Sin[3*(c + d*x)]))/
(42*d*e^4)
```

Maple [A] time = 0.196, size = 187, normalized size = 1.5

$$\frac{2a}{21d(\cos(dx+c))^4}\left(-3i(\cos(dx+c))^4+5i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(\cos(d}{\sin(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x)
```

```
[Out] 2/21*a/d*(-3*I*cos(d*x+c)^4+5*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+3*cos(d
*x+c)^3*sin(d*x+c)+5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+5*cos(d*x+c)*sin(d*x+c))/co
s(d*x+c)^4/(e/cos(d*x+c))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i a\tan(dx+c)+a}{(e\sec(dx+c))^{\frac{7}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(7/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(84\,de^{4}e^{(2i\,dx+2i\,c)}\text{integral}\left(-\frac{5i\,\sqrt{2}a\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{21\,de^{4}},x\right)+\sqrt{2}\left(-3i\,ae^{(6i\,dx+6i\,c)}-19i\,ae^{(4i\,dx+4i\,c)}-9i\,ae^{(2i\,dx+2i\,c)}+7i\,a\right)\cdots}{84\,de^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")

[Out] 1/84*(84*d*e^4*e^(2*I*d*x + 2*I*c)*integral(-5/21*I*sqrt(2)*a*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^4), x) + sqrt(2)*(-3*I*a*e^(6*I*d*x + 6*I*c) - 19*I*a*e^(4*I*d*x + 4*I*c) - 9*I*a*e^(2*I*d*x + 2*I*c) + 7*I*a)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(d*e^4)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\sec\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(7/2), x)

3.193 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=138

$$-\frac{14a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d}+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\ldots}{5d}$$

```
[Out] (-14*a^2*e^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((14*I)/15)*a^2*(e*Sec[c + d*x])^(3/2))/d + (14*a^2*e*Sqrt[e*
Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2
+ I*a^2*Tan[c + d*x]))/d
```

Rubi [A] time = 0.117203, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3498, 3486, 3768, 3771, 2639}

$$-\frac{14a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d}+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\ldots}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-14*a^2*e^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((14*I)/15)*a^2*(e*Sec[c + d*x])^(3/2))/d + (14*a^2*e*Sqrt[e*
Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2
+ I*a^2*Tan[c + d*x]))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}{5d}+\frac{1}{5}(7a)\int (e\sec(c+dx))^{3/2}(a+$$
$$=\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}{5d}+\frac{1}{5}\,(7a$$
$$=\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2i(e\sec(c+dx$$
$$=\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2i(e\sec(c+dx$$
$$=-\frac{14a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sqrt{e\sec(c$$

Mathematica [C] time = 2.50229, size = 267, normalized size = 1.93

$$\frac{(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}\left(\frac{1}{2}\csc(c)(\cos(2c)-i\sin(2c))\sec^{\frac{5}{2}}(c+dx)(20i\sin(2c+dx)+27\cos(2c+dx)+21\right.}{15d\sec^{\frac{7}{2}}(c+dx)(\cos(dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Sec[c + d*x])^(3/2)*(((-14*I)*Sqrt[2]*(3*Sqrt[1 + E^((2*I)*(c + d*x))]
- E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
I)*(c + d*x))]))/((-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]) + (Csc[c]*Sec[c + d*x]^(5/2)*(Cos[
2*c] - I*Sin[2*c])*(36*Cos[d*x] + 27*Cos[2*c + d*x] + 21*Cos[2*c + 3*d*x] -
 (20*I)*Sin[d*x] + (20*I)*Sin[2*c + d*x]))/2)*(a + I*a*Tan[c + d*x])^2)/(15
*d*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [B] time = 0.266, size = 374, normalized size = 2.7

$$-\frac{2a^2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2}{15d(\sin(dx+c))^5\cos(dx+c)}\left(21i(\cos(dx+c))^3\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(\cos}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -2/15*a^2/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(21*I*cos(d*x+c)^3*(1/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)
```

```
/sin(d*x+c),I)*sin(d*x+c)-21*I*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*
x+c)+21*I*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-21*I*cos(d*x+c)^
2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(c
os(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+21*cos(d*x+c)^3-10*I*cos(d*x+c)*sin(d
*x+c)-24*cos(d*x+c)^2+3)*(e/cos(d*x+c))^(3/2)/sin(d*x+c)^5/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{3}{2}}(ia\tan(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-42i\,a^2ee^{(5i\,dx+5i\,c)}-32i\,a^2ee^{(3i\,dx+3i\,c)}-14i\,a^2ee^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+\right.}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(sqrt(2)*(-42*I*a^2*e*e^(5*I*d*x + 5*I*c) - 32*I*a^2*e*e^(3*I*d*x + 3*
I*c) - 14*I*a^2*e*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2
*I*d*x + 1/2*I*c) + 15*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d
)*integral(7/5*I*sqrt(2)*a^2*e*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d
*x + 1/2*I*c)/d, x))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{3}{2}}(ia\tan(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2, x)
```

3.194 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=106

$$\frac{10ia^2\sqrt{e\sec(c+dx)}}{3d}+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{10a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3d}$$

```
[Out] (((10*I)/3)*a^2*Sqrt[e*Sec[c + d*x]])/d + (10*a^2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (((2*I)/3)*Sqrt[e*Sec[c +
 d*x]]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rubi [A] time = 0.0894341, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3498, 3486, 3771, 2641}

$$\frac{10ia^2\sqrt{e\sec(c+dx)}}{3d}+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{3d}+\frac{10a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((10*I)/3)*a^2*Sqrt[e*Sec[c + d*x]])/d + (10*a^2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (((2*I)/3)*Sqrt[e*Sec[c +
 d*x]]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \frac{2i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d} + \frac{1}{3}(5a)\int \sqrt{e\sec(c+dx)}(a+ia\tan$$

$$= \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d} + \frac{2i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d} + \frac{1}{3}\left(5a^2\right)\int$$

$$= \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d} + \frac{2i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d} + \frac{1}{3}\left(5a^2\sqrt{\cos}\right.$$

$$= \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d} + \frac{10a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3d} + \frac{2}{}$$

Mathematica [A] time = 0.614256, size = 67, normalized size = 0.63

$$\frac{2a^2(e\sec(c+dx))^{3/2}\left(-\sin(c+dx)+6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*a^2*(e*Sec[c + d*x])^(3/2)*((6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*El
lipticF[(c + d*x)/2, 2] - Sin[c + d*x]))/(3*d*e)
```

Maple [A] time = 0.237, size = 201, normalized size = 1.9

$$\frac{2a^2(\cos(dx+c)-1)^2(\cos(dx+c)+1)^2}{3d\cos(dx+c)(\sin(dx+c))^4}\sqrt{\frac{e}{\cos(dx+c)}}\left(5i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c))^2\,\mathrm{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/3*a^2/d*(e/cos(d*x+c))^(1/2)*(cos(d*x+c)-1)^2*(5*I*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticF(I*(cos(d*x+c)-1
)/sin(d*x+c),I)+5*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+6*I*cos(d*x+c)-sin(
d*x+c))*(cos(d*x+c)+1)^2/cos(d*x+c)/sin(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(14i\,a^{2}e^{(2i\,dx+2i\,c)}+10i\,a^{2}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\left(de^{(2i\,dx+2i\,c)}+d\right)\mathrm{integral}\left(-\frac{5i\,\sqrt{2}a^{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,d},x\right)}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(14*I*a^2*e^(2*I*d*x + 2*I*c) + 10*I*a^2)*sqrt(e/(e^(2*I*d*x +
 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*(d*e^(2*I*d*x + 2*I*c) + d)*integ
ral(-5/3*I*sqrt(2)*a^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/
2*I*c)/d, x))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int\sqrt{e\sec\left(c+dx\right)}\,dx+\int-\sqrt{e\sec\left(c+dx\right)}\tan^{2}\left(c+dx\right)dx+\int 2i\sqrt{e\sec\left(c+dx\right)}\tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sqrt(e*sec(c + d*x)), x) + Integral(-sqrt(e*sec(c + d*x))*ta
n(c + d*x)**2, x) + Integral(2*I*sqrt(e*sec(c + d*x))*tan(c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^2, x)
```

3.195 $\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=107

$$-\frac{6a^2\sin(c+dx)\sqrt{e\sec(c+dx)}}{de}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}+\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

```
[Out] (6*a^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]
]) - (6*a^2*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(d*e) - ((4*I)*(a^2 + I*a^2*
Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.0806821, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3496, 3768, 3771, 2639}

$$-\frac{6a^2\sin(c+dx)\sqrt{e\sec(c+dx)}}{de}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}+\frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (6*a^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]
]) - (6*a^2*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(d*e) - ((4*I)*(a^2 + I*a^2*
Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx &= -\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}-\frac{\left(3a^2\right)\int(e\sec(c+dx))^{3/2}\,dx}{e^2}\\
&= -\frac{6a^2\sqrt{e\sec(c+dx)}\sin(c+dx)}{de}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}+\left(3a^2\right)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx\\
&= -\frac{6a^2\sqrt{e\sec(c+dx)}\sin(c+dx)}{de}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}+\frac{\left(3a^2\right)\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= \frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{6a^2\sqrt{e\sec(c+dx)}\sin(c+dx)}{de}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 0.978643, size = 132, normalized size = 1.23

$$
-\frac{2i\sqrt{2}a^2e^{2i(c+dx)}\left(\left(1+e^{2i(c+dx)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-\sqrt{1+e^{2i(c+dx)}}\right)}{d\left(1+e^{2i(c+dx)}\right)^{3/2}\sqrt{\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a^2*E^((2*I)*(c + d*x))*(-Sqrt[1 + E^((2*I)*(c + d*x))] + (
1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))]))/(d*Sqrt[(e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)
*(c + d*x)))^(3/2))
```

Maple [B] time = 0.266, size = 1099, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x)
```

```
[Out] a^2/d*(cos(d*x+c)-1)*(I*ln(-2*(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d
*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)
/sin(d*x+c)^2)*cos(d*x+c)^2*sin(d*x+c)+12*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^
(3/2)*cos(d*x+c)^2*sin(d*x+c)+12*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos
(d*x+c)^3*sin(d*x+c)-6*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(co
s(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/si
n(d*x+c),I)*cos(d*x+c)^3*sin(d*x+c)-I*cos(d*x+c)^2*ln(-(2*(-cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+6*I*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c
),I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)*sin(d*x+c)+4*(-cos(d*x
+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^5+4*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)
^(3/2)*cos(d*x+c)*sin(d*x+c)+12*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d
*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x
+c)-1)/sin(d*x+c),I)*cos(d*x+c)^2*sin(d*x+c)-6*I*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d
*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+6*(-cos(d*x+c)/(c
```

```
os(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^4+6*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(co
s(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)^3*sin(d*x+c)-4*(-cos(d*x+c)/(cos(d*x+c
)+1)^2)^(3/2)*cos(d*x+c)^3+4*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x
+c)^4*sin(d*x+c)-12*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d
*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d
*x+c),I)*cos(d*x+c)^2*sin(d*x+c)-8*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos
(d*x+c)^2+2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2))/(cos(d*x+c)+1)^2/(-cos(d*
x+c)/(cos(d*x+c)+1)^2)^(3/2)/cos(d*x+c)/sin(d*x+c)^3/(e/cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^2}{\sqrt{e\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/sqrt(e*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-4i\,a^2e^{(2i\,dx+2i\,c)}-2i\,a^2e^{(i\,dx+i\,c)}-6i\,a^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\left(dee^{(i\,dx+i\,c)}-de\right)\text{integral}\left(\frac{\sqrt{2}\left(-3i\,a^2e^{(2i\,dx+2i\,c)}-6i\,a^2e^{(i\,dx+i}\right.}{dee^{(3i\,dx+3i\,c)}-2\,dee}\right.}{dee^{(i\,dx+i\,c)}-de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(-4*I*a^2*e^(2*I*d*x + 2*I*c) - 2*I*a^2*e^(I*d*x + I*c) - 6*I*a^2)
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + (d*e*e^(I*d*x
+ I*c) - d*e)*integral(sqrt(2)*(-3*I*a^2*e^(2*I*d*x + 2*I*c) - 6*I*a^2*e^(I
*d*x + I*c) - 3*I*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2
*I*c)/(d*e*e^(3*I*d*x + 3*I*c) - 2*d*e*e^(2*I*d*x + 2*I*c) + d*e*e^(I*d*x +
 I*c)), x))/(d*e*e^(I*d*x + I*c) - d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx+\int -\frac{\tan^2(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx+\int \frac{2i\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*sec(c + d*x)), x) + Integral(-tan(c + d*x)**2/sqrt(
e*sec(c + d*x)), x) + Integral(2*I*tan(c + d*x)/sqrt(e*sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan{(dx+c)}+a)^2}{\sqrt{e\sec{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/sqrt(e*sec(d*x + c)), x)
```

3.196 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] (-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*d*e^2) - (((4*I)/3)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(3/2
))
```

Rubi [A] time = 0.0713038, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3496, 3771, 2641}

$$-\frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(3/2),x]
```


```
[Out] (-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*d*e^2) - (((4*I)/3)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(3/2
))
```


Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}} - \frac{a^2\int\sqrt{e\sec(c+dx)}\,dx}{3e^2}$$
$$= -\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}} - \frac{\left(a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2}$$
$$= -\frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{3d(e\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.437026, size = 114, normalized size = 1.34

$$-\frac{2a^2\sec^2(c+dx)(\cos(c+3dx)+i\sin(c+3dx))\left(2i\cos(c+dx)+\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)-i\sin(c\right.}{3d(\cos(dx)+i\sin(dx))^2(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*Sec[c + d*x]^2*((2*I)*Cos[c + d*x] + Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]))*(Cos[c + 3*d*x] + I*Sin[c +
 3*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.224, size = 173, normalized size = 2.

$$-\frac{2a^2}{3d(\cos(dx+c))^2}\left(i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)+i\sqrt{(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3*a^2/d*(I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+I*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I
)+2*I*cos(d*x+c)^2-2*cos(d*x+c)*sin(d*x+c))/(e/cos(d*x+c))^(3/2)/cos(d*x+c)
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,de^2\text{integral}\left(\frac{i\sqrt{2}a^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,de^2}, x\right)+\sqrt{2}\left(-2i\,a^2e^{(2i\,dx+2i\,c)}-2i\,a^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{3\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*e^2*integral(1/3*I*sqrt(2)*a^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e
^(-1/2*I*d*x - 1/2*I*c)/(d*e^2), x) + sqrt(2)*(-2*I*a^2*e^(2*I*d*x + 2*I*c)
 - 2*I*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e
^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{1}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int-\frac{\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\frac{2i\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*sec(c + d*x))**(-3/2), x) + Integral(-tan(c + d*x)**2/(e*
sec(c + d*x))**(3/2), x) + Integral(2*I*tan(c + d*x)/(e*sec(c + d*x))**(3/2
), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(3/2), x)
```

3.197 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] (2*a^2*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) - (((4*I)/5)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(5/2)
)
```

Rubi [A] time = 0.0715836, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3496, 3771, 2639}

$$\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) - (((4*I)/5)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(5/2)
)
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}} + \frac{a^2\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}$$
$$= -\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}} + \frac{a^2\int \sqrt{\cos(c+dx)}\,dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{5d(e\sec(c+dx))^{5/2}}$$

Mathematica [C] time = 0.982941, size = 114, normalized size = 1.34

$$-\frac{i\sqrt{2}a^2\left(1+e^{2i(c+dx)}\right)^{3/2}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{3/2}\left(2\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+3\sqrt{1+e^{2i(c+dx)}}\right)}{15de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-I/15)*Sqrt[2]*a^2*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(3/2)*
(1 + E^((2*I)*(c + d*x)))^(3/2)*(3*Sqrt[1 + E^((2*I)*(c + d*x))] + 2*Hyperg
eometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(d*e^4)
```

Maple [B] time = 0.203, size = 343, normalized size = 4.

$$-\frac{2a^2}{5d(\cos(dx+c))^3\sin(dx+c)}\left(i\text{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\text{co}}{\cos(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/5*a^2/d*(I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*EllipticF(I*
(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),
I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*
EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*I*sin(d*x+c)*cos(d*x+c)^3+2*cos(d*x+c
)^4-cos(d*x+c)^2-cos(d*x+c))/cos(d*x+c)^3/sin(d*x+c)/(e/cos(d*x+c))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-i\,a^2e^{(4i\,dx+4i\,c)}+i\,a^2e^{(3i\,dx+3i\,c)}-3i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2e^{(i\,dx+i\,c)}-2i\,a^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\left(de^3e^{(i\,dx+i\,c)}-de^3\right.}{5\left(de^3e^{(i\,dx+i\,c)}-de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-I*a^2*e^(4*I*d*x + 4*I*c) + I*a^2*e^(3*I*d*x + 3*I*c) - 3*I*
a^2*e^(2*I*d*x + 2*I*c) + I*a^2*e^(I*d*x + I*c) - 2*I*a^2)*sqrt(e/(e^(2*I*d
*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*(d*e^3*e^(I*d*x + I*c) - d*e^
3)*integral(1/5*sqrt(2)*(-I*a^2*e^(2*I*d*x + 2*I*c) - 2*I*a^2*e^(I*d*x + I*
c) - I*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^
3*e^(3*I*d*x + 3*I*c) - 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3*e^(I*d*x + I*c)
), x))/(d*e^3*e^(I*d*x + I*c) - d*e^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{1}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx+\int-\frac{\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx+\int\frac{2i\tan(c+dx)}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] a**2*(Integral((e*sec(c + d*x))**(-5/2), x) + Integral(-tan(c + d*x)**2/(e*
sec(c + d*x))**(5/2), x) + Integral(2*I*tan(c + d*x)/(e*sec(c + d*x))**(5/2
), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(5/2), x)
```

3.198 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}}+\frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7de^4}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
7*d*e^4) + (2*a^2*Sin[c + d*x])/(7*d*e^3*Sqrt[e*Sec[c + d*x]]) - (((4*I)/7)
*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(7/2))
```

Rubi [A] time = 0.0873508, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3496, 3769, 3771, 2641}

$$\frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}}+\frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7de^4}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
7*d*e^4) + (2*a^2*Sin[c + d*x])/(7*d*e^3*Sqrt[e*Sec[c + d*x]]) - (((4*I)/7)
*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(7/2))
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}+\frac{\left(3a^2\right)\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}$$
$$=\frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}+\frac{a^2\int\sqrt{e\sec(c+dx)}\,dx}{7e^4}$$
$$=\frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{7d(e\sec(c+dx))^{7/2}}+\frac{\left(a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{}{\sqrt{c}}}{7e^4}$$
$$=\frac{2a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7de^4}+\frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\text{ta}\right.}{7d(e\sec(c+}$$

Mathematica [A] time = 0.877063, size = 133, normalized size = 1.15

$$\frac{a^2\sqrt{e\sec(c+dx)}(\cos(2(c+2dx))+i\sin(2(c+2dx)))\left(-\sin(2(c+dx))-2i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.\right.}{7de^4(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (a^2*Sqrt[e*Sec[c + d*x]]*(-2*I - (2*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]) - Si
n[2*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(7*d*e^4*(Cos[
d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.203, size = 189, normalized size = 1.6

$$-\frac{2a^2}{7d(\cos(dx+c))^4}\left(2i(\cos(dx+c))^4-i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i(\cos(dx+}{\sin(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x)
```

```
[Out] -2/7*a^2/d*(2*I*cos(d*x+c)^4-I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-I*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c
)-1)/sin(d*x+c),I)-2*cos(d*x+c)^3*sin(d*x+c)-cos(d*x+c)*sin(d*x+c))/cos(d*x
+c)^4/(e/cos(d*x+c))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{14\,de^4\text{integral}\left(-\frac{i\sqrt{2}a^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{7\,de^4},x\right)+\sqrt{2}\left(-i\,a^2e^{(4i\,dx+4i\,c)}-4i\,a^2e^{(2i\,dx+2i\,c)}-3i\,a^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{14\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/14*(14*d*e^4*integral(-1/7*I*sqrt(2)*a^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^4), x) + sqrt(2)*(-I*a^2*e^(4*I*d*x + 4*I*c
) - 4*I*a^2*e^(2*I*d*x + 2*I*c) - 3*I*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(7/2), x)
```

3.199 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}+\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}$$

```
[Out] (2*a^2*EllipticE[(c + d*x)/2, 2])/(3*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) + (2*a^2*Sin[c + d*x])/(9*d*e^3*(e*Sec[c + d*x])^(3/2)) - (((4*I)/
9)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(9/2))
```

Rubi [A] time = 0.0871165, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3496, 3769, 3771, 2639}

$$\frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}+\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (2*a^2*EllipticE[(c + d*x)/2, 2])/(3*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) + (2*a^2*Sin[c + d*x])/(9*d*e^3*(e*Sec[c + d*x])^(3/2)) - (((4*I)/
9)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(9/2))
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}+\frac{\left(5a^2\right)\int \frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{9e^2}$$

$$=\frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}+\frac{a^2\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{3e^4}$$

$$=\frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}+\frac{a^2\int \sqrt{\cos(c+dx)}\,dx}{3e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$=\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{9d(e\sec(c+dx))^{9/2}}$$

Mathematica [C] time = 1.70416, size = 133, normalized size = 1.15

$$\frac{ia^2\left(-\frac{8e^{2i(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-4e^{2i(c+dx)}-e^{4i(c+dx)}+9\right)}{18\sqrt{2}de^4\sqrt{\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] ((I/18)*a^2*(9 - 4*E^((2*I)*(c + d*x)) - E^((4*I)*(c + d*x)) - (8*E^((2*I)*
(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/Sqrt[1 +
 E^((2*I)*(c + d*x))]))/(Sqrt[2]*d*e^4*Sqrt[(e*E^(I*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))])
```

Maple [B] time = 0.254, size = 353, normalized size = 3.

$$-\frac{2a^2}{9d(\cos(dx+c))^5\sin(dx+c)}\left(2i(\cos(dx+c))^5\sin(dx+c)+3i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticE}\left(\frac{i}{\ }\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x)
```

```
[Out] -2/9*a^2/d*(2*I*cos(d*x+c)^5*sin(d*x+c)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*
x+c)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+2*cos(d*
x+c)^6+3*I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*sin(d*x+c)*EllipticF(I*(
cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)-cos(d*x+c)^4+2*cos(d*x+c)^2-3*cos(d*x+c))/cos(d*x+c)^5/sin(d*x+
c)/(e/cos(d*x+c))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan (dx + c) + a)^2}{(e \sec (dx + c))^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-i a^2 e^{(7i dx+7i c)} + i a^2 e^{(6i dx+6i c)} - 5i a^2 e^{(5i dx+5i c)} + 5i a^2 e^{(4i dx+4i c)} - 19i a^2 e^{(3i dx+3i c)} - 5i a^2 e^{(2i dx+2i c)} - 15i a^2 e^{(i dx+i c}\right.}{36\left(de^5 e^{(2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/36*(sqrt(2)*(-I*a^2*e^(7*I*d*x + 7*I*c) + I*a^2*e^(6*I*d*x + 6*I*c) - 5*I
*a^2*e^(5*I*d*x + 5*I*c) + 5*I*a^2*e^(4*I*d*x + 4*I*c) - 19*I*a^2*e^(3*I*d*
x + 3*I*c) - 5*I*a^2*e^(2*I*d*x + 2*I*c) - 15*I*a^2*e^(I*d*x + I*c) - 9*I*a
^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 36*(d*e^5*e
^(2*I*d*x + 2*I*c) - d*e^5*e^(I*d*x + I*c))*integral(1/3*sqrt(2)*(-I*a^2*e^
(2*I*d*x + 2*I*c) - 2*I*a^2*e^(I*d*x + I*c) - I*a^2)*sqrt(e/(e^(2*I*d*x + 2
*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^5*e^(3*I*d*x + 3*I*c) - 2*d*e^5*e^
(2*I*d*x + 2*I*c) + d*e^5*e^(I*d*x + I*c)), x))/(d*e^5*e^(2*I*d*x + 2*I*c)
- d*e^5*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan (dx + c) + a)^2}{(e \sec (dx + c))^{\frac{9}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(9/2), x)
```

3.200 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=147

$$\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}+\frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}+\frac{10a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33de^6}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

```
[Out] (10*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(33*d*e^6) + (2*a^2*Sin[c + d*x])/(11*d*e^3*(e*Sec[c + d*x])^(5/2)) + (10*a
^2*Sin[c + d*x])/(33*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((4*I)/11)*(a^2 + I*a^2
*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(11/2))
```

Rubi [A] time = 0.105537, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3496, 3769, 3771, 2641}

$$\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}+\frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}+\frac{10a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33de^6}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (10*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(33*d*e^6) + (2*a^2*Sin[c + d*x])/(11*d*e^3*(e*Sec[c + d*x])^(5/2)) + (10*a
^2*Sin[c + d*x])/(33*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((4*I)/11)*(a^2 + I*a^2
*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(11/2))
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}+\frac{\left(7a^2\right)\int\frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{11e^2}$$

$$= \frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}+\frac{\left(5a^2\right)\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{11e^4}$$

$$= \frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}+\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}+\frac{\left(5a^2\right)\int\sqrt{e}\ldots}{\ldots}$$

$$= \frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}+\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}-\frac{4i\left(a^2+ia^2\tan(c+dx)\right)}{11d(e\sec(c+dx))^{11/2}}+\frac{\left(5a^2\sqrt{\cos(\ldots}\right.}{\ldots}$$

$$= \frac{10a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33de^6}+\frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}+\frac{10a^2\sin\ldots}{33de^5\sqrt{e\ldots}}$$

Mathematica [A] time = 1.24247, size = 155, normalized size = 1.05

$$\frac{a^2\sqrt{e\sec(c+dx)}(\cos(2(c+2dx))+i\sin(2(c+2dx)))\left(-6\sin(2(c+dx))+7\sin(4(c+dx))-24i\cos(2(c+dx))+4i\,c\ldots\right.}{132de^6(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (a^2*Sqrt[e*Sec[c + d*x]]*(-28*I - (24*I)*Cos[2*(c + d*x)] + (4*I)*Cos[4*(c
 + d*x)] + 40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)
] - I*Sin[2*(c + d*x)]) - 6*Sin[2*(c + d*x)] + 7*Sin[4*(c + d*x)])*(Cos[2*(
c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(132*d*e^6*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.283, size = 205, normalized size = 1.4

$$-\frac{2a^2}{33d(\cos(dx+c))^6}\left(6i(\cos(dx+c))^6-6(\cos(dx+c))^5\sin(dx+c)-5i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos\ldots}{\cos(\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x)
```

```
[Out] -2/33*a^2/d*(6*I*cos(d*x+c)^6-6*cos(d*x+c)^5*sin(d*x+c)-5*I*cos(d*x+c)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x
+c)-1)/sin(d*x+c),I)-5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-3*cos(d*x+c)^3*sin(d*x+c)
-5*cos(d*x+c)*sin(d*x+c))/cos(d*x+c)^6/(e/cos(d*x+c))^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(264\,de^6e^{(2i\,dx+2i\,c)}\text{integral}\left(-\frac{5i\sqrt{2}a^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{33\,de^6},x\right)+\sqrt{2}\left(-3i\,a^2e^{(8i\,dx+8i\,c)}-18i\,a^2e^{(6i\,dx+6i\,c)}-56i\,a^2e^{(4i\,dx+4i\,c)}-\right.\right.}{264\,de^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/264*(264*d*e^6*e^(2*I*d*x + 2*I*c)*integral(-5/33*I*sqrt(2)*a^2*sqrt(e/(e
^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^6), x) + sqrt(2)*(-3
*I*a^2*e^(8*I*d*x + 8*I*c) - 18*I*a^2*e^(6*I*d*x + 6*I*c) - 56*I*a^2*e^(4*I
*d*x + 4*I*c) - 30*I*a^2*e^(2*I*d*x + 2*I*c) + 11*I*a^2)*sqrt(e/(e^(2*I*d*x
 + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(d*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(11/2), x)
```

3.201 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=202

$$\frac{2a^3e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2a^3e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2a^3e\sin(c+dx)(e\sec(c+}{3d}$$

```
[Out] (-2*a^3*e^4*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((10*I)/21)*a^3*(e*Sec[c + d*x])^(7/2))/d + (2*a^3*e^3*Sqrt[e*Se
c[c + d*x]]*Sin[c + d*x])/d + (2*a^3*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])
/(3*d) + (((2*I)/11)*a*(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2)/d +
 (((10*I)/33)*(e*Sec[c + d*x])^(7/2)*(a^3 + I*a^3*Tan[c + d*x]))/d
```

Rubi [A] time = 0.201336, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3498, 3486, 3768, 3771, 2639}

$$\frac{2a^3e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}-\frac{2a^3e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2a^3e\sin(c+dx)(e\sec(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-2*a^3*e^4*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((10*I)/21)*a^3*(e*Sec[c + d*x])^(7/2))/d + (2*a^3*e^3*Sqrt[e*Se
c[c + d*x]]*Sin[c + d*x])/d + (2*a^3*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])
/(3*d) + (((2*I)/11)*a*(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2)/d +
 (((10*I)/33)*(e*Sec[c + d*x])^(7/2)*(a^3 + I*a^3*Tan[c + d*x]))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx &= \frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} + \frac{1}{11}(15a)\int (e\sec(c+dx))^{7/2}(a+\\
&= \frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} + \frac{10i(e\sec(c+dx))^{7/2}\left(a^3+ia^3\text{t}\right.}{33d}\\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d} + \frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} + \frac{10i(e}{}\\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d} + \frac{2a^3e(e\sec(c+dx))^{5/2}\sin(c+dx)}{3d} + \frac{2ia(e\sec(c+}{}\\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d} + \frac{2a^3e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^3e(e\sec(c+}{}\\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d} + \frac{2a^3e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^3e(e\sec(c+}{}\\
&= -\frac{2a^3e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d} + \frac{2a^3e^3\sqrt{e\sec(c+}}{}
\end{aligned}
$$

Mathematica [C] time = 7.55664, size = 442, normalized size = 2.19

$$
\frac{2i\sqrt{2}e^{-i(2c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\left(-1+e^{2ic}\right)e^{2idx}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)(a+i}{3\left(-1+e^{2ic}\right)d\sec^{\frac{13}{2}}(c+dx)(\cos(dx)+i\sin(dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((2*I)/3)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 +
 E^((2*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1
 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(e*
Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3)/(d*E^(I*(2*c + d*x))*(-1 + E^
((2*I)*c))*Sec[c + d*x]^(13/2)*(Cos[d*x] + I*Sin[d*x])^3) + (Cos[c + d*x]^6
*(e*Sec[c + d*x])^(7/2)*(Sec[c + d*x]^5*(((-2*I)/11)*Cos[3*c] - (2*Sin[3*c]
)/11) + Cos[d*x]*Csc[c]*(2*Cos[3*c] - (2*I)*Sin[3*c]) + Sec[c]*Sec[c + d*x]
^3*(12*Cos[c] + (7*I)*Sin[c])*(((2*I)/21)*Cos[3*c] + (2*Sin[3*c])/21) + Sec
[c]*Sec[c + d*x]^2*((2*Cos[3*c])/3 - ((2*I)/3)*Sin[3*c])*Sin[d*x] + Sec[c]*
Sec[c + d*x]^4*((-2*Cos[3*c])/3 + ((2*I)/3)*Sin[3*c])*Sin[d*x] + Sec[c + d*
x]*((2*Cos[3*c])/3 - ((2*I)/3)*Sin[3*c])*Tan[c])*(a + I*a*Tan[c + d*x])^3)/
(d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] time = 0.371, size = 402, normalized size = 2.

$$\frac{2\,a^{3}\left(\cos\left(dx+c\right)+1\right)^{2}\left(\cos\left(dx+c\right)-1\right)^{2}}{231\,d\left(\cos\left(dx+c\right)\right)^{2}\left(\sin\left(dx+c\right)\right)^{5}}\left(231\,i\left(\cos\left(dx+c\right)\right)^{6}\sqrt{\left(\cos\left(dx+c\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(dx+c\right)}{\cos\left(dx+c\right)+1}}\mathrm{EllipticE}\left(\frac{i\,(}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/231*a^3/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(231*I*cos(d*x+c)^6*(1/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1
)/sin(d*x+c),I)*sin(d*x+c)-231*I*cos(d*x+c)^6*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(
d*x+c)+231*I*cos(d*x+c)^5*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-231*I*cos(d*x
+c)^5*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(
I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-231*cos(d*x+c)^6+154*cos(d*x+c)^5
+132*I*cos(d*x+c)^2*sin(d*x+c)+154*cos(d*x+c)^3-21*I*sin(d*x+c)-77*cos(d*x+
c))*(e/cos(d*x+c))^(7/2)/cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-462i\,a^{3}e^{3}e^{(11i\,dx+11i\,c)}-2618i\,a^{3}e^{3}e^{(9i\,dx+9i\,c)}-1892i\,a^{3}e^{3}e^{(7i\,dx+7i\,c)}-1740i\,a^{3}e^{3}e^{(5i\,dx+5i\,c)}-814i\,a^{3}e^{3}e^{(3i\,dx+3i\,c)}-1\right.}{231\left(de^{(10i\,dx+10i\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/231*(sqrt(2)*(-462*I*a^3*e^3*e^(11*I*d*x + 11*I*c) - 2618*I*a^3*e^3*e^(9*
I*d*x + 9*I*c) - 1892*I*a^3*e^3*e^(7*I*d*x + 7*I*c) - 1740*I*a^3*e^3*e^(5*I
*d*x + 5*I*c) - 814*I*a^3*e^3*e^(3*I*d*x + 3*I*c) - 154*I*a^3*e^3*e^(I*d*x
+ I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*(d*
e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c)
+ 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)*integral(I*sqrt(2
)*a^3*e^3*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/d, x))/
(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*
c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec (dx+c))^{\frac{7}{2}}(ia\tan (dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^3, x)
```

3.202 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=175

$$\frac{26a^3e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21d}+\frac{26ia^3(e\sec(c+dx))^{5/2}}{35d}+\frac{26a^3e\sin(c+dx)(e\sec(c+dx))^{3/2}}{21d}+$$

```
[Out] (26*a^3*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x
]])/(21*d) + (((26*I)/35)*a^3*(e*Sec[c + d*x])^(5/2))/d + (26*a^3*e*(e*Sec[
c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (((2*I)/9)*a*(e*Sec[c + d*x])^(5/2)*
(a + I*a*Tan[c + d*x])^2)/d + (((26*I)/63)*(e*Sec[c + d*x])^(5/2)*(a^3 + I*
a^3*Tan[c + d*x]))/d
```

Rubi [A] time = 0.187791, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3498, 3486, 3768, 3771, 2641}

$$\frac{26a^3e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21d}+\frac{26ia^3(e\sec(c+dx))^{5/2}}{35d}+\frac{26a^3e\sin(c+dx)(e\sec(c+dx))^{3/2}}{21d}+$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```


```
[Out] (26*a^3*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x
]])/(21*d) + (((26*I)/35)*a^3*(e*Sec[c + d*x])^(5/2))/d + (26*a^3*e*(e*Sec[
c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (((2*I)/9)*a*(e*Sec[c + d*x])^(5/2)*
(a + I*a*Tan[c + d*x])^2)/d + (((26*I)/63)*(e*Sec[c + d*x])^(5/2)*(a^3 + I*
a^3*Tan[c + d*x]))/d
```


Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} + \frac{1}{9}(13a)\int (e\sec(c+dx))^{5/2}(a+$$

$$= \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} + \frac{26i(e\sec(c+dx))^{5/2}\left(a^3+ia^3\mathrm{t}\right.}{63d}$$

$$= \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} + \frac{26i(e}{}$$

$$= \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{26a^3e(e\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2ia(e\sec(c+}{}$$

$$= \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{26a^3e(e\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2ia(e\sec(c+}{}$$

$$= \frac{26a^3e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21d} + \frac{26ia^3(e\sec(c+dx))^5}{35d}$$

Mathematica [A] time = 1.69703, size = 89, normalized size = 0.51

$$\frac{a^3\sec^2(c+dx)(e\sec(c+dx))^{5/2}\left(-150\sin(2(c+dx))+195\sin(4(c+dx))+1008i\cos(2(c+dx))+1560\cos^{\frac{9}{2}}(c+dx)F\left(\right.\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(e*Sec[c + d*x])^(5/2)*(728*I + (1008*I)*Cos[2*(c + d*x
)] + 1560*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2, 2] - 150*Sin[2*(c + d*x
)] + 195*Sin[4*(c + d*x)]))/(1260*d)
```

Maple [A] time = 0.292, size = 229, normalized size = 1.3

$$\frac{2a^3(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2}{315d(\cos(dx+c))^2(\sin(dx+c))^4}\left(195i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c))^5\,\mathrm{EllipticF}\left(\frac{i(\cos}{\mathrm{si}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/315*a^3/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(195*I*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*EllipticF(I*(cos(d*x+c)-1
)/sin(d*x+c),I)+195*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*cos(d*x+c)^4*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+195*cos(d*x+c)^
```

```
3*sin(d*x+c)+252*I*cos(d*x+c)^2-135*cos(d*x+c)*sin(d*x+c)-35*I)*(e/cos(d*x+
c))^(5/2)/cos(d*x+c)^2/sin(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e \sec(dx + c))^{\frac{5}{2}} (i a \tan(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-390i\,a^3e^2e^{(8i\,dx+8i\,c)} + 2316i\,a^3e^2e^{(6i\,dx+6i\,c)} + 2912i\,a^3e^2e^{(4i\,dx+4i\,c)} + 1716i\,a^3e^2e^{(2i\,dx+2i\,c)} + 390i\,a^3e^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e}{315\left(de^{(8i\,dx+8i\,c)} + 4\,de^{(6i\,dx+6i}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/315*(sqrt(2)*(-390*I*a^3*e^2*e^(8*I*d*x + 8*I*c) + 2316*I*a^3*e^2*e^(6*I*
d*x + 6*I*c) + 2912*I*a^3*e^2*e^(4*I*d*x + 4*I*c) + 1716*I*a^3*e^2*e^(2*I*d
*x + 2*I*c) + 390*I*a^3*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
 + 1/2*I*c) + 315*(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^
(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)*integral(-13/21*I*sqrt(2)*
a^3*e^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/d, x))/(
d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) +
 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e \sec(dx + c))^{\frac{5}{2}} (i a \tan(dx + c) + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^3, x)
```

3.203 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=175

$$-\frac{22a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}+\frac{22a^3e\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d}+\frac{22i\left(a^3+ia^3\tan(c+\ldots\right.}{35\ldots}$$

```
[Out] (-22*a^3*e^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((22*I)/15)*a^3*(e*Sec[c + d*x])^(3/2))/d + (22*a^3*e*Sqrt[e*
Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (((2*I)/7)*a*(e*Sec[c + d*x])^(3/2)*(a
+ I*a*Tan[c + d*x])^2)/d + (((22*I)/35)*(e*Sec[c + d*x])^(3/2)*(a^3 + I*a^3
*Tan[c + d*x]))/d
```

Rubi [A] time = 0.185427, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3498, 3486, 3768, 3771, 2639}

$$-\frac{22a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}+\frac{22a^3e\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d}+\frac{22i\left(a^3+ia^3\tan(c+\ldots\right.}{35\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-22*a^3*e^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((22*I)/15)*a^3*(e*Sec[c + d*x])^(3/2))/d + (22*a^3*e*Sqrt[e*
Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (((2*I)/7)*a*(e*Sec[c + d*x])^(3/2)*(a
+ I*a*Tan[c + d*x])^2)/d + (((22*I)/35)*(e*Sec[c + d*x])^(3/2)*(a^3 + I*a^3
*Tan[c + d*x]))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d} + \frac{1}{7}(11a)\int (e\sec(c+dx))^{3/2}(a+$$
$$= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d} + \frac{22i(e\sec(c+dx))^{3/2}\left(a^3+ia^3\text{t}\right.}{35d}$$
$$= \frac{22ia^3(e\sec(c+dx))^{3/2}}{15d} + \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d} + \frac{22i(e}{}$$
$$= \frac{22ia^3(e\sec(c+dx))^{3/2}}{15d} + \frac{22a^3e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2ia(e\sec(c+}{}$$
$$= \frac{22ia^3(e\sec(c+dx))^{3/2}}{15d} + \frac{22a^3e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2ia(e\sec(c+}{}$$
$$= -\frac{22a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{22ia^3(e\sec(c+dx))^{3/2}}{15d} + \frac{22a^3e\sqrt{e\sec(c}}{}$$

Mathematica [C] time = 2.46012, size = 129, normalized size = 0.74

$$\frac{a^3(1+i\tan(c+dx))(e\sec(c+dx))^{3/2}\left(77ie^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-308i\cos(\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(e*Sec[c + d*x])^(3/2)*(1 + I*Tan[c + d*x])*(-116*I - (308*I)*Cos[2*(c
 + d*x)] + ((77*I)*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 77*Sec[c + d*x]*Sin[3
*(c + d*x)] + 17*Tan[c + d*x]))/(210*d)
```

Maple [B] time = 0.281, size = 392, normalized size = 2.2

$$\frac{2a^3(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2}{105d(\sin(dx+c))^5(\cos(dx+c))^2}\left(231i\sin(dx+c)(\cos(dx+c))^4\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/105*a^3/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(231*I*sin(d*x+c)*cos(d*x+c)^
4*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(c
os(d*x+c)-1)/sin(d*x+c),I)-231*I*sin(d*x+c)*cos(d*x+c)^4*(1/(cos(d*x+c)+1))
```

```
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x
+c),I)+231*I*sin(d*x+c)*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-231*I*sin(d*x
+c)*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+140*I*cos(d*x+c)^2*sin(d*x+c)-231
*cos(d*x+c)^4+294*cos(d*x+c)^3-15*I*sin(d*x+c)-63*cos(d*x+c))*(e/cos(d*x+c)
)^(3/2)/sin(d*x+c)^5/cos(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{3}{2}} (ia\tan(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-462i\,a^3ee^{(7i\,dx+7i\,c)} - 574i\,a^3ee^{(5i\,dx+5i\,c)} - 506i\,a^3ee^{(3i\,dx+3i\,c)} - 154i\,a^3ee^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + 105\left(de^{(6\cdots\right.}}{105\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4i\,dx+4i\,c)} + 3\,de^{(2i\,dx\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(sqrt(2)*(-462*I*a^3*e*e^(7*I*d*x + 7*I*c) - 574*I*a^3*e*e^(5*I*d*x +
 5*I*c) - 506*I*a^3*e*e^(3*I*d*x + 3*I*c) - 154*I*a^3*e*e^(I*d*x + I*c))*sq
rt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 105*(d*e^(6*I*d*x
 + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*integral
(11/5*I*sqrt(2)*a^3*e*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*
I*c)/d, x))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d
*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{3}{2}}(ia\tan(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3, x)
```

3.204 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=139

$$\frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{6i\left(a^3+ia^3\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{5d}+\frac{6a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2i}{}$$

```
[Out] ((6*I)*a^3*Sqrt[e*Sec[c + d*x]])/d + (6*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d + (((2*I)/5)*a*Sqrt[e*Sec[c + d*x]]*(
a + I*a*Tan[c + d*x])^2)/d + (((6*I)/5)*Sqrt[e*Sec[c + d*x]]*(a^3 + I*a^3*T
an[c + d*x]))/d
```

Rubi [A] time = 0.139344, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3498, 3486, 3771, 2641}

$$\frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{6i\left(a^3+ia^3\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{5d}+\frac{6a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2i}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((6*I)*a^3*Sqrt[e*Sec[c + d*x]])/d + (6*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d + (((2*I)/5)*a*Sqrt[e*Sec[c + d*x]]*(
a + I*a*Tan[c + d*x])^2)/d + (((6*I)/5)*Sqrt[e*Sec[c + d*x]]*(a^3 + I*a^3*T
an[c + d*x]))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d} + \frac{1}{5}(9a)\int \sqrt{e\sec(c+dx)}(a+ia\tan($$

$$= \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d} + \frac{6i\sqrt{e\sec(c+dx)}\left(a^3+ia^3\tan(c+\right.}{5d}$$

$$= \frac{6ia^3\sqrt{e\sec(c+dx)}}{d} + \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d} + \frac{6i\sqrt{e\sec(c+}}{}$$

$$= \frac{6ia^3\sqrt{e\sec(c+dx)}}{d} + \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d} + \frac{6i\sqrt{e\sec(c+}}{}$$

$$= \frac{6ia^3\sqrt{e\sec(c+dx)}}{d} + \frac{6a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{d} + \frac{2ia}{}$$

Mathematica [A] time = 1.24938, size = 79, normalized size = 0.57

$$\frac{a^3\sec^2(c+dx)\sqrt{e\sec(c+dx)}\left(-5\sin(2(c+dx))+20i\cos(2(c+dx))+30\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+18i\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(18*I + (20*I)*Cos[2*(c + d*x)] +
30*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[2*(c + d*x)]))/(5*d
)
```

Maple [A] time = 0.276, size = 213, normalized size = 1.5

$$\frac{2a^3(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2}{5d(\cos(dx+c))^2(\sin(dx+c))^4}\left(15i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c))^3\,\mathrm{EllipticF}\left(\frac{i(\cos(}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/5*a^3/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(15*I*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticF(I*(cos(d*x+c)-1)/s
in(d*x+c),I)+15*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*cos(d*x+c)^2*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+20*I*cos(d*x+c)^2-5
*cos(d*x+c)*sin(d*x+c)-I)*(e/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(50 i a^3 e^{(4 i dx+4 i c)} + 72 i a^3 e^{(2 i dx+2 i c)} + 30 i a^3\right)\sqrt{\frac{e}{e^{(2 i dx+2 i c)}+1}}e^{\left(\frac{1}{2} i dx+\frac{1}{2} i c\right)} + 5\left(de^{(4 i dx+4 i c)} + 2 de^{(2 i dx+2 i c)} + d\right)\mathrm{integral}\left(-\right.}{5\left(de^{(4 i dx+4 i c)} + 2 de^{(2 i dx+2 i c)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(50*I*a^3*e^(4*I*d*x + 4*I*c) + 72*I*a^3*e^(2*I*d*x + 2*I*c) +
 30*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*(d
*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*integral(-3*I*sqrt(2)*a
^3*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/d, x))/(d*e^(
4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int \sqrt{e\sec(c+dx)}\,dx + \int -3\sqrt{e\sec(c+dx)}\tan^2(c+dx)\,dx + \int 3i\sqrt{e\sec(c+dx)}\tan(c+dx)\,dx + \int -i\sqrt{e\sec}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sqrt(e*sec(c + d*x)), x) + Integral(-3*sqrt(e*sec(c + d*x))*
tan(c + d*x)**2, x) + Integral(3*I*sqrt(e*sec(c + d*x))*tan(c + d*x), x) +
Integral(-I*sqrt(e*sec(c + d*x))*tan(c + d*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\sec(dx+c)}(i a\tan(dx+c)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^3, x)
```

3.205 $\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=124

$$-\frac{26ia^3}{3d\sqrt{e\sec(c+dx)}}-\frac{2ia^3\tan^2(c+dx)}{3d\sqrt{e\sec(c+dx)}}-\frac{6a^3\tan(c+dx)}{d\sqrt{e\sec(c+dx)}}+\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

```
[Out] (((-26*I)/3)*a^3)/(d*Sqrt[e*Sec[c + d*x]]) + (14*a^3*EllipticE[(c + d*x)/2,
 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) - (6*a^3*Tan[c + d*x])/(d*
Sqrt[e*Sec[c + d*x]]) - (((2*I)/3)*a^3*Tan[c + d*x]^2)/(d*Sqrt[e*Sec[c + d*
x]])
```

Rubi [A] time = 0.133643, antiderivative size = 146, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3498, 3496, 3768, 3771, 2639}

$$-\frac{14a^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{de}-\frac{28i\left(a^3+ia^3\tan(c+dx)\right)}{3d\sqrt{e\sec(c+dx)}}+\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (14*a^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x
]]) - (14*a^3*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(d*e) + (((2*I)/3)*a*(a +
I*a*Tan[c + d*x])^2)/(d*Sqrt[e*Sec[c + d*x]]) - (((28*I)/3)*(a^3 + I*a^3*Ta
n[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx &= \frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}+\frac{1}{3}(7a)\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx\\ &= \frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}-\frac{28i\left(a^3+ia^3\tan(c+dx)\right)}{3d\sqrt{e\sec(c+dx)}}-\frac{\left(7a^3\right)\int (e\sec(c+dx))^{3/2}\,dx}{e^2}\\ &= -\frac{14a^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{de}+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}-\frac{28i\left(a^3+ia^3\tan(c+dx)\right)}{3d\sqrt{e\sec(c+dx)}}\\ &= -\frac{14a^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{de}+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}-\frac{28i\left(a^3+ia^3\tan(c+dx)\right)}{3d\sqrt{e\sec(c+dx)}}\\ &= \frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{14a^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{de}+\frac{2ia(a+ia\tan(c+dx}{3d\sqrt{e\sec(c+dx)}}\end{aligned}$$

Mathematica [C] time = 1.56091, size = 101, normalized size = 0.81

$$\frac{2a^3(\cos(c)+i\sin(c))(\sin(dx)-i\cos(dx))\sqrt{e\sec(c+dx)}\left(7\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-i\right.}{3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (2*a^3*Sqrt[e*Sec[c + d*x]]*(Cos[c] + I*Sin[c])*((-I)*Cos[d*x] + Sin[d*x])*
(-8 + 7*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))] - I*Tan[c + d*x]))/(3*d*e)
```

Maple [B] time = 0.317, size = 2552, normalized size = 20.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/3*a^3/d*(cos(d*x+c)-1)*(-18*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/
2)+24*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^7+6*(-cos(d*x+c)/(cos
(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^6+147*I*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)
/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-2*I*(-cos(d*x+c)/(cos(d*x
+c)+1)^2)^(3/2)*sin(d*x+c)+2*I*cos(d*x+c)^4*sin(d*x+c)*(-cos(d*x+c)/(cos(d*
```

x+c)+1)^2)^(3/2)-44*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^3*sin
(d*x+c)+6*I*cos(d*x+c)^4*sin(d*x+c)*ln(-2*(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*c
os(d*x+c)-1)/sin(d*x+c)^2)-6*I*cos(d*x+c)^4*sin(d*x+c)*ln(-(2*(-cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c
)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)-24*I*cos(d*x+c)^2*sin(d*x+c)*(-
cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-4*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)
*cos(d*x+c)*sin(d*x+c)-6*I*cos(d*x+c)^3*sin(d*x+c)*ln(-2*(2*(-cos(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+
1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)+6*I*cos(d*x+c)^3*sin(d*x+c)*ln(-(
2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)+24*I*(-cos(d*x+c
)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^6*sin(d*x+c)-66*(-cos(d*x+c)/(cos(d*x+
c)+1)^2)^(3/2)*cos(d*x+c)^5-12*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x
+c)^4+60*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^3+6*(-cos(d*x+c)/(
cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^2+48*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/
2)*cos(d*x+c)^5*sin(d*x+c)+21*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*sin(d*x+c
)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-42*I*cos(d*x+c)*sin(d*x+c)*(-cos
(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+42*I*cos(d*x+c)*si
n(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-21*I
*cos(d*x+c)^5*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/si
n(d*x+c),I)+21*I*cos(d*x+c)^5*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
cos(d*x+c)-1)/sin(d*x+c),I)-21*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*sin(d*x+
c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+21*I*(-cos(d*x+c)/(cos(d*x+c)+1
)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)^6*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-21*I*(-cos(d*x+
c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*cos(d*x+c)^6*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)
+42*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-
1)/sin(d*x+c),I)-42*I*cos(d*x+c)^4*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2
)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elliptic
E(I*(cos(d*x+c)-1)/sin(d*x+c),I)-42*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*sin
(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-42*I*(-cos(d*x+c)/(cos(d*x
+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
cos(d*x+c)^4*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+42*I*(-cos
(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*cos(d*x+c)^4*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+
c),I)-168*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*EllipticE(I*(cos(d
*x+c)-1)/sin(d*x+c),I)+168*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d
x+c)+1))^(1/2)(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*E
llipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+42*I*cos(d*x+c)^4*sin(d*x+c)*(-cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-147*I*cos(d*x+c)^2*
sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I))*(
cos(d*x+c)+1)^5*(e/cos(d*x+c))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)/c
os(d*x+c)^4/e/sin(d*x+c)^5

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^3}{\sqrt{e\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/sqrt(e*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-24i\,a^3e^{(4i\,dx+4i\,c)}-18i\,a^3e^{(3i\,dx+3i\,c)}-70i\,a^3e^{(2i\,dx+2i\,c)}-14i\,a^3e^{(i\,dx+i\,c)}-42i\,a^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\left(dee^{(3i\,d}\right.}{3\left(dee^{(3i\,dx+3i\,c)}-dee^{(2i\,dx+2i\,c)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-24*I*a^3*e^(4*I*d*x + 4*I*c) - 18*I*a^3*e^(3*I*d*x + 3*I*c)
- 70*I*a^3*e^(2*I*d*x + 2*I*c) - 14*I*a^3*e^(I*d*x + I*c) - 42*I*a^3)*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*(d*e*e^(3*I*d*x +
3*I*c) - d*e*e^(2*I*d*x + 2*I*c) + d*e*e^(I*d*x + I*c) - d*e)*integral(sqrt
(2)*(-7*I*a^3*e^(2*I*d*x + 2*I*c) - 14*I*a^3*e^(I*d*x + I*c) - 7*I*a^3)*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e*e^(3*I*d*x + 3*
I*c) - 2*d*e*e^(2*I*d*x + 2*I*c) + d*e*e^(I*d*x + I*c)), x))/(d*e*e^(3*I*d*
x + 3*I*c) - d*e*e^(2*I*d*x + 2*I*c) + d*e*e^(I*d*x + I*c) - d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx+\int -\frac{3\tan^2(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx+\int \frac{3i\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx+\int -\frac{i\tan^3(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] a**3*(Integral(1/sqrt(e*sec(c + d*x)), x) + Integral(-3*tan(c + d*x)**2/sqr
t(e*sec(c + d*x)), x) + Integral(3*I*tan(c + d*x)/sqrt(e*sec(c + d*x)), x)
+ Integral(-I*tan(c + d*x)**3/sqrt(e*sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^3}{\sqrt{e\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((I*a*tan(d*x + c) + a)^3/sqrt(e*sec(d*x + c)), x)
```

3.206 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=111

$$-\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2}-\frac{10a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] (((-10*I)/3)*a^3*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (10*a^3*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) - (((4*I)/3)*a*(
a + I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.102424, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3496, 3486, 3771, 2641}

$$-\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2}-\frac{10a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((-10*I)/3)*a^3*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (10*a^3*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) - (((4*I)/3)*a*(
a + I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(3/2))
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}} - \frac{\left(5a^2\right)\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx}{3e^2}$$

$$= -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}} - \frac{\left(5a^3\right)\int \sqrt{e\sec(c+dx)}\,dx}{3e^2}$$

$$= -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}} - \frac{\left(5a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{}{\sqrt{\text{co}}}}{3e^2}$$

$$= -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{10a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4ia(a+ia\tan}{3d(e\sec(c+}$$

Mathematica [A] time = 0.654838, size = 123, normalized size = 1.11

$$-\frac{2a^3\sec^2(c+dx)(\cos(c+4dx)+i\sin(c+4dx))\left(3\sin(c+dx)+7i\cos(c+dx)+5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+\right.}{3d(\cos(dx)+i\sin(dx))^3(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^3*Sec[c + d*x]^2*((7*I)*Cos[c + d*x] + 5*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + 3*Sin[c + d*x])*(Cos[c +
 4*d*x] + I*Sin[c + 4*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[
d*x])^3)
```

Maple [A] time = 0.274, size = 175, normalized size = 1.6

$$-\frac{2a^3}{3d(\cos(dx+c))^2}\left(5i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)+5i\sqrt{(\text{co}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3*a^3/d*(5*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+5*I*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+
c),I)+4*I*cos(d*x+c)^2-4*cos(d*x+c)*sin(d*x+c)+3*I)/(e/cos(d*x+c))^(3/2)/co
s(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\,de^2\text{integral}\left(\frac{5i\,\sqrt{2}a^3\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,de^2},x\right)+\sqrt{2}\left(-4i\,a^3e^{(2i\,dx+2i\,c)}-10i\,a^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{3\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*e^2*integral(5/3*I*sqrt(2)*a^3*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e
^(-1/2*I*d*x - 1/2*I*c)/(d*e^2), x) + sqrt(2)*(-4*I*a^3*e^(2*I*d*x + 2*I*c)
 - 10*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*
e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int\frac{1}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int-\frac{3\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\frac{3i\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int-\frac{i\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] a**3*(Integral((e*sec(c + d*x))**(-3/2), x) + Integral(-3*tan(c + d*x)**2/(
e*sec(c + d*x))**(3/2), x) + Integral(3*I*tan(c + d*x)/(e*sec(c + d*x))**(3
/2), x) + Integral(-I*tan(c + d*x)**3/(e*sec(c + d*x))**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(3/2), x)
```

3.207 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=111

$$\frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}}-\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] (((6*I)/5)*a^3)/(d*e^2*Sqrt[e*Sec[c + d*x]]) - (6*a^3*EllipticE[(c + d*x)/2
, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) - (((4*I)/5)*a*(a +
 I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(5/2))
```

Rubi [A] time = 0.101171, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3496, 3486, 3771, 2639}

$$\frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}}-\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(5/2),x]
```


```
[Out] (((6*I)/5)*a^3)/(d*e^2*Sqrt[e*Sec[c + d*x]]) - (6*a^3*EllipticE[(c + d*x)/2
, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) - (((4*I)/5)*a*(a +
 I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(5/2))
```


Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}} - \frac{\left(3a^2\right)\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}$$

$$= \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}} - \frac{\left(3a^3\right)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}$$

$$= \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}} - \frac{\left(3a^3\right)\int \sqrt{\cos(c+dx)}\,dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}} - \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

Mathematica [C] time = 1.25711, size = 108, normalized size = 0.97

$$-\frac{4ia^3e^{2i(c+dx)}\left(-\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+e^{2i(c+dx)}+1\right)}{5de^2\left(1+e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/5)*a^3*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)) - Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/
(d*e^2*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Sec[c + d*x]])
```

Maple [B] time = 0.245, size = 1086, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/10*a^3/d*(cos(d*x+c)+1)*(cos(d*x+c)-1)^2*(-12*I*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*Ell
ipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)^2*sin(d*x+c)+12*I*(-cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-12*I*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+
c)-1)/sin(d*x+c),I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*sin(d*x+c)+24*I*co
s(d*x+c)*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(-cos(d*x+c)/(
cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)+16*I*cos(d*x+c)^3*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-20
*I*cos(d*x+c)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-24*I*cos(d*x+
c)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)
+16*I*cos(d*x+c)^4*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+16*cos(d
*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-20*I*cos(d*x+c)^2*sin(d*x+c)*(
-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+16*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c
)+1)^2)^(1/2)+12*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+
```

```
c),I)*cos(d*x+c)^2*sin(d*x+c)+5*I*ln(-2*(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos
(d*x+c)-1)/sin(d*x+c)^2)*cos(d*x+c)*sin(d*x+c)-5*I*cos(d*x+c)*ln(-(2*(-cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(co
s(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)-28*cos(d*x+c)
^3*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-16*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(
1/2)*cos(d*x+c)^2+12*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2))/cos(d
*x+c)^3/sin(d*x+c)^5/(e/cos(d*x+c))^(5/2)/(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-2i\,a^3e^{(4i\,dx+4i\,c)}+2i\,a^3e^{(3i\,dx+3i\,c)}+4i\,a^3e^{(2i\,dx+2i\,c)}+2i\,a^3e^{(i\,dx+i\,c)}+6i\,a^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\left(de^3e^{(i\,dx+i\,c)}-de\cdots\right.}{5\left(de^3e^{(i\,dx+i\,c)}-de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-2*I*a^3*e^(4*I*d*x + 4*I*c) + 2*I*a^3*e^(3*I*d*x + 3*I*c) +
4*I*a^3*e^(2*I*d*x + 2*I*c) + 2*I*a^3*e^(I*d*x + I*c) + 6*I*a^3)*sqrt(e/(e^
(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*(d*e^3*e^(I*d*x + I*c)
- d*e^3)*integral(1/5*sqrt(2)*(3*I*a^3*e^(2*I*d*x + 2*I*c) + 6*I*a^3*e^(I*d
*x + I*c) + 3*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I
*c)/(d*e^3*e^(3*I*d*x + 3*I*c) - 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3*e^(I*d
*x + I*c)), x))/(d*e^3*e^(I*d*x + I*c) - d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan (dx + c) + a)^3}{(e \sec (dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(5/2), x)
```

3.208 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{4i\left(a^3+ia^3\tan(c+dx)\right)}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] (-2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*d*e^4) - (((2*I)/7)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/2)
) - (((4*I)/21)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.126506, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3497, 3496, 3771, 2641}

$$-\frac{4i\left(a^3+ia^3\tan(c+dx)\right)}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}-\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*d*e^4) - (((2*I)/7)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/2)
) - (((4*I)/21)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(3/2))
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{a\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}$$
$$= -\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} - \frac{4i\left(a^3+ia^3\tan(c+dx)\right)}{21de^2(e\sec(c+dx))^{3/2}} - \frac{a^3\int \sqrt{e\sec(c+dx)}\,dx}{21e^4}$$
$$= -\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} - \frac{4i\left(a^3+ia^3\tan(c+dx)\right)}{21de^2(e\sec(c+dx))^{3/2}} - \frac{\left(a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)}{21e^4}$$
$$= -\frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4} - \frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} - \frac{4i\left(a^3+i\right.}{21de^2(e}$$

Mathematica [A] time = 0.932838, size = 133, normalized size = 1.07

$$-\frac{a^3\sqrt{e\sec(c+dx)}(\cos(2c+5dx)+i\sin(2c+5dx))\left(-\sin(2(c+dx))+5i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.\right.}{21de^4(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] -(a^3*Sqrt[e*Sec[c + d*x]]*(5*I + (5*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]) - Si
n[2*(c + d*x)])*(Cos[2*c + 5*d*x] + I*Sin[2*c + 5*d*x]))/(21*d*e^4*(Cos[d*x
] + I*Sin[d*x])^3)
```

Maple [A] time = 0.231, size = 199, normalized size = 1.6

$$-\frac{2a^3}{21d(\cos(dx+c))^4}\left(i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)+12i(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x)
```

```
[Out] -2/21*a^3/d*(I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+12*I*cos(d*x+c)^4+I*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x
+c)-1)/sin(d*x+c),I)-12*cos(d*x+c)^3*sin(d*x+c)-7*I*cos(d*x+c)^2+cos(d*x+c)
*sin(d*x+c))/(e/cos(d*x+c))^(7/2)/cos(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{21\,de^4\text{integral}\left(\frac{i\sqrt{2}a^3\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{21\,de^4},x\right)+\sqrt{2}\left(-3i\,a^3e^{(4i\,dx+4i\,c)}-5i\,a^3e^{(2i\,dx+2i\,c)}-2i\,a^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{21\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(21*d*e^4*integral(1/21*I*sqrt(2)*a^3*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^4), x) + sqrt(2)*(-3*I*a^3*e^(4*I*d*x + 4*I
*c) - 5*I*a^3*e^(2*I*d*x + 2*I*c) - 2*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) +
1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(7/2), x)
```

3.209 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{4i\left(a^3+ia^3\tan(c+dx)\right)}{15de^2(e\sec(c+dx))^{5/2}}+\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

```
[Out] (2*a^3*EllipticE[(c + d*x)/2, 2])/(15*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/2))
 - (((4*I)/15)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(5/2))
```

Rubi [A] time = 0.126562, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3497, 3496, 3771, 2639}

$$-\frac{4i\left(a^3+ia^3\tan(c+dx)\right)}{15de^2(e\sec(c+dx))^{5/2}}+\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (2*a^3*EllipticE[(c + d*x)/2, 2])/(15*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/2))
 - (((4*I)/15)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(5/2))
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}dx &= -\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}+\frac{a\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}dx}{3e^2}\\ &= -\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{4i\left(a^3+ia^3\tan(c+dx)\right)}{15de^2(e\sec(c+dx))^{5/2}}+\frac{a^3\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{15e^4}\\ &= -\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{4i\left(a^3+ia^3\tan(c+dx)\right)}{15de^2(e\sec(c+dx))^{5/2}}+\frac{a^3\int \sqrt{\cos(c+dx)}\,dx}{15e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\ &= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{4i\left(a^3+ia^3\tan(c+dx)\right)}{15de^2(e\sec(c+dx))^{5/2}}\end{aligned}$$

Mathematica [C] time = 2.40854, size = 118, normalized size = 0.95

$$-\frac{a^3e^{-2i(c+dx)}(\tan(c+dx)-i)^3\left(4\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+16e^{2i(c+dx)}+5e^{4i(c+dx)}+11\right)}{90de^2(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] -(a^3*(11 + 16*E^((2*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 4*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(
-I + Tan[c + d*x])^3)/(90*d*e^2*E^((2*I)*(c + d*x))*(e*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.283, size = 370, normalized size = 3.

$$-\frac{2\,a^3}{45\,d\,(\cos(dx+c))^5\sin(dx+c)}\left(20\,i\,(\cos(dx+c))^5\sin(dx+c)-3\,i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x)
```

```
[Out] -2/45*a^3/d*(20*I*cos(d*x+c)^5*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(
d*x+c)*sin(d*x+c)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+20*cos
(d*x+c)^6-9*I*cos(d*x+c)^3*sin(d*x+c)-3*I*sin(d*x+c)*EllipticF(I*(cos(d*x+c
)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)+3*I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-19*cos(d*x+c)^4+2*cos(d*x+c)^2-3
*cos(d*x+c))/cos(d*x+c)^5/sin(d*x+c)/(e/cos(d*x+c))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan(dx+c)+a)^3}{(e \sec(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-5i a^3 e^{(6i dx+6i c)} + 5i a^3 e^{(5i dx+5i c)} - 16i a^3 e^{(4i dx+4i c)} + 16i a^3 e^{(3i dx+3i c)} - 23i a^3 e^{(2i dx+2i c)} + 11i a^3 e^{(i dx+i c)} - 12i a^3\right)\sqrt{}}{90\left(de^5 e^{(i dx+i c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/90*(sqrt(2)*(-5*I*a^3*e^(6*I*d*x + 6*I*c) + 5*I*a^3*e^(5*I*d*x + 5*I*c) -
 16*I*a^3*e^(4*I*d*x + 4*I*c) + 16*I*a^3*e^(3*I*d*x + 3*I*c) - 23*I*a^3*e^(
2*I*d*x + 2*I*c) + 11*I*a^3*e^(I*d*x + I*c) - 12*I*a^3)*sqrt(e/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 90*(d*e^5*e^(I*d*x + I*c) - d*e^5)
*integral(1/15*sqrt(2)*(-I*a^3*e^(2*I*d*x + 2*I*c) - 2*I*a^3*e^(I*d*x + I*c
) - I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^5
*e^(3*I*d*x + 3*I*c) - 2*d*e^5*e^(2*I*d*x + 2*I*c) + d*e^5*e^(I*d*x + I*c))
, x))/(d*e^5*e^(I*d*x + I*c) - d*e^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan(dx+c)+a)^3}{(e \sec(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(9/2), x)
```

3.210 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=155

$$\frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}-\frac{20i\left(a^3+ia^3\tan(c+dx)\right)}{77de^2(e\sec(c+dx))^{7/2}}+\frac{10a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6}-\frac{2i(a+ia\tan(c+}{11d(e\sec(c+d}$$

```
[Out] (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*d*e^6) + (10*a^3*Sin[c + d*x])/(77*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((2*I
)/11)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) - (((20*I)/77)*
(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(7/2))
```

Rubi [A] time = 0.15469, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3497, 3496, 3769, 3771, 2641}

$$\frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}-\frac{20i\left(a^3+ia^3\tan(c+dx)\right)}{77de^2(e\sec(c+dx))^{7/2}}+\frac{10a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6}-\frac{2i(a+ia\tan(c+}{11d(e\sec(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*d*e^6) + (10*a^3*Sin[c + d*x])/(77*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((2*I
)/11)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) - (((20*I)/77)*
(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(7/2))
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{(5a)\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx}{11e^2}$$

$$= -\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} - \frac{20i\left(a^3+ia^3\tan(c+dx)\right)}{77de^2(e\sec(c+dx))^{7/2}} + \frac{\left(15a^3\right)\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{77e^4}$$

$$= \frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} - \frac{20i\left(a^3+ia^3\tan(c+dx)\right)}{77de^2(e\sec(c+dx))^{7/2}} + \frac{\left(5a^3\right)\int \sqrt{\ldots}}{\ldots}$$

$$= \frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} - \frac{20i\left(a^3+ia^3\tan(c+dx)\right)}{77de^2(e\sec(c+dx))^{7/2}} + \frac{\left(5a^3\sqrt{\cos\ldots}\right.}{\ldots}$$

$$= \frac{10a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6} + \frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan\ldots}{11d(e\sec(c\ldots}$$

Mathematica [A] time = 1.25003, size = 148, normalized size = 0.95

$$\frac{a^3\sqrt{e\sec(c+dx)}(\cos(3(c+2dx))+i\sin(3(c+2dx)))\left(-15\sin(c+dx)-15\sin(3(c+dx))-46i\cos(c+dx)-22i\cos\ldots\right.}{154de^6(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (a^3*Sqrt[e*Sec[c + d*x]]*((-46*I)*Cos[c + d*x] - (22*I)*Cos[3*(c + d*x)] -
 15*Sin[c + d*x] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[3*(
c + d*x)] - I*Sin[3*(c + d*x)]) - 15*Sin[3*(c + d*x)])*(Cos[3*(c + 2*d*x)]
+ I*Sin[3*(c + 2*d*x)]))/(154*d*e^6*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] time = 0.287, size = 216, normalized size = 1.4

$$-\frac{2a^3}{77d(\cos(dx+c))^6}\left(28i(\cos(dx+c))^6-28(\cos(dx+c))^5\sin(dx+c)-11i(\cos(dx+c))^4-5i\cos(dx+c)\sqrt{(\cos\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x)
```

```
[Out] -2/77*a^3/d*(28*I*cos(d*x+c)^6-28*cos(d*x+c)^5*sin(d*x+c)-11*I*cos(d*x+c)^4
-5*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
```

```
EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-3*cos
(d*x+c)^3*sin(d*x+c)-5*cos(d*x+c)*sin(d*x+c))/cos(d*x+c)^6/(e/cos(d*x+c))^(
11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan(dx+c)+a)^3}{(e \sec(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{308\, de^6 \text{integral}\left(-\frac{5i\sqrt{2}a^3\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{77\,de^6}, x\right) + \sqrt{2}\left(-7i\,a^3e^{(6i\,dx+6i\,c)} - 31i\,a^3e^{(4i\,dx+4i\,c)} - 61i\,a^3e^{(2i\,dx+2i\,c)} - 37i\,a^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{308\,de^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/308*(308*d*e^6*integral(-5/77*I*sqrt(2)*a^3*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^6), x) + sqrt(2)*(-7*I*a^3*e^(6*I*d*x +
6*I*c) - 31*I*a^3*e^(4*I*d*x + 4*I*c) - 61*I*a^3*e^(2*I*d*x + 2*I*c) - 37*I
*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan(dx+c)+a)^3}{(e \sec(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(11/2), x)
```

3.211 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx$

Optimal. Leaf size=155

$$\frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{28i\left(a^3+ia^3\tan(c+dx)\right)}{117de^2(e\sec(c+dx))^{9/2}}+\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

```
[Out] (14*a^3*EllipticE[(c + d*x)/2, 2])/(39*d*e^6*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (14*a^3*Sin[c + d*x])/(117*d*e^5*(e*Sec[c + d*x])^(3/2)) - (((
2*I)/13)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) - (((28*I)/1
17)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(9/2))
```

Rubi [A] time = 0.147979, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3497, 3496, 3769, 3771, 2639}

$$\frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{28i\left(a^3+ia^3\tan(c+dx)\right)}{117de^2(e\sec(c+dx))^{9/2}}+\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(13/2),x]
```


```
[Out] (14*a^3*EllipticE[(c + d*x)/2, 2])/(39*d*e^6*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (14*a^3*Sin[c + d*x])/(117*d*e^5*(e*Sec[c + d*x])^(3/2)) - (((
2*I)/13)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) - (((28*I)/1
17)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(9/2))
```


Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx &= -\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}+\frac{(7a)\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx}{13e^2}\\
&= -\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{28i\left(a^3+ia^3\tan(c+dx)\right)}{117de^2(e\sec(c+dx))^{9/2}}+\frac{\left(35a^3\right)\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{117e^4}\\
&= \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{28i\left(a^3+ia^3\tan(c+dx)\right)}{117de^2(e\sec(c+dx))^{9/2}}+\frac{\left(7a^3\right)\ldots}{}\\
&= \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{28i\left(a^3+ia^3\tan(c+dx)\right)}{117de^2(e\sec(c+dx))^{9/2}}+\frac{(7\ldots}{39e^6\sqrt{\ldots}}\\
&= \frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}\ldots
\end{aligned}
$$

Mathematica [C] time = 6.36057, size = 155, normalized size = 1.

$$
-\frac{a^3e^{-4i(c+dx)}(\tan(c+dx)-i)^3\left(112e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-34e^{2i(c+dx)}+124e\ldots\right.}{936de^4(e\sec(c+dx))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(13/2),x]
```

```
[Out] -(a^3*(-117 - 34*E^((2*I)*(c + d*x)) + 124*E^((4*I)*(c + d*x)) + 50*E^((6*I
)*(c + d*x)) + 9*E^((8*I)*(c + d*x)) + 112*E^((2*I)*(c + d*x))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(
-I + Tan[c + d*x])^3)/(936*d*e^4*E^((4*I)*(c + d*x))*(e*Sec[c + d*x])^(5/2)
)
```

Maple [B] time = 0.37, size = 380, normalized size = 2.5

$$
\frac{2a^3}{117d(\cos(dx+c))^7\sin(dx+c)}\left(-36i(\cos(dx+c))^7\sin(dx+c)-36(\cos(dx+c))^8+13i(\cos(dx+c))^5\sin(dx+\ldots\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x)
```

```
[Out] 2/117*a^3/d*(-36*I*cos(d*x+c)^7*sin(d*x+c)-36*cos(d*x+c)^8+13*I*cos(d*x+c)^
5*sin(d*x+c)+21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
```

```
)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-21*I*Ellip
ticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+31*cos(d*x+c)^6+21*I*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/s
in(d*x+c),I)*sin(d*x+c)-21*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d
*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*cos(d*x+
c)^4-14*cos(d*x+c)^2+21*cos(d*x+c))/cos(d*x+c)^7/sin(d*x+c)/(e/cos(d*x+c))^
(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan (dx+c)+a)^3}{(e \sec (dx+c))^{\frac{13}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-9i\,a^3e^{(9i\,dx+9i\,c)} + 9i\,a^3e^{(8i\,dx+8i\,c)} - 50i\,a^3e^{(7i\,dx+7i\,c)} + 50i\,a^3e^{(6i\,dx+6i\,c)} - 124i\,a^3e^{(5i\,dx+5i\,c)} + 124i\,a^3e^{(4i\,dx+4i\,c)} - 302i\,a^3 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="fricas")
```

```
[Out] 1/936*(sqrt(2)*(-9*I*a^3*e^(9*I*d*x + 9*I*c) + 9*I*a^3*e^(8*I*d*x + 8*I*c)
- 50*I*a^3*e^(7*I*d*x + 7*I*c) + 50*I*a^3*e^(6*I*d*x + 6*I*c) - 124*I*a^3*e
^(5*I*d*x + 5*I*c) + 124*I*a^3*e^(4*I*d*x + 4*I*c) - 302*I*a^3*e^(3*I*d*x +
 3*I*c) - 34*I*a^3*e^(2*I*d*x + 2*I*c) - 219*I*a^3*e^(I*d*x + I*c) - 117*I*
a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 936*(d*e^7
*e^(2*I*d*x + 2*I*c) - d*e^7*e^(I*d*x + I*c))*integral(1/39*sqrt(2)*(-7*I*a
^3*e^(2*I*d*x + 2*I*c) - 14*I*a^3*e^(I*d*x + I*c) - 7*I*a^3)*sqrt(e/(e^(2*I
*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^7*e^(3*I*d*x + 3*I*c) - 2*
d*e^7*e^(2*I*d*x + 2*I*c) + d*e^7*e^(I*d*x + I*c)), x))/(d*e^7*e^(2*I*d*x +
 2*I*c) - d*e^7*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(13/2), x)
```

3.212 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}}+\frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}-\frac{12i\left(a^3+ia^3\tan(c+dx)\right)}{55de^2(e\sec(c+dx))^{11/2}}+\frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+}}{11de^8}$$

```
[Out] (2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
11*d*e^8) + (6*a^3*Sin[c + d*x])/(55*d*e^5*(e*Sec[c + d*x])^(5/2)) + (2*a^3
*Sin[c + d*x])/(11*d*e^7*Sqrt[e*Sec[c + d*x]]) - (((2*I)/15)*(a + I*a*Tan[c
 + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) - (((12*I)/55)*(a^3 + I*a^3*Tan[c +
 d*x]))/(d*e^2*(e*Sec[c + d*x])^(11/2))
```

Rubi [A] time = 0.173392, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3497, 3496, 3769, 3771, 2641}

$$\frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}}+\frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}-\frac{12i\left(a^3+ia^3\tan(c+dx)\right)}{55de^2(e\sec(c+dx))^{11/2}}+\frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+}}{11de^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] (2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
11*d*e^8) + (6*a^3*Sin[c + d*x])/(55*d*e^5*(e*Sec[c + d*x])^(5/2)) + (2*a^3
*Sin[c + d*x])/(11*d*e^7*Sqrt[e*Sec[c + d*x]]) - (((2*I)/15)*(a + I*a*Tan[c
 + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) - (((12*I)/55)*(a^3 + I*a^3*Tan[c +
 d*x]))/(d*e^2*(e*Sec[c + d*x])^(11/2))
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} + \frac{(3a)\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx}{5e^2}$$

$$= -\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{12i\left(a^3+ia^3\tan(c+dx)\right)}{55de^2(e\sec(c+dx))^{11/2}} + \frac{\left(21a^3\right)\int \frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{55e^4}$$

$$= \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{12i\left(a^3+ia^3\tan(c+dx)\right)}{55de^2(e\sec(c+dx))^{11/2}} + \frac{\left(3a^3\right)\int}{}$$

$$= \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{12i\left(a^3+ia^3\right.}{55de^2(e\sec}$$

$$= \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{12i\left(a^3+ia^3\right.}{55de^2(e\sec}$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{11de^8} + \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^3\sin(}{11de^7\sqrt{e\sec}}$$

Mathematica [A] time = 1.85125, size = 170, normalized size = 0.91

$$\frac{a^3\sqrt{e\sec(c+dx)}(\cos(3(c+2dx))+i\sin(3(c+2dx)))\left(-114\sin(c+dx)-81\sin(3(c+dx))+33\sin(5(c+dx))-332i\right.}{1320de^8(\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] (a^3*Sqrt[e*Sec[c + d*x]]*((-332*I)*Cos[c + d*x] - (154*I)*Cos[3*(c + d*x)]
 + (22*I)*Cos[5*(c + d*x)] - 114*Sin[c + d*x] + 240*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*(Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]) - 81*Sin[3*(c
 + d*x)] + 33*Sin[5*(c + d*x)])*(Cos[3*(c + 2*d*x)] + I*Sin[3*(c + 2*d*x)])
)/(1320*d*e^8*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] time = 0.378, size = 232, normalized size = 1.3

$$\frac{2a^3}{165d(\cos(dx+c))^8}\left(-44i(\cos(dx+c))^8+44\sin(dx+c)(\cos(dx+c))^7+15i(\cos(dx+c))^6+7(\cos(dx+c))^5\mathrm{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x)
```

```
[Out] 2/165*a^3/d*(-44*I*cos(d*x+c)^8+44*sin(d*x+c)*cos(d*x+c)^7+15*I*cos(d*x+c)^
6+7*cos(d*x+c)^5*sin(d*x+c)+15*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+15*I*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(
d*x+c)-1)/sin(d*x+c),I)+9*cos(d*x+c)^3*sin(d*x+c)+15*cos(d*x+c)*sin(d*x+c))
/cos(d*x+c)^8/(e/cos(d*x+c))^(15/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan(dx+c)+a)^3}{(e \sec(dx+c))^{\frac{15}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2640\, de^{8} e^{(2i\,dx+2i\,c)} \mathrm{integral}\left(-\frac{i\sqrt{2}a^{3}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{11\, de^{8}}, x\right) + \sqrt{2}\left(-11i\,a^{3}e^{(10i\,dx+10i\,c)} - 73i\,a^{3}e^{(8i\,dx+8i\,c)} - 218i\,a^{3}e^{(6i\,dx+6}\right.\right.}{2640\, de^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="fricas")
```

```
[Out] 1/2640*(2640*d*e^8*e^(2*I*d*x + 2*I*c)*integral(-1/11*I*sqrt(2)*a^3*sqrt(e/
(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^8), x) + sqrt(2)*(
-11*I*a^3*e^(10*I*d*x + 10*I*c) - 73*I*a^3*e^(8*I*d*x + 8*I*c) - 218*I*a^3*
e^(6*I*d*x + 6*I*c) - 446*I*a^3*e^(4*I*d*x + 4*I*c) - 235*I*a^3*e^(2*I*d*x
+ 2*I*c) + 55*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I
*c))*e^(-2*I*d*x - 2*I*c)/(d*e^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(15/2), x)
```

3.213 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=215

$$-\frac{22a^4e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^4(e\sec(c+dx))^{3/2}}{9d}+\frac{22a^4e\sin(c+dx)\sqrt{e\sec(c+dx)}}{3d}+\frac{10i\left(a^2+ia^2\tan(c+dx)\right)}{21d}$$

```
[Out] (-22*a^4*e^2*EllipticE[(c + d*x)/2, 2])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((22*I)/9)*a^4*(e*Sec[c + d*x])^(3/2))/d + (22*a^4*e*Sqrt[e*S
ec[c + d*x]]*Sin[c + d*x])/(3*d) + (((2*I)/9)*a*(e*Sec[c + d*x])^(3/2)*(a +
 I*a*Tan[c + d*x])^3)/d + (((10*I)/21)*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*
Tan[c + d*x])^2)/d + (((22*I)/21)*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*Tan[c
 + d*x]))/d
```

Rubi [A] time = 0.256976, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3498, 3486, 3768, 3771, 2639}

$$-\frac{22a^4e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^4(e\sec(c+dx))^{3/2}}{9d}+\frac{22a^4e\sin(c+dx)\sqrt{e\sec(c+dx)}}{3d}+\frac{10i\left(a^2+ia^2\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4,x]
```


```
[Out] (-22*a^4*e^2*EllipticE[(c + d*x)/2, 2])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((22*I)/9)*a^4*(e*Sec[c + d*x])^(3/2))/d + (22*a^4*e*Sqrt[e*S
ec[c + d*x]]*Sin[c + d*x])/(3*d) + (((2*I)/9)*a*(e*Sec[c + d*x])^(3/2)*(a +
 I*a*Tan[c + d*x])^3)/d + (((10*I)/21)*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*
Tan[c + d*x])^2)/d + (((22*I)/21)*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*Tan[c
 + d*x]))/d
```


Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} + \frac{1}{3}(5a)\int (e\sec(c+dx))^{3/2}(a$$

$$= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} + \frac{10i(e\sec(c+dx))^{3/2}\left(a^2+ia\right.}{21d}$$

$$= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} + \frac{10i(e\sec(c+dx))^{3/2}\left(a^2+ia\right.}{21d}$$

$$= \frac{22ia^4(e\sec(c+dx))^{3/2}}{9d} + \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} + \frac{10}{}$$

$$= \frac{22ia^4(e\sec(c+dx))^{3/2}}{9d} + \frac{22a^4e\sqrt{e\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2ia(e\sec(c}{}$$

$$= \frac{22ia^4(e\sec(c+dx))^{3/2}}{9d} + \frac{22a^4e\sqrt{e\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2ia(e\sec(c}{}$$

$$= -\frac{22a^4e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{22ia^4(e\sec(c+dx))^{3/2}}{9d} + \frac{22a^4e\sqrt{e\,\mathrm{se}}}{}$$

Mathematica [C] time = 7.48129, size = 429, normalized size = 2.

$$\frac{22i\sqrt{2}e^{-i(3c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\left(-1+e^{2ic}\right)e^{2idx}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+e^{2i(c+dx)}}\right)(}{9\left(-1+e^{2ic}\right)d\sec^{\frac{11}{2}}(c+dx)(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (((22*I)/9)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1
+ E^((2*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*(e
*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4)/(d*E^(I*(3*c + d*x))*(-1 + E
^((2*I)*c))*Sec[c + d*x]^(11/2)*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[c + d*x]^
5*(e*Sec[c + d*x])^(3/2)*(Sec[c]*Sec[c + d*x]^3*(36*Cos[c] + (7*I)*Sin[c])*
(((-2*I)/63)*Cos[4*c] - (2*Sin[4*c])/63) + Cos[d*x]*Csc[c]*((22*Cos[4*c])/3
 - ((22*I)/3)*Sin[4*c]) + Sec[c]*Sec[c + d*x]*(24*Cos[c] + (13*I)*Sin[c])*(
((2*I)/9)*Cos[4*c] + (2*Sin[4*c])/9) + Sec[c]*Sec[c + d*x]^4*((2*Cos[4*c])/
9 - ((2*I)/9)*Sin[4*c])*Sin[d*x] + Sec[c]*Sec[c + d*x]^2*((-26*Cos[4*c])/9
+ ((26*I)/9)*Sin[4*c])*Sin[d*x])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I
*Sin[d*x])^4)
```

Maple [A] time = 0.325, size = 401, normalized size = 1.9

$$-\frac{2a^4(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2}{63d(\sin(dx+c))^5(\cos(dx+c))^3}\left(231i\sin(dx+c)(\cos(dx+c))^5\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] -2/63*a^4/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(231*I*sin(d*x+c)*cos(d*x+c)^
5*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(c
os(d*x+c)-1)/sin(d*x+c),I)-231*I*sin(d*x+c)*cos(d*x+c)^5*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x
+c),I)+231*I*sin(d*x+c)*cos(d*x+c)^4*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-231*I*sin(d*x
+c)*cos(d*x+c)^4*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-168*I*sin(d*x+c)*cos(d*x+c)^3+231
*cos(d*x+c)^5-322*cos(d*x+c)^4+36*I*cos(d*x+c)*sin(d*x+c)+98*cos(d*x+c)^2-7
)*(e/cos(d*x+c))^(3/2)/sin(d*x+c)^5/cos(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-462i\,a^4ee^{(9i\,dx+9i\,c)}-812i\,a^4ee^{(7i\,dx+7i\,c)}-1080i\,a^4ee^{(5i\,dx+5i\,c)}-660i\,a^4ee^{(3i\,dx+3i\,c)}-154i\,a^4ee^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,c\right.}}{63\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/63*(sqrt(2)*(-462*I*a^4*e*e^(9*I*d*x + 9*I*c) - 812*I*a^4*e*e^(7*I*d*x +
7*I*c) - 1080*I*a^4*e*e^(5*I*d*x + 5*I*c) - 660*I*a^4*e*e^(3*I*d*x + 3*I*c)
 - 154*I*a^4*e*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*
d*x + 1/2*I*c) + 63*(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*
e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)*integral(11/3*I*sqrt(2)*
a^4*e*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/d, x))/(d*e
^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*
d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{3}{2}} (ia\tan(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^4, x)
```

3.214 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx$

Optimal. Leaf size=183

$$\frac{78ia^4\sqrt{e\sec(c+dx)}}{7d}+\frac{26i\left(a^2+ia^2\tan(c+dx)\right)^2\sqrt{e\sec(c+dx)}}{35d}+\frac{78i\left(a^4+ia^4\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{35d}+\frac{78a^4\sqrt{\mathrm{co}}}{}$$

```
[Out] (((78*I)/7)*a^4*Sqrt[e*Sec[c + d*x]])/d + (78*a^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*d) + (((2*I)/7)*a*Sqrt[e*Sec[c
 + d*x]]*(a + I*a*Tan[c + d*x])^3)/d + (((26*I)/35)*Sqrt[e*Sec[c + d*x]]*(a
^2 + I*a^2*Tan[c + d*x])^2)/d + (((78*I)/35)*Sqrt[e*Sec[c + d*x]]*(a^4 + I*
a^4*Tan[c + d*x]))/d
```

Rubi [A] time = 0.201563, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3498, 3486, 3771, 2641}

$$\frac{78ia^4\sqrt{e\sec(c+dx)}}{7d}+\frac{26i\left(a^2+ia^2\tan(c+dx)\right)^2\sqrt{e\sec(c+dx)}}{35d}+\frac{78i\left(a^4+ia^4\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{35d}+\frac{78a^4\sqrt{\mathrm{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^4,x]
```


```
[Out] (((78*I)/7)*a^4*Sqrt[e*Sec[c + d*x]])/d + (78*a^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*d) + (((2*I)/7)*a*Sqrt[e*Sec[c
 + d*x]]*(a + I*a*Tan[c + d*x])^3)/d + (((26*I)/35)*Sqrt[e*Sec[c + d*x]]*(a
^2 + I*a^2*Tan[c + d*x])^2)/d + (((78*I)/35)*Sqrt[e*Sec[c + d*x]]*(a^4 + I*
a^4*Tan[c + d*x]))/d
```


Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{1}{7}(13a)\int \sqrt{e\sec(c+dx)}(a+ia$$

$$= \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{26i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan\right.}{35d}$$

$$= \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{26i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan\right.}{35d}$$

$$= \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d} + \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{26i\sqrt{e\,\mathrm{se}}}{}$$

$$= \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d} + \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{26i\sqrt{e\,\mathrm{se}}}{}$$

$$= \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d} + \frac{78a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7d}$$

Mathematica [A] time = 1.46473, size = 101, normalized size = 0.55

$$\frac{a^4\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(-150\sin(2(c+dx))-85\sin(4(c+dx))+1008i\cos(2(c+dx))+280i\cos(4(c+dx))+\right.}{140d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(728*I + (1008*I)*Cos[2*(c + d*x)]
 + (280*I)*Cos[4*(c + d*x)] + 1560*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2
, 2] - 150*Sin[2*(c + d*x)] - 85*Sin[4*(c + d*x)]))/(140*d)
```

Maple [A] time = 0.318, size = 230, normalized size = 1.3

$$\frac{2a^4(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2}{35d(\sin(dx+c))^4(\cos(dx+c))^3}\left(195i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c))^4\,\mathrm{EllipticF}\left(\frac{i(}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/35*a^4/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(195*I*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4*EllipticF(I*(cos(d*x+c)-1)
/sin(d*x+c),I)+195*I*cos(d*x+c)^3*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+280*I*cos(d*x+c)
^3-85*cos(d*x+c)^2*sin(d*x+c)-28*I*cos(d*x+c)+5*sin(d*x+c))*(e/cos(d*x+c))^
(1/2)/sin(d*x+c)^4/cos(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\sec(dx+c)}(ia\tan(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(730i\,a^4e^{(6i\,dx+6i\,c)} + 1586i\,a^4e^{(4i\,dx+4i\,c)} + 1326i\,a^4e^{(2i\,dx+2i\,c)} + 390i\,a^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + 35\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4}\right.}{35\left(de^{(6i\,dx+6i\,c)} + 3\,de^{(4i\,dx+4i\,c)} + 3\,de^{(2i\,dx+2i\,c)} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(sqrt(2)*(730*I*a^4*e^(6*I*d*x + 6*I*c) + 1586*I*a^4*e^(4*I*d*x + 4*I*
c) + 1326*I*a^4*e^(2*I*d*x + 2*I*c) + 390*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 35*(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*
x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)*integral(-39/7*I*sqrt(2)*a^4*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/d, x))/(d*e^(6*I*d*x
 + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int \sqrt{e\sec(c+dx)}\,dx + \int -6\sqrt{e\sec(c+dx)}\tan^2(c+dx)\,dx + \int \sqrt{e\sec(c+dx)}\tan^4(c+dx)\,dx + \int 4i\sqrt{e\sec(c+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sqrt(e*sec(c + d*x)), x) + Integral(-6*sqrt(e*sec(c + d*x))*
tan(c + d*x)**2, x) + Integral(sqrt(e*sec(c + d*x))*tan(c + d*x)**4, x) + I
ntegral(4*I*sqrt(e*sec(c + d*x))*tan(c + d*x), x) + Integral(-4*I*sqrt(e*se
c(c + d*x))*tan(c + d*x)**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^4, x)
```

3.215 $\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=178

$$-\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2}-\frac{22i\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5de^2}-\frac{154a^4\sin(c+dx)\sqrt{e\sec(c+dx)}}{5de}+\frac{154}{5d\sqrt{\cos}}$$

```
[Out] (154*a^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) - (((154*I)/15)*a^4*(e*Sec[c + d*x])^(3/2))/(d*e^2) - (154*a^4*Sqrt[
e*Sec[c + d*x]]*Sin[c + d*x])/(5*d*e) - ((4*I)*a*(a + I*a*Tan[c + d*x])^3)/
(d*Sqrt[e*Sec[c + d*x]]) - (((22*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*
Tan[c + d*x]))/(d*e^2)
```

Rubi [A] time = 0.186339, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3496, 3498, 3486, 3768, 3771, 2639}

$$-\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2}-\frac{22i\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5de^2}-\frac{154a^4\sin(c+dx)\sqrt{e\sec(c+dx)}}{5de}+\frac{154}{5d\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (154*a^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) - (((154*I)/15)*a^4*(e*Sec[c + d*x])^(3/2))/(d*e^2) - (154*a^4*Sqrt[
e*Sec[c + d*x]]*Sin[c + d*x])/(5*d*e) - ((4*I)*a*(a + I*a*Tan[c + d*x])^3)/
(d*Sqrt[e*Sec[c + d*x]]) - (((22*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*
Tan[c + d*x]))/(d*e^2)
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
```

```
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} - \frac{\left(11a^2\right)\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx}{e^2}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} - \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{5de^2} - \frac{\left(77a^3\right)\int (e\sec(c$$

$$= -\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} - \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c\right.}{5de^2}$$

$$= -\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{154a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de} - \frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} -$$

$$= -\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{154a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de} - \frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} -$$

$$= \frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{154a^4\sqrt{e\sec(c+dx)}\sin(c+dx}{5de}$$

Mathematica [C] time = 3.98108, size = 123, normalized size = 0.69

$$-\frac{2ia^4e^{i(c+dx)}\left(77\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-176e^{2i(c+dx)}-111e^{4i(c+dx)}-77\right)\sqrt{e\sec(c+d}}{15de\left(1+e^{2i(c+dx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (((-2*I)/15)*a^4*E^(I*(c + d*x))*(-77 - 176*E^((2*I)*(c + d*x)) - 111*E^((4
*I)*(c + d*x)) + 77*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(d*e*(1 + E^((2*I)*(
c + d*x)))^2)
```

Maple [B] time = 0.337, size = 1618, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x)
```

```
[Out] 2/15*a^4/d*(cos(d*x+c)-1)^3*(-9*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(
3/2)+120*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^7+129*(-cos(d*x+c)
/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^6-3*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2
)+120*I*sin(d*x+c)*cos(d*x+c)^6*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)+360*I*
sin(d*x+c)*cos(d*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)+380*I*sin(d*x+
c)*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)+180*I*sin(d*x+c)*cos(d
*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)+30*I*sin(d*x+c)*cos(d*x+c)^4*l
n(-2*(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)-30*I*sin(d
*x+c)*cos(d*x+c)^4*ln(-(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2
-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*
x+c)^2)+60*I*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)+2
0*I*sin(d*x+c)*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)+1386*I*(-cos
(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+
c),I)-1386*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticF(I*(cos(
d*x+c)-1)/sin(d*x+c),I)+231*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^6*
EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-219*(-cos(d*x+c)/(cos(d*x+c)+1)^2)
^(3/2)*cos(d*x+c)^5-231*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^4+1
08*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^3+105*(-cos(d*x+c)/(cos(
d*x+c)+1)^2)^(3/2)*cos(d*x+c)^2-231*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d
*x+c)^6*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+924*I*(-cos(d*x+c)/(cos(d*
x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)*cos(d*x+c)^5*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-924*I*(-c
os(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticF(I*(cos(d*x+c)-1)/sin(d*
x+c),I)+924*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^3*EllipticE(I*(cos
(d*x+c)-1)/sin(d*x+c),I)-924*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^3
*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+231*I*sin(d*x+c)*cos(d*x+c)^2*(-c
os(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-231*I*sin(d*x+c)
*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)
)/sin(d*x+c)^7/cos(d*x+c)^3/(e/cos(d*x+c))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1
)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan{(dx+c)}+a)^4}{\sqrt{e\sec{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/sqrt(e*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-240i\,a^4e^{(6i\,dx+6i\,c)} - 222i\,a^4e^{(5i\,dx+5i\,c)} - 1034i\,a^4e^{(4i\,dx+4i\,c)} - 352i\,a^4e^{(3i\,dx+3i\,c)} - 1232i\,a^4e^{(2i\,dx+2i\,c)} - 154i\,a^4e^{(i\,dx+i\,c)} -\right.}{15\left(dee^{(5\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(sqrt(2)*(-240*I*a^4*e^(6*I*d*x + 6*I*c) - 222*I*a^4*e^(5*I*d*x + 5*I*
c) - 1034*I*a^4*e^(4*I*d*x + 4*I*c) - 352*I*a^4*e^(3*I*d*x + 3*I*c) - 1232*
I*a^4*e^(2*I*d*x + 2*I*c) - 154*I*a^4*e^(I*d*x + I*c) - 462*I*a^4)*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 15*(d*e*e^(5*I*d*x + 5*
I*c) - d*e*e^(4*I*d*x + 4*I*c) + 2*d*e*e^(3*I*d*x + 3*I*c) - 2*d*e*e^(2*I*d
*x + 2*I*c) + d*e*e^(I*d*x + I*c) - d*e)*integral(1/5*sqrt(2)*(-77*I*a^4*e^
(2*I*d*x + 2*I*c) - 154*I*a^4*e^(I*d*x + I*c) - 77*I*a^4)*sqrt(e/(e^(2*I*d*
x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e*e^(3*I*d*x + 3*I*c) - 2*d*e*e
^(2*I*d*x + 2*I*c) + d*e*e^(I*d*x + I*c)), x))/(d*e*e^(5*I*d*x + 5*I*c) - d
*e*e^(4*I*d*x + 4*I*c) + 2*d*e*e^(3*I*d*x + 3*I*c) - 2*d*e*e^(2*I*d*x + 2*I
*c) + d*e*e^(I*d*x + I*c) - d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx + \int -\frac{6\tan^2(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx + \int \frac{\tan^4(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx + \int \frac{4i\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx + \int -\frac{4i\tan^3(c+dx}{\sqrt{e\sec(c+dx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] a**4*(Integral(1/sqrt(e*sec(c + d*x)), x) + Integral(-6*tan(c + d*x)**2/sqr
t(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**4/sqrt(e*sec(c + d*x)), x) +
 Integral(4*I*tan(c + d*x)/sqrt(e*sec(c + d*x)), x) + Integral(-4*I*tan(c +
 d*x)**3/sqrt(e*sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^4}{\sqrt{e\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/sqrt(e*sec(d*x + c)), x)
```

3.216 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2}-\frac{2i\left(a^4+ia^4\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{de^2}-\frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{de^2}-$$

```
[Out] ((-10*I)*a^4*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (10*a^4*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (((4*I)/3)*a*(a + I*
a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(3/2)) - ((2*I)*Sqrt[e*Sec[c + d*x]]
*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2)
```

Rubi [A] time = 0.149021, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3496, 3498, 3486, 3771, 2641}

$$-\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2}-\frac{2i\left(a^4+ia^4\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}{de^2}-\frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{de^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(3/2),x]
```


```
[Out] ((-10*I)*a^4*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (10*a^4*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (((4*I)/3)*a*(a + I*
a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(3/2)) - ((2*I)*Sqrt[e*Sec[c + d*x]]
*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2)
```


Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}} - \frac{\left(3a^2\right)\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx}{e^2}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}} - \frac{2i\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)}{de^2} - \frac{\left(5a^3\right)\int \sqrt{e\sec(c+}$$

$$= -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2} - \frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}} - \frac{2i\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)}{de^2}$$

$$= -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2} - \frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}} - \frac{2i\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)}{de^2}$$

$$= -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2} - \frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{de^2} - \frac{4ia(a+ia\tan}{3d(e\sec(c+}$$

Mathematica [A] time = 1.16543, size = 130, normalized size = 0.89

$$\frac{a^4\sec^3(c+dx)(\sin(c+5dx)-i\cos(c+5dx))\left(-11i\sin(2(c+dx))+19\cos(2(c+dx))-30i\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right.}{3d(\cos(dx)+i\sin(dx))^4(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a^4*Sec[c + d*x]^3*(21 + 19*Cos[2*(c + d*x)] - (30*I)*Cos[c + d*x]^(3/2)*E
llipticF[(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) - (11*I)*Sin[2*(c
+ d*x)])*((-I)*Cos[c + 5*d*x] + Sin[c + 5*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2
)*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A] time = 0.332, size = 198, normalized size = 1.4

$$\frac{2a^4}{3d(\cos(dx+c))^3}\left(-15i(\cos(dx+c))^2\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)-15i\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/3*a^4/d*(-15*I*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-15*I*cos(d*x+c)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x
```

```
+c)-1)/sin(d*x+c),I)-8*I*cos(d*x+c)^3+8*cos(d*x+c)^2*sin(d*x+c)-12*I*cos(d*
x+c)+sin(d*x+c))/cos(d*x+c)^3/(e/cos(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-8i\,a^4e^{(4i\,dx+4i\,c)}-42i\,a^4e^{(2i\,dx+2i\,c)}-30i\,a^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\left(de^2e^{(2i\,dx+2i\,c)}+de^2\right)\text{integral}\left(\frac{5i\sqrt{2}a^4\sqrt{\frac{e}{e^{(2i\,dx}}}}{}\right.}{3\left(de^2e^{(2i\,dx+2i\,c)}+de^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-8*I*a^4*e^(4*I*d*x + 4*I*c) - 42*I*a^4*e^(2*I*d*x + 2*I*c) -
 30*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*(d
*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)*integral(5*I*sqrt(2)*a^4*sqrt(e/(e^(2*I*d
*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^2), x))/(d*e^2*e^(2*I*d*x +
 2*I*c) + d*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int\frac{1}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int-\frac{6\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\frac{\tan^4(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int\frac{4i\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx+\int-\frac{4i\tan}{(e\sec(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] a**4*(Integral((e*sec(c + d*x))**(-3/2), x) + Integral(-6*tan(c + d*x)**2/(
e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**4/(e*sec(c + d*x))**(3/
2), x) + Integral(4*I*tan(c + d*x)/(e*sec(c + d*x))**(3/2), x) + Integral(-
4*I*tan(c + d*x)**3/(e*sec(c + d*x))**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(3/2), x)
```

3.217 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$\frac{42a^4\sin(c+dx)\sqrt{e\sec(c+dx)}}{5de^3}+\frac{28i\left(a^4+ia^4\tan(c+dx)\right)}{5de^2\sqrt{e\sec(c+dx)}}-\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+}{5d(e\sec(c+dx))}$$

```
[Out] (-42*a^4*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (42*a^4*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*d*e^3) - (((4*I)
/5)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(5/2)) + (((28*I)/5)*(a
^4 + I*a^4*Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.138728, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3496, 3768, 3771, 2639}

$$\frac{42a^4\sin(c+dx)\sqrt{e\sec(c+dx)}}{5de^3}+\frac{28i\left(a^4+ia^4\tan(c+dx)\right)}{5de^2\sqrt{e\sec(c+dx)}}-\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+}{5d(e\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(5/2),x]
```


```
[Out] (-42*a^4*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (42*a^4*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*d*e^3) - (((4*I)
/5)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(5/2)) + (((28*I)/5)*(a
^4 + I*a^4*Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]])
```


Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}dx = -\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}} - \frac{(7a^2)\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}dx}{5e^2}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}} + \frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}} + \frac{(21a^4)\int(e\sec(c+dx))^{3/2}dx}{5e^4}$$

$$= \frac{42a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de^3} - \frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}} + \frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}} - \frac{(}{}$$

$$= \frac{42a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de^3} - \frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}} + \frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}} - \frac{}{5}$$

$$= -\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{42a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de^3} - \frac{4ia(a+ia\tan(c+}{5d(e\sec(c+dx))}$$

Mathematica [C] time = 2.51534, size = 110, normalized size = 0.71

$$-\frac{4ia^4e^{2i(c+dx)}\left(-7\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+2e^{2i(c+dx)}+7\right)}{5de^2\left(1+e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/5)*a^4*E^((2*I)*(c + d*x))*(7 + 2*E^((2*I)*(c + d*x)) - 7*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))
]))/(d*e^2*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Sec[c + d*x]])
```

Maple [B] time = 0.336, size = 3762, normalized size = 24.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x)
```

```
[Out] -1/10*a^4/d*(cos(d*x+c)-1)*(-64*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(
3/2)+160*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^7+76*(-cos(d*x+c)/
(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^6-63*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*
(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^5+63*I*(-cos(d*x+c)/(cos
(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^5+504*I*
(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*co
s(d*x+c)^2-504*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c)
,I)*sin(d*x+c)*cos(d*x+c)^2-168*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(
```

cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^4+168*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^4+168*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)-168*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)+168*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^2-168*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^2+84*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)*sin(d*x+c)-84*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)*sin(d*x+c)+21*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^8-21*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^8+84*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^7-84*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^7+21*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^7-21*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^7-42*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^6+42*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^6-32*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^9-32*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^8+20*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-32*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^8-32*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^7+144*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^6-192*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^3+80*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^2-20*I*cos(d*x+c)^4*ln(-2*(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+20*I*cos(d*x+c)^4*ln(-(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+80*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)+40*I*cos(d*x+c)^3*ln(-2*(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)-40*I*cos(d*x+c)^3*ln(-(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)-20*I*cos(d*x+c)^2*ln(-2*(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+20*I*cos(d*x+c)^2*ln(-(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)-420*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^5+420*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))

```
^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^5-315
*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)
*cos(d*x+c)^4+315*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x
+c),I)*sin(d*x+c)*cos(d*x+c)^4+42*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d
*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^6-42*I*(-cos(d*x+c)/(cos(d*x+c
)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)^6+336*I*(-cos(
d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+
c)^3-336*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*si
n(d*x+c)*cos(d*x+c)^3-288*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^5
-36*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^4+224*(-cos(d*x+c)/(cos
(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^3-28*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*co
s(d*x+c)^2+144*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)
^5-192*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^4)/(-co
s(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)/sin(d*x+c)^7/cos(d*x+c)^3/(e/cos(d*x+c))^(
5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan(dx+c) + a)^4}{(e \sec(dx+c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-4i a^4 e^{(4i dx+4i c)} + 4i a^4 e^{(3i dx+3i c)} + 28i a^4 e^{(2i dx+2i c)} + 14i a^4 e^{(i dx+i c)} + 42i a^4\right)\sqrt{\frac{e}{e^{(2i dx+2i c)}+1}}\, e^{\left(\frac{1}{2} i dx+\frac{1}{2} i c\right)} + 5\left(de^3 e^{(i dx+i c)} \cdots\right.}{5\left(de^3 e^{(i dx+i c)} - de^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-4*I*a^4*e^(4*I*d*x + 4*I*c) + 4*I*a^4*e^(3*I*d*x + 3*I*c) +
28*I*a^4*e^(2*I*d*x + 2*I*c) + 14*I*a^4*e^(I*d*x + I*c) + 42*I*a^4)*sqrt(e/
(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*(d*e^3*e^(I*d*x + I*
c) - d*e^3)*integral(1/5*sqrt(2)*(21*I*a^4*e^(2*I*d*x + 2*I*c) + 42*I*a^4*e
^(I*d*x + I*c) + 21*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x +
 1/2*I*c)/(d*e^3*e^(3*I*d*x + 3*I*c) - 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3*
e^(I*d*x + I*c)), x))/(d*e^3*e^(I*d*x + I*c) - d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(5/2), x)
```

3.218 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=125

$$\frac{20i\left(a^4+ia^4\tan(c+dx)\right)}{21de^2(e\sec(c+dx))^{3/2}}+\frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] (10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*d*e^4) - (((4*I)/7)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/
2)) + (((20*I)/21)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(3/2
))
```

Rubi [A] time = 0.133925, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3496, 3771, 2641}

$$\frac{20i\left(a^4+ia^4\tan(c+dx)\right)}{21de^2(e\sec(c+dx))^{3/2}}+\frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4}-\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*d*e^4) - (((4*I)/7)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/
2)) + (((20*I)/21)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(3/2
))
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} - \frac{\left(5a^2\right)\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{20i\left(a^4+ia^4\tan(c+dx)\right)}{21de^2(e\sec(c+dx))^{3/2}} + \frac{\left(5a^4\right)\int\sqrt{e\sec(c+dx)}\,dx}{21e^4}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{20i\left(a^4+ia^4\tan(c+dx)\right)}{21de^2(e\sec(c+dx))^{3/2}} + \frac{\left(5a^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+\right.}{21e^4}$$

$$= \frac{10a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21de^4} - \frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{20i\left(a^4+\right.}{21de^2(}$$

Mathematica [A] time = 1.03143, size = 133, normalized size = 1.06

$$\frac{2a^4\sqrt{e\sec(c+dx)}(\cos(2(c+3dx))+i\sin(2(c+3dx)))\left(8\sin(2(c+dx))+2i\cos(2(c+dx))+5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.\right.}{21de^4(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (2*a^4*Sqrt[e*Sec[c + d*x]]*(2*I + (2*I)*Cos[2*(c + d*x)] + 5*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]) + 8
*Sin[2*(c + d*x)])*(Cos[2*(c + 3*d*x)] + I*Sin[2*(c + 3*d*x)]))/(21*d*e^4*(
Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A] time = 0.245, size = 200, normalized size = 1.6

$$\frac{2\,a^4}{21\,d\,(\cos(dx+c))^4}\left(-24\,i\,(\cos(dx+c))^4+5\,i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i\,(\cos(}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x)
```

```
[Out] 2/21*a^4/d*(-24*I*cos(d*x+c)^4+5*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+24*c
os(d*x+c)^3*sin(d*x+c)+5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+28*I*cos(d*x+c)^2-16*co
s(d*x+c)*sin(d*x+c))/(e/cos(d*x+c))^(7/2)/cos(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{21\,de^4\text{integral}\left(-\frac{5i\sqrt{2}a^4\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{21\,de^4},x\right)+\sqrt{2}\left(-6i\,a^4e^{(4i\,dx+4i\,c)}+4i\,a^4e^{(2i\,dx+2i\,c)}+10i\,a^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{21\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(21*d*e^4*integral(-5/21*I*sqrt(2)*a^4*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1
))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^4), x) + sqrt(2)*(-6*I*a^4*e^(4*I*d*x + 4*
I*c) + 4*I*a^4*e^(2*I*d*x + 2*I*c) + 10*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(7/2), x)
```

3.219 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=125

$$\frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{15de^2(e\sec(c+dx))^{5/2}}-\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

```
[Out] (-2*a^4*EllipticE[(c + d*x)/2, 2])/(15*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (((4*I)/9)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/
2)) + (((4*I)/15)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(5/2)
)
```

Rubi [A] time = 0.131385, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3496, 3771, 2639}

$$\frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{15de^2(e\sec(c+dx))^{5/2}}-\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (-2*a^4*EllipticE[(c + d*x)/2, 2])/(15*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (((4*I)/9)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/
2)) + (((4*I)/15)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(5/2)
)
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} - \frac{a^2\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx}{3e^2}$$
$$= -\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} + \frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{15de^2(e\sec(c+dx))^{5/2}} - \frac{a^4\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{15e^4}$$
$$= -\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} + \frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{15de^2(e\sec(c+dx))^{5/2}} - \frac{a^4\int \sqrt{\cos(c+dx)}\,dx}{15e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= -\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} + \frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{15de^2(e\sec(c+dx))^{5/2}}$$

Mathematica [C] time = 3.667, size = 108, normalized size = 0.86

$$-\frac{ia^4e^{i(c+dx)}\left(-2\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+7e^{2i(c+dx)}+5e^{4i(c+dx)}+2\right)\sqrt{e\sec(c+dx)}}{45de^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] ((-I/45)*a^4*E^(I*(c + d*x))*(2 + 7*E^((2*I)*(c + d*x)) + 5*E^((4*I)*(c + d
*x)) - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(d*e^5)
```

Maple [B] time = 0.285, size = 370, normalized size = 3.

$$\frac{2a^4}{45d\sin(dx+c)(\cos(dx+c))^5}\left(-40i\sin(dx+c)(\cos(dx+c))^5+3i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticE}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x)
```

```
[Out] 2/45*a^4/d*(-40*I*sin(d*x+c)*cos(d*x+c)^5+3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(
d*x+c)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-40*cos
(d*x+c)^6+3*I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*sin(d*x+c)*EllipticF(
I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)+36*I*sin(d*x+c)*cos(d*x+c)^3+56*cos(d*x+c)^4-13*cos(d*x+c)^2
-3*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^5/(e/cos(d*x+c))^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")

[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-5i\,a^4e^{(6i\,dx+6i\,c)} + 5i\,a^4e^{(5i\,dx+5i\,c)} - 7i\,a^4e^{(4i\,dx+4i\,c)} + 7i\,a^4e^{(3i\,dx+3i\,c)} + 4i\,a^4e^{(2i\,dx+2i\,c)} + 2i\,a^4e^{(i\,dx+i\,c)} + 6i\,a^4\right)\sqrt{\frac{\cdots}{e^{(2i\,dx+\cdots}}}}{45\left(de^5e^{(i\,dx+i\,c)} - \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")

[Out] 1/45*(sqrt(2)*(-5*I*a^4*e^(6*I*d*x + 6*I*c) + 5*I*a^4*e^(5*I*d*x + 5*I*c) -
 7*I*a^4*e^(4*I*d*x + 4*I*c) + 7*I*a^4*e^(3*I*d*x + 3*I*c) + 4*I*a^4*e^(2*I
*d*x + 2*I*c) + 2*I*a^4*e^(I*d*x + I*c) + 6*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 45*(d*e^5*e^(I*d*x + I*c) - d*e^5)*inte
gral(1/15*sqrt(2)*(I*a^4*e^(2*I*d*x + 2*I*c) + 2*I*a^4*e^(I*d*x + I*c) + I*
a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^5*e^(3*
I*d*x + 3*I*c) - 2*d*e^5*e^(2*I*d*x + 2*I*c) + d*e^5*e^(I*d*x + I*c)), x))/
(d*e^5*e^(I*d*x + I*c) - d*e^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(9/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="giac")

[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(9/2), x)
```

3.220 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}+\frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{77de^2(e\sec(c+dx))^{7/2}}-\frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6}-\frac{4ia(a+ia\tan(c-}{11d(e\sec(c+dx}$$

```
[Out] (-2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*d*e^6) - (2*a^4*Sin[c + d*x])/(77*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((4*I)
/11)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) + (((4*I)/77)*
(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(7/2))
```

Rubi [A] time = 0.147023, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3496, 3769, 3771, 2641}

$$-\frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}+\frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{77de^2(e\sec(c+dx))^{7/2}}-\frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6}-\frac{4ia(a+ia\tan(c-}{11d(e\sec(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (-2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*d*e^6) - (2*a^4*Sin[c + d*x])/(77*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((4*I)
/11)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) + (((4*I)/77)*
(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(7/2))
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}dx = -\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} - \frac{a^2\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}dx}{11e^2}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{77de^2(e\sec(c+dx))^{7/2}} - \frac{\left(3a^4\right)\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{77e^4}$$

$$= -\frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{77de^2(e\sec(c+dx))^{7/2}} - \frac{a^4\int \sqrt{e}$$

$$= -\frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{77de^2(e\sec(c+dx))^{7/2}} - \frac{\left(a^4\sqrt{\cos}\right.}{}$$

$$= -\frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77de^6} - \frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan}{11d(e\sec(}$$

Mathematica [A] time = 1.38942, size = 148, normalized size = 0.95

$$-\frac{a^4\sqrt{e\sec(c+dx)}(\cos(3c+7dx)+i\sin(3c+7dx))\left(-3\sin(c+dx)-3\sin(3(c+dx))+37i\cos(c+dx)+11i\cos(3(c\right.}{154de^6(\cos(dx)+i\sin(dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] -(a^4*Sqrt[e*Sec[c + d*x]]*((37*I)*Cos[c + d*x] + (11*I)*Cos[3*(c + d*x)] -
 3*Sin[c + d*x] + 4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[3*(c
+ d*x)] - I*Sin[3*(c + d*x)]) - 3*Sin[3*(c + d*x)])*(Cos[3*c + 7*d*x] + I*S
in[3*c + 7*d*x]))/(154*d*e^6*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A] time = 0.306, size = 215, normalized size = 1.4

$$-\frac{2a^4}{77d(\cos(dx+c))^6}\left(56i(\cos(dx+c))^6 - 56(\cos(dx+c))^5\sin(dx+c) + i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\text{c}}{\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x)
```

```
[Out] -2/77*a^4/d*(56*I*cos(d*x+c)^6-56*cos(d*x+c)^5*sin(d*x+c)+I*cos(d*x+c)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x
+c)-1)/sin(d*x+c),I)-44*I*cos(d*x+c)^4+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+16*cos(d*
x+c)^3*sin(d*x+c)+cos(d*x+c)*sin(d*x+c))/cos(d*x+c)^6/(e/cos(d*x+c))^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{154\,de^6\text{integral}\left(\frac{i\sqrt{2}a^4\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{77\,de^6},x\right)+\sqrt{2}\left(-7i\,a^4e^{(6i\,dx+6i\,c)}-20i\,a^4e^{(4i\,dx+4i\,c)}-17i\,a^4e^{(2i\,dx+2i\,c)}-4i\,a^4\right)\sqrt{\frac{}{e^{(2i\,dx}}}}{154\,de^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/154*(154*d*e^6*integral(1/77*I*sqrt(2)*a^4*sqrt(e/(e^(2*I*d*x + 2*I*c) +
1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^6), x) + sqrt(2)*(-7*I*a^4*e^(6*I*d*x + 6
*I*c) - 20*I*a^4*e^(4*I*d*x + 4*I*c) - 17*I*a^4*e^(2*I*d*x + 2*I*c) - 4*I*a
^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(11/2), x)
```

3.221 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx$

Optimal. Leaf size=156

$$\frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{117de^2(e\sec(c+dx))^{9/2}}+\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

```
[Out] (2*a^4*EllipticE[(c + d*x)/2, 2])/(39*d*e^6*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (2*a^4*Sin[c + d*x])/(117*d*e^5*(e*Sec[c + d*x])^(3/2)) - (((4*
I)/13)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) - (((4*I)/11
7)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(9/2))
```

Rubi [A] time = 0.146584, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3496, 3769, 3771, 2639}

$$\frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{117de^2(e\sec(c+dx))^{9/2}}+\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(13/2),x]
```

```
[Out] (2*a^4*EllipticE[(c + d*x)/2, 2])/(39*d*e^6*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (2*a^4*Sin[c + d*x])/(117*d*e^5*(e*Sec[c + d*x])^(3/2)) - (((4*
I)/13)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) - (((4*I)/11
7)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(9/2))
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}+\frac{a^2\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx}{13e^2}$$
$$= -\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{117de^2(e\sec(c+dx))^{9/2}}+\frac{(5a^4)\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{117e^4}$$
$$= \frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{117de^2(e\sec(c+dx))^{9/2}}+\frac{a^4\int\frac{1}{\sqrt{e\sec}}}{39}$$
$$= \frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{117de^2(e\sec(c+dx))^{9/2}}+\frac{a^4\int}{39e^6\sqrt{\cos}}$$
$$= \frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-$$

Mathematica [C] time = 6.92916, size = 450, normalized size = 2.88

$$\frac{\sec^3(c+dx)(a+ia\tan(c+dx))^4\left(\left(-\frac{59\sin(c)}{468}-\frac{59}{468}i\cos(c)\right)\cos(3dx)+\left(\frac{37\sin(c)}{468}-\frac{37}{468}i\cos(c)\right)\cos(5dx)+\left(\frac{1}{52}\sin(3c)-\frac{1}{52}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(13/2),x]
```

```
[Out] (((-2*I)/117)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[
1 + E^((2*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*
(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*
Sec[c + d*x]^(5/2)*(a + I*a*Tan[c + d*x])^4)/(d*E^(I*(3*c + d*x))*(-1 + E^(
(2*I)*c))*(e*Sec[c + d*x])^(13/2)*(Cos[d*x] + I*Sin[d*x])^4) + (Sec[c + d*x
]^3*(Cos[3*d*x]*(((-59*I)/468)*Cos[c] - (59*Sin[c])/468) + Cos[5*d*x]*(((-3
7*I)/468)*Cos[c] + (37*Sin[c])/468) + Cos[d*x]*Csc[c]*(24*Cos[c] + (31*I)*S
in[c])*(-Cos[3*c]/468 + (I/468)*Sin[3*c]) + Cos[7*d*x]*((-I/52)*Cos[3*c] +
Sin[3*c]/52) + ((55*Cos[3*c])/468 - ((55*I)/468)*Sin[3*c])*Sin[d*x] + ((59*
Cos[c])/468 - ((59*I)/468)*Sin[c])*Sin[3*d*x] + ((37*Cos[c])/468 + ((37*I)/
468)*Sin[c])*Sin[5*d*x] + (Cos[3*c]/52 + (I/52)*Sin[3*c])*Sin[7*d*x])*(a +
I*a*Tan[c + d*x])^4)/(d*(e*Sec[c + d*x])^(13/2)*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [B] time = 0.357, size = 380, normalized size = 2.4

$$\frac{2\,a^4}{117\,d\sin(dx+c)(\cos(dx+c))^7}\left(-72\,i\sin(dx+c)(\cos(dx+c))^7-72\,(\cos(dx+c))^8+52\,i\sin(dx+c)(\cos(dx+c))^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x)
```

```
[Out] 2/117*a^4/d*(-72*I*sin(d*x+c)*cos(d*x+c)^7-72*cos(d*x+c)^8+52*I*sin(d*x+c)*
cos(d*x+c)^5+3*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*
x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*Ellipti
cE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+88*cos(d*x+c)^6+3*I*EllipticF(I*(co
s(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)-3*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-17*cos(d*x+c)^
4-2*cos(d*x+c)^2+3*cos(d*x+c))/sin(d*x+c)/cos(d*x+c)^7/(e/cos(d*x+c))^(13/2
)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{2}\left(-9i\,a^4e^{(8i\,dx+8i\,c)} + 9i\,a^4e^{(7i\,dx+7i\,c)} - 37i\,a^4e^{(6i\,dx+6i\,c)} + 37i\,a^4e^{(5i\,dx+5i\,c)} - 59i\,a^4e^{(4i\,dx+4i\,c)} + 59i\,a^4e^{(3i\,dx+3i\,c)} - 55i\,a^4e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="fricas")
```

```
[Out] 1/468*(sqrt(2)*(-9*I*a^4*e^(8*I*d*x + 8*I*c) + 9*I*a^4*e^(7*I*d*x + 7*I*c)
- 37*I*a^4*e^(6*I*d*x + 6*I*c) + 37*I*a^4*e^(5*I*d*x + 5*I*c) - 59*I*a^4*e^
(4*I*d*x + 4*I*c) + 59*I*a^4*e^(3*I*d*x + 3*I*c) - 55*I*a^4*e^(2*I*d*x + 2*
I*c) + 31*I*a^4*e^(I*d*x + I*c) - 24*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1
))*e^(1/2*I*d*x + 1/2*I*c) + 468*(d*e^7*e^(I*d*x + I*c) - d*e^7)*integral(1
/39*sqrt(2)*(-I*a^4*e^(2*I*d*x + 2*I*c) - 2*I*a^4*e^(I*d*x + I*c) - I*a^4)*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^7*e^(3*I*d*x
 + 3*I*c) - 2*d*e^7*e^(2*I*d*x + 2*I*c) + d*e^7*e^(I*d*x + I*c)), x))/(d*e^
7*e^(I*d*x + I*c) - d*e^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan (dx+c)+a)^4}{(e \sec (dx+c))^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(13/2), x)
```

3.222 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx$

Optimal. Leaf size=187

$$\frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}}+\frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}-\frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{55de^2(e\sec(c+dx))^{11/2}}+\frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(}}{33de^8}$$

```
[Out] (2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
33*d*e^8) + (2*a^4*Sin[c + d*x])/(55*d*e^5*(e*Sec[c + d*x])^(5/2)) + (2*a^4
*Sin[c + d*x])/(33*d*e^7*Sqrt[e*Sec[c + d*x]]) - (((4*I)/15)*a*(a + I*a*Tan
[c + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) - (((4*I)/55)*(a^4 + I*a^4*Tan[c
+ d*x]))/(d*e^2*(e*Sec[c + d*x])^(11/2))
```

Rubi [A] time = 0.166563, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3496, 3769, 3771, 2641}

$$\frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}}+\frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}-\frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{55de^2(e\sec(c+dx))^{11/2}}+\frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(}}{33de^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] (2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
33*d*e^8) + (2*a^4*Sin[c + d*x])/(55*d*e^5*(e*Sec[c + d*x])^(5/2)) + (2*a^4
*Sin[c + d*x])/(33*d*e^7*Sqrt[e*Sec[c + d*x]]) - (((4*I)/15)*a*(a + I*a*Tan
[c + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) - (((4*I)/55)*(a^4 + I*a^4*Tan[c
+ d*x]))/(d*e^2*(e*Sec[c + d*x])^(11/2))
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} + \frac{a^2\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx}{5e^2}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{55de^2(e\sec(c+dx))^{11/2}} + \frac{\left(7a^4\right)\int \frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{55e^4}$$

$$= \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i\left(a^4+ia^4\tan(c+dx)\right)}{55de^2(e\sec(c+dx))^{11/2}} + \frac{a^4\int \frac{}{(e\sec(}}{11}$$

$$= \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i\left(a^4+ia^4\tan\right.}{55de^2(e\sec(c}$$

$$= \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i\left(a^4+ia^4\tan\right.}{55de^2(e\sec(c}$$

$$= \frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33de^8} + \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^4\sin(c+}{33de^7\sqrt{e\sec(}}$$

Mathematica [A] time = 2.04776, size = 155, normalized size = 0.83

$$-\frac{ia^4\sqrt{e\sec(c+dx)}(\cos(4(c+2dx))+i\sin(4(c+2dx)))\left(-54i\sin(2(c+dx))-37i\sin(4(c+dx))+112\cos(2(c+dx))+\right.}{660de^8(\cos(dx)+i\sin(dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] ((-I/660)*a^4*Sqrt[e*Sec[c + d*x]]*(64 + 112*Cos[2*(c + d*x)] + 48*Cos[4*(c
 + d*x)] - (54*I)*Sin[2*(c + d*x)] - (37*I)*Sin[4*(c + d*x)] + 40*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])
)*(Cos[4*(c + 2*d*x)] + I*Sin[4*(c + 2*d*x)]))/(d*e^8*(Cos[d*x] + I*Sin[d*x
])^4)
```

Maple [A] time = 0.364, size = 232, normalized size = 1.2

$$\frac{2a^4}{165d(\cos(dx+c))^8}\left(-88i(\cos(dx+c))^8+88\sin(dx+c)(\cos(dx+c))^7+60i(\cos(dx+c))^6-16(\cos(dx+c))^5\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x)
```

```
[Out] 2/165*a^4/d*(-88*I*cos(d*x+c)^8+88*sin(d*x+c)*cos(d*x+c)^7+60*I*cos(d*x+c)^
6-16*cos(d*x+c)^5*sin(d*x+c)+5*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+5*I*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d
```

```
*x+c)-1)/sin(d*x+c),I)+3*cos(d*x+c)^3*sin(d*x+c)+5*cos(d*x+c)*sin(d*x+c))/c
os(d*x+c)^8/(e/cos(d*x+c))^(15/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{1320\,de^{8}\mathrm{integral}\left(-\frac{i\sqrt{2}a^{4}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{33\,de^{8}},x\right)+\sqrt{2}\left(-11i\,a^{4}e^{(8i\,dx+8i\,c)}-58i\,a^{4}e^{(6i\,dx+6i\,c)}-128i\,a^{4}e^{(4i\,dx+4i\,c)}-166i\right.}{1320\,de^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="fricas")
```

```
[Out] 1/1320*(1320*d*e^8*integral(-1/33*I*sqrt(2)*a^4*sqrt(e/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^8), x) + sqrt(2)*(-11*I*a^4*e^(8*I*d*x
 + 8*I*c) - 58*I*a^4*e^(6*I*d*x + 6*I*c) - 128*I*a^4*e^(4*I*d*x + 4*I*c) -
166*I*a^4*e^(2*I*d*x + 2*I*c) - 85*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))
*e^(1/2*I*d*x + 1/2*I*c))/(d*e^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(15/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^{4}}{(e\sec(dx+c))^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(15/2), x)
```

3.223 $\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}+\frac{6e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5ad}+\frac{2e^3\sin(c+dx)(e\sec(c+dx))^{5/2}}{5ad}-\frac{6e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+}}$$

```
[Out] (-6*e^6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) - (((2*I)/7)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d) + (6*e^5*Sqrt[e*Sec[
c + d*x]]*Sin[c + d*x])/(5*a*d) + (2*e^3*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x
])/(5*a*d)
```

Rubi [A] time = 0.10816, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3501, 3768, 3771, 2639}

$$-\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}+\frac{6e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5ad}+\frac{2e^3\sin(c+dx)(e\sec(c+dx))^{5/2}}{5ad}-\frac{6e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x]),x]
```


```
[Out] (-6*e^6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) - (((2*I)/7)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d) + (6*e^5*Sqrt[e*Sec[
c + d*x]]*Sin[c + d*x])/(5*a*d) + (2*e^3*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x
])/(5*a*d)
```


Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{e^2\int(e\sec(c+dx))^{7/2}\,dx}{a}$$

$$= -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{2e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{5ad} + \frac{\left(3e^4\right)\int(e\sec(c+dx))^{3/2}\,dx}{5a}$$

$$= -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{6e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5ad} + \frac{2e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{5ad}$$

$$= -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{6e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5ad} + \frac{2e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{5ad}$$

$$= -\frac{6e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{6e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5ad}$$

Mathematica [C] time = 1.43315, size = 128, normalized size = 0.94

$$\frac{e^4(\tan(c+dx)-i)(e\sec(c+dx))^{3/2}\left(-7e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+28\cos(2\right.}{70ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (e^4*(e*Sec[c + d*x])^(3/2)*(76 + 28*Cos[2*(c + d*x)] - (7*(1 + E^((2*I)*(c
 + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(
(2*I)*(c + d*x)) + (7*I)*Sec[c + d*x]*Sin[3*(c + d*x)] - (13*I)*Tan[c + d*x
])*(-I + Tan[c + d*x]))/(70*a*d)
```

Maple [B] time = 0.274, size = 375, normalized size = 2.8

$$-\frac{2\left(\cos(dx+c)+1\right)^2\left(\cos(dx+c)-1\right)^2\left(\cos(dx+c)\right)^2}{35\,ad\left(\sin(dx+c)\right)^5}\left(21\,i\left(\cos(dx+c)\right)^4\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -2/35/a/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(21*I*cos(d*x+c)^4*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/s
in(d*x+c),I)*sin(d*x+c)-21*I*cos(d*x+c)^4*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+
c)+21*I*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-21*I*cos(d*x+c)^3*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos
(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+21*cos(d*x+c)^4-14*cos(d*x+c)^3+5*I*sin
(d*x+c)-7*cos(d*x+c))*(e/cos(d*x+c))^(11/2)*cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-42i\,e^5e^{(7i\,dx+7i\,c)}-154i\,e^5e^{(5i\,dx+5i\,c)}-206i\,e^5e^{(3i\,dx+3i\,c)}-14i\,e^5e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+35\left(ade^{(6i\,dx+6i\,c)}+3\cdots\right.}{35\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx+2i\,c)}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/35*(sqrt(2)*(-42*I*e^5*e^(7*I*d*x + 7*I*c) - 154*I*e^5*e^(5*I*d*x + 5*I*c
) - 206*I*e^5*e^(3*I*d*x + 3*I*c) - 14*I*e^5*e^(I*d*x + I*c))*sqrt(e/(e^(2*
I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 35*(a*d*e^(6*I*d*x + 6*I*c)
+ 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x + 2*I*c) + a*d)*integral(3/5
*I*sqrt(2)*e^5*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(a
*d), x))/(a*d*e^(6*I*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*
I*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec{(dx+c)})^{\frac{11}{2}}}{i\,a\tan{(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a), x)
```

3.224 $\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}+\frac{2e^3\sin(c+dx)(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3ad}$$

```
[Out] (2*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
3*a*d) - (((2*I)/5)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d) + (2*e^3*(e*Sec[c + d
*x])^(3/2)*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.0891565, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3501, 3768, 3771, 2641}

$$-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}+\frac{2e^3\sin(c+dx)(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
3*a*d) - (((2*I)/5)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d) + (2*e^3*(e*Sec[c + d
*x])^(3/2)*Sin[c + d*x])/(3*a*d)
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad} + \frac{e^2\int (e\sec(c+dx))^{5/2}\,dx}{a}$$
$$= -\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad} + \frac{2e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3ad} + \frac{e^4\int \sqrt{e\sec(c+dx)}\,dx}{3a}$$
$$= -\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad} + \frac{2e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3ad} + \frac{\left(e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)}{3a}$$
$$= \frac{2e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3ad} - \frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad} + \frac{2e^3(e\sec(c+dx))^{3}}{3ad}$$

Mathematica [A] time = 0.584932, size = 62, normalized size = 0.59

$$\frac{e^2(e\sec(c+dx))^{5/2}\left(5\sin(2(c+dx))+10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6i\right)}{15ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (e^2*(e*Sec[c + d*x])^(5/2)*(-6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*
x)/2, 2] + 5*Sin[2*(c + d*x)]))/(15*a*d)
```

Maple [A] time = 0.239, size = 202, normalized size = 1.9

$$\frac{2\,(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2(\cos(dx+c))^2}{15\,ad\,(\sin(dx+c))^4}\left(5\,i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c))^3\,\mathrm{Ellipt}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/15/a/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticF(I*(cos(d*x+c)-1)/sin
(d*x+c),I)+5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)^2*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+5*cos(d*x+c)*sin(d*x+c
)-3*I)*(e/cos(d*x+c))^(9/2)*cos(d*x+c)^2/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-10i\,e^4e^{(4i\,dx+4i\,c)} - 24i\,e^4e^{(2i\,dx+2i\,c)} + 10i\,e^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + 15\left(ade^{(4i\,dx+4i\,c)} + 2\,ade^{(2i\,dx+2i\,c)} + ad\right)\mathrm{integ}}{15\left(ade^{(4i\,dx+4i\,c)} + 2\,ade^{(2i\,dx+2i\,c)} + ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(sqrt(2)*(-10*I*e^4*e^(4*I*d*x + 4*I*c) - 24*I*e^4*e^(2*I*d*x + 2*I*c)
 + 10*I*e^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 15
*(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)*integral(-1/3*
I*sqrt(2)*e^4*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a
*d), x))/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a), x)
```

3.225 $\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=101

$$-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{ad}-\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

```
[Out] (-2*e^4*EllipticE[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d
*x]]) - (((2*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d) + (2*e^3*Sqrt[e*Sec[c
+ d*x]]*Sin[c + d*x])/(a*d)
```

Rubi [A] time = 0.0878165, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3501, 3768, 3771, 2639}

$$-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{ad}-\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-2*e^4*EllipticE[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d
*x]]) - (((2*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d) + (2*e^3*Sqrt[e*Sec[c
+ d*x]]*Sin[c + d*x])/(a*d)
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad} + \frac{e^2\int(e\sec(c+dx))^{3/2}\,dx}{a}$$
$$= -\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad} + \frac{2e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{ad} - \frac{e^4\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{a}$$
$$= -\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad} + \frac{2e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{ad} - \frac{e^4\int\sqrt{\cos(c+dx)}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= -\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad} + \frac{2e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{ad}$$

Mathematica [C] time = 0.721794, size = 102, normalized size = 1.01

$$\frac{2ie^3(\cos(c)+i\sin(c))(\cos(dx)+i\sin(dx))\sqrt{e\sec(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+i\,t\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((2*I)/3)*e^3*Sqrt[e*Sec[c + d*x]]*(Cos[c] + I*Sin[c])*(Cos[d*x] + I*Sin[d
*x])*(-4 + Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))] + I*Tan[c + d*x]))/(a*d)
```

Maple [B] time = 0.233, size = 361, normalized size = 3.6

$$-\frac{2\left(\cos(dx+c)+1\right)^2\left(\cos(dx+c)-1\right)^2\left(\cos(dx+c)\right)^2}{3\,ad\left(\sin(dx+c)\right)^5}\left(3\,i\left(\cos(dx+c)\right)^2\sin(dx+c)\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos}{\cos(d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -2/3/a/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(3*I*cos(d*x+c)^2*sin(d*x+c)*(1/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*
x+c)-1)/sin(d*x+c),I)-3*I*cos(d*x+c)^2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+
3*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*EllipticE(I*(cos(d*x
+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)^2+I*sin(d*x+c)-3*cos(d*x+c))*(e/cos
(d*x+c))^(7/2)*cos(d*x+c)^2/sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-6i\,e^{3}e^{(3i\,dx+3i\,c)}-10i\,e^{3}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\left(ade^{(2i\,dx+2i\,c)}+ad\right)\text{integral}\left(\frac{i\sqrt{2}e^{3}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{ad},x\right.}{3\left(ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-6*I*e^3*e^(3*I*d*x + 3*I*c) - 10*I*e^3*e^(I*d*x + I*c))*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*(a*d*e^(2*I*d*x +
 2*I*c) + a*d)*integral(I*sqrt(2)*e^3*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(
1/2*I*d*x + 1/2*I*c)/(a*d), x))/(a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a), x)
```

3.226 $\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{ad}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

```
[Out] ((-2*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(a*d)
```

Rubi [A] time = 0.0736854, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3501, 3771, 2641}

$$\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{ad}-\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-2*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(a*d)
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] && !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx &= -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}+\frac{e^2\int \sqrt{e\sec(c+dx)}\,dx}{a}\\
&= -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}+\frac{\left(e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a}\\
&= -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}+\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{ad}
\end{aligned}$$

Mathematica [A] time = 0.302151, size = 49, normalized size = 0.7

$$\frac{2e^2\sqrt{e\sec(c+dx)}\left(\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-i\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*e^2*(-I + Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[e*Sec[c + d
*x]])/(a*d)
```

Maple [A] time = 0.237, size = 174, normalized size = 2.5

$$\frac{2\,i\left(\cos\left(dx+c\right)+1\right)^{2}\left(\cos\left(dx+c\right)-1\right)^{2}\left(\cos\left(dx+c\right)\right)^{2}}{ad\left(\sin\left(dx+c\right)\right)^{4}}\left(\cos\left(dx+c\right)\mathrm{EllipticF}\left(\frac{i\left(\cos\left(dx+c\right)-1\right)}{\sin\left(dx+c\right)},i\right)\sqrt{\left(\cos\left(dx+c\right)+1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2*I/a/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(cos(d*x+c)*EllipticF(I*(cos(d*x+
c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)+EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)-1)*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^2/sin(d*x+c
)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{-2i\sqrt{2}e^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+ad\mathrm{integral}\left(-\frac{i\sqrt{2}e^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{ad},x\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-2*I*sqrt(2)*e^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
 + a*d*integral(-I*sqrt(2)*e^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*
```

```
d*x - 1/2*I*c)/(a*d), x))/(a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a), x)
```

3.227 $\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

```
[Out] ((2*I)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]) + (2*e^2*EllipticE[(c + d*x)/2, 2])/
(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.0728845, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3501, 3771, 2639}

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((2*I)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]) + (2*e^2*EllipticE[(c + d*x)/2, 2])/
(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \frac{2ie^2}{ad\sqrt{e\sec(c+dx)}} + \frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{a}$$
$$= \frac{2ie^2}{ad\sqrt{e\sec(c+dx)}} + \frac{e^2\int \sqrt{\cos(c+dx)}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= \frac{2ie^2}{ad\sqrt{e\sec(c+dx)}} + \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

Mathematica [C] time = 0.383024, size = 74, normalized size = 1.06

$$\frac{2iee^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((2*I)*e*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E
^((2*I)*(c + d*x))]*Sqrt[e*Sec[c + d*x]])/(a*d*E^(I*(c + d*x)))
```

Maple [B] time = 0.248, size = 916, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -1/2/a/d*(e/cos(d*x+c))^(3/2)*(cos(d*x+c)+1)^3*(cos(d*x+c)-1)^2*(4*I*(-cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*sin(d*x+c)*cos(d*x+c)^2*EllipticF(I*(cos(d*x
+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)-4*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*sin(d*x+c)*cos(d*x+c)^2*Ellip
ticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)+8*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*sin(d*x+c)*cos
(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-8*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*s
in(d*x+c)*cos(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4*I*(-cos(d*x+c)/(cos(d*x+c)+
1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elli
pticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-4*I*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*El
lipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+4*I*cos(d*x+c)^2*sin(d*x+
c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-4*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)+4*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)*sin(d*
x+c)+I*ln(-(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^
2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*cos(
d*x+c)*sin(d*x+c)-I*ln(-2*(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c
)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin
(d*x+c)^2)*cos(d*x+c)*sin(d*x+c)+4*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2
)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)/sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(ade^{(i\,dx+i\,c)}\text{integral}\left(-\frac{i\,\sqrt{2}e\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{ad},x\right)+\sqrt{2}\left(2i\,ee^{(2i\,dx+2i\,c)}+2i\,e\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right)e^{(-i\,dx-i\,c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*d*e^(I*d*x + I*c)*integral(-I*sqrt(2)*e*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(1/2*I*d*x + 1/2*I*c)/(a*d), x) + sqrt(2)*(2*I*e*e^(2*I*d*x + 2*I*c) +
2*I*e)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x
 - I*c)/(a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a), x)
```

3.228 $\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3ad}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*a*
d) + (((2*I)/3)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0666296, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3502, 3771, 2641}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3ad}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*a*
d) + (((2*I)/3)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx &= \frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}+\frac{\int \sqrt{e\sec(c+dx)}\,dx}{3a}\\ &= \frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a}\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3ad}+\frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.305043, size = 83, normalized size = 1.04

$$\frac{2(e\sec(c+dx))^{3/2}\left(\cos(c+dx)+\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(\sin(c+dx)-i\cos(c+dx))\right)}{3ade(\tan(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*(e*Sec[c + d*x])^(3/2)*(Cos[c + d*x] + Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*((-I)*Cos[c + d*x] + Sin[c + d*x])))/(3*a*d*e*(-I + Tan[c + d*x
]))
```

Maple [B] time = 0.319, size = 192, normalized size = 2.4

$$\frac{2\left(\cos(dx+c)-1\right)^{2}\left(\cos(dx+c)+1\right)^{2}}{3\,ad\left(\sin(dx+c)\right)^{4}}\sqrt{\frac{e}{\cos(dx+c)}}\left(i\cos(dx+c)\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/3/a/d*(e/cos(d*x+c))^(1/2)*(cos(d*x+c)-1)^2*(cos(d*x+c)+1)^2*(I*cos(d*x+c
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(c
os(d*x+c)-1)/sin(d*x+c),I)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+I*cos(d*x+c)^2+cos(d*
x+c)*sin(d*x+c))/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,ade^{(2i\,dx+2i\,c)}\mathrm{integral}\left(-\frac{i\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,ad},x\right)+\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right)e^{(-2i\,dx-2i\,c)}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*d*e^(2*I*d*x + 2*I*c)*integral(-1/3*I*sqrt(2)*sqrt(e/(e^(2*I*d*x +
 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a*d), x) + sqrt(2)*sqrt(e/(e^(2*I*d
*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1/2*I*c))*e^(-
2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e \sec(dx + c)}}{i a \tan(dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a), x)
```

3.229 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=80

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]
]) + ((2*I)/5)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0696769, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3502, 3771, 2639}

$$\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]
]) + ((2*I)/5)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx &= \frac{2i}{5d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}+\frac{3\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a}\\
&= \frac{2i}{5d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}+\frac{3\int \sqrt{\cos(c+dx)}\,dx}{5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}
\end{aligned}$$

Mathematica [C] time = 0.799048, size = 109, normalized size = 1.36

$$\frac{(\tan(c+dx)+i)\left(-2e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+3i\sin(2(c+dx))+4\cos(2(c+\right.}{5ad\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((4 + 4*Cos[2*(c + d*x)] - 2*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c
 + d*x)])*(I + Tan[c + d*x]))/(5*a*d*Sqrt[e*Sec[c + d*x]])
```

Maple [B] time = 0.444, size = 358, normalized size = 4.5

$$\frac{2\left(\cos(dx+c)+1\right)^{2}\left(\cos(dx+c)-1\right)^{2}}{5ad\left(\sin(dx+c)\right)^{5}e}\left(3i\text{EllipticF}\left(\frac{i\left(\cos(dx+c)-1\right)}{\sin(dx+c)},i\right)\sin(dx+c)\cos(dx+c)\sqrt{(\cos(dx+c)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/5/a/d*(3*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*EllipticE(I
*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*sin(d*x+c)*cos(d*x+c)^3+3*I*EllipticF
(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)-3*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin
(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-cos(d*x+
c)^4-2*cos(d*x+c)^2+3*cos(d*x+c))*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(e/cos(
d*x+c))^(1/2)/sin(d*x+c)^5/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(5i\,dx+5i\,c)}-7i\,e^{(4i\,dx+4i\,c)}-4i\,e^{(3i\,dx+3i\,c)}-8i\,e^{(2i\,dx+2i\,c)}+i\,e^{(i\,dx+i\,c)}-i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+10\left(adee^{(4i\,dx+4\right.}{10\left(adee^{(4i\,dx+4i\,c)}-adee^{(3i\,dx+3i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-5*I*e^(5*I*d*x + 5*I*c) -
 7*I*e^(4*I*d*x + 4*I*c) - 4*I*e^(3*I*d*x + 3*I*c) - 8*I*e^(2*I*d*x + 2*I*c
) + I*e^(I*d*x + I*c) - I)*e^(1/2*I*d*x + 1/2*I*c) + 10*(a*d*e*e^(4*I*d*x +
 4*I*c) - a*d*e*e^(3*I*d*x + 3*I*c))*integral(1/5*sqrt(2)*sqrt(e/(e^(2*I*d*
x + 2*I*c) + 1))*(-3*I*e^(2*I*d*x + 2*I*c) - 6*I*e^(I*d*x + I*c) - 3*I)*e^(
1/2*I*d*x + 1/2*I*c)/(a*d*e*e^(3*I*d*x + 3*I*c) - 2*a*d*e*e^(2*I*d*x + 2*I*
c) + a*d*e*e^(I*d*x + I*c)), x))/(a*d*e*e^(4*I*d*x + 4*I*c) - a*d*e*e^(3*I*
d*x + 3*I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)), x)
```

3.230 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=114

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21ade^2}+\frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(21*
a*d*e^2) + (10*Sin[c + d*x])/(21*a*d*e*Sqrt[e*Sec[c + d*x]]) + ((2*I)/7)/(d
*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0930508, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3502, 3769, 3771, 2641}

$$\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21ade^2}+\frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}}+\frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(21*
a*d*e^2) + (10*Sin[c + d*x])/(21*a*d*e*Sqrt[e*Sec[c + d*x]]) + ((2*I)/7)/(d
*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}dx = \frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))} + \frac{5\int \frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a}$$
$$= \frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}} + \frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))} + \frac{5\int \sqrt{e\,\mathrm{se}}}{2}$$
$$= \frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}} + \frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))} + \frac{\left(5\sqrt{\cos(}\right.}{}$$
$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21ade^2} + \frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}} + \frac{}{7}$$

Mathematica [A] time = 0.487852, size = 125, normalized size = 1.1

$$-\frac{\sec^3(c+dx)\left(5i\sin(c+dx)+5i\sin(3(c+dx))-14\cos(c+dx)+2\cos(3(c+dx))+20i\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(c\right.}{42ad(\tan(c+dx)-i)(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] -(Sec[c + d*x]^3*(-14*Cos[c + d*x] + 2*Cos[3*(c + d*x)] + (20*I)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]) + (5*I)*
Sin[c + d*x] + (5*I)*Sin[3*(c + d*x)]))/(42*a*d*(e*Sec[c + d*x])^(3/2)*(-I
+ Tan[c + d*x]))
```

Maple [A] time = 0.336, size = 218, normalized size = 1.9

$$\frac{2\left(\cos(dx+c)-1\right)^2\left(\cos(dx+c)+1\right)^2\cos(dx+c)}{21\,ade^3\left(\sin(dx+c)\right)^4}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(5\,i\cos(dx+c)\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+}{\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/21/a/d*(e/cos(d*x+c))^(3/2)*(cos(d*x+c)-1)^2*(cos(d*x+c)+1)^2*cos(d*x+c)*
(5*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+3*I*cos(d*x+c)^4+5*I*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/si
n(d*x+c),I)+3*cos(d*x+c)^3*sin(d*x+c)+5*cos(d*x+c)*sin(d*x+c))/e^3/sin(d*x+
c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

[Out] Exception raised: RuntimeError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(84\,ade^2e^{(4i\,dx+4i\,c)}\text{integral}\left(-\frac{5i\,\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{21\,ade^2},x\right)+\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(6i\,dx+6i\,c)}+9i\,e^{(4i\,dx+4i\,c)}+19i\,e^{(2i\,dx}\right.\right.}{84\,ade^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/84*(84*a*d*e^2*e^(4*I*d*x + 4*I*c)*integral(-5/21*I*sqrt(2)*sqrt(e/(e^(2*
I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a*d*e^2), x) + sqrt(2)*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*(-7*I*e^(6*I*d*x + 6*I*c) + 9*I*e^(4*I*d*x + 4
*I*c) + 19*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(1/2*I*d*x + 1/2*I*c))*e^(-4*I*d*
x - 4*I*c)/(a*d*e^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)), x)
```

3.231 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=114

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*a*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) + (14*Sin[c + d*x])/(45*a*d*e*(e*Sec[c + d*x])^(3/2)) + ((2*I)/9)/
(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0919698, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3502, 3769, 3771, 2639}

$$\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}}+\frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*a*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) + (14*Sin[c + d*x])/(45*a*d*e*(e*Sec[c + d*x])^(3/2)) + ((2*I)/9)/
(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))} + \frac{7\int \frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{9a}$$

$$= \frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}} + \frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))} + \frac{7\int}{}$$

$$= \frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}} + \frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))} + \frac{}{15a}$$

$$= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}} + \frac{}{9d(e\sec}$$

Mathematica [C] time = 0.982436, size = 134, normalized size = 1.18

$$\frac{(\tan(c+dx)+i)\left(-56e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+70i\sin(2(c+dx))-7i\sin(4(c\right.}{180ade^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((106 + 104*Cos[2*(c + d*x)] - 2*Cos[4*(c + d*x)] - 56*E^((2*I)*(c + d*x))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))] + (70*I)*Sin[2*(c + d*x)] - (7*I)*Sin[4*(c + d*x)])*(I + Tan[c +
d*x]))/(180*a*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [B] time = 0.408, size = 376, normalized size = 3.3

$$-\frac{2\,(\cos(dx+c))^2(\cos(dx+c)-1)^2(\cos(dx+c)+1)^2}{45\,ade^5\,(\sin(dx+c))^5}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(-5\,i\,(\cos(dx+c))^5\sin(dx+c)+5\,(\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -2/45/a/d*cos(d*x+c)^2*(e/cos(d*x+c))^(5/2)*(cos(d*x+c)-1)^2*(cos(d*x+c)+1)
^2*(-5*I*cos(d*x+c)^5*sin(d*x+c)+5*cos(d*x+c)^6-21*I*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),
I)*cos(d*x+c)*sin(d*x+c)+21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c
)-21*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+21*I*EllipticE(I*(cos(d*x+c)-1)/
sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)+2*cos(d*x+c)^4+14*cos(d*x+c)^2-21*cos(d*x+c))/e^5/sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-9i\,e^{(9i\,dx+9i\,c)}+9i\,e^{(8i\,dx+8i\,c)}-162i\,e^{(7i\,dx+7i\,c)}-174i\,e^{(6i\,dx+6i\,c)}-124i\,e^{(5i\,dx+5i\,c)}-212i\,e^{(4i\,dx+4i\,c)}+34i\,e^{(3}\right.$$

36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/360*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-9*I*e^(9*I*d*x + 9*I*c)
+ 9*I*e^(8*I*d*x + 8*I*c) - 162*I*e^(7*I*d*x + 7*I*c) - 174*I*e^(6*I*d*x +
6*I*c) - 124*I*e^(5*I*d*x + 5*I*c) - 212*I*e^(4*I*d*x + 4*I*c) + 34*I*e^(3*
I*d*x + 3*I*c) - 34*I*e^(2*I*d*x + 2*I*c) + 5*I*e^(I*d*x + I*c) - 5*I)*e^(1
/2*I*d*x + 1/2*I*c) + 360*(a*d*e^3*e^(6*I*d*x + 6*I*c) - a*d*e^3*e^(5*I*d*x
 + 5*I*c))*integral(1/15*sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-7*I*e^
(2*I*d*x + 2*I*c) - 14*I*e^(I*d*x + I*c) - 7*I)*e^(1/2*I*d*x + 1/2*I*c)/(a*
d*e^3*e^(3*I*d*x + 3*I*c) - 2*a*d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3*e^(I*d*
x + I*c)), x))/(a*d*e^3*e^(6*I*d*x + 6*I*c) - a*d*e^3*e^(5*I*d*x + 5*I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{5}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)), x)
```

3.232 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx$

Optimal. Leaf size=145

$$\frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}}+\frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77ade^4}+\frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}}+\frac{}{11d(a+ia\tan(c}$$

```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(77*
a*d*e^4) + (18*Sin[c + d*x])/(77*a*d*e*(e*Sec[c + d*x])^(5/2)) + (30*Sin[c
+ d*x])/(77*a*d*e^3*Sqrt[e*Sec[c + d*x]]) + ((2*I)/11)/(d*(e*Sec[c + d*x])^
(7/2)*(a + I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.109497, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3502, 3769, 3771, 2641}

$$\frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}}+\frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77ade^4}+\frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}}+\frac{}{11d(a+ia\tan(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(77*
a*d*e^4) + (18*Sin[c + d*x])/(77*a*d*e*(e*Sec[c + d*x])^(5/2)) + (30*Sin[c
+ d*x])/(77*a*d*e^3*Sqrt[e*Sec[c + d*x]]) + ((2*I)/11)/(d*(e*Sec[c + d*x])^
(7/2)*(a + I*a*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))} + \frac{9\int \frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{11a}$$
$$= \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}} + \frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))} + \frac{45\int$$
$$= \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}} + \frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}} + \frac{2i}{11d(e\sec(c+dx))^{7/2}(a+$$
$$= \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}} + \frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}} + \frac{2i}{11d(e\sec(c+dx))^{7/2}(a+$$
$$= \frac{30\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77ade^4} + \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}} +$$

Mathematica [A] time = 0.720903, size = 142, normalized size = 0.98

$$-\frac{(e\sec(c+dx))^{3/2}\left(78i\sin(c+dx)+87i\sin(3(c+dx))+9i\sin(5(c+dx))-148\cos(c+dx)+34\cos(3(c+dx))+2\cos(}{616ade^5(\tan(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] -((e*Sec[c + d*x])^(3/2)*(-148*Cos[c + d*x] + 34*Cos[3*(c + d*x)] + 2*Cos[5
*(c + d*x)] + (240*I)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[c +
 d*x] + I*Sin[c + d*x]) + (78*I)*Sin[c + d*x] + (87*I)*Sin[3*(c + d*x)] + (
9*I)*Sin[5*(c + d*x)]))/(616*a*d*e^5*(-I + Tan[c + d*x]))
```

Maple [A] time = 0.397, size = 236, normalized size = 1.6

$$\frac{2\left(\cos(dx+c)-1\right)^2\left(\cos(dx+c)+1\right)^2\left(\cos(dx+c)\right)^3}{77\,ade^7\left(\sin(dx+c)\right)^4}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}\left(7\,i\left(\cos(dx+c)\right)^6+7\left(\cos(dx+c)\right)^5\sin(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/77/a/d*(e/cos(d*x+c))^(7/2)*(cos(d*x+c)-1)^2*(cos(d*x+c)+1)^2*cos(d*x+c)^
3*(7*I*cos(d*x+c)^6+7*cos(d*x+c)^5*sin(d*x+c)+15*I*cos(d*x+c)*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/si
n(d*x+c),I)+15*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+9*cos(d*x+c)^3*sin(d*x+c)+15*cos(
d*x+c)*sin(d*x+c))/e^7/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(1232\, ade^4 e^{(6i\,dx+6i\,c)}\text{integral}\left(-\frac{15i\,\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{77\,ade^4}, x\right) + \sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-11i\,e^{(10i\,dx+10i\,c)} - 121i\,e^{(8i\,dx+8i\,c)} + \right.\right.}{1232\,ade^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1232*(1232*a*d*e^4*e^(6*I*d*x + 6*I*c)*integral(-15/77*I*sqrt(2)*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a*d*e^4), x) + sqrt(2)*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(10*I*d*x + 10*I*c) - 121*I*e^(8
*I*d*x + 8*I*c) + 70*I*e^(6*I*d*x + 6*I*c) + 226*I*e^(4*I*d*x + 4*I*c) + 53
*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(1/2*I*d*x + 1/2*I*c))*e^(-6*I*d*x - 6*I*c)
/(a*d*e^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e \sec(dx+c))^{\frac{7}{2}}\,(i\,a\tan(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)), x)
```

3.233 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=183

$$\frac{22e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{15a^2d}+\frac{22e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{45a^2d}+\frac{22e^3\sin(c+dx)(e\sec(c+dx))^{9/2}}{63a^2d}-\frac{4ie^2(e\sec(c+}{7d\left(a^2+ia^2\tan\right.}$$

```
[Out] (-22*e^8*EllipticE[(c + d*x)/2, 2])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec
[c + d*x]]) + (22*e^7*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d) + (22*e
^5*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(45*a^2*d) + (22*e^3*(e*Sec[c + d*x
])^(9/2)*Sin[c + d*x])/(63*a^2*d) - (((4*I)/7)*e^2*(e*Sec[c + d*x])^(11/2))
/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.127274, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3768, 3771, 2639}

$$\frac{22e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{15a^2d}+\frac{22e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{45a^2d}+\frac{22e^3\sin(c+dx)(e\sec(c+dx))^{9/2}}{63a^2d}-\frac{4ie^2(e\sec(c+}{7d\left(a^2+ia^2\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-22*e^8*EllipticE[(c + d*x)/2, 2])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec
[c + d*x]]) + (22*e^7*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d) + (22*e
^5*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(45*a^2*d) + (22*e^3*(e*Sec[c + d*x
])^(9/2)*Sin[c + d*x])/(63*a^2*d) - (((4*I)/7)*e^2*(e*Sec[c + d*x])^(11/2))
/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(11e^2\right)\int(e\sec(c+dx))^{11/2}\,dx}{7a^2}$$

$$= \frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(11e^4\right)\int(e\sec(c+dx))^{7}}{9a^2}$$

$$= \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d} + \frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d} - \frac{4ie^2(e\sec(c+}{7d\left(a^2+ia^2\tan\right.}$$

$$= \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d} + \frac{22e^3(e\sec(c+dx}{63a}$$

$$= \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d} + \frac{22e^3(e\sec(c+dx}{63a}$$

$$= -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{22e^5(e\sec(c+}{4}$$

Mathematica [C] time = 2.28387, size = 302, normalized size = 1.65

$$\frac{(\cos(dx)+i\sin(dx))^2(e\sec(c+dx))^{15/2}\left(\frac{22i\sqrt{2}e^{3ic-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(\left(-1+e^{2ic}\right)e^{2idx}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3\sqrt{1+}\right.}{-1+e^{2ic}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Sec[c + d*x])^(15/2)*(Cos[d*x] + I*Sin[d*x])^2*(((22*I)*Sqrt[2]*E^((3*I
)*c - I*d*x)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^(
(2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(-1 + E^
((2*I)*c)) + (Csc[c]*Sec[c + d*x]^(9/2)*(Cos[2*c] + I*Sin[2*c])*(1260*Cos[d
*x] + 1050*Cos[2*c + d*x] + 1078*Cos[2*c + 3*d*x] + 77*Cos[4*c + 3*d*x] + 2
31*Cos[4*c + 5*d*x] + (720*I)*Sin[d*x] - (720*I)*Sin[2*c + d*x]))/56))/(45*
d*Sec[c + d*x]^(11/2)*(a + I*a*Tan[c + d*x])^2)
```

Maple [B] time = 0.312, size = 384, normalized size = 2.1

$$\frac{2\left(\cos(dx+c)+1\right)^2\left(\cos(dx+c)-1\right)^2\left(\cos(dx+c)\right)^3}{315\,a^2d\left(\sin(dx+c)\right)^5}\left(231\,i\left(\cos(dx+c)\right)^5\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{E}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/315/a^2/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(231*I*cos(d*x+c)^5*(1/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1
)/sin(d*x+c),I)*sin(d*x+c)-231*I*cos(d*x+c)^5*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(
d*x+c)+231*I*sin(d*x+c)*cos(d*x+c)^4*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-231*I*sin(d*x
+c)*cos(d*x+c)^4*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-231*cos(d*x+c)^5+154*cos(d*x+c)^4
-90*I*cos(d*x+c)*sin(d*x+c)+112*cos(d*x+c)^2-35)*(e/cos(d*x+c))^(15/2)*cos(
d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-462 i e^{7} e^{(9 i dx+9 i c)}-2156 i e^{7} e^{(7 i dx+7 i c)}-3960 i e^{7} e^{(5 i dx+5 i c)}-3540 i e^{7} e^{(3 i dx+3 i c)}-154 i e^{7} e^{(i dx+i c)}\right)\sqrt{\frac{e}{e^{(2 i dx+2 i c)}+1}}\, e^{\left(\frac{1}{2} i dx+\frac{1}{2}\right.}}{315\left(a^{2} d e^{(8 i dx+8 i c)}+4 a^{2} d e^{(6 i dx+6 i c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/315*(sqrt(2)*(-462*I*e^7*e^(9*I*d*x + 9*I*c) - 2156*I*e^7*e^(7*I*d*x + 7*
I*c) - 3960*I*e^7*e^(5*I*d*x + 5*I*c) - 3540*I*e^7*e^(3*I*d*x + 3*I*c) - 15
4*I*e^7*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1
/2*I*c) + 315*(a^2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*
a^2*d*e^(4*I*d*x + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*integral(1
1/15*I*sqrt(2)*e^7*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
)/(a^2*d), x))/(a^2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6
*a^2*d*e^(4*I*d*x + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec(dx+c))^{\frac{15}{2}}}{(i a \tan(dx+c)+a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.234 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$\frac{6e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{7a^2d}+\frac{18e^3\sin(c+dx)(e\sec(c+dx))^{7/2}}{35a^2d}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{}$$

```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
7*a^2*d) + (6*e^5*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*a^2*d) + (18*e^3*
(e*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(35*a^2*d) - (((4*I)/5)*e^2*(e*Sec[c +
 d*x])^(9/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.108869, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3768, 3771, 2641}

$$\frac{6e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{7a^2d}+\frac{18e^3\sin(c+dx)(e\sec(c+dx))^{7/2}}{35a^2d}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
7*a^2*d) + (6*e^5*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*a^2*d) + (18*e^3*
(e*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(35*a^2*d) - (((4*I)/5)*e^2*(e*Sec[c +
 d*x])^(9/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}dx = -\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(9e^2\right)\int (e\sec(c+dx))^{9/2}dx}{5a^2}$$

$$= \frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(9e^4\right)\int (e\sec(c+dx))^{5/\cdots}}{7a^2}$$

$$= \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d} - \frac{4ie^2(e\sec(c+\cdots}{5d\left(a^2+ia^2\tan(\cdots\right.}$$

$$= \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d} - \frac{4ie^2(e\sec(c+\cdots}{5d\left(a^2+ia^2\tan(\cdots\right.}$$

$$= \frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d} + \frac{18\cdots}{\cdots}$$

Mathematica [A] time = 0.486882, size = 85, normalized size = 0.56

$$\frac{e^6\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(-5\sin(c+dx)+15\sin(3(c+dx))-56i\cos(c+dx)+60\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right.}{70a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (e^6*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((-56*I)*Cos[c + d*x] + 60*Cos[c +
 d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[c + d*x] + 15*Sin[3*(c + d*x)
]))/(70*a^2*d)
```

Maple [A] time = 0.279, size = 219, normalized size = 1.4

$$\frac{2\left(\cos(dx+c)+1\right)^2\left(\cos(dx+c)-1\right)^2\left(\cos(dx+c)\right)^3}{35a^2d\left(\sin(dx+c)\right)^4}\left(15i\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(\cos\cdots}{\sin\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/35/a^2/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(15*I*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)
*cos(d*x+c)^4+15*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)^3*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+15*cos(d*x+c)^2*si
n(d*x+c)-14*I*cos(d*x+c)-5*sin(d*x+c))*(e/cos(d*x+c))^(13/2)*cos(d*x+c)^3/s
in(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-30i\,e^{6}e^{(6i\,dx+6i\,c)}-102i\,e^{6}e^{(4i\,dx+4i\,c)}-122i\,e^{6}e^{(2i\,dx+2i\,c)}+30i\,e^{6}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+35\left(a^{2}de^{(6i\,dx+6i\,c)}+3\,a^{2}de^{(4}\right.}{35\left(a^{2}de^{(6i\,dx+6i\,c)}+3\,a^{2}de^{(4i\,dx+4i\,c)}+3\,a^{2}de^{(2i\,dx+2i\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/35*(sqrt(2)*(-30*I*e^6*e^(6*I*d*x + 6*I*c) - 102*I*e^6*e^(4*I*d*x + 4*I*c
) - 122*I*e^6*e^(2*I*d*x + 2*I*c) + 30*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(1/2*I*d*x + 1/2*I*c) + 35*(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4
*I*d*x + 4*I*c) + 3*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*integral(-3/7*I*sqrt
(2)*e^6*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^2*d),
 x))/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^(
2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{13}{2}}}{(i\,a\tan(dx+c)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.235 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$\frac{14e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^2d}+\frac{14e^3\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^2d}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{14e^6E\left(\frac{1}{2}(c+\right.}{5a^2d\sqrt{\cos(c+dx)}\sqrt{}}$$

```
[Out] (-14*e^6*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (14*e^5*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*d) + (14*e^3
*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(15*a^2*d) - (((4*I)/3)*e^2*(e*Sec[c
+ d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.107401, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3768, 3771, 2639}

$$\frac{14e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^2d}+\frac{14e^3\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^2d}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{14e^6E\left(\frac{1}{2}(c+\right.}{5a^2d\sqrt{\cos(c+dx)}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^2,x]
```


```
[Out] (-14*e^6*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (14*e^5*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*d) + (14*e^3
*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(15*a^2*d) - (((4*I)/3)*e^2*(e*Sec[c
+ d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}dx = -\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(7e^2\right)\int(e\sec(c+dx))^{7/2}dx}{3a^2}$$

$$= \frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d} - \frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(7e^4\right)\int(e\sec(c+dx))^{3/2}dx}{5a^2}$$

$$= \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^2d} + \frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d} - \frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$= \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^2d} + \frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d} - \frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

$$= -\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^2d} + \frac{14e^3(e\sec(c+dx))^{5/2}}{15a^2d}$$

Mathematica [C] time = 0.870055, size = 123, normalized size = 0.81

$$\frac{2ie^5e^{i(c+dx)}\left(7\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-56e^{2i(c+dx)}-21e^{4i(c+dx)}-47\right)\sqrt{e\sec(c+dx)}}{15a^2d\left(1+e^{2i(c+dx)}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((2*I)/15)*e^5*E^(I*(c + d*x))*(-47 - 56*E^((2*I)*(c + d*x)) - 21*E^((4*I)
*(c + d*x)) + 7*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4,
 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(a^2*d*(1 + E^((2*I)*(c
+ d*x)))^2)
```

Maple [B] time = 0.284, size = 374, normalized size = 2.5

$$-\frac{2\,(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2(\cos(dx+c))^3}{15\,a^2d(\sin(dx+c))^5}\left(21\,i(\cos(dx+c))^3\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -2/15/a^2/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(21*I*cos(d*x+c)^3*(1/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)
/sin(d*x+c),I)*sin(d*x+c)-21*I*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*
x+c)+21*I*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-21*I*cos(d*x+c)^
2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(c
os(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+21*cos(d*x+c)^3+10*I*cos(d*x+c)*sin(d
```

```
*x+c)-24*cos(d*x+c)^2+3)*(e/cos(d*x+c))^(11/2)*cos(d*x+c)^3/sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-42i\,e^5e^{(5i\,dx+5i\,c)}-112i\,e^5e^{(3i\,dx+3i\,c)}-94i\,e^5e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}\right.}{15\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(sqrt(2)*(-42*I*e^5*e^(5*I*d*x + 5*I*c) - 112*I*e^5*e^(3*I*d*x + 3*I*c
) - 94*I*e^5*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*
x + 1/2*I*c) + 15*(a^2*d*e^(4*I*d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c)
+ a^2*d)*integral(7/5*I*sqrt(2)*e^5*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/
2*I*d*x + 1/2*I*c)/(a^2*d), x))/(a^2*d*e^(4*I*d*x + 4*I*c) + 2*a^2*d*e^(2*I
*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{11}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.236 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=119

$$\frac{10e^3\sin(c+dx)(e\sec(c+dx))^{3/2}}{3a^2d}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^2d}$$

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*a^2*d) + (10*e^3*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*a^2*d) - ((4*I)
*e^2*(e*Sec[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.0889732, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3768, 3771, 2641}

$$\frac{10e^3\sin(c+dx)(e\sec(c+dx))^{3/2}}{3a^2d}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*a^2*d) + (10*e^3*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*a^2*d) - ((4*I)
*e^2*(e*Sec[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(5e^2\right)\int (e\sec(c+dx))^{5/2}\,dx}{a^2}$$
$$= \frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d} - \frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(5e^4\right)\int \sqrt{e\sec(c+dx)}\,d}{3a^2}$$
$$= \frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d} - \frac{4ie^2(e\sec(c+dx))^{5/2}}{d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(5e^4\sqrt{\cos(c+dx)}\sqrt{e\sec}\right.}{3a}$$
$$= \frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^2d} + \frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d} -$$

Mathematica [A] time = 0.301563, size = 67, normalized size = 0.56

$$\frac{2e^3(e\sec(c+dx))^{3/2}\left(-\sin(c+dx)-6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*e^3*(e*Sec[c + d*x])^(3/2)*((-6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*E
llipticF[(c + d*x)/2, 2] - Sin[c + d*x]))/(3*a^2*d)
```

Maple [A] time = 0.248, size = 201, normalized size = 1.7

$$\frac{2\,(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2(\cos(dx+c))^3}{3\,a^2d\,(\sin(dx+c))^4}\left(5\,i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}(\cos(dx+c))^2\,\mathrm{Ell}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/3/a^2/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(5*I*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*EllipticF(I*(cos(d*x+c)-1)/si
n(d*x+c),I)+5*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-6*I*cos(d*x+c)-sin(d*x+
c))*(e/cos(d*x+c))^(9/2)*cos(d*x+c)^3/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

[Out] Exception raised: RuntimeError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-10i\,e^4e^{(2i\,dx+2i\,c)}-14i\,e^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\text{integral}\left(-\frac{5i\sqrt{2}e^4\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,a^2d},x\right)}{3\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-10*I*e^4*e^(2*I*d*x + 2*I*c) - 14*I*e^4)*sqrt(e/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2
*d)*integral(-5/3*I*sqrt(2)*e^4*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I
*d*x - 1/2*I*c)/(a^2*d), x))/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.237 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$-\frac{6e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{a^2d}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

```
[Out] (6*e^4*EllipticE[(c + d*x)/2, 2])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) - (6*e^3*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + ((4*I)*e^2*(e*
Sec[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.0888349, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3768, 3771, 2639}

$$-\frac{6e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{a^2d}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (6*e^4*EllipticE[(c + d*x)/2, 2])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) - (6*e^3*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + ((4*I)*e^2*(e*
Sec[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)} - \frac{\left(3e^2\right)\int (e\sec(c+dx))^{3/2}\,dx}{a^2}$$
$$= -\frac{6e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(3e^4\right)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{a^2}$$
$$= -\frac{6e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(3e^4\right)\int \sqrt{\cos(c+dx)}\,dx}{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= \frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{6e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [C] time = 0.473509, size = 80, normalized size = 0.7

$$\frac{2ie^3e^{-i(c+dx)}\left(-1+3\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\right)\sqrt{e\sec(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((2*I)*e^3*(-1 + 3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/
2, 3/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(a^2*d*E^(I*(c + d*x))
)
```

Maple [B] time = 0.239, size = 1093, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 1/a^2/d*(e/cos(d*x+c))^(7/2)*(cos(d*x+c)-1)^4*(cos(d*x+c)+1)^7*(-12*I*cos(d
*x+c)^3*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-12*I*(-cos(d*x+c)/(
cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^2*sin(d*x+c)-4*I*(-cos(d*x+c)/(cos(d*x+c)
+1)^2)^(3/2)*cos(d*x+c)*sin(d*x+c)+4*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*c
os(d*x+c)^5-6*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/
sin(d*x+c),I)*sin(d*x+c)-12*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-
1)/sin(d*x+c),I)*cos(d*x+c)^2*sin(d*x+c)+I*cos(d*x+c)^2*ln(-(2*(-cos(d*x+c)
/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+6*(-cos(d*x+c)/(cos
(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^4+6*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-co
s(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(
d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)^3*sin(d*x+c)+12*I*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)^2*sin(d*x+c)-I*ln(-2*(2*
(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*cos(d*x+c)^2*sin(d
```

```
*x+c)-4*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^3+6*I*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/si
n(d*x+c),I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)*sin(d*x+c)-6*I*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)^3*
sin(d*x+c)-4*I*cos(d*x+c)^4*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)
-8*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^2+2*(-cos(d*x+c)/(cos(d*
x+c)+1)^2)^(3/2))*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)/sin(d*x+c)^9
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a^{2}de^{(i\,dx+i\,c)}\mathrm{integral}\left(-\frac{3i\,\sqrt{2}e^{3}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{a^{2}d},x\right)+\sqrt{2}\left(6i\,e^{3}e^{(2i\,dx+2i\,c)}+4i\,e^{3}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right)e^{(-i\,dx-i\,c)}}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^(I*d*x + I*c)*integral(-3*I*sqrt(2)*e^3*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(a^2*d), x) + sqrt(2)*(6*I*e^3*e^(2*I*d*x +
 2*I*c) + 4*I*e^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
))*e^(-I*d*x - I*c)/(a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.238 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=90

$$-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*a^2*d) + (((4*I)/3)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rubi [A] time = 0.073417, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3500, 3771, 2641}

$$-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*a^2*d) + (((4*I)/3)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)} - \frac{e^2\int\sqrt{e\sec(c+dx)}\,dx}{3a^2}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)} - \frac{\left(e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^2}$$

$$= -\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^2d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [A] time = 0.36464, size = 101, normalized size = 1.12

$$\frac{2(e\sec(c+dx))^{5/2}(\cos(c+dx)+i\sin(c+dx))\left(\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(c+dx)+i\sin(c+dx))-2i\cos(c+dx\right.}{3a^2d(\tan(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*(e*Sec[c + d*x])^(5/2)*((-2*I)*Cos[c + d*x] + Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]))*(Cos[c + d*x] + I*Sin[c
 + d*x]))/(3*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A] time = 0.24, size = 173, normalized size = 1.9

$$-\frac{2\,(\cos(dx+c))^2}{3\,a^2d}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -2/3/a^2/d*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(I*cos(d*x+c)*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d
*x+c),I)+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellip
ticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-2*I*cos(d*x+c)^2-2*cos(d*x+c)*sin(d*x+c
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,a^2de^{(2i\,dx+2i\,c)}\mathrm{integral}\left(\frac{i\,\sqrt{2}e^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,a^2d},x\right)+\sqrt{2}\left(2i\,e^2e^{(2i\,dx+2i\,c)}+2i\,e^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right)e^{(-2i\,dx-2i}}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*d*e^(2*I*d*x + 2*I*c)*integral(1/3*I*sqrt(2)*e^2*sqrt(e/(e^(2*I*
d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^2*d), x) + sqrt(2)*(2*I*e^2*
e^(2*I*d*x + 2*I*c) + 2*I*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d
*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.239 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=90

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) + (((4*I)/5)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^2 + I*a^2*Tan[c + d*x
]))
```

Rubi [A] time = 0.0746756, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3500, 3771, 2639}

$$\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) + (((4*I)/5)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^2 + I*a^2*Tan[c + d*x
]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a^2}$$
$$= \frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{e^2\int \sqrt{\cos(c+dx)}\,dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [C] time = 0.495644, size = 102, normalized size = 1.13

$$\frac{iee^{-3i(c+dx)}\left(2e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+e^{2i(c+dx)}+1\right)\sqrt{e\sec(c+dx)}}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/5)*e*(1 + E^((2*I)*(c + d*x)) + 2*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)
*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])*Sqrt[
e*Sec[c + d*x]])/(a^2*d*E^((3*I)*(c + d*x)))
```

Maple [B] time = 0.228, size = 341, normalized size = 3.8

$$-\frac{2\cos(dx+c)}{5a^2d\sin(dx+c)}\left(i\text{EllipticE}\left(\frac{i(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\cos(dx+c)\sin(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -2/5/a^2/d*(I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*EllipticF(I*
(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),
I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-I*
EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*I*cos(d*x+c)^3*sin(d*x+c)+2*cos(d*x+c
)^4-cos(d*x+c)^2-cos(d*x+c))*(e/cos(d*x+c))^(3/2)*cos(d*x+c)/sin(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(5\,a^{2}de^{(3i\,dx+3i\,c)}\text{integral}\left(-\frac{i\,\sqrt{2}e\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{5\,a^{2}d},x\right)+\sqrt{2}\left(2i\,ee^{(4i\,dx+4i\,c)}+3i\,ee^{(2i\,dx+2i\,c)}+i\,e\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right.}{5\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5*(5*a^2*d*e^(3*I*d*x + 3*I*c)*integral(-1/5*I*sqrt(2)*e*sqrt(e/(e^(2*I*d
*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(a^2*d), x) + sqrt(2)*(2*I*e*e^(4
*I*d*x + 4*I*c) + 3*I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.240 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}+\frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2d}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*a^
2*d) + (2*e*Sin[c + d*x])/(7*a^2*d*Sqrt[e*Sec[c + d*x]]) + (((4*I)/7)*e^2)/
(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.0832255, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3769, 3771, 2641}

$$\frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}+\frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*a^
2*d) + (2*e*Sin[c + d*x])/(7*a^2*d*Sqrt[e*Sec[c + d*x]]) + (((4*I)/7)*e^2)/
(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(3e^2\right)\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7a^2}$$

$$= \frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\int \sqrt{e\sec(c+dx)}\,dx}{7a^2}$$

$$= \frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{e\sec(c\right.}{7a}$$

$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}} + \frac{4}{7d(e\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.384495, size = 112, normalized size = 0.97

$$-\frac{\sec^2(c+dx)\sqrt{e\sec(c+dx)}\left(-\sin(2(c+dx))+2i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))+i\sin\right.}{7a^2d(\tan(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(2*I + (2*I)*Cos[2*(c + d*x)] + 2*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c +
d*x)]) - Sin[2*(c + d*x)])))/(7*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A] time = 0.325, size = 208, normalized size = 1.8

$$\frac{2\,(\cos(dx+c)-1)^2(\cos(dx+c)+1)^2}{7\,a^2d\,(\sin(dx+c))^4}\sqrt{\frac{e}{\cos(dx+c)}}\left(i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/7/a^2/d*(e/cos(d*x+c))^(1/2)*(cos(d*x+c)-1)^2*(cos(d*x+c)+1)^2*(I*cos(d*x
+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*
(cos(d*x+c)-1)/sin(d*x+c),I)+2*I*cos(d*x+c)^4+I*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+2*
cos(d*x+c)^3*sin(d*x+c)+cos(d*x+c)*sin(d*x+c))/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(14\,a^{2}de^{(4i\,dx+4i\,c)}\mathrm{integral}\left(-\frac{i\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{7\,a^{2}d},x\right)+\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(3i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right)e^{(-4i\,dx-4i\,c)}}{14\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/14*(14*a^2*d*e^(4*I*d*x + 4*I*c)*integral(-1/7*I*sqrt(2)*sqrt(e/(e^(2*I*d
*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^2*d), x) + sqrt(2)*sqrt(e/(e^
(2*I*d*x + 2*I*c) + 1))*(3*I*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c)
+ I)*e^(1/2*I*d*x + 1/2*I*c))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^2, x)
```

3.241 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}+\frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*
x]]) + (2*e*Sin[c + d*x])/(9*a^2*d*(e*Sec[c + d*x])^(3/2)) + (((4*I)/9)*e^2
)/(d*(e*Sec[c + d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.0843109, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3769, 3771, 2639}

$$\frac{4ie^2}{9d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}+\frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*
x]]) + (2*e*Sin[c + d*x])/(9*a^2*d*(e*Sec[c + d*x])^(3/2)) + (((4*I)/9)*e^2
)/(d*(e*Sec[c + d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(5e^2\right)\int \frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{9a^2}$$

$$= \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\int \frac{1}{}}{}$$

$$= \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{}{3a^2}$$

$$= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}} + \frac{}{9d(e\sec(c+}$$

Mathematica [C] time = 1.26681, size = 123, normalized size = 1.06

$$\frac{(\sin(2(c+dx))+i\cos(2(c+dx)))\left(2(7i\sin(2(c+dx))+8\cos(2(c+dx))+2)-\frac{8e^{4i(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right.}{18a^2d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (((-8*E^((4*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*(2 + 8*Cos[2*(c + d*x)] + (7*I)*Si
n[2*(c + d*x)]))*(I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(18*a^2*d*Sqrt[e*
Sec[c + d*x]])
```

Maple [B] time = 0.386, size = 366, normalized size = 3.2

$$\frac{2\,(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2}{9\,a^2d\,(\sin(dx+c))^5\,e}\left(2\,i\,(\cos(dx+c))^5\sin(dx+c)+3\,i\text{EllipticF}\left(\frac{i\,(\cos(dx+c)-1)}{\sin(dx+c)},i\right)\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/9/a^2/d*(2*I*cos(d*x+c)^5*sin(d*x+c)+3*I*EllipticF(I*(cos(d*x+c)-1)/sin(d
*x+c),I)*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)-3*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(
d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*cos(d*x
+c)^6+3*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*EllipticE(I*(cos(d*x+c)-1
)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)+cos(d*x+c)^4-2*cos(d*x+c)^2+3*cos(d*x+c))*(cos(d*x+c)+1)^2*(cos(
d*x+c)-1)^2*(e/cos(d*x+c))^(1/2)/sin(d*x+c)^5/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-9i\,e^{(7i\,dx+7i\,c)}-15i\,e^{(6i\,dx+6i\,c)}-5i\,e^{(5i\,dx+5i\,c)}-19i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(3i\,dx+3i\,c)}-5i\,e^{(2i\,dx+2i\,c)}+i\,e^{(i\,dx+i\,c)}-i\right)}{36\left(a^2dee^{(6i\,dx+6i\,c)}-a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/36*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-9*I*e^(7*I*d*x + 7*I*c) -
 15*I*e^(6*I*d*x + 6*I*c) - 5*I*e^(5*I*d*x + 5*I*c) - 19*I*e^(4*I*d*x + 4*I
*c) + 5*I*e^(3*I*d*x + 3*I*c) - 5*I*e^(2*I*d*x + 2*I*c) + I*e^(I*d*x + I*c)
 - I)*e^(1/2*I*d*x + 1/2*I*c) + 36*(a^2*d*e*e^(6*I*d*x + 6*I*c) - a^2*d*e*e
^(5*I*d*x + 5*I*c))*integral(1/3*sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-I*e^(2*I*d*x + 2*I*c) - 2*I*e^(I*d*x + I*c) - I)*e^(1/2*I*d*x + 1/2*I*c)/
(a^2*d*e*e^(3*I*d*x + 3*I*c) - 2*a^2*d*e*e^(2*I*d*x + 2*I*c) + a^2*d*e*e^(I
*d*x + I*c)), x))/(a^2*d*e*e^(6*I*d*x + 6*I*c) - a^2*d*e*e^(5*I*d*x + 5*I*c
))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^2), x)
```

3.242 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33a^2de^2}+\frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}}$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(33*
a^2*d*e^2) + (2*e*Sin[c + d*x])/(11*a^2*d*(e*Sec[c + d*x])^(5/2)) + (10*Sin
[c + d*x])/(33*a^2*d*e*Sqrt[e*Sec[c + d*x]]) + (((4*I)/11)*e^2)/(d*(e*Sec[c
 + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.109988, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3769, 3771, 2641}

$$\frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33a^2de^2}+\frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(33*
a^2*d*e^2) + (2*e*Sin[c + d*x])/(11*a^2*d*(e*Sec[c + d*x])^(5/2)) + (10*Sin
[c + d*x])/(33*a^2*d*e*Sqrt[e*Sec[c + d*x]]) + (((4*I)/11)*e^2)/(d*(e*Sec[c
 + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(7e^2\right)\int \frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{11a^2}$$

$$= \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{5 \ldots}{\ldots}$$

$$= \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2 \ldots\right.}$$

$$= \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2 \ldots\right.}$$

$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33a^2de^2} + \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} \ldots$$

Mathematica [A] time = 0.484103, size = 134, normalized size = 0.89

$$-\frac{\sec^4(c+dx)\left(-6\sin(2(c+dx))+7\sin(4(c+dx))+24i\cos(2(c+dx))-4i\cos(4(c+dx))+40\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d \ldots\right.\right.}{132a^2d(\tan(c+dx)-i)^2(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] -(Sec[c + d*x]^4*(28*I + (24*I)*Cos[2*(c + d*x)] - (4*I)*Cos[4*(c + d*x)] +
 40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[
2*(c + d*x)]) - 6*Sin[2*(c + d*x)] + 7*Sin[4*(c + d*x)]))/(132*a^2*d*(e*Sec
[c + d*x])^(3/2)*(-I + Tan[c + d*x])^2)
```

Maple [A] time = 0.339, size = 234, normalized size = 1.6

$$\frac{2\cos(dx+c)(\cos(dx+c)-1)^2(\cos(dx+c)+1)^2}{33a^2de^3(\sin(dx+c))^4}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(6i(\cos(dx+c))^6+6(\cos(dx+c))^5\sin(dx+c)+ \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/33/a^2/d*cos(d*x+c)*(e/cos(d*x+c))^(3/2)*(cos(d*x+c)-1)^2*(cos(d*x+c)+1)^
2*(6*I*cos(d*x+c)^6+6*cos(d*x+c)^5*sin(d*x+c)+5*I*cos(d*x+c)*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin
(d*x+c),I)+5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+3*cos(d*x+c)^3*sin(d*x+c)+5*cos(d*x
+c)*sin(d*x+c))/e^3/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(264\,a^2de^2e^{(6i\,dx+6i\,c)}\text{integral}\left(-\frac{5i\,\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{33\,a^2de^2},x\right)+\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-11i\,e^{(8i\,dx+8i\,c)}+30i\,e^{(6i\,dx+6i\,c)}+56i\right.\right.}{264\,a^2de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/264*(264*a^2*d*e^2*e^(6*I*d*x + 6*I*c)*integral(-5/33*I*sqrt(2)*sqrt(e/(e
^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^2*d*e^2), x) + sqrt(2)
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(8*I*d*x + 8*I*c) + 30*I*e^(6*I
*d*x + 6*I*c) + 56*I*e^(4*I*d*x + 4*I*c) + 18*I*e^(2*I*d*x + 2*I*c) + 3*I)*
e^(1/2*I*d*x + 1/2*I*c))*e^(-6*I*d*x - 6*I*c)/(a^2*d*e^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.243 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=150

$$\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}+\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{14\,\mathrm{si}}{65a^2de(e}$$

```
[Out] (42*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (2*e*Sin[c + d*x])/(13*a^2*d*(e*Sec[c + d*x])^(7/2)) + (14*Sin
[c + d*x])/(65*a^2*d*e*(e*Sec[c + d*x])^(3/2)) + (((4*I)/13)*e^2)/(d*(e*Sec
[c + d*x])^(9/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.109295, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3769, 3771, 2639}

$$\frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}}+\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{14\,\mathrm{si}}{65a^2de(e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (42*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (2*e*Sin[c + d*x])/(13*a^2*d*(e*Sec[c + d*x])^(7/2)) + (14*Sin
[c + d*x])/(65*a^2*d*e*(e*Sec[c + d*x])^(3/2)) + (((4*I)/13)*e^2)/(d*(e*Sec
[c + d*x])^(9/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{13d(e\sec(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\left(9e^2\right)\int \frac{1}{(e\sec(c+dx))^{9/2}}\,dx}{13a^2}$$

$$= \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{4ie^2}{13d(e\sec(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}+$$

$$= \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}}+\frac{}{13d(e\sec(c+dx))}$$

$$= \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}}+\frac{}{13d(e\sec(c+dx))}$$

$$= \frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{1}{65a^2d}$$

Mathematica [C] time = 2.02242, size = 149, normalized size = 0.99

$$\frac{(\cos(2(c+dx))-i\sin(2(c+dx)))\left(-\frac{224ie^{4i(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}-356\sin(2(c+dx))+18\sin(4(c+dx))\right.}{520a^2de^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(88*I + (416*I)*Cos[2*(c + d*x)] -
 (8*I)*Cos[4*(c + d*x)] - ((224*I)*E^((4*I)*(c + d*x))*Hypergeometric2F1[1/
2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - 356*Sin
[2*(c + d*x)] + 18*Sin[4*(c + d*x)]))/(520*a^2*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [B] time = 0.723, size = 386, normalized size = 2.6

$$\frac{2\left(\cos(dx+c)\right)^2\left(\cos(dx+c)-1\right)^2\left(\cos(dx+c)+1\right)^2}{65a^2de^5\left(\sin(dx+c)\right)^5}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(10i\left(\cos(dx+c)\right)^7\sin(dx+c)-10\left(\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/65/a^2/d*cos(d*x+c)^2*(e/cos(d*x+c))^(5/2)*(cos(d*x+c)-1)^2*(cos(d*x+c)+1
)^2*(10*I*cos(d*x+c)^7*sin(d*x+c)-10*cos(d*x+c)^8+21*I*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c
),I)*cos(d*x+c)*sin(d*x+c)-21*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*co
s(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)+5*cos(d*x+c)^6+21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-21*I*EllipticE(
```

```
I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)-2*cos(d*x+c)^4-14*cos(d*x+c)^2+21*cos(d*x+c))/e^5
/sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-13i\,e^{(11i\,dx+11i\,c)}+13i\,e^{(10i\,dx+10i\,c)}-299i\,e^{(9i\,dx+9i\,c)}-373i\,e^{(8i\,dx+8i\,c)}-198i\,e^{(7i\,dx+7i\,c)}-474i\,e^{(6i\,dx+6i\,c)}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/1040*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-13*I*e^(11*I*d*x + 11*I
*c) + 13*I*e^(10*I*d*x + 10*I*c) - 299*I*e^(9*I*d*x + 9*I*c) - 373*I*e^(8*I
*d*x + 8*I*c) - 198*I*e^(7*I*d*x + 7*I*c) - 474*I*e^(6*I*d*x + 6*I*c) + 118
*I*e^(5*I*d*x + 5*I*c) - 118*I*e^(4*I*d*x + 4*I*c) + 35*I*e^(3*I*d*x + 3*I*
c) - 35*I*e^(2*I*d*x + 2*I*c) + 5*I*e^(I*d*x + I*c) - 5*I)*e^(1/2*I*d*x + 1
/2*I*c) + 1040*(a^2*d*e^3*e^(8*I*d*x + 8*I*c) - a^2*d*e^3*e^(7*I*d*x + 7*I*
c))*integral(1/65*sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-21*I*e^(2*I*d
*x + 2*I*c) - 42*I*e^(I*d*x + I*c) - 21*I)*e^(1/2*I*d*x + 1/2*I*c)/(a^2*d*e
^3*e^(3*I*d*x + 3*I*c) - 2*a^2*d*e^3*e^(2*I*d*x + 2*I*c) + a^2*d*e^3*e^(I*d
*x + I*c)), x))/(a^2*d*e^3*e^(8*I*d*x + 8*I*c) - a^2*d*e^3*e^(7*I*d*x + 7*I
*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\sec(dx+c))^{\frac{5}{2}}(ia\tan(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.244 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=181

$$\frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2de^4}+$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*a^
2*d*e^4) + (2*e*Sin[c + d*x])/(15*a^2*d*(e*Sec[c + d*x])^(9/2)) + (6*Sin[c
+ d*x])/(35*a^2*d*e*(e*Sec[c + d*x])^(5/2)) + (2*Sin[c + d*x])/(7*a^2*d*e^3
*Sqrt[e*Sec[c + d*x]]) + (((4*I)/15)*e^2)/(d*(e*Sec[c + d*x])^(11/2)*(a^2 +
 I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.130622, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3769, 3771, 2641}

$$\frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2de^4}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```


```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*a^
2*d*e^4) + (2*e*Sin[c + d*x])/(15*a^2*d*(e*Sec[c + d*x])^(9/2)) + (6*Sin[c
+ d*x])/(35*a^2*d*e*(e*Sec[c + d*x])^(5/2)) + (2*Sin[c + d*x])/(7*a^2*d*e^3
*Sqrt[e*Sec[c + d*x]]) + (((4*I)/15)*e^2)/(d*(e*Sec[c + d*x])^(11/2)*(a^2 +
 I*a^2*Tan[c + d*x]))
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{15d(e\sec(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(11e^2\right)\int \frac{1}{(e\sec(c+dx))^{11/2}}\,d}{15a^2}$$

$$= \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{4ie^2}{15d(e\sec(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

$$= \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}} + \frac{}{15d(e\sec(c+dx))}$$

$$= \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+}}$$

$$= \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+}}$$

$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{7a^2de^4} + \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/}}$$

Mathematica [A] time = 0.873756, size = 151, normalized size = 0.83

$$-\frac{(e\sec(c+dx))^{5/2}\left(-17\sin(2(c+dx))+128\sin(4(c+dx))+11\sin(6(c+dx))+228i\cos(2(c+dx))-72i\cos(4(c+\right.}{1680a^2de^6(\tan(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] -((e*Sec[c + d*x])^(5/2)*(296*I + (228*I)*Cos[2*(c + d*x)] - (72*I)*Cos[4*(
c + d*x)] - (4*I)*Cos[6*(c + d*x)] + 480*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - 17*Sin[2*(c + d*x)] +
128*Sin[4*(c + d*x)] + 11*Sin[6*(c + d*x)]))/(1680*a^2*d*e^6*(-I + Tan[c +
d*x])^2)
```

Maple [A] time = 0.492, size = 252, normalized size = 1.4

$$\frac{2\,(\cos(dx+c)-1)^2(\cos(dx+c)+1)^2(\cos(dx+c))^3}{105\,a^2de^7(\sin(dx+c))^4}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}\left(14\,i\,(\cos(dx+c))^8+14\,\sin(dx+c)\,(\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/105/a^2/d*(e/cos(d*x+c))^(7/2)*(cos(d*x+c)-1)^2*(cos(d*x+c)+1)^2*cos(d*x+
c)^3*(14*I*cos(d*x+c)^8+14*sin(d*x+c)*cos(d*x+c)^7+7*cos(d*x+c)^5*sin(d*x+c
)+15*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+15*I*(1/(cos(d*x+c)+1))^(1/2)*(c
```

```
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+9*
cos(d*x+c)^3*sin(d*x+c)+15*cos(d*x+c)*sin(d*x+c))/e^7/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3360\,a^2de^4e^{(8i\,dx+8i\,c)}\mathrm{integral}\left(-\frac{i\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{7\,a^2de^4},x\right)+\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-15i\,e^{(12i\,dx+12i\,c)}-200i\,e^{(10i\,dx+10i\,c)}+24\right.\right.}{3360\,a^2de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/3360*(3360*a^2*d*e^4*e^(8*I*d*x + 8*I*c)*integral(-1/7*I*sqrt(2)*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^2*d*e^4), x) + sqrt(2
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-15*I*e^(12*I*d*x + 12*I*c) - 200*I*e^
(10*I*d*x + 10*I*c) + 245*I*e^(8*I*d*x + 8*I*c) + 592*I*e^(6*I*d*x + 6*I*c)
 + 211*I*e^(4*I*d*x + 4*I*c) + 56*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(1/2*I*d*x
 + 1/2*I*c))*e^(-8*I*d*x - 8*I*c)/(a^2*d*e^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{7}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.245 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$-\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d}+\frac{22e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^3d}+\frac{22e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^3d}-\frac{22e^8E\left(\frac{1}{2}(c+dx)\right|}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\,\mathrm{se}}}$$

```
[Out] (-22*e^8*EllipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (((22*I)/21)*e^4*(e*Sec[c + d*x])^(7/2))/(a^3*d) + (22*e^7*Sqr
t[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*d) + (22*e^5*(e*Sec[c + d*x])^(5/2)*
Sin[c + d*x])/(15*a^3*d) - (((4*I)/3)*e^2*(e*Sec[c + d*x])^(11/2))/(a*d*(a
+ I*a*Tan[c + d*x])^2)
```

Rubi [A] time = 0.170492, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3500, 3501, 3768, 3771, 2639}

$$-\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d}+\frac{22e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^3d}+\frac{22e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^3d}-\frac{22e^8E\left(\frac{1}{2}(c+dx)\right|}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\,\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-22*e^8*EllipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (((22*I)/21)*e^4*(e*Sec[c + d*x])^(7/2))/(a^3*d) + (22*e^7*Sqr
t[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*d) + (22*e^5*(e*Sec[c + d*x])^(5/2)*
Sin[c + d*x])/(15*a^3*d) - (((4*I)/3)*e^2*(e*Sec[c + d*x])^(11/2))/(a*d*(a
+ I*a*Tan[c + d*x])^2)
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2} + \frac{\left(11e^2\right)\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx}{3a^2}$$
$$= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2} + \frac{\left(11e^4\right)\int (e\sec(c+dx))^{7/2}\,dx}{3a^3}$$
$$= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^3d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$
$$= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} + \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^3d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c}{15a^3d}$$
$$= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} + \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^3d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c}{15a^3d}$$
$$= -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} + \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c}{5a^3d}$$

Mathematica [C] time = 1.54052, size = 128, normalized size = 0.72

$$-\frac{e^6(\tan(c+dx)-i)(e\sec(c+dx))^{3/2}\left(77e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-868\cos\right.}{210a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -(e^6*(e*Sec[c + d*x])^(3/2)*(-556 - 868*Cos[2*(c + d*x)] + (77*(1 + E^((2*
I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
)/E^((2*I)*(c + d*x)) + (203*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (143*I)*Tan
[c + d*x])*(-I + Tan[c + d*x]))/(210*a^3*d)
```

Maple [B] time = 0.323, size = 392, normalized size = 2.2

$$-\frac{2(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2(\cos(dx+c))^4}{105\,da^3(\sin(dx+c))^5}\left(231\,i\sin(dx+c)(\cos(dx+c))^4\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\text{co}}{\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -2/105/a^3/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(231*I*sin(d*x+c)*cos(d*x+c)
^4*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
cos(d*x+c)-1)/sin(d*x+c),I)-231*I*sin(d*x+c)*cos(d*x+c)^4*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*
x+c),I)+231*I*cos(d*x+c)^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-231*I*cos(d*
x+c)^3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+231*cos(d*x+c)^4+140*I*cos(d*x+c
)^2*sin(d*x+c)-294*cos(d*x+c)^3-15*I*sin(d*x+c)+63*cos(d*x+c))*(e/cos(d*x+c
))^(15/2)*cos(d*x+c)^4/sin(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-462i\,e^{7}e^{(7i\,dx+7i\,c)} - 1694i\,e^{7}e^{(5i\,dx+5i\,c)} - 2266i\,e^{7}e^{(3i\,dx+3i\,c)} - 1274i\,e^{7}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + 105\left(a^{3}de^{(6i\,dx}\right.}{105\left(a^{3}de^{(6i\,dx+6i\,c)} + 3\,a^{3}de^{(4i\,dx+4i\,c)} + 3\,a^{3}de^{(2i}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/105*(sqrt(2)*(-462*I*e^7*e^(7*I*d*x + 7*I*c) - 1694*I*e^7*e^(5*I*d*x + 5*
I*c) - 2266*I*e^7*e^(3*I*d*x + 3*I*c) - 1274*I*e^7*e^(I*d*x + I*c))*sqrt(e/
(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 105*(a^3*d*e^(6*I*d*x
+ 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*
d)*integral(11/5*I*sqrt(2)*e^7*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d
*x + 1/2*I*c)/(a^3*d), x))/(a^3*d*e^(6*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x
+ 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{15}{2}}}{(ia\tan(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.246 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=141

$$-\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d}+\frac{6e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{a^3d}+\frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{a^3d}-\frac{4ie^2(e\,\mathrm{s}}{ad(a+i\mathrm{a}}$$

```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
a^3*d) - (((18*I)/5)*e^4*(e*Sec[c + d*x])^(5/2))/(a^3*d) + (6*e^5*(e*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(a^3*d) - ((4*I)*e^2*(e*Sec[c + d*x])^(9/2))/(a
*d*(a + I*a*Tan[c + d*x])^2)
```

Rubi [A] time = 0.150456, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3500, 3501, 3768, 3771, 2641}

$$-\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d}+\frac{6e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{a^3d}+\frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{a^3d}-\frac{4ie^2(e\,\mathrm{s}}{ad(a+i\mathrm{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
a^3*d) - (((18*I)/5)*e^4*(e*Sec[c + d*x])^(5/2))/(a^3*d) + (6*e^5*(e*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(a^3*d) - ((4*I)*e^2*(e*Sec[c + d*x])^(9/2))/(a
*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2} + \frac{\left(9e^2\right)\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx}{a^2}$$
$$= -\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2} + \frac{\left(9e^4\right)\int (e\sec(c+dx))^{5/2}\,dx}{a^3}$$
$$= -\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^3d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2} +$$
$$= -\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^3d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2} +$$
$$= \frac{6e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{a^3d} - \frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} + \frac{6e^5(e\sec(c\cdot}{}$$

Mathematica [A] time = 0.57678, size = 74, normalized size = 0.52

$$\frac{e^4(e\sec(c+dx))^{5/2}\left(-5\sin(2(c+dx))-20i\cos(2(c+dx))+30\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-18i\right)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (e^4*(e*Sec[c + d*x])^(5/2)*(-18*I - (20*I)*Cos[2*(c + d*x)] + 30*Cos[c + d
*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[2*(c + d*x)]))/(5*a^3*d)
```

Maple [A] time = 0.285, size = 213, normalized size = 1.5

$$\frac{2\left(\cos(dx+c)+1\right)^2\left(\cos(dx+c)-1\right)^2\left(\cos(dx+c)\right)^4}{5\,da^3\left(\sin(dx+c)\right)^4}\left(15\,i\sqrt{\left(\cos(dx+c)+1\right)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\left(\cos(dx+c)\right)^3\mathrm{El}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/5/a^3/d*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(15*I*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*EllipticF(I*(cos(d*x+c)-1)/s
in(d*x+c),I)+15*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*cos(d*x+c)^2*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-20*I*cos(d*x+c)^2-5
```

```
*cos(d*x+c)*sin(d*x+c)+I)*(e/cos(d*x+c))^(13/2)*cos(d*x+c)^4/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(-30i\,e^6e^{(4i\,dx+4i\,c)} - 72i\,e^6e^{(2i\,dx+2i\,c)} - 50i\,e^6\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + 5\left(a^3de^{(4i\,dx+4i\,c)} + 2\,a^3de^{(2i\,dx+2i\,c)} + a^3d\right)\text{integr}}{5\left(a^3de^{(4i\,dx+4i\,c)} + 2\,a^3de^{(2i\,dx+2i\,c)} + a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(-30*I*e^6*e^(4*I*d*x + 4*I*c) - 72*I*e^6*e^(2*I*d*x + 2*I*c)
- 50*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*(
a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*integral(-
3*I*sqrt(2)*e^6*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/
(a^3*d), x))/(a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c) + a^3
*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{13}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.247 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=141

$$\frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d}-\frac{14e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{a^3d}+\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}$$

```
[Out] (14*e^6*EllipticE[(c + d*x)/2, 2])/(a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((14*I)/3)*e^4*(e*Sec[c + d*x])^(3/2))/(a^3*d) - (14*e^5*Sqrt[e*
Sec[c + d*x]]*Sin[c + d*x])/(a^3*d) + ((4*I)*e^2*(e*Sec[c + d*x])^(7/2))/(a
*d*(a + I*a*Tan[c + d*x])^2)
```

Rubi [A] time = 0.14939, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3500, 3501, 3768, 3771, 2639}

$$\frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d}-\frac{14e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{a^3d}+\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (14*e^6*EllipticE[(c + d*x)/2, 2])/(a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((14*I)/3)*e^4*(e*Sec[c + d*x])^(3/2))/(a^3*d) - (14*e^5*Sqrt[e*
Sec[c + d*x]]*Sin[c + d*x])/(a^3*d) + ((4*I)*e^2*(e*Sec[c + d*x])^(7/2))/(a
*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx &= \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2} - \frac{\left(7e^2\right)\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx}{a^2} \\
&= \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2} - \frac{\left(7e^4\right)\int (e\sec(c+dx))^{3/2}\,dx}{a^3} \\
&= \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} - \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^3d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2} + \frac{\left(7e^6\right)\ldots}{\ldots} \\
&= \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} - \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^3d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2} + \frac{(7\ldots}{a^3\sqrt{c\ldots}} \\
&= \frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} - \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^3d}\ldots
\end{aligned}$$

Mathematica [C] time = 0.918654, size = 93, normalized size = 0.66

$$\frac{ie^4(e\sec(c+dx))^{3/2}\left(-7\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+9i\sin(2(c+dx))+33\cos(2(c+dx)\ldots}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/3)*e^4*(e*Sec[c + d*x])^(3/2)*(35 + 33*Cos[2*(c + d*x)] - 7*(1 + E^((2*
I)*(c + d*x)))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
 + (9*I)*Sin[2*(c + d*x)]))/(a^3*d)
```

Maple [B] time = 0.298, size = 1562, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -2/3/a^3/d*(cos(d*x+c)+1)^7*(cos(d*x+c)-1)^4*(-3*I*cos(d*x+c)^3*sin(d*x+c)*
ln(-(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-c
os(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)+3*I*cos(d*x
+c)^3*sin(d*x+c)*ln(-2*(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2
-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*
```

```
x+c)^2)-84*I*cos(d*x+c)^4*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),
I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)-84*I*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+
c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+37*I*cos(d*x+c)^3*sin(d*x+c)*(-cos(
d*x+c)/(cos(d*x+c)+1)^2)^(3/2)+126*I*cos(d*x+c)^3*sin(d*x+c)*EllipticF(I*(c
os(d*x+c)-1)/sin(d*x+c),I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+21*I*cos(d*x+c)^5*sin(d*x+c
)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(
3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+84*I*cos(d*
x+c)^4*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(-cos(d*x+c)/(co
s(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)+12*I*cos(d*x+c)^5*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)+I*(-
cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)+21*I*cos(d*x+c)*sin(d*x+c)*(-
cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-12*(-cos(d*x+c)
/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^6-126*I*cos(d*x+c)^3*sin(d*x+c)*Ellipti
cE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-15*(-cos(d*x+c)/(cos
(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^5+3*I*cos(d*x+c)*sin(d*x+c)*(-cos(d*x+c)/(co
s(d*x+c)+1)^2)^(3/2)+18*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^4+8
4*I*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)
/sin(d*x+c),I)+24*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^3-21*I*co
s(d*x+c)*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x
+c),I)-21*I*cos(d*x+c)^5*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I
)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)+36*I*cos(d*x+c)^4*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c
)+1)^2)^(3/2)-6*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^2+15*I*cos(
d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-9*cos(d*x+c)*(-cos
(d*x+c)/(cos(d*x+c)+1)^2)^(3/2))*(e/cos(d*x+c))^(11/2)*cos(d*x+c)*(-cos(d*x
+c)/(cos(d*x+c)+1)^2)^(3/2)/sin(d*x+c)^9
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(42i\,e^5e^{(4i\,dx+4i\,c)}+70i\,e^5e^{(2i\,dx+2i\,c)}+24i\,e^5\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\left(a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}\right)\text{integral}\left(-\frac{7i\,\sqrt{}}{}\right.}{3\left(a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(42*I*e^5*e^(4*I*d*x + 4*I*c) + 70*I*e^5*e^(2*I*d*x + 2*I*c) +
 24*I*e^5)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*(a
^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))*integral(-7*I*sqrt(2)*e^5
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(a^3*d), x))/(a^
3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{11}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.248 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=116

$$\frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d}-\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^3d}+\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}$$

```
[Out] (((10*I)/3)*e^4*Sqrt[e*Sec[c + d*x]])/(a^3*d) - (10*e^4*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*a^3*d) + (((4*I)/3)*e^2*
(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rubi [A] time = 0.133941, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3501, 3771, 2641}

$$\frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d}-\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^3d}+\frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((10*I)/3)*e^4*Sqrt[e*Sec[c + d*x]])/(a^3*d) - (10*e^4*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*a^3*d) + (((4*I)/3)*e^2*
(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2} - \frac{\left(5e^2\right)\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx}{3a^2}$$
$$= \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2} - \frac{\left(5e^4\right)\int \sqrt{e\sec(c+dx)}\,dx}{3a^3}$$
$$= \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2} - \frac{\left(5e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos}}}{3a^3}$$
$$= \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} - \frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{3a^3d} + \frac{4ie^2(e\sec(c+}{3ad(a+ia\tan(}$$

Mathematica [A] time = 0.383465, size = 125, normalized size = 1.08

$$\frac{2e^4\sec^3(c+dx)\sqrt{e\sec(c+dx)}(\sin(2(c+dx))-i\cos(2(c+dx)))\left(3\sin(c+dx)-7i\cos(c+dx)+5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.\right.}{3a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (2*e^4*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((-7*I)*Cos[c + d*x] + 5*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]) + 3*S
in[c + d*x])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(3*a^3*d*(-I + Tan
[c + d*x])^3)
```

Maple [A] time = 0.272, size = 175, normalized size = 1.5

$$-\frac{2\,(\cos(dx+c))^4}{3\,da^3}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{9}{2}}\left(5\,i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-1}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -2/3/a^3/d*(e/cos(d*x+c))^(9/2)*cos(d*x+c)^4*(5*I*cos(d*x+c)*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin
(d*x+c),I)+5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-4*I*cos(d*x+c)^2-4*cos(d*x+c)*sin(d
*x+c)-3*I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

[Out] Exception raised: RuntimeError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,a^{3}de^{(2i\,dx+2i\,c)}\text{integral}\left(\frac{5i\,\sqrt{2}e^{4}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,a^{3}d},x\right)+\sqrt{2}\left(10i\,e^{4}e^{(2i\,dx+2i\,c)}+4i\,e^{4}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right)e^{(-2i\,dx}}{3\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/3*(3*a^3*d*e^(2*I*d*x + 2*I*c)*integral(5/3*I*sqrt(2)*e^4*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^3*d), x) + sqrt(2)*(10*I*e^4*e^(2*I*d*x + 2*I*c) + 4*I*e^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^3*d)

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**3,x)

[Out] Exception raised: AttributeError

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")

[Out] integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^3, x)

3.249 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=116

$$-\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}}-\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}$$

```
[Out] (((-6*I)/5)*e^4)/(a^3*d*Sqrt[e*Sec[c + d*x]]) - (6*e^4*EllipticE[(c + d*x)/
2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (((4*I)/5)*e^2*(
e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rubi [A] time = 0.132058, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3501, 3771, 2639}

$$-\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}}-\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-6*I)/5)*e^4)/(a^3*d*Sqrt[e*Sec[c + d*x]]) - (6*e^4*EllipticE[(c + d*x)/
2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (((4*I)/5)*e^2*(
e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx &= \frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{\left(3e^2\right)\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx}{5a^2}\\
&= -\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{\left(3e^4\right)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a^3}\\
&= -\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{\left(3e^4\right)\int \sqrt{\cos(c+dx)}\,dx}{5a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= -\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}}-\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}
\end{aligned}$$

Mathematica [C] time = 0.575989, size = 117, normalized size = 1.01

$$\frac{2ee^{-idx}\left(-2+\frac{6e^{2i(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)(e\sec(c+dx))^{5/2}(\cos(c+2dx)+i\sin(c+2dx))}{5a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (2*e*(-2 + (6*E^((2*I)*(c + d*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*
I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))])*(e*Sec[c + d*x])^(5/2)*(Cos[
c + 2*d*x] + I*Sin[c + 2*d*x]))/(5*a^3*d*E^(I*d*x)*(-I + Tan[c + d*x])^3)
```

Maple [B] time = 0.243, size = 1086, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 1/10/a^3/d*(e/cos(d*x+c))^(7/2)*(cos(d*x+c)-1)^2*cos(d*x+c)^3*(cos(d*x+c)+1
)*(-12*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF(I*(cos(d*x+
c)-1)/sin(d*x+c),I)-5*I*cos(d*x+c)*ln(-(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1
/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(
d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)-20*I*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c
)/(cos(d*x+c)+1)^2)^(1/2)+5*I*ln(-2*(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)
*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x
+c)-1)/sin(d*x+c)^2)*cos(d*x+c)*sin(d*x+c)-20*I*cos(d*x+c)*sin(d*x+c)*(-cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+12*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(
d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+16*I*cos(d*x+c)^4*sin(d*x+c
)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-16*cos(d*x+c)^5*(-cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)+12*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1
```

```
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d
*x+c),I)*sin(d*x+c)-16*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+16
*I*cos(d*x+c)^3*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+28*cos(d*x+
c)^3*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-24*I*(-cos(d*x+c)/(cos(d*x+c)+1)^
2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+24*I*cos(d*x+c)*sin
(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-12*I*
(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)+16
*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-12*cos(d*x+c)*(-cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2))/(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)/sin(d*x+c)
^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(5\,a^3 d e^{(3i\,dx+3i\,c)}\text{integral}\left(\frac{3i\,\sqrt{2}e^3\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{5\,a^3 d},x\right)+\sqrt{2}\left(-6i\,e^3e^{(4i\,dx+4i\,c)}-4i\,e^3e^{(2i\,dx+2i\,c)}+2i\,e^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx\right.}}{5\,a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(5*a^3*d*e^(3*I*d*x + 3*I*c)*integral(3/5*I*sqrt(2)*e^3*sqrt(e/(e^(2*I*
d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(a^3*d), x) + sqrt(2)*(-6*I*e^3*
e^(4*I*d*x + 4*I*c) - 4*I*e^3*e^(2*I*d*x + 2*I*c) + 2*I*e^3)*sqrt(e/(e^(2*I
*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(ia\tan(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.250 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=132

$$-\frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\left(a^3+ia^3\tan(c+dx)\right)}-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^3d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}$$

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*a^3*d) + (((4*I)/7)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x
])^2) - (((2*I)/21)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^3 + I*a^3*Tan[c + d*x])
)
```

Rubi [A] time = 0.127377, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3502, 3771, 2641}

$$-\frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\left(a^3+ia^3\tan(c+dx)\right)}-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^3d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*a^3*d) + (((4*I)/7)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x
])^2) - (((2*I)/21)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^3 + I*a^3*Tan[c + d*x])
)
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{e^2\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx}{7a^2}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\left(a^3+ia^3\tan(c+dx)\right)} - \frac{e^2\int\sqrt{e\sec(c+dx)}\,dx}{21a^3}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\left(a^3+ia^3\tan(c+dx)\right)} - \frac{\left(e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right.}{21a^3}$$

$$= -\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^3d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{2ie^2}{21d\left(a^3\right.}$$

Mathematica [A] time = 0.573827, size = 104, normalized size = 0.79

$$\frac{(e\sec(c+dx))^{5/2}\left(-\sin(2(c+dx)) - 5i\cos(2(c+dx)) + 2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx)) + i\sin(2(c+d\right.}{21a^3d(\tan(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((e*Sec[c + d*x])^(5/2)*(-5*I - (5*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - Sin[
2*(c + d*x)]))/(21*a^3*d*(-I + Tan[c + d*x])^2)
```

Maple [A] time = 0.257, size = 199, normalized size = 1.5

$$-\frac{2\,(\cos(dx+c))^2}{21\,da^3}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -2/21/a^3/d*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(I*cos(d*x+c)*(1/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(
d*x+c),I)-12*I*cos(d*x+c)^4+I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-12*cos(d*x+c)^3*sin(
d*x+c)+7*I*cos(d*x+c)^2+cos(d*x+c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(21\,a^3de^{(4i\,dx+4i\,c)}\text{integral}\left(\frac{i\,\sqrt{2}e^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{21\,a^3d},x\right)+\sqrt{2}\left(2i\,e^2e^{(4i\,dx+4i\,c)}+5i\,e^2e^{(2i\,dx+2i\,c)}+3i\,e^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx\right.}\right.}{21\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/21*(21*a^3*d*e^(4*I*d*x + 4*I*c)*integral(1/21*I*sqrt(2)*e^2*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^3*d), x) + sqrt(2)*(2*I*e
^2*e^(4*I*d*x + 4*I*c) + 5*I*e^2*e^(2*I*d*x + 2*I*c) + 3*I*e^2)*sqrt(e/(e^(
2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-4*I*d*x - 4*I*c)/(a^3*d
)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec (dx + c))^{\frac{5}{2}}}{(i\,a \tan (dx + c) + a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.251 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=132

$$\frac{2ie^2}{45d\left(a^3+ia^3\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}+\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (((4*I)/9)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]
)^2) + (((2*I)/45)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^3 + I*a^3*Tan[c + d*x]))
```

Rubi [A] time = 0.128976, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3502, 3771, 2639}

$$\frac{2ie^2}{45d\left(a^3+ia^3\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}+\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^3,x]
```


```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (((4*I)/9)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]
)^2) + (((2*I)/45)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^3 + I*a^3*Tan[c + d*x]))
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2} + \frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx}{9a^2}$$

$$= \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2} + \frac{2ie^2}{45d\sqrt{e\sec(c+dx)}\left(a^3+ia^3\tan(c+dx)\right)} + \frac{e^2\int$$

$$= \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2} + \frac{2ie^2}{45d\sqrt{e\sec(c+dx)}\left(a^3+ia^3\tan(c+dx)\right)} + \frac{}{15a^3$$

$$= \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2} + \frac{}{45d\sqrt{e\sec(}$$

Mathematica [C] time = 0.743169, size = 140, normalized size = 1.06

$$-\frac{e^{-idx}\sec^2(c+dx)(\cos(dx)+i\sin(dx))(e\sec(c+dx))^{3/2}\left(6e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx}\right.\right.}{45a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -(Sec[c + d*x]^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])*(8 + 8*Cos[
2*(c + d*x)] + 6*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeo
metric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c + d*x)]))/
(45*a^3*d*E^(I*d*x)*(-I + Tan[c + d*x])^3)
```

Maple [B] time = 0.299, size = 368, normalized size = 2.8

$$-\frac{2\cos(dx+c)}{45\,da^3\sin(dx+c)}\left(-20\,i\,(\cos(dx+c))^5\sin(dx+c)+3\,i\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\text{EllipticE}\left(\frac{i\,(\cos(dx}{\sin(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -2/45/a^3/d*(-20*I*cos(d*x+c)^5*sin(d*x+c)+3*I*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos
(d*x+c)*sin(d*x+c)-3*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)+20*co
s(d*x+c)^6+3*I*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*sin(d*x+c)*EllipticF
(I*(cos(d*x+c)-1)/sin(d*x+c),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)+9*I*cos(d*x+c)^3*sin(d*x+c)-19*cos(d*x+c)^4+2*cos(d*x+c)^2-
3*cos(d*x+c))*(e/cos(d*x+c))^(3/2)*cos(d*x+c)/sin(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(90\,a^3de^{(5i\,dx+5i\,c)}\mathrm{integral}\left(-\frac{i\sqrt{2}e\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{15\,a^3d},x\right)+\sqrt{2}\left(12i\,ee^{(6i\,dx+6i\,c)}+23i\,ee^{(4i\,dx+4i\,c)}+16i\,ee^{(2i\,dx+2i\,c)}+5i\,e\right.}{90\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/90*(90*a^3*d*e^(5*I*d*x + 5*I*c)*integral(-1/15*I*sqrt(2)*e*sqrt(e/(e^(2*
I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(a^3*d), x) + sqrt(2)*(12*I*e*
e^(6*I*d*x + 6*I*c) + 23*I*e*e^(4*I*d*x + 4*I*c) + 16*I*e*e^(2*I*d*x + 2*I*
c) + 5*I*e)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-
5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^3, x)
```

3.252 $\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=152

$$\frac{20ie^2}{77d\left(a^3+ia^3\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}+\frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77a^3d}+$$

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(77*
a^3*d) + (10*e*Sin[c + d*x])/(77*a^3*d*Sqrt[e*Sec[c + d*x]]) + (((2*I)/11)*
Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^3) + (((20*I)/77)*e^2)/(d*(
e*Sec[c + d*x])^(3/2)*(a^3 + I*a^3*Tan[c + d*x]))
```

Rubi [A] time = 0.139877, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3502, 3500, 3769, 3771, 2641}

$$\frac{20ie^2}{77d\left(a^3+ia^3\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}+\frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}}+\frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77a^3d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```


```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(77*
a^3*d) + (10*e*Sin[c + d*x])/(77*a^3*d*Sqrt[e*Sec[c + d*x]]) + (((2*I)/11)*
Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^3) + (((20*I)/77)*e^2)/(d*(
e*Sec[c + d*x])^(3/2)*(a^3 + I*a^3*Tan[c + d*x]))
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{5\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx}{11a}$$

$$= \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{20ie^2}{77d(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c+dx)\right)} + \frac{\left(15e^2\right)\int \frac{}{(e\sec}}{77a}$$

$$= \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{20ie^2}{77d(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c\right.}$$

$$= \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{20ie^2}{77d(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c\right.}$$

$$= \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77a^3d} + \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(}}{11d(a+ia\tan}$$

Mathematica [A] time = 0.425284, size = 129, normalized size = 0.85

$$\frac{i\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(-15\sin(c+dx)-15\sin(3(c+dx))+46i\cos(c+dx)+22i\cos(3(c+dx))+20\sqrt{\cos(c+\right.}{154a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/154)*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((46*I)*Cos[c + d*x] + (22*I)*
Cos[3*(c + d*x)] - 15*Sin[c + d*x] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - 15*Sin[3*(c + d*x)]))/(
a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] time = 0.39, size = 236, normalized size = 1.6

$$\frac{2\left(\cos(dx+c)-1\right)^2\left(\cos(dx+c)+1\right)^2}{77\,da^3\left(\sin(dx+c)\right)^4}\sqrt{\frac{e}{\cos(dx+c)}}\left(28\,i\left(\cos(dx+c)\right)^6+28\left(\cos(dx+c)\right)^5\sin(dx+c)+5\,i\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/77/a^3/d*(e/cos(d*x+c))^(1/2)*(cos(d*x+c)-1)^2*(cos(d*x+c)+1)^2*(28*I*cos
(d*x+c)^6+28*cos(d*x+c)^5*sin(d*x+c)+5*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
```

```
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I
)-11*I*cos(d*x+c)^4+5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+3*cos(d*x+c)^3*sin(d*x+c)+
5*cos(d*x+c)*sin(d*x+c))/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(308\,a^{3}de^{(6i\,dx+6i\,c)}\mathrm{integral}\left(-\frac{5i\,\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{77\,a^{3}d},x\right)+\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(37i\,e^{(6i\,dx+6i\,c)}+61i\,e^{(4i\,dx+4i\,c)}+31i\,e^{(2i\,dx+2}\right.\right.}{308\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/308*(308*a^3*d*e^(6*I*d*x + 6*I*c)*integral(-5/77*I*sqrt(2)*sqrt(e/(e^(2*
I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^3*d), x) + sqrt(2)*sqrt(e/
(e^(2*I*d*x + 2*I*c) + 1))*(37*I*e^(6*I*d*x + 6*I*c) + 61*I*e^(4*I*d*x + 4*
I*c) + 31*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(1/2*I*d*x + 1/2*I*c))*e^(-6*I*d*x
 - 6*I*c)/(a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^3, x)
```

3.253 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=152

$$\frac{28ie^2}{117d\left(a^3+ia^3\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}+\frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{}{13d(a+i}$$

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(39*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) + (14*e*Sin[c + d*x])/(117*a^3*d*(e*Sec[c + d*x])^(3/2)) + ((2*I)/13
)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (((28*I)/117)*e^2)/(d
*(e*Sec[c + d*x])^(5/2)*(a^3 + I*a^3*Tan[c + d*x]))
```

Rubi [A] time = 0.144562, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3502, 3500, 3769, 3771, 2639}

$$\frac{28ie^2}{117d\left(a^3+ia^3\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}+\frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{}{13d(a+i}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(39*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) + (14*e*Sin[c + d*x])/(117*a^3*d*(e*Sec[c + d*x])^(3/2)) + ((2*I)/13
)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (((28*I)/117)*e^2)/(d
*(e*Sec[c + d*x])^(5/2)*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{7\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx}{13a}$$

$$= \frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{28ie^2}{117d(e\sec(c+dx))^{5/2}\left(a^3+ia^3\right.}$$

$$= \frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}} + \frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{}{117}$$

$$= \frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}} + \frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{}{117}$$

$$= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}} + \frac{}{13d\sqrt{e\,\mathrm{se}}}$$

Mathematica [C] time = 1.3221, size = 145, normalized size = 0.95

$$\frac{\sqrt{e\sec(c+dx)}(\sin(3(c+dx))+i\cos(3(c+dx)))\left(-56e^{4i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right.}{468a^3de}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (Sqrt[e*Sec[c + d*x]]*(I*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(62 + 176*Cos
[2*(c + d*x)] + 114*Cos[4*(c + d*x)] - 56*E^((4*I)*(c + d*x))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + (
126*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(468*a^3*d*e)
```

Maple [B] time = 0.394, size = 395, normalized size = 2.6

$$\frac{2\,(\cos(dx+c)+1)^2(\cos(dx+c)-1)^2}{117\,da^3(\sin(dx+c))^5e}\left(36\,i(\cos(dx+c))^7\sin(dx+c)-36\,(\cos(dx+c))^8-13\,i(\cos(dx+c))^5\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/117/a^3/d*(36*I*cos(d*x+c)^7*sin(d*x+c)-36*cos(d*x+c)^8-13*I*cos(d*x+c)^5
*sin(d*x+c)+21*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
```

```
*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)-21*I*Ellipt
icE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+31*cos(d*x+c)^6+21*I*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/si
n(d*x+c),I)*sin(d*x+c)-21*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*
x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*cos(d*x+c
)^4-14*cos(d*x+c)^2+21*cos(d*x+c))*(cos(d*x+c)+1)^2*(cos(d*x+c)-1)^2*(e/cos
(d*x+c))^(1/2)/sin(d*x+c)^5/e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-117i\,e^{(9i\,dx+9i\,c)}-219i\,e^{(8i\,dx+8i\,c)}-34i\,e^{(7i\,dx+7i\,c)}-302i\,e^{(6i\,dx+6i\,c)}+124i\,e^{(5i\,dx+5i\,c)}-124i\,e^{(4i\,dx+4i\,c)}+50\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/936*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-117*I*e^(9*I*d*x + 9*I*c
) - 219*I*e^(8*I*d*x + 8*I*c) - 34*I*e^(7*I*d*x + 7*I*c) - 302*I*e^(6*I*d*x
 + 6*I*c) + 124*I*e^(5*I*d*x + 5*I*c) - 124*I*e^(4*I*d*x + 4*I*c) + 50*I*e^
(3*I*d*x + 3*I*c) - 50*I*e^(2*I*d*x + 2*I*c) + 9*I*e^(I*d*x + I*c) - 9*I)*e
^(1/2*I*d*x + 1/2*I*c) + 936*(a^3*d*e*e^(8*I*d*x + 8*I*c) - a^3*d*e*e^(7*I*
d*x + 7*I*c))*integral(1/39*sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-7*I
*e^(2*I*d*x + 2*I*c) - 14*I*e^(I*d*x + I*c) - 7*I)*e^(1/2*I*d*x + 1/2*I*c)/
(a^3*d*e*e^(3*I*d*x + 3*I*c) - 2*a^3*d*e*e^(2*I*d*x + 2*I*c) + a^3*d*e*e^(I
*d*x + I*c)), x))/(a^3*d*e*e^(8*I*d*x + 8*I*c) - a^3*d*e*e^(7*I*d*x + 7*I*c
))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\sec(dx+c)}(ia\tan(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^3), x)
```

3.254 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=186

$$\frac{12ie^2}{55d\left(a^3+ia^3\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{11a^3de^2}+\frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}}+$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(11*a
^3*d*e^2) + (6*e*Sin[c + d*x])/(55*a^3*d*(e*Sec[c + d*x])^(5/2)) + (2*Sin[c
 + d*x])/(11*a^3*d*e*Sqrt[e*Sec[c + d*x]]) + ((2*I)/15)/(d*(e*Sec[c + d*x])
^(3/2)*(a + I*a*Tan[c + d*x])^3) + (((12*I)/55)*e^2)/(d*(e*Sec[c + d*x])^(7
/2)*(a^3 + I*a^3*Tan[c + d*x]))
```

Rubi [A] time = 0.18169, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3502, 3500, 3769, 3771, 2641}

$$\frac{12ie^2}{55d\left(a^3+ia^3\tan(c+dx)\right)(e\sec(c+dx))^{7/2}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{11a^3de^2}+\frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```


```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(11*a
^3*d*e^2) + (6*e*Sin[c + d*x])/(55*a^3*d*(e*Sec[c + d*x])^(5/2)) + (2*Sin[c
 + d*x])/(11*a^3*d*e*Sqrt[e*Sec[c + d*x]]) + ((2*I)/15)/(d*(e*Sec[c + d*x])
^(3/2)*(a + I*a*Tan[c + d*x])^3) + (((12*I)/55)*e^2)/(d*(e*Sec[c + d*x])^(7
/2)*(a^3 + I*a^3*Tan[c + d*x]))
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3} + \frac{3\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+d}}{5a}$$

$$= \frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3} + \frac{12ie^2}{55d(e\sec(c+dx))^{7/2}\left(a^3+\right.}$$

$$= \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}} + \frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3} +$$

$$= \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}} + \frac{2}{15d(e\sec(c+dx))^{3/2}}$$

$$= \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}} + \frac{2}{15d(e\sec(c+dx))^{3/2}}$$

$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{11a^3de^2} + \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/}}$$

Mathematica [A] time = 0.582232, size = 151, normalized size = 0.81

$$\frac{\sec^5(c+dx)\left(-114i\sin(c+dx)-81i\sin(3(c+dx))+33i\sin(5(c+dx))-332\cos(c+dx)-154\cos(3(c+dx))+22\,\mathrm{c}\right.}{1320a^3d(\tan(c+dx)-i)^3(e\sec(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (Sec[c + d*x]^5*(-332*Cos[c + d*x] - 154*Cos[3*(c + d*x)] + 22*Cos[5*(c + d
*x)] - (114*I)*Sin[c + d*x] + (240*I)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - (81*I)*Sin[3*(c + d*x)] +
 (33*I)*Sin[5*(c + d*x)]))/(1320*a^3*d*(e*Sec[c + d*x])^(3/2)*(-I + Tan[c +
 d*x])^3)
```

Maple [A] time = 0.411, size = 261, normalized size = 1.4

$$\frac{2\left(\cos(dx+c)-1\right)^2\left(\cos(dx+c)+1\right)^2\cos(dx+c)}{165\,da^3e^3\left(\sin(dx+c)\right)^4}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\left(44\,i\left(\cos(dx+c)\right)^8+44\,\sin(dx+c)\left(\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] 2/165/a^3/d*(e/cos(d*x+c))^(3/2)*(cos(d*x+c)-1)^2*(cos(d*x+c)+1)^2*cos(d*x+
c)*(44*I*cos(d*x+c)^8+44*sin(d*x+c)*cos(d*x+c)^7-15*I*cos(d*x+c)^6+7*cos(d*
x+c)^5*sin(d*x+c)+15*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+9*cos(d*x+c)^3*s
in(d*x+c)+15*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+15*cos(d*x+c)*sin(d*x+c))/e^3/sin(d
*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2640\,a^3de^2e^{(8i\,dx+8i\,c)}\text{integral}\left(-\frac{i\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{11\,a^3de^2},x\right)+\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-55i\,e^{(10i\,dx+10i\,c)}+235i\,e^{(8i\,dx+8i\,c)}+446\right.\right.}{2640\,a^3de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/2640*(2640*a^3*d*e^2*e^(8*I*d*x + 8*I*c)*integral(-1/11*I*sqrt(2)*sqrt(e/
(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^3*d*e^2), x) + sqrt(
2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-55*I*e^(10*I*d*x + 10*I*c) + 235*I*e
^(8*I*d*x + 8*I*c) + 446*I*e^(6*I*d*x + 6*I*c) + 218*I*e^(4*I*d*x + 4*I*c)
+ 73*I*e^(2*I*d*x + 2*I*c) + 11*I)*e^(1/2*I*d*x + 1/2*I*c))*e^(-8*I*d*x - 8
*I*c)/(a^3*d*e^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\sec(dx+c))^{\frac{3}{2}}(ia\tan(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3), x)
```

3.255 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=192

$$-\frac{154e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^4d}-\frac{154e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^4d}+\frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{154e^8E\left(\frac{1}{2}(c\right.}{5a^4d\sqrt{\cos(c+dx)}}$$

```
[Out] (154*e^8*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (154*e^7*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^4*d) - (154*e
^5*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(15*a^4*d) + ((4*I)*e^2*(e*Sec[c +
d*x])^(11/2))/(a*d*(a + I*a*Tan[c + d*x])^3) + (((44*I)/3)*e^4*(e*Sec[c + d
*x])^(7/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.174935, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3768, 3771, 2639}

$$-\frac{154e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^4d}-\frac{154e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^4d}+\frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{154e^8E\left(\frac{1}{2}(c\right.}{5a^4d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^4,x]
```


```
[Out] (154*e^8*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (154*e^7*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^4*d) - (154*e
^5*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(15*a^4*d) + ((4*I)*e^2*(e*Sec[c +
d*x])^(11/2))/(a*d*(a + I*a*Tan[c + d*x])^3) + (((44*I)/3)*e^4*(e*Sec[c + d
*x])^(7/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3} - \frac{\left(11e^2\right)\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx}{a^2}$$

$$= \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3} + \frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{\left(77e^4\right)\int (e\sec(c+dx))^{7/2}\,dx}{3a^4}$$

$$= -\frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d} + \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3} + \frac{44ie^4(e\sec(c+dx))^{7/}}{3d\left(a^4+ia^4\tan(c+d\right.}$$

$$= -\frac{154e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} - \frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d} + \frac{4ie^2(e\sec(c+}{ad(a+ia\tan(}$$

$$= -\frac{154e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} - \frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d} + \frac{4ie^2(e\sec(c+}{ad(a+ia\tan(}$$

$$= \frac{154e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{154e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} - \frac{154e^5(e\sec(c+d}{15}$$

Mathematica [C] time = 1.33692, size = 124, normalized size = 0.65

$$-\frac{ie^5(e\sec(c+dx))^{5/2}\left(77e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-1133\cos(c+dx)-3(33}{30a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-I/30)*e^5*(e*Sec[c + d*x])^(5/2)*(-1133*Cos[c + d*x] + (77*(1 + E^((2*I)
*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/
E^(I*(c + d*x)) - 3*(117*Cos[3*(c + d*x)] + (33*I)*Sin[c + d*x] + (37*I)*Si
n[3*(c + d*x)])))/(a^4*d)
```

Maple [B] time = 0.339, size = 1628, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/15/a^4/d*(cos(d*x+c)+1)^7*(cos(d*x+c)-1)^4*(-9*cos(d*x+c)*(-cos(d*x+c)/(c
os(d*x+c)+1)^2)^(3/2)+120*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^7
+129*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^6-60*I*(-cos(d*x+c)/(c
os(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^2*sin(d*x+c)-30*I*cos(d*x+c)^4*ln(-2*(2*(-
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)
/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+30*I*cos(
d*x+c)^4*ln(-(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c
)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*si
```

```
n(d*x+c)-20*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)*sin(d*x+c)-12
0*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^6*sin(d*x+c)-360*I*(-co
s(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^5*sin(d*x+c)-380*I*(-cos(d*x+c)
/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^4*sin(d*x+c)-180*I*(-cos(d*x+c)/(cos(d*
x+c)+1)^2)^(3/2)*cos(d*x+c)^3*sin(d*x+c)-3*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(
3/2)+1386*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*EllipticE(I*(cos(d
*x+c)-1)/sin(d*x+c),I)-1386*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^4*
EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+231*I*(-cos(d*x+c)/(cos(d*x+c)+1)^
2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x
+c)*cos(d*x+c)^6*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-219*(-cos(d*x+c)/
(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^5-231*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/
2)*cos(d*x+c)^4+108*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^3+105*(
-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^2-231*I*(-cos(d*x+c)/(cos(d*
x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)*cos(d*x+c)^6*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+924*I*(-c
os(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticE(I*(cos(d*x+c)-1)/sin(d*
x+c),I)-924*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^5*EllipticF(I*(cos
(d*x+c)-1)/sin(d*x+c),I)+924*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)^3
*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-924*I*(-cos(d*x+c)/(cos(d*x+c)+1)
^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*
x+c)*cos(d*x+c)^3*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+231*I*sin(d*x+c)
*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)
-231*I*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)
-1)/sin(d*x+c),I))*(e/cos(d*x+c))^(15/2)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x
+c)+1)^2)^(3/2)/sin(d*x+c)^9
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(462i\,e^{7}e^{(6i\,dx+6i\,c)} + 1232i\,e^{7}e^{(4i\,dx+4i\,c)} + 1034i\,e^{7}e^{(2i\,dx+2i\,c)} + 240i\,e^{7}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + 15\left(a^{4}de^{(5i\,dx+5i\,c)} + 2\,a^{4}\right.}{15\left(a^{4}de^{(5i\,dx+5i\,c)} + 2\,a^{4}de^{(3i\,dx+3i\,c)} + a^{4}de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/15*(sqrt(2)*(462*I*e^7*e^(6*I*d*x + 6*I*c) + 1232*I*e^7*e^(4*I*d*x + 4*I*
c) + 1034*I*e^7*e^(2*I*d*x + 2*I*c) + 240*I*e^7)*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 15*(a^4*d*e^(5*I*d*x + 5*I*c) + 2*a^4*d*e
^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))*integral(-77/5*I*sqrt(2)*e^7*sq
rt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(a^4*d), x))/(a^4*d
*e^(5*I*d*x + 5*I*c) + 2*a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{15}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.256 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=157

$$-\frac{10e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{a^4d}+\frac{12ie^4(e\sec(c+dx))^{5/2}}{d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{10e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{a^4d}+\frac{4ie}{3ad(}$$

```
[Out] (-10*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(a^4*d) - (10*e^5*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(a^4*d) + (((4*I)/3
)*e^2*(e*Sec[c + d*x])^(9/2))/(a*d*(a + I*a*Tan[c + d*x])^3) + ((12*I)*e^4*
(e*Sec[c + d*x])^(5/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.154751, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3768, 3771, 2641}

$$-\frac{10e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{a^4d}+\frac{12ie^4(e\sec(c+dx))^{5/2}}{d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{10e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{a^4d}+\frac{4ie}{3ad(}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^4,x]
```


```
[Out] (-10*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(a^4*d) - (10*e^5*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(a^4*d) + (((4*I)/3
)*e^2*(e*Sec[c + d*x])^(9/2))/(a*d*(a + I*a*Tan[c + d*x])^3) + ((12*I)*e^4*
(e*Sec[c + d*x])^(5/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3} - \frac{(3e^2)\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx}{a^2}$$

$$= \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3} + \frac{12ie^4(e\sec(c+dx))^{5/2}}{d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{(15e^4)\int (e\sec(c+dx))^{5/2}\,dx}{a^4}$$

$$= -\frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d} + \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3} + \frac{12ie^4(e\sec(c+dx))^{5/2}}{d\left(a^4+ia^4\tan(c+dx)\right)}$$

$$= -\frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d} + \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3} + \frac{12ie^4(e\sec(c+dx))^{5/2}}{d\left(a^4+ia^4\tan(c+dx)\right)}$$

$$= -\frac{10e^6\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{a^4d} - \frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d}$$

Mathematica [A] time = 0.500015, size = 134, normalized size = 0.85

$$\frac{ie^6\sec^5(c+dx)\sqrt{e\sec(c+dx)}(\cos(3(c+dx))+i\sin(3(c+dx)))\left(11i\sin(2(c+dx))+19\cos(2(c+dx))+30i\cos^{\frac{3}{2}}(c+\right.}{3a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/3)*e^6*Sec[c + d*x]^5*Sqrt[e*Sec[c + d*x]]*(21 + 19*Cos[2*(c + d*x)] +
(30*I)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c
 + d*x]) + (11*I)*Sin[2*(c + d*x)])*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])
)/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.3, size = 198, normalized size = 1.3

$$\frac{2\,(\cos(dx+c))^5}{3\,a^4d}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{13}{2}}\left(-15\,i\,(\cos(dx+c))^2\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i\,(\cos(dx}{\sin(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/3/a^4/d*(e/cos(d*x+c))^(13/2)*(-15*I*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I
)-15*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+8*I*cos(d*x+c)^3+8*cos(d*x+c)^2*
sin(d*x+c)+12*I*cos(d*x+c)+sin(d*x+c))*cos(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{2}\left(30i\,e^6e^{(4i\,dx+4i\,c)}+42i\,e^6e^{(2i\,dx+2i\,c)}+8i\,e^6\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\left(a^4de^{(4i\,dx+4i\,c)}+a^4de^{(2i\,dx+2i\,c)}\right)\text{integral}\left(\frac{5i\sqrt{2}e^6\cdots}{\cdots}\right.}{3\left(a^4de^{(4i\,dx+4i\,c)}+a^4de^{(2i\,dx+2i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(30*I*e^6*e^(4*I*d*x + 4*I*c) + 42*I*e^6*e^(2*I*d*x + 2*I*c) +
 8*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*(a^
4*d*e^(4*I*d*x + 4*I*c) + a^4*d*e^(2*I*d*x + 2*I*c))*integral(5*I*sqrt(2)*e
^6*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^4*d), x))/
(a^4*d*e^(4*I*d*x + 4*I*c) + a^4*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{13}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.257 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=163

$$\frac{42e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^4d}-\frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{42e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))}{5ad(a+ia\tan(c+}$$

```
[Out] (-42*e^6*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (42*e^5*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^4*d) + (((4*I)
/5)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (((28*I)/5
)*e^4*(e*Sec[c + d*x])^(3/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.154252, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3768, 3771, 2639}

$$\frac{42e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^4d}-\frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{42e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))}{5ad(a+ia\tan(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-42*e^6*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (42*e^5*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^4*d) + (((4*I)
/5)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (((28*I)/5
)*e^4*(e*Sec[c + d*x])^(3/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{\left(7e^2\right)\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx}{5a^2}$$
$$= \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{\left(21e^4\right)\int (e\sec(c+dx))^{3/2}\,dx}{5a^4}$$
$$= \frac{42e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)} - \ldots$$
$$= \frac{42e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)} - \ldots$$
$$= -\frac{42e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{42e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} + \frac{4ie^2(e\sec(c+dx)\ldots}{5ad(a+ia\tan(c+\ldots}$$

Mathematica [C] time = 0.563876, size = 106, normalized size = 0.65

$$-\frac{2ie^5e^{-3i(c+dx)}\left(21e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)-7e^{2i(c+dx)}-2\right)\sqrt{e\sec(c+dx)}}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (((-2*I)/5)*e^5*(-2 - 7*E^((2*I)*(c + d*x)) + 21*E^((2*I)*(c + d*x))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*
x))])*Sqrt[e*Sec[c + d*x]])/(a^4*d*E^((3*I)*(c + d*x)))
```

Maple [B] time = 0.335, size = 3582, normalized size = 22.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/10/a^4/d*(e/cos(d*x+c))^(11/2)*(cos(d*x+c)-1)^2*cos(d*x+c)^2*(cos(d*x+c)+
1)^7*(64*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-160*(-cos(d*x+c)/(
cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^7-76*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)
*cos(d*x+c)^6+42*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+
c),I)*sin(d*x+c)+32*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^9+32*(-
cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^8-42*I*(-cos(d*x+c)/(cos(d*x+
c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)-20*(-cos(d*x+c)/(cos(d*x
+c)+1)^2)^(3/2)-32*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*
x+c)^8-32*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^7+14
```

4*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^6-192*I*(-co
s(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^3+80*I*(-cos(d*x+c)/
(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^2-20*I*cos(d*x+c)^4*ln(-2*(2*
(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+20*I*co
s(d*x+c)^4*ln(-(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x
+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*
sin(d*x+c)+80*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)+
40*I*cos(d*x+c)^3*ln(-2*(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^
2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d
*x+c)^2)*sin(d*x+c)-40*I*cos(d*x+c)^3*ln(-(2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)
^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*c
os(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)-20*I*cos(d*x+c)^2*ln(-2*(2*(-cos(d*x+
c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-cos(d*x+c)/(cos(d*
x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)+20*I*cos(d*x+c)^2
ln(-(2(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)^2-cos(d*x+c)^2-2*(-
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*cos(d*x+c)-1)/sin(d*x+c)^2)*sin(d*x+c)
+288*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^5+36*(-cos(d*x+c)/(cos
(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^4-224*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*c
os(d*x+c)^3+28*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*cos(d*x+c)^2-21*I*cos(d
*x+c)^5*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+
c),I)+21*I*cos(d*x+c)^5*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*
x+c)-1)/sin(d*x+c),I)+144*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)
*cos(d*x+c)^5-294*I*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^
(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(
I*(cos(d*x+c)-1)/sin(d*x+c),I)+294*I*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(
cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-42*I*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),
I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)*sin(d*x+c)+42*I*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-
1)/sin(d*x+c),I)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*cos(d*x+c)*sin(d*x+c)
-21*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^8*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-
1)/sin(d*x+c),I)+21*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^8*sin(d*x+c)*Elliptic
E(I*(cos(d*x+c)-1)/sin(d*x+c),I)-21*I*cos(d*x+c)^7*sin(d*x+c)*(-cos(d*x+c)/
(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+21*I*cos(d*x+c)^7*sin(d*x+
c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+252*I*(-co
s(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x
+c),I)-252*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^5*sin(d*x+c)*EllipticE(I*(cos(
d*x+c)-1)/sin(d*x+c),I)-42*I*cos(d*x+c)^6*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+42*I*cos(d*x+c)^6*sin(d*x+c)*(-cos(
d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+105*I*cos(d*x+c)^4*
sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-10
5*I*cos(d*x+c)^4*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*
x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)
/sin(d*x+c),I)-336*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*sin(d*x+c)*EllipticF
(I*(cos(d*x+c)-1)/sin(d*x+c),I)+336*I*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*

```
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*sin
(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-84*I*(-cos(d*x+c)/(cos(d*x
+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
cos(d*x+c)^2*sin(d*x+c)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+84*I*(-cos
(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+
c),I)-84*I*cos(d*x+c)^7*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*(1/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*
x+c)-1)/sin(d*x+c),I)+84*I*cos(d*x+c)^7*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)
+1)^2)^(3/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)+84*I*cos(d*x+c)^4*sin(d*x+c)*(-cos(d*
x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-84*I*cos(d*x+c)^4*sin
(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)-192*I
*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)*sin(d*x+c)*cos(d*x+c)^4)*(-cos(d*x+c)
/(cos(d*x+c)+1)^2)^(3/2)/sin(d*x+c)^9
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(5\,a^4 d e^{(3i\,dx+3i\,c)}\mathrm{integral}\left(\frac{21i\,\sqrt{2}e^5\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{5\,a^4 d}, x\right) + \sqrt{2}\left(-42i\,e^5 e^{(4i\,dx+4i\,c)} - 28i\,e^5 e^{(2i\,dx+2i\,c)} + 4i\,e^5\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(}\right.}{5\,a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/5*(5*a^4*d*e^(3*I*d*x + 3*I*c)*integral(21/5*I*sqrt(2)*e^5*sqrt(e/(e^(2*I
*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(a^4*d), x) + sqrt(2)*(-42*I*e^
5*e^(4*I*d*x + 4*I*c) - 28*I*e^5*e^(2*I*d*x + 2*I*c) + 4*I*e^5)*sqrt(e/(e^(
2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^4*d
)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec (dx + c))^{\frac{11}{2}}}{(i\,a \tan (dx + c) + a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.258 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=132

$$-\frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^4d}+\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}$$

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*a^4*d) + (((4*I)/7)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d
*x])^3) - (((20*I)/21)*e^4*Sqrt[e*Sec[c + d*x]])/(d*(a^4 + I*a^4*Tan[c + d*
x]))
```

Rubi [A] time = 0.137437, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3500, 3771, 2641}

$$-\frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^4d}+\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^4,x]
```


```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*a^4*d) + (((4*I)/7)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d
*x])^3) - (((20*I)/21)*e^4*Sqrt[e*Sec[c + d*x]])/(d*(a^4 + I*a^4*Tan[c + d*
x]))
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}dx = \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3} - \frac{(5e^2)\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}dx}{7a^2}$$
$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3} - \frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{(5e^4)\int\sqrt{e\sec(c+dx)}dx}{21a^4}$$
$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3} - \frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{(5e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+d}}{21a^4}$$
$$= \frac{10e^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{21a^4d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3} - \frac{20ie^4}{21d\left(a^4\right.}$$

Mathematica [A] time = 0.494361, size = 137, normalized size = 1.04

$$\frac{2e^4\sec^4(c+dx)\sqrt{e\sec(c+dx)}(\cos(2(c+dx))+i\sin(2(c+dx)))\left(5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)(\cos(2(c+dx))+i\,\mathrm{si}\right.}{21a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (2*e^4*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(5*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - (2*I)*(1 + Cos[2*
(c + d*x)] + (4*I)*Sin[2*(c + d*x)]))*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)
]))/(21*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.251, size = 200, normalized size = 1.5

$$\frac{2\,(\cos(dx+c))^4}{21\,a^4d}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{9}{2}}\left(5\,i\cos(dx+c)\sqrt{(\cos(dx+c)+1)^{-1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\mathrm{EllipticF}\left(\frac{i(\cos(dx+c)-}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/21/a^4/d*(e/cos(d*x+c))^(9/2)*cos(d*x+c)^4*(5*I*cos(d*x+c)*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin
(d*x+c),I)+24*I*cos(d*x+c)^4+5*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+24*cos(d*x+c)^3*s
in(d*x+c)-28*I*cos(d*x+c)^2-16*cos(d*x+c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(21\,a^4de^{(4i\,dx+4i\,c)}\text{integral}\left(-\frac{5i\,\sqrt{2}e^4\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{21\,a^4d},x\right)+\sqrt{2}\left(-10i\,e^4e^{(4i\,dx+4i\,c)}-4i\,e^4e^{(2i\,dx+2i\,c)}+6i\,e^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e\right.}{21\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/21*(21*a^4*d*e^(4*I*d*x + 4*I*c)*integral(-5/21*I*sqrt(2)*e^4*sqrt(e/(e^(
2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^4*d), x) + sqrt(2)*(-10*
I*e^4*e^(4*I*d*x + 4*I*c) - 4*I*e^4*e^(2*I*d*x + 2*I*c) + 6*I*e^4)*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-4*I*d*x - 4*I*c)/(a^
4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.259 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=132

$$-\frac{4ie^4}{15d\left(a^4+ia^4\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}-\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}$$

```
[Out] (-2*e^4*EllipticE[(c + d*x)/2, 2])/(15*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((4*I)/9)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d
*x])^3) - (((4*I)/15)*e^4)/(d*Sqrt[e*Sec[c + d*x]]*(a^4 + I*a^4*Tan[c + d*x
]))
```

Rubi [A] time = 0.137749, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3500, 3771, 2639}

$$-\frac{4ie^4}{15d\left(a^4+ia^4\tan(c+dx)\right)\sqrt{e\sec(c+dx)}}-\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^4,x]
```


```
[Out] (-2*e^4*EllipticE[(c + d*x)/2, 2])/(15*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((4*I)/9)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d
*x])^3) - (((4*I)/15)*e^4)/(d*Sqrt[e*Sec[c + d*x]]*(a^4 + I*a^4*Tan[c + d*x
]))
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3} - \frac{e^2\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}dx}{3a^2}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)} - \frac{e^4\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{15a^4}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)} - \frac{e^4\int \sqrt{\cos(c+}}{15a^4\sqrt{\cos(c+dx)}\sqrt{}}$$

$$= -\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\left(a^4+\right.}$$

Mathematica [C] time = 0.739934, size = 149, normalized size = 1.13

$$\frac{e^3e^{-idx}\sec^4(c+dx)\sqrt{e\sec(c+dx)}(\sin(c+2dx)-i\cos(c+2dx))\left(6e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\right.\right.}{45a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (e^3*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(-7 - 7*Cos[2*(c + d*x)] + 6*E^((2
*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c + d*x)])*((-I)*Cos[c + 2*d*x] +
Sin[c + 2*d*x]))/(45*a^4*d*E^(I*d*x)*(-I + Tan[c + d*x])^4)
```

Maple [B] time = 0.269, size = 370, normalized size = 2.8

$$-\frac{2\,(\cos(dx+c))^3}{45\,a^4d\sin(dx+c)}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{7}{2}}\left(-40\,i\sin(dx+c)(\cos(dx+c))^5+40\,(\cos(dx+c))^6+3\,i\text{EllipticF}\left(\frac{i(\cos(dx+c}{\sin(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] -2/45/a^4/d*(e/cos(d*x+c))^(7/2)*cos(d*x+c)^3*(-40*I*sin(d*x+c)*cos(d*x+c)^
5+40*cos(d*x+c)^6+3*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*c
os(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*El
lipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+36*I*sin(d*x+c)*cos(d*x+c)^3+3
*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*EllipticE(I*(cos(d*x+c)-1)/sin(d
*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)-56*cos(d*x+c)^4+13*cos(d*x+c)^2+3*cos(d*x+c))/sin(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(45\, a^4 d e^{(5i\, dx+5i\, c)}\text{integral}\left(\frac{i\sqrt{2}e^3\sqrt{\frac{e}{e^{(2i\, dx+2i\, c)}+1}}e^{\left(\frac{1}{2}i\, dx+\frac{1}{2}i\, c\right)}}{15\, a^4 d}, x\right) + \sqrt{2}\left(-6i\, e^3 e^{(6i\, dx+6i\, c)} - 4i\, e^3 e^{(4i\, dx+4i\, c)} + 7i\, e^3 e^{(2i\, dx+2i\, c)} + 5i\, e\right.}{45\, a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/45*(45*a^4*d*e^(5*I*d*x + 5*I*c)*integral(1/15*I*sqrt(2)*e^3*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(a^4*d), x) + sqrt(2)*(-6*I*e
^3*e^(6*I*d*x + 6*I*c) - 4*I*e^3*e^(4*I*d*x + 4*I*c) + 7*I*e^3*e^(2*I*d*x +
 2*I*c) + 5*I*e^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
))*e^(-5*I*d*x - 5*I*c)/(a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\, a\tan(dx+c)+a)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.260 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=163

$$-\frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}}-\frac{4ie^4}{77d\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77a^4d}-$$

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*a^4*d) - (2*e^3*Sin[c + d*x])/(77*a^4*d*Sqrt[e*Sec[c + d*x]]) + (((4*I)
/11)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^3) - (((4*I)/77)
*e^4)/(d*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.143827, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3769, 3771, 2641}

$$-\frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}}-\frac{4ie^4}{77d\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}-\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77a^4d}-$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^4,x]
```


```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*a^4*d) - (2*e^3*Sin[c + d*x])/(77*a^4*d*Sqrt[e*Sec[c + d*x]]) + (((4*I)
/11)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^3) - (((4*I)/77)
*e^4)/(d*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*Tan[c + d*x]))
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{e^2\int\frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx}{11a^2}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{77d(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)} - \frac{\left(3e^4\right)\int\frac{}{(e\sec(}}{77a}$$

$$= -\frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{77d(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan\right.}$$

$$= -\frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{77d(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan\right.}$$

$$= -\frac{2e^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{77a^4d} - \frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} + \frac{4ie^2\sqrt{e\sec}}{11ad(a+ia}$$

Mathematica [A] time = 0.556159, size = 144, normalized size = 0.88

$$\frac{\sec^2(c+dx)(e\sec(c+dx))^{5/2}(\cos(c+dx)+i\sin(c+dx))\left(3\sin(c+dx)+3\sin(3(c+dx))+37i\cos(c+dx)+11i\cos\right.}{154a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^2*(e*Sec[c + d*x])^(5/2)*(Cos[c + d*x] + I*Sin[c + d*x])*((37
*I)*Cos[c + d*x] + (11*I)*Cos[3*(c + d*x)] + 3*Sin[c + d*x] - 4*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) +
 3*Sin[3*(c + d*x)]))/(154*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.329, size = 216, normalized size = 1.3

$$\frac{2\,(\cos(dx+c))^2}{77\,a^4d}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\left(56\,i\,(\cos(dx+c))^6+56\,(\cos(dx+c))^5\sin(dx+c)-44\,i\,(\cos(dx+c))^4-i\cos(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/77/a^4/d*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(56*I*cos(d*x+c)^6+56*cos(d*x+
c)^5*sin(d*x+c)-44*I*cos(d*x+c)^4-I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)-I*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(
d*x+c)-1)/sin(d*x+c),I)-16*cos(d*x+c)^3*sin(d*x+c)-cos(d*x+c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(154\, a^4 d e^{(6i\,dx+6i\,c)} \text{integral}\left(\frac{i\,\sqrt{2} e^2 \sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\, e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{77\, a^4 d}, x\right) + \sqrt{2}\left(4i\, e^2 e^{(6i\,dx+6i\,c)} + 17i\, e^2 e^{(4i\,dx+4i\,c)} + 20i\, e^2 e^{(2i\,dx+2i\,c)} + 7i\, e\right.}{154\, a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/154*(154*a^4*d*e^(6*I*d*x + 6*I*c)*integral(1/77*I*sqrt(2)*e^2*sqrt(e/(e^
(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^4*d), x) + sqrt(2)*(4*I
*e^2*e^(6*I*d*x + 6*I*c) + 17*I*e^2*e^(4*I*d*x + 4*I*c) + 20*I*e^2*e^(2*I*d
*x + 2*I*c) + 7*I*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2
*I*c))*e^(-6*I*d*x - 6*I*c)/(a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec (dx + c))^{\frac{5}{2}}}{(i\, a \tan (dx + c) + a)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.261 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=163

$$\frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}}+\frac{4ie^4}{117d\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}+\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{}{13ad(}$$

```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(39*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (2*e^3*Sin[c + d*x])/(117*a^4*d*(e*Sec[c + d*x])^(3/2)) + (((4*
I)/13)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (((4*I)/1
17)*e^4)/(d*(e*Sec[c + d*x])^(5/2)*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.146264, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3500, 3769, 3771, 2639}

$$\frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}}+\frac{4ie^4}{117d\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{5/2}}+\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{}{13ad(}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^4,x]
```


```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(39*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (2*e^3*Sin[c + d*x])/(117*a^4*d*(e*Sec[c + d*x])^(3/2)) + (((4*
I)/13)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (((4*I)/1
17)*e^4)/(d*(e*Sec[c + d*x])^(5/2)*(a^4 + I*a^4*Tan[c + d*x]))
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}dx = \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}dx}{13a^2}$$

$$= \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{4ie^4}{117d(e\sec(c+dx))^{5/2}\left(a^4+ia^4\tan(c+dx)\right)} +$$

$$= \frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{}{117d(e\sec(c+dx))^{5/}}$$

$$= \frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{}{117d(e\sec(c+dx))^{5/}}$$

$$= \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia}$$

Mathematica [C] time = 1.4279, size = 142, normalized size = 0.87

$$\frac{ie^{-idx}\sec^2(c+dx)(\cos(dx)+i\sin(dx))(e\sec(c+dx))^{3/2}\left(\frac{24e^{4i(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+22i\sin(2(c+dx))+4\right.}{234a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/234)*Sec[c + d*x]^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])*(28
+ 40*Cos[2*(c + d*x)] + (24*E^((4*I)*(c + d*x))*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + (22*I)*Sin[2*
(c + d*x)]))/(a^4*d*E^(I*d*x)*(-I + Tan[c + d*x])^4)
```

Maple [B] time = 0.381, size = 378, normalized size = 2.3

$$\frac{2\cos(dx+c)}{117a^4d\sin(dx+c)}\left(72i(\cos(dx+c))^7\sin(dx+c)-72(\cos(dx+c))^8-52i\sin(dx+c)(\cos(dx+c))^5-3i\sqrt{(\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/117/a^4/d*(72*I*cos(d*x+c)^7*sin(d*x+c)-72*cos(d*x+c)^8-52*I*cos(d*x+c)^5
*sin(d*x+c)-3*I*EllipticE(I*(cos(d*x+c)-1)/sin(d*x+c),I)*cos(d*x+c)*sin(d*x
+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*I*Elliptic
F(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+88*cos(d*x+c)^6-3*I*EllipticE(I*(cos
(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)+3*I*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)*sin(d*x+c)
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-17*cos(d*x+c)^4
```

```
-2*cos(d*x+c)^2+3*cos(d*x+c))*(e/cos(d*x+c))^(3/2)*cos(d*x+c)/sin(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(468\,a^4de^{(7i\,dx+7i\,c)}\mathrm{integral}\left(-\frac{i\sqrt{2}e\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{39\,a^4d},x\right)+\sqrt{2}\left(24i\,ee^{(8i\,dx+8i\,c)}+55i\,ee^{(6i\,dx+6i\,c)}+59i\,ee^{(4i\,dx+4i\,c)}+3\right.\right.}{468\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/468*(468*a^4*d*e^(7*I*d*x + 7*I*c)*integral(-1/39*I*sqrt(2)*e*sqrt(e/(e^(
2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(a^4*d), x) + sqrt(2)*(24*I*
e*e^(8*I*d*x + 8*I*c) + 55*I*e*e^(6*I*d*x + 6*I*c) + 59*I*e*e^(4*I*d*x + 4*
I*c) + 37*I*e*e^(2*I*d*x + 2*I*c) + 9*I*e)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(1/2*I*d*x + 1/2*I*c))*e^(-7*I*d*x - 7*I*c)/(a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^4, x)
```

3.262 $\int \frac{\sqrt{e \sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal. Leaf size=191

$$\frac{4ie^2}{33d\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}+\frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33a^4d}+\ldots$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(33*a
^4*d) + (2*e*Sin[c + d*x])/(33*a^4*d*Sqrt[e*Sec[c + d*x]]) + (((2*I)/15)*Sq
rt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^4) + (((14*I)/165)*Sqrt[e*Sec
[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^3) + (((4*I)/33)*e^2)/(d*(e*Sec[c +
 d*x])^(3/2)*(a^4 + I*a^4*Tan[c + d*x]))
```

Rubi [A] time = 0.191723, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3502, 3500, 3769, 3771, 2641}

$$\frac{4ie^2}{33d\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}+\frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33a^4d}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^4,x]
```


```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(33*a
^4*d) + (2*e*Sin[c + d*x])/(33*a^4*d*Sqrt[e*Sec[c + d*x]]) + (((2*I)/15)*Sq
rt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^4) + (((14*I)/165)*Sqrt[e*Sec
[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^3) + (((4*I)/33)*e^2)/(d*(e*Sec[c +
 d*x])^(3/2)*(a^4 + I*a^4*Tan[c + d*x]))
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} + \frac{7\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx}{15a}$$

$$= \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} + \frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3} + \frac{7\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx}{33a^2}$$

$$= \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} + \frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3} + \frac{4ie^2}{33d(e\sec(c+dx))^{3/2}\left(a^4+ia^4\right.}$$

$$= \frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} + \frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3} + \frac{}{33d(e}$$

$$= \frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} + \frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3} + \frac{}{33d(e}$$

$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\sec(c+dx)}}{33a^4d} + \frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c}{15d(a+ia\tan}$$

Mathematica [A] time = 0.486795, size = 137, normalized size = 0.72

$$\frac{\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(i(54i\sin(2(c+dx))+37i\sin(4(c+dx))+112\cos(2(c+dx))+48\cos(4(c+dx))+64)+\right.}{660a^4d(\tan(c+dx)-i)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]) + I*(64 + 112*Cos[2*(c +
 d*x)] + 48*Cos[4*(c + d*x)] + (54*I)*Sin[2*(c + d*x)] + (37*I)*Sin[4*(c +
d*x)])))/(660*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] time = 0.356, size = 252, normalized size = 1.3

$$\frac{2\,(\cos(dx+c)-1)^2(\cos(dx+c)+1)^2}{165\,a^4d\,(\sin(dx+c))^4}\sqrt{\frac{e}{\cos(dx+c)}}\left(88\,i\,(\cos(dx+c))^8+88\,\sin(dx+c)\,(\cos(dx+c))^7-60\,i\,(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 2/165/a^4/d*(e/cos(d*x+c))^(1/2)*(cos(d*x+c)-1)^2*(cos(d*x+c)+1)^2*(88*I*co
s(d*x+c)^8+88*sin(d*x+c)*cos(d*x+c)^7-60*I*cos(d*x+c)^6-16*cos(d*x+c)^5*sin
(d*x+c)+5*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I)+5*I*(1/(cos(d*x+c)+1))^(1/2
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cos(d*x+c)-1)/sin(d*x+c),I
)+3*cos(d*x+c)^3*sin(d*x+c)+5*cos(d*x+c)*sin(d*x+c))/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(1320\,a^4de^{(8i\,dx+8i\,c)}\text{integral}\left(-\frac{i\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{33\,a^4d},x\right)+\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(85i\,e^{(8i\,dx+8i\,c)}+166i\,e^{(6i\,dx+6i\,c)}+128i\,e^{(4i\,d}\right.\right.}{1320\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/1320*(1320*a^4*d*e^(8*I*d*x + 8*I*c)*integral(-1/33*I*sqrt(2)*sqrt(e/(e^(
2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a^4*d), x) + sqrt(2)*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*(85*I*e^(8*I*d*x + 8*I*c) + 166*I*e^(6*I*d*x +
 6*I*c) + 128*I*e^(4*I*d*x + 4*I*c) + 58*I*e^(2*I*d*x + 2*I*c) + 11*I)*e^(1
/2*I*d*x + 1/2*I*c))*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^4, x)
```

3.263 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx$

Optimal. Leaf size=69

$$\frac{6i2^{5/6}a(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

```
[Out] (((6*I)/5)*2^(5/6)*a*Hypergeometric2F1[-5/6, 5/6, 11/6, (1 - I*Tan[e + f*x]
)/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

Rubi [A] time = 0.166016, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3505, 3523, 70, 69}

$$\frac{6i2^{5/6}a(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (((6*I)/5)*2^(5/6)*a*Hypergeometric2F1[-5/6, 5/6, 11/6, (1 - I*Tan[e + f*x]
)/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx &= \frac{(d\sec(e+fx))^{5/3}\int (a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{11/6}\,dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}\\
&= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\operatorname{Subst}\left(\int \frac{(a+iax)^{5/6}}{\sqrt[6]{a-iax}}\,dx,x,\tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}\\
&= \frac{\left(2^{5/6}a^2(d\sec(e+fx))^{5/3}\right)\operatorname{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{5/6}}{\sqrt[6]{a-iax}}\,dx,x,\tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}}\\
&= \frac{6i2^{5/6}a\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}
\end{aligned}$$

Mathematica [A] time = 1.2436, size = 104, normalized size = 1.51

$$\frac{3ade^{-2ifx}(d\sec(e+fx))^{2/3}(\cos(e+3fx)+i\sin(e+3fx))\left(i\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(e+fx)}\right)\right.}{10f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (3*a*d*(d*Sec[e + f*x])^(2/3)*(Cos[e + 3*f*x] + I*Sin[e + 3*f*x])*(-3*I + I
*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I
)*(e + f*x))] + 2*Tan[e + f*x]))/(10*E^((2*I)*f*x)*f)
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(a+ia\tan\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2^{\frac{2}{3}}\left(-15i\,ade^{(3i\,fx+3i\,e)}-3i\,ade^{(i\,fx+i\,e)}\right)\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}e^{\left(\frac{2}{3}i\,fx+\frac{2}{3}i\,e\right)}+10\left(fe^{(2i\,fx+2i\,e)}+f\right)\text{integral}\left(\frac{i\cdot 2^{\frac{2}{3}}ad\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}e^{\left(\frac{2}{3}i\,fx+\right.}}{2f}\right.}{10\left(fe^{(2i\,fx+2i\,e)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/10*(2^(2/3)*(-15*I*a*d*e^(3*I*f*x + 3*I*e) - 3*I*a*d*e^(I*f*x + I*e))*(d/
(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e) + 10*(f*e^(2*I*f*x
 + 2*I*e) + f)*integral(1/2*I*2^(2/3)*a*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/
3)*e^(2/3*I*f*x + 2/3*I*e)/f, x))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a), x)
```

3.264 $\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx$

Optimal. Leaf size=67

$$\frac{6i\sqrt[6]{2}a\sqrt[3]{d\sec(e+fx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[6]{1+i\tan(e+fx)}}$$

```
[Out] ((6*I)*2^(1/6)*a*Hypergeometric2F1[-1/6, 1/6, 7/6, (1 - I*Tan[e + f*x])/2]*
(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

Rubi [A] time = 0.14672, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3505, 3523, 70, 69}

$$\frac{6i\sqrt[6]{2}a\sqrt[3]{d\sec(e+fx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[6]{1+i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((6*I)*2^(1/6)*a*Hypergeometric2F1[-1/6, 1/6, 7/6, (1 - I*Tan[e + f*x])/2]*
(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))^{7/6}\,dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$
$$= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int \frac{\sqrt[6]{a+iax}}{(a-iax)^{5/6}}\,dx, x, \tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$
$$= \frac{\left(\sqrt[6]{2}a^2\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int \frac{\sqrt[6]{\frac{1}{2}+\frac{ix}{2}}}{(a-iax)^{5/6}}\,dx, x, \tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}}$$
$$= \frac{6i\sqrt[6]{2}a\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}$$

Mathematica [A] time = 0.555025, size = 92, normalized size = 1.37

$$\frac{3ade^{-ie}(\tan(e+fx)-i)(\cos(fx)-i\sin(fx))\left(-1+\sqrt[3]{1+e^{2i(e+fx)}}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)\right)}{f(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (3*a*d*(-1 + (1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/
6, -E^((2*I)*(e + f*x))])*(Cos[f*x] - I*Sin[f*x])*(-I + Tan[e + f*x]))/(E^(
I*e)*f*(d*Sec[e + f*x])^(2/3))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \sqrt[3]{d\sec(fx+e)}\left(a+ia\tan(fx+e)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec(fx+e)\right)^{\frac{1}{3}}\left(ia\tan(fx+e)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{3i\cdot 2^{\frac{1}{3}}a\left(\frac{d}{e^{(2ifx+2ie)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{1}{3}ifx+\frac{1}{3}ie\right)}+f\mathrm{integral}\left(-\frac{i\cdot 2^{\frac{1}{3}}a\left(\frac{d}{e^{(2ifx+2ie)}+1}\right)^{\frac{1}{3}}e^{\left(-\frac{2}{3}ifx-\frac{2}{3}ie\right)}}{f},x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] (3*I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e)
+ f*integral(-I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x
 - 2/3*I*e)/f, x))/f
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sqrt[3]{d\sec\left(e+fx\right)}\,dx+\int i\sqrt[3]{d\sec\left(e+fx\right)}\tan\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*(a+I*a*tan(f*x+e)),x)
```

```
[Out] a*(Integral((d*sec(e + f*x))**(1/3), x) + Integral(I*(d*sec(e + f*x))**(1/3
)*tan(e + f*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(ia\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a), x)
```

3.265 $\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=67

$$-\frac{3i2^{5/6}a\sqrt[6]{1+i\tan(e+fx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] ((-3*I)*2^(5/6)*a*Hypergeometric2F1[-1/6, 1/6, 5/6, (1 - I*Tan[e + f*x])/2]
*(1 + I*Tan[e + f*x])^(1/6))/(f*(d*Sec[e + f*x])^(1/3))
```

Rubi [A] time = 0.154474, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3505, 3523, 70, 69}

$$-\frac{3i2^{5/6}a\sqrt[6]{1+i\tan(e+fx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((-3*I)*2^(5/6)*a*Hypergeometric2F1[-1/6, 1/6, 5/6, (1 - I*Tan[e + f*x])/2]
*(1 + I*Tan[e + f*x])^(1/6))/(f*(d*Sec[e + f*x])^(1/3))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\int \frac{(a+ia\tan(e+fx))^{5/6}}{\sqrt[6]{a-ia\tan(e+fx)}}\,dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{(a-iax)^{7/6}\sqrt[6]{a+iax}}\,dx, x, \tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{\frac{1}{2}+\frac{ix}{2}}(a-iax)^{7/6}}\,dx, x, \tan(e+fx)\right)}{\sqrt[6]{2}f\sqrt[3]{d\sec(e+fx)}}$$

$$= -\frac{3i2^{5/6}a\,{}_2F_1\left(-\frac{1}{6},\frac{1}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A] time = 0.440552, size = 98, normalized size = 1.46

$$-\frac{3i2^{2/3}ae^{2i(e+fx)}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(e+fx)}\right)}{5f\sqrt[3]{1+e^{2i(e+fx)}}\sqrt[3]{\frac{de^{i(e+fx)}}{1+e^{2i(e+fx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (((-3*I)/5)*2^(2/3)*a*E^((2*I)*(e + f*x))*Hypergeometric2F1[2/3, 5/6, 11/6,
 -E^((2*I)*(e + f*x))])/(((d*E^(I*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(1
/3)*(1 + E^((2*I)*(e + f*x)))^(1/3)*f)
```

Maple [F] time = 0.143, size = 0, normalized size = 0.

$$\int (a+ia\tan(fx+e))\frac{1}{\sqrt[3]{d\sec(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,a\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2^{\frac{2}{3}}\left(-3i\,ae^{(2i\,fx+2i\,e)}-3i\,a\right)\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}e^{\left(\frac{2}{3}i\,fx+\frac{2}{3}i\,e\right)}+\left(dfe^{(i\,fx+i\,e)}-df\right)\mathrm{integral}\left(\frac{2^{\frac{2}{3}}\left(-2i\,ae^{(2i\,fx+2i\,e)}-2i\,ae^{(i\,fx+i\,e)}-2i\,a\right)\left(\frac{d}{e^{(2i\,fx+2i\,e)}\cdots}\right.}{dfe^{(3i\,fx+3i\,e)}-2\,dfe^{(2i\,fx+2i\,e)}+dfe^{(i\,f}\cdots}\right.}{dfe^{(i\,fx+i\,e)}-df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] (2^(2/3)*(-3*I*a*e^(2*I*f*x + 2*I*e) - 3*I*a)*(d/(e^(2*I*f*x + 2*I*e) + 1))
^(2/3)*e^(2/3*I*f*x + 2/3*I*e) + (d*f*e^(I*f*x + I*e) - d*f)*integral(2^(2/
3)*(-2*I*a*e^(2*I*f*x + 2*I*e) - 2*I*a*e^(I*f*x + I*e) - 2*I*a)*(d/(e^(2*I*
f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e)/(d*f*e^(3*I*f*x + 3*I*e) -
 2*d*f*e^(2*I*f*x + 2*I*e) + d*f*e^(I*f*x + I*e)), x))/(d*f*e^(I*f*x + I*e)
 - d*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\frac{1}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx+\int\frac{i\tan\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))**(1/3),x)
```

```
[Out] a*(Integral((d*sec(e + f*x))**(-1/3), x) + Integral(I*tan(e + f*x)/(d*sec(e
 + f*x))**(1/3), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{i\,a\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

3.266 $\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=69

$$-\frac{3i\sqrt[6]{2}a(1+i\tan(e+fx))^{5/6}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(d\sec(e+fx))^{5/3}}$$

```
[Out] (((-3*I)/5)*2^(1/6)*a*Hypergeometric2F1[-5/6, 5/6, 1/6, (1 - I*Tan[e + f*x]
)/2]*(1 + I*Tan[e + f*x])^(5/6))/(f*(d*Sec[e + f*x])^(5/3))
```

Rubi [A] time = 0.152235, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3505, 3523, 70, 69}

$$-\frac{3i\sqrt[6]{2}a(1+i\tan(e+fx))^{5/6}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (((-3*I)/5)*2^(1/6)*a*Hypergeometric2F1[-5/6, 5/6, 1/6, (1 - I*Tan[e + f*x]
)/2]*(1 + I*Tan[e + f*x])^(5/6))/(f*(d*Sec[e + f*x])^(5/3))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \frac{\left((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\int \frac{\sqrt[6]{a+ia\tan(e+fx)}}{(a-ia\tan(e+fx))^{5/6}}\,dx}{(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\text{Subst}\left(\int \frac{1}{(a-iax)^{11/6}(a+iax)^{5/6}}\,dx, x, \tan(e+fx)\right)}{f(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{5/6}(a-iax)^{11/6}}\,dx, x, \tan(e+fx)\right)}{2^{5/6}f(d\sec(e+fx))^{5/3}}$$

$$= -\frac{3i\sqrt[6]{2}a\,{}_2F_1\left(-\frac{5}{6},\frac{5}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{5f(d\sec(e+fx))^{5/3}}$$

Mathematica [A] time = 0.443457, size = 106, normalized size = 1.54

$$-\frac{3iae^{i(e+fx)}\left(4\sqrt[3]{1+e^{2i(e+fx)}}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)+e^{2i(e+fx)}+1\right)}{5df\left(1+e^{2i(e+fx)}\right)(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (((-3*I)/5)*a*E^(I*(e + f*x))*(1 + E^((2*I)*(e + f*x)) + 4*(1 + E^((2*I)*(e
 + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))]))/(d
*(1 + E^((2*I)*(e + f*x)))*f*(d*Sec[e + f*x])^(2/3))
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int (a+ia\tan(fx+e))\left(d\sec(fx+e)\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

```
[Out] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,a\tan(fx+e)+a}{\left(d\sec(fx+e)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{10\, d^2 f \text{integral}\left(-\frac{2i \cdot 2^{\frac{1}{3}} a \left(\frac{d}{e^{(2i\, fx+2i\, e)}+1}\right)^{\frac{1}{3}} e^{\left(-\frac{2}{3}i\, fx-\frac{2}{3}i\, e\right)}}{5\, d^2 f}, x\right) + 2^{\frac{1}{3}}\left(-3i\, a e^{(2i\, fx+2i\, e)} - 3i\, a\right)\left(\frac{d}{e^{(2i\, fx+2i\, e)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{1}{3}i\, fx+\frac{1}{3}i\, e\right)}}{10\, d^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] 1/10*(10*d^2*f*integral(-2/5*I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3
)*e^(-2/3*I*f*x - 2/3*I*e)/(d^2*f), x) + 2^(1/3)*(-3*I*a*e^(2*I*f*x + 2*I*e
) - 3*I*a)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e))/(d^
2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}}\, dx + \int \frac{i\tan\left(e+fx\right)}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))**(5/3),x)
```

```
[Out] a*(Integral((d*sec(e + f*x))**(-5/3), x) + Integral(I*tan(e + f*x)/(d*sec(e
 + f*x))**(5/3), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\, a\tan\left(fx+e\right) + a}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

3.267 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx$

Optimal. Leaf size=71

$$\frac{12i2^{5/6}a^2(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

```
[Out] (((12*I)/5)*2^(5/6)*a^2*Hypergeometric2F1[-11/6, 5/6, 11/6, (1 - I*Tan[e +
f*x])/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

Rubi [A] time = 0.176646, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{12i2^{5/6}a^2(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((12*I)/5)*2^(5/6)*a^2*Hypergeometric2F1[-11/6, 5/6, 11/6, (1 - I*Tan[e +
f*x])/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx &= \frac{(d\sec(e+fx))^{5/3}\int (a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{17/6}\,dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}\\ &= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{(a+iax)^{11/6}}{\sqrt[6]{a-iax}}\,dx,x,\tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}\\ &= \frac{\left(2\,2^{5/6}a^3(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{11/6}}{\sqrt[6]{a-iax}}\,dx,x,\tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}}\\ &= \frac{12i2^{5/6}a^2\,{}_2F_1\left(-\frac{11}{6},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}\end{aligned}$$

Mathematica [B] time = 2.71808, size = 267, normalized size = 3.76

$$\frac{(a+ia\tan(e+fx))^2(d\sec(e+fx))^{5/3}\left(\frac{3}{4}\csc(e)(\cos(2e)-i\sin(2e))\sec^{\frac{8}{3}}(e+fx)(64i\sin(2e+fx)+75\cos(2e+fx)+\right.}{80f\sec^{\frac{11}{3}}(e+fx)(\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^(5/3)*(((-33*I)*2^(2/3)*(5*(1 + E^((2*I)*(e + f*x)))^(1/3
) - E^((2*I)*f*x)*(-1 + E^((2*I)*e))*Hypergeometric2F1[2/3, 5/6, 11/6, -E^(
(2*I)*(e + f*x))]))/((-1 + E^((2*I)*e))*(E^(I*(e + f*x))/(1 + E^((2*I)*(e +
 f*x))))^(1/3)*(1 + E^((2*I)*(e + f*x)))^(1/3)) + (3*Csc[e]*Sec[e + f*x]^(8
/3)*(Cos[2*e] - I*Sin[2*e])*(90*Cos[f*x] + 75*Cos[2*e + f*x] + 55*Cos[2*e +
 3*f*x] - (64*I)*Sin[f*x] + (64*I)*Sin[2*e + f*x]))/4)*(a + I*a*Tan[e + f*x
])^2)/(80*f*Sec[e + f*x]^(11/3)*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(a+ia\tan\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2^{\frac{2}{3}}\left(-165i\,a^2de^{\left(5i\,fx+5i\,e\right)}-78i\,a^2de^{\left(3i\,fx+3i\,e\right)}-33i\,a^2de^{\left(i\,fx+i\,e\right)}\right)\left(\frac{d}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{2}{3}}e^{\left(\frac{2}{3}i\,fx+\frac{2}{3}i\,e\right)}+80\left(fe^{\left(4i\,fx+4i\,e\right)}+2\,fe^{\left(2i\,fx+2i\,e\right)}\right.}{80\left(fe^{\left(4i\,fx+4i\,e\right)}+2\,fe^{\left(2i\,fx+2i\,e\right)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/80*(2^(2/3)*(-165*I*a^2*d*e^(5*I*f*x + 5*I*e) - 78*I*a^2*d*e^(3*I*f*x + 3
*I*e) - 33*I*a^2*d*e^(I*f*x + I*e))*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(
2/3*I*f*x + 2/3*I*e) + 80*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e)
+ f)*integral(11/16*I*2^(2/3)*a^2*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(
2/3*I*f*x + 2/3*I*e)/f, x))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e
) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2, x)
```

3.268 $\int \sqrt[3]{d \sec(e+fx)}(a+ia\tan(e+fx))^2\,dx$

Optimal. Leaf size=69

$$\frac{12i\sqrt[6]{2}a^2\sqrt[3]{d\sec(e+fx)}\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[6]{1+i\tan(e+fx)}}$$

```
[Out] ((12*I)*2^(1/6)*a^2*Hypergeometric2F1[-7/6, 1/6, 7/6, (1 - I*Tan[e + f*x])/
2]*(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

Rubi [A] time = 0.157643, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{12i\sqrt[6]{2}a^2\sqrt[3]{d\sec(e+fx)}\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[6]{1+i\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((12*I)*2^(1/6)*a^2*Hypergeometric2F1[-7/6, 1/6, 7/6, (1 - I*Tan[e + f*x])/
2]*(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$
\begin{aligned}
\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx &= \frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))^{13/6}\,dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}\\
&= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\mathrm{Subst}\left(\int \frac{(a+iax)^{7/6}}{(a-iax)^{5/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}\\
&= \frac{\left(2\sqrt[6]{2}a^3\sqrt[3]{d\sec(e+fx)}\right)\mathrm{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{7/6}}{(a-iax)^{5/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}}\\
&= \frac{12i\sqrt[6]{2}a^2\,{}_2F_1\left(-\frac{7}{6},\frac{1}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}
\end{aligned}
$$

Mathematica [A] time = 0.965874, size = 128, normalized size = 1.86

$$
-\frac{3a^2e^{-2ie}\sqrt[3]{d\sec(e+fx)}(\cos(2(e+fx))+i\sin(2(e+fx)))\left(7i\sqrt[3]{1+e^{2i(e+fx)}}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)+\text{s}\right.}{4f(\cos(fx)+i\sin(fx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (-3*a^2*(d*Sec[e + f*x])^(1/3)*(Cos[2*(e + f*x)] + I*Sin[2*(e + f*x)])*(-8*
I + (7*I)*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6,
-E^((2*I)*(e + f*x))] + Sec[e]*Sec[e + f*x]*Sin[f*x] + Tan[e]))/(4*E^((2*I)
*e)*f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [F] time = 0.132, size = 0, normalized size = 0.

$$
\int \sqrt[3]{d\sec\left(fx+e\right)}\left(a+ia\tan\left(fx+e\right)\right)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2^{\frac{1}{3}}\left(27i\,a^2e^{\left(2i\,fx+2i\,e\right)}+21i\,a^2\right)\left(\frac{d}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{1}{3}i\,fx+\frac{1}{3}i\,e\right)}+4\left(fe^{\left(2i\,fx+2i\,e\right)}+f\right)\mathrm{integral}\left(-\frac{7i\cdot 2^{\frac{1}{3}}a^2\left(\frac{d}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{1}{3}}e^{\left(-\frac{2}{3}i\,fx-\frac{2}{3}i\,e\right)}}{4\,f}\right.}{4\left(fe^{\left(2i\,fx+2i\,e\right)}+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2^(1/3)*(27*I*a^2*e^(2*I*f*x + 2*I*e) + 21*I*a^2)*(d/(e^(2*I*f*x + 2*I
*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e) + 4*(f*e^(2*I*f*x + 2*I*e) + f)*int
egral(-7/4*I*2^(1/3)*a^2*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x
- 2/3*I*e)/f, x))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \sqrt[3]{d\sec\left(e+fx\right)}\,dx+\int -\sqrt[3]{d\sec\left(e+fx\right)}\tan^2\left(e+fx\right)dx+\int 2i\sqrt[3]{d\sec\left(e+fx\right)}\tan\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] a**2*(Integral((d*sec(e + f*x))**(1/3), x) + Integral(-(d*sec(e + f*x))**(1
/3)*tan(e + f*x)**2, x) + Integral(2*I*(d*sec(e + f*x))**(1/3)*tan(e + f*x)
, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2, x)
```

3.269 $\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=83

$$-\frac{6i2^{5/6}\left(a^2+ia^2\tan(e+fx)\right)\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] ((-6*I)*2^(5/6)*Hypergeometric2F1[-5/6, -1/6, 5/6, (1 - I*Tan[e + f*x])/2]*
(a^2 + I*a^2*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^
(5/6))
```

Rubi [A] time = 0.165748, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$-\frac{6i2^{5/6}\left(a^2+ia^2\tan(e+fx)\right)\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((-6*I)*2^(5/6)*Hypergeometric2F1[-5/6, -1/6, 5/6, (1 - I*Tan[e + f*x])/2]*
(a^2 + I*a^2*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^
(5/6))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\int \frac{(a+ia\tan(e+fx))^{11/6}}{\sqrt[6]{a-ia\tan(e+fx)}}\,dx}{\sqrt[3]{d\sec(e+fx)}}$$
$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\text{Subst}\left(\int \frac{(a+iax)^{5/6}}{(a-iax)^{7/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}}$$
$$= \frac{\left(2^{5/6}a^2\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{5/6}}{(a-iax)^{7/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}}$$
$$= -\frac{6i2^{5/6}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}$$

Mathematica [A] time = 1.11866, size = 132, normalized size = 1.59

$$-\frac{3ia^2e^{2i(e+fx)}\left(\left(1+e^{2i(e+fx)}\right)\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(e+fx)}\right)-\sqrt[3]{1+e^{2i(e+fx)}}\right)}{\sqrt[3]{2}f\left(1+e^{2i(e+fx)}\right)^{4/3}\sqrt[3]{\frac{de^{i(e+fx)}}{1+e^{2i(e+fx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((-3*I)*a^2*E^((2*I)*(e + f*x))*(-(1 + E^((2*I)*(e + f*x)))^(1/3) + (1 + E^
((2*I)*(e + f*x)))*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))])
)/(2^(1/3)*((d*E^(I*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(1/3)*(1 + E^((2
*I)*(e + f*x)))^(4/3)*f)
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int \left(a+ia\tan\left(fx+e\right)\right)^2\frac{1}{\sqrt[3]{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(i\,a\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2^{\frac{2}{3}}\left(-12i\,a^2e^{\left(2i\,fx+2i\,e\right)}-3i\,a^2e^{\left(i\,fx+i\,e\right)}-15i\,a^2\right)\left(\frac{d}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{2}{3}}e^{\left(\frac{2}{3}i\,fx+\frac{2}{3}i\,e\right)}+2\left(dfe^{\left(i\,fx+i\,e\right)}-df\right)\text{integral}\left(\frac{2^{\frac{2}{3}}\left(-5i\,a^2e^{\left(2i\,fx+2i\,e\right)}-\dots\right.}{dfe^{\left(3i\,f\dots\right.}}\right.}{2\left(dfe^{\left(i\,fx+i\,e\right)}-df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2^(2/3)*(-12*I*a^2*e^(2*I*f*x + 2*I*e) - 3*I*a^2*e^(I*f*x + I*e) - 15*
I*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e) + 2*(d*f
*e^(I*f*x + I*e) - d*f)*integral(2^(2/3)*(-5*I*a^2*e^(2*I*f*x + 2*I*e) - 5*
I*a^2*e^(I*f*x + I*e) - 5*I*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3
*I*f*x + 2/3*I*e)/(d*f*e^(3*I*f*x + 3*I*e) - 2*d*f*e^(2*I*f*x + 2*I*e) + d*
f*e^(I*f*x + I*e)), x))/(d*f*e^(I*f*x + I*e) - d*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{1}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx+\int -\frac{\tan^2\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx+\int \frac{2i\tan\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(d*sec(f*x+e))**(1/3),x)
```

```
[Out] a**2*(Integral((d*sec(e + f*x))**(-1/3), x) + Integral(-tan(e + f*x)**2/(d*
sec(e + f*x))**(1/3), x) + Integral(2*I*tan(e + f*x)/(d*sec(e + f*x))**(1/3
), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(i\,a\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

3.270 $\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=85

$$-\frac{6i\sqrt[6]{2}\left(a^2+ia^2\tan(e+fx)\right)\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}}$$

```
[Out] (((-6*I)/5)*2^(1/6)*Hypergeometric2F1[-5/6, -1/6, 1/6, (1 - I*Tan[e + f*x])
/2]*(a^2 + I*a^2*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(5/3)*(1 + I*Tan[e + f*
x])^(1/6))
```

Rubi [A] time = 0.180333, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$-\frac{6i\sqrt[6]{2}\left(a^2+ia^2\tan(e+fx)\right)\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```


```
[Out] (((-6*I)/5)*2^(1/6)*Hypergeometric2F1[-5/6, -1/6, 1/6, (1 - I*Tan[e + f*x])
/2]*(a^2 + I*a^2*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(5/3)*(1 + I*Tan[e + f*
x])^(1/6))
```


Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \frac{\left((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\int \frac{(a+ia\tan(e+fx))^{7/6}}{(a-ia\tan(e+fx))^{5/6}}\,dx}{(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a+iax}}{(a-iax)^{11/6}}\,dx,x,\tan(e+fx)\right)}{f(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(\sqrt[6]{2}a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))\right)\text{Subst}\left(\int \frac{\sqrt[6]{\frac{1}{2}+\frac{ix}{2}}}{(a-iax)^{11/6}}\,dx,x,\tan(e+fx)\right)}{f(d\sec(e+fx))^{5/3}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}}$$

$$= -\frac{6i\sqrt[6]{2}\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{5f(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}$$

Mathematica [A] time = 0.635907, size = 105, normalized size = 1.24

$$-\frac{12ia^2e^{2i(e+fx)}\left(-\sqrt[3]{1+e^{2i(e+fx)}}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)+e^{2i(e+fx)}+1\right)}{5f\left(1+e^{2i(e+fx)}\right)^2(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (((-12*I)/5)*a^2*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)) - (1 + E^((2*
I)*(e + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))]
))/((1 + E^((2*I)*(e + f*x)))^2*f*(d*Sec[e + f*x])^(5/3))
```

Maple [F] time = 0.159, size = 0, normalized size = 0.

$$\int \left(a+ia\tan\left(fx+e\right)\right)^2\left(d\sec\left(fx+e\right)\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(i\,a\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{5\,d^2 f \text{integral}\left(\frac{i\cdot 2^{\frac{1}{3}} a^2 \left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{1}{3}} e^{\left(-\frac{2}{3}i\,fx-\frac{2}{3}i\,e\right)}}{5\,d^2 f}, x\right) + 2^{\frac{1}{3}}\left(-3i\,a^2 e^{(2i\,fx+2i\,e)} - 3i\,a^2\right)\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{1}{3}} e^{\left(\frac{1}{3}i\,fx+\frac{1}{3}i\,e\right)}}{5\,d^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] 1/5*(5*d^2*f*integral(1/5*I*2^(1/3)*a^2*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)
*e^(-2/3*I*f*x - 2/3*I*e)/(d^2*f), x) + 2^(1/3)*(-3*I*a^2*e^(2*I*f*x + 2*I*
e) - 3*I*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e))/
(d^2*f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}}\,dx + \int -\frac{\tan^2\left(e+fx\right)}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}}\,dx + \int \frac{2i\tan\left(e+fx\right)}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))**2/(d*sec(f*x+e))**(5/3),x)
```

```
[Out] a**2*(Integral((d*sec(e + f*x))**(-5/3), x) + Integral(-tan(e + f*x)**2/(d*
sec(e + f*x))**(5/3), x) + Integral(2*I*tan(e + f*x)/(d*sec(e + f*x))**(5/3
), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(i\,a\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

3.271 $\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=83

$$\frac{3i\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{7}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5\sqrt[6]{2}f(a+ia\tan(e+fx))}$$

```
[Out] (((3*I)/5)*Hypergeometric2F1[5/6, 7/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*Sec
[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*f*(a + I*a*Tan[e + f*
x]))
```

Rubi [A] time = 0.175058, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{3i\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{7}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5\sqrt[6]{2}f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (((3*I)/5)*Hypergeometric2F1[5/6, 7/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*Sec
[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*f*(a + I*a*Tan[e + f*
x]))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}dx = \frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{\sqrt[6]{a+ia\tan(e+fx)}}dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$
$$= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[6]{a-iax}(a+iax)^{7/6}}dx, x, \tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$
$$= \frac{\left(a(d\sec(e+fx))^{5/3}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\mathrm{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{7/6}\sqrt[6]{a-iax}}dx, x, \tan(e+fx)\right)}{2\sqrt[6]{2}f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}$$
$$= \frac{3i\,{}_2F_1\left(\frac{5}{6},\frac{7}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}{5\sqrt[6]{2}f(a+ia\tan(e+fx))}$$

Mathematica [A] time = 0.460657, size = 84, normalized size = 1.01

$$\frac{6de^{i(e+fx)}\mathrm{Hypergeometric2F1}\left(-\frac{1}{6},\frac{2}{3},\frac{5}{6},-e^{2i(e+fx)}\right)(d\sec(e+fx))^{2/3}}{af\sqrt[3]{1+e^{2i(e+fx)}}(\tan(e+fx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (6*d*E^(I*(e + f*x))*Hypergeometric2F1[-1/6, 2/3, 5/6, -E^((2*I)*(e + f*x))
]*(d*Sec[e + f*x])^(2/3))/(a*(1 + E^((2*I)*(e + f*x)))^(1/3)*f*(-I + Tan[e
+ f*x]))
```

Maple [F] time = 0.136, size = 0, normalized size = 0.

$$\int \frac{1}{a+ia\tan(fx+e)}\left(d\sec(fx+e)\right)^{\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(afe^{(ifx+ie)}\text{integral}\left(-\frac{i\cdot 2^{\frac{2}{3}}d\left(\frac{d}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}e^{\left(\frac{2}{3}ifx+\frac{2}{3}ie\right)}}{af},x\right)+2^{\frac{2}{3}}\left(3ide^{(2ifx+2ie)}+3id\right)\left(\frac{d}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}e^{\left(\frac{2}{3}ifx+\frac{2}{3}ie\right)}\right)e^{(-ifx-ie)}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] (a*f*e^(I*f*x + I*e)*integral(-I*2^(2/3)*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2
/3)*e^(2/3*I*f*x + 2/3*I*e)/(a*f), x) + 2^(2/3)*(3*I*d*e^(2*I*f*x + 2*I*e)
+ 3*I*d)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e))*e^(-I
*f*x - I*e)/(a*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}{ia\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)/(I*a*tan(f*x + e) + a), x)
```

3.272 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx$

Optimal. Leaf size=81

$$\frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{11}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2^{5/6}f(a+ia\tan(e+fx))}$$

```
[Out] ((3*I)*Hypergeometric2F1[1/6, 11/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Sec[e +
 f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*f*(a + I*a*Tan[e + f*x]))
```

Rubi [A] time = 0.169389, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{11}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2^{5/6}f(a+ia\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((3*I)*Hypergeometric2F1[1/6, 11/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Sec[e +
 f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*f*(a + I*a*Tan[e + f*x]))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/6}}\,dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a-iax)^{5/6}(a+iax)^{11/6}}\,dx, x, \tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(a\sqrt[3]{d\sec(e+fx)}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{11/6}(a-iax)^{5/6}}\,dx, x, \tan(e+fx)\right)}{2\,2^{5/6}f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}$$

$$= \frac{3i\,{}_2F_1\left(\frac{1}{6},\frac{11}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}{2^{5/6}f(a+ia\tan(e+fx))}$$

Mathematica [A] time = 0.42943, size = 103, normalized size = 1.27

$$-\frac{3ie^{-2i(e+fx)}\left(4e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)-e^{2i(e+fx)}-1\right)\sqrt[3]{d\sec(e+fx)}}{10af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (((-3*I)/10)*(-1 - E^((2*I)*(e + f*x)) + 4*E^((2*I)*(e + f*x))*(1 + E^((2*I
)*(e + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))])
*(d*Sec[e + f*x])^(1/3))/(a*E^((2*I)*(e + f*x))*f)
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int \frac{1}{a+ia\tan\left(fx+e\right)}\sqrt[3]{d\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(10\,afe^{\left(2i\,fx+2i\,e\right)}\text{integral}\left(-\frac{2i\cdot 2^{\frac{1}{3}}\left(\frac{d}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{1}{3}}e^{\left(-\frac{2}{3}i\,fx-\frac{2}{3}i\,e\right)}}{5\,af},x\right)+2^{\frac{1}{3}}\left(\frac{d}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{1}{3}}\left(3i\,e^{\left(2i\,fx+2i\,e\right)}+3i\right)e^{\left(\frac{1}{3}i\,fx+\frac{1}{3}i\,e\right)}\right)e^{\left(-2i\,fx-\right.}}{10\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/10*(10*a*f*e^(2*I*f*x + 2*I*e)*integral(-2/5*I*2^(1/3)*(d/(e^(2*I*f*x + 2
*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a*f), x) + 2^(1/3)*(d/(e^(2*I*f
*x + 2*I*e) + 1))^(1/3)*(3*I*e^(2*I*f*x + 2*I*e) + 3*I)*e^(1/3*I*f*x + 1/3*
I*e))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}{i\,a\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)/(I*a*tan(f*x + e) + a), x)
```

3.273 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx$

Optimal. Leaf size=71

$$-\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{13}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] (((-3*I)/2)*Hypergeometric2F1[-1/6, 13/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1 +
 I*Tan[e + f*x])^(1/6))/(2^(1/6)*a*f*(d*Sec[e + f*x])^(1/3))
```

Rubi [A] time = 0.183121, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$-\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{13}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (((-3*I)/2)*Hypergeometric2F1[-1/6, 13/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1 +
 I*Tan[e + f*x])^(1/6))/(2^(1/6)*a*f*(d*Sec[e + f*x])^(1/3))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rubi steps

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}dx = \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))^{7/6}}$$
$$\sqrt[3]{d\sec(e+fx)}$$
$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a-iax)^{7/6}(a+iax)^{13/6}}d\right.}{f\sqrt[3]{d\sec(e+fx)}}$$
$$= \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{13/6}(a-iax)^{7/6}}dx,x,\tan\right.}{4\sqrt[6]{2}f\sqrt[3]{d\sec(e+fx)}}$$
$$= -\frac{3i\,{}_2F_1\left(-\frac{1}{6},\frac{13}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{2\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A] time = 0.90771, size = 112, normalized size = 1.58

$$\frac{3(\tan(e+fx)+i)\left(5(4i\sin(2(e+fx))+5\cos(2(e+fx))+5)-8e^{2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\right.\right.}{70af\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (3*(-8*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2F
1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))] + 5*(5 + 5*Cos[2*(e + f*x)] + (4*I)
*Sin[2*(e + f*x)]))*(I + Tan[e + f*x]))/(70*a*f*(d*Sec[e + f*x])^(1/3))
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \frac{1}{a+ia\tan(fx+e)}\frac{1}{\sqrt[3]{d\sec(fx+e)}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2^{\frac{2}{3}}\left(\frac{d}{e^{(2i fx+2i e)}+1}\right)^{\frac{2}{3}}\left(-21i e^{(5i fx+5i e)}-27i e^{(4i fx+4i e)}-18i e^{(3i fx+3i e)}-30i e^{(2i fx+2i e)}+3i e^{(i fx+i e)}-3i\right)e^{\left(\frac{2}{3}i fx+\frac{2}{3}i e\right)}+28\left(ad\right.}{28\left(adfe^{(4i fx+4i e)}-adfe^{(3i fx+3i\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/28*(2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(-21*I*e^(5*I*f*x + 5*I*e
) - 27*I*e^(4*I*f*x + 4*I*e) - 18*I*e^(3*I*f*x + 3*I*e) - 30*I*e^(2*I*f*x +
 2*I*e) + 3*I*e^(I*f*x + I*e) - 3*I)*e^(2/3*I*f*x + 2/3*I*e) + 28*(a*d*f*e^
(4*I*f*x + 4*I*e) - a*d*f*e^(3*I*f*x + 3*I*e))*integral(1/7*2^(2/3)*(d/(e^(
2*I*f*x + 2*I*e) + 1))^(2/3)*(-8*I*e^(2*I*f*x + 2*I*e) - 8*I*e^(I*f*x + I*e
) - 8*I)*e^(2/3*I*f*x + 2/3*I*e)/(a*d*f*e^(3*I*f*x + 3*I*e) - 2*a*d*f*e^(2*
I*f*x + 2*I*e) + a*d*f*e^(I*f*x + I*e)), x))/(a*d*f*e^(4*I*f*x + 4*I*e) - a
*d*f*e^(3*I*f*x + 3*I*e))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(i a\tan\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)), x)
```

3.274 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx$

Optimal. Leaf size=71

$$-\frac{3i(1+i\tan(e+fx))^{5/6}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{17}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{10\,2^{5/6}af(d\sec(e+fx))^{5/3}}$$

```
[Out] (((-3*I)/10)*Hypergeometric2F1[-5/6, 17/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(5/6))/(2^(5/6)*a*f*(d*Sec[e + f*x])^(5/3))
```

Rubi [A] time = 0.20907, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$-\frac{3i(1+i\tan(e+fx))^{5/6}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{17}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{10\,2^{5/6}af(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (((-3*I)/10)*Hypergeometric2F1[-5/6, 17/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(5/6))/(2^(5/6)*a*f*(d*Sec[e + f*x])^(5/3))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = \frac{\left((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+f}}{(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\text{Subst}\left(\int \frac{1}{(a-iax)^{11/6}(a+iax)^{17}}\right.}{f(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left((a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{17/6}(a-iax)^{11/6}}\,dx,x\right.}{4\,2^{5/6}f(d\sec(e+fx))^{5/3}}$$

$$= -\frac{3i\,{}_2F_1\left(-\frac{5}{6},\frac{17}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{10\,2^{5/6}af(d\sec(e+fx))^{5/3}}$$

Mathematica [A] time = 0.861761, size = 119, normalized size = 1.68

$$-\frac{3\sec^2(e+fx)\left(\frac{128e^{2i(e+fx)}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^{2/3}}+16i\sin(2(e+fx))+6\cos(2(e+fx))-26\right)}{220af(\tan(e+fx)-i)(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (-3*Sec[e + f*x]^2*(-26 + 6*Cos[2*(e + f*x)] + (128*E^((2*I)*(e + f*x))*Hyp
ergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))])/(1 + E^((2*I)*(e + f*x
)))^(2/3) + (16*I)*Sin[2*(e + f*x)]))/(220*a*f*(d*Sec[e + f*x])^(5/3)*(-I +
 Tan[e + f*x]))
```

Maple [F] time = 0.133, size = 0, normalized size = 0.

$$\int \frac{1}{a+ia\tan\left(fx+e\right)}\left(d\sec\left(fx+e\right)\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(440\,ad^2 f e^{(4i\,fx+4i\,e)}\text{integral}\left(-\frac{16i\cdot 2^{\frac{1}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{1}{3}} e^{\left(-\frac{2}{3}i\,fx-\frac{2}{3}i\,e\right)}}{55\,ad^2 f}, x\right) + 2^{\frac{1}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{1}{3}}\left(-33i\,e^{(6i\,fx+6i\,e)} + 45i\,e^{(4i\,fx+4i\,e)} \cdots\right.\right.}{440\,ad^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/440*(440*a*d^2*f*e^(4*I*f*x + 4*I*e)*integral(-16/55*I*2^(1/3)*(d/(e^(2*I
*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a*d^2*f), x) + 2^(1/3)*
(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(-33*I*e^(6*I*f*x + 6*I*e) + 45*I*e^(4*
I*f*x + 4*I*e) + 93*I*e^(2*I*f*x + 2*I*e) + 15*I)*e^(1/3*I*f*x + 1/3*I*e))*
e^(-4*I*f*x - 4*I*e)/(a*d^2*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}(i\,a\tan(fx+e)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)), x)
```

3.275 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=87

$$\frac{3i\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{13}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{10\sqrt[6]{2}f\left(a^2+ia^2\tan(e+fx)\right)}$$

```
[Out] (((3*I)/10)*Hypergeometric2F1[5/6, 13/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*S
ec[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*f*(a^2 + I*a^2*Tan[
e + f*x]))
```

Rubi [A] time = 0.185338, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{3i\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{13}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{10\sqrt[6]{2}f\left(a^2+ia^2\tan(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((3*I)/10)*Hypergeometric2F1[5/6, 13/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*S
ec[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*f*(a^2 + I*a^2*Tan[
e + f*x]))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{(a+ia\tan(e+fx))^{7/6}}\,dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$
$$= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{a-iax}(a+iax)^{13/6}}\,dx,x,\tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$
$$= \frac{\left((d\sec(e+fx))^{5/3}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{13/6}\sqrt[6]{a-iax}}\,dx,x,\tan(e+fx)\right)}{4\sqrt[6]{2}f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}$$
$$= \frac{3i\,{}_2F_1\left(\frac{5}{6},\frac{13}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}{10\sqrt[6]{2}f\left(a^2+ia^2\tan(e+fx)\right)}$$

Mathematica [A] time = 0.704386, size = 128, normalized size = 1.47

$$-\frac{3e^{-i(4e+5fx)}\left(1+e^{2i(e+fx)}\right)(\sin(fx)-i\cos(fx))\left(2e^{2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{2}{3},\frac{5}{6},-e^{2i(e+fx)}\right)\right.}{28a^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (-3*(1 + E^((2*I)*(e + f*x)))*(1 + E^((2*I)*(e + f*x)) + 2*E^((2*I)*(e + f*
x))*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -E^((
2*I)*(e + f*x))])*(d*Sec[e + f*x])^(5/3)*((-I)*Cos[f*x] + Sin[f*x]))/(28*a^
2*E^(I*(4*e + 5*f*x))*f)
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+ia\tan\left(fx+e\right)\right)^2}\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(14\,a^{2}fe^{\left(3i\,fx+3i\,e\right)}\text{integral}\left(-\frac{i\cdot 2^{\frac{2}{3}}d\left(\frac{d}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{2}{3}}e^{\left(\frac{2}{3}i\,fx+\frac{2}{3}i\,e\right)}}{7\,a^{2}f},x\right)+2^{\frac{2}{3}}\left(6i\,de^{\left(4i\,fx+4i\,e\right)}+9i\,de^{\left(2i\,fx+2i\,e\right)}+3i\,d\right)\left(\frac{d}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{2}{3}}e^{\left(\frac{2}{3}\right.}\cdots}{14\,a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/14*(14*a^2*f*e^(3*I*f*x + 3*I*e)*integral(-1/7*I*2^(2/3)*d*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e)/(a^2*f), x) + 2^(2/3)*(6*I*d*
e^(4*I*f*x + 4*I*e) + 9*I*d*e^(2*I*f*x + 2*I*e) + 3*I*d)*(d/(e^(2*I*f*x + 2
*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e))*e^(-3*I*f*x - 3*I*e)/(a^2*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}{\left(i\,a\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)/(I*a*tan(f*x + e) + a)^2, x)
```

3.276 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=87

$$\frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{17}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2\,2^{5/6}f\left(a^2+ia^2\tan(e+fx)\right)}$$

```
[Out] (((3*I)/2)*Hypergeometric2F1[1/6, 17/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Sec
[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*f*(a^2 + I*a^2*Tan[e
+ f*x]))
```

Rubi [A] time = 0.170317, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{17}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2\,2^{5/6}f\left(a^2+ia^2\tan(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((3*I)/2)*Hypergeometric2F1[1/6, 17/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Sec
[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*f*(a^2 + I*a^2*Tan[e
+ f*x]))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx &= \frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(a+ia\tan(e+fx))^{11/6}}\,dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}\\
&= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{(a-iax)^{5/6}(a+iax)^{17/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}\\
&= \frac{\left(\sqrt[3]{d\sec(e+fx)}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{17/6}(a-iax)^{5/6}}\,dx,x,\tan(e+fx)\right)}{4\,2^{5/6}f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}\\
&= \frac{3i\,{}_2F_1\left(\frac{1}{6},\frac{17}{6};\frac{7}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}{2\,2^{5/6}f\left(a^2+ia^2\tan(e+fx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.562393, size = 121, normalized size = 1.39

$$\frac{3\sec^2(e+fx)\sqrt[3]{d\sec(e+fx)}\left(4ie^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)+\sin(2(e+fx))-2i\cos\right.}{22a^2f(\tan(e+fx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (3*Sec[e + f*x]^2*(d*Sec[e + f*x])^(1/3)*(-2*I - (2*I)*Cos[2*(e + f*x)] + (
4*I)*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F1[
1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))] + Sin[2*(e + f*x)]))/(22*a^2*f*(-I + T
an[e + f*x])^2)
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+ia\tan\left(fx+e\right)\right)^2}\sqrt[3]{d\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(44\,a^2 f e^{\left(4i\,fx+4i\,e\right)}\text{integral}\left(-\frac{2i\cdot 2^{\frac{1}{3}}\left(\frac{d}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{1}{3}} e^{\left(-\frac{2}{3}i\,fx-\frac{2}{3}i\,e\right)}}{11\,a^2 f},x\right)+2^{\frac{1}{3}}\left(\frac{d}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{\frac{1}{3}}\left(9i\,e^{\left(4i\,fx+4i\,e\right)}+12i\,e^{\left(2i\,fx+2i\,e\right)}+3i\right)e^{\left(\cdots\right.}}{44\,a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/44*(44*a^2*f*e^(4*I*f*x + 4*I*e)*integral(-2/11*I*2^(1/3)*(d/(e^(2*I*f*x
+ 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a^2*f), x) + 2^(1/3)*(d/(e^(
2*I*f*x + 2*I*e) + 1))^(1/3)*(9*I*e^(4*I*f*x + 4*I*e) + 12*I*e^(2*I*f*x + 2
*I*e) + 3*I)*e^(1/3*I*f*x + 1/3*I*e))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}{\left(i\,a\tan\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)/(I*a*tan(f*x + e) + a)^2, x)
```

3.277 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=71

$$-\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{19}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{4\sqrt[6]{2}a^2f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] (((-3*I)/4)*Hypergeometric2F1[-1/6, 19/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1 +
 I*Tan[e + f*x])^(1/6))/(2^(1/6)*a^2*f*(d*Sec[e + f*x])^(1/3))
```

Rubi [A] time = 0.179847, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$-\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{19}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{4\sqrt[6]{2}a^2f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] (((-3*I)/4)*Hypergeometric2F1[-1/6, 19/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1 +
 I*Tan[e + f*x])^(1/6))/(2^(1/6)*a^2*f*(d*Sec[e + f*x])^(1/3))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))^{13}}}{\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{(a-iax)^{7/6}(a+iax)^{19/6}}\, d\right.}{f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{19/6}(a-iax)^{7/6}}\,dx, x, \text{ta}\right.}{8\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

$$= -\frac{3i\,{}_2F_1\left(-\frac{1}{6},\frac{19}{6};\frac{5}{6};\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{4\sqrt[6]{2}a^2f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A] time = 1.35768, size = 141, normalized size = 1.99

$$\frac{(d\sec(e+fx))^{2/3}(-3\sin(2(e+fx))-3i\cos(2(e+fx)))\left(16e^{3i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-\right.\right.}{260a^2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] ((d*Sec[e + f*x])^(2/3)*(16*E^((3*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(
2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))] - 10*(7*Cos[e
+ f*x] + 5*Cos[3*(e + f*x)] + (18*I)*Cos[e + f*x]^2*Sin[e + f*x]))*((-3*I)*
Cos[2*(e + f*x)] - 3*Sin[2*(e + f*x)]))/(260*a^2*d*f)
```

Maple [F] time = 0.15, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+ia\tan\left(fx+e\right)\right)^2}\frac{1}{\sqrt[3]{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2^{\frac{2}{3}}\left(\frac{d}{e^{(2i fx+2ie)}+1}\right)^{\frac{2}{3}}\left(-39ie^{(7ifx+7ie)} - 57ie^{(6ifx+6ie)} - 27ie^{(5ifx+5ie)} - 69ie^{(4ifx+4ie)} + 15ie^{(3ifx+3ie)} - 15ie^{(2ifx+2ie)} + 3ie^{(}\right.}{104\left(a^2dfe^{(6ifx+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/104*(2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(-39*I*e^(7*I*f*x + 7*I*
e) - 57*I*e^(6*I*f*x + 6*I*e) - 27*I*e^(5*I*f*x + 5*I*e) - 69*I*e^(4*I*f*x
+ 4*I*e) + 15*I*e^(3*I*f*x + 3*I*e) - 15*I*e^(2*I*f*x + 2*I*e) + 3*I*e^(I*f
*x + I*e) - 3*I)*e^(2/3*I*f*x + 2/3*I*e) + 104*(a^2*d*f*e^(6*I*f*x + 6*I*e)
 - a^2*d*f*e^(5*I*f*x + 5*I*e))*integral(1/13*2^(2/3)*(d/(e^(2*I*f*x + 2*I*
e) + 1))^(2/3)*(-8*I*e^(2*I*f*x + 2*I*e) - 8*I*e^(I*f*x + I*e) - 8*I)*e^(2/
3*I*f*x + 2/3*I*e)/(a^2*d*f*e^(3*I*f*x + 3*I*e) - 2*a^2*d*f*e^(2*I*f*x + 2*
I*e) + a^2*d*f*e^(I*f*x + I*e)), x))/(a^2*d*f*e^(6*I*f*x + 6*I*e) - a^2*d*f
*e^(5*I*f*x + 5*I*e))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(ia\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2), x)
```

3.278 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx$

Optimal. Leaf size=71

$$-\frac{3i(1+i\tan(e+fx))^{5/6}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{23}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{20\,2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

```
[Out] (((-3*I)/20)*Hypergeometric2F1[-5/6, 23/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(5/6))/(2^(5/6)*a^2*f*(d*Sec[e + f*x])^(5/3))
```

Rubi [A] time = 0.190183, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$-\frac{3i(1+i\tan(e+fx))^{5/6}\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{23}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{20\,2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] (((-3*I)/20)*Hypergeometric2F1[-5/6, 23/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(5/6))/(2^(5/6)*a^2*f*(d*Sec[e + f*x])^(5/3))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \frac{\left((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+}}{(d\sec(e+fx))^{5/3}}$$
$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\text{Subst}\left(\int \frac{1}{(a-iax)^{11/6}(a+iax)}\right.}{f(d\sec(e+fx))^{5/3}}$$
$$= \frac{\left((a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{23/6}(a-iax)^{11/6}}\,dx,\right.}{8\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$
$$= -\frac{3i\,{}_2F_1\left(-\frac{5}{6},\frac{23}{6};\frac{1}{6};\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{20\ 2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

Mathematica [B] time = 0.809679, size = 143, normalized size = 2.01

$$\frac{3i\sec^4(e+fx)\left(128e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)-10i\sin(2(e+fx))+11i\sin(4(e+f\right.}{680a^2f(\tan(e+fx)-i)^2(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] (((3*I)/680)*Sec[e + f*x]^4*(-46 - 40*Cos[2*(e + f*x)] + 6*Cos[4*(e + f*x)]
 + 128*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F
1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))] - (10*I)*Sin[2*(e + f*x)] + (11*I)*S
in[4*(e + f*x)]))/(a^2*f*(d*Sec[e + f*x])^(5/3)*(-I + Tan[e + f*x])^2)
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+ia\tan\left(fx+e\right)\right)^2}\left(d\sec\left(fx+e\right)\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

[Out] Exception raised: RuntimeError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(1360\,a^2d^2fe^{(6i\,fx+6i\,e)}\text{integral}\left(-\frac{16i\cdot 2^{\frac{1}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{1}{3}}e^{\left(-\frac{2}{3}i\,fx-\frac{2}{3}i\,e\right)}}{85\,a^2d^2f},x\right)+2^{\frac{1}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{1}{3}}\left(-51i\,e^{(8i\,fx+8i\,e)}+150i\,e^{(6i\,fx+6i}\right.\right.}{1360\,a^2d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/1360*(1360*a^2*d^2*f*e^(6*I*f*x + 6*I*e)*integral(-16/85*I*2^(1/3)*(d/(e^
(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a^2*d^2*f), x) + 2^
(1/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(-51*I*e^(8*I*f*x + 8*I*e) + 150*
I*e^(6*I*f*x + 6*I*e) + 276*I*e^(4*I*f*x + 4*I*e) + 90*I*e^(2*I*f*x + 2*I*e
) + 15*I)*e^(1/3*I*f*x + 1/3*I*e))*e^(-6*I*f*x - 6*I*e)/(a^2*d^2*f)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2), x)
```

3.279 $\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^7d}-\frac{12i(a+ia\tan(c+dx))^{13/2}}{13a^6d}+\frac{24i(a+ia\tan(c+dx))^{11/2}}{11a^5d}-\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^4d}$$

```
[Out] (((-16*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^4*d) + (((24*I)/11)*(a + I*a*
Tan[c + d*x])^(11/2))/(a^5*d) - (((12*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))
/(a^6*d) + (((2*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/(a^7*d)
```

Rubi [A] time = 0.0754535, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^7d}-\frac{12i(a+ia\tan(c+dx))^{13/2}}{13a^6d}+\frac{24i(a+ia\tan(c+dx))^{11/2}}{11a^5d}-\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-16*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^4*d) + (((24*I)/11)*(a + I*a*
Tan[c + d*x])^(11/2))/(a^5*d) - (((12*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))
/(a^6*d) + (((2*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^3(a+x)^{7/2}\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(8a^3(a+x)^{7/2}-12a^2(a+x)^{9/2}+6a(a+x)^{11/2}-(a+x)^{13/2}\right)dx,x,i\right.}{a^7d}\\ &= -\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^4d}+\frac{24i(a+ia\tan(c+dx))^{11/2}}{11a^5d}-\frac{12i(a+ia\tan(c+d}{13a^6d}\end{aligned}$$

Mathematica [A] time = 0.818673, size = 95, normalized size = 0.81

$$\frac{2\sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}(-3i(90\sin(c+dx)+233\sin(3(c+dx)))+510\cos(c+dx)+731\cos(3(c+dx)))(\sin(4}{6435d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^7*(510*Cos[c + d*x] + 731*Cos[3*(c + d*x)] - (3*I)*(90*Sin[
c + d*x] + 233*Sin[3*(c + d*x)]))*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)]
)*Sqrt[a + I*a*Tan[c + d*x]])/(6435*d)
```

Maple [A] time = 2.026, size = 141, normalized size = 1.2

$$\frac{-2048\,i\,(\cos(dx+c))^7+2048\,(\cos(dx+c))^6\sin(dx+c)-256\,i\,(\cos(dx+c))^5+1280\,\sin(dx+c)\,(\cos(dx+c))^4-}{6435\,d\,(\cos(dx+c))^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/6435/d*(-1024*I*cos(d*x+c)^7+1024*cos(d*x+c)^6*sin(d*x+c)-128*I*cos(d*x+c
)^5+640*sin(d*x+c)*cos(d*x+c)^4-56*I*cos(d*x+c)^3+504*cos(d*x+c)^2*sin(d*x+
c)-33*I*cos(d*x+c)+429*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))
^(1/2)/cos(d*x+c)^7
```

Maxima [A] time = 0.979973, size = 103, normalized size = 0.88

$$\frac{2i\left(429\,(i\,a\tan(dx+c)+a)^{\frac{15}{2}}-2970\,(i\,a\tan(dx+c)+a)^{\frac{13}{2}}a+7020\,(i\,a\tan(dx+c)+a)^{\frac{11}{2}}a^2-5720\,(i\,a\tan(dx+c}{6435\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/6435*I*(429*(I*a*tan(d*x + c) + a)^(15/2) - 2970*(I*a*tan(d*x + c) + a)^(
13/2)*a + 7020*(I*a*tan(d*x + c) + a)^(11/2)*a^2 - 5720*(I*a*tan(d*x + c) +
 a)^(9/2)*a^3)/(a^7*d)
```

Fricas [A] time = 2.37633, size = 522, normalized size = 4.46

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-4096i\,e^{(14i\,dx+14i\,c)}-30720i\,e^{(12i\,dx+12i\,c)}-99840i\,e^{(10i\,dx+10i\,c)}-183040i\,e^{(8i\,dx+8i\,c)}\right)e^{(i\,dx+i\,c)}}{6435\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4i\,c)}+7\,de^{(2i\,dx+2i\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6435*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-4096*I*e^(14*I*d*x + 14*
I*c) - 30720*I*e^(12*I*d*x + 12*I*c) - 99840*I*e^(10*I*d*x + 10*I*c) - 1830
40*I*e^(8*I*d*x + 8*I*c))*e^(I*d*x + I*c)/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^
(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c)
 + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2
```

```
*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)^{8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^8, x)
```

3.280 $\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=88

$$-\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^5d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^4d}-\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^3d}$$

```
[Out] (((-8*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^3*d) + (((8*I)/9)*(a + I*a*Tan
[c + d*x])^(9/2))/(a^4*d) - (((2*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^5
*d)
```

Rubi [A] time = 0.0690455, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$-\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^5d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^4d}-\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-8*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^3*d) + (((8*I)/9)*(a + I*a*Tan
[c + d*x])^(9/2))/(a^4*d) - (((2*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^5
*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{i\,\mathrm{Subst}\left(\int (a-x)^2(a+x)^{5/2}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{i\,\mathrm{Subst}\left(\int \left(4a^2(a+x)^{5/2}-4a(a+x)^{7/2}+(a+x)^{9/2}\right)dx, x, ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^3d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^4d}-\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^5d}
\end{aligned}$$

Mathematica [A] time = 0.407978, size = 77, normalized size = 0.88

$$\frac{2\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}(-91i\sin(2(c+dx))+107\cos(2(c+dx))+44)(\sin(3(c+dx))-i\cos(3(c+dx)))}{693d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^5*(44 + 107*Cos[2*(c + d*x)] - (91*I)*Sin[2*(c + d*x)])*((-
I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(693*d)
```

Maple [A] time = 0.536, size = 114, normalized size = 1.3

$$-\frac{256\,i\,(\cos(dx+c))^5-256\,\sin(dx+c)\,(\cos(dx+c))^4+32\,i\,(\cos(dx+c))^3-160\,(\cos(dx+c))^2\sin(dx+c)+14\,i\cos}{693\,d\,(\cos(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -2/693/d*(128*I*cos(d*x+c)^5-128*sin(d*x+c)*cos(d*x+c)^4+16*I*cos(d*x+c)^3-
80*cos(d*x+c)^2*sin(d*x+c)+7*I*cos(d*x+c)-63*sin(d*x+c))*(a*(I*sin(d*x+c)+c
os(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5
```

Maxima [A] time = 0.999381, size = 78, normalized size = 0.89

$$-\frac{2i\left(63\,(i\,a\tan(dx+c)+a)^{\frac{11}{2}}-308\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a+396\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^2\right)}{693\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/693*I*(63*(I*a*tan(d*x + c) + a)^(11/2) - 308*(I*a*tan(d*x + c) + a)^(9/
2)*a + 396*(I*a*tan(d*x + c) + a)^(7/2)*a^2)/(a^5*d)
```

Fricas [B] time = 2.31028, size = 392, normalized size = 4.45

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-512i\,e^{(10i\,dx+10i\,c)}-2816i\,e^{(8i\,dx+8i\,c)}-6336i\,e^{(6i\,dx+6i\,c)}\right)e^{(i\,dx+i\,c)}}{693\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/693*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-512*I*e^(10*I*d*x + 10*I*
c) - 2816*I*e^(8*I*d*x + 8*I*c) - 6336*I*e^(6*I*d*x + 6*I*c))*e^(I*d*x + I*
c)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6
*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^6, x)
```

3.281 $\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^3d}-\frac{4i(a+ia\tan(c+dx))^{5/2}}{5a^2d}$$

```
[Out] (((-4*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^2*d) + (((2*I)/7)*(a + I*a*Tan
[c + d*x])^(7/2))/(a^3*d)
```

Rubi [A] time = 0.0622915, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^3d}-\frac{4i(a+ia\tan(c+dx))^{5/2}}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-4*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^2*d) + (((2*I)/7)*(a + I*a*Tan
[c + d*x])^(7/2))/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)(a+x)^{3/2}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(2a(a+x)^{3/2}-(a+x)^{5/2}\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{5/2}}{5a^2d}+\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^3d}\end{aligned}$$

Mathematica [A] time = 0.267442, size = 58, normalized size = 0.98

$$\frac{2\sqrt{a+ia\tan(c+dx)}\left(8(\tan(c+dx)-i)+(5\tan(c+dx)-i)\sec^2(c+dx)\right)}{35d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + I*a*Tan[c + d*x]]*(8*(-I + Tan[c + d*x]) + Sec[c + d*x]^2*(-I +
 5*Tan[c + d*x])))/(35*d)
```

Maple [A] time = 0.34, size = 87, normalized size = 1.5

$$-\frac{16\,i\left(\cos\left(dx+c\right)\right)^{3}-16\,\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)+2\,i\cos\left(dx+c\right)-10\,\sin\left(dx+c\right)}{35\,d\left(\cos\left(dx+c\right)\right)^{3}}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -2/35/d*(8*I*cos(d*x+c)^3-8*cos(d*x+c)^2*sin(d*x+c)+I*cos(d*x+c)-5*sin(d*x+
c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3
```

Maxima [A] time = 0.947808, size = 54, normalized size = 0.92

$$\frac{2i\left(5\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}-14\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a\right)}{35\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*I*(5*(I*a*tan(d*x + c) + a)^(7/2) - 14*(I*a*tan(d*x + c) + a)^(5/2)*a)
/(a^3*d)
```

Fricas [B] time = 2.32816, size = 270, normalized size = 4.58

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-32i\,e^{(6i\,dx+6i\,c)}-112i\,e^{(4i\,dx+4i\,c)}\right)e^{(i\,dx+i\,c)}}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-32*I*e^(6*I*d*x + 6*I*c) -
 112*I*e^(4*I*d*x + 4*I*c))*e^(I*d*x + I*c)/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^
(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(i\tan\left(c+dx\right)+1\right)}\sec^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(I*tan(c + d*x) + 1))*sec(c + d*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\tan\left(dx+c\right)+a}\,\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^4, x)
```

3.282 $\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=29

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a*d)
```

Rubi [A] time = 0.0556067, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 32}

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{i\,\text{Subst}\left(\int \sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}\end{aligned}$$

Mathematica [A] time = 0.122623, size = 34, normalized size = 1.17

$$\frac{2(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d)
```

Maple [A] time = 0.043, size = 24, normalized size = 0.8

$$\frac{-\frac{2i}{3}}{ad}(a+ia\tan(dx+c))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

Maxima [A] time = 0.971031, size = 28, normalized size = 0.97

$$-\frac{2i\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*I*(I*a*tan(d*x + c) + a)^(3/2)/(a*d)
```

Fricas [B] time = 2.27043, size = 132, normalized size = 4.55

$$-\frac{4i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(3i\,dx+3i\,c)}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/3*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3*I*d*x + 3*I*c)/(d*e^(
2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\,(i\tan(c+dx)+1)}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(I*tan(c + d*x) + 1))*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{i\,a\tan(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^2, x)
```

3.283 $\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=120

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}+\frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{3i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d}$$

```
[Out] (((-3*I)/4)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/
(Sqrt[2]*d) + (((3*I)/4)*a)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^2)/(d
*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.0981145, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}+\frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{3i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-3*I)/4)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/
(Sqrt[2]*d) + (((3*I)/4)*a)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^2)/(d
*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} - \frac{(3ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx$$

$$= \frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} - \frac{(3ia)\,\text{S}$$

$$= \frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} - \frac{(3ia)\,\text{S}$$

$$= -\frac{3i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} + \frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{}{2d(a-ia\tan(c+}$$

Mathematica [A] time = 0.43948, size = 105, normalized size = 0.88

$$-\frac{ie^{-2i(c+dx)}\left(-e^{2i(c+dx)}+e^{4i(c+dx)}+3e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)-2\right)\sqrt{a+ia\tan(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I/8)*(-2 - E^((2*I)*(c + d*x)) + E^((4*I)*(c + d*x)) + 3*E^(I*(c + d*x))
*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c
 + d*x]])/(d*E^((2*I)*(c + d*x)))
```

Maple [B] time = 0.39, size = 397, normalized size = 3.3

$$\frac{1}{16\,d\,(i\sin(dx+c)+\cos(dx+c)-1)\cos(dx+c)}\left(3\,i\cos(dx+c)\sin(dx+c)\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{\frac{3}{2}}\text{Artanh}\left(\frac{\sqrt{2}\sin}{2\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d*(3*I*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arct
anh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))
*2^(1/2)+3*I*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+3
*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctan(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)+3*2^(1/2)*arctan(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3
/2)*sin(d*x+c)-8*I*cos(d*x+c)^4-4*I*cos(d*x+c)^3+8*cos(d*x+c)^3*sin(d*x+c)+
12*I*cos(d*x+c)^2-12*cos(d*x+c)^2*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/
cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.69812, size = 772, normalized size = 6.43

$$\frac{\left(3\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{1}{3}\left(6i\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(2i\,dx+2i\,c)}+3\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3\sqrt{\frac{1}{2}}d\sqrt{-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*x + 2*I*c)*log(1/3*(6*I*sqrt(1/2)*
d*sqrt(-a/d^2)*e^(2*I*d*x + 2*I*c) + 3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*sqrt
(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*x + 2*I*c)*log(1/3*(-6*I*sqrt(1/2)*d*sqrt(-a/
d^2)*e^(2*I*d*x + 2*I*c) + 3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(
2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c) +
 2*I)*e^(I*d*x + I*c))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\,(i\tan\,(c+dx)+1)}\cos^2\,(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(I*tan(c + d*x) + 1))*cos(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{i\,a\tan\,(dx+c)+a}\cos\,(dx+c)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^2, x)
```

3.284 $\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=193

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}-\frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}+\frac{35ia^2}{96d(a+ia\tan(c+$$

```
[Out] (((-35*I)/64)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/(Sqrt[2]*d) + (((35*I)/96)*a^2)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I/4)
*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) - (((7*I)/1
6)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (((35*I)/
64)*a)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.112087, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}-\frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}+\frac{35ia^2}{96d(a+ia\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-35*I)/64)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/(Sqrt[2]*d) + (((35*I)/96)*a^2)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I/4)
*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) - (((7*I)/1
6)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (((35*I)/
64)*a)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{(7ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{5/2}}\,d\right.}{8d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia}$$

$$= \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{}{16}$$

$$= \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{}{16}$$

$$= \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{}{16}$$

$$= -\frac{35i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} + \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{}{4d(a-ia\tan(c}$$

Mathematica [A] time = 0.370383, size = 133, normalized size = 0.69

$$-\frac{ie^{-4i(c+dx)}\left(-88e^{2i(c+dx)}-41e^{4i(c+dx)}+45e^{6i(c+dx)}+6e^{8i(c+dx)}+105e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)-8\right)\sqrt{a+ia\tan}}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I/384)*(-8 - 88*E^((2*I)*(c + d*x)) - 41*E^((4*I)*(c + d*x)) + 45*E^((6*
I)*(c + d*x)) + 6*E^((8*I)*(c + d*x)) + 105*E^((3*I)*(c + d*x))*Sqrt[1 + E^
((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c + d*x]])/(d
*E^((4*I)*(c + d*x)))
```

Maple [B] time = 0.355, size = 741, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/3072/d*(-768*I*cos(d*x+c)^8-224*I*cos(d*x+c)^6+105*2^(1/2)*arctan(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
7/2)*cos(d*x+c)^3*sin(d*x+c)+105*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
7/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/co
```

```
s(d*x+c))*cos(d*x+c)^3*sin(d*x+c)+315*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)
^2*sin(d*x+c)+105*I*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*sin(d
*x+c)+315*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)*sin(d*x+c)+105*2^(1/2)*arct
an(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(7/2)*sin(d*x+c)+315*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2
)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d
*x+c))*cos(d*x+c)*sin(d*x+c)-128*I*cos(d*x+c)^7+768*sin(d*x+c)*cos(d*x+c)^7
-560*I*cos(d*x+c)^5-896*cos(d*x+c)^6*sin(d*x+c)+1680*I*cos(d*x+c)^4+1120*co
s(d*x+c)^5*sin(d*x+c)+315*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c
))*cos(d*x+c)^2*sin(d*x+c)-1680*sin(d*x+c)*cos(d*x+c)^4)*(a*(I*sin(d*x+c)+c
os(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.84487, size = 868, normalized size = 4.5

$$\left(105\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(4idx+4ic)}\log\left(\frac{1}{35}\left(70i\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(2idx+2ic)}+35\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-1\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(105*sqrt(1/2)*d*sqrt(-a/d^2)*e^(4*I*d*x + 4*I*c)*log(1/35*(70*I*sqrt
(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*x + 2*I*c) + 35*sqrt(2)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) -
 105*sqrt(1/2)*d*sqrt(-a/d^2)*e^(4*I*d*x + 4*I*c)*log(1/35*(-70*I*sqrt(1/2)
*d*sqrt(-a/d^2)*e^(2*I*d*x + 2*I*c) + 35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt
(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-6*I*e^(8*I*d*x + 8*I*c) - 45*I*e^(6
*I*d*x + 6*I*c) + 41*I*e^(4*I*d*x + 4*I*c) + 88*I*e^(2*I*d*x + 2*I*c) + 8*I
)*e^(I*d*x + I*c))*e^(-4*I*d*x - 4*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i a \tan(dx + c) + a} \cos(dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^4, x)
```

3.285 $\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=266

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}}-\frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}-\frac{}{64d(a-ia\tan(c+}$$

```
[Out] (((-231*I)/512)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a]
)])/(Sqrt[2]*d) + (((231*I)/640)*a^3)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((
I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(5/2)) - (((11
*I)/48)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) - ((
(33*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (
((77*I)/256)*a^2)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((231*I)/512)*a)/(d*S
qrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.14195, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}}-\frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}-\frac{}{64d(a-ia\tan(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```


```
[Out] (((-231*I)/512)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a]
)])/(Sqrt[2]*d) + (((231*I)/640)*a^3)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((
I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(5/2)) - (((11
*I)/48)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) - ((
(33*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (
((77*I)/256)*a^2)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((231*I)/512)*a)/(d*S
qrt[a + I*a*Tan[c + d*x]])
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{\left(ia^7\right)\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{7/2}}\,dx,x,ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{\left(11ia^6\right)\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{7/2}}\right.}{12d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+i}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+i}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{}{4}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{}{4}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{}{4}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{}{4}$$

$$= -\frac{231i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}d} + \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{}{6d(a-ia\tan}$$

Mathematica [A] time = 0.609072, size = 159, normalized size = 0.6

$$-\frac{ie^{-6i(c+dx)}\left(-464e^{2i(c+dx)}-3184e^{4i(c+dx)}-1433e^{6i(c+dx)}+1645e^{8i(c+dx)}+350e^{10i(c+dx)}+40e^{12i(c+dx)}+3465e^{5i(c+dx)}\sqrt{1+e^{\cdots}}\right.}{15360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I/15360)*(-48 - 464*E^((2*I)*(c + d*x)) - 3184*E^((4*I)*(c + d*x)) - 143
3*E^((6*I)*(c + d*x)) + 1645*E^((8*I)*(c + d*x)) + 350*E^((10*I)*(c + d*x))
 + 40*E^((12*I)*(c + d*x)) + 3465*E^((5*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
+ d*x))]*ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((6*I)*
(c + d*x)))
```

Maple [B] time = 0.427, size = 1085, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/491520/d*(3465*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*2^(1/2)+17325*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)+34650*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)+34650*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11
/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*s
in(d*x+c)*2^(1/2)+17325*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)+3
465*I*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*sin(d*x+c)+101376*
sin(d*x+c)*cos(d*x+c)^9+81920*sin(d*x+c)*cos(d*x+c)^11-90112*sin(d*x+c)*cos
(d*x+c)^10-81920*I*cos(d*x+c)^12-8192*I*cos(d*x+c)^11-11264*I*cos(d*x+c)^10
-16896*I*cos(d*x+c)^9-29568*I*cos(d*x+c)^8-73920*I*cos(d*x+c)^7+221760*I*co
s(d*x+c)^6-118272*sin(d*x+c)*cos(d*x+c)^8+147840*sin(d*x+c)*cos(d*x+c)^7-22
1760*cos(d*x+c)^6*sin(d*x+c)+3465*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*cos(d*x+c)^5*sin(d*x+c)*2^(1/2)+17325*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/
cos(d*x+c))*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)+34650*I*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)/cos(d*x+c))*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)+34650*I*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+17325*I*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)*sin(d*x+c)*2^(1/2)+3465*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c)
)^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.93715, size = 973, normalized size = 3.66

$$\left(3465\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(6i\,dx+6i\,c)}\log\left(\frac{1}{231}\left(462i\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(2i\,dx+2i\,c)}+231\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15360*(3465*sqrt(1/2)*d*sqrt(-a/d^2)*e^(6*I*d*x + 6*I*c)*log(1/231*(462*I
*sqrt(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*x + 2*I*c) + 231*sqrt(2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)) - 3465*sqrt(1/2)*d*sqrt(-a/d^2)*e^(6*I*d*x + 6*I*c)*log(1/231*(-462*I*
sqrt(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*x + 2*I*c) + 231*sqrt(2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*
c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-40*I*e^(12*I*d*x + 12*I*c
) - 350*I*e^(10*I*d*x + 10*I*c) - 1645*I*e^(8*I*d*x + 8*I*c) + 1433*I*e^(6*
I*d*x + 6*I*c) + 3184*I*e^(4*I*d*x + 4*I*c) + 464*I*e^(2*I*d*x + 2*I*c) + 4
8*I)*e^(I*d*x + I*c))*e^(-6*I*d*x - 6*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^6, x)
```

3.286 $\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}}+\frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}}+\frac{256ia^4\sec^7(c+dx)}{3003d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (((256*I)/3003)*a^4*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((6
4*I)/429)*a^3*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((24*I)/1
43)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/13)*a*Se
c[c + d*x]^7)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.249842, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}}+\frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}}+\frac{256ia^4\sec^7(c+dx)}{3003d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((256*I)/3003)*a^4*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((6
4*I)/429)*a^3*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((24*I)/1
43)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/13)*a*Se
c[c + d*x]^7)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= \frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{13}(12a)\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\ &= \frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{143}\left(96a^2\right)\int \frac{}{(a+\\ &= \frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}}+\frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^7(c+}{13d\sqrt{a+ia\tan}}\\ &= \frac{256ia^4\sec^7(c+dx)}{3003d(a+ia\tan(c+dx))^{7/2}}+\frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}}+\frac{24ia^2\sec^7}{143d(a+ia\,\mathrm{ta}}\end{aligned}$$

Mathematica [A] time = 0.586104, size = 95, normalized size = 0.65

$$\frac{2\sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}(7i(26\sin(c+dx)+59\sin(3(c+dx)))+390\cos(c+dx)+445\cos(3(c+dx)))(\sin(4(c}{3003d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^6*(390*Cos[c + d*x] + 445*Cos[3*(c + d*x)] + (7*I)*(26*Sin[
c + d*x] + 59*Sin[3*(c + d*x)]))*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*Sq
rt[a + I*a*Tan[c + d*x]])/(3003*d)
```

Maple [A] time = 0.841, size = 141, normalized size = 1.

$$\frac{2048\,i\,(\cos(dx+c))^7+2048\,(\cos(dx+c))^6\sin(dx+c)-256\,i\,(\cos(dx+c))^5+768\,\sin(dx+c)\,(\cos(dx+c))^4-80\,i\,(c}{3003\,d\,(\cos(dx+c))^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/3003/d*(1024*I*cos(d*x+c)^7+1024*cos(d*x+c)^6*sin(d*x+c)-128*I*cos(d*x+c)
^5+384*sin(d*x+c)*cos(d*x+c)^4-40*I*cos(d*x+c)^3+280*cos(d*x+c)^2*sin(d*x+c
)-21*I*cos(d*x+c)+231*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^
(1/2)/cos(d*x+c)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.30954, size = 467, normalized size = 3.18

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(54912i\,e^{(6i\,dx+6i\,c)}+36608i\,e^{(4i\,dx+4i\,c)}+13312i\,e^{(2i\,dx+2i\,c)}+2048i\right)e^{(i\,dx+i\,c)}}{3003\left(de^{(13i\,dx+13i\,c)}+6\,de^{(11i\,dx+11i\,c)}+15\,de^{(9i\,dx+9i\,c)}+20\,de^{(7i\,dx+7i\,c)}+15\,de^{(5i\,dx+5i\,c)}+6\,de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3003*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(54912*I*e^(6*I*d*x + 6*I*
c) + 36608*I*e^(4*I*d*x + 4*I*c) + 13312*I*e^(2*I*d*x + 2*I*c) + 2048*I)*e^
(I*d*x + I*c)/(d*e^(13*I*d*x + 13*I*c) + 6*d*e^(11*I*d*x + 11*I*c) + 15*d*e
^(9*I*d*x + 9*I*c) + 20*d*e^(7*I*d*x + 7*I*c) + 15*d*e^(5*I*d*x + 5*I*c) +
```

```
6*d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)^{7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^7, x)
```

3.287 $\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}}+\frac{64ia^3\sec^5(c+dx)}{315d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (((64*I)/315)*a^3*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((16*
I)/63)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/9)*a*
Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.177871, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}}+\frac{64ia^3\sec^5(c+dx)}{315d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((64*I)/315)*a^3*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((16*
I)/63)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/9)*a*
Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= \frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{9}(8a)\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\ &= \frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{63}\left(32a^2\right)\int \frac{\sec^5(c}{(a+ia\tan(}\\ &= \frac{64ia^3\sec^5(c+dx)}{315d(a+ia\tan(c+dx))^{5/2}}+\frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+d}\end{aligned}$$

Mathematica [A] time = 0.369822, size = 77, normalized size = 0.7

$$\frac{2\sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}(55i\sin(2(c+dx))+71\cos(2(c+dx))+36)(\sin(3(c+dx))+i\cos(3(c+dx)))}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^4*(36 + 71*Cos[2*(c + d*x)] + (55*I)*Sin[2*(c + d*x)])*(I*C
os[3*(c + d*x)] + Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(315*d)
```

Maple [A] time = 0.385, size = 114, normalized size = 1.

$$\frac{256\,i\,(\cos(dx+c))^5+256\,\sin(dx+c)\,(\cos(dx+c))^4-32\,i\,(\cos(dx+c))^3+96\,(\cos(dx+c))^2\sin(dx+c)-10\,i\cos}{315\,d\,(\cos(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/315/d*(128*I*cos(d*x+c)^5+128*sin(d*x+c)*cos(d*x+c)^4-16*I*cos(d*x+c)^3+4
8*cos(d*x+c)^2*sin(d*x+c)-5*I*cos(d*x+c)+35*sin(d*x+c))*(a*(I*sin(d*x+c)+co
s(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.23593, size = 342, normalized size = 3.11

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2016i\,e^{(4i\,dx+4i\,c)}+1152i\,e^{(2i\,dx+2i\,c)}+256i\right)e^{(i\,dx+i\,c)}}{315\left(de^{(9i\,dx+9i\,c)}+4\,de^{(7i\,dx+7i\,c)}+6\,de^{(5i\,dx+5i\,c)}+4\,de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2016*I*e^(4*I*d*x + 4*I*c)
 + 1152*I*e^(2*I*d*x + 2*I*c) + 256*I)*e^(I*d*x + I*c)/(d*e^(9*I*d*x + 9*I*
c) + 4*d*e^(7*I*d*x + 7*I*c) + 6*d*e^(5*I*d*x + 5*I*c) + 4*d*e^(3*I*d*x + 3
*I*c) + d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i a \tan(dx+c) + a} \sec(dx+c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^5, x)
```

3.288 $\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (((8*I)/15)*a^2*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/
5)*a*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.107429, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((8*I)/15)*a^2*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/
5)*a*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= \frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{5}(4a)\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\ &= \frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.217766, size = 63, normalized size = 0.86

$$-\frac{2(3\tan(c+dx)-7i)\sec(c+dx)\sqrt{a+ia\tan(c+dx)}(\cos(2(c+dx))-i\sin(2(c+dx)))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*Sec[c + d*x]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-7*I + 3*Tan[c +
d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(15*d)
```

Maple [A] time = 0.317, size = 87, normalized size = 1.2

$$\frac{16\,i\,(\cos(dx+c))^3+16\,(\cos(dx+c))^2\sin(dx+c)-2\,i\cos(dx+c)+6\,\sin(dx+c)}{15\,d\,(\cos(dx+c))^2}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d*(8*I*cos(d*x+c)^3+8*cos(d*x+c)^2*sin(d*x+c)-I*cos(d*x+c)+3*sin(d*x+c
))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2
```

Maxima [B] time = 116.376, size = 304, normalized size = 4.16

$$-\frac{}{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{4}}\left((225\,\cos(4\,dx+4\,c)+450\,\cos(2\,dx+2\,c)+225i\,\sin(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -(-600*I*sqrt(2)*cos(2*d*x + 2*c) + 600*sqrt(2)*sin(2*d*x + 2*c) - 240*I*sq
rt(2))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*((225*cos(4*d*x + 4*c) + 450*cos(2*d*x + 2*c) + 225*I*sin(4*d
*x + 4*c) + 450*I*sin(2*d*x + 2*c) + 225)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) - (-225*I*cos(4*d*x + 4*c) - 450*I*cos(2*d*x + 2*c)
 + 225*sin(4*d*x + 4*c) + 450*sin(2*d*x + 2*c) - 225*I)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*d)
```

Fricas [A] time = 2.33966, size = 227, normalized size = 3.11

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(40i\,e^{(2i\,dx+2i\,c)}+16i\right)e^{(i\,dx+i\,c)}}{15\left(de^{(5i\,dx+5i\,c)}+2\,de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(40*I*e^(2*I*d*x + 2*I*c) +
16*I)*e^(I*d*x + I*c)/(d*e^(5*I*d*x + 5*I*c) + 2*d*e^(3*I*d*x + 3*I*c) + d*
e^(I*d*x + I*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(i\tan\left(c+dx\right)+1\right)}\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(I*tan(c + d*x) + 1))*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.289 $\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

[Out] ((2*I)*a*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]])

Rubi [A] time = 0.0281013, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3493}

$$\frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]

[Out] ((2*I)*a*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]])

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 0.143904, size = 39, normalized size = 1.26

$$\frac{2\sqrt{a+ia\tan(c+dx)}(\sin(c+dx)+i\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]

[Out] (2*(I*Cos[c + d*x] + Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/d

---

Maple [A] time = 0.242, size = 50, normalized size = 1.6

$$2\,\frac{i\cos(dx+c)+\sin(dx+c)}{d}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(I*cos(d*x+c)+sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i a \tan(dx + c) + a} \sec(dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [A] time = 2.34026, size = 66, normalized size = 2.13

$$\frac{2i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(i\tan(c + dx) + 1)} \sec(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(I*tan(c + d*x) + 1))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i a \tan(dx + c) + a} \sec(dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c), x)
```

3.290 $\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] (I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.0937708, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3490, 3489, 206}

$$\frac{i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{1}{2}a\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= -\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{(ia)\,\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\\
&= \frac{i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.517113, size = 87, normalized size = 1.05

$$-\frac{ie^{-i(c+dx)}\left(e^{2i(c+dx)}-\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)+1\right)\sqrt{a+ia\tan(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I/2)*(1 + E^((2*I)*(c + d*x)) - Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sq
rt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)
))
```

Maple [B] time = 0.322, size = 217, normalized size = 2.6

$$-\frac{1}{2\,d\,(i\sin(dx+c)+\cos(dx+c)-1)}\left(i\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}\sin(dx+c)}{2\cos(dx+c)}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*(I*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-2^(1
/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*I*cos(d*x+c)^2-2*I*cos(d*x+c)-2*cos(d*x
+c)*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c
)+cos(d*x+c)-1)
```

Maxima [B] time = 1.97359, size = 1045, normalized size = 12.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*(-4*I*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a
) - (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
```

```
 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/d
```

Fricas [B] time = 2.35262, size = 690, normalized size = 8.31

$$-\frac{\left(\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(idx+ic)}\log\left(\left(2i\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(idx+ic)}+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(idx+ic}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*d*sqrt(-a/d^2)*e^(I*d*x + I*c)*log((2*I*sqrt(1/2)*d*sqrt(-a
/d^2)*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d
*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - sqrt(1/2)*d*sqrt(-a/d
^2)*e^(I*d*x + I*c)*log((-2*I*sqrt(1/2)*d*sqrt(-a/d^2)*e^(I*d*x + I*c) + sq
rt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x
+ I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e
^(2*I*d*x + 2*I*c) - I)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(i\tan\left(c+dx\right)+1\right)}\cos\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(I*tan(c + d*x) + 1))*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c), x)
```

3.291 $\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=154

$$-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}-\frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}+\frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}+\frac{5i\sqrt{a}\tanh^{-1}\Big(}{8\sqrt{}}$$

```
[Out] (((5*I)/8)*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan
[c + d*x]])])/(Sqrt[2]*d) + (((5*I)/12)*a*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan
[c + d*x]]) - (((5*I)/8)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/3
)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.223128, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3497, 3502, 3490, 3489, 206}

$$-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}-\frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}+\frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}+\frac{5i\sqrt{a}\tanh^{-1}\Big(}{8\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```


```
[Out] (((5*I)/8)*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan
[c + d*x]])])/(Sqrt[2]*d) + (((5*I)/12)*a*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan
[c + d*x]]) - (((5*I)/8)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/3
)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d
```


Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx &= -\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{1}{6}(5a)\int\frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\ &= \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{5}{8}\int\cos(c+dx)\\ &= \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}-\frac{i\cos^3(c+dx)\sqrt{\ }}{}\\ &= \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}-\frac{i\cos^3(c+dx)\sqrt{\ }}{}\\ &= \frac{5i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d}+\frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{5i\cos(c+dx)\sqrt{a-}}{8d}\end{aligned}$$

Mathematica [A] time = 0.462658, size = 126, normalized size = 0.82

$$-\frac{ie^{-3i(c+dx)}\left(11e^{2i(c+dx)}+16e^{4i(c+dx)}+2e^{6i(c+dx)}-15e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)-3\right)\sqrt{a+ia\tan(c+dx)}}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I/48)*(-3 + 11*E^((2*I)*(c + d*x)) + 16*E^((4*I)*(c + d*x)) + 2*E^((6*I)
*(c + d*x)) - 15*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[
Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((3*I)*(c
+ d*x)))
```

Maple [B] time = 0.387, size = 569, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/192/d*(15*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)
^2*sin(d*x+c)+30*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*
x+c)*sin(d*x+c)-15*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
```

```
)^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c)+15*I*
2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-30*2^(1/2)*arc
tan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)-15*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x
+c)+64*I*cos(d*x+c)^6+16*I*cos(d*x+c)^5-64*cos(d*x+c)^5*sin(d*x+c)+40*I*cos
(d*x+c)^4+80*sin(d*x+c)*cos(d*x+c)^4-120*I*cos(d*x+c)^3-120*cos(d*x+c)^3*si
n(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(
d*x+c)-1)/cos(d*x+c)^2
```

Maxima [B] time = 2.2328, size = 1261, normalized size = 8.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/192*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
3/4)*(-8*I*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 8*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt
(a) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*((12*I*sqrt(2)*cos(2*d*x + 2*c) + 12*sqrt(2)*sin(2*d*x + 2*c) - 48*I*sq
rt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 12*(sqrt(
2)*cos(2*d*x + 2*c) - I*sqrt(2)*sin(2*d*x + 2*c) - 4*sqrt(2))*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) - (30*sqrt(2)*arctan2(
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 30*sqrt(2)*arctan2((cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - 1) - 15*I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) +
15*I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + s
qrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/d
```

Fricas [B] time = 2.38646, size = 813, normalized size = 5.28

$$-\frac{\left(15\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(3idx+3ic)}\log\left(\frac{1}{5}\left(10i\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(idx+ic)}+5\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-15\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/48*(15*sqrt(1/2)*d*sqrt(-a/d^2)*e^(3*I*d*x + 3*I*c)*log(1/5*(10*I*sqrt(1
/2)*d*sqrt(-a/d^2)*e^(I*d*x + I*c) + 5*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 15*sqr
t(1/2)*d*sqrt(-a/d^2)*e^(3*I*d*x + 3*I*c)*log(1/5*(-10*I*sqrt(1/2)*d*sqrt(-
a/d^2)*e^(I*d*x + I*c) + 5*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*
I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(6*I*d*x + 6*I*c) - 16*I*e^(4*I*d*x + 4*I*c
) - 11*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(I*d*x + I*c))*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^3, x)
```

3.292 $\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=223

$$-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d}+\frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{63i\cos(c+d}{}$$

```
[Out] (((63*I)/128)*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*
Tan[c + d*x]])])/(Sqrt[2]*d) + (((21*I)/64)*a*Cos[c + d*x])/(d*Sqrt[a + I*a
*Tan[c + d*x]]) + (((9*I)/40)*a*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x
]]) - (((63*I)/128)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d - (((21*I)/8
0)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/5)*Cos[c + d*x]^5*Sqr
t[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.390931, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3497, 3502, 3490, 3489, 206}

$$-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}-\frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d}+\frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{63i\cos(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```


```
[Out] (((63*I)/128)*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*
Tan[c + d*x]])])/(Sqrt[2]*d) + (((21*I)/64)*a*Cos[c + d*x])/(d*Sqrt[a + I*a
*Tan[c + d*x]]) + (((9*I)/40)*a*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x
]]) - (((63*I)/128)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d - (((21*I)/8
0)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/5)*Cos[c + d*x]^5*Sqr
t[a + I*a*Tan[c + d*x]])/d
```


Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
```

```
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d} + \frac{1}{10}(9a)\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d} + \frac{63}{80}\int \cos^3(c+$$

$$= \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}} - \frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d} - \frac{i\cos^5(c+dx)}{}$$

$$= \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}} + \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}} - \frac{21i\cos^3(c+dx)\sqrt{a+ia\,\mathrm{ta}}}{80d}$$

$$= \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}} + \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}} - \frac{63i\cos(c+dx)\sqrt{a+ia\,\mathrm{ta}}}{128d}$$

$$= \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}} + \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}} - \frac{63i\cos(c+dx)\sqrt{a+ia\,\mathrm{ta}}}{128d}$$

$$= \frac{63i\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}d} + \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}} + \frac{9ia\cos^3(c+}{40d\sqrt{a+ia\tan(}}$$

Mathematica [A] time = 0.555925, size = 152, normalized size = 0.68

$$-\frac{ie^{-5i(c+dx)}\left(-95e^{2i(c+dx)} + 203e^{4i(c+dx)} + 344e^{6i(c+dx)} + 64e^{8i(c+dx)} + 8e^{10i(c+dx)} - 315e^{4i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2}}\right.\right.}{1280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I/1280)*(-10 - 95*E^((2*I)*(c + d*x)) + 203*E^((4*I)*(c + d*x)) + 344*E^
((6*I)*(c + d*x)) + 64*E^((8*I)*(c + d*x)) + 8*E^((10*I)*(c + d*x)) - 315*E
^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*
(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((5*I)*(c + d*x)))
```

Maple [B] time = 0.352, size = 913, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/20480/d*(768*I*cos(d*x+c)^8+3360*I*cos(d*x+c)^6-315*arctan(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2
^(1/2)*cos(d*x+c)^4*sin(d*x+c)+4096*I*cos(d*x+c)^10-1260*arctan(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)
*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+315*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)
*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*
x+c))*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)-1890*arctan(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(d
*x+c)^2*sin(d*x+c)+1344*I*cos(d*x+c)^7-1260*arctan(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(
d*x+c)*sin(d*x+c)-315*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+512*I*cos(d*x+c
)^9+1260*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)^3*s
in(d*x+c)-4096*sin(d*x+c)*cos(d*x+c)^9+315*I*2^(1/2)*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(9/2)*sin(d*x+c)+4608*sin(d*x+c)*cos(d*x+c)^8+1260*I*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)*sin(d*x+c)-5376*sin(d*
x+c)*cos(d*x+c)^7-10080*I*cos(d*x+c)^5+6720*cos(d*x+c)^6*sin(d*x+c)+1890*I*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-1
0080*cos(d*x+c)^5*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^4
```

Maxima [B] time = 2.80716, size = 2990, normalized size = 13.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5120*((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((60*I*sqrt(2)*cos(4*d*x + 4*c) + 6
0*sqrt(2)*sin(4*d*x + 4*c) + 160*I*sqrt(2))*cos(3/2*arctan2(sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))) + 1)) - 20*(3*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*sin(4
*d*x + 4*c) + 8*sqrt(2))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)))*sqrt(a) + (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((32*I*sqrt(2)*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 32*I*sqrt(2)*sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 64*I*sqrt(2)*cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))) + 32*I*sqrt(2))*cos(5/2*arctan2(sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)) + (-100*I*sqrt(2)*cos(4*d*x + 4*c) - 400*I*sqrt(2)*
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 100*sqrt(2)*sin(4*d*
x + 4*c) - 400*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 960*I*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - 3
2*(sqrt(2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sqrt(2)
*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*sqrt(2)*cos(1/2
```

```
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + sqrt(2))*sin(5/2*arctan2(si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)) + (100*sqrt(2)*cos(4*d*x + 4*c) + 400*s
qrt(2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 100*I*sqrt(2)
*sin(4*d*x + 4*c) - 400*I*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) - 960*sqrt(2))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
+ 1)))*sqrt(a) + (630*sqrt(2)*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/
2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(
1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) - 630*sqrt(
2)*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + 1)) - 1) - 315*I*sqrt(2)*log(sqrt(cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))^2 + sqrt(c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
)^2 + 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))) + 1)) + 1) + 315*I*sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos
(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))^2 - 2*(cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)) + 1))*sqrt(a))/d
```

Fricas [A] time = 2.44112, size = 906, normalized size = 4.06

$$-\frac{\left(315\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(5idx+5ic)}\log\left(\frac{1}{63}\left(126i\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(idx+ic)}+63\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-315\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/1280*(315*sqrt(1/2)*d*sqrt(-a/d^2)*e^(5*I*d*x + 5*I*c)*log(1/63*(126*I*s
qrt(1/2)*d*sqrt(-a/d^2)*e^(I*d*x + I*c) + 63*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) -
315*sqrt(1/2)*d*sqrt(-a/d^2)*e^(5*I*d*x + 5*I*c)*log(1/63*(-126*I*sqrt(1/2)
*d*sqrt(-a/d^2)*e^(I*d*x + I*c) + 63*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(10*I*d*x + 10*I*c) - 64*I*e^(8*I
*d*x + 8*I*c) - 344*I*e^(6*I*d*x + 6*I*c) - 203*I*e^(4*I*d*x + 4*I*c) + 95*
I*e^(2*I*d*x + 2*I*c) + 10*I)*e^(I*d*x + I*c))*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i a \tan(dx + c) + a} \cos(dx + c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^5, x)
```

3.293 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=117

$$\frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^7d}-\frac{4i(a+ia\tan(c+dx))^{15/2}}{5a^6d}+\frac{24i(a+ia\tan(c+dx))^{13/2}}{13a^5d}-\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^4d}$$

```
[Out] (((-16*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^4*d) + (((24*I)/13)*(a + I*
a*Tan[c + d*x])^(13/2))/(a^5*d) - (((4*I)/5)*(a + I*a*Tan[c + d*x])^(15/2))
/(a^6*d) + (((2*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))/(a^7*d)
```

Rubi [A] time = 0.0893523, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^7d}-\frac{4i(a+ia\tan(c+dx))^{15/2}}{5a^6d}+\frac{24i(a+ia\tan(c+dx))^{13/2}}{13a^5d}-\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-16*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^4*d) + (((24*I)/13)*(a + I*
a*Tan[c + d*x])^(13/2))/(a^5*d) - (((4*I)/5)*(a + I*a*Tan[c + d*x])^(15/2))
/(a^6*d) + (((2*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= -\frac{i\,\mathrm{Subst}\left(\int (a-x)^3(a+x)^{9/2}\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\mathrm{Subst}\left(\int \left(8a^3(a+x)^{9/2}-12a^2(a+x)^{11/2}+6a(a+x)^{13/2}-(a+x)^{15/2}\right)dx,\right.}{a^7d}\\ &= -\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^4d}+\frac{24i(a+ia\tan(c+dx))^{13/2}}{13a^5d}-\frac{4i(a+ia\tan(c+\right.}{5a^6d}\end{aligned}$$

Mathematica [A] time = 1.12643, size = 111, normalized size = 0.95

$$\frac{2a\sec^8(c+dx)(\cos(dx)-i\sin(dx))\sqrt{a+ia\tan(c+dx)}(-11i(34\sin(c+dx)+99\sin(3(c+dx)))+646\cos(c+dx)+112}{12155d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^8*(Cos[d*x] - I*Sin[d*x])*(646*Cos[c + d*x] + 1121*Cos[3*
(c + d*x)] - (11*I)*(34*Sin[c + d*x] + 99*Sin[3*(c + d*x)]))*((-I)*Cos[5*c
+ 6*d*x] + Sin[5*c + 6*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(12155*d)
```

Maple [A] time = 5.216, size = 152, normalized size = 1.3

$$-\frac{2a\left(2048i(\cos(dx+c))^8-2048\sin(dx+c)(\cos(dx+c))^7+256i(\cos(dx+c))^6-1280(\cos(dx+c))^5\sin(dx+\right.}{12155d(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -2/12155/d*a*(2048*I*cos(d*x+c)^8-2048*sin(d*x+c)*cos(d*x+c)^7+256*I*cos(d*
x+c)^6-1280*cos(d*x+c)^5*sin(d*x+c)+112*I*cos(d*x+c)^4-1008*cos(d*x+c)^3*si
n(d*x+c)+66*I*cos(d*x+c)^2-858*cos(d*x+c)*sin(d*x+c)-715*I)*(a*(I*sin(d*x+c
)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^8
```

Maptima [A] time = 1.04075, size = 103, normalized size = 0.88

$$\frac{2i\left(715(ia\tan(dx+c)+a)^{\frac{17}{2}}-4862(ia\tan(dx+c)+a)^{\frac{15}{2}}a+11220(ia\tan(dx+c)+a)^{\frac{13}{2}}a^2-8840(ia\tan(dx+\right.}{12155a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/12155*I*(715*(I*a*tan(d*x + c) + a)^(17/2) - 4862*(I*a*tan(d*x + c) + a)^
(15/2)*a + 11220*(I*a*tan(d*x + c) + a)^(13/2)*a^2 - 8840*(I*a*tan(d*x + c)
 + a)^(11/2)*a^3)/(a^7*d)
```

Fricas [B] time = 2.59432, size = 578, normalized size = 4.94

$$\frac{\sqrt{2}\left(-8192ia e^{(16idx+16ic)}-69632ia e^{(14idx+14ic)}-261120ia e^{(12idx+12ic)}-565760ia e^{(10idx+10ic)}\right)\sqrt{\frac{a}{e^{(2idx+2i}\cdots}}}{12155\left(de^{(16idx+16ic)}+8de^{(14idx+14ic)}+28de^{(12idx+12ic)}+56de^{(10idx+10ic)}+70de^{(8idx+8ic)}+56de^{(6idx+6ic)}+28de^{(4id}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12155*sqrt(2)*(-8192*I*a*e^(16*I*d*x + 16*I*c) - 69632*I*a*e^(14*I*d*x +
14*I*c) - 261120*I*a*e^(12*I*d*x + 12*I*c) - 565760*I*a*e^(10*I*d*x + 10*I*
c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(d*e^(16*I*d*x + 16*I
*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I
*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d
```

```
*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan(dx+c) + a)^{\frac{3}{2}} \sec(dx+c)^8 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^8, x)
```

3.294 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=88

$$-\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^5d}+\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^4d}-\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^3d}$$

```
[Out] (((-8*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^3*d) + (((8*I)/11)*(a + I*a*Ta
n[c + d*x])^(11/2))/(a^4*d) - (((2*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a
^5*d)
```

Rubi [A] time = 0.0768244, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$-\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^5d}+\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^4d}-\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```



```
[Out] (((-8*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^3*d) + (((8*I)/11)*(a + I*a*Ta
n[c + d*x])^(11/2))/(a^4*d) - (((2*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a
^5*d)
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^2(a+x)^{7/2}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a+x)^{7/2}-4a(a+x)^{9/2}+(a+x)^{11/2}\right)dx, x, ia\tan(c+dx\right.}{a^5d}\\ &= -\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^3d}+\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^4d}-\frac{2i(a+ia\tan(c+}{13a^5d}\end{aligned}$$

Mathematica [A] time = 0.491337, size = 93, normalized size = 1.06

$$\frac{2a\sec^6(c+dx)(\cos(dx)-i\sin(dx))\sqrt{a+ia\tan(c+dx)}(-135i\sin(2(c+dx))+151\cos(2(c+dx))+52)(\sin(4c+5dx}{1287d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^6*(Cos[d*x] - I*Sin[d*x])*(52 + 151*Cos[2*(c + d*x)] - (1
35*I)*Sin[2*(c + d*x)])*((-I)*Cos[4*c + 5*d*x] + Sin[4*c + 5*d*x])*Sqrt[a +
 I*a*Tan[c + d*x]])/(1287*d)
```

Maple [A] time = 0.335, size = 125, normalized size = 1.4

$$-\frac{2\,a\left(256\,i\left(\cos\left(dx+c\right)\right)^{6}-256\,\left(\cos\left(dx+c\right)\right)^{5}\sin\left(dx+c\right)+32\,i\left(\cos\left(dx+c\right)\right)^{4}-160\,\left(\cos\left(dx+c\right)\right)^{3}\sin\left(dx+c\right)+14\,i\right.}{1287\,d\left(\cos\left(dx+c\right)\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -2/1287/d*a*(256*I*cos(d*x+c)^6-256*cos(d*x+c)^5*sin(d*x+c)+32*I*cos(d*x+c)
^4-160*cos(d*x+c)^3*sin(d*x+c)+14*I*cos(d*x+c)^2-126*cos(d*x+c)*sin(d*x+c)-
99*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^6
```

Maxima [A] time = 1.00145, size = 78, normalized size = 0.89

$$-\frac{2i\left(99\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{13}{2}}-468\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}a+572\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}a^{2}\right)}{1287\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1287*I*(99*(I*a*tan(d*x + c) + a)^(13/2) - 468*(I*a*tan(d*x + c) + a)^(1
1/2)*a + 572*(I*a*tan(d*x + c) + a)^(9/2)*a^2)/(a^5*d)
```

Fricas [B] time = 2.3263, size = 446, normalized size = 5.07

$$\frac{\sqrt{2}\left(-1024i\,ae^{(12i\,dx+12i\,c)}-6656i\,ae^{(10i\,dx+10i\,c)}-18304i\,ae^{(8i\,dx+8i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{1287\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1287*sqrt(2)*(-1024*I*a*e^(12*I*d*x + 12*I*c) - 6656*I*a*e^(10*I*d*x + 10
*I*c) - 18304*I*a*e^(8*I*d*x + 8*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^
(I*d*x + I*c)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e
^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) +
6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^6, x)
```

3.295 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^3d}-\frac{4i(a+ia\tan(c+dx))^{7/2}}{7a^2d}$$

```
[Out] (((-4*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^2*d) + (((2*I)/9)*(a + I*a*Tan
[c + d*x])^(9/2))/(a^3*d)
```

Rubi [A] time = 0.0687435, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^3d}-\frac{4i(a+ia\tan(c+dx))^{7/2}}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-4*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^2*d) + (((2*I)/9)*(a + I*a*Tan
[c + d*x])^(9/2))/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)(a+x)^{5/2}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(2a(a+x)^{5/2}-(a+x)^{7/2}\right)\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{7/2}}{7a^2d}+\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^3d}\end{aligned}$$

Mathematica [A] time = 0.473612, size = 81, normalized size = 1.37

$$-\frac{2a(7\tan(c+dx)+11i)\sec^3(c+dx)(\cos(dx)-i\sin(dx))\sqrt{a+ia\tan(c+dx)}(\cos(3c+4dx)+i\sin(3c+4dx))}{63d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*Sec[c + d*x]^3*(Cos[d*x] - I*Sin[d*x])*(Cos[3*c + 4*d*x] + I*Sin[3*c
+ 4*d*x])*(11*I + 7*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(63*d)
```

Maple [B] time = 0.289, size = 98, normalized size = 1.7

$$-\frac{2a\left(16i(\cos(dx+c))^4-16(\cos(dx+c))^3\sin(dx+c)+2i(\cos(dx+c))^2-10\cos(dx+c)\sin(dx+c)-7i\right)}{63d(\cos(dx+c))^4}\sqrt{a(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -2/63/d*a*(16*I*cos(d*x+c)^4-16*cos(d*x+c)^3*sin(d*x+c)+2*I*cos(d*x+c)^2-10
*cos(d*x+c)*sin(d*x+c)-7*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/
cos(d*x+c)^4
```

Maxima [A] time = 1.02943, size = 54, normalized size = 0.92

$$\frac{2i\left(7\left(ia\tan(dx+c)+a\right)^{\frac{9}{2}}-18\left(ia\tan(dx+c)+a\right)^{\frac{7}{2}}a\right)}{63a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/63*I*(7*(I*a*tan(d*x + c) + a)^(9/2) - 18*(I*a*tan(d*x + c) + a)^(7/2)*a)
/(a^3*d)
```

Fricas [B] time = 2.35138, size = 311, normalized size = 5.27

$$\frac{\sqrt{2}\left(-64i\,ae^{(8i\,dx+8i\,c)}-288i\,ae^{(6i\,dx+6i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{63\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/63*sqrt(2)*(-64*I*a*e^(8*I*d*x + 8*I*c) - 288*I*a*e^(6*I*d*x + 6*I*c))*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(d*e^(8*I*d*x + 8*I*c) + 4*
d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ia\tan(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

3.296 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=29

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a*d)
```

Rubi [A] time = 0.060965, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 32}

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= -\frac{i\,\text{Subst}\left(\int (a+x)^{3/2}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}\end{aligned}$$

Mathematica [B] time = 0.265127, size = 69, normalized size = 2.38

$$\frac{2a\sec^2(c+dx)(\cos(dx)-i\sin(dx))\sqrt{a+ia\tan(c+dx)}(\sin(2c+3dx)-i\cos(2c+3dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^2*(Cos[d*x] - I*Sin[d*x])*((-I)*Cos[2*c + 3*d*x] + Sin[2*
c + 3*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(5*d)
```

Maple [A] time = 0.03, size = 24, normalized size = 0.8

$$\frac{-\frac{2i}{5}}{ad}(a+ia\tan(dx+c))^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a/d
```

Maxima [A] time = 1.10683, size = 28, normalized size = 0.97

$$-\frac{2i\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{5\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*I*(I*a*tan(d*x + c) + a)^(5/2)/(a*d)
```

Fricas [B] time = 2.34836, size = 170, normalized size = 5.86

$$-\frac{8i\,\sqrt{2}a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(5i\,dx+5i\,c)}}{5\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -8/5*I*sqrt(2)*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(5*I*d*x + 5*I*c)/(d*e
^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

3.297 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=93

$$-\frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))}-\frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}$$

```
[Out] ((-I/2)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqr
t[2]*d) - ((I/2)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0834103, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))}-\frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I/2)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqr
t[2]*d) - ((I/2)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))} - \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{4d}$$
$$= -\frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))} - \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{2d}$$
$$= -\frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d} - \frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))}$$

Mathematica [A] time = 0.580332, size = 97, normalized size = 1.04

$$-\frac{iae^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}+\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I/4)*a*Sqrt[1 + E^((2*I)*(c + d*x))]*(E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*
(c + d*x))] + ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I
*(c + d*x)))
```

Maple [B] time = 0.332, size = 396, normalized size = 4.3

$$\frac{a}{8\,d\,(i\sin(dx+c)+\cos(dx+c)-1)\cos(dx+c)}\left(i\sqrt{2}\sin(dx+c)\cos(dx+c)\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{\frac{3}{2}}\text{Artanh}\left(\frac{\sqrt{2}\,\text{si}}{2\,\text{co}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/8/d*a*(I*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/
2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
d*x+c))+I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+cos(
d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctan(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)+2^(1/2)*arctan(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin
(d*x+c)-8*I*cos(d*x+c)^4+4*I*cos(d*x+c)^3+8*cos(d*x+c)^3*sin(d*x+c)+4*I*cos
(d*x+c)^2-4*cos(d*x+c)^2*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c
))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.36565, size = 671, normalized size = 7.22

$$\frac{\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}+\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}d\log\left(\frac{\left(2i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(2i\,dx+2i\,c)}+\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}\right)e^{(-2i\,dx}}{a}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(-I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*e^(I*d*x + I*c) + sqrt(1/2)*sqrt(-a^3/d^2)*d*log((2*I*sqrt(1/2)*sqrt(-
a^3/d^2)*d*e^(2*I*d*x + 2*I*c) + sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-2*I*d*x - 2*I*c)/a) - sqrt
(1/2)*sqrt(-a^3/d^2)*d*log((-2*I*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(2*I*d*x + 2*
I*c) + sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(I*d*x + I*c))*e^(-2*I*d*x - 2*I*c)/a))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.298 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=166

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^3}{16d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}+\frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (((-15*I)/32)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/(Sqrt[2]*d) + (((15*I)/32)*a^2)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/4)*a
^4)/(d*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((5*I)/16)*a
^3)/(d*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.105757, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^3}{16d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}+\frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] (((-15*I)/32)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/(Sqrt[2]*d) + (((15*I)/32)*a^2)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/4)*a
^4)/(d*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((5*I)/16)*a
^3)/(d*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{\left(ia^5\right)\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} - \frac{\left(5ia^4\right)\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{3/2}}\,d\right.}{8d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^3}{16d(a-ia\tan(c+dx))\sqrt{a+i}}$$

$$= \frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} - \frac{}{16d(}$$

$$= \frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} - \frac{}{16d(}$$

$$= -\frac{15ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d} + \frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}} - \frac{}{4d(a-ia\tan(c}$$

Mathematica [A] time = 0.730225, size = 143, normalized size = 0.86

$$\frac{ae^{-2i(c+dx)}\cos^2(c+dx)(\tan(c+dx)-i)\left(\sqrt{1+e^{2i(c+dx)}}\left(9e^{2i(c+dx)}+2e^{4i(c+dx)}-8\right)+15e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan}}{32d\sqrt{1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-8 + 9*E^((2*I)*(c + d*x)) + 2*E^((4*I)*
(c + d*x))) + 15*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))])*Cos[c + d*x]^2*(
-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(32*d*E^((2*I)*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))])
```

Maple [B] time = 0.313, size = 742, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/512/d*a*(-256*I*cos(d*x+c)^8-32*I*cos(d*x+c)^6+15*2^(1/2)*arctan(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7
/2)*cos(d*x+c)^3*sin(d*x+c)+45*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*
sin(d*x+c)*cos(d*x+c)^2*2^(1/2)+45*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^2*
sin(d*x+c)+15*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*arctanh(1/2*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*
```

```
x+c)^3*2^(1/2)+45*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)*sin(d*x+c)+15*2^(1/
2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(7/2)*sin(d*x+c)+45*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*
arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x
+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+128*I*cos(d*x+c)^7+256*sin(d*x+c)*cos(d*
x+c)^7+15*I*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*sin(d*x+c)-12
8*cos(d*x+c)^6*sin(d*x+c)-80*I*cos(d*x+c)^5+160*cos(d*x+c)^5*sin(d*x+c)+240
*I*cos(d*x+c)^4-240*sin(d*x+c)*cos(d*x+c)^4)*(a*(I*sin(d*x+c)+cos(d*x+c))/c
os(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66866, size = 856, normalized size = 5.16

$$\left(15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(2i\,dx+2i\,c)}\log\left(\frac{\left(30i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(2i\,dx+2i\,c)}+15\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{15a}\right)-15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(2i\,dx+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(15*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(2*I*d*x + 2*I*c)*log(1/15*(30*I*sqrt
(1/2)*sqrt(-a^3/d^2)*d*e^(2*I*d*x + 2*I*c) + 15*sqrt(2)*(a*e^(2*I*d*x + 2*I
*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c
)/a) - 15*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(2*I*d*x + 2*I*c)*log(1/15*(-30*I*sq
rt(1/2)*sqrt(-a^3/d^2)*d*e^(2*I*d*x + 2*I*c) + 15*sqrt(2)*(a*e^(2*I*d*x + 2
*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)/a) + sqrt(2)*(-2*I*a*e^(6*I*d*x + 6*I*c) - 11*I*a*e^(4*I*d*x + 4*I*c) -
 I*a*e^(2*I*d*x + 2*I*c) + 8*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*
x + I*c))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.299 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=239

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}-\frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}-\frac{}{64d(a-ia\tan(c+}$$

```
[Out] (((-105*I)/256)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a]
)])/(Sqrt[2]*d) + (((35*I)/128)*a^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I
/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(3/2)) - (((3*I
)/16)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) - (((2
1*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (((
105*I)/256)*a^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.134344, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}-\frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}-\frac{}{64d(a-ia\tan(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] (((-105*I)/256)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a]
)])/(Sqrt[2]*d) + (((35*I)/128)*a^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I
/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(3/2)) - (((3*I
)/16)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) - (((2
1*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (((
105*I)/256)*a^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{\left(ia^7\right)\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} - \frac{\left(3ia^6\right)\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{5/2}}\right.}{4d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} - \frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} - \frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a}$$

$$= \frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} -$$

$$= \frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} -$$

$$= \frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} -$$

$$= -\frac{105ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}d} + \frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}} - \frac{}{6d(a-ia\,\mathrm{t}}$$

Mathematica [A] time = 0.981902, size = 169, normalized size = 0.71

$$\frac{ae^{-4i(c+dx)}\cos^2(c+dx)(\tan(c+dx)-i)\left(\sqrt{1+e^{2i(c+dx)}}\left(-208e^{2i(c+dx)}+165e^{4i(c+dx)}+50e^{6i(c+dx)}+8e^{8i(c+dx)}-16\right)+315e^{3}\right.}{768d\sqrt{1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-16 - 208*E^((2*I)*(c + d*x)) + 165*E^((
4*I)*(c + d*x)) + 50*E^((6*I)*(c + d*x)) + 8*E^((8*I)*(c + d*x))) + 315*E^(
(3*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))])*Cos[c + d*x]^2*(-I + Tan[c + d*x
])*Sqrt[a + I*a*Tan[c + d*x]])/(768*d*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)
*(c + d*x))])
```

Maple [B] time = 0.365, size = 1086, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/49152/d*a*(315*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*2^(1/2)+1575*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)+3150*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)+3150*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)
*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(
d*x+c)*2^(1/2)+1575*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)+315*I
*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*sin(d*x+c)+1575*I*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^4*sin(d*x+c)+315*I
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^5*sin(d*x+c)
+9216*sin(d*x+c)*cos(d*x+c)^9+16384*sin(d*x+c)*cos(d*x+c)^11-8192*sin(d*x+c
)*cos(d*x+c)^10-2688*I*cos(d*x+c)^8-6720*I*cos(d*x+c)^7+20160*I*cos(d*x+c)^
6-16384*I*cos(d*x+c)^12+8192*I*cos(d*x+c)^11-1024*I*cos(d*x+c)^10-1536*I*co
s(d*x+c)^9-10752*sin(d*x+c)*cos(d*x+c)^8+13440*sin(d*x+c)*cos(d*x+c)^7-2016
0*cos(d*x+c)^6*sin(d*x+c)+315*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*2^(1/2)
*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+3150*I
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^3*sin(d*x+c)
+3150*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^2*sin
(d*x+c)+1575*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c
)*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+
cos(d*x+c)-1)/cos(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.76755, size = 959, normalized size = 4.01

$$\left(315\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(4i\,dx+4i\,c)}\log\left(\frac{\left(210i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(2i\,dx+2i\,c)}+105\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{105\,a}\right)-315\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(4i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(315*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(4*I*d*x + 4*I*c)*log(1/105*(210*I
*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(2*I*d*x + 2*I*c) + 105*sqrt(2)*(a*e^(2*I*d*x
```

```
 + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x
 - I*c)/a) - 315*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(4*I*d*x + 4*I*c)*log(1/105*(
-210*I*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(2*I*d*x + 2*I*c) + 105*sqrt(2)*(a*e^(2
*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(
-I*d*x - I*c)/a) + sqrt(2)*(-8*I*a*e^(10*I*d*x + 10*I*c) - 58*I*a*e^(8*I*d*
x + 8*I*c) - 215*I*a*e^(6*I*d*x + 6*I*c) + 43*I*a*e^(4*I*d*x + 4*I*c) + 224
*I*a*e^(2*I*d*x + 2*I*c) + 16*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d
*x + I*c))*e^(-4*I*d*x - 4*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.300 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=147

$$\frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}}+\frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}}+\frac{256ia^4\sec^5(c+dx)}{1155d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(}}{11d}$$

```
[Out] (((256*I)/1155)*a^4*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((6
4*I)/231)*a^3*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((8*I)/33
)*a^2*Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/11)*a*Sec[c
+ d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.240723, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}}+\frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}}+\frac{256ia^4\sec^5(c+dx)}{1155d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((256*I)/1155)*a^4*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((6
4*I)/231)*a^3*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((8*I)/33
)*a^2*Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/11)*a*Sec[c
+ d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= \frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}+\frac{1}{11}(12a)\int \sec^5(c+dx)\sqrt{a+ia\tan}\\ &= \frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}+\frac{1}{33}(32a\\ &= \frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}}+\frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec^5(c+dx)}{1}\\ &= \frac{256ia^4\sec^5(c+dx)}{1155d(a+ia\tan(c+dx))^{5/2}}+\frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}}+\frac{8ia^2\sec^5}{33d\sqrt{a+ia\tan}}\end{aligned}$$

Mathematica [A] time = 0.954824, size = 109, normalized size = 0.74

$$\frac{2a\sec^4(c+dx)(\cos(dx)-i\sin(dx))\sqrt{a+ia\tan(c+dx)}(\sin(3c+2dx)+i\cos(3c+2dx))(494\cos(2(c+dx))+110i\tan(c}{1155d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^4*(Cos[d*x] - I*Sin[d*x])*(I*Cos[3*c + 2*d*x] + Sin[3*c +
 2*d*x])*(39 + 494*Cos[2*(c + d*x)] + (215*I)*Sec[c + d*x]*Sin[3*(c + d*x)]
 + (110*I)*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(1155*d)
```

Maple [A] time = 0.321, size = 125, normalized size = 0.9

$$\frac{2a\left(512i(\cos(dx+c))^6+512(\cos(dx+c))^5\sin(dx+c)-64i(\cos(dx+c))^4+192(\cos(dx+c))^3\sin(dx+c)-20i(c}{1155d(\cos(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2/1155/d*a*(512*I*cos(d*x+c)^6+512*cos(d*x+c)^5*sin(d*x+c)-64*I*cos(d*x+c)^
4+192*cos(d*x+c)^3*sin(d*x+c)-20*I*cos(d*x+c)^2+140*cos(d*x+c)*sin(d*x+c)+1
05*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5
```

Maxima [B] time = 69.6067, size = 1345, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -(-17075520*I*sqrt(2)*a*cos(6*d*x + 6*c) - 14636160*I*sqrt(2)*a*cos(4*d*x +
 4*c) - 6504960*I*sqrt(2)*a*cos(2*d*x + 2*c) + 17075520*sqrt(2)*a*sin(6*d*x
 + 6*c) + 14636160*sqrt(2)*a*sin(4*d*x + 4*c) + 6504960*sqrt(2)*a*sin(2*d*x
 + 2*c) - 1182720*I*sqrt(2)*a)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)/(((5336100*cos(2*d*x + 2*c)^3 + 133402
5*(4*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 5336100*I*sin(2*d*x + 2*c)^
3 + 1334025*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*cos(8*d*x + 8*c) + 5336100*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 8004150*(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 12006225*cos(2
*d*x + 2*c)^2 - (-1334025*I*cos(2*d*x + 2*c)^2 - 1334025*I*sin(2*d*x + 2*c)
^2 - 2668050*I*cos(2*d*x + 2*c) - 1334025*I)*sin(8*d*x + 8*c) - (-5336100*I
*cos(2*d*x + 2*c)^2 - 5336100*I*sin(2*d*x + 2*c)^2 - 10672200*I*cos(2*d*x +
 2*c) - 5336100*I)*sin(6*d*x + 6*c) - (-8004150*I*cos(2*d*x + 2*c)^2 - 8004
150*I*sin(2*d*x + 2*c)^2 - 16008300*I*cos(2*d*x + 2*c) - 8004150*I)*sin(4*d
*x + 4*c) - (-5336100*I*cos(2*d*x + 2*c)^2 - 10672200*I*cos(2*d*x + 2*c) -
5336100*I)*sin(2*d*x + 2*c) + 8004150*cos(2*d*x + 2*c) + 1334025)*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (-5336100*I*cos(2*d*x + 2
*c)^3 + (-5336100*I*cos(2*d*x + 2*c) - 1334025*I)*sin(2*d*x + 2*c)^2 + 5336
100*sin(2*d*x + 2*c)^3 + (-1334025*I*cos(2*d*x + 2*c)^2 - 1334025*I*sin(2*d
```

```
*x + 2*c)^2 - 2668050*I*cos(2*d*x + 2*c) - 1334025*I)*cos(8*d*x + 8*c) + (-
5336100*I*cos(2*d*x + 2*c)^2 - 5336100*I*sin(2*d*x + 2*c)^2 - 10672200*I*co
s(2*d*x + 2*c) - 5336100*I)*cos(6*d*x + 6*c) + (-8004150*I*cos(2*d*x + 2*c)
^2 - 8004150*I*sin(2*d*x + 2*c)^2 - 16008300*I*cos(2*d*x + 2*c) - 8004150*I
)*cos(4*d*x + 4*c) - 12006225*I*cos(2*d*x + 2*c)^2 + 1334025*(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8*c) + 533
6100*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin
(6*d*x + 6*c) + 8004150*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*sin(4*d*x + 4*c) + 5336100*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)*sin(2*d*x + 2*c) - 8004150*I*cos(2*d*x + 2*c) - 1334025*I)*si
n(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*d)
```

Fricas [A] time = 2.33687, size = 437, normalized size = 2.97

$$\frac{\sqrt{2}\left(14784i\,ae^{(6i\,dx+6i\,c)}+12672i\,ae^{(4i\,dx+4i\,c)}+5632i\,ae^{(2i\,dx+2i\,c)}+1024i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{1155\left(de^{(11i\,dx+11i\,c)}+5\,de^{(9i\,dx+9i\,c)}+10\,de^{(7i\,dx+7i\,c)}+10\,de^{(5i\,dx+5i\,c)}+5\,de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*sqrt(2)*(14784*I*a*e^(6*I*d*x + 6*I*c) + 12672*I*a*e^(4*I*d*x + 4*I*
c) + 5632*I*a*e^(2*I*d*x + 2*I*c) + 1024*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*e^(I*d*x + I*c)/(d*e^(11*I*d*x + 11*I*c) + 5*d*e^(9*I*d*x + 9*I*c) + 1
0*d*e^(7*I*d*x + 7*I*c) + 10*d*e^(5*I*d*x + 5*I*c) + 5*d*e^(3*I*d*x + 3*I*c
) + d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^5, x)
```

3.301 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=110

$$\frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{64ia^3\sec^3(c+dx)}{105d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

```
[Out] (((64*I)/105)*a^3*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((16*
I)/35)*a^2*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/7)*a*Se
c[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.176691, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{64ia^3\sec^3(c+dx)}{105d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] (((64*I)/105)*a^3*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((16*
I)/35)*a^2*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/7)*a*Se
c[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d
```


Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] && !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= \frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}+\frac{1}{7}(8a)\int \sec^3(c+dx)\sqrt{a+ia\tan(c+\\ &= \frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}+\frac{1}{35}\left(32a^2\right)\\ &= \frac{64ia^3\sec^3(c+dx)}{105d(a+ia\tan(c+dx))^{3/2}}+\frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec^3(c+dx)\sqrt{a-}}{7d}\end{aligned}$$

Mathematica [A] time = 0.414873, size = 91, normalized size = 0.83

$$\frac{2a\sec^3(c+dx)(\cos(dx)-i\sin(dx))\sqrt{a+ia\tan(c+dx)}(27i\sin(2(c+dx))+43\cos(2(c+dx))+28)(\sin(2c+dx)+i\cos}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^3*(Cos[d*x] - I*Sin[d*x])*(28 + 43*Cos[2*(c + d*x)] + (27
*I)*Sin[2*(c + d*x)])*(I*Cos[2*c + d*x] + Sin[2*c + d*x])*Sqrt[a + I*a*Tan[
c + d*x]])/(105*d)
```

Maple [A] time = 0.286, size = 98, normalized size = 0.9

$$\frac{2\,a\left(64\,i\,(\cos(dx+c))^{4}+64\,(\cos(dx+c))^{3}\sin(dx+c)-8\,i\,(\cos(dx+c))^{2}+24\,\cos(dx+c)\sin(dx+c)+15\,i\right)}{105\,d\,(\cos(dx+c))^{3}}\sqrt{a\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2/105/d*a*(64*I*cos(d*x+c)^4+64*cos(d*x+c)^3*sin(d*x+c)-8*I*cos(d*x+c)^2+24
*cos(d*x+c)*sin(d*x+c)+15*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
/cos(d*x+c)^3
```

Maxima [B] time = 3.53031, size = 788, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -(-560*I*sqrt(2)*a*cos(4*d*x + 4*c) - 448*I*sqrt(2)*a*cos(2*d*x + 2*c) + 56
0*sqrt(2)*a*sin(4*d*x + 4*c) + 448*sqrt(2)*a*sin(2*d*x + 2*c) - 128*I*sqrt(
2)*a)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sqrt(a)/(((210*cos(2*d*x + 2*c)^3 + 105*(2*cos(2*d*x + 2*c) + 1)*sin(2*
d*x + 2*c)^2 + 210*I*sin(2*d*x + 2*c)^3 + 105*(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 525*cos(2*d*x + 2*
c)^2 - (-105*I*cos(2*d*x + 2*c)^2 - 105*I*sin(2*d*x + 2*c)^2 - 210*I*cos(2*
d*x + 2*c) - 105*I)*sin(4*d*x + 4*c) - (-210*I*cos(2*d*x + 2*c)^2 - 420*I*c
os(2*d*x + 2*c) - 210*I)*sin(2*d*x + 2*c) + 420*cos(2*d*x + 2*c) + 105)*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (-210*I*cos(2*d*x +
 2*c)^3 + (-210*I*cos(2*d*x + 2*c) - 105*I)*sin(2*d*x + 2*c)^2 + 210*sin(2*
d*x + 2*c)^3 + (-105*I*cos(2*d*x + 2*c)^2 - 105*I*sin(2*d*x + 2*c)^2 - 210*
I*cos(2*d*x + 2*c) - 105*I)*cos(4*d*x + 4*c) - 525*I*cos(2*d*x + 2*c)^2 + 1
05*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4
*d*x + 4*c) + 210*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x +
 2*c) - 420*I*cos(2*d*x + 2*c) - 105*I)*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)))*d)
```

Fricas [A] time = 2.21079, size = 312, normalized size = 2.84

$$\frac{\sqrt{2}\left(560i\,ae^{(4i\,dx+4i\,c)}+448i\,ae^{(2i\,dx+2i\,c)}+128i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{105\left(de^{(7i\,dx+7i\,c)}+3\,de^{(5i\,dx+5i\,c)}+3\,de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*sqrt(2)*(560*I*a*e^(4*I*d*x + 4*I*c) + 448*I*a*e^(2*I*d*x + 2*I*c) +
128*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(d*e^(7*I*d*x +
7*I*c) + 3*d*e^(5*I*d*x + 5*I*c) + 3*d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I
*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

3.302 $\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

```
[Out] (((8*I)/3)*a^2*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/3)*a*
Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.0643351, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3494, 3493}

$$\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((8*I)/3)*a^2*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/3)*a*
Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= \frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{1}{3}(4a)\int \sec(c+dx)\sqrt{a+ia\tan(c+\\ &= \frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\end{aligned}$$

Mathematica [A] time = 0.24852, size = 57, normalized size = 0.83

$$-\frac{2a(\cos(c)-i\sin(c))(\tan(c+dx)-5i)(\cos(dx)-i\sin(dx))\sqrt{a+ia\tan(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(Cos[c] - I*Sin[c])*(Cos[d*x] - I*Sin[d*x])*(-5*I + Tan[c + d*x])*Sqr
t[a + I*a*Tan[c + d*x]])/(3*d)
```

Maple [A] time = 0.237, size = 71, normalized size = 1.

$$\frac{2\,a\left(4\,i\left(\cos\left(dx+c\right)\right)^{2}+4\,\cos\left(dx+c\right)\sin\left(dx+c\right)+i\right)}{3\,d\cos\left(dx+c\right)}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2/3/d*a*(4*I*cos(d*x+c)^2+4*cos(d*x+c)*sin(d*x+c)+I)*(a*(I*sin(d*x+c)+cos(d
*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [A] time = 2.30756, size = 194, normalized size = 2.81

$$\frac{\sqrt{2}\left(12i\,ae^{(2i\,dx+2i\,c)}+8i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{3\left(de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*(12*I*a*e^(2*I*d*x + 2*I*c) + 8*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*e^(I*d*x + I*c)/(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan(dx + c) + a)^{\frac{3}{2}} \sec(dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.303 $\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=31

$$-\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] ((-2*I)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.0490911, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3493}

$$-\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-2*I)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Mathematica [A] time = 0.135678, size = 31, normalized size = 1.

$$-\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-2*I)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A] time = 0.259, size = 42, normalized size = 1.4

$$\frac{-2\,ia\cos(dx+c)}{d}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -2*I/d*a*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)
```

Maxima [B] time = 1.87649, size = 271, normalized size = 8.74

$$\frac{2\left(i\,a^{\frac{3}{2}}-\frac{2i\,a^{\frac{3}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{i\,a^{\frac{3}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{3}{2}}\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}-\frac{2i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(I*a^(3/2) - 2*I*a^(3/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + I*a^(3/2)*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1)
+ sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)/(d*(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)*(-2*I*sin(
d*x + c)/(cos(d*x + c) + 1) - 2*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1))
```

Fricas [A] time = 2.20871, size = 105, normalized size = 3.39

$$\frac{\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(-I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/
d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

3.304 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=122

$$\frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}d}-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

```
[Out] ((I/2)*a^(3/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*d) - ((I/2)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
 - ((I/3)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.149419, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3490, 3489, 206}

$$\frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}d}-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] ((I/2)*a^(3/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*d) - ((I/2)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
 - ((I/3)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```


Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{1}{2}a\int \cos(c+dx)\sqrt{a+ia\tan(c+dx}$$

$$= -\frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} - \frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} +$$

$$= -\frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} - \frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} +$$

$$= \frac{ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}d} - \frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} - \frac{i\cos^3(}{}$$

Mathematica [A] time = 0.881703, size = 101, normalized size = 0.83

$$-\frac{iae^{-i(c+dx)}\left(5e^{2i(c+dx)}+e^{4i(c+dx)}-3\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)+4\right)\sqrt{a+ia\tan(c+dx)}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I/12)*a*(4 + 5*E^((2*I)*(c + d*x)) + E^((4*I)*(c + d*x)) - 3*Sqrt[1 + E^
((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan
[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] time = 0.36, size = 570, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/48/d*a*(-3*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)
*cos(d*x+c)^2+3*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*cos(d*x+c)^2*sin(d*x+c)-6*I*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*sin(d*x+c)*cos(d*x+c)+6*2^(1/
2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(5/2)*cos(d*x+c)*sin(d*x+c)-3*I*2^(1/2)*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+3*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)-3
2*I*cos(d*x+c)^6+32*cos(d*x+c)^5*sin(d*x+c)+16*I*cos(d*x+c)^5-16*sin(d*x+c)
*cos(d*x+c)^4-8*I*cos(d*x+c)^4+24*cos(d*x+c)^3*sin(d*x+c)+24*I*cos(d*x+c)^3
)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1
)/cos(d*x+c)^2
```

Maxima [B] time = 2.21607, size = 1192, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/48*(4*(I*sqrt(2)*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqr
t(a) + 12*(I*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sq
rt(a) + (6*sqrt(2)*a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 6*s
qrt(2)*a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - 3*I*sqrt(2)*a*l
og(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + 1) + 3*I*sqrt(2)*a*log(sqrt(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))
*sqrt(a))/d
```

Fricas [B] time = 2.37164, size = 764, normalized size = 6.26

$$-\frac{\left(3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(idx+ic)}\log\left(\frac{\left(2i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(idx+ic)}+\sqrt{2}\left(ae^{(2idx+2ic)}+a\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{(idx+ic)}\right)e^{(-idx-ic)}}{a}\right)-3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(idx+ic)}\log\left(\left(\cdots\right)\right)\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(I*d*x + I*c)*log((2*I*sqrt(1/2)*sqrt
(-a^3/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 3*sqrt(1/2
)*sqrt(-a^3/d^2)*d*e^(I*d*x + I*c)*log((-2*I*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(
I*d*x + I*c) + sqrt(2)*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(2)*(-I*a*e^(4*I*d*x +
 4*I*c) - 5*I*a*e^(2*I*d*x + 2*I*c) - 4*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan (dx+c)+a)^{\frac{3}{2}} \cos (dx+c)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

3.305 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=192

$$\frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}}+\frac{7ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}d}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}-\frac{7ia\cos^3(c+dx)\cdots}{3\cdots}$$

```
[Out] (((7*I)/16)*a^(3/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Ta
n[c + d*x]])])/(Sqrt[2]*d) + (((7*I)/24)*a^2*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) - (((7*I)/16)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d -
 (((7*I)/30)*a*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/5)*Cos[c
+ d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.270018, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3497, 3502, 3490, 3489, 206}

$$\frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}}+\frac{7ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}d}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}-\frac{7ia\cos^3(c+dx)\cdots}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] (((7*I)/16)*a^(3/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Ta
n[c + d*x]])])/(Sqrt[2]*d) + (((7*I)/24)*a^2*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) - (((7*I)/16)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d -
 (((7*I)/30)*a*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/5)*Cos[c
+ d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d
```


Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}+\frac{1}{10}(7a)\int \cos^3(c+dx)\sqrt{a+ia\tan(c$$

$$= -\frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

$$= \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}}-\frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d}-\frac{i\cos^5(c+d$$

$$= \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}}-\frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d}-\frac{7ia\cos^3(c+$$

$$= \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}}-\frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d}-\frac{7ia\cos^3(c+$$

$$= \frac{7ia^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}d}+\frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}}-\frac{7ia\cos(c+dx)$$

Mathematica [A] time = 1.37671, size = 160, normalized size = 0.83

$$-\frac{iae^{-3i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(101e^{2i(c+dx)}+148e^{4i(c+dx)}+38e^{6i(c+dx)}+6e^{8i(c+dx)}-105e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+}\right.\right.}{240\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I/240)*a*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-15 +
101*E^((2*I)*(c + d*x)) + 148*E^((4*I)*(c + d*x)) + 38*E^((6*I)*(c + d*x))
+ 6*E^((8*I)*(c + d*x)) - 105*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*
x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*E^((3*I)*(c + d*x)
))
```

Maple [B] time = 0.313, size = 914, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -1/7680/d*a*(256*I*cos(d*x+c)^8+1120*I*cos(d*x+c)^6-105*arctan(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*
2^(1/2)*cos(d*x+c)^4*sin(d*x+c)+3072*I*cos(d*x+c)^10-420*arctan(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)
*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+105*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)
*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)/cos(d*x+c))*sin(d*x+c)-630*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(d*x+c)^2*sin(d*
x+c)+420*I*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arcta
nh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*
2^(1/2)-420*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(d*x+c)*sin(d*x+c)-105*2^(1/2)*ar
ctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(9/2)*sin(d*x+c)+105*I*cos(d*x+c)^4*sin(d*x+c)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+448*I*cos(d*x+c)^7-3072*sin(d*x+c)*cos(d*x
+c)^9-1536*I*cos(d*x+c)^9+1536*sin(d*x+c)*cos(d*x+c)^8+630*I*cos(d*x+c)^2*s
in(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)-1792*sin(d*x+c)
*cos(d*x+c)^7-3360*I*cos(d*x+c)^5+2240*cos(d*x+c)^6*sin(d*x+c)+420*I*cos(d*
x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)-3360*co
s(d*x+c)^5*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*si
n(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.39696, size = 892, normalized size = 4.65

$$-\frac{\left(105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(3i\,dx+3i\,c)}\log\left(\frac{\left(14i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(i\,dx+i\,c)}+7\sqrt{2}\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{7\,a}\right)-105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(3i\,dx\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/480*(105*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(3*I*d*x + 3*I*c)*log(1/7*(14*I*sq
rt(1/2)*sqrt(-a^3/d^2)*d*e^(I*d*x + I*c) + 7*sqrt(2)*(a*e^(2*I*d*x + 2*I*c)
 + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a
) - 105*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(3*I*d*x + 3*I*c)*log(1/7*(-14*I*sqrt(
1/2)*sqrt(-a^3/d^2)*d*e^(I*d*x + I*c) + 7*sqrt(2)*(a*e^(2*I*d*x + 2*I*c) +
a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) -
 sqrt(2)*(-6*I*a*e^(8*I*d*x + 8*I*c) - 38*I*a*e^(6*I*d*x + 6*I*c) - 148*I*a
*e^(4*I*d*x + 4*I*c) - 101*I*a*e^(2*I*d*x + 2*I*c) + 15*I*a)*sqrt(a/(e^(2*I
```

```
*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^5\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c)^5, x)
```

3.306 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=117

$$\frac{2i(a+ia\tan(c+dx))^{19/2}}{19a^7d}-\frac{12i(a+ia\tan(c+dx))^{17/2}}{17a^6d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{5a^5d}-\frac{16i(a+ia\tan(c+dx))^{13/2}}{13a^4d}$$

```
[Out] (((-16*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^4*d) + (((8*I)/5)*(a + I*a*
Tan[c + d*x])^(15/2))/(a^5*d) - (((12*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))
/(a^6*d) + (((2*I)/19)*(a + I*a*Tan[c + d*x])^(19/2))/(a^7*d)
```

Rubi [A] time = 0.0860853, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{19/2}}{19a^7d}-\frac{12i(a+ia\tan(c+dx))^{17/2}}{17a^6d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{5a^5d}-\frac{16i(a+ia\tan(c+dx))^{13/2}}{13a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-16*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^4*d) + (((8*I)/5)*(a + I*a*
Tan[c + d*x])^(15/2))/(a^5*d) - (((12*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))
/(a^6*d) + (((2*I)/19)*(a + I*a*Tan[c + d*x])^(19/2))/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^3(a+x)^{11/2}\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(8a^3(a+x)^{11/2}-12a^2(a+x)^{13/2}+6a(a+x)^{15/2}-(a+x)^{17/2}\right)\right.}{a^7d}\\ &= -\frac{16i(a+ia\tan(c+dx))^{13/2}}{13a^4d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{5a^5d}-\frac{12i(a+ia\tan(c}{17a^6d}\end{aligned}$$

Mathematica [A] time = 1.41642, size = 113, normalized size = 0.97

$$-\frac{2a^2\sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}(\cos(6c+8dx)+i\sin(6c+8dx))(3262i\cos(2(c+dx))+494\tan(c+dx)+159}{20995d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*Sec[c + d*x]^8*(Cos[6*c + 8*d*x] + I*Sin[6*c + 8*d*x])*(-833*I + (3
262*I)*Cos[2*(c + d*x)] + 1599*Sec[c + d*x]*Sin[3*(c + d*x)] + 494*Tan[c +
d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(20995*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 15.969, size = 171, normalized size = 1.5

$$-\frac{2a^2\left(4096\,i\,(\cos(dx+c))^9 - 4096\,\sin(dx+c)\,(\cos(dx+c))^8 + 512\,i\,(\cos(dx+c))^7 - 2560\,(\cos(dx+c))^6\sin(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -2/20995/d*a^2*(4096*I*cos(d*x+c)^9-4096*sin(d*x+c)*cos(d*x+c)^8+512*I*cos(
d*x+c)^7-2560*cos(d*x+c)^6*sin(d*x+c)+224*I*cos(d*x+c)^5-2016*sin(d*x+c)*co
s(d*x+c)^4+132*I*cos(d*x+c)^3-1716*cos(d*x+c)^2*sin(d*x+c)-2535*I*cos(d*x+c
)+1105*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c
)^9
```

Maxima [A] time = 1.11067, size = 103, normalized size = 0.88

$$\frac{2i\left(1105\,(i\,a\tan(dx+c)+a)^{\frac{19}{2}} - 7410\,(i\,a\tan(dx+c)+a)^{\frac{17}{2}}a + 16796\,(i\,a\tan(dx+c)+a)^{\frac{15}{2}}a^2 - 12920\,(i\,a\tan(dx+c\right.}{20995\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/20995*I*(1105*(I*a*tan(d*x + c) + a)^(19/2) - 7410*(I*a*tan(d*x + c) + a)
^(17/2)*a + 16796*(I*a*tan(d*x + c) + a)^(15/2)*a^2 - 12920*(I*a*tan(d*x +
c) + a)^(13/2)*a^3)/(a^7*d)
```

Fricas [B] time = 2.52358, size = 635, normalized size = 5.43

$$\frac{\sqrt{2}\left(-16384i\,a^2e^{(18i\,dx+18i\,c)} - 155648i\,a^2e^{(16i\,dx+16i\,c)} - 661504i\,a^2e^{(14i\,dx+14i\,c)} - 1653760i\,a^2e^{(12i\,dx+12i\right.}{20995\left(de^{(18i\,dx+18i\,c)} + 9\,de^{(16i\,dx+16i\,c)} + 36\,de^{(14i\,dx+14i\,c)} + 84\,de^{(12i\,dx+12i\,c)} + 126\,de^{(10i\,dx+10i\,c)} + 126\,de^{(8i\,dx+8i\,c)} + 84\,de^{(6i\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20995*sqrt(2)*(-16384*I*a^2*e^(18*I*d*x + 18*I*c) - 155648*I*a^2*e^(16*I*
d*x + 16*I*c) - 661504*I*a^2*e^(14*I*d*x + 14*I*c) - 1653760*I*a^2*e^(12*I*
d*x + 12*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(d*e^(18*I
*d*x + 18*I*c) + 9*d*e^(16*I*d*x + 16*I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 8
```

```
4*d*e^(12*I*d*x + 12*I*c) + 126*d*e^(10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x
+ 8*I*c) + 84*d*e^(6*I*d*x + 6*I*c) + 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I
*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^8, x)
```

3.307 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=88

$$-\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^5d}+\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^4d}-\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^3d}$$

```
[Out] (((-8*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^3*d) + (((8*I)/13)*(a + I*a*
Tan[c + d*x])^(13/2))/(a^4*d) - (((2*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/
(a^5*d)
```

Rubi [A] time = 0.0771402, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$-\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^5d}+\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^4d}-\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-8*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^3*d) + (((8*I)/13)*(a + I*a*
Tan[c + d*x])^(13/2))/(a^4*d) - (((2*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/
(a^5*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^2(a+x)^{9/2}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a+x)^{9/2}-4a(a+x)^{11/2}+(a+x)^{13/2}\right)dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^3d}+\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^4d}-\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^5d}\end{aligned}$$

Mathematica [A] time = 0.745479, size = 97, normalized size = 1.1

$$\frac{2a^2\sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}(-187i\sin(2(c+dx))+203\cos(2(c+dx))+60)(\sin(5c+7dx)-i\cos(5c+7dx))}{2145d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sec[c + d*x]^7*(60 + 203*Cos[2*(c + d*x)] - (187*I)*Sin[2*(c + d*x)]
)*((-I)*Cos[5*c + 7*d*x] + Sin[5*c + 7*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(2
145*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [B] time = 0.581, size = 144, normalized size = 1.6

$$-\frac{2\,a^{2}\left(512\,i\left(\cos\left(dx+c\right)\right)^{7}-512\,\left(\cos\left(dx+c\right)\right)^{6}\sin\left(dx+c\right)+64\,i\left(\cos\left(dx+c\right)\right)^{5}-320\,\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{4}+\right.}{2145\,d\left(\cos\left(dx+c\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -2/2145/d*a^2*(512*I*cos(d*x+c)^7-512*cos(d*x+c)^6*sin(d*x+c)+64*I*cos(d*x+
c)^5-320*sin(d*x+c)*cos(d*x+c)^4+28*I*cos(d*x+c)^3-252*cos(d*x+c)^2*sin(d*x
+c)-341*I*cos(d*x+c)+143*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c
))^(1/2)/cos(d*x+c)^7
```

Maxima [A] time = 1.09649, size = 78, normalized size = 0.89

$$-\frac{2i\left(143\,(i\,a\tan\left(dx+c\right)+a)^{\frac{15}{2}}-660\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}a+780\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}a^{2}\right)}{2145\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/2145*I*(143*(I*a*tan(d*x + c) + a)^(15/2) - 660*(I*a*tan(d*x + c) + a)^(
13/2)*a + 780*(I*a*tan(d*x + c) + a)^(11/2)*a^2)/(a^5*d)
```

Fricas [B] time = 2.61204, size = 497, normalized size = 5.65

$$\frac{\sqrt{2}\left(-2048i\,a^{2}e^{(14i\,dx+14i\,c)}-15360i\,a^{2}e^{(12i\,dx+12i\,c)}-49920i\,a^{2}e^{(10i\,dx+10i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{(i\,dx+i\,c)}}{2145\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4i\,c)}+7\,de^{(2i\,dx+2i\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2145*sqrt(2)*(-2048*I*a^2*e^(14*I*d*x + 14*I*c) - 15360*I*a^2*e^(12*I*d*x
 + 12*I*c) - 49920*I*a^2*e^(10*I*d*x + 10*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(I*d*x + I*c)/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c)
 + 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x
+ 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^6, x)
```

3.308 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=59

$$\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^3d}-\frac{4i(a+ia\tan(c+dx))^{9/2}}{9a^2d}$$

```
[Out] (((-4*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^2*d) + (((2*I)/11)*(a + I*a*Ta
n[c + d*x])^(11/2))/(a^3*d)
```

Rubi [A] time = 0.0705784, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^3d}-\frac{4i(a+ia\tan(c+dx))^{9/2}}{9a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^2*d) + (((2*I)/11)*(a + I*a*Ta
n[c + d*x])^(11/2))/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx&=-\frac{i\,\text{Subst}\left(\int(a-x)(a+x)^{7/2}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\&=-\frac{i\,\text{Subst}\left(\int\left(2a(a+x)^{7/2}-(a+x)^{9/2}\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\&=-\frac{4i(a+ia\tan(c+dx))^{9/2}}{9a^2d}+\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^3d}\end{aligned}$$

Mathematica [A] time = 0.531602, size = 85, normalized size = 1.44

$$-\frac{2a^2(9\tan(c+dx)+13i)\sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}(\cos(4c+6dx)+i\sin(4c+6dx))}{99d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*Sec[c + d*x]^4*(Cos[4*c + 6*d*x] + I*Sin[4*c + 6*d*x])*(13*I + 9*Ta
n[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(99*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [B] time = 0.327, size = 117, normalized size = 2.

$$-\frac{2 a^{2} \left(32 i \left(\cos \left(dx+c\right)\right)^{5}-32 \sin \left(dx+c\right) \left(\cos \left(dx+c\right)\right)^{4}+4 i \left(\cos \left(dx+c\right)\right)^{3}-20 \left(\cos \left(dx+c\right)\right)^{2} \sin \left(dx+c\right)-23 i \cos}{99 d \left(\cos \left(dx+c\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -2/99/d*a^2*(32*I*cos(d*x+c)^5-32*sin(d*x+c)*cos(d*x+c)^4+4*I*cos(d*x+c)^3-
20*cos(d*x+c)^2*sin(d*x+c)-23*I*cos(d*x+c)+9*sin(d*x+c))*(a*(I*sin(d*x+c)+c
os(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5
```

Maxima [A] time = 1.07692, size = 54, normalized size = 0.92

$$\frac{2i \left(9 \left(i a \tan \left(dx+c\right)+a\right)^{\frac{11}{2}}-22 \left(i a \tan \left(dx+c\right)+a\right)^{\frac{9}{2}} a\right)}{99 a^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/99*I*(9*(I*a*tan(d*x + c) + a)^(11/2) - 22*(I*a*tan(d*x + c) + a)^(9/2)*a
)/(a^3*d)
```

Fricas [B] time = 2.61219, size = 360, normalized size = 6.1

$$\frac{\sqrt{2} \left(-128 i a^{2} e^{\left(10 i dx+10 i c\right)}-704 i a^{2} e^{\left(8 i dx+8 i c\right)}\right) \sqrt{\frac{a}{e^{\left(2 i dx+2 i c\right)}+1}} e^{\left(i dx+i c\right)}}{99 \left(d e^{\left(10 i dx+10 i c\right)}+5 d e^{\left(8 i dx+8 i c\right)}+10 d e^{\left(6 i dx+6 i c\right)}+10 d e^{\left(4 i dx+4 i c\right)}+5 d e^{\left(2 i dx+2 i c\right)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/99*sqrt(2)*(-128*I*a^2*e^(10*I*d*x + 10*I*c) - 704*I*a^2*e^(8*I*d*x + 8*I
*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(d*e^(10*I*d*x + 10*
I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x
 + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\sec\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

3.309 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=29

$$-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

```
[Out] (((-2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a*d)
```

Rubi [A] time = 0.0628827, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 32}

$$-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= -\frac{i\,\text{Subst}\left(\int (a+x)^{5/2}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}\end{aligned}$$

Mathematica [B] time = 0.313834, size = 73, normalized size = 2.52

$$\frac{2a^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}(\sin(3c+5dx)-i\cos(3c+5dx))}{7d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sec[c + d*x]^3*((-I)*Cos[3*c + 5*d*x] + Sin[3*c + 5*d*x])*Sqrt[a + I
*a*Tan[c + d*x]])/(7*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.032, size = 24, normalized size = 0.8

$$\frac{-\frac{2i}{7}}{ad}\left(a+ia\tan\left(dx+c\right)\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x)

[Out] -2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a/d
```

Maxima [A] time = 1.12718, size = 28, normalized size = 0.97

$$-\frac{2i\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}{7\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")

[Out] -2/7*I*(I*a*tan(d*x + c) + a)^(7/2)/(a*d)
```

Fricas [B] time = 2.61548, size = 209, normalized size = 7.21

$$-\frac{16i\sqrt{2}a^{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(7i\,dx+7i\,c)}}{7\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")

[Out] -16/7*I*sqrt(2)*a^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(7*I*d*x + 7*I*c)/(
d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}\sec{(dx+c)}^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

3.310 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}-\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))}$$

```
[Out] (I*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*
d) - (I*a^3*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0827533, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 47, 63, 206}

$$\frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d}-\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*
d) - (I*a^3*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{(ia^3)\operatorname{Subst}\left(\int \frac{\sqrt{a+x}}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))} + \frac{(ia^3)\operatorname{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{2d}$$
$$= -\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))} + \frac{(ia^3)\operatorname{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{d}$$
$$= \frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} - \frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))}$$

Mathematica [A] time = 0.596658, size = 116, normalized size = 1.3

$$-\frac{ie^{-5i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\left(\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{5/2}\left(e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}-\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*((a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(5/2)*(1 + E^((2*
I)*(c + d*x)))^(5/2)*(E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))] - ArcSi
nh[E^(I*(c + d*x))]))/(Sqrt[2]*d*E^((5*I)*(c + d*x)))
```

Maple [B] time = 0.332, size = 398, normalized size = 4.5

$$-\frac{a^2}{4\,d\,(i\sin(dx+c)+\cos(dx+c)-1)\cos(dx+c)}\left(i\sqrt{2}\sin(dx+c)\cos(dx+c)\left(-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}\right)^{\frac{3}{2}}\operatorname{Artanh}\left(\frac{\sqrt{2}\sin}{2\cos(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -1/4/d*a^2*(I*2^(1/2)*sin(d*x+c)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/c
os(d*x+c))+I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)+c
os(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctan(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)+2^(1/2)*arctan(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*
sin(d*x+c)+8*I*cos(d*x+c)^4-8*cos(d*x+c)^3*sin(d*x+c)-4*I*cos(d*x+c)^3+4*co
s(d*x+c)^2*sin(d*x+c)-4*I*cos(d*x+c)^2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*
x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.4937, size = 693, normalized size = 7.79

$$\frac{\sqrt{2}\left(-i\,a^2e^{(2i\,dx+2i\,c)} - i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)} - \sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{\left(2i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(2i\,dx+2i\,c)}+\sqrt{2}\left(a^2e^{(2i\,dx+2i\,c)}+a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i}\right.}{a^2}\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(-I*a^2*e^(2*I*d*x + 2*I*c) - I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*e^(I*d*x + I*c) - sqrt(1/2)*sqrt(-a^5/d^2)*d*log((2*I*sqrt(1/2)*sq
rt(-a^5/d^2)*d*e^(2*I*d*x + 2*I*c) + sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-2*I*d*x - 2*I*c)/a
^2) + sqrt(1/2)*sqrt(-a^5/d^2)*d*log((-2*I*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(2*
I*d*x + 2*I*c) + sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-2*I*d*x - 2*I*c)/a^2))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.311 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=137

$$-\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2}-\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))}-\frac{3ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d}$$

```
[Out] (((-3*I)/16)*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])
/(Sqrt[2]*d) - ((I/4)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x])^2) - (((3*I)/16)*a^3*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x]))
```

Rubi [A] time = 0.0949402, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2}-\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))}-\frac{3ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-3*I)/16)*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])
/(Sqrt[2]*d) - ((I/4)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x])^2) - (((3*I)/16)*a^3*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{\left(ia^5\right)\text{Subst}\left(\int \frac{1}{(a-x)^3\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{\left(3ia^4\right)\text{Subst}\left(\int \frac{1}{(a-x)^2\sqrt{a+x}}\,dx, x, ia\tan(c+dx\right)}{8d}$$

$$= -\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))} - \frac{\left(3ia^3\right)\text{Subst}\left(\int \frac{}{(a-}\right.}{}$$

$$= -\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))} - \frac{\left(3ia^3\right)\text{Subst}\left(\int \frac{}{2a-}\right.}{}$$

$$= -\frac{3ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d} - \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{3ia^3\sqrt{a+ia\,t}}{16d(a-ia\tan}$$

Mathematica [A] time = 0.704104, size = 116, normalized size = 0.85

$$-\frac{ia^2e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(5+2e^{2i(c+dx)}\right)+3\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx)}}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I/32)*a^2*Sqrt[1 + E^((2*I)*(c + d*x))]*(E^(I*(c + d*x))*Sqrt[1 + E^((2*
I)*(c + d*x))]*(5 + 2*E^((2*I)*(c + d*x))) + 3*ArcSinh[E^(I*(c + d*x))])*Sq
rt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] time = 0.336, size = 744, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/256/d*a^2*(-256*I*cos(d*x+c)^8+96*I*cos(d*x+c)^6+3*2^(1/2)*arctan(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
7/2)*cos(d*x+c)^3*sin(d*x+c)+9*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/
2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(
d*x+c))*sin(d*x+c)*cos(d*x+c)^2+9*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^2*s
in(d*x+c)+3*I*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*sin(d*x+c)+
9*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)*sin(d*x+c)+3*2^(1/2)*arctan(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
7/2)*sin(d*x+c)+3*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d
```

```
*x+c)*cos(d*x+c)^3+128*I*cos(d*x+c)^7+256*sin(d*x+c)*cos(d*x+c)^7-16*I*cos(
d*x+c)^5-128*cos(d*x+c)^6*sin(d*x+c)+48*I*cos(d*x+c)^4+32*cos(d*x+c)^5*sin(
d*x+c)+9*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*arctanh(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos
(d*x+c)-48*sin(d*x+c)*cos(d*x+c)^4)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c)
)^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.4793, size = 761, normalized size = 5.55

$$\frac{\sqrt{2}\left(-2i\,a^2e^{(4i\,dx+4i\,c)}-7i\,a^2e^{(2i\,dx+2i\,c)}-5i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}+3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{\left(6i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(2i\,dx+2i\,c)}+3\sqrt{2}\left(a^2e^{(2i\,dx+2i\,c)}+\right.\right.}{3\,a^2}\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(2)*(-2*I*a^2*e^(4*I*d*x + 4*I*c) - 7*I*a^2*e^(2*I*d*x + 2*I*c) -
 5*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c) + 3*sqrt(1/2)*s
qrt(-a^5/d^2)*d*log(1/3*(6*I*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(2*I*d*x + 2*I*c)
 + 3*sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*e^(I*d*x + I*c))*e^(-2*I*d*x - 2*I*c)/a^2) - 3*sqrt(1/2)*sqrt(-a^5/d^2)
*d*log(1/3*(-6*I*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(2*I*d*x + 2*I*c) + 3*sqrt(2)
*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x
 + I*c))*e^(-2*I*d*x - 2*I*c)/a^2))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.312 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=210

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}-\frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}-\frac{35ia^4}{192d(a-ia\tan(c+dx))\sqrt{}}$$

```
[Out] (((-35*I)/128)*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/(Sqrt[2]*d) + (((35*I)/128)*a^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/6)
*a^6)/(d*(a - I*a*Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]) - (((7*I)/48)
*a^5)/(d*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((35*I)/19
2)*a^4)/(d*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.119865, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}-\frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}-\frac{35ia^4}{192d(a-ia\tan(c+dx))\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```


```
[Out] (((-35*I)/128)*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/(Sqrt[2]*d) + (((35*I)/128)*a^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/6)
*a^6)/(d*(a - I*a*Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]) - (((7*I)/48)
*a^5)/(d*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((35*I)/19
2)*a^4)/(d*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{\left(ia^7\right)\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} - \frac{\left(7ia^6\right)\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{3}}\right.}{12d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} - \frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{}}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} - \frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{}}$$

$$= \frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} -$$

$$= \frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} -$$

$$= -\frac{35ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}d} + \frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}} - \frac{}{6d(a-ia\,\mathrm{t}}$$

Mathematica [A] time = 0.825375, size = 142, normalized size = 0.68

$$-\frac{ia^2e^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(\sqrt{1+e^{2i(c+dx)}}\left(87e^{2i(c+dx)}+38e^{4i(c+dx)}+8e^{6i(c+dx)}-48\right)+105e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a}}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I/768)*a^2*Sqrt[1 + E^((2*I)*(c + d*x))]*(Sqrt[1 + E^((2*I)*(c + d*x))]*
(-48 + 87*E^((2*I)*(c + d*x)) + 38*E^((4*I)*(c + d*x)) + 8*E^((6*I)*(c + d*
x))) + 105*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c + d
*x]])/(d*E^((2*I)*(c + d*x)))
```

Maple [B] time = 0.402, size = 1088, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/24576/d*a^2*(5120*I*cos(d*x+c)^10-512*I*cos(d*x+c)^9-896*I*cos(d*x+c)^8-2
240*I*cos(d*x+c)^7+6720*I*cos(d*x+c)^6+105*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
11/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^5
```

```
*sin(d*x+c)*2^(1/2)+525*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)
+1050*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)+1050*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+525*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(11/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*
sin(d*x+c)*2^(1/2)+525*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arct
anh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))
*sin(d*x+c)*cos(d*x+c)+1050*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)
*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*
x+c))*sin(d*x+c)*cos(d*x+c)^2+3072*sin(d*x+c)*cos(d*x+c)^9+16384*sin(d*x+c)
*cos(d*x+c)^11-8192*sin(d*x+c)*cos(d*x+c)^10+105*I*2^(1/2)*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(11/2)*sin(d*x+c)-16384*I*cos(d*x+c)^12+8192*I*cos(d*x+
c)^11-3584*sin(d*x+c)*cos(d*x+c)^8+4480*sin(d*x+c)*cos(d*x+c)^7-6720*cos(d*
x+c)^6*sin(d*x+c)+105*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*2^(1/2)*arctan(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+105*I*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^5+525*I*2^
(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^4+10
50*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+
c)^3)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+
c)-1)/cos(d*x+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.17728, size = 936, normalized size = 4.46

$$\left(105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(2i\,dx+2i\,c)}\log\left(\frac{\left(70i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(2i\,dx+2i\,c)}+35\sqrt{2}\left(a^2e^{(2i\,dx+2i\,c)}+a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{35\,a^2}\right)-105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(2i\,dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/768*(105*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(2*I*d*x + 2*I*c)*log(1/35*(70*I*sq
rt(1/2)*sqrt(-a^5/d^2)*d*e^(2*I*d*x + 2*I*c) + 35*sqrt(2)*(a^2*e^(2*I*d*x +
 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x
 - I*c)/a^2) - 105*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(2*I*d*x + 2*I*c)*log(1/35*
(-70*I*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(2*I*d*x + 2*I*c) + 35*sqrt(2)*(a^2*e^(
2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*
```

```
e^(-I*d*x - I*c)/a^2) + sqrt(2)*(-8*I*a^2*e^(8*I*d*x + 8*I*c) - 46*I*a^2*e^
(6*I*d*x + 6*I*c) - 125*I*a^2*e^(4*I*d*x + 4*I*c) - 39*I*a^2*e^(2*I*d*x + 2
*I*c) + 48*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-2*
I*d*x - 2*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.313 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=147

$$\frac{256ia^4\sec^3(c+dx)}{315d(a+ia\tan(c+dx))^{3/2}}+\frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}}+\frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d}+\frac{2ia\sec^3(c+dx)(a+ia}{9d}$$

```
[Out] (((256*I)/315)*a^4*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((64
*I)/105)*a^3*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((8*I)/21)*a
^2*Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (((2*I)/9)*a*Sec[c + d*x]
^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.240449, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{256ia^4\sec^3(c+dx)}{315d(a+ia\tan(c+dx))^{3/2}}+\frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}}+\frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d}+\frac{2ia\sec^3(c+dx)(a+ia}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((256*I)/315)*a^4*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((64
*I)/105)*a^3*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((8*I)/21)*a
^2*Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (((2*I)/9)*a*Sec[c + d*x]
^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] && !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= \frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}+\frac{1}{3}(4a)\int \sec^3(c+dx)(a+ia\tan(c+\\ &= \frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d}+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/}}{9d}\\ &= \frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}}+\frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d}+\frac{2ia\sec^3(}{}\\ &= \frac{256ia^4\sec^3(c+dx)}{315d(a+ia\tan(c+dx))^{3/2}}+\frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}}+\frac{8ia^2\sec^3(c+dx)\sqrt{}}{21}\end{aligned}$$

Mathematica [A] time = 0.680422, size = 103, normalized size = 0.7

$$\frac{2a^2(\sin(2c)+i\cos(2c))\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}(242\cos(2(c+dx))+54i\tan(c+dx)+89i\sin(3(c+dx))\sec}{315d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sec[c + d*x]^3*(I*Cos[2*c] + Sin[2*c])*(77 + 242*Cos[2*(c + d*x)] +
(89*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (54*I)*Tan[c + d*x])*Sqrt[a + I*a*Ta
n[c + d*x]])/(315*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.293, size = 117, normalized size = 0.8

$$\frac{2\,a^2\left(256\,i\,(\cos(dx+c))^5+256\,\sin(dx+c)\,(\cos(dx+c))^4-32\,i\,(\cos(dx+c))^3+96\,(\cos(dx+c))^2\sin(dx+c)+95}{315\,d\,(\cos(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/315/d*a^2*(256*I*cos(d*x+c)^5+256*sin(d*x+c)*cos(d*x+c)^4-32*I*cos(d*x+c)
^3+96*cos(d*x+c)^2*sin(d*x+c)+95*I*cos(d*x+c)-35*sin(d*x+c))*(a*(I*sin(d*x+
c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.48567, size = 402, normalized size = 2.73

$$\frac{\sqrt{2}\left(3360i\,a^2e^{(6i\,dx+6i\,c)}+4032i\,a^2e^{(4i\,dx+4i\,c)}+2304i\,a^2e^{(2i\,dx+2i\,c)}+512i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{315\left(de^{(9i\,dx+9i\,c)}+4\,de^{(7i\,dx+7i\,c)}+6\,de^{(5i\,dx+5i\,c)}+4\,de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*sqrt(2)*(3360*I*a^2*e^(6*I*d*x + 6*I*c) + 4032*I*a^2*e^(4*I*d*x + 4*I
*c) + 2304*I*a^2*e^(2*I*d*x + 2*I*c) + 512*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*e^(I*d*x + I*c)/(d*e^(9*I*d*x + 9*I*c) + 4*d*e^(7*I*d*x + 7*I*c) +
 6*d*e^(5*I*d*x + 5*I*c) + 4*d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan(dx+c) + a)^{\frac{5}{2}} \sec(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

3.314 $\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=104

$$\frac{64ia^3\sec(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}}+\frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d}+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

```
[Out] (((64*I)/15)*a^3*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I)/15
)*a^2*Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((2*I)/5)*a*Sec[c + d*x
]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.101283, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3494, 3493}

$$\frac{64ia^3\sec(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}}+\frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d}+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((64*I)/15)*a^3*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I)/15
)*a^2*Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((2*I)/5)*a*Sec[c + d*x
]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}+\frac{1}{5}(8a)\int \sec(c+dx)(a+ia\tan(c+\\ &= \frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d}+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3}}{5d}\\ &= \frac{64ia^3\sec(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}}+\frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d}+\frac{2ia\sec(c\end{aligned}$$

Mathematica [A] time = 0.314176, size = 93, normalized size = 0.89

$$\frac{2a^2\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}(\sin(c-dx)+i\cos(c-dx))(7i\sin(2(c+dx))+23\cos(2(c+dx))+20)}{15d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sec[c + d*x]^2*(I*Cos[c - d*x] + Sin[c - d*x])*(20 + 23*Cos[2*(c + d
*x)] + (7*I)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(15*d*(Cos[d*x]
+ I*Sin[d*x])^2)
```

Maple [A] time = 0.247, size = 90, normalized size = 0.9

$$\frac{2\,a^{2}\left(32\,i\left(\cos\left(dx+c\right)\right)^{3}+32\,\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)+11\,i\cos\left(dx+c\right)-3\,\sin\left(dx+c\right)\right)}{15\,d\left(\cos\left(dx+c\right)\right)^{2}}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx\right.\right.}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/15/d*a^2*(32*I*cos(d*x+c)^3+32*cos(d*x+c)^2*sin(d*x+c)+11*I*cos(d*x+c)-3*
sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Fricas [A] time = 2.03447, size = 282, normalized size = 2.71

$$\frac{\sqrt{2}\left(120i\,a^{2}e^{(4i\,dx+4i\,c)}+160i\,a^{2}e^{(2i\,dx+2i\,c)}+64i\,a^{2}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{15\left(de^{(5i\,dx+5i\,c)}+2\,de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*sqrt(2)*(120*I*a^2*e^(4*I*d*x + 4*I*c) + 160*I*a^2*e^(2*I*d*x + 2*I*c)
 + 64*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(d*e^(5*I*d*
x + 5*I*c) + 2*d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.315 $\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=65

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}-\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] ((-8*I)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((2*I)*a*Cos[c + d
*x]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.0966806, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3494, 3493}

$$\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}-\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-8*I)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((2*I)*a*Cos[c + d
*x]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx &= \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}+(4a)\int \cos(c+dx)(a+ia\tan(c+dx)) \\ &= -\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}\end{aligned}$$

Mathematica [A] time = 0.253252, size = 46, normalized size = 0.71

$$-\frac{2ia^2\sqrt{a+ia\tan(c+dx)}(3\cos(c+dx)-i\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]

[Out] ((-2*I)*a^2*(3*Cos[c + d*x] - I*Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A] time = 0.279, size = 53, normalized size = 0.8

$$-2\,\frac{a^2\left(3\,i\cos\left(dx+c\right)+\sin\left(dx+c\right)\right)}{d}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x)

[Out] -2/d*a^2*(3*I*cos(d*x+c)+sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c
))^(1/2)
```

Maxima [B] time = 1.93972, size = 447, normalized size = 6.88

$$\frac{2\left(-3i\,a^{\frac{5}{2}}-\frac{2\,a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{9i\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4\,a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{9i\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2\,a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3i\,a^{\frac{5}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\right.}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}\left(\frac{4i\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 2*(-3*I*a^(5/2) - 2*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 9*I*a^(5/2)*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 9*I*a^(5/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*a^(5/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 + 3*I*a^(5/2)*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 1)^(5/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)^(5/2)*(4*I*sin(d*x + c)/(cos(d*x + c) + 1) - 5
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 - 4*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + sin(d*x + c)^6/(cos(d*x + c)
+ 1)^6 + 1))
```

Fricas [A] time = 1.97035, size = 116, normalized size = 1.78

$$\frac{\sqrt{2}\left(-2i\,a^2e^{(2i\,dx+2i\,c)}-4i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")

[Out] sqrt(2)*(-2*I*a^2*e^(2*I*d*x + 2*I*c) - 4*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan (dx + c) + a)^{\frac{5}{2}} \cos (dx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

3.316 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=35

$$-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

```
[Out] (((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.0598799, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3493}

$$-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

Mathematica [A] time = 0.323248, size = 69, normalized size = 1.97

$$\frac{2a^2\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}(\sin(c+3dx)-i\cos(c+3dx))}{3d(\cos(dx)+i\sin(dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Cos[c + d*x]^2*((-I)*Cos[c + 3*d*x] + Sin[c + 3*d*x])*Sqrt[a + I*a*T
an[c + d*x]])/(3*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [B] time = 0.293, size = 63, normalized size = 1.8

$$-\frac{2\,a^2\,(i\cos(dx+c)-\sin(dx+c))\,(\cos(dx+c))^2}{3\,d}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d*a^2*(I*cos(d*x+c)-sin(d*x+c))*cos(d*x+c)^2*(a*(I*sin(d*x+c)+cos(d*x+
c))/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.14478, size = 443, normalized size = 12.66

$$\frac{2\left(i\,a^{\frac{5}{2}}-\frac{4i\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6i\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4i\,a^{\frac{5}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{i\,a^{\frac{5}{2}}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\right.}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}\left(-\frac{6i\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{18i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{18i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{6\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6i\sin(dx}{(\cos(dx+c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*(I*a^(5/2) - 4*I*a^(5/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*I*a^(5/2
)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*I*a^(5/2)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 + I*a^(5/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(-2*I*sin(d*x
 + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2)/(
d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) +
 1) - 1)^(5/2)*(-6*I*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 18*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 18*I*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 + 6*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*I*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 3))
```

Fricas [B] time = 2.04885, size = 157, normalized size = 4.49

$$\frac{\sqrt{2}\left(-i\,a^2e^{(4i\,dx+4i\,c)}-2i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*(-I*a^2*e^(4*I*d*x + 4*I*c) - 2*I*a^2*e^(2*I*d*x + 2*I*c) - I*a
^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan (dx + c) + a)^{\frac{5}{2}} \cos (dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

3.317 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=159

$$-\frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}+\frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}d}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}-\frac{ia\cos^3(c$$

```
[Out] ((I/4)*a^(5/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*d) - ((I/4)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d - ((I/6)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/5)*Cos[c
 + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rubi [A] time = 0.217917, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3490, 3489, 206}

$$-\frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}+\frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}d}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}-\frac{ia\cos^3(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2),x]
```


```
[Out] ((I/4)*a^(5/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*d) - ((I/4)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d - ((I/6)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/5)*Cos[c
 + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d
```


Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d} + \frac{1}{2}a\int \cos^3(c+dx)(a+ia\tan(c+$$

$$= -\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{6d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx)}{5d}$$

$$= -\frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))}{6d}$$

$$= -\frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))}{6d}$$

$$= \frac{ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}d} - \frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - ia$$

Mathematica [A] time = 1.01094, size = 118, normalized size = 0.74

$$-\frac{ia^2e^{-i(c+dx)}\left(34e^{2i(c+dx)}+14e^{4i(c+dx)}+3e^{6i(c+dx)}-15\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)+23\right)\sqrt{a+ia\tan(c+dx)}}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I/120)*a^2*(23 + 34*E^((2*I)*(c + d*x)) + 14*E^((4*I)*(c + d*x)) + 3*E^(
(6*I)*(c + d*x)) - 15*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*
I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] time = 0.324, size = 916, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -1/1920/d*a^2*(-512*I*cos(d*x+c)^8+160*I*cos(d*x+c)^6-15*arctan(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)
*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)+1536*I*cos(d*x+c)^10-60*arctan(1/2*2^(1/2)
*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)
*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+60*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*
arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x
+c))*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)-90*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(d*x+
c)^2*sin(d*x+c)+60*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(
d*x+c)*sin(d*x+c)-60*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*2^(1/2)*cos(d*x+c)*sin(d*x+c)-15*2^
(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(9/2)*sin(d*x+c)+64*I*cos(d*x+c)^7-768*I*cos(d*x+c)^9-15
36*sin(d*x+c)*cos(d*x+c)^9+15*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(9/2)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2
^(1/2)*cos(d*x+c)^4*sin(d*x+c)+768*sin(d*x+c)*cos(d*x+c)^8-480*I*cos(d*x+c)
^5-256*sin(d*x+c)*cos(d*x+c)^7+15*I*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)
```

```
+1))^(9/2)*sin(d*x+c)+320*cos(d*x+c)^6*sin(d*x+c)+90*I*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(9/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-480*cos(d*x+c)^5*sin(
d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*
x+c)-1)/cos(d*x+c)^4
```

Maxima [B] time = 2.29853, size = 1451, normalized size = 9.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/480*(20*(I*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) - sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/
4)*sqrt(a) - (-60*I*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 60*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + (-12*I*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 - 12*I*sqrt(2)*a^2*sin
(2*d*x + 2*c)^2 - 24*I*sqrt(2)*a^2*cos(2*d*x + 2*c) - 12*I*sqrt(2)*a^2)*cos
(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 12*(sqrt(2)*a^2*cos
(2*d*x + 2*c)^2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x
+ 2*c) + sqrt(2)*a^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*sqrt(a) + (30*sqrt(2)*a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1) - 30*sqrt(2)*a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - 15*I
*sqrt(2)*a^2*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + s
qrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + 15*I*sqrt(2)*a^2*log(sqrt(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + 1))*sqrt(a))/d
```

Fricas [B] time = 2.2251, size = 840, normalized size = 5.28

$$-\frac{\left(15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(idx+ic)}\log\left(\frac{\left(2i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(idx+ic)}+\sqrt{2}\left(a^2e^{(2idx+2ic)}+a^2\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{(idx+ic)}\right)e^{(-idx-ic)}}{a^2}\right)-15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(idx+ic)}\log\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/120*(15*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(I*d*x + I*c)*log((2*I*sqrt(1/2)*sq
rt(-a^5/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) - 15
*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(I*d*x + I*c)*log((-2*I*sqrt(1/2)*sqrt(-a^5/d
^2)*d*e^(I*d*x + I*c) + sqrt(2)*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^2) - sqrt(2)*(-3
*I*a^2*e^(6*I*d*x + 6*I*c) - 14*I*a^2*e^(4*I*d*x + 4*I*c) - 34*I*a^2*e^(2*I
*d*x + 2*I*c) - 23*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)
)*e^(-I*d*x - I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan(dx + c) + a)^{\frac{5}{2}} \cos(dx + c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^5, x)
```

3.318 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=231

$$-\frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d}-\frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d}+\frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}+\frac{9ia^{5/2}\tanh^{-1}}{3}$$

```
[Out] (((9*I)/32)*a^(5/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Ta
n[c + d*x]])])/(Sqrt[2]*d) + (((3*I)/16)*a^3*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) - (((9*I)/32)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
 - (((3*I)/20)*a^2*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((9*I)/7
0)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/7)*Cos[c + d*x]^7
*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rubi [A] time = 0.338896, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3497, 3502, 3490, 3489, 206}

$$-\frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d}-\frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d}+\frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}+\frac{9ia^{5/2}\tanh^{-1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2),x]
```


```
[Out] (((9*I)/32)*a^(5/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Ta
n[c + d*x]])])/(Sqrt[2]*d) + (((3*I)/16)*a^3*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) - (((9*I)/32)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
 - (((3*I)/20)*a^2*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((9*I)/7
0)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/7)*Cos[c + d*x]^7
*(a + I*a*Tan[c + d*x])^(5/2))/d
```


Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
```

```
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}+\frac{1}{14}(9a)\int \cos^5(c+dx)(a+ia\tan$$

$$= -\frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx$$

$$= -\frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d}-\frac{9ia\cos^5(c+dx)(a+ia\tan(c+d$$

$$= \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}-\frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d}-\frac{9ia\cos$$

$$= \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}-\frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d}-\frac{3ia^2\cos$$

$$= \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}-\frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d}-\frac{3ia^2\cos$$

$$= \frac{9ia^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}d}+\frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}-\frac{9ia^2\cos(c+$$

Mathematica [A] time = 1.04712, size = 155, normalized size = 0.67

$$-\frac{ia^2e^{-3i(c+dx)}\left(353e^{2i(c+dx)}+544e^{4i(c+dx)}+214e^{6i(c+dx)}+68e^{8i(c+dx)}+10e^{10i(c+dx)}-315e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{\right.}\right.}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I/2240)*a^2*(-35 + 353*E^((2*I)*(c + d*x)) + 544*E^((4*I)*(c + d*x)) + 2
14*E^((6*I)*(c + d*x)) + 68*E^((8*I)*(c + d*x)) + 10*E^((10*I)*(c + d*x)) -
 315*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^(
(2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((3*I)*(c + d*x)))
```

Maple [B] time = 0.484, size = 1260, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -1/143360/d*a^2*(1890*I*2^(1/2)*cos(d*x+c)^5*sin(d*x+c)*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(13/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)/cos(d*x+c))+315*I*2^(1/2)*cos(d*x+c)^6*sin(d*x+c)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(13/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))-81920*sin(d*x+c)*cos(d*x+c)^13+40960*sin(d*x+
c)*cos(d*x+c)^12-21504*sin(d*x+c)*cos(d*x+c)^9-16384*sin(d*x+c)*cos(d*x+c)^
11+18432*sin(d*x+c)*cos(d*x+c)^10+26880*sin(d*x+c)*cos(d*x+c)^8-40320*sin(d
*x+c)*cos(d*x+c)^7+81920*I*cos(d*x+c)^14-40960*I*cos(d*x+c)^13-24576*I*cos(
d*x+c)^12+2048*I*cos(d*x+c)^11+5376*I*cos(d*x+c)^9+13440*I*cos(d*x+c)^8-403
20*I*cos(d*x+c)^7+315*I*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)*
sin(d*x+c)-315*2^(1/2)*cos(d*x+c)^6*sin(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)-1890*2^(
1/2)*cos(d*x+c)^5*sin(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)-4725*2^(1/2)*cos(d*x+c)^4*
sin(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(13/2)-6300*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arctan(
1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(13/2)-4725*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)-1890*
2^(1/2)*cos(d*x+c)*sin(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+3072*I*cos(d*x+c)^10-315*
2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(13/2)*sin(d*x+c)+4725*I*2^(1/2)*cos(d*x+c)^4*sin(d*x+
c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+6300*I*2^(1/2)*cos(d*x+c)^3*s
in(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)*arctanh(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+4725*I*2^(1/2)*cos(d*x
+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)*arctanh(1/2*2^(1/2)*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))+1890*I*2^(1/2)*
cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)*arctanh(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)))*(a*(I*sin
(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c
)^6
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.04606, size = 971, normalized size = 4.2

$$-\frac{\left(315 \sqrt{\frac{1}{2}} \sqrt{-\frac{a^{5}}{d^{2}}} d e^{\left(3 i d x+3 i c\right)} \log\left(\frac{\left(18 i \sqrt{\frac{1}{2}} \sqrt{-\frac{a^{5}}{d^{2}}} d e^{\left(i d x+i c\right)}+9 \sqrt{2} \left(a^{2} e^{\left(2 i d x+2 i c\right)}+a^{2}\right) \sqrt{\frac{a}{e^{\left(2 i d x+2 i c\right)}+1}} e^{\left(i d x+i c\right)}\right) e^{\left(-i d x-i c\right)}}{9 a^{2}}\right)-315 \sqrt{\frac{1}{2}} \sqrt{-\frac{a^{5}}{d^{2}}} d e^{\left(3 i d x+3\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2240*(315*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(3*I*d*x + 3*I*c)*log(1/9*(18*I*s
qrt(1/2)*sqrt(-a^5/d^2)*d*e^(I*d*x + I*c) + 9*sqrt(2)*(a^2*e^(2*I*d*x + 2*I
*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)/a^2) - 315*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(3*I*d*x + 3*I*c)*log(1/9*(-18*
I*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(I*d*x + I*c) + 9*sqrt(2)*(a^2*e^(2*I*d*x +
2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x
- I*c)/a^2) - sqrt(2)*(-10*I*a^2*e^(10*I*d*x + 10*I*c) - 68*I*a^2*e^(8*I*d*
x + 8*I*c) - 214*I*a^2*e^(6*I*d*x + 6*I*c) - 544*I*a^2*e^(4*I*d*x + 4*I*c)
- 353*I*a^2*e^(2*I*d*x + 2*I*c) + 35*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*e^(I*d*x + I*c))*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan (dx + c) + a)^{\frac{5}{2}} \cos (dx + c)^7 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^7, x)
```

3.319 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=117

$$\frac{2i(a+ia\tan(c+dx))^{21/2}}{21a^7d}-\frac{12i(a+ia\tan(c+dx))^{19/2}}{19a^6d}+\frac{24i(a+ia\tan(c+dx))^{17/2}}{17a^5d}-\frac{16i(a+ia\tan(c+dx))^{15/2}}{15a^4d}$$

```
[Out] (((-16*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/(a^4*d) + (((24*I)/17)*(a + I*
a*Tan[c + d*x])^(17/2))/(a^5*d) - (((12*I)/19)*(a + I*a*Tan[c + d*x])^(19/2
))/(a^6*d) + (((2*I)/21)*(a + I*a*Tan[c + d*x])^(21/2))/(a^7*d)
```

Rubi [A] time = 0.0839321, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{21/2}}{21a^7d}-\frac{12i(a+ia\tan(c+dx))^{19/2}}{19a^6d}+\frac{24i(a+ia\tan(c+dx))^{17/2}}{17a^5d}-\frac{16i(a+ia\tan(c+dx))^{15/2}}{15a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-16*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/(a^4*d) + (((24*I)/17)*(a + I*
a*Tan[c + d*x])^(17/2))/(a^5*d) - (((12*I)/19)*(a + I*a*Tan[c + d*x])^(19/2
))/(a^6*d) + (((2*I)/21)*(a + I*a*Tan[c + d*x])^(21/2))/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{i\,\text{Subst}\left(\int (a-x)^3(a+x)^{13/2}\,dx,x,ia\tan(c+dx)\right)}{a^7d}$$
$$=-\frac{i\,\text{Subst}\left(\int \left(8a^3(a+x)^{13/2}-12a^2(a+x)^{15/2}+6a(a+x)^{17/2}-(a+x)^{19/2}\right)dx\right.}{a^7d}$$
$$=-\frac{16i(a+ia\tan(c+dx))^{15/2}}{15a^4d}+\frac{24i(a+ia\tan(c+dx))^{17/2}}{17a^5d}-\frac{12i(a+ia\tan(c+}{19a^6d}$$

Mathematica [A] time = 1.69881, size = 113, normalized size = 0.97

$$-\frac{2a^3\sec^9(c+dx)\sqrt{a+ia\tan(c+dx)}(\cos(7c+10dx)+i\sin(7c+10dx))(4554i\cos(2(c+dx))+630\tan(c+dx)+2245}{33915d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Sec[c + d*x]^9*(Cos[7*c + 10*d*x] + I*Sin[7*c + 10*d*x])*(-1311*I +
 (4554*I)*Cos[2*(c + d*x)] + 2245*Sec[c + d*x]*Sin[3*(c + d*x)] + 630*Tan[c
 + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(33915*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] time = 47.629, size = 181, normalized size = 1.6

$$-\frac{2a^3\left(8192\,i\left(\cos\left(dx+c\right)\right)^{10}-8192\,\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{9}+1024\,i\left(\cos\left(dx+c\right)\right)^{8}-5120\,\sin\left(dx+c\right)\left(\cos\left(dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -2/33915/d*a^3*(8192*I*cos(d*x+c)^10-8192*sin(d*x+c)*cos(d*x+c)^9+1024*I*co
s(d*x+c)^8-5120*sin(d*x+c)*cos(d*x+c)^7+448*I*cos(d*x+c)^6-4032*cos(d*x+c)^
5*sin(d*x+c)+264*I*cos(d*x+c)^4-3432*cos(d*x+c)^3*sin(d*x+c)-8300*I*cos(d*x
+c)^2+5440*cos(d*x+c)*sin(d*x+c)+1615*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d
*x+c))^(1/2)/cos(d*x+c)^10
```

Maxima [A] time = 1.10873, size = 103, normalized size = 0.88

$$\frac{2i\left(1615\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{21}{2}}-10710\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{19}{2}}a+23940\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{17}{2}}a^2-18088\left(i\,a\tan\left(d\right.\right.\right.}{33915\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/33915*I*(1615*(I*a*tan(d*x + c) + a)^(21/2) - 10710*(I*a*tan(d*x + c) + a
)^(19/2)*a + 23940*(I*a*tan(d*x + c) + a)^(17/2)*a^2 - 18088*(I*a*tan(d*x +
 c) + a)^(15/2)*a^3)/(a^7*d)
```

Fricas [B] time = 2.32793, size = 682, normalized size = 5.83

$$\frac{\sqrt{2}\left(-32768i\,a^3e^{(20i\,dx+20i\,c)}-344064i\,a^3e^{(18i\,dx+18i\,c)}-1634304i\,a^3e^{(16i\,dx+16i\,c)}-46305\right.}{33915\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de^{(10i\,dx+10i\,c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/33915*sqrt(2)*(-32768*I*a^3*e^(20*I*d*x + 20*I*c) - 344064*I*a^3*e^(18*I*
d*x + 18*I*c) - 1634304*I*a^3*e^(16*I*d*x + 16*I*c) - 4630528*I*a^3*e^(14*I
*d*x + 14*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(d*e^(20*
I*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) + 45*d*e^(16*I*d*x + 16*I*c) +
```

```
 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d*x + 12*I*c) + 252*d*e^(10*I*
d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 120*d*e^(6*I*d*x + 6*I*c) + 45*
d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ia\tan(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^8\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^8, x)
```

3.320 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=88

$$-\frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^5d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{15a^4d}-\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^3d}$$

```
[Out] (((-8*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^3*d) + (((8*I)/15)*(a + I*a*
Tan[c + d*x])^(15/2))/(a^4*d) - (((2*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))/
(a^5*d)
```

Rubi [A] time = 0.0766982, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$-\frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^5d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{15a^4d}-\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-8*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^3*d) + (((8*I)/15)*(a + I*a*
Tan[c + d*x])^(15/2))/(a^4*d) - (((2*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))/
(a^5*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^2(a+x)^{11/2}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a+x)^{11/2}-4a(a+x)^{13/2}+(a+x)^{15/2}\right)dx,x,ia\tan(c+\right.}{a^5d}\\ &= -\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^3d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{15a^4d}-\frac{2i(a+ia\tan(c+}{17a^5d}\end{aligned}$$

Mathematica [A] time = 0.858398, size = 97, normalized size = 1.1

$$\frac{2a^3\sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}(-247i\sin(2(c+dx))+263\cos(2(c+dx))+68)(\sin(6c+9dx)-i\cos(6c+9dx))}{3315d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Sec[c + d*x]^8*(68 + 263*Cos[2*(c + d*x)] - (247*I)*Sin[2*(c + d*x)]
)*((-I)*Cos[6*c + 9*d*x] + Sin[6*c + 9*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3
315*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [B] time = 0.628, size = 154, normalized size = 1.8

$$-\frac{2a^3\left(1024i(\cos(dx+c))^8-1024\sin(dx+c)(\cos(dx+c))^7+128i(\cos(dx+c))^6-640(\cos(dx+c))^5\sin(dx+c)+\right.}{3315d(\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -2/3315/d*a^3*(1024*I*cos(d*x+c)^8-1024*sin(d*x+c)*cos(d*x+c)^7+128*I*cos(d
*x+c)^6-640*cos(d*x+c)^5*sin(d*x+c)+56*I*cos(d*x+c)^4-504*cos(d*x+c)^3*sin(
d*x+c)-1072*I*cos(d*x+c)^2+676*cos(d*x+c)*sin(d*x+c)+195*I)*(a*(I*sin(d*x+c
)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^8
```

Maxima [A] time = 1.11964, size = 78, normalized size = 0.89

$$-\frac{2i\left(195(ia\tan(dx+c)+a)^{\frac{17}{2}}-884(ia\tan(dx+c)+a)^{\frac{15}{2}}a+1020(ia\tan(dx+c)+a)^{\frac{13}{2}}a^2\right)}{3315a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3315*I*(195*(I*a*tan(d*x + c) + a)^(17/2) - 884*(I*a*tan(d*x + c) + a)^(
15/2)*a + 1020*(I*a*tan(d*x + c) + a)^(13/2)*a^2)/(a^5*d)
```

Fricas [B] time = 2.05958, size = 537, normalized size = 6.1

$$\frac{\sqrt{2}\left(-4096ia^3e^{(16idx+16ic)}-34816ia^3e^{(14idx+14ic)}-130560ia^3e^{(12idx+12ic)}\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{(idx+ic)}}{3315\left(de^{(16idx+16ic)}+8de^{(14idx+14ic)}+28de^{(12idx+12ic)}+56de^{(10idx+10ic)}+70de^{(8idx+8ic)}+56de^{(6idx+6ic)}+28de^{(4idx+4ic}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3315*sqrt(2)*(-4096*I*a^3*e^(16*I*d*x + 16*I*c) - 34816*I*a^3*e^(14*I*d*x
 + 14*I*c) - 130560*I*a^3*e^(12*I*d*x + 12*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*e^(I*d*x + I*c)/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x + 14*I*c
) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e^(8*I*d
*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(
2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan(dx+c)+a)^{\frac{7}{2}} \sec(dx+c)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^6, x)
```

3.321 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=59

$$\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^3d}-\frac{4i(a+ia\tan(c+dx))^{11/2}}{11a^2d}$$

```
[Out] (((-4*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^2*d) + (((2*I)/13)*(a + I*a*
Tan[c + d*x])^(13/2))/(a^3*d)
```

Rubi [A] time = 0.0679677, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^3d}-\frac{4i(a+ia\tan(c+dx))^{11/2}}{11a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-4*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^2*d) + (((2*I)/13)*(a + I*a*
Tan[c + d*x])^(13/2))/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)(a+x)^{9/2}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\
&= -\frac{i\,\text{Subst}\left(\int \left(2a(a+x)^{9/2}-(a+x)^{11/2}\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\
&= -\frac{4i(a+ia\tan(c+dx))^{11/2}}{11a^2d}+\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^3d}
\end{aligned}$$

Mathematica [A] time = 0.590619, size = 85, normalized size = 1.44

$$-\frac{2a^3(11\tan(c+dx)+15i)\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}(\cos(5c+8dx)+i\sin(5c+8dx))}{143d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Sec[c + d*x]^5*(Cos[5*c + 8*d*x] + I*Sin[5*c + 8*d*x])*(15*I + 11*T
an[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(143*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [B] time = 0.365, size = 127, normalized size = 2.2

$$\frac{2\,a^{3}\left(-64\,i\,(\cos\,(dx+c))^{6}+64\,\,(\cos\,(dx+c))^{5}\sin\,(dx+c)-8\,i\,(\cos\,(dx+c))^{4}+40\,\,(\cos\,(dx+c))^{3}\sin\,(dx+c)+68\,i}{143\,d\,(\cos\,(dx+c))^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 2/143/d*a^3*(-64*I*cos(d*x+c)^6+64*cos(d*x+c)^5*sin(d*x+c)-8*I*cos(d*x+c)^4
+40*cos(d*x+c)^3*sin(d*x+c)+68*I*cos(d*x+c)^2-40*cos(d*x+c)*sin(d*x+c)-11*I
)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^6
```

Maxima [A] time = 1.09262, size = 54, normalized size = 0.92

$$\frac{2i\left(11\,(i\,a\tan\,(dx+c)+a)^{\frac{13}{2}}-26\,(i\,a\tan\,(dx+c)+a)^{\frac{11}{2}}\,a\right)}{143\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/143*I*(11*(I*a*tan(d*x + c) + a)^(13/2) - 26*(I*a*tan(d*x + c) + a)^(11/2
)*a)/(a^3*d)
```

Fricas [B] time = 1.98042, size = 405, normalized size = 6.86

$$\frac{\sqrt{2}\left(-256i\,a^{3}e^{(12i\,dx+12i\,c)}-1664i\,a^{3}e^{(10i\,dx+10i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{143\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/143*sqrt(2)*(-256*I*a^3*e^(12*I*d*x + 12*I*c) - 1664*I*a^3*e^(10*I*d*x +
10*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(d*e^(12*I*d*x +
 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6
*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan (dx+c)+a)^{\frac{7}{2}} \sec (dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^4, x)
```

3.322 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=29

$$-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

```
[Out] (((-2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a*d)
```

Rubi [A] time = 0.0615514, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 32}

$$-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= -\frac{i\,\text{Subst}\left(\int (a+x)^{7/2}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}\end{aligned}$$

Mathematica [B] time = 0.339362, size = 73, normalized size = 2.52

$$\frac{2a^3\sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}(\sin(4c+7dx)-i\cos(4c+7dx))}{9d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Sec[c + d*x]^4*((-I)*Cos[4*c + 7*d*x] + Sin[4*c + 7*d*x])*Sqrt[a + I
*a*Tan[c + d*x]])/(9*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] time = 0.03, size = 24, normalized size = 0.8

$$\frac{-\frac{2i}{9}}{ad}\left(a+ia\tan\left(dx+c\right)\right)^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a/d
```

Maxima [A] time = 1.18487, size = 28, normalized size = 0.97

$$-\frac{2i\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}}{9\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/9*I*(I*a*tan(d*x + c) + a)^(9/2)/(a*d)
```

Fricas [B] time = 2.14733, size = 244, normalized size = 8.41

$$-\frac{32i\,\sqrt{2}a^{3}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(9i\,dx+9i\,c)}}{9\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -32/9*I*sqrt(2)*a^3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(9*I*d*x + 9*I*c)/(
d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) +
 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^2, x)
```

3.323 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=116

$$-\frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))}-\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d}+\frac{3i\sqrt{2}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

```
[Out] ((3*I)*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/d - ((3*I)*a^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (I*a^3*(a + I*a*Tan[c + d
*x])^(3/2))/(d*(a - I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0884955, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3487, 47, 50, 63, 206}

$$-\frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))}-\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d}+\frac{3i\sqrt{2}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((3*I)*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/d - ((3*I)*a^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (I*a^3*(a + I*a*Tan[c + d
*x])^(3/2))/(d*(a - I*a*Tan[c + d*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{\left(ia^3\right)\text{Subst}\left(\int \frac{(a+x)^{3/2}}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))} + \frac{\left(3ia^3\right)\text{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx, x, ia\tan(c+dx)\right)}{2d}$$

$$= -\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d} - \frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))} + \frac{\left(3ia^4\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d} - \frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))} + \frac{\left(6ia^4\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{3i\sqrt{2}a^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d} - \frac{ia^3(a+ia\,\mathrm{t}}{d(a-ia\,\mathrm{t}}$$

Mathematica [A] time = 1.24282, size = 137, normalized size = 1.18

$$-\frac{i\sqrt{2}e^{-4i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3e^{i(c+dx)}+e^{3i(c+dx)}-3\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)(a+ia\tan(c+dx))^{7/2}}{d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(3*E^(I*(c +
d*x)) + E^((3*I)*(c + d*x)) - 3*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*
(c + d*x))])*(a + I*a*Tan[c + d*x])^(7/2))/(d*E^((4*I)*(c + d*x))*Sec[c + d
*x]^(7/2))
```

Maple [B] time = 0.374, size = 412, normalized size = 3.6

$$-\frac{a^3}{2d(i\sin(dx+c)+\cos(dx+c)-1)\cos(dx+c)}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(3i\cos(dx+c)\sin(dx+c)\left(-2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -1/2/d*a^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*I*cos(d*x+c)*s
in(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctanh(1/2*2^(1/2)*(-2*cos(
```

```
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)+3*I*2^(1/2)*arc
tanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))*2^(1/2)+3*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(d*x+c)+8*I*cos(d
*x+c)^4-4*I*cos(d*x+c)^3-8*cos(d*x+c)^3*sin(d*x+c)+4*cos(d*x+c)^2*sin(d*x+c
)-4*I*cos(d*x+c)-4*cos(d*x+c)*sin(d*x+c))/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d
*x+c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.15239, size = 709, normalized size = 6.11

$$\frac{\sqrt{2}\left(-2i a^3 e^{(2i dx+2ic)} - 6i a^3\right)\sqrt{\frac{a}{e^{(2i dx+2ic)}+1}}\, e^{(i dx+ic)} - 3\sqrt{2}\sqrt{-\frac{a^7}{d^2}}\, d \log\left(\frac{\left(3i\sqrt{2}\sqrt{-\frac{a^7}{d^2}}\, d e^{(2i dx+2ic)} + 3\sqrt{2}\left(a^3 e^{(2i dx+2ic)} + a^3\right)\sqrt{\frac{a}{e^{(2i dx+2ic)}+1}}\, e^{(i dx+}\right.}{3a^3}\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(-2*I*a^3*e^(2*I*d*x + 2*I*c) - 6*I*a^3)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*e^(I*d*x + I*c) - 3*sqrt(2)*sqrt(-a^7/d^2)*d*log(1/3*(3*I*sqrt
(2)*sqrt(-a^7/d^2)*d*e^(2*I*d*x + 2*I*c) + 3*sqrt(2)*(a^3*e^(2*I*d*x + 2*I*
c) + a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-2*I*d*x -
2*I*c)/a^3) + 3*sqrt(2)*sqrt(-a^7/d^2)*d*log(1/3*(-3*I*sqrt(2)*sqrt(-a^7/d^
2)*d*e^(2*I*d*x + 2*I*c) + 3*sqrt(2)*(a^3*e^(2*I*d*x + 2*I*c) + a^3)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-2*I*d*x - 2*I*c)/a^3))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan (dx + c) + a)^{\frac{7}{2}} \cos (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^2, x)
```

3.324 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=137

$$-\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2}+\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))}+\frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d}$$

```
[Out] ((I/8)*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*d) - ((I/2)*a^5*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x])^2
) + ((I/8)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]))
```

Rubi [A] time = 0.0937976, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3487, 47, 51, 63, 206}

$$-\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2}+\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))}+\frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```


```
[Out] ((I/8)*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*d) - ((I/2)*a^5*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x])^2
) + ((I/8)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]))
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{\left(ia^5\right)\text{Subst}\left(\int \frac{\sqrt{a+x}}{(a-x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{\left(ia^5\right)\text{Subst}\left(\int \frac{1}{(a-x)^2\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{4d}$$

$$= -\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))} + \frac{\left(ia^4\right)\text{Subst}\left(\int \frac{1}{(a-x)\cdots}\right.}{\cdots}$$

$$= -\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))} + \frac{\left(ia^4\right)\text{Subst}\left(\int \frac{1}{2a-x^2}\cdots\right.}{\cdots}$$

$$= \frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d} - \frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^4\sqrt{a+ia\tan(c\cdots}}{8d(a-ia\tan(c\cdots}$$

Mathematica [A] time = 1.34732, size = 152, normalized size = 1.11

$$-\frac{ie^{-4i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(e^{i(c+dx)}+3e^{3i(c+dx)}+2e^{5i(c+dx)}-\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)(a+ia\tan(c+dx))^{7/2}}{8\sqrt{2}d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I/8)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(E^(I*(c + d*x)) +
3*E^((3*I)*(c + d*x)) + 2*E^((5*I)*(c + d*x)) - Sqrt[1 + E^((2*I)*(c + d*x)
)]*ArcSinh[E^(I*(c + d*x))])*(a + I*a*Tan[c + d*x])^(7/2))/(Sqrt[2]*d*E^((4
*I)*(c + d*x))*Sec[c + d*x]^(7/2))
```

Maple [B] time = 0.344, size = 742, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -1/128/d*a^3*(256*I*cos(d*x+c)^8+2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^3*si
n(d*x+c)-224*I*cos(d*x+c)^6+3*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(co
```

```
s(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)^2*sin(d
*x+c)+3*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*arctanh(1/2*2^(1/2)*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)*sin(
d*x+c)+3*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*cos(d*x+c)*sin(d*x+c)-128*I*cos(d*x+c)^
7+2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(7/2)*sin(d*x+c)+I*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(7/2)*sin(d*x+c)+3*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(7/2)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2
^(1/2)*cos(d*x+c)^2*sin(d*x+c)-256*sin(d*x+c)*cos(d*x+c)^7+I*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(7/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)+128*cos(d*x+c)^
6*sin(d*x+c)+80*I*cos(d*x+c)^5+96*cos(d*x+c)^5*sin(d*x+c)+16*I*cos(d*x+c)^4
-16*sin(d*x+c)*cos(d*x+c)^4)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27462, size = 737, normalized size = 5.38

$$\frac{\sqrt{2}\left(-2i a^3 e^{(4i dx+4i c)} - 3i a^3 e^{(2i dx+2i c)} - i a^3\right)\sqrt{\frac{a}{e^{(2i dx+2i c)}+1}}\, e^{(i dx+i c)} - \sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\, d \log\left(\frac{\left(2i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\, d e^{(2i dx+2i c)} + \sqrt{2}\left(a^3 e^{(2i dx+2i c)}+a^3\right)\sqrt{\cdots}\right.}{a^3}\right.}{16 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*(-2*I*a^3*e^(4*I*d*x + 4*I*c) - 3*I*a^3*e^(2*I*d*x + 2*I*c) -
 I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c) - sqrt(1/2)*sqrt(
-a^7/d^2)*d*log((2*I*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(2*I*d*x + 2*I*c) + sqrt(
2)*(a^3*e^(2*I*d*x + 2*I*c) + a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d
*x + I*c))*e^(-2*I*d*x - 2*I*c)/a^3) + sqrt(1/2)*sqrt(-a^7/d^2)*d*log((-2*I
*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(2*I*d*x + 2*I*c) + sqrt(2)*(a^3*e^(2*I*d*x +
 2*I*c) + a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-2*I*d
*x - 2*I*c)/a^3))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.325 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=181

$$-\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3}-\frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2}-\frac{5ia^4\sqrt{a+ia\tan(c+dx)}}{64d(a-ia\tan(c+dx))}-\frac{5ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}$$

```
[Out] (((-5*I)/64)*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])
/(Sqrt[2]*d) - ((I/6)*a^6*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x])^3) - (((5*I)/48)*a^5*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x])^2) - (((5*I)/64)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x]))
```

Rubi [A] time = 0.108335, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3}-\frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2}-\frac{5ia^4\sqrt{a+ia\tan(c+dx)}}{64d(a-ia\tan(c+dx))}-\frac{5ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-5*I)/64)*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])
/(Sqrt[2]*d) - ((I/6)*a^6*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x])^3) - (((5*I)/48)*a^5*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x])^2) - (((5*I)/64)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{\left(ia^7\right)\text{Subst}\left(\int \frac{1}{(a-x)^4\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{\left(5ia^6\right)\text{Subst}\left(\int \frac{1}{(a-x)^3\sqrt{a+x}}\,dx, x, ia\tan(c+dx\right.}{12d}$$

$$= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{\left(5ia^5\right)\text{Subst}\left(\int \frac{}{(a-}\right.}{}$$

$$= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{5ia^4\sqrt{a+ia\tan(c}}{64d(a-ia\tan(c+}$$

$$= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{5ia^4\sqrt{a+ia\tan(c}}{64d(a-ia\tan(c+}$$

$$= -\frac{5ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} - \frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\,\text{t}}}{48d(a-ia\,\text{tan}}$$

Mathematica [A] time = 1.00398, size = 129, normalized size = 0.71

$$-\frac{ia^3e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(26e^{2i(c+dx)}+8e^{4i(c+dx)}+33\right)+15\sinh^{-1}\left(e^{i(c+dx)}\right)\right)\sqrt{a+ia\tan(c+dx}}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I/384)*a^3*Sqrt[1 + E^((2*I)*(c + d*x))]*(E^(I*(c + d*x))*Sqrt[1 + E^((2
*I)*(c + d*x))]*(33 + 26*E^((2*I)*(c + d*x)) + 8*E^((4*I)*(c + d*x))) + 15*
ArcSinh[E^(I*(c + d*x))])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] time = 0.402, size = 1088, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -1/12288/d*a^3*(-15*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*2^(1/2)-75*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*2^(1/2)-150*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*cos(d*x+c)^3*sin(d*x+c)*2^(1/2)-150*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*
arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d
*x+c)*2^(1/2)-75*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctan(1/2*2^(1/2)*(
```

```
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)-75*I*cos
(d*x+c)^4*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)-15*
I*cos(d*x+c)^5*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2
)+3072*sin(d*x+c)*cos(d*x+c)^9-16384*sin(d*x+c)*cos(d*x+c)^11+8192*sin(d*x+
c)*cos(d*x+c)^10-8192*I*cos(d*x+c)^11-11264*I*cos(d*x+c)^10+512*sin(d*x+c)*
cos(d*x+c)^8-640*sin(d*x+c)*cos(d*x+c)^7+960*cos(d*x+c)^6*sin(d*x+c)+16384*
I*cos(d*x+c)^12+3584*I*cos(d*x+c)^9+128*I*cos(d*x+c)^8+320*I*cos(d*x+c)^7-9
60*I*cos(d*x+c)^6-15*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*2^(1/2)*arctan(1
/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-15*I*2^(1/2)*ar
ctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c
))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*sin(d*x+c)-150*I*cos(d*x+c)^3*sin(
d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)-150*I*cos(d*x+c)^
2*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2)-75*I*cos(d*
x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(11/2)*arctanh(1/2*2^(1/2)*(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*2^(1/2))*(a*(I*s
in(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x
+c)^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.24483, size = 813, normalized size = 4.49

$$\sqrt{2}\left(-8i\,a^3e^{(6i\,dx+6i\,c)}-34i\,a^3e^{(4i\,dx+4i\,c)}-59i\,a^3e^{(2i\,dx+2i\,c)}-33i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}+15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{\left(10i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(sqrt(2)*(-8*I*a^3*e^(6*I*d*x + 6*I*c) - 34*I*a^3*e^(4*I*d*x + 4*I*c)
 - 59*I*a^3*e^(2*I*d*x + 2*I*c) - 33*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*e^(I*d*x + I*c) + 15*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(1/5*(10*I*sqrt(1/2)*
sqrt(-a^7/d^2)*d*e^(2*I*d*x + 2*I*c) + 5*sqrt(2)*(a^3*e^(2*I*d*x + 2*I*c) +
 a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-2*I*d*x - 2*I*
c)/a^3) - 15*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(1/5*(-10*I*sqrt(1/2)*sqrt(-a^7/
d^2)*d*e^(2*I*d*x + 2*I*c) + 5*sqrt(2)*(a^3*e^(2*I*d*x + 2*I*c) + a^3)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-2*I*d*x - 2*I*c)/a^3))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.326 $\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=139

$$\frac{256ia^4\sec(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}+\frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d}+\frac{2ia\sec(c+$$

```
[Out] (((256*I)/35)*a^4*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((64*I)/3
5)*a^3*Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((24*I)/35)*a^2*Sec[c
+ d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (((2*I)/7)*a*Sec[c + d*x]*(a + I*a
*Tan[c + d*x])^(5/2))/d
```

Rubi [A] time = 0.139905, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3494, 3493}

$$\frac{256ia^4\sec(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}+\frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d}+\frac{2ia\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((256*I)/35)*a^4*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((64*I)/3
5)*a^3*Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((24*I)/35)*a^2*Sec[c
+ d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (((2*I)/7)*a*Sec[c + d*x]*(a + I*a
*Tan[c + d*x])^(5/2))/d
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}+\frac{1}{7}(12a)\int \sec(c+dx)(a+ia\tan(c+d\\ &= \frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d}+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/}}{7d}\\ &= \frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}+\frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/}}{35d}\\ &= \frac{256ia^4\sec(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}+\frac{24ia^2\sec(c\end{aligned}$$

Mathematica [A] time = 0.641958, size = 109, normalized size = 0.78

$$\frac{2a^3\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}(\sin(c-2dx)+i\cos(c-2dx))(102\cos(2(c+dx))+14i\tan(c+dx)+19i\sin(3(c}{35d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Sec[c + d*x]^2*(I*Cos[c - 2*d*x] + Sin[c - 2*d*x])*(75 + 102*Cos[2*(
c + d*x)] + (19*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (14*I)*Tan[c + d*x])*Sqr
t[a + I*a*Tan[c + d*x]])/(35*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] time = 0.253, size = 100, normalized size = 0.7

$$\frac{2a^3\left(128i(\cos(dx+c))^4+128(\cos(dx+c))^3\sin(dx+c)+54i(\cos(dx+c))^2-22\cos(dx+c)\sin(dx+c)-5i\right)}{35d(\cos(dx+c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 2/35/d*a^3*(128*I*cos(d*x+c)^4+128*cos(d*x+c)^3*sin(d*x+c)+54*I*cos(d*x+c)^
2-22*cos(d*x+c)*sin(d*x+c)-5*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1
/2)/cos(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(ia\tan(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c), x)
```

Fricas [A] time = 2.04188, size = 363, normalized size = 2.61

$$\frac{\sqrt{2}\left(560i\,a^3e^{(6i\,dx+6i\,c)}+1120i\,a^3e^{(4i\,dx+4i\,c)}+896i\,a^3e^{(2i\,dx+2i\,c)}+256i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{35\left(de^{(7i\,dx+7i\,c)}+3\,de^{(5i\,dx+5i\,c)}+3\,de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/35*sqrt(2)*(560*I*a^3*e^(6*I*d*x + 6*I*c) + 1120*I*a^3*e^(4*I*d*x + 4*I*c
) + 896*I*a^3*e^(2*I*d*x + 2*I*c) + 256*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(I*d*x + I*c)/(d*e^(7*I*d*x + 7*I*c) + 3*d*e^(5*I*d*x + 5*I*c) + 3*
d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c), x)
```

3.327 $\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=104

$$-\frac{64ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx}{3d}$$

```
[Out] (((-64*I)/3)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((16*I)/3)*a
^2*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (((2*I)/3)*a*Cos[c + d*x]
*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rubi [A] time = 0.153474, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3494, 3493}

$$-\frac{64ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-64*I)/3)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((16*I)/3)*a
^2*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (((2*I)/3)*a*Cos[c + d*x]
*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}+\frac{1}{3}(8a)\int \cos(c+dx)(a+ia\tan(c+\\ &= \frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx}{3d}\\ &= -\frac{64ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{16ia^2\cos(c+dx)(a+ia\tan(c+}{3d}\end{aligned}$$

Mathematica [A] time = 0.357459, size = 59, normalized size = 0.57

$$-\frac{2ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}(-5i\sin(2(c+dx))+11\cos(2(c+dx))+12)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/3)*a^3*Sec[c + d*x]*(12 + 11*Cos[2*(c + d*x)] - (5*I)*Sin[2*(c + d
*x)])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A] time = 0.283, size = 73, normalized size = 0.7

$$-\frac{2\,a^{3}\left(22\,i\left(\cos\left(dx+c\right)\right)^{2}+10\,\cos\left(dx+c\right)\sin\left(dx+c\right)+i\right)}{3\,d\cos\left(dx+c\right)}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -2/3/d*a^3*(22*I*cos(d*x+c)^2+10*cos(d*x+c)*sin(d*x+c)+I)*(a*(I*sin(d*x+c)+
cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)
```

Maxima [B] time = 2.0588, size = 564, normalized size = 5.42

$$\frac{2\left(23i\,a^{\frac{7}{2}}+\frac{20\,a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{88i\,a^{\frac{7}{2}}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{60\,a^{\frac{7}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{130i\,a^{\frac{7}{2}}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{60\,a^{\frac{7}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{88i\,a^{\frac{7}{2}}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{20\,a^{\frac{7}{2}}\sin(dx+\cdots}{(\cos(dx+c)+\cdots}\right.}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}\left(-\frac{18i\sin(dx+c)}{\cos(dx+c)+1}+\frac{42\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{42i\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{42i\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{42\sin(d\cdots}{(\cos(dx+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2*(23*I*a^(7/2) + 20*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 88*I*a^(7/2)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 60*a^(7/2)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 130*I*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 60*a^(7/2)
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 88*I*a^(7/2)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 - 20*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 23*I*a^(7/2
)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)*(-18*I*s
in(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4
2*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 42*I*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 42*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18*I*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 3))
```

Fricas [A] time = 1.97789, size = 244, normalized size = 2.35

$$\frac{\sqrt{2}\left(-12i\,a^{3}e^{(4i\,dx+4i\,c)}-48i\,a^{3}e^{(2i\,dx+2i\,c)}-32i\,a^{3}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{3\left(de^{(3i\,dx+3i\,c)}+de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*(-12*I*a^3*e^(4*I*d*x + 4*I*c) - 48*I*a^3*e^(2*I*d*x + 2*I*c) -
 32*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(d*e^(3*I*d*x
+ 3*I*c) + d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c), x)
```

3.328 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=71

$$\frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

```
[Out] (((8*I)/3)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((2*I)*a*Co
s[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rubi [A] time = 0.121975, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((8*I)/3)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((2*I)*a*Co
s[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}-(4a)\int \cos^3(c+dx)(a+ia\tan(c+\\ &= \frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.346773, size = 86, normalized size = 1.21

$$\frac{2a^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}(3\sin(c+dx)+i\cos(c+dx))(\cos(c+4dx)+i\sin(c+4dx))}{3d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Cos[c + d*x]*(I*Cos[c + d*x] + 3*Sin[c + d*x])*(Cos[c + 4*d*x] + I*S
in[c + 4*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] time = 0.278, size = 71, normalized size = 1.

$$-\frac{2\,a^{3}\left(2\,i\left(\cos\left(dx+c\right)\right)^{2}-2\,\cos\left(dx+c\right)\sin\left(dx+c\right)-3\,i\right)\cos\left(dx+c\right)}{3\,d}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -2/3/d*a^3*(2*I*cos(d*x+c)^2-2*cos(d*x+c)*sin(d*x+c)-3*I)*cos(d*x+c)*(a*(I*
sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.17934, size = 680, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2*(-I*a^(7/2) - 6*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 5*I*a^(7/2)*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 24*a^(7/2)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 10*I*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^(7/2)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*I*a^(7/2)*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 24*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5*I*a^(7/2)*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^(7/2)*sin(d*x + c)^9/(cos(d*x + c) +
 1)^9 + I*a^(7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*(-2*I*sin(d*x + c)
/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)^(7/2)*(12*I*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 24*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 42*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 42*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 24*I*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7 - 9*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 12
*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 3*sin(d*x + c)^10/(cos(d*x + c) +
1)^10 + 3))
```

Fricas [A] time = 1.9417, size = 157, normalized size = 2.21

$$\frac{\sqrt{2}\left(-i\,a^{3}e^{\left(4i\,dx+4i\,c\right)}+i\,a^{3}e^{\left(2i\,dx+2i\,c\right)}+2i\,a^{3}\right)\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*(-I*a^3*e^(4*I*d*x + 4*I*c) + I*a^3*e^(2*I*d*x + 2*I*c) + 2*I*a
^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan (dx+c)+a)^{\frac{7}{2}}\cos (dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^3, x)
```

3.329 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=35

$$-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

```
[Out] (((-2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rubi [A] time = 0.0586437, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3493}

$$-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

Mathematica [B] time = 0.514704, size = 73, normalized size = 2.09

$$\frac{2a^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}(\sin(2c+5dx)-i\cos(2c+5dx))}{5d(\cos(dx)+i\sin(dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Cos[c + d*x]^3*((-I)*Cos[2*c + 5*d*x] + Sin[2*c + 5*d*x])*Sqrt[a + I
*a*Tan[c + d*x]])/(5*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [B] time = 0.349, size = 73, normalized size = 2.1

$$-\frac{2\,a^3\left(2\,i\left(\cos\left(dx+c\right)\right)^2-2\,\cos\left(dx+c\right)\sin\left(dx+c\right)-i\right)\left(\cos\left(dx+c\right)\right)^3}{5\,d}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -2/5/d*a^3*(2*I*cos(d*x+c)^2-2*cos(d*x+c)*sin(d*x+c)-I)*cos(d*x+c)^3*(a*(I*
sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.14331, size = 613, normalized size = 17.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2*(I*a^(7/2) - 6*I*a^(7/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*I*a^(7/
2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*I*a^(7/2)*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 15*I*a^(7/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*I*a^(
7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + I*a^(7/2)*sin(d*x + c)^12/(cos
(d*x + c) + 1)^12)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/
2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)*(-10*I*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 50*I*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 - 25*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 100*I*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5 - 100*I*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 +
 25*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 50*I*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 + 20*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 10*I*sin(d*x + c)^11/(
cos(d*x + c) + 1)^11 + 5*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 5))
```

Fricas [B] time = 2.22783, size = 198, normalized size = 5.66

$$\frac{\sqrt{2}\left(-i\,a^3e^{(6i\,dx+6i\,c)}-3i\,a^3e^{(4i\,dx+4i\,c)}-3i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(2)*(-I*a^3*e^(6*I*d*x + 6*I*c) - 3*I*a^3*e^(4*I*d*x + 4*I*c) - 3*
I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^5, x)
```

3.330 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=196

$$-\frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d}-\frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}+\frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d}-\frac{i\cos^7(}{}$$

```
[Out] ((I/8)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*d) - ((I/8)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d - ((I/12)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/10)*a
*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/7)*Cos[c + d*x]^7*(a
+ I*a*Tan[c + d*x])^(7/2))/d
```

Rubi [A] time = 0.286647, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3490, 3489, 206}

$$-\frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d}-\frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}+\frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d}-\frac{i\cos^7(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((I/8)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*d) - ((I/8)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d - ((I/12)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/10)*a
*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/7)*Cos[c + d*x]^7*(a
+ I*a*Tan[c + d*x])^(7/2))/d
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d} + \frac{1}{2}a\int \cos^5(c+dx)(a+ia\tan(c+$$
$$= -\frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx)}{7d}$$
$$= -\frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d} - \frac{ia\cos^5(c+dx)(a+ia\tan(c+d}{10d}$$
$$= -\frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx)}{12d}$$
$$= -\frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx)}{12d}$$
$$= \frac{ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d} - \frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{ia^2}{}$$

Mathematica [A] time = 1.78194, size = 131, normalized size = 0.67

$$-\frac{ia^3e^{-i(c+dx)}\left(298e^{2i(c+dx)}+188e^{4i(c+dx)}+81e^{6i(c+dx)}+15e^{8i(c+dx)}-105\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)+176\right)\sqrt{a}}{1680d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I/1680)*a^3*(176 + 298*E^((2*I)*(c + d*x)) + 188*E^((4*I)*(c + d*x)) + 8
1*E^((6*I)*(c + d*x)) + 15*E^((8*I)*(c + d*x)) - 105*Sqrt[1 + E^((2*I)*(c +
 d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])
/(d*E^(I*(c + d*x)))
```

Maple [B] time = 0.422, size = 1260, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 1/107520/d*a^3*(-630*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)*arctan
h(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*c
os(d*x+c)*sin(d*x+c)-1575*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*cos(d*x+c)^2*sin(d*x+c)+122880*sin(d*x+c)*cos(d*x+c)^13-61440*sin(d*x+c
)*cos(d*x+c)^12+7168*sin(d*x+c)*cos(d*x+c)^9-18432*sin(d*x+c)*cos(d*x+c)^11
-6144*sin(d*x+c)*cos(d*x+c)^10-105*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(13/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*cos(d*x+c)^6*sin(d*x+c)-630*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(13/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(
d*x+c)/cos(d*x+c))*cos(d*x+c)^5*sin(d*x+c)-1575*I*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(13/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^4*sin(d*x+c)-2100*I*2^(1/2)*(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(13/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)/cos(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-1024*I*cos(d*x+c)^
10-8960*sin(d*x+c)*cos(d*x+c)^8+13440*sin(d*x+c)*cos(d*x+c)^7-122880*I*cos(
```

```
d*x+c)^14+61440*I*cos(d*x+c)^13+79872*I*cos(d*x+c)^12-24576*I*cos(d*x+c)^11
-1792*I*cos(d*x+c)^9-4480*I*cos(d*x+c)^8+13440*I*cos(d*x+c)^7+105*2^(1/2)*c
os(d*x+c)^6*sin(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+630*2^(1/2)*cos(d*x+c)^5*sin(d*x
+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(13/2)+1575*2^(1/2)*cos(d*x+c)^4*sin(d*x+c)*arctan(1/2*2^(
1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
13/2)+2100*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+1575*2^(1/2)
*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)+630*2^(1/2)*cos(d*x+c)*sin(d*x
+c)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(13/2)+105*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)*sin(d*x+c)-105*I*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(13/2)*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c))*(a*(I*sin(d*x+
c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^6
```

Maxima [B] time = 2.46521, size = 1688, normalized size = 8.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/6720*((-140*I*sqrt(2)*a^3*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 140*sqrt(2)*a^3*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + (-60*I*sqrt(2)*a^3*cos(2*d*x + 2*c)^2 - 60*I*sqrt(2)*a^3*sin(2*
d*x + 2*c)^2 - 120*I*sqrt(2)*a^3*cos(2*d*x + 2*c) - 60*I*sqrt(2)*a^3)*cos(7
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 60*(sqrt(2)*a^3*cos(2
*d*x + 2*c)^2 + sqrt(2)*a^3*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x +
2*c) + sqrt(2)*a^3)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)
*sqrt(a) + (-420*I*sqrt(2)*a^3*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + 420*sqrt(2)*a^3*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + (-84*I*sqrt(2)*a^3*cos(2*d*x + 2*c)^2 - 84*I*sqrt(2)*a^3*sin
(2*d*x + 2*c)^2 - 168*I*sqrt(2)*a^3*cos(2*d*x + 2*c) - 84*I*sqrt(2)*a^3)*co
s(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 84*(sqrt(2)*a^3*co
s(2*d*x + 2*c)^2 + sqrt(2)*a^3*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x
 + 2*c) + sqrt(2)*a^3)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sqrt(a) - (210*sqrt(2)*a^3*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) - 210*sqrt(2)*a^3*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - 1
05*I*sqrt(2)*a^3*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2
 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + 105*I*sqrt(2)*a^3*log(sqrt
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*
```

```
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) + 1))*sqrt(a))/d
```

Fricas [B] time = 2.26525, size = 891, normalized size = 4.55

$$-\frac{\left(105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(idx+ic)}\log\left(\frac{\left(2i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(idx+ic)}+\sqrt{2}\left(a^3e^{(2idx+2ic)}+a^3\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{(idx+ic)}\right)e^{(-idx-ic)}}{a^3}\right)-105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(idx+ic)}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/1680*(105*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(I*d*x + I*c)*log((2*I*sqrt(1/2)*
sqrt(-a^7/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*(a^3*e^(2*I*d*x + 2*I*c) + a^3)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^3) -
105*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(I*d*x + I*c)*log((-2*I*sqrt(1/2)*sqrt(-a^
7/d^2)*d*e^(I*d*x + I*c) + sqrt(2)*(a^3*e^(2*I*d*x + 2*I*c) + a^3)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a^3) - sqrt(2)*
(-15*I*a^3*e^(8*I*d*x + 8*I*c) - 81*I*a^3*e^(6*I*d*x + 6*I*c) - 188*I*a^3*e
^(4*I*d*x + 4*I*c) - 298*I*a^3*e^(2*I*d*x + 2*I*c) - 176*I*a^3)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.331 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=268

$$-\frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d}-\frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d}-\frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+d}}{64d}$$

```
[Out] (((11*I)/64)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*T
an[c + d*x]])])/(Sqrt[2]*d) + (((11*I)/96)*a^4*Cos[c + d*x])/(d*Sqrt[a + I*
a*Tan[c + d*x]]) - (((11*I)/64)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]]
)/d - (((11*I)/120)*a^3*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((1
1*I)/140)*a^2*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d - (((11*I)/126
)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/9)*Cos[c + d*x]^9*
(a + I*a*Tan[c + d*x])^(7/2))/d
```

Rubi [A] time = 0.411437, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3497, 3502, 3490, 3489, 206}

$$-\frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d}-\frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d}-\frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+d}}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2),x]
```


```
[Out] (((11*I)/64)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*T
an[c + d*x]])])/(Sqrt[2]*d) + (((11*I)/96)*a^4*Cos[c + d*x])/(d*Sqrt[a + I*
a*Tan[c + d*x]]) - (((11*I)/64)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]]
)/d - (((11*I)/120)*a^3*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((1
1*I)/140)*a^2*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d - (((11*I)/126
)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/9)*Cos[c + d*x]^9*
(a + I*a*Tan[c + d*x])^(7/2))/d
```


Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
```

```
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d} + \frac{1}{18}(11a)\int \cos^7(c+dx)(a+ia\tan$$

$$= -\frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+d}{9d}$$

$$= -\frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d} - \frac{11ia\cos^7(c+dx)(a+ia\tan(}{126d}$$

$$= -\frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} - \frac{11ia^2\cos^5(c+dx)(a+ia\tan(c}{140d}$$

$$= \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} - \frac{11ia^2 c}{}$$

$$= \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d} - \frac{11ia^3 c}{}$$

$$= \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d} - \frac{11ia^3 c}{}$$

$$= \frac{11ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}d} + \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos(c}{}$$

Mathematica [A] time = 3.20505, size = 188, normalized size = 0.7

$$-\frac{ia^3e^{-3i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(4303e^{2i(c+dx)}+7034e^{4i(c+dx)}+3754e^{6i(c+dx)}+1798e^{8i(c+dx)}+530e^{10i(c+dx)}+70e^{12i(c+dx)}-34\right.}{20160\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I/20160)*a^3*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-3
15 + 4303*E^((2*I)*(c + d*x)) + 7034*E^((4*I)*(c + d*x)) + 3754*E^((6*I)*(c
 + d*x)) + 1798*E^((8*I)*(c + d*x)) + 530*E^((10*I)*(c + d*x)) + 70*E^((12*
I)*(c + d*x)) - 3465*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcT
anh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*E^((3*I)*(c + d*x)))
```

Maple [B] time = 0.615, size = 1604, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -1/10321920/d*a^3*(97020*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(17/2)*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d
*x+c)*cos(d*x+c)^6*2^(1/2)+194040*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(17/2)*a
rctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+
c))*sin(d*x+c)*cos(d*x+c)^5*2^(1/2)+242550*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(17/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)
/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)+194040*I*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(17/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*s
in(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)+97020*I*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(17/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)+27720*I*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(17/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+3465*I*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(17/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^8*2^(1/2)+
983040*sin(d*x+c)*cos(d*x+c)^15-3465*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(17/2)*sin(d*x+c)
+655360*sin(d*x+c)*cos(d*x+c)^14-720896*sin(d*x+c)*cos(d*x+c)^13-9175040*si
n(d*x+c)*cos(d*x+c)^17+811008*sin(d*x+c)*cos(d*x+c)^12-1774080*sin(d*x+c)*c
os(d*x+c)^9-946176*sin(d*x+c)*cos(d*x+c)^11+1182720*sin(d*x+c)*cos(d*x+c)^1
0+27720*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(17/2)*arctanh(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*sin(d*x+c)*cos(d*x+c)^
7*2^(1/2)+3465*I*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(17/2)*sin(d*x
+c)-3465*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d
*x+c)/(cos(d*x+c)+1))^(17/2)*sin(d*x+c)*cos(d*x+c)^8*2^(1/2)-27720*arctan(1
/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(17/2)*sin(d*x+c)*cos(d*x+c)^7*2^(1/2)-97020*arctan(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(17/2)*sin(d
*x+c)*cos(d*x+c)^6*2^(1/2)-194040*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(17/2)*sin(d*x+c)*cos(d*x+c)
^5*2^(1/2)-242550*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(17/2)*sin(d*x+c)*cos(d*x+c)^4*2^(1/2)-19404
0*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(17/2)*sin(d*x+c)*cos(d*x+c)^3*2^(1/2)-97020*arctan(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
7/2)*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)-27720*arctan(1/2*2^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(17/2)*sin(d*x+c)*c
os(d*x+c)*2^(1/2)+4587520*sin(d*x+c)*cos(d*x+c)^16+9175040*I*cos(d*x+c)^18-
4587520*I*cos(d*x+c)^17-5570560*I*cos(d*x+c)^16+1638400*I*cos(d*x+c)^15+655
36*I*cos(d*x+c)^14+90112*I*cos(d*x+c)^13+135168*I*cos(d*x+c)^12+236544*I*co
s(d*x+c)^11+591360*I*cos(d*x+c)^10-1774080*I*cos(d*x+c)^9)*(a*(I*sin(d*x+c)
+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^8
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.25117, size = 1034, normalized size = 3.86

$$-\frac{\left(3465\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(3i\,dx+3i\,c)}\log\left(\frac{\left(22i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(i\,dx+i\,c)}+11\sqrt{2}\left(a^3e^{(2i\,dx+2i\,c)}+a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{11\,a^3}\right)-3465\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/40320*(3465*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(3*I*d*x + 3*I*c)*log(1/11*(22*
I*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(I*d*x + I*c) + 11*sqrt(2)*(a^3*e^(2*I*d*x +
 2*I*c) + a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x
 - I*c)/a^3) - 3465*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(3*I*d*x + 3*I*c)*log(1/11
*(-22*I*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(I*d*x + I*c) + 11*sqrt(2)*(a^3*e^(2*I
*d*x + 2*I*c) + a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(
-I*d*x - I*c)/a^3) - sqrt(2)*(-70*I*a^3*e^(12*I*d*x + 12*I*c) - 530*I*a^3*e
^(10*I*d*x + 10*I*c) - 1798*I*a^3*e^(8*I*d*x + 8*I*c) - 3754*I*a^3*e^(6*I*d
*x + 6*I*c) - 7034*I*a^3*e^(4*I*d*x + 4*I*c) - 4303*I*a^3*e^(2*I*d*x + 2*I*
c) + 315*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-3*I*
d*x - 3*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.332 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=342

$$-\frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d}-\frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d}-\frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d}$$

```
[Out] (((195*I)/1024)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*
a*Tan[c + d*x]])])/(Sqrt[2]*d) + (((65*I)/512)*a^4*Cos[c + d*x])/(d*Sqrt[a
+ I*a*Tan[c + d*x]]) + (((39*I)/448)*a^4*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Ta
n[c + d*x]]) - (((195*I)/1024)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d - (((13*I)/128)*a^3*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((13
*I)/168)*a^3*Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d - (((65*I)/924)*a
^2*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(3/2))/d - (((5*I)/66)*a*Cos[c + d
*x]^9*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/11)*Cos[c + d*x]^11*(a + I*a*Ta
n[c + d*x])^(7/2))/d
```

Rubi [A] time = 0.56461, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3497, 3502, 3490, 3489, 206}

$$-\frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d}-\frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d}-\frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2),x]
```


```
[Out] (((195*I)/1024)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*
a*Tan[c + d*x]])])/(Sqrt[2]*d) + (((65*I)/512)*a^4*Cos[c + d*x])/(d*Sqrt[a
+ I*a*Tan[c + d*x]]) + (((39*I)/448)*a^4*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Ta
n[c + d*x]]) - (((195*I)/1024)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d - (((13*I)/128)*a^3*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((13
*I)/168)*a^3*Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d - (((65*I)/924)*a
^2*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(3/2))/d - (((5*I)/66)*a*Cos[c + d
*x]^9*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/11)*Cos[c + d*x]^11*(a + I*a*Ta
n[c + d*x])^(7/2))/d
```


Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx &= -\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}+\frac{1}{22}(15a)\int \cos^9(c+dx)(a+ia\,\mathrm{t}\\
&= -\frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d}-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+}{11d}\\
&= -\frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d}-\frac{5ia\cos^9(c+dx)(a+ia\tan(}{66d}\\
&= -\frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d}-\frac{65ia^2\cos^7(c+dx)(a+ia\tan(}{924d}\\
&= \frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}}-\frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d}-\frac{65ia}{}\\
&= \frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}}-\frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d}-\frac{13ia}{}\\
&= \frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}}+\frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}}-\frac{13ia^3\cos^3(c+dx}{1}\\
&= \frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}}+\frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}}-\frac{195ia^3\cos(c+dx}{1}\\
&= \frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}}+\frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}}-\frac{195ia^3\cos(c+dx}{1}\\
&= \frac{195ia^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{1024\sqrt{2}d}+\frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}}+\frac{39ia}{448d\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 6.32674, size = 194, normalized size = 0.57

$$-\frac{ia^3e^{-5i(c+dx)}\left(-7161e^{2i(c+dx)}+47413e^{4i(c+dx)}+78800e^{6i(c+dx)}+38512e^{8i(c+dx)}+19552e^{10i(c+dx)}+7184e^{12i(c+dx)}+162\right.}{473088d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I/473088)*a^3*(-462 - 7161*E^((2*I)*(c + d*x)) + 47413*E^((4*I)*(c + d*x
)) + 78800*E^((6*I)*(c + d*x)) + 38512*E^((8*I)*(c + d*x)) + 19552*E^((10*I
)*(c + d*x)) + 7184*E^((12*I)*(c + d*x)) + 1624*E^((14*I)*(c + d*x)) + 168*
E^((16*I)*(c + d*x)) - 45045*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x
))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*
E^((5*I)*(c + d*x)))
```

Maple [B] time = 0.816, size = 1948, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -1/484442112/d*a^3*(5405400*I*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*2^
(1/2)*sin(d*x+c)*cos(d*x+c)^7-37486592*sin(d*x+c)*cos(d*x+c)^15+42172416*si
n(d*x+c)*cos(d*x+c)^14-49201152*sin(d*x+c)*cos(d*x+c)^13-31457280*sin(d*x+c
)*cos(d*x+c)^17+61501440*sin(d*x+c)*cos(d*x+c)^12-92252160*sin(d*x+c)*cos(d
*x+c)^11+11351340*I*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*2^(1/2)*sin(
d*x+c)*cos(d*x+c)^5+9459450*I*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*2^
(1/2)*sin(d*x+c)*cos(d*x+c)^4+5405400*I*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(21/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3+2027025*I*arctanh(1/2*2^(1/2)*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(
d*x+c)+1))^(21/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2+450450*I*arctanh(1/2*2^(1
/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(21/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)+45045*I*arctanh(1/
2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(21/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^10+450450*I*
arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)/cos(d*x
+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^9+
2027025*I*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+
c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*2^(1/2)*sin(d*x+c)*cos
(d*x+c)^8+9459450*I*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*2^(1/2)*sin(
d*x+c)*cos(d*x+c)^6-45045*2^(1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*sin(d*x+c)+352321536*
I*cos(d*x+c)^22-176160768*I*cos(d*x+c)^21-205520896*I*cos(d*x+c)^20+5872025
6*I*cos(d*x+c)^19+2097152*I*cos(d*x+c)^18+2621440*I*cos(d*x+c)^17+3407872*I
*cos(d*x+c)^16+4685824*I*cos(d*x+c)^15+7028736*I*cos(d*x+c)^14+12300288*I*c
os(d*x+c)^13+30750720*I*cos(d*x+c)^12-92252160*I*cos(d*x+c)^11-352321536*si
n(d*x+c)*cos(d*x+c)^21+176160768*sin(d*x+c)*cos(d*x+c)^20+29360128*sin(d*x+
c)*cos(d*x+c)^19+29360128*sin(d*x+c)*cos(d*x+c)^18+34078720*sin(d*x+c)*cos(
d*x+c)^16+45045*I*2^(1/2)*arctanh(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*sin(d*x+c)/cos(d*x+c))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*sin(d*
x+c)-45045*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*arctan(1/2*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^10-450450*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^9-2027025*(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(21/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*2^(1/2)*sin(d*x+c)*cos(d*x+c)^8-5405400*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21
/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*
x+c)*cos(d*x+c)^7-9459450*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*arctan(1/2*
```

```
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)
^6-11351340*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*arctan(1/2*2^(1/2)*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^5-9459450*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)^4-5405400*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(21/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3-2027025*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(2
1/2)*arctan(1/2*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d
*x+c)*cos(d*x+c)^2-450450*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(21/2)*arctan(1/2*
2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)*cos(d*x+c)
)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1
)/cos(d*x+c)^10
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.26877, size = 1149, normalized size = 3.36

$$-\left(45045\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(5idx+5ic)}\log\left(\frac{\left(390i\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(idx+ic)}+195\sqrt{2}\left(a^3e^{(2idx+2ic)}+a^3\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{(idx+ic)}\right)e^{(-idx-ic)}}{195a^3}\right)-45045\sqrt{\frac{1}{2}}\sqrt{-\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/473088*(45045*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(5*I*d*x + 5*I*c)*log(1/195*(
390*I*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(I*d*x + I*c) + 195*sqrt(2)*(a^3*e^(2*I*
d*x + 2*I*c) + a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-
I*d*x - I*c)/a^3) - 45045*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(5*I*d*x + 5*I*c)*lo
g(1/195*(-390*I*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(I*d*x + I*c) + 195*sqrt(2)*(a
^3*e^(2*I*d*x + 2*I*c) + a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x +
I*c))*e^(-I*d*x - I*c)/a^3) - sqrt(2)*(-168*I*a^3*e^(16*I*d*x + 16*I*c) - 1
624*I*a^3*e^(14*I*d*x + 14*I*c) - 7184*I*a^3*e^(12*I*d*x + 12*I*c) - 19552*
I*a^3*e^(10*I*d*x + 10*I*c) - 38512*I*a^3*e^(8*I*d*x + 8*I*c) - 78800*I*a^3
*e^(6*I*d*x + 6*I*c) - 47413*I*a^3*e^(4*I*d*x + 4*I*c) + 7161*I*a^3*e^(2*I*
d*x + 2*I*c) + 462*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)
)*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.333 $\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=117

$$\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^7d}-\frac{12i(a+ia\tan(c+dx))^{11/2}}{11a^6d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{3a^5d}-\frac{16i(a+ia\tan(c+dx))^{7/2}}{7a^4d}$$

```
[Out] (((-16*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^4*d) + (((8*I)/3)*(a + I*a*Ta
n[c + d*x])^(9/2))/(a^5*d) - (((12*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a
^6*d) + (((2*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^7*d)
```

Rubi [A] time = 0.0797237, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^7d}-\frac{12i(a+ia\tan(c+dx))^{11/2}}{11a^6d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{3a^5d}-\frac{16i(a+ia\tan(c+dx))^{7/2}}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-16*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^4*d) + (((8*I)/3)*(a + I*a*Ta
n[c + d*x])^(9/2))/(a^5*d) - (((12*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a
^6*d) + (((2*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^3(a+x)^{5/2}\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(8a^3(a+x)^{5/2}-12a^2(a+x)^{7/2}+6a(a+x)^{9/2}-(a+x)^{11/2}\right)dx, x, ia\tan(c+\right.}{a^7d}\\ &= -\frac{16i(a+ia\tan(c+dx))^{7/2}}{7a^4d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{3a^5d}-\frac{12i(a+ia\tan(c+dx))^{11/2}}{11a^6d}+\frac{2}{}\end{aligned}$$

Mathematica [A] time = 0.466889, size = 95, normalized size = 0.81

$$\frac{2\sec^7(c+dx)(-7i(26\sin(c+dx)+59\sin(3(c+dx)))+390\cos(c+dx)+445\cos(3(c+dx)))(\sin(4(c+dx))-i\cos(}{3003d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^7*(390*Cos[c + d*x] + 445*Cos[3*(c + d*x)] - (7*I)*(26*Sin[
c + d*x] + 59*Sin[3*(c + d*x)]))*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])
)/(3003*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.583, size = 127, normalized size = 1.1

$$-\frac{1024\,i\left(\cos\left(dx+c\right)\right)^{6}-1024\,\left(\cos\left(dx+c\right)\right)^{5}\sin\left(dx+c\right)+128\,i\left(\cos\left(dx+c\right)\right)^{4}-640\,\left(\cos\left(dx+c\right)\right)^{3}\sin\left(dx+c\right)+56\,i\left(\ldots\right.}{3003\,ad\left(\cos\left(dx+c\right)\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -2/3003/d/a*(512*I*cos(d*x+c)^6-512*cos(d*x+c)^5*sin(d*x+c)+64*I*cos(d*x+c)
^4-320*cos(d*x+c)^3*sin(d*x+c)+28*I*cos(d*x+c)^2-252*cos(d*x+c)*sin(d*x+c)+
231*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^6
```

Maxima [B] time = 1.12363, size = 401, normalized size = 3.43

$$-\frac{2i\left(15015\,\sqrt{i\,a\tan\left(dx+c\right)+a}-\frac{3003\left(3\left(i\,a\tan(dx+c)+a\right)^{\frac{5}{2}}-10\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}a+15\,\sqrt{i\,a\tan(dx+c)+a}\,a^{2}\right)}{a^{2}}+\frac{143\left(35\left(i\,a\tan(dx+c)+a\right)^{\frac{9}{2}}-180\ldots\right.}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15015*I*(15015*sqrt(I*a*tan(d*x + c) + a) - 3003*(3*(I*a*tan(d*x + c) +
a)^(5/2) - 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a
)*a^2)/a^2 + 143*(35*(I*a*tan(d*x + c) + a)^(9/2) - 180*(I*a*tan(d*x + c) +
 a)^(7/2)*a + 378*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 420*(I*a*tan(d*x + c)
+ a)^(3/2)*a^3 + 315*sqrt(I*a*tan(d*x + c) + a)*a^4)/a^4 - 5*(231*(I*a*tan(
d*x + c) + a)^(13/2) - 1638*(I*a*tan(d*x + c) + a)^(11/2)*a + 5005*(I*a*tan
(d*x + c) + a)^(9/2)*a^2 - 8580*(I*a*tan(d*x + c) + a)^(7/2)*a^3 + 9009*(I*
a*tan(d*x + c) + a)^(5/2)*a^4 - 6006*(I*a*tan(d*x + c) + a)^(3/2)*a^5 + 300
3*sqrt(I*a*tan(d*x + c) + a)*a^6)/a^6)/(a*d)
```

Fricas [A] time = 2.09395, size = 498, normalized size = 4.26

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-2048i\,e^{(12i\,dx+12i\,c)}-13312i\,e^{(10i\,dx+10i\,c)}-36608i\,e^{(8i\,dx+8i\,c)}-54912i\,e^{(6i\,dx+6i\,c)}\right)e^{(i\,dx+i\,c)}}{3003\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3003*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2048*I*e^(12*I*d*x + 12*
I*c) - 13312*I*e^(10*I*d*x + 10*I*c) - 36608*I*e^(8*I*d*x + 8*I*c) - 54912*
I*e^(6*I*d*x + 6*I*c))*e^(I*d*x + I*c)/(a*d*e^(12*I*d*x + 12*I*c) + 6*a*d*e
^(10*I*d*x + 10*I*c) + 15*a*d*e^(8*I*d*x + 8*I*c) + 20*a*d*e^(6*I*d*x + 6*I
*c) + 15*a*d*e^(4*I*d*x + 4*I*c) + 6*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^8}{\sqrt{i a \tan (dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^8/sqrt(I*a*tan(d*x + c) + a), x)
```

3.334 $\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=88

$$-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^5d}+\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^4d}-\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^3d}$$

```
[Out] (((-8*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^3*d) + (((8*I)/7)*(a + I*a*Tan
[c + d*x])^(7/2))/(a^4*d) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^5*d
)
```

Rubi [A] time = 0.0720852, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^5d}+\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^4d}-\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-8*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^3*d) + (((8*I)/7)*(a + I*a*Tan
[c + d*x])^(7/2))/(a^4*d) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^5*d
)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^2(a+x)^{3/2}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a+x)^{3/2}-4a(a+x)^{5/2}+(a+x)^{7/2}\right)dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^3d}+\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^4d}-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^5d}\end{aligned}$$

Mathematica [A] time = 0.279938, size = 77, normalized size = 0.88

$$\frac{2\sec^5(c+dx)(-55i\sin(2(c+dx))+71\cos(2(c+dx))+36)(\sin(3(c+dx))-i\cos(3(c+dx)))}{315d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^5*(36 + 71*Cos[2*(c + d*x)] - (55*I)*Sin[2*(c + d*x)])*((-I
)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)]))/(315*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.375, size = 100, normalized size = 1.1

$$-\frac{128\,i\,(\cos(dx+c))^4-128\,(\cos(dx+c))^3\sin(dx+c)+16\,i\,(\cos(dx+c))^2-80\,\cos(dx+c)\sin(dx+c)+70\,i}{315\,ad\,(\cos(dx+c))^4}\sqrt{a(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -2/315/d/a*(64*I*cos(d*x+c)^4-64*cos(d*x+c)^3*sin(d*x+c)+8*I*cos(d*x+c)^2-4
0*cos(d*x+c)*sin(d*x+c)+35*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)/cos(d*x+c)^4
```

Maxima [B] time = 1.13443, size = 228, normalized size = 2.59

$$-\frac{2i\left(315\sqrt{ia\tan(dx+c)+a}-\frac{42\left(3\,(ia\tan(dx+c)+a)^{\frac{5}{2}}-10\,(ia\tan(dx+c)+a)^{\frac{3}{2}}a+15\,\sqrt{ia\tan(dx+c)+a}a^2\right)}{a^2}+\frac{35\,(ia\tan(dx+c)+a)^{\frac{9}{2}}-180\,(ia\,\mathrm{ta}}{}\right.}{315\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/315*I*(315*sqrt(I*a*tan(d*x + c) + a) - 42*(3*(I*a*tan(d*x + c) + a)^(5/
2) - 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a)*a^2)
/a^2 + (35*(I*a*tan(d*x + c) + a)^(9/2) - 180*(I*a*tan(d*x + c) + a)^(7/2)*
a + 378*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 420*(I*a*tan(d*x + c) + a)^(3/2)
*a^3 + 315*sqrt(I*a*tan(d*x + c) + a)*a^4)/a^4)/(a*d)
```

Fricas [A] time = 2.21809, size = 362, normalized size = 4.11

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-256i\,e^{(8i\,dx+8i\,c)}-1152i\,e^{(6i\,dx+6i\,c)}-2016i\,e^{(4i\,dx+4i\,c)}\right)e^{(i\,dx+i\,c)}}{315\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-256*I*e^(8*I*d*x + 8*I*c)
 - 1152*I*e^(6*I*d*x + 6*I*c) - 2016*I*e^(4*I*d*x + 4*I*c))*e^(I*d*x + I*c)
/(a*d*e^(8*I*d*x + 8*I*c) + 4*a*d*e^(6*I*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x +
```

```
4*I*c) + 4*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(1/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^6}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^6/sqrt(I*a*tan(d*x + c) + a), x)
```

3.335 $\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=59

$$\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^3d}-\frac{4i(a+ia\tan(c+dx))^{3/2}}{3a^2d}$$

```
[Out] (((-4*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^2*d) + (((2*I)/5)*(a + I*a*Tan
[c + d*x])^(5/2))/(a^3*d)
```

Rubi [A] time = 0.0643235, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^3d}-\frac{4i(a+ia\tan(c+dx))^{3/2}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-4*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^2*d) + (((2*I)/5)*(a + I*a*Tan
[c + d*x])^(5/2))/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx&=-\frac{i\,\text{Subst}\left(\int (a-x)\sqrt{a+x}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\&=-\frac{i\,\text{Subst}\left(\int \left(2a\sqrt{a+x}-(a+x)^{3/2}\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\&=-\frac{4i(a+ia\tan(c+dx))^{3/2}}{3a^2d}+\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^3d}\end{aligned}$$

Mathematica [A] time = 0.175313, size = 65, normalized size = 1.1

$$-\frac{2(3\tan(c+dx)+7i)\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))}{15d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*(7*I + 3*Tan[c +
 d*x]))/(15*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.319, size = 73, normalized size = 1.2

$$-\frac{8\,i\,(\cos(dx+c))^2-8\,\cos(dx+c)\sin(dx+c)+6\,i}{15\,ad\,(\cos(dx+c))^2}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/a*(4*I*cos(d*x+c)^2-4*cos(d*x+c)*sin(d*x+c)+3*I)*(a*(I*sin(d*x+c)+c
os(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2
```

Maxima [A] time = 1.09604, size = 107, normalized size = 1.81

$$-\frac{2i\left(15\,\sqrt{i\,a\tan(dx+c)+a}-\frac{3\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}-10\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a+15\,\sqrt{i\,a\tan(dx+c)+a}a^2}{a^2}\right)}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15*I*(15*sqrt(I*a*tan(d*x + c) + a) - (3*(I*a*tan(d*x + c) + a)^(5/2) -
10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a)*a^2)/a^2)
/(a*d)
```

Fricas [A] time = 1.91261, size = 242, normalized size = 4.1

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-16i\,e^{(4i\,dx+4i\,c)}-40i\,e^{(2i\,dx+2i\,c)}\right)e^{(i\,dx+i\,c)}}{15\left(ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-16*I*e^(4*I*d*x + 4*I*c) -
 40*I*e^(2*I*d*x + 2*I*c))*e^(I*d*x + I*c)/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d
*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^4(c+dx)}{\sqrt{a\,(i\tan(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(I*a*tan(d*x + c) + a), x)
```

3.336 $\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=27

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

```
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rubi [A] time = 0.0566053, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 32}

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1}{\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}\end{aligned}$$

Mathematica [A] time = 0.143902, size = 32, normalized size = 1.19

$$\frac{2(\tan(c+dx)-i)}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*(-I + Tan[c + d*x]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.043, size = 24, normalized size = 0.9

$$\frac{-2\,i}{ad}\sqrt{a+ia\tan\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -2*I*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Maxima [A] time = 1.05822, size = 28, normalized size = 1.04

$$-\frac{2i\,\sqrt{i\,a\tan\left(dx+c\right)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*I*sqrt(I*a*tan(d*x + c) + a)/(a*d)
```

Fricas [A] time = 1.92316, size = 95, normalized size = 3.52

$$-\frac{2i\,\sqrt{2}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}e^{\left(i\,dx+i\,c\right)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{2}\left(c+dx\right)}{\sqrt{a\left(i\tan\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [A] time = 1.67694, size = 53, normalized size = 1.96

$$-\frac{2i\,\sqrt{a-\frac{2i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*I*sqrt(a - 2*I*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/(a*d
)
```

3.337 $\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=146

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}+\frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}}+\frac{5i}{8d\sqrt{a+ia\tan(c+dx)}}-\frac{5i\tanh^{-1}\left(\sqrt{a\cdots}\right.}{8\sqrt{2}\cdots}$$

```
[Out] (((-5*I)/8)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]
*Sqrt[a]*d) + (((5*I)/12)*a)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I/2)*a^2)
/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + ((5*I)/8)/(d*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.0981909, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}+\frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}}+\frac{5i}{8d\sqrt{a+ia\tan(c+dx)}}-\frac{5i\tanh^{-1}\left(\sqrt{a\cdots}\right.}{8\sqrt{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-5*I)/8)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]
*Sqrt[a]*d) + (((5*I)/12)*a)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I/2)*a^2)
/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + ((5*I)/8)/(d*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\left(ia^3\right)\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} - \frac{\left(5ia^2\right)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c\right.}{4d}$$

$$= \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} - \frac{(5ia)\text{Subst}\left(\int\right.}{}$$

$$= \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{5i}{8d\sqrt{a+ia\tan(}}$$

$$= \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{5i}{8d\sqrt{a+ia\tan(}}$$

$$= -\frac{5i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d} + \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan}$$

Mathematica [A] time = 0.4769, size = 126, normalized size = 0.86

$$-\frac{ie^{-2i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\left(-14e^{2i(c+dx)}+3e^{4i(c+dx)}-2\right)+15e^{3i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{24d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I/24)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-2 - 14*E^((2*I)*(c + d*x)) + 3*E^
((4*I)*(c + d*x))) + 15*E^((3*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))]))/(d*E
^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]]
)
```

Maple [B] time = 0.334, size = 341, normalized size = 2.3

$$\frac{1}{96\,ad}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(15\,i\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(i\cos(dx+c)-i-\sin(dx+c))}{2\,\sin(dx+c)}\frac{1}{\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{-2\,\frac{\text{co}}{\cos(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/96/d/a*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*I*2^(1/2)*arcta
n(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+32*I*cos(d*
```

```
x+c)^4+15*I*2^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+
c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)+15*2^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin
(d*x+c)+32*cos(d*x+c)^3*sin(d*x+c)+20*I*cos(d*x+c)^2+60*cos(d*x+c)*sin(d*x+
c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.46901, size = 837, normalized size = 5.73

$$\left(-15i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(4idx+4ic)}\log\left(\left(2\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(2idx+2ic)}+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)+15i\sqrt{\frac{1}{2}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(-15*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(4*I*d*x + 4*I*c)*log((2*sqrt(1
/2)*a*d*sqrt(1/(a*d^2))*e^(2*I*d*x + 2*I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
 15*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(4*I*d*x + 4*I*c)*log(-(2*sqrt(1/2)*a
*d*sqrt(1/(a*d^2))*e^(2*I*d*x + 2*I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt
(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-3*I*e^(6*I*d*x + 6*I*c) + 11*I*e^(4
*I*d*x + 4*I*c) + 16*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(I*d*x + I*c))*e^(-4*I*
d*x - 4*I*c)/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a(i\tan(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{\sqrt{i a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(I*a*tan(d*x + c) + a), x)
```

3.338 $\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=219

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}-\frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}+\frac{63ia^2}{160d(a+ia\tan(c+$$

```
[Out] (((-63*I)/128)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*Sqrt[a]*d) + (((63*I)/160)*a^2)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((I/
4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) - (((9*I)
/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (((21*I
)/64)*a)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + ((63*I)/128)/(d*Sqrt[a + I*a*Ta
n[c + d*x]])
```

Rubi [A] time = 0.12402, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}-\frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}+\frac{63ia^2}{160d(a+ia\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```


```
[Out] (((-63*I)/128)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*Sqrt[a]*d) + (((63*I)/160)*a^2)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((I/
4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) - (((9*I)
/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (((21*I
)/64)*a)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + ((63*I)/128)/(d*Sqrt[a + I*a*Ta
n[c + d*x]])
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\left(ia^5\right)\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{7/2}}\,dx,x,ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{\left(9ia^4\right)\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{7/2}}\,dx,x,ia\tan(\right.}{8d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx)}$$

$$= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{}{16d(a-ia\,\text{ta}}$$

$$= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{}{16d(a-ia\,\text{ta}}$$

$$= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{}{16d(a-ia\,\text{ta}}$$

$$= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{}{16d(a-ia\,\text{ta}}$$

$$= -\frac{63i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d} + \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia}$$

Mathematica [A] time = 0.627008, size = 152, normalized size = 0.69

$$-\frac{ie^{-4i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\left(-56e^{2i(c+dx)}-288e^{4i(c+dx)}+85e^{6i(c+dx)}+10e^{8i(c+dx)}-8\right)+315e^{5i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)\right)}{640d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I/640)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-8 - 56*E^((2*I)*(c + d*x)) - 288
*E^((4*I)*(c + d*x)) + 85*E^((6*I)*(c + d*x)) + 10*E^((8*I)*(c + d*x))) + 3
15*E^((5*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))]))/(d*E^((4*I)*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.361, size = 368, normalized size = 1.7

$$\frac{1}{2560\,ad}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(512\,i\,(\cos(dx+c))^6+512\,(\cos(dx+c))^5\sin(dx+c)+315\,i\sqrt{2}\sqrt{-2\,\frac{\cos(d}{\cos(dx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/2560/d/a*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(512*I*cos(d*x+c)
^6+512*cos(d*x+c)^5*sin(d*x+c)+315*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+96*I*cos(d*x+c)^4+315*I*2^(1/2)*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x
+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+315*2^(1/2)*arctan(1/
2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+672*cos(d*x+c)^
3*sin(d*x+c)+420*I*cos(d*x+c)^2+1260*cos(d*x+c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.57692, size = 922, normalized size = 4.21

$$\frac{\left(-315i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(6i\,dx+6i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(2i\,dx+2i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+315i\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(-315*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(6*I*d*x + 6*I*c)*log((2*sqr
t(1/2)*a*d*sqrt(1/(a*d^2))*e^(2*I*d*x + 2*I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
) + 315*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(6*I*d*x + 6*I*c)*log(-(2*sqrt(1/
2)*a*d*sqrt(1/(a*d^2))*e^(2*I*d*x + 2*I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-10*I*e^(10*I*d*x + 10*I*c) - 95
*I*e^(8*I*d*x + 8*I*c) + 203*I*e^(6*I*d*x + 6*I*c) + 344*I*e^(4*I*d*x + 4*I
*c) + 64*I*e^(2*I*d*x + 2*I*c) + 8*I)*e^(I*d*x + I*c))*e^(-6*I*d*x - 6*I*c)
/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(I*a*tan(d*x + c) + a), x)
```

3.339 $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=292

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}-\frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}-\frac{}{192d(a-ia\tan(c}$$

```
[Out] (((-429*I)/1024)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*Sqrt[a]*d) + (((429*I)/896)*a^3)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - (
(I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(7/2)) - (((1
3*I)/48)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) - (
((143*I)/192)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2))
+ (((429*I)/1280)*a^2)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((143*I)/512)*a)
/(d*(a + I*a*Tan[c + d*x])^(3/2)) + ((429*I)/1024)/(d*Sqrt[a + I*a*Tan[c +
d*x]])
```

Rubi [A] time = 0.153432, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}-\frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}-\frac{}{192d(a-ia\tan(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-429*I)/1024)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*Sqrt[a]*d) + (((429*I)/896)*a^3)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - (
(I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(7/2)) - (((1
3*I)/48)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) - (
((143*I)/192)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2))
+ (((429*I)/1280)*a^2)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((143*I)/512)*a)
/(d*(a + I*a*Tan[c + d*x])^(3/2)) + ((429*I)/1024)/(d*Sqrt[a + I*a*Tan[c +
d*x]])
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\left(ia^7\right)\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{\left(13ia^6\right)\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{9/2}}\,dx, x, ia\tan\right.}{12d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx}$$

$$= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{}{48d(a-ia\tan}$$

$$= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{}{48d(a-ia\tan}$$

$$= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{}{48d(a-ia\tan}$$

$$= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{}{48d(a-ia\tan}$$

$$= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{}{48d(a-ia\tan}$$

$$= -\frac{429i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\sqrt{a}d} + \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+}$$

Mathematica [A] time = 0.958909, size = 178, normalized size = 0.61

$$-\frac{ie^{-6i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\left(-2064e^{2i(c+dx)} - 9008e^{4i(c+dx)} - 40784e^{6i(c+dx)} + 13755e^{8i(c+dx)} + 2590e^{10i(c+dx)} + 280e^{12i(c+dx)}\right.\right.}{107520d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I/107520)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(-240 - 2064*E^((2*I)*(c + d*x)
) - 9008*E^((4*I)*(c + d*x)) - 40784*E^((6*I)*(c + d*x)) + 13755*E^((8*I)*(
```

```
c + d*x)) + 2590*E^((10*I)*(c + d*x)) + 280*E^((12*I)*(c + d*x))) + 45045*E
^((7*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))]))/(d*E^((6*I)*(c + d*x))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.506, size = 395, normalized size = 1.4

$$\frac{1}{430080\,ad}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(61440\,i\,(\cos(dx+c))^{8}+61440\,\sin(dx+c)\,(\cos(dx+c))^{7}+6656\,i\,(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/430080/d/a*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(61440*I*cos(d*
x+c)^8+61440*sin(d*x+c)*cos(d*x+c)^7+6656*I*cos(d*x+c)^6+73216*cos(d*x+c)^5
*sin(d*x+c)+45045*I*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))+13728*I*cos(d*x+c)^4+45045*I*(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*2^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x
+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+45045*2^(1/2)*arctan(1/2*2^(1/2)*
(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+96096*cos(d*x+c)^3*sin(d*
x+c)+60060*I*cos(d*x+c)^2+180180*cos(d*x+c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.03972, size = 1031, normalized size = 3.53

$$\left(-45045i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^{2}}}e^{(8i\,dx+8i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^{2}}}e^{(2i\,dx+2i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+45\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/215040*(-45045*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(8*I*d*x + 8*I*c)*log((2
*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(2*I*d*x + 2*I*c) + sqrt(2)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x -
I*c)) + 45045*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(8*I*d*x + 8*I*c)*log(-(2*s
qrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(2*I*d*x + 2*I*c) - sqrt(2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*
```

```
c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-280*I*e^(14*I*d*x + 14*I*
c) - 2870*I*e^(12*I*d*x + 12*I*c) - 16345*I*e^(10*I*d*x + 10*I*c) + 27029*I
*e^(8*I*d*x + 8*I*c) + 49792*I*e^(6*I*d*x + 6*I*c) + 11072*I*e^(4*I*d*x + 4
*I*c) + 2304*I*e^(2*I*d*x + 2*I*c) + 240*I)*e^(I*d*x + I*c))*e^(-8*I*d*x -
8*I*c)/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/sqrt(I*a*tan(d*x + c) + a), x)
```

3.340 $\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=147

$$\frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}}+\frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}}+\frac{256ia^4\sec^9(c+dx)}{6435d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] (((256*I)/6435)*a^4*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((6
4*I)/715)*a^3*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((8*I)/65
)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/15)*a*Sec[
c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.258464, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}}+\frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}}+\frac{256ia^4\sec^9(c+dx)}{6435d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((256*I)/6435)*a^4*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((6
4*I)/715)*a^3*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((8*I)/65
)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/15)*a*Sec[
c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{1}{5}(4a)\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx\\ &= \frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{1}{65}\left(32a^2\right)\int \frac{\sec^9(c+dx}{(a+ia\tan(c+d}\\ &= \frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}}+\frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\\ &= \frac{256ia^4\sec^9(c+dx)}{6435d(a+ia\tan(c+dx))^{9/2}}+\frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}}+\frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.501341, size = 95, normalized size = 0.65

$$\frac{2\sec^8(c+dx)(3i(90\sin(c+dx)+233\sin(3(c+dx)))+510\cos(c+dx)+731\cos(3(c+dx)))(\sin(4(c+dx))+i\cos(4(c}{6435d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^8*(510*Cos[c + d*x] + 731*Cos[3*(c + d*x)] + (3*I)*(90*Sin[
c + d*x] + 233*Sin[3*(c + d*x)]))*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)]))/
(6435*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 1.273, size = 154, normalized size = 1.1

$$\frac{4096\,i\,(\cos(dx+c))^8+4096\,\sin(dx+c)\,(\cos(dx+c))^7-512\,i\,(\cos(dx+c))^6+1536\,(\cos(dx+c))^5\sin(dx+c)-160\,i}{6435\,ad\,(\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/6435/d/a*(2048*I*cos(d*x+c)^8+2048*sin(d*x+c)*cos(d*x+c)^7-256*I*cos(d*x+
c)^6+768*cos(d*x+c)^5*sin(d*x+c)-80*I*cos(d*x+c)^4+560*cos(d*x+c)^3*sin(d*x
+c)-42*I*cos(d*x+c)^2+462*cos(d*x+c)*sin(d*x+c)-429*I)*(a*(I*sin(d*x+c)+cos
(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^7
```

Maxima [B] time = 2.05545, size = 821, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/6435*(-1241*I*sqrt(a) - 5194*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 6
090*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2490*sqrt(a)*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 - 14430*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 33618*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 13442*I*sqrt(a)*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18590*sqrt(a)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 18590*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 13442*I*
sqrt(a)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 33618*sqrt(a)*sin(d*x + c)^
11/(cos(d*x + c) + 1)^11 + 14430*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) +
1)^12 + 2490*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 6090*I*sqrt(a)
*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 5194*sqrt(a)*sin(d*x + c)^15/(cos(
d*x + c) + 1)^15 + 1241*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*sq
rt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1
) - 1)/((a - 8*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 - 56*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(
d*x + c)^8/(cos(d*x + c) + 1)^8 - 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^1
0 + 28*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a*sin(d*x + c)^14/(cos(d
*x + c) + 1)^14 + a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*sqrt(-2*I*sin(
d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Fricas [A] time = 2.10861, size = 529, normalized size = 3.6

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(183040i\,e^{(6i\,dx+6i\,c)}+99840i\,e^{(4i\,dx+4i\,c)}+30720i\,e^{(2i\,dx+2i\,c)}+4096i\right)e^{(i\,dx+i\,c)}}{6435\left(ade^{(15i\,dx+15i\,c)}+7\,ade^{(13i\,dx+13i\,c)}+21\,ade^{(11i\,dx+11i\,c)}+35\,ade^{(9i\,dx+9i\,c)}+35\,ade^{(7i\,dx+7i\,c)}+21\,ade^{(5i\,dx+5i\,c)}+7\,ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6435*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(183040*I*e^(6*I*d*x + 6*I
*c) + 99840*I*e^(4*I*d*x + 4*I*c) + 30720*I*e^(2*I*d*x + 2*I*c) + 4096*I)*e
^(I*d*x + I*c)/(a*d*e^(15*I*d*x + 15*I*c) + 7*a*d*e^(13*I*d*x + 13*I*c) + 2
1*a*d*e^(11*I*d*x + 11*I*c) + 35*a*d*e^(9*I*d*x + 9*I*c) + 35*a*d*e^(7*I*d*
x + 7*I*c) + 21*a*d*e^(5*I*d*x + 5*I*c) + 7*a*d*e^(3*I*d*x + 3*I*c) + a*d*e
^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^9}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^9/sqrt(I*a*tan(d*x + c) + a), x)
```

3.341 $\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=110

$$\frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}}+\frac{64ia^3\sec^7(c+dx)}{693d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] (((64*I)/693)*a^3*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((16*
I)/99)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/11)*a
*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.187032, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}}+\frac{64ia^3\sec^7(c+dx)}{693d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((64*I)/693)*a^3*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((16*
I)/99)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/11)*a
*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}+\frac{1}{11}(8a)\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx\\ &= \frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}+\frac{1}{99}\left(32a^2\right)\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\\ &= \frac{64ia^3\sec^7(c+dx)}{693d(a+ia\tan(c+dx))^{7/2}}+\frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.378569, size = 77, normalized size = 0.7

$$\frac{2\sec^6(c+dx)(91i\sin(2(c+dx))+107\cos(2(c+dx))+44)(\sin(3(c+dx))+i\cos(3(c+dx)))}{693d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^6*(44 + 107*Cos[2*(c + d*x)] + (91*I)*Sin[2*(c + d*x)])*(I*
Cos[3*(c + d*x)] + Sin[3*(c + d*x)]))/(693*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.388, size = 127, normalized size = 1.2

$$\frac{512\,i\,(\cos(dx+c))^6+512\,(\cos(dx+c))^5\sin(dx+c)-64\,i\,(\cos(dx+c))^4+192\,(\cos(dx+c))^3\sin(dx+c)-20\,i\,(\cos}{693\,ad\,(\cos(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/693/d/a*(256*I*cos(d*x+c)^6+256*cos(d*x+c)^5*sin(d*x+c)-32*I*cos(d*x+c)^4
+96*cos(d*x+c)^3*sin(d*x+c)-10*I*cos(d*x+c)^2+70*cos(d*x+c)*sin(d*x+c)-63*I
)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5
```

Maxima [B] time = 1.90074, size = 640, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/693*(-151*I*sqrt(a) - 542*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 484*
I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 22*sqrt(a)*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 - 627*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1
452*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1452*sqrt(a)*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 + 627*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 22*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 484*I*sqrt(a)*sin(d*x +
 c)^10/(cos(d*x + c) + 1)^10 - 542*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) +
1)^11 + 151*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*sqrt(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/((a -
 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a*sin(d*x + c)^8/(
cos(d*x + c) + 1)^8 - 6*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a*sin(d*x
 + c)^12/(cos(d*x + c) + 1)^12)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1)
 + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Fricas [A] time = 2.02557, size = 398, normalized size = 3.62

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(6336i\,e^{(4i\,dx+4i\,c)}+2816i\,e^{(2i\,dx+2i\,c)}+512i\right)e^{(i\,dx+i\,c)}}{693\left(ade^{(11i\,dx+11i\,c)}+5\,ade^{(9i\,dx+9i\,c)}+10\,ade^{(7i\,dx+7i\,c)}+10\,ade^{(5i\,dx+5i\,c)}+5\,ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/693*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(6336*I*e^(4*I*d*x + 4*I*c)
 + 2816*I*e^(2*I*d*x + 2*I*c) + 512*I)*e^(I*d*x + I*c)/(a*d*e^(11*I*d*x + 1
1*I*c) + 5*a*d*e^(9*I*d*x + 9*I*c) + 10*a*d*e^(7*I*d*x + 7*I*c) + 10*a*d*e^
(5*I*d*x + 5*I*c) + 5*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^7}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^7/sqrt(I*a*tan(d*x + c) + a), x)
```

3.342 $\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=73

$$\frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] (((8*I)/35)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/
7)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.118018, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((8*I)/35)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/
7)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}+\frac{1}{7}(4a)\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx\\ &= \frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.213615, size = 65, normalized size = 0.89

$$-\frac{2(5\tan(c+dx)-9i)\sec^3(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))}{35d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*Sec[c + d*x]^3*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-9*I + 5*Tan[c
+ d*x]))/(35*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.322, size = 100, normalized size = 1.4

$$\frac{32\,i\,(\cos(dx+c))^4+32\,(\cos(dx+c))^3\sin(dx+c)-4\,i\,(\cos(dx+c))^2+12\,\cos(dx+c)\sin(dx+c)-10\,i}{35\,ad\,(\cos(dx+c))^3}\sqrt{\frac{a\,(i\sin(dx}{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/35/d/a*(16*I*cos(d*x+c)^4+16*cos(d*x+c)^3*sin(d*x+c)-2*I*cos(d*x+c)^2+6*c
os(d*x+c)*sin(d*x+c)-5*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/co
s(d*x+c)^3
```

Maxima [B] time = 1.64271, size = 459, normalized size = 6.29

$$-\frac{2\left(-9i\sqrt{a}-\frac{26\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{14i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{14\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{26\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9i\sqrt{a}\sin(dx}{(\cos(dx+c)+}\right.}{35\left(a-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(d}{(\cos(dx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/35*(-9*I*sqrt(a) - 26*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 14*I*sqr
t(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 14*sqrt(a)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 14*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*I*sqrt
(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 26*sqrt(a)*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7 + 9*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*sqrt(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/(
(a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Fricas [A] time = 2.10777, size = 274, normalized size = 3.75

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(112i\,e^{(2i\,dx+2i\,c)}+32i\right)e^{(i\,dx+i\,c)}}{35\left(ade^{(7i\,dx+7i\,c)}+3\,ade^{(5i\,dx+5i\,c)}+3\,ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(112*I*e^(2*I*d*x + 2*I*c) +
 32*I)*e^(I*d*x + I*c)/(a*d*e^(7*I*d*x + 7*I*c) + 3*a*d*e^(5*I*d*x + 5*I*c)
 + 3*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c+dx)}{\sqrt{a(i\tan(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(I*a*tan(d*x + c) + a), x)
```

3.343 $\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=35

$$\frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] (((2*I)/3)*a*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.0544807, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3493}

$$\frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((2*I)/3)*a*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] time = 0.119705, size = 40, normalized size = 1.14

$$\frac{2(\tan(c+dx)+i)\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]*(I + Tan[c + d*x]))/(3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.298, size = 73, normalized size = 2.1

$$\frac{4\,i\left(\cos\left(dx+c\right)\right)^{2}+4\,\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,i}{3\,ad\cos\left(dx+c\right)}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d/a*(2*I*cos(d*x+c)^2+2*cos(d*x+c)*sin(d*x+c)-I)*(a*(I*sin(d*x+c)+cos(d
*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)
```

Maxima [B] time = 1.53481, size = 278, normalized size = 7.94

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}-1}}{3\left(a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sqrt(a)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*
x + c) + 1) - 1)/((a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) +
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Fricas [B] time = 1.99937, size = 153, normalized size = 4.37

$$\frac{4i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{3\left(ade^{(3i\,dx+3i\,c)}+ade^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(a*d*e^(3*I
*d*x + 3*I*c) + a*d*e^(I*d*x + I*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a(i\tan(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(I*a*tan(d*x + c) + a), x)
```

3.344 $\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=52

$$\frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] (I*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.0410734, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3489, 206}

$$\frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{(2i)\,\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\\ &= \frac{i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] time = 0.334691, size = 70, normalized size = 1.35

$$\frac{2ie^{i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((2*I)*E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/(d*Sqrt[1 +
E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.255, size = 137, normalized size = 2.6

$$-\frac{\sqrt{2}\sin(dx+c)}{ad\,(i\sin(dx+c)+\cos(dx+c)-1)}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}\,(i\cos(dx+c)-}{2\sin(d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/d*2^(1/2)/a*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(
d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)/(I*sin(d*x+c)+cos(d
*x+c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [B] time = 2.20008, size = 502, normalized size = 9.65

$$\frac{1}{2}i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(\left(\sqrt{2}ad\sqrt{\frac{1}{ad^2}}e^{(i\,dx+i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-\frac{1}{2}i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\left(\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*I*sqrt(2)*sqrt(1/(a*d^2))*log((sqrt(2)*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I
*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e
^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 1/2*I*sqrt(2)*sqrt(1/(a*d^2))*log(-(sqr
t(2)*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{\sqrt{a\,(i\tan(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.345 $\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=122

$$-\frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}+\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d}$$

```
[Out] (((3*I)/4)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[2]*Sqrt[a]*d) + ((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*
x]]) - (((3*I)/4)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rubi [A] time = 0.133504, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3502, 3490, 3489, 206}

$$-\frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}+\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/4)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[2]*Sqrt[a]*d) + ((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*
x]]) - (((3*I)/4)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} + \frac{3\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{4a}$$
$$= \frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad} + \frac{3}{8}\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}$$
$$= \frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad} + \frac{(3i)\operatorname{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x,\right.}{4d}$$
$$= \frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d} + \frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+}}{4ad}$$

Mathematica [A] time = 0.455174, size = 96, normalized size = 0.79

$$\frac{\sec(c+dx)\left(3i\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right) - i(3i\sin(2(c+dx)) + \cos(2(c+dx)) + 1)\right)}{8d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]*((3*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I
)*(c + d*x))]] - I*(1 + Cos[2*(c + d*x)] + (3*I)*Sin[2*(c + d*x)])))/(8*d*S
qrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.325, size = 319, normalized size = 2.6

$$\frac{1}{16\,ad}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(3\,i\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}\,(i\cos(dx+c)-i+\sin(dx+c))}{2\,\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/16/d/a*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*I*(-2*cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d
*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*cos(d*x+c)+3*I*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))
/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)+8*I*cos(d*x+c)^3+
3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+s
in(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x
+c)+8*cos(d*x+c)^2*sin(d*x+c)-12*I*cos(d*x+c))
```

Maxima [B] time = 2.17792, size = 1130, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*((4*I*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c) - 8*I*sqrt(
2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*(sqrt(2)*c
os(2*d*x + 2*c) - I*sqrt(2)*sin(2*d*x + 2*c) - 2*sqrt(2))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) - (6*sqrt(2)*arctan2((cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 6*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - 1) - 3*I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^
2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + 3*I*sqr
t(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/(a*d)
```

Fricas [B] time = 2.14608, size = 778, normalized size = 6.38

$$\left(3i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(3i\,dx+3i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(i\,dx+i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-3i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(3*I*d*x + 3*I*c)*log((2*sqrt(1/2)
*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*I*sq
rt(1/2)*a*d*sqrt(1/(a*d^2))*e^(3*I*d*x + 3*I*c)*log(-(2*sqrt(1/2)*a*d*sqrt(
1/(a*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(
2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(4*I*d*x + 4*I*c) - I*e^(2*I*d*x + 2*I*c)
 + I)*e^(I*d*x + I*c))*e^(-3*I*d*x - 3*I*c)/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{\sqrt{a(i\tan(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)}{\sqrt{i a \tan (dx+c)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.346 $\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=193

$$-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad}+\frac{35i\cos(c+d}{96d\sqrt{a+ia\tan(c}$$

```
[Out] (((35*I)/64)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d
*x]])])/(Sqrt[2]*Sqrt[a]*d) + (((35*I)/96)*Cos[c + d*x])/(d*Sqrt[a + I*a*Ta
n[c + d*x]]) + ((I/4)*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((3
5*I)/64)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) - (((7*I)/24)*Cos[c
 + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rubi [A] time = 0.266155, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3502, 3497, 3490, 3489, 206}

$$-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad}+\frac{35i\cos(c+d}{96d\sqrt{a+ia\tan(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((35*I)/64)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d
*x]])])/(Sqrt[2]*Sqrt[a]*d) + (((35*I)/96)*Cos[c + d*x])/(d*Sqrt[a + I*a*Ta
n[c + d*x]]) + ((I/4)*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((3
5*I)/64)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) - (((7*I)/24)*Cos[c
 + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{7\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{8a}$$
$$= \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad} + \frac{35}{48}\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+d}}$$
$$= \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}$$
$$= \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad}$$
$$= \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad}$$
$$= \frac{35i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}d} + \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - 35i$$

Mathematica [A] time = 0.527769, size = 117, normalized size = 0.61

$$\frac{\sec(c+dx)\left(133\sin(2(c+dx)) + 14\sin(4(c+dx)) - 43i\cos(2(c+dx)) - 2i\cos(4(c+dx)) + 105i\sqrt{1+e^{2i(c+dx)}}\tanh^{-}\right.}{384d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]*(-41*I + (105*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
 + E^((2*I)*(c + d*x))]] - (43*I)*Cos[2*(c + d*x)] - (2*I)*Cos[4*(c + d*x)]
 + 133*Sin[2*(c + d*x)] + 14*Sin[4*(c + d*x)]))/(384*d*Sqrt[a + I*a*Tan[c +
 d*x]])
```

Maple [B] time = 0.329, size = 346, normalized size = 1.8

$$\frac{1}{768\,ad}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(192\,i\,(\cos(dx+c))^5 + 105\,i\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}\,(i\cos(dx-}{2\,\mathrm{s}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/768/d/a*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(192*I*cos(d*x+c)^
5+105*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*
cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*c
os(d*x+c)+192*sin(d*x+c)*cos(d*x+c)^4+105*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+56*I*cos(d*x+c)^3+105*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c
)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)+280*cos(d*x+c)^2
*sin(d*x+c)-420*I*cos(d*x+c))
```

Maxima [B] time = 2.63402, size = 2618, normalized size = 13.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/1536*((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((12*I*sqrt(2)*cos(4*d*x + 4*c) + 12
*sqrt(2)*sin(4*d*x + 4*c) - 32*I*sqrt(2))*cos(3/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)) - 4*(3*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*sin(4*d*
x + 4*c) - 8*sqrt(2))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))
)*sqrt(a) + (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((12*I*sqrt(2)*cos(4*d*x + 4*c)
+ 144*I*sqrt(2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 12*s
qrt(2)*sin(4*d*x + 4*c) + 144*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) - 288*I*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)) - (12*sqrt(2)*cos(4*d*x + 4*c) + 144*sqrt(2)*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) - 12*I*sqrt(2)*sin(4*d*x + 4*c) - 144*I*
sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 288*sqrt(2))
*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - (210*sqrt
(2)*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) + 1)) + 1) - 210*sqrt(2)*arctan2((cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
```

```
+ 1)) - 1) - 105*I*sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2
(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin
(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))^2 + 2*(cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + 1
05*I*sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1))^2 - 2*(cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*c
os(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1))*sqrt(a))/(a*d)
```

Fricas [B] time = 2.03375, size = 865, normalized size = 4.48

$$\left(105i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(5idx+5ic)}\log\left(\left(2\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(idx+ic)}+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-105i\sqrt{\frac{1}{2}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(105*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(5*I*d*x + 5*I*c)*log((2*sqrt(
1/2)*a*d*sqrt(1/(a*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 10
5*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(5*I*d*x + 5*I*c)*log(-(2*sqrt(1/2)*a*d
*sqrt(1/(a*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(8*I*d*x + 8*I*c) - 88*I*e^(6*I*d*x
 + 6*I*c) - 41*I*e^(4*I*d*x + 4*I*c) + 45*I*e^(2*I*d*x + 2*I*c) + 6*I)*e^(I
*d*x + I*c))*e^(-5*I*d*x - 5*I*c)/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(I*a*tan(d*x + c) + a), x)
```

3.347 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^7d}-\frac{4i(a+ia\tan(c+dx))^{9/2}}{3a^6d}+\frac{24i(a+ia\tan(c+dx))^{7/2}}{7a^5d}-\frac{16i(a+ia\tan(c+dx))^{5/2}}{5a^4d}$$

```
[Out] (((-16*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^4*d) + (((24*I)/7)*(a + I*a*T
an[c + d*x])^(7/2))/(a^5*d) - (((4*I)/3)*(a + I*a*Tan[c + d*x])^(9/2))/(a^6
*d) + (((2*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^7*d)
```

Rubi [A] time = 0.0869304, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^7d}-\frac{4i(a+ia\tan(c+dx))^{9/2}}{3a^6d}+\frac{24i(a+ia\tan(c+dx))^{7/2}}{7a^5d}-\frac{16i(a+ia\tan(c+dx))^{5/2}}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-16*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^4*d) + (((24*I)/7)*(a + I*a*T
an[c + d*x])^(7/2))/(a^5*d) - (((4*I)/3)*(a + I*a*Tan[c + d*x])^(9/2))/(a^6
*d) + (((2*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^3(a+x)^{3/2}\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(8a^3(a+x)^{3/2}-12a^2(a+x)^{5/2}+6a(a+x)^{7/2}-(a+x)^{9/2}\right)dx,x,ia\tan(c\right.}{a^7d}\\ &= -\frac{16i(a+ia\tan(c+dx))^{5/2}}{5a^4d}+\frac{24i(a+ia\tan(c+dx))^{7/2}}{7a^5d}-\frac{4i(a+ia\tan(c+dx))^{9/2}}{3a^6d}+\end{aligned}$$

Mathematica [A] time = 0.669403, size = 110, normalized size = 0.94

$$\frac{2i\sec^6(c+dx)(\cos(4(c+dx))+i\sin(4(c+dx)))(494i\cos(2(c+dx))+110\tan(c+dx)+215\sin(3(c+dx))\sec(c+d}{1155ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/1155)*Sec[c + d*x]^6*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)])*(39*I
+ (494*I)*Cos[2*(c + d*x)] + 215*Sec[c + d*x]*Sin[3*(c + d*x)] + 110*Tan[c
+ d*x]))/(a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.318, size = 117, normalized size = 1.

$$-\frac{512\,i\left(\cos\left(dx+c\right)\right)^{5}-512\,\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{4}+64\,i\left(\cos\left(dx+c\right)\right)^{3}-320\,\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)+490\,i\cos}{1155\,a^{2}d\left(\cos\left(dx+c\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -2/1155/d/a^2*(256*I*cos(d*x+c)^5-256*sin(d*x+c)*cos(d*x+c)^4+32*I*cos(d*x+
c)^3-160*cos(d*x+c)^2*sin(d*x+c)+245*I*cos(d*x+c)+105*sin(d*x+c))*(a*(I*sin
(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5
```

Maxima [A] time = 1.10234, size = 103, normalized size = 0.88

$$\frac{2i\left(105\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}-770\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}a+1980\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}a^{2}-1848\left(i\,a\tan\left(dx+c\right)+a\right)}{1155\,a^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*I*(105*(I*a*tan(d*x + c) + a)^(11/2) - 770*(I*a*tan(d*x + c) + a)^(9
/2)*a + 1980*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 1848*(I*a*tan(d*x + c) + a)
^(5/2)*a^3)/(a^7*d)
```

Fricas [A] time = 2.06482, size = 468, normalized size = 4.

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-1024i\,e^{(10i\,dx+10i\,c)}-5632i\,e^{(8i\,dx+8i\,c)}-12672i\,e^{(6i\,dx+6i\,c)}-14784i\,e^{(4i\,dx+4i\,c)}\right)e^{(i\,dx+i\,c)}}{1155\left(a^{2}de^{(10i\,dx+10i\,c)}+5\,a^{2}de^{(8i\,dx+8i\,c)}+10\,a^{2}de^{(6i\,dx+6i\,c)}+10\,a^{2}de^{(4i\,dx+4i\,c)}+5\,a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1024*I*e^(10*I*d*x + 10*
I*c) - 5632*I*e^(8*I*d*x + 8*I*c) - 12672*I*e^(6*I*d*x + 6*I*c) - 14784*I*e
^(4*I*d*x + 4*I*c))*e^(I*d*x + I*c)/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d*
e^(8*I*d*x + 8*I*c) + 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x +
4*I*c) + 5*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^8}{(i a \tan (dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.348 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^5d}+\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^4d}-\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^3d}$$

```
[Out] (((-8*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^3*d) + (((8*I)/5)*(a + I*a*Tan
[c + d*x])^(5/2))/(a^4*d) - (((2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^5*d
)
```

Rubi [A] time = 0.078767, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^5d}+\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^4d}-\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-8*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^3*d) + (((8*I)/5)*(a + I*a*Tan
[c + d*x])^(5/2))/(a^4*d) - (((2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^5*d
)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^2\sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2\sqrt{a+x}-4a(a+x)^{3/2}+(a+x)^{5/2}\right)dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^3d}+\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^4d}-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^5d}\end{aligned}$$

Mathematica [A] time = 0.284994, size = 92, normalized size = 1.05

$$-\frac{2\sec^5(c+dx)(-27i\sin(2(c+dx))+43\cos(2(c+dx))+28)(\cos(3(c+dx))+i\sin(3(c+dx)))}{105ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sec[c + d*x]^5*(28 + 43*Cos[2*(c + d*x)] - (27*I)*Sin[2*(c + d*x)])*(Co
s[3*(c + d*x)] + I*Sin[3*(c + d*x)]))/(105*a*d*(-I + Tan[c + d*x])*Sqrt[a +
 I*a*Tan[c + d*x]])
```

Maple [A] time = 0.279, size = 90, normalized size = 1.

$$\frac{-64\,i\,(\cos(dx+c))^3+64\,(\cos(dx+c))^2\sin(dx+c)-78\,i\cos(dx+c)-30\,\sin(dx+c)}{105\,a^2d\,(\cos(dx+c))^3}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2/105/d/a^2*(-32*I*cos(d*x+c)^3+32*cos(d*x+c)^2*sin(d*x+c)-39*I*cos(d*x+c)-
15*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3
```

Maxima [A] time = 1.08481, size = 78, normalized size = 0.89

$$-\frac{2i\left(15\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}-84\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a+140\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^2\right)}{105\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/105*I*(15*(I*a*tan(d*x + c) + a)^(7/2) - 84*(I*a*tan(d*x + c) + a)^(5/2)
*a + 140*(I*a*tan(d*x + c) + a)^(3/2)*a^2)/(a^5*d)
```

Fricas [A] time = 2.23479, size = 332, normalized size = 3.77

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-128i\,e^{(6i\,dx+6i\,c)}-448i\,e^{(4i\,dx+4i\,c)}-560i\,e^{(2i\,dx+2i\,c)}\right)e^{(i\,dx+i\,c)}}{105\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-128*I*e^(6*I*d*x + 6*I*c)
 - 448*I*e^(4*I*d*x + 4*I*c) - 560*I*e^(2*I*d*x + 2*I*c))*e^(I*d*x + I*c)/(
a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^(2*I*d*
x + 2*I*c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^6}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.349 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}} dx$

Optimal. Leaf size=57

$$\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^3d} - \frac{4i\sqrt{a+ia\tan(c+dx)}}{a^2d}$$

```
[Out] ((-4*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d) + (((2*I)/3)*(a + I*a*Tan[c + d
*x])^(3/2))/(a^3*d)
```

Rubi [A] time = 0.0716966, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^3d} - \frac{4i\sqrt{a+ia\tan(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d) + (((2*I)/3)*(a + I*a*Tan[c + d
*x])^(3/2))/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{a-x}{\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(\frac{2a}{\sqrt{a+x}}-\sqrt{a+x}\right)dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{4i\sqrt{a+ia\tan(c+dx)}}{a^2d}+\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^3d}\end{aligned}$$

Mathematica [A] time = 0.191338, size = 80, normalized size = 1.4

$$\frac{2i(\tan(c+dx)+5i)\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))}{3ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/3)*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*(5*I + Ta
n[c + d*x]))/(a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.262, size = 61, normalized size = 1.1

$$-\frac{10\,i\cos\left(dx+c\right)+2\,\sin\left(dx+c\right)}{3\,a^{2}d\cos\left(dx+c\right)}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d/a^2*(5*I*cos(d*x+c)+sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x
+c))^(1/2)/cos(d*x+c)
```

Maxima [A] time = 1.05252, size = 51, normalized size = 0.89

$$\frac{2i\left(\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}-6\,\sqrt{i\,a\tan\left(dx+c\right)+a}a\right)}{3\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*I*((I*a*tan(d*x + c) + a)^(3/2) - 6*sqrt(I*a*tan(d*x + c) + a)*a)/(a^3*
d)
```

Fricas [A] time = 2.07209, size = 180, normalized size = 3.16

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-8i\,e^{(2i\,dx+2i\,c)}-12i\right)e^{(i\,dx+i\,c)}}{3\left(a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(2*I*d*x + 2*I*c) - 1
2*I)*e^(I*d*x + I*c)/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{4}\left(c+dx\right)}{\left(a\left(i\tan\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*(I*tan(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^4}{(i\,a\tan (dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.350 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (2*I)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.0646933, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 32}

$$\frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*I)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.143462, size = 27, normalized size = 1.

$$\frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*I)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.032, size = 24, normalized size = 0.9

$$\frac{2i}{ad}\frac{1}{\sqrt{a+ia\tan(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2*I/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Maxima [A] time = 1.09316, size = 28, normalized size = 1.04

$$\frac{2i}{\sqrt{i\,a\tan(dx+c)+a}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*I/(sqrt(I*a*tan(d*x + c) + a)*a*d)
```

Fricas [B] time = 2.12964, size = 130, normalized size = 4.81

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I)*e^(-I
*d*x - I*c)/(a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a(i\tan(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(I*tan(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.351 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=175

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}-\frac{7i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d}+\frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}}+\frac{}{24d(a+ia}$$

```
[Out] (((-7*I)/16)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2
]*a^(3/2)*d) + (((7*I)/20)*a)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((I/2)*a^2
)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + ((7*I)/24)/(d*(
a + I*a*Tan[c + d*x])^(3/2)) + ((7*I)/16)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.118418, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}-\frac{7i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d}+\frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}}+\frac{}{24d(a+ia}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-7*I)/16)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2
]*a^(3/2)*d) + (((7*I)/20)*a)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((I/2)*a^2
)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + ((7*I)/24)/(d*(
a + I*a*Tan[c + d*x])^(3/2)) + ((7*I)/16)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\left(ia^3\right)\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} - \frac{\left(7ia^2\right)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, ia\tan(\right.}{4d}$$

$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} - \frac{(7ia)\,\text{Subst}\left(\right.}{}$$

$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{7}{24d(a+ia\,\text{ta}}$$

$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{7}{24d(a+ia\,\text{ta}}$$

$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{7}{24d(a+ia\,\text{ta}}$$

$$= -\frac{7i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d} + \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\,\text{t}}$$

Mathematica [A] time = 0.7801, size = 142, normalized size = 0.81

$$-\frac{ie^{-5i(c+dx)}\sec(c+dx)\left(-38e^{2i(c+dx)} - 148e^{4i(c+dx)} - 101e^{6i(c+dx)} + 15e^{8i(c+dx)} + 105e^{5i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right) - \right.}{480ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I/480)*(-6 - 38*E^((2*I)*(c + d*x)) - 148*E^((4*I)*(c + d*x)) - 101*E^((
6*I)*(c + d*x)) + 15*E^((8*I)*(c + d*x)) + 105*E^((5*I)*(c + d*x))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))])*Sec[c + d*x])/(a*d*E^((5*I)
*(c + d*x))*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.276, size = 368, normalized size = 2.1

$$\frac{1}{960\,a^2d}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(384\,i\,(\cos(dx+c))^6 + 384\,(\cos(dx+c))^5\sin(dx+c) + 105\,i\sqrt{-2\,\frac{\cos(dx+}{\cos(dx+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/960/d/a^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(384*I*cos(d*x+c
)^6+384*cos(d*x+c)^5*sin(d*x+c)+105*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))*2^(1/2)*cos(d*x+c)+32*I*cos(d*x+c)^4+105*I*(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/si
n(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)+105*2^(1/2)*arctan(1
/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+224*cos(d*x+c)
^3*sin(d*x+c)+140*I*cos(d*x+c)^2+420*cos(d*x+c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.49285, size = 906, normalized size = 5.18

$$\frac{\left(-105i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(6i\,dx+6i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(2i\,dx+2i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/480*(-105*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(6*I*d*x + 6*I*c)*log((2*
sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c) + sqrt(2)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x
 - I*c)) + 105*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(6*I*d*x + 6*I*c)*log(
-(2*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c) - sqrt(2)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I
*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-15*I*e^(8*I*d*x
+ 8*I*c) + 101*I*e^(6*I*d*x + 6*I*c) + 148*I*e^(4*I*d*x + 4*I*c) + 38*I*e^(
2*I*d*x + 2*I*c) + 6*I)*e^(I*d*x + I*c))*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{(a(i\tan(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*(I*tan(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(ia\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.352 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}-\frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{99ia^2}{224d(a+ia\tan(c+$$

```
[Out] (((-99*I)/256)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*a^(3/2)*d) + (((99*I)/224)*a^2)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - ((I/
4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) - (((11*I
)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + (((99*
I)/320)*a)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + ((33*I)/128)/(d*(a + I*a*Tan[
c + d*x])^(3/2)) + ((99*I)/256)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.143786, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}-\frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{99ia^2}{224d(a+ia\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] (((-99*I)/256)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*a^(3/2)*d) + (((99*I)/224)*a^2)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - ((I/
4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) - (((11*I
)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + (((99*
I)/320)*a)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + ((33*I)/128)/(d*(a + I*a*Tan[
c + d*x])^(3/2)) + ((99*I)/256)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{(11ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{9/2}}\,dx, x, ia\,\text{ta}\right.}{8d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+a}$$

$$= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{}{16d(a-ia}$$

$$= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{}{16d(a-ia}$$

$$= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{}{16d(a-ia}$$

$$= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{}{16d(a-ia}$$

$$= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{}{16d(a-ia}$$

$$= -\frac{99i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+}$$

Mathematica [A] time = 1.11543, size = 168, normalized size = 0.68

$$-\frac{ie^{-7i(c+dx)}\sec(c+dx)\left(-328e^{2i(c+dx)} - 1304e^{4i(c+dx)} - 4584e^{6i(c+dx)} - 2833e^{8i(c+dx)} + 805e^{10i(c+dx)} + 70e^{12i(c+dx)} + 3465e^{7i}\right.}{17920ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I/17920)*(-40 - 328*E^((2*I)*(c + d*x)) - 1304*E^((4*I)*(c + d*x)) - 458
4*E^((6*I)*(c + d*x)) - 2833*E^((8*I)*(c + d*x)) + 805*E^((10*I)*(c + d*x))
 + 70*E^((12*I)*(c + d*x)) + 3465*E^((7*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
+ d*x))]*ArcSinh[E^(I*(c + d*x))])*Sec[c + d*x])/(a*d*E^((7*I)*(c + d*x))*S
qrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.398, size = 395, normalized size = 1.6

$$\frac{1}{35840\,a^2d}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}\left(10240\,i\left(\cos\left(dx+c\right)\right)^8+10240\,\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^7+512\,i\left(\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/35840/d/a^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(10240*I*cos(d
*x+c)^8+10240*sin(d*x+c)*cos(d*x+c)^7+512*I*cos(d*x+c)^6+5632*cos(d*x+c)^5*
sin(d*x+c)+3465*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(
I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*cos(d*x+c)*2^(1/2)+1056*I*cos(d*x+c)^4+3465*I*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*2^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/
(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+3465*2^(1/2)*arctan(1/2*2^(1/2)*(I*co
s(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+7392*cos(d*x+c)^3*sin(d*x+c)+4
620*I*cos(d*x+c)^2+13860*cos(d*x+c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.73794, size = 999, normalized size = 4.03

$$\left(-3465i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(8i\,dx+8i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(2i\,dx+2i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/17920*(-3465*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(8*I*d*x + 8*I*c)*log(
(2*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c) + sqrt(2)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*
d*x - I*c)) + 3465*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(8*I*d*x + 8*I*c)*
log(-(2*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c) - sqrt(2)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e
^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-70*I*e^(12*I
*d*x + 12*I*c) - 805*I*e^(10*I*d*x + 10*I*c) + 2833*I*e^(8*I*d*x + 8*I*c) +
 4584*I*e^(6*I*d*x + 6*I*c) + 1304*I*e^(4*I*d*x + 4*I*c) + 328*I*e^(2*I*d*x
 + 2*I*c) + 40*I)*e^(I*d*x + I*c))*e^(-8*I*d*x - 8*I*c)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.353 $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=321

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}}-\frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}-\frac{}{64d(a-ia\tan(c+}$$

```
[Out] (((-715*I)/2048)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*a^(3/2)*d) + (((715*I)/1152)*a^3)/(d*(a + I*a*Tan[c + d*x])^(9/2)) -
((I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(9/2)) - (((
5*I)/16)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) - (
((65*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) +
(((715*I)/1792)*a^2)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((143*I)/512)*a)/(
d*(a + I*a*Tan[c + d*x])^(5/2)) + ((715*I)/3072)/(d*(a + I*a*Tan[c + d*x])^
(3/2)) + ((715*I)/2048)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.180061, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}}-\frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}-\frac{}{64d(a-ia\tan(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] (((-715*I)/2048)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*a^(3/2)*d) + (((715*I)/1152)*a^3)/(d*(a + I*a*Tan[c + d*x])^(9/2)) -
((I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(9/2)) - (((
5*I)/16)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) - (
((65*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) +
(((715*I)/1792)*a^2)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((143*I)/512)*a)/(
d*(a + I*a*Tan[c + d*x])^(5/2)) + ((715*I)/3072)/(d*(a + I*a*Tan[c + d*x])^
(3/2)) + ((715*I)/2048)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\left(ia^7\right)\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{\left(5ia^6\right)\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{11/2}}\,dx, x, ia\,\text{ta}\right.}{4d}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= -\frac{715i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}a^{3/2}d} + \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a}$$

Mathematica [A] time = 1.76946, size = 203, normalized size = 0.63

$$\frac{ie^{-8i(c+dx)}\left(1136e^{2i(c+dx)} + 5440e^{4i(c+dx)} + 17344e^{6i(c+dx)} + 57632e^{8i(c+dx)} + 33301e^{10i(c+dx)} - 13209e^{12i(c+dx)} - 1974e^{14i(c+dx)}\right.}{129024ad\left(1+e^{2i(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/129024)*(112 + 1136*E^((2*I)*(c + d*x)) + 5440*E^((4*I)*(c + d*x)) + 17
344*E^((6*I)*(c + d*x)) + 57632*E^((8*I)*(c + d*x)) + 33301*E^((10*I)*(c +
d*x)) - 13209*E^((12*I)*(c + d*x)) - 1974*E^((14*I)*(c + d*x)) - 168*E^((16
*I)*(c + d*x)) - 45045*E^((9*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Ar
cSinh[E^(I*(c + d*x))]))/(a*d*E^((8*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))
*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.648, size = 422, normalized size = 1.3

$$\frac{1}{516096\,a^2d}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(114688\,i\,(\cos(dx+c))^{10}+114688\,\sin(dx+c)\,(\cos(dx+c))^9+4096\,i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/516096/d/a^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(114688*I*cos
(d*x+c)^10+114688*sin(d*x+c)*cos(d*x+c)^9+4096*I*cos(d*x+c)^8+61440*sin(d*x
+c)*cos(d*x+c)^7+6656*I*cos(d*x+c)^6+73216*cos(d*x+c)^5*sin(d*x+c)+45045*I*
2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*
(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)+13728*I*cos(d*x+c)^4+45045*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)
*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))+45045*2^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(
d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*sin(d*x+c)+96096*cos(d*x+c)^3*sin(d*x+c)+60060*I*cos(d*x
+c)^2+180180*cos(d*x+c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.26408, size = 1106, normalized size = 3.45

$$\left(-45045i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(10i\,dx+10i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(2i\,dx+2i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/258048*(-45045*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(10*I*d*x + 10*I*c)*
log((2*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c) + sqrt(2)*sqrt
```

```
(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^
(-I*d*x - I*c)) + 45045*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(10*I*d*x + 1
0*I*c)*log(-(2*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c) - sqrt
(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x +
I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-168*I
*e^(16*I*d*x + 16*I*c) - 1974*I*e^(14*I*d*x + 14*I*c) - 13209*I*e^(12*I*d*x
 + 12*I*c) + 33301*I*e^(10*I*d*x + 10*I*c) + 57632*I*e^(8*I*d*x + 8*I*c) +
17344*I*e^(6*I*d*x + 6*I*c) + 5440*I*e^(4*I*d*x + 4*I*c) + 1136*I*e^(2*I*d*
x + 2*I*c) + 112*I)*e^(I*d*x + I*c))*e^(-10*I*d*x - 10*I*c)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^6}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.354 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}}+\frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}}+\frac{256ia^4\sec^{11}(c+dx)}{12155d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] (((256*I)/12155)*a^4*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (
((64*I)/1105)*a^3*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((8*
I)/85)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/17)*
a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] time = 0.262362, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}}+\frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}}+\frac{256ia^4\sec^{11}(c+dx)}{12155d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((256*I)/12155)*a^4*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (
((64*I)/1105)*a^3*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((8*
I)/85)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/17)*
a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}+\frac{1}{17}(12a)\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx\\ &= \frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}+\frac{1}{85}\left(32a^2\right)\int \frac{\sec^{11}(c+\\ &= \frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}}+\frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}\\ &= \frac{256ia^4\sec^{11}(c+dx)}{12155d(a+ia\tan(c+dx))^{11/2}}+\frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}}+\frac{8ia^2\sec^{11}(c+dx}{85d(a+ia\tan(c+dx}\end{aligned}$$

Mathematica [A] time = 0.770109, size = 108, normalized size = 0.73

$$\frac{2\sec^9(c+dx)(\sin(4(c+dx))+i\cos(4(c+dx)))(-2242i\cos(2(c+dx))+374\tan(c+dx)+1089\sin(3(c+dx))\sec(c+dx)}{12155ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sec[c + d*x]^9*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*(475*I - (2242*I)
*Cos[2*(c + d*x)] + 1089*Sec[c + d*x]*Sin[3*(c + d*x)] + 374*Tan[c + d*x]))
/(12155*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 3.044, size = 171, normalized size = 1.2

$$\frac{8192\,i\,(\cos(dx+c))^9+8192\,\sin(dx+c)\,(\cos(dx+c))^8-1024\,i\,(\cos(dx+c))^7+3072\,(\cos(dx+c))^6\sin(dx+c)-320}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2/12155/d/a^2*(4096*I*cos(d*x+c)^9+4096*sin(d*x+c)*cos(d*x+c)^8-512*I*cos(d
*x+c)^7+1536*cos(d*x+c)^6*sin(d*x+c)-160*I*cos(d*x+c)^5+1120*sin(d*x+c)*cos
(d*x+c)^4-84*I*cos(d*x+c)^3+924*cos(d*x+c)^2*sin(d*x+c)-1573*I*cos(d*x+c)-7
15*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^8
```

Maxima [B] time = 2.40486, size = 1031, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/12155*(-1767*I*sqrt(a) - 6854*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) +
2088*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 16438*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 - 5661*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 - 56984*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 13328*I*sqrt(a)
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 129336*sqrt(a)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 7514*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15646
8*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 156468*sqrt(a)*sin(d*x + c)
^11/(cos(d*x + c) + 1)^11 - 7514*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) +
1)^12 - 129336*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 13328*I*sqrt
(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 56984*sqrt(a)*sin(d*x + c)^15/(
cos(d*x + c) + 1)^15 + 5661*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
 - 16438*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 2088*I*sqrt(a)*sin
(d*x + c)^18/(cos(d*x + c) + 1)^18 - 6854*sqrt(a)*sin(d*x + c)^19/(cos(d*x
+ c) + 1)^19 + 1767*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
^(3/2)/((a^2 - 10*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 45*a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 120*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
 210*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 252*a^2*sin(d*x + c)^10/(cos
```

```
(d*x + c) + 1)^10 + 210*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 120*a^2
*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45*a^2*sin(d*x + c)^16/(cos(d*x +
c) + 1)^16 - 10*a^2*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + a^2*sin(d*x + c
)^20/(cos(d*x + c) + 1)^20)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Fricas [A] time = 2.09942, size = 598, normalized size = 4.07

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(565760i\,e^{(6i\,dx+6i\,c)} + 261120i\,e^{(4i\,dx+4i\,c)} + 69632i\,e^{(2i\,dx+2i\,c)} + 8\right.}{12155\left(a^2de^{(17i\,dx+17i\,c)} + 8\,a^2de^{(15i\,dx+15i\,c)} + 28\,a^2de^{(13i\,dx+13i\,c)} + 56\,a^2de^{(11i\,dx+11i\,c)} + 70\,a^2de^{(9i\,dx+9i\,c)} + 56\,a^2de^{(7i\,dx+7\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12155*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(565760*I*e^(6*I*d*x + 6*
I*c) + 261120*I*e^(4*I*d*x + 4*I*c) + 69632*I*e^(2*I*d*x + 2*I*c) + 8192*I)
*e^(I*d*x + I*c)/(a^2*d*e^(17*I*d*x + 17*I*c) + 8*a^2*d*e^(15*I*d*x + 15*I*
c) + 28*a^2*d*e^(13*I*d*x + 13*I*c) + 56*a^2*d*e^(11*I*d*x + 11*I*c) + 70*a
^2*d*e^(9*I*d*x + 9*I*c) + 56*a^2*d*e^(7*I*d*x + 7*I*c) + 28*a^2*d*e^(5*I*d
*x + 5*I*c) + 8*a^2*d*e^(3*I*d*x + 3*I*c) + a^2*d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{11}}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.355 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}}+\frac{64ia^3\sec^9(c+dx)}{1287d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] (((64*I)/1287)*a^3*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((16
*I)/143)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/13)
*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] time = 0.191073, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}}+\frac{64ia^3\sec^9(c+dx)}{1287d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((64*I)/1287)*a^3*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((16
*I)/143)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/13)
*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] && !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}+\frac{1}{13}(8a)\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx\\ &= \frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}+\frac{1}{143}\left(32a^2\right)\int \frac{\sec^9(c+d}{(a+ia\tan(c+}\\ &= \frac{64ia^3\sec^9(c+dx)}{1287d(a+ia\tan(c+dx))^{9/2}}+\frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.404781, size = 92, normalized size = 0.84

$$\frac{2\sec^8(c+dx)(135i\sin(2(c+dx))+151\cos(2(c+dx))+52)(\cos(3(c+dx))-i\sin(3(c+dx)))}{1287ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sec[c + d*x]^8*(52 + 151*Cos[2*(c + d*x)] + (135*I)*Sin[2*(c + d*x)])*(C
os[3*(c + d*x)] - I*Sin[3*(c + d*x)]))/(1287*a*d*(-I + Tan[c + d*x])*Sqrt[a
 + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.342, size = 144, normalized size = 1.3

$$\frac{1024\,i\,(\cos(dx+c))^7+1024\,(\cos(dx+c))^6\sin(dx+c)-128\,i\,(\cos(dx+c))^5+384\,\sin(dx+c)\,(\cos(dx+c))^4-40}{1287\,a^2d\,(\cos(dx+c))^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2/1287/d/a^2*(512*I*cos(d*x+c)^7+512*cos(d*x+c)^6*sin(d*x+c)-64*I*cos(d*x+c
)^5+192*sin(d*x+c)*cos(d*x+c)^4-20*I*cos(d*x+c)^3+140*cos(d*x+c)^2*sin(d*x+
c)-225*I*cos(d*x+c)-99*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))
^(1/2)/cos(d*x+c)^6
```

Maxima [B] time = 2.36937, size = 845, normalized size = 7.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1287*(-203*I*sqrt(a) - 678*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*I
*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1802*sqrt(a)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 - 26*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3
614*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 858*I*sqrt(a)*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 - 6578*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 - 6578*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 858*I*sqrt(a)*sin(d*x
 + c)^10/(cos(d*x + c) + 1)^10 - 3614*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c)
 + 1)^11 + 26*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 1802*sqrt(a
)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 2*I*sqrt(a)*sin(d*x + c)^14/(cos(
d*x + c) + 1)^14 - 678*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 203*
I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((a^2 - 8*a^2
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 - 56*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a
^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^2*sin(d*x + c)^14/(cos(d*x +
 c) + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d*x +
c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Fricas [A] time = 2.31033, size = 463, normalized size = 4.21

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(18304i\,e^{(4i\,dx+4i\,c)}+6656i\,e^{(2i\,dx+2i\,c)}+1024i\right)e^{(i\,dx+i\,c)}}{1287\left(a^2de^{(13i\,dx+13i\,c)}+6\,a^2de^{(11i\,dx+11i\,c)}+15\,a^2de^{(9i\,dx+9i\,c)}+20\,a^2de^{(7i\,dx+7i\,c)}+15\,a^2de^{(5i\,dx+5i\,c)}+6\,a^2de^{(3i\,dx+3i\,c)}+a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1287*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(18304*I*e^(4*I*d*x + 4*I*
c) + 6656*I*e^(2*I*d*x + 2*I*c) + 1024*I)*e^(I*d*x + I*c)/(a^2*d*e^(13*I*d*
x + 13*I*c) + 6*a^2*d*e^(11*I*d*x + 11*I*c) + 15*a^2*d*e^(9*I*d*x + 9*I*c)
+ 20*a^2*d*e^(7*I*d*x + 7*I*c) + 15*a^2*d*e^(5*I*d*x + 5*I*c) + 6*a^2*d*e^(
3*I*d*x + 3*I*c) + a^2*d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^9}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.356 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] (((8*I)/63)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/
9)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] time = 0.128931, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((8*I)/63)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/
9)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}+\frac{1}{9}(4a)\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx\\ &= \frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.275391, size = 80, normalized size = 1.1

$$\frac{2(7\tan(c+dx)-11i)\sec^5(c+dx)(\sin(2(c+dx))+i\cos(2(c+dx)))}{63ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sec[c + d*x]^5*(I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(-11*I + 7*Tan[c
+ d*x]))/(63*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.295, size = 117, normalized size = 1.6

$$\frac{64\,i\left(\cos\left(dx+c\right)\right)^{5}+64\,\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{4}-8\,i\left(\cos\left(dx+c\right)\right)^{3}+24\,\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)-34\,i\cos\left(dx+\right.}{63\,a^{2}d\left(\cos\left(dx+c\right)\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2/63/d/a^2*(32*I*cos(d*x+c)^5+32*sin(d*x+c)*cos(d*x+c)^4-4*I*cos(d*x+c)^3+1
2*cos(d*x+c)^2*sin(d*x+c)-17*I*cos(d*x+c)-7*sin(d*x+c))*(a*(I*sin(d*x+c)+co
s(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4
```

Maxima [B] time = 1.72257, size = 659, normalized size = 9.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/63*(-11*I*sqrt(a) - 30*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 12*I*sq
rt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 86*sqrt(a)*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 9*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 108*sqr
t(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 108*sqrt(a)*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7 - 9*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 86*sqrt
(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 12*I*sqrt(a)*sin(d*x + c)^10/(cos
(d*x + c) + 1)^10 - 30*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 11*I
*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*(sin(d*x + c)/(cos(d*x + c)
 + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((a^2 - 6*a^2*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 - 20*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8 - 6*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*si
n(d*x + c)^12/(cos(d*x + c) + 1)^12)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Fricas [B] time = 2.08432, size = 325, normalized size = 4.45

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(288i\,e^{(2i\,dx+2i\,c)}+64i\right)e^{(i\,dx+i\,c)}}{63\left(a^{2}de^{(9i\,dx+9i\,c)}+4\,a^{2}de^{(7i\,dx+7i\,c)}+6\,a^{2}de^{(5i\,dx+5i\,c)}+4\,a^{2}de^{(3i\,dx+3i\,c)}+a^{2}de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/63*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(288*I*e^(2*I*d*x + 2*I*c) +
 64*I)*e^(I*d*x + I*c)/(a^2*d*e^(9*I*d*x + 9*I*c) + 4*a^2*d*e^(7*I*d*x + 7*
I*c) + 6*a^2*d*e^(5*I*d*x + 5*I*c) + 4*a^2*d*e^(3*I*d*x + 3*I*c) + a^2*d*e^
(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^7}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.357 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] (((2*I)/5)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] time = 0.0615494, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3493}

$$\frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/5)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A] time = 0.192013, size = 59, normalized size = 1.69

$$\frac{2(1-i\tan(c+dx))\sec^3(c+dx)}{5ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sec[c + d*x]^3*(1 - I*Tan[c + d*x]))/(5*a*d*(-I + Tan[c + d*x])*Sqrt[a +
 I*a*Tan[c + d*x]])
```

Maple [B] time = 0.267, size = 90, normalized size = 2.6

$$\frac{8\,i\left(\cos\left(dx+c\right)\right)^3+8\,\left(\cos\left(dx+c\right)\right)^2\sin\left(dx+c\right)-6\,i\cos\left(dx+c\right)-2\,\sin\left(dx+c\right)}{5\,a^2d\left(\cos\left(dx+c\right)\right)^2}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2/5/d/a^2*(4*I*cos(d*x+c)^3+4*cos(d*x+c)^2*sin(d*x+c)-3*I*cos(d*x+c)-sin(d*
x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2
```

Maxima [B] time = 1.50598, size = 473, normalized size = 13.51

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{6\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{i\sqrt{a}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)}{5\left(a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+}{(\cos(dx+c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*I*sqrt(a)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 6*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*I*sqrt(a)*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 + I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((a^2 -
4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Fricas [B] time = 2.15081, size = 198, normalized size = 5.66

$$\frac{8i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{5\left(a^2de^{(5i\,dx+5i\,c)}+2\,a^2de^{(3i\,dx+3i\,c)}+a^2de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 8/5*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(a^2*d*e^(5
*I*d*x + 5*I*c) + 2*a^2*d*e^(3*I*d*x + 3*I*c) + a^2*d*e^(I*d*x + I*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5(c+dx)}{(a(i\tan(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a*(I*tan(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^5}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.358 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{2i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] ((2*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(3/2)*d) - ((2*I)*Sec[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]
])
```

Rubi [A] time = 0.101564, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3491, 3489, 206}

$$\frac{2i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(3/2)*d) - ((2*I)*Sec[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]
])
```

Rule 3491

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e
+ f*x])^(n + 1))/(b*f*(m - 2)), x] + Dist[(2*d^2)/a, Int[(d*Sec[e + f*x])^(
m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{2\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a}$$

$$= -\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(4i)\,\mathrm{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{ad}$$

$$= \frac{2i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} - \frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 0.72189, size = 101, normalized size = 1.17

$$\frac{8e^{3i(c+dx)}\left(-1+\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{ad\left(1+e^{2i(c+dx)}\right)^2(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (8*E^((3*I)*(c + d*x))*(-1 + Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 +
 E^((2*I)*(c + d*x))]]))/(a*d*(1 + E^((2*I)*(c + d*x)))^2*(-I + Tan[c + d*x
])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.238, size = 158, normalized size = 1.8

$$-2\,\frac{1}{a^2d\,(i\sin(dx+c)+\cos(dx+c)-1)}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(1/2\,\frac{\sqrt{2}\,(i\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -2/d/a^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*((-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c
)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)-I*cos(d*x+c)+I-s
in(d*x+c))/(I*sin(d*x+c)+cos(d*x+c)-1)
```

Maxima [B] time = 2.0252, size = 1099, normalized size = 12.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*(2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
```

```
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a) - (-4*I
*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*sqrt(
2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))/((cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*a^2*d)
```

Fricas [B] time = 2.10683, size = 690, normalized size = 8.02

$$\left(i\sqrt{2}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(i\,dx+i\,c)}\log\left(\left(\sqrt{2}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(i\,dx+i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-i\sqrt{2}a^2d\sqrt{\frac{1}{a^3d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*a^2*d*sqrt(1/(a^3*d^2))*e^(I*d*x + I*c)*log((sqrt(2)*a^2*d*sqrt(
1/(a^3*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e
^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - I*sqrt(2)*a^2*
d*sqrt(1/(a^3*d^2))*e^(I*d*x + I*c)*log(-(sqrt(2)*a^2*d*sqrt(1/(a^3*d^2))*e
^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*
I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 2*I*sqrt(2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/(a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a\,(i\tan(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(I*tan(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.359 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] ((I/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])]
)/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2
))
```

Rubi [A] time = 0.0752919, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3502, 3489, 206}

$$\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])]
)/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2
))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{4a}$$
$$= \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}} + \frac{i\,\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{2ad}$$
$$= \frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] time = 0.605281, size = 95, normalized size = 1.09

$$\frac{\sec(c+dx)\left(2+\frac{2e^{2i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)}{4ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((2 + (2*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/Sqrt[1
 + E^((2*I)*(c + d*x))])*Sec[c + d*x])/(4*a*d*(-I + Tan[c + d*x])*Sqrt[a +
I*a*Tan[c + d*x]])
```

Maple [B] time = 0.258, size = 318, normalized size = 3.7

$$\frac{1}{8\,a^2d}\left(i\cos(dx+c)\sqrt{2}\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}\,(i\cos(dx+c)-i+\sin(dx+c))}{2\sin(dx+c)}\frac{1}{\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+\sqrt{-2\frac{\mathrm{co}}{\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/8/d/a^2*(I*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))+(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*c
os(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^
(1/2)*sin(d*x+c)+8*I*cos(d*x+c)^3+I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))+8*cos(d*x+c)^2*sin(d*x+c)-4*I*cos(d*x+c))*(a*(I*
sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 2.13831, size = 760, normalized size = 8.74

$$\frac{\left(i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(3i\,dx+3i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(i\,dx+i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-i\sqrt{\frac{1}{2}}a^2d\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log((2*sqrt(1/
2)*a^2*d*sqrt(1/(a^3*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) -
I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(3*I*d*x + 3*I*c)*log(-(2*sqrt(1/2)*a
^2*d*sqrt(1/(a^3*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt
(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I)*e^(I*d*x
+ I*c))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{\left(a\left(i\tan(c+dx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(I*tan(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.360 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}} dx$

Optimal. Leaf size=157

$$-\frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}+\frac{15i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d}+\frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx$$

```
[Out] (((15*I)/32)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d
*x]])])/(Sqrt[2]*a^(3/2)*d) + ((I/4)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x]
)^(3/2)) + (((5*I)/16)*Cos[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((
15*I)/32)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d)
```

Rubi [A] time = 0.192411, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3502, 3490, 3489, 206}

$$-\frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}+\frac{15i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d}+\frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}}+\frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((15*I)/32)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d
*x]])])/(Sqrt[2]*a^(3/2)*d) + ((I/4)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x]
)^(3/2)) + (((5*I)/16)*Cos[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((
15*I)/32)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d)
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{8a} \\ &= \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} + \frac{15\int \cos(c+dx)\sqrt{a+ia\tan(c}{32a^2} \\ &= \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+}{32a^2d} \\ &= \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+}{32a^2d} \\ &= \frac{15i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d} + \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.978652, size = 120, normalized size = 0.76

$$\frac{\sec(c+dx)\left(\frac{30e^{2i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2(10i\sin(2(c+dx)) + 6\cos(2(c+dx)) - 9)\right)}{64ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]*((30*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))
]])/Sqrt[1 + E^((2*I)*(c + d*x))] - 2*(-9 + 6*Cos[2*(c + d*x)] + (10*I)*Sin
[2*(c + d*x)])))/(64*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.246, size = 346, normalized size = 2.2

$$\frac{1}{128\,a^2d}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(64\,i\,(\cos(dx+c))^5 + 15\,i\cos(dx+c)\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/128/d/a^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(64*I*cos(d*x+c)
^5+15*I*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*
2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))+64*sin(d*x+c)*cos(d*x+c)^4+15*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))+8*I*cos(d*x+c)^3+15*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)+40*cos(d*x+c)^2*sin(
```

```
d*x+c)-60*I*cos(d*x+c))
```

Maxima [B] time = 2.16221, size = 2457, normalized size = 15.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/256*((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((36*I*sqrt(2)*cos(4*d*x + 4*c) + 36*
sqrt(2)*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)) - 36*(sqrt(2)*cos(4*d*x + 4*c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(3/2*a
rctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + (cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 1)^(1/4)*((-28*I*sqrt(2)*cos(4*d*x + 4*c) - 28*sqrt(2)*sin(4*d*x + 4*c)
- 32*I*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 4*(
7*sqrt(2)*cos(4*d*x + 4*c) - 7*I*sqrt(2)*sin(4*d*x + 4*c) + 8*sqrt(2))*sin(
1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - (30*sqrt(2)*ar
ctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)) + 1) - 30*sqrt(2)*arctan2((cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) -
 1) - 15*I*sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arc
tan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))^2 + 2*(cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(
1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + 15*I*sqrt
(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1
```

```
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))) + 1))^2 - 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*
d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2
*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1))*sqrt(a))/(a^2*d)
```

Fricas [B] time = 2.08285, size = 845, normalized size = 5.38

$$\left(15i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(5i\,dx+5i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(i\,dx+i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-15i\sqrt{\frac{1}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(15*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(5*I*d*x + 5*I*c)*log((2*sqr
t(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
) - 15*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(5*I*d*x + 5*I*c)*log(-(2*sqrt
(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
 + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(6*I*d*x + 6*I*c) + I*
e^(4*I*d*x + 4*I*c) + 11*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(I*d*x + I*c))*e^(-
5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{(a(i\tan(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)/(a*(I*tan(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.361 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}-\frac{105i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{256a^2d}+\frac{105i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d}+\frac{}{16ad}$$

```
[Out] (((105*I)/256)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*a^(3/2)*d) + ((I/6)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c +
d*x])^(3/2)) + (((35*I)/128)*Cos[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
 + (((3*I)/16)*Cos[c + d*x]^3)/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((105*I)
/256)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d) - (((7*I)/32)*Cos[c
+ d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d)
```

Rubi [A] time = 0.337251, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3502, 3497, 3490, 3489, 206}

$$-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}-\frac{105i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{256a^2d}+\frac{105i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d}+\frac{}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] (((105*I)/256)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*a^(3/2)*d) + ((I/6)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c +
d*x])^(3/2)) + (((35*I)/128)*Cos[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
 + (((3*I)/16)*Cos[c + d*x]^3)/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((105*I)
/256)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d) - (((7*I)/32)*Cos[c
+ d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d)
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
```

```
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{3\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{4a}$$

$$= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} + \frac{21\int \cos^3(c+dx)\sqrt{a+ia\tan(c+}}{32a^2}$$

$$= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx}}{32a^2d}$$

$$= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}} + \frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{7i\,\mathrm{cc}}{}$$

$$= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}} + \frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{105i}{}$$

$$= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}} + \frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{105i}{}$$

$$= \frac{105i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}} -$$

Mathematica [A] time = 1.5202, size = 145, normalized size = 0.62

$$\frac{\sec(c+dx)\left(\frac{630e^{2i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2(3i(86\sin(2(c+dx))+8\sin(4(c+dx))+55i)+158\cos(2(c+dx))+8\cos(4(\right.}{1536ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]*((630*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x)
)]])/Sqrt[1 + E^((2*I)*(c + d*x))] - 2*(158*Cos[2*(c + d*x)] + 8*Cos[4*(c +
 d*x)] + (3*I)*(55*I + 86*Sin[2*(c + d*x)] + 8*Sin[4*(c + d*x)]))))/(1536*a
*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.301, size = 373, normalized size = 1.6

$$\frac{1}{3072\,a^2d}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(1024\,i\,(\cos(dx+c))^7+1024\,(\cos(dx+c))^6\sin(dx+c)+64\,i\,(\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/3072/d/a^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(1024*I*cos(d*x
+c)^7+1024*cos(d*x+c)^6*sin(d*x+c)+64*I*cos(d*x+c)^5+315*I*cos(d*x+c)*2^(1/
2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+
sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+576*sin(d*x+c)
*cos(d*x+c)^4+315*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2
*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2))+168*I*cos(d*x+c)^3+315*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
an(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)+840*cos(d*x+c)^2*sin(d*x+c)-1260*I*cos(
d*x+c))
```

Maxima [B] time = 2.44383, size = 3553, normalized size = 15.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6144*((cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*a
rctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x
 + 6*c), cos(6*d*x + 6*c))) + 1)^(3/4)*((32*I*sqrt(2)*cos(6*d*x + 6*c) + 36
0*I*sqrt(2)*cos(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 32*sqrt(
2)*sin(6*d*x + 6*c) + 360*sqrt(2)*sin(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d
*x + 6*c))) - 64*I*sqrt(2))*cos(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c
), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
 + 1)) - (32*sqrt(2)*cos(6*d*x + 6*c) + 360*sqrt(2)*cos(2/3*arctan2(sin(6*d
*x + 6*c), cos(6*d*x + 6*c))) - 32*I*sqrt(2)*sin(6*d*x + 6*c) - 360*I*sqrt(
2)*sin(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 64*sqrt(2))*sin(3
/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*ar
ctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + (cos(1/3*arctan
2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c)
, cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*
c))) + 1)^(1/4)*(((12*I*sqrt(2)*cos(6*d*x + 6*c) + 12*sqrt(2)*sin(6*d*x + 6
*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (12*I*sqrt(2)
*cos(6*d*x + 6*c) + 12*sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*d*x
+ 6*c), cos(6*d*x + 6*c)))^2 + (24*I*sqrt(2)*cos(6*d*x + 6*c) + 24*sqrt(2)*
sin(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 12
*I*sqrt(2)*cos(6*d*x + 6*c) + 12*sqrt(2)*sin(6*d*x + 6*c))*cos(5/2*arctan2(
sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + (-12*I*sqrt(2)*cos(6*d*x + 6*c) - 2
16*I*sqrt(2)*cos(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 288*I*s
qrt(2)*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 12*sqrt(2)*si
n(6*d*x + 6*c) - 216*sqrt(2)*sin(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
6*c))) + 288*sqrt(2)*sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) -
```

```
 768*I*sqrt(2))*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
 + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) - 12*
((sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*cos(1/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2
)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2
+ 2*(sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*cos(1/3*arctan2
(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2
)*sin(6*d*x + 6*c))*sin(5/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6
*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) +
 (12*sqrt(2)*cos(6*d*x + 6*c) + 216*sqrt(2)*cos(2/3*arctan2(sin(6*d*x + 6*c
), cos(6*d*x + 6*c))) - 288*sqrt(2)*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6
*d*x + 6*c))) - 12*I*sqrt(2)*sin(6*d*x + 6*c) - 216*I*sqrt(2)*sin(2/3*arcta
n2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 288*I*sqrt(2)*sin(1/3*arctan2(sin
(6*d*x + 6*c), cos(6*d*x + 6*c))) + 768*sqrt(2))*sin(1/2*arctan2(sin(1/3*ar
ctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c
), cos(6*d*x + 6*c))) + 1)))*sqrt(a) - (630*sqrt(2)*arctan2((cos(1/3*arctan
2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c)
, cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*
c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
 + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)), (cos
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos
(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c)
, cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
+ 1)) + 1) - 630*sqrt(2)*arctan2((cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d
*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*
cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*sin(1/2*arc
tan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(
sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)), (cos(1/3*arctan2(sin(6*d*x + 6*
c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*
c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*
cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) - 1) - 315*I*sqrt(2)*
log(sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*a
rctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x
 + 6*c), cos(6*d*x + 6*c))) + 1)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x
+ 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6
*c))) + 1))^2 + sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2
 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arcta
n2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*sin(1/2*arctan2(sin(1/3*arctan
2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), c
os(6*d*x + 6*c))) + 1))^2 + 2*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
+ 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan
2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin
(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + 1) + 315*I*sqrt(2)*log(sqrt(cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d
*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6
*d*x + 6*c))) + 1)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*
d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1))^2
+ sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arc
tan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x +
 6*c), cos(6*d*x + 6*c))) + 1)*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x +
6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c
))) + 1))^2 - 2*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + s
in(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(s
in(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/3*arct
an2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c),
 cos(6*d*x + 6*c))) + 1)) + 1))*sqrt(a))/(a^2*d)
```

Fricas [A] time = 2.23513, size = 934, normalized size = 4.01

$$\left(315i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(7i\,dx+7i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(i\,dx+i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-315i\,\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(315*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(7*I*d*x + 7*I*c)*log((2*
sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)) - 315*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(7*I*d*x + 7*I*c)*log(-(2*
sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-16*I*e^(10*I*d*x + 10*I*
c) - 224*I*e^(8*I*d*x + 8*I*c) - 43*I*e^(6*I*d*x + 6*I*c) + 215*I*e^(4*I*d*
x + 4*I*c) + 58*I*e^(2*I*d*x + 2*I*c) + 8*I)*e^(I*d*x + I*c))*e^(-7*I*d*x -
 7*I*c)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^3}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.362 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^9d}+\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^8d}-\frac{16i(a+ia\tan(c+dx))^{9/2}}{3a^7d}+\frac{64i(a+ia\tan(c+dx))^{7/2}}{7a^6d}-32i(a$$

```
[Out] (((-32*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^5*d) + (((64*I)/7)*(a + I*a*T
an[c + d*x])^(7/2))/(a^6*d) - (((16*I)/3)*(a + I*a*Tan[c + d*x])^(9/2))/(a^
7*d) + (((16*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^8*d) - (((2*I)/13)*(a
 + I*a*Tan[c + d*x])^(13/2))/(a^9*d)
```

Rubi [A] time = 0.0931937, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$-\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^9d}+\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^8d}-\frac{16i(a+ia\tan(c+dx))^{9/2}}{3a^7d}+\frac{64i(a+ia\tan(c+dx))^{7/2}}{7a^6d}-32i(a$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-32*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^5*d) + (((64*I)/7)*(a + I*a*T
an[c + d*x])^(7/2))/(a^6*d) - (((16*I)/3)*(a + I*a*Tan[c + d*x])^(9/2))/(a^
7*d) + (((16*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^8*d) - (((2*I)/13)*(a
 + I*a*Tan[c + d*x])^(13/2))/(a^9*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^4(a+x)^{3/2}\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(16a^4(a+x)^{3/2}-32a^3(a+x)^{5/2}+24a^2(a+x)^{7/2}-8a(a+x)^{9/2}+(a+x)^{11/2}\right)\right.}{a^9d}\\ &= -\frac{32i(a+ia\tan(c+dx))^{5/2}}{5a^5d}+\frac{64i(a+ia\tan(c+dx))^{7/2}}{7a^6d}-\frac{16i(a+ia\tan(c+dx))^{9/2}}{3a^7d}+16\end{aligned}$$

Mathematica [A] time = 0.692974, size = 116, normalized size = 0.79

$$\frac{2\sec^9(c+dx)(2600\sin(2(c+dx))+2875\sin(4(c+dx))+4264i\cos(2(c+dx))+3131i\cos(4(c+dx))+2288i)(\cos(5(c}{15015a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*Sec[c + d*x]^9*(2288*I + (4264*I)*Cos[2*(c + d*x)] + (3131*I)*Cos[4*(c +
 d*x)] + 2600*Sin[2*(c + d*x)] + 2875*Sin[4*(c + d*x)])*(Cos[5*(c + d*x)] +
 I*Sin[5*(c + d*x)]))/(15015*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c
 + d*x]])
```

Maple [A] time = 0.349, size = 127, normalized size = 0.9

$$\frac{-8192\,i\left(\cos\left(dx+c\right)\right)^{6}+8192\,\left(\cos\left(dx+c\right)\right)^{5}\sin\left(dx+c\right)-1024\,i\left(\cos\left(dx+c\right)\right)^{4}+5120\,\left(\cos\left(dx+c\right)\right)^{3}\sin\left(dx+c\right)-}{15015\,da^{3}\left(\cos\left(dx+c\right)\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/15015/d/a^3*(-4096*I*cos(d*x+c)^6+4096*cos(d*x+c)^5*sin(d*x+c)-512*I*cos(
d*x+c)^4+2560*cos(d*x+c)^3*sin(d*x+c)-6230*I*cos(d*x+c)^2-3990*cos(d*x+c)*s
in(d*x+c)+1155*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)
^6
```

Maxima [A] time = 0.97875, size = 127, normalized size = 0.87

$$-\frac{2i\left(1155\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{13}{2}}-10920\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}a+40040\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}a^{2}-68640\left(i\,a\tan\left(\right.\right.}{15015\,a^{9}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15015*I*(1155*(I*a*tan(d*x + c) + a)^(13/2) - 10920*(I*a*tan(d*x + c) +
a)^(11/2)*a + 40040*(I*a*tan(d*x + c) + a)^(9/2)*a^2 - 68640*(I*a*tan(d*x +
 c) + a)^(7/2)*a^3 + 48048*(I*a*tan(d*x + c) + a)^(5/2)*a^4)/(a^9*d)
```

Fricas [A] time = 2.27935, size = 566, normalized size = 3.88

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-16384i\,e^{(12i\,dx+12i\,c)}-106496i\,e^{(10i\,dx+10i\,c)}-292864i\,e^{(8i\,dx+8i\,c)}-439296i\,e^{(6i\,dx+6i\,c)}-384384i\,e^{(4i\,dx+}\right.}{15015\left(a^{3}de^{(12i\,dx+12i\,c)}+6\,a^{3}de^{(10i\,dx+10i\,c)}+15\,a^{3}de^{(8i\,dx+8i\,c)}+20\,a^{3}de^{(6i\,dx+6i\,c)}+15\,a^{3}de^{(4i\,dx+4i\,c)}+6\,a^{3}de^{(2i\,dx+2i\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15015*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-16384*I*e^(12*I*d*x + 1
2*I*c) - 106496*I*e^(10*I*d*x + 10*I*c) - 292864*I*e^(8*I*d*x + 8*I*c) - 43
9296*I*e^(6*I*d*x + 6*I*c) - 384384*I*e^(4*I*d*x + 4*I*c))*e^(I*d*x + I*c)/
(a^3*d*e^(12*I*d*x + 12*I*c) + 6*a^3*d*e^(10*I*d*x + 10*I*c) + 15*a^3*d*e^(
```

```
8*I*d*x + 8*I*c) + 20*a^3*d*e^(6*I*d*x + 6*I*c) + 15*a^3*d*e^(4*I*d*x + 4*I
*c) + 6*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**(5/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{10}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^10/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.363 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^7d}-\frac{12i(a+ia\tan(c+dx))^{7/2}}{7a^6d}+\frac{24i(a+ia\tan(c+dx))^{5/2}}{5a^5d}-\frac{16i(a+ia\tan(c+dx))^{3/2}}{3a^4d}$$

```
[Out] (((-16*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^4*d) + (((24*I)/5)*(a + I*a*T
an[c + d*x])^(5/2))/(a^5*d) - (((12*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^
6*d) + (((2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^7*d)
```

Rubi [A] time = 0.0856808, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^7d}-\frac{12i(a+ia\tan(c+dx))^{7/2}}{7a^6d}+\frac{24i(a+ia\tan(c+dx))^{5/2}}{5a^5d}-\frac{16i(a+ia\tan(c+dx))^{3/2}}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-16*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^4*d) + (((24*I)/5)*(a + I*a*T
an[c + d*x])^(5/2))/(a^5*d) - (((12*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^
6*d) + (((2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{i\,\mathrm{Subst}\left(\int (a-x)^3\sqrt{a+x}\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\mathrm{Subst}\left(\int \left(8a^3\sqrt{a+x}-12a^2(a+x)^{3/2}+6a(a+x)^{5/2}-(a+x)^{7/2}\right)dx,x,ia\tan(c+d\right.}{a^7d}\\ &= -\frac{16i(a+ia\tan(c+dx))^{3/2}}{3a^4d}+\frac{24i(a+ia\tan(c+dx))^{5/2}}{5a^5d}-\frac{12i(a+ia\tan(c+dx))^{7/2}}{7a^6d}+\end{aligned}$$

Mathematica [A] time = 0.494226, size = 108, normalized size = 0.92

$$\frac{2\sec^6(c+dx)(\cos(4(c+dx))+i\sin(4(c+dx)))(242i\cos(2(c+dx))+54\tan(c+dx)+89\sin(3(c+dx))\sec(c+dx)}{315a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*Sec[c + d*x]^6*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)])*(77*I + (242*I)*C
os[2*(c + d*x)] + 89*Sec[c + d*x]*Sin[3*(c + d*x)] + 54*Tan[c + d*x]))/(315
*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.3, size = 100, normalized size = 0.9

$$-\frac{256\,i\,(\cos(dx+c))^4-256\,(\cos(dx+c))^3\sin(dx+c)+452\,i\,(\cos(dx+c))^2+260\,\cos(dx+c)\sin(dx+c)-70\,i}{315\,da^3\,(\cos(dx+c))^4}\sqrt{a\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -2/315/d/a^3*(128*I*cos(d*x+c)^4-128*cos(d*x+c)^3*sin(d*x+c)+226*I*cos(d*x+
c)^2+130*cos(d*x+c)*sin(d*x+c)-35*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c
))^(1/2)/cos(d*x+c)^4
```

Maxima [A] time = 1.18941, size = 103, normalized size = 0.88

$$\frac{2i\left(35\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}-270\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a+756\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^2-840\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^3\right)}{315\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/315*I*(35*(I*a*tan(d*x + c) + a)^(9/2) - 270*(I*a*tan(d*x + c) + a)^(7/2)
*a + 756*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 840*(I*a*tan(d*x + c) + a)^(3/2
)*a^3)/(a^7*d)
```

Fricas [A] time = 2.21845, size = 414, normalized size = 3.54

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-512i\,e^{(8i\,dx+8i\,c)}-2304i\,e^{(6i\,dx+6i\,c)}-4032i\,e^{(4i\,dx+4i\,c)}-3360i\,e^{(2i\,dx+2i\,c)}\right)e^{(i\,dx+i\,c)}}{315\left(a^3de^{(8i\,dx+8i\,c)}+4\,a^3de^{(6i\,dx+6i\,c)}+6\,a^3de^{(4i\,dx+4i\,c)}+4\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-512*I*e^(8*I*d*x + 8*I*c)
 - 2304*I*e^(6*I*d*x + 6*I*c) - 4032*I*e^(4*I*d*x + 4*I*c) - 3360*I*e^(2*I*
d*x + 2*I*c))*e^(I*d*x + I*c)/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d
*x + 6*I*c) + 6*a^3*d*e^(4*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a
^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^8}{(i\,a\tan (dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.364 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^5d}+\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^4d}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^3d}$$

```
[Out] ((-8*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d) + (((8*I)/3)*(a + I*a*Tan[c + d
*x])^(3/2))/(a^4*d) - (((2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^5*d)
```

Rubi [A] time = 0.0793853, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^5d}+\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^4d}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-8*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d) + (((8*I)/3)*(a + I*a*Tan[c + d
*x])^(3/2))/(a^4*d) - (((2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^5*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{(a-x)^2}{\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{i\,\text{Subst}\left(\int \left(\frac{4a^2}{\sqrt{a+x}}-4a\sqrt{a+x}+(a+x)^{3/2}\right)dx, x, ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^3d}+\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^4d}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^5d}
\end{aligned}$$

Mathematica [A] time = 0.263347, size = 94, normalized size = 1.09

$$\frac{2\sec^5(c+dx)(7\sin(2(c+dx))+23i\cos(2(c+dx))+20i)(\cos(3(c+dx))+i\sin(3(c+dx)))}{15a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*Sec[c + d*x]^5*(20*I + (23*I)*Cos[2*(c + d*x)] + 7*Sin[2*(c + d*x)])*(Co
s[3*(c + d*x)] + I*Sin[3*(c + d*x)]))/(15*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[
a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.273, size = 73, normalized size = 0.9

$$-\frac{92\,i\,(\cos(dx+c))^2+28\,\cos(dx+c)\sin(dx+c)-6\,i}{15\,da^3\,(\cos(dx+c))^2}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -2/15/d/a^3*(46*I*cos(d*x+c)^2+14*cos(d*x+c)*sin(d*x+c)-3*I)*(a*(I*sin(d*x+
c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2
```

Maxima [A] time = 1.06895, size = 78, normalized size = 0.91

$$-\frac{2i\left(3\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}-20\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a+60\,\sqrt{i\,a\tan(dx+c)+a}a^2\right)}{15\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*I*(3*(I*a*tan(d*x + c) + a)^(5/2) - 20*(I*a*tan(d*x + c) + a)^(3/2)*a
 + 60*sqrt(I*a*tan(d*x + c) + a)*a^2)/(a^5*d)
```

Fricas [A] time = 2.11693, size = 262, normalized size = 3.05

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-64i\,e^{(4i\,dx+4i\,c)}-160i\,e^{(2i\,dx+2i\,c)}-120i\right)e^{(i\,dx+i\,c)}}{15\left(a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-64*I*e^(4*I*d*x + 4*I*c) -
 160*I*e^(2*I*d*x + 2*I*c) - 120*I)*e^(I*d*x + I*c)/(a^3*d*e^(4*I*d*x + 4*I
*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^6}{(ia\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.365 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=55

$$\frac{2i\sqrt{a+ia\tan(c+dx)}}{a^3d}+\frac{4i}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (4*I)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*Sqrt[a + I*a*Tan[c + d*x]
])/(a^3*d)
```

Rubi [A] time = 0.0701862, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i\sqrt{a+ia\tan(c+dx)}}{a^3d}+\frac{4i}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (4*I)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*Sqrt[a + I*a*Tan[c + d*x]
])/(a^3*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{a-x}{(a+x)^{3/2}}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(\frac{2a}{(a+x)^{3/2}}-\frac{1}{\sqrt{a+x}}\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= \frac{4i}{a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{2i\sqrt{a+ia\tan(c+dx)}}{a^3d}\end{aligned}$$

Mathematica [A] time = 0.254093, size = 36, normalized size = 0.65

$$\frac{-2\tan(c+dx)+6i}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (6*I - 2*Tan[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.266, size = 65, normalized size = 1.2

$$2\,{\frac {2\,\cos \left(dx+c \right) \sin \left(dx+c \right) +i+2\,i \left(\cos \left(dx+c \right) \right) ^{2}}{d{a}^{3}}\sqrt {{\frac {a \left(i\sin \left(dx+c \right) +\cos \left(dx+c \right) \right) }{\cos \left(dx+c \right) }}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/d/a^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(2*cos(d*x+c)*sin(d*
x+c)+I+2*I*cos(d*x+c)^2)
```

Maxima [A] time = 1.11321, size = 59, normalized size = 1.07

$$\frac{2i\left(\frac{\sqrt{i\,a\tan(dx+c)+a}}{a^2}+\frac{2}{\sqrt{i\,a\tan(dx+c)+a}a}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*I*(sqrt(I*a*tan(d*x + c) + a)/a^2 + 2/(sqrt(I*a*tan(d*x + c) + a)*a))/(a*
d)
```

Fricas [A] time = 2.14453, size = 135, normalized size = 2.45

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(4i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-i\,dx-i\,c)}}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(4*I*e^(2*I*d*x + 2*I*c) + 2*I)*e
^(-I*d*x - I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.366 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] ((2*I)/3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.0628563, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 32}

$$\frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((2*I)/3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.213731, size = 39, normalized size = 1.34

$$\frac{2}{3a^2d(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] 2/(3*a^2*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.03, size = 24, normalized size = 0.8

$$\frac{\frac{2i}{3}}{ad}(a+ia\tan(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/3*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Maxima [A] time = 1.08588, size = 28, normalized size = 0.97

$$\frac{2i}{3(ia\tan(dx+c)+a)^{\frac{3}{2}}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*I/((I*a*tan(d*x + c) + a)^(3/2)*a*d)
```

Fricas [B] time = 2.16009, size = 176, normalized size = 6.07

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i dx+2i c)}+1}}\left(i e^{(4i dx+4i c)}+2i e^{(2i dx+2i c)}+i\right)e^{(-3i dx-3i c)}}{6a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(4*I*d*x + 4*I*c) + 2*I*
e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(dx+c)^2}{(ia\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.367 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{9i}{32a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{9i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d}+\frac{}{28d(a+ia}$$

```
[Out] (((-9*I)/32)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2
]*a^(5/2)*d) + (((9*I)/28)*a)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - ((I/2)*a^2
)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + ((9*I)/40)/(d*(
a + I*a*Tan[c + d*x])^(5/2)) + ((3*I)/16)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)
) + ((9*I)/32)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.126773, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{9i}{32a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{9i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d}+\frac{}{28d(a+ia}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-9*I)/32)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2
]*a^(5/2)*d) + (((9*I)/28)*a)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - ((I/2)*a^2
)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + ((9*I)/40)/(d*(
a + I*a*Tan[c + d*x])^(5/2)) + ((3*I)/16)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)
) + ((9*I)/32)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\left(ia^3\right)\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{9/2}}\,dx,x,ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} - \frac{\left(9ia^2\right)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{9/2}}\,dx,x,ia\tan(\right.}{4d}$$

$$= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} - \frac{(9ia)\text{Subst}\left(\right.}{}$$

$$= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{9}{40d(a+ia\,\text{ta}}$$

$$= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{9}{40d(a+ia\,\text{ta}}$$

$$= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{9}{40d(a+ia\,\text{ta}}$$

$$= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{9}{40d(a+ia\,\text{ta}}$$

$$= -\frac{9i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d} + \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\,\text{t}}$$

Mathematica [A] time = 1.00374, size = 163, normalized size = 0.8

$$-\frac{ie^{-8i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\sec^2(c+dx)\left(\sqrt{1+e^{2i(c+dx)}}\left(-58e^{2i(c+dx)}-156e^{4i(c+dx)}-388e^{6i(c+dx)}+35e^{8i(c+dx)}-10\right)+315e\right.}{4480a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I/4480)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(Sqrt[1 + E^((2*I)*(c + d*x))]*(
-10 - 58*E^((2*I)*(c + d*x)) - 156*E^((4*I)*(c + d*x)) - 388*E^((6*I)*(c +
d*x)) + 35*E^((8*I)*(c + d*x))) + 315*E^((7*I)*(c + d*x))*ArcSinh[E^(I*(c +
 d*x))])*Sec[c + d*x]^2)/(a^2*d*E^((8*I)*(c + d*x))*Sqrt[a + I*a*Tan[c + d*
x]])
```

Maple [B] time = 0.302, size = 395, normalized size = 1.9

$$\frac{1}{4480\,da^3}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(2560\,i\,(\cos(dx+c))^8+2560\,\sin(dx+c)\,(\cos(dx+c))^7-768\,i\,(\cos(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/4480/d/a^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(2560*I*cos(d*x
+c)^8+2560*sin(d*x+c)*cos(d*x+c)^7-768*I*cos(d*x+c)^6+512*cos(d*x+c)^5*sin(
d*x+c)+315*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2
)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))*cos(d*x+c)+96*I*cos(d*x+c)^4+315*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))+315*2^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-
I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+672*cos(d*x+c)^3*sin(d*x+c)+420*I*cos(d
*x+c)^2+1260*cos(d*x+c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.53606, size = 949, normalized size = 4.65

$$\left(-315i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(8i\,dx+8i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(2i\,dx+2i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+31$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2240*(-315*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(8*I*d*x + 8*I*c)*log((2
*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(2*I*d*x + 2*I*c) + sqrt(2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*
x - I*c)) + 315*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(8*I*d*x + 8*I*c)*log
(-(2*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(2*I*d*x + 2*I*c) - sqrt(2)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-
I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-35*I*e^(10*I*d*
x + 10*I*c) + 353*I*e^(8*I*d*x + 8*I*c) + 544*I*e^(6*I*d*x + 6*I*c) + 214*I
*e^(4*I*d*x + 4*I*c) + 68*I*e^(2*I*d*x + 2*I*c) + 10*I)*e^(I*d*x + I*c))*e^
(-8*I*d*x - 8*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.368 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}-\frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}+\frac{143ia^2}{288d(a+ia\tan(c+$$

```
[Out] (((-143*I)/512)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqr
t[2]*a^(5/2)*d) + (((143*I)/288)*a^2)/(d*(a + I*a*Tan[c + d*x])^(9/2)) - ((
I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) - (((13
*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + (((1
43*I)/448)*a)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + ((143*I)/640)/(d*(a + I*a*
Tan[c + d*x])^(5/2)) + ((143*I)/768)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (
(143*I)/512)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.157167, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}-\frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}+\frac{143ia^2}{288d(a+ia\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```


```
[Out] (((-143*I)/512)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqr
t[2]*a^(5/2)*d) + (((143*I)/288)*a^2)/(d*(a + I*a*Tan[c + d*x])^(9/2)) - ((
I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) - (((13
*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + (((1
43*I)/448)*a)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + ((143*I)/640)/(d*(a + I*a*
Tan[c + d*x])^(5/2)) + ((143*I)/768)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (
(143*I)/512)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\left(ia^5\right)\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{\left(13ia^4\right)\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{11/2}}\,dx, x, ia\,\text{t}\right.}{8d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{}{16d(a-ia}$$

$$= -\frac{143i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}a^{5/2}d} + \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a}$$

Mathematica [A] time = 1.46397, size = 189, normalized size = 0.68

$$-\frac{ie^{-10i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\sec^2(c+dx)\left(\sqrt{1+e^{2i(c+dx)}}\left(-2200e^{2i(c+dx)}-7944e^{4i(c+dx)}-18808e^{6i(c+dx)}-50584e^{8i(c+dx)}+\right.\right.}{645120a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I/645120)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(Sqrt[1 + E^((2*I)*(c + d*x))]
*(-280 - 2200*E^((2*I)*(c + d*x)) - 7944*E^((4*I)*(c + d*x)) - 18808*E^((6*
I)*(c + d*x)) - 50584*E^((8*I)*(c + d*x)) + 7875*E^((10*I)*(c + d*x)) + 630
*E^((12*I)*(c + d*x))) + 45045*E^((9*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))]
```

```
)*Sec[c + d*x]^2)/(a^2*d*E^((10*I)*(c + d*x))*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.452, size = 422, normalized size = 1.5

$$\frac{1}{645120\,da^3}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}\left(286720\,i\left(\cos\left(dx+c\right)\right)^{10}+286720\,\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{9}-81920\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/645120/d/a^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(286720*I*cos
(d*x+c)^10+286720*sin(d*x+c)*cos(d*x+c)^9-81920*I*cos(d*x+c)^8+61440*sin(d*
x+c)*cos(d*x+c)^7+6656*I*cos(d*x+c)^6+73216*cos(d*x+c)^5*sin(d*x+c)+45045*I
*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)
*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))+13728*I*cos(d*x+c)^4+45045*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2
)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2))+45045*2^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin
(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)+96096*cos(d*x+c)^3*sin(d*x+c)+60060*I*cos(d*
x+c)^2+180180*cos(d*x+c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.85249, size = 1064, normalized size = 3.84

$$\frac{\left(-45045i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(10i\,dx+10i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(2i\,dx+2i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/322560*(-45045*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(10*I*d*x + 10*I*c)*
log((2*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(2*I*d*x + 2*I*c) + sqrt(2)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^
(-I*d*x - I*c)) + 45045*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(10*I*d*x + 1
0*I*c)*log(-(2*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(2*I*d*x + 2*I*c) - sqrt
(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x +
I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-630*I
```

```
*e^(14*I*d*x + 14*I*c) - 8505*I*e^(12*I*d*x + 12*I*c) + 42709*I*e^(10*I*d*x
 + 10*I*c) + 69392*I*e^(8*I*d*x + 8*I*c) + 26752*I*e^(6*I*d*x + 6*I*c) + 10
144*I*e^(4*I*d*x + 4*I*c) + 2480*I*e^(2*I*d*x + 2*I*c) + 280*I)*e^(I*d*x +
I*c))*e^(-10*I*d*x - 10*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.369 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=147

$$\frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}}+\frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}}+\frac{256ia^4\sec^{13}(c+dx)}{20995d(a+ia\tan(c+dx))^{13/2}}+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+d}$$

```
[Out] (((256*I)/20995)*a^4*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (
((64*I)/1615)*a^3*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2
4*I)/323)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/1
9)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rubi [A] time = 0.265408, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}}+\frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}}+\frac{256ia^4\sec^{13}(c+dx)}{20995d(a+ia\tan(c+dx))^{13/2}}+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((256*I)/20995)*a^4*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (
((64*I)/1615)*a^3*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2
4*I)/323)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/1
9)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}+\frac{1}{19}(12a)\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx\\ &= \frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}+\frac{1}{323}\left(96a^2\right)\int \frac{\sec^{13}(c}{(a+ia\tan(}\\ &= \frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}}+\frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))}\\ &= \frac{256ia^4\sec^{13}(c+dx)}{20995d(a+ia\tan(c+dx))^{13/2}}+\frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}}+\frac{24ia^2\sec^{13}(c+}{323d(a+ia\tan(c+}\end{aligned}$$

Mathematica [A] time = 0.952615, size = 112, normalized size = 0.76

$$\frac{\sec^{12}(c+dx)(13i(38\sin(c+dx)+123\sin(3(c+dx)))+798\cos(c+dx)+1631\cos(3(c+dx)))(-2\sin(4(c+dx))-2i\cos}{20995a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^12*(798*Cos[c + d*x] + 1631*Cos[3*(c + d*x)] + (13*I)*(38*Sin
[c + d*x] + 123*Sin[3*(c + d*x)]))*((-2*I)*Cos[4*(c + d*x)] - 2*Sin[4*(c +
d*x)]))/(20995*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 15.287, size = 181, normalized size = 1.2

$$16384\,i(\cos(dx+c))^{10}+16384\,\sin(dx+c)(\cos(dx+c))^{9}-2048\,i(\cos(dx+c))^{8}+6144\,\sin(dx+c)(\cos(dx+c))^{7}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/20995/d/a^3*(8192*I*cos(d*x+c)^10+8192*sin(d*x+c)*cos(d*x+c)^9-1024*I*cos
(d*x+c)^8+3072*sin(d*x+c)*cos(d*x+c)^7-320*I*cos(d*x+c)^6+2240*cos(d*x+c)^5
*sin(d*x+c)-168*I*cos(d*x+c)^4+1848*cos(d*x+c)^3*sin(d*x+c)-5356*I*cos(d*x+
c)^2-3640*cos(d*x+c)*sin(d*x+c)+1105*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*
x+c))^(1/2)/cos(d*x+c)^9
```

Maxima [B] time = 2.87335, size = 1218, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/20995*(-2429*I*sqrt(a) - 8850*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) -
5122*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 45190*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 - 12924*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - 152478*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 40470*I*sqrt(
a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 397594*sqrt(a)*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 - 50065*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 72
2228*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 19380*I*sqrt(a)*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10 - 936700*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c
) + 1)^11 - 936700*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 19380*I*
sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 722228*sqrt(a)*sin(d*x + c)
^15/(cos(d*x + c) + 1)^15 + 50065*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) +
 1)^16 - 397594*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 40470*I*sqr
t(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 152478*sqrt(a)*sin(d*x + c)^19
/(cos(d*x + c) + 1)^19 + 12924*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)
^20 - 45190*sqrt(a)*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 + 5122*I*sqrt(a)*
sin(d*x + c)^22/(cos(d*x + c) + 1)^22 - 8850*sqrt(a)*sin(d*x + c)^23/(cos(d
*x + c) + 1)^23 + 2429*I*sqrt(a)*sin(d*x + c)^24/(cos(d*x + c) + 1)^24)*(si
```

```
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)^(5/2)/((a^3 - 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 66*a^3*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 220*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 495*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 792*a^3*sin(d*x + c)^10/(
cos(d*x + c) + 1)^10 + 924*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 792*
a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 495*a^3*sin(d*x + c)^16/(cos(d*
x + c) + 1)^16 - 220*a^3*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 66*a^3*sin
(d*x + c)^20/(cos(d*x + c) + 1)^20 - 12*a^3*sin(d*x + c)^22/(cos(d*x + c) +
 1)^22 + a^3*sin(d*x + c)^24/(cos(d*x + c) + 1)^24)*d*(-2*I*sin(d*x + c)/(c
os(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2))
```

Fricas [A] time = 2.24708, size = 649, normalized size = 4.41

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(1653760i\,e^{(6i\,dx+6i\,c)}+661504i\,e^{(4i\,dx+4i\,c)}+155648i\,e\right.}{20995\left(a^3de^{(19i\,dx+19i\,c)}+9\,a^3de^{(17i\,dx+17i\,c)}+36\,a^3de^{(15i\,dx+15i\,c)}+84\,a^3de^{(13i\,dx+13i\,c)}+126\,a^3de^{(11i\,dx+11i\,c)}+126\,a^3de^{(9i\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20995*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(1653760*I*e^(6*I*d*x + 6
*I*c) + 661504*I*e^(4*I*d*x + 4*I*c) + 155648*I*e^(2*I*d*x + 2*I*c) + 16384
*I)*e^(I*d*x + I*c)/(a^3*d*e^(19*I*d*x + 19*I*c) + 9*a^3*d*e^(17*I*d*x + 17
*I*c) + 36*a^3*d*e^(15*I*d*x + 15*I*c) + 84*a^3*d*e^(13*I*d*x + 13*I*c) + 1
26*a^3*d*e^(11*I*d*x + 11*I*c) + 126*a^3*d*e^(9*I*d*x + 9*I*c) + 84*a^3*d*e
^(7*I*d*x + 7*I*c) + 36*a^3*d*e^(5*I*d*x + 5*I*c) + 9*a^3*d*e^(3*I*d*x + 3*
I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{13}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^13/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.370 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=110

$$\frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}}+\frac{64ia^3\sec^{11}(c+dx)}{2145d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

```
[Out] (((64*I)/2145)*a^3*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((
16*I)/195)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/
15)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rubi [A] time = 0.192844, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}}+\frac{64ia^3\sec^{11}(c+dx)}{2145d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(5/2),x]
```



```
[Out] (((64*I)/2145)*a^3*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((
16*I)/195)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/
15)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```


Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}+\frac{1}{15}(8a)\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx\\ &= \frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}+\frac{1}{195}\left(32a^2\right)\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{9/2}}\,dx\\ &= \frac{64ia^3\sec^{11}(c+dx)}{2145d(a+ia\tan(c+dx))^{11/2}}+\frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.591508, size = 94, normalized size = 0.85

$$\frac{\sec^{10}(c+dx)(187i\sin(2(c+dx))+203\cos(2(c+dx))+60)(-2\sin(3(c+dx))-2i\cos(3(c+dx)))}{2145a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^10*(60 + 203*Cos[2*(c + d*x)] + (187*I)*Sin[2*(c + d*x)])*((-
2*I)*Cos[3*(c + d*x)] - 2*Sin[3*(c + d*x)]))/(2145*a^2*d*(-I + Tan[c + d*x]
)^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.819, size = 154, normalized size = 1.4

$$\frac{2048\,i\,(\cos(dx+c))^8 + 2048\,\sin(dx+c)\,(\cos(dx+c))^7 - 256\,i\,(\cos(dx+c))^6 + 768\,(\cos(dx+c))^5\sin(dx+c) - 80}{2145\,da^3\,(\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/2145/d/a^3*(1024*I*cos(d*x+c)^8+1024*sin(d*x+c)*cos(d*x+c)^7-128*I*cos(d*
x+c)^6+384*cos(d*x+c)^5*sin(d*x+c)-40*I*cos(d*x+c)^4+280*cos(d*x+c)^3*sin(d
*x+c)-736*I*cos(d*x+c)^2-484*cos(d*x+c)*sin(d*x+c)+143*I)*(a*(I*sin(d*x+c)+
cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^7
```

Maxima [B] time = 2.50036, size = 1031, normalized size = 9.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/2145*(-263*I*sqrt(a) - 830*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 760
*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4270*sqrt(a)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 - 1085*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - 11576*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2000*I*sqrt(a)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - 23000*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 - 2470*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 33540*sqrt(a
)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 33540*sqrt(a)*sin(d*x + c)^11/(cos(
d*x + c) + 1)^11 + 2470*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 2
3000*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 2000*I*sqrt(a)*sin(d*x
 + c)^14/(cos(d*x + c) + 1)^14 - 11576*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c
) + 1)^15 + 1085*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 4270*sqr
t(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 760*I*sqrt(a)*sin(d*x + c)^18/
(cos(d*x + c) + 1)^18 - 830*sqrt(a)*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 +
 263*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 -
10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 45*a^3*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 - 120*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 210*a^3*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 252*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^
10 + 210*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 120*a^3*sin(d*x + c)^1
4/(cos(d*x + c) + 1)^14 + 45*a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 10
*a^3*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + a^3*sin(d*x + c)^20/(cos(d*x +
 c) + 1)^20)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1)^(5/2))
```

Fricas [B] time = 2.21804, size = 509, normalized size = 4.63

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(49920i\,e^{(4i\,dx+4i\,c)}+15360i\,e^{(2i\,dx+2i\,c)}+2048i\right)e^{(i\,dx+i\,c)}}{2145\left(a^3de^{(15i\,dx+15i\,c)}+7\,a^3de^{(13i\,dx+13i\,c)}+21\,a^3de^{(11i\,dx+11i\,c)}+35\,a^3de^{(9i\,dx+9i\,c)}+35\,a^3de^{(7i\,dx+7i\,c)}+21\,a^3de^{(5i\,dx+5i\,c)}+7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2145*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(49920*I*e^(4*I*d*x + 4*I*
c) + 15360*I*e^(2*I*d*x + 2*I*c) + 2048*I)*e^(I*d*x + I*c)/(a^3*d*e^(15*I*d
*x + 15*I*c) + 7*a^3*d*e^(13*I*d*x + 13*I*c) + 21*a^3*d*e^(11*I*d*x + 11*I*
c) + 35*a^3*d*e^(9*I*d*x + 9*I*c) + 35*a^3*d*e^(7*I*d*x + 7*I*c) + 21*a^3*d
*e^(5*I*d*x + 5*I*c) + 7*a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{11}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.371 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

```
[Out] (((8*I)/99)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/
11)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rubi [A] time = 0.125539, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((8*I)/99)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/
11)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}+\frac{1}{11}(4a)\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx\\ &= \frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.434218, size = 80, normalized size = 1.1

$$\frac{2(9\tan(c+dx)-13i)\sec^7(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))}{99a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*Sec[c + d*x]^7*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-13*I + 9*Tan[c
+ d*x]))/(99*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.416, size = 127, normalized size = 1.7

$$\frac{128\,i\left(\cos\left(dx+c\right)\right)^{6}+128\,\left(\cos\left(dx+c\right)\right)^{5}\sin\left(dx+c\right)-16\,i\left(\cos\left(dx+c\right)\right)^{4}+48\,\left(\cos\left(dx+c\right)\right)^{3}\sin\left(dx+c\right)-104\,i\left(\cos\left(\right.\right.}{99\,da^{3}\left(\cos\left(dx+c\right)\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/99/d/a^3*(64*I*cos(d*x+c)^6+64*cos(d*x+c)^5*sin(d*x+c)-8*I*cos(d*x+c)^4+2
4*cos(d*x+c)^3*sin(d*x+c)-52*I*cos(d*x+c)^2-32*cos(d*x+c)*sin(d*x+c)+9*I)*(
a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5
```

Maxima [B] time = 2.17587, size = 845, normalized size = 11.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/99*(-13*I*sqrt(a) - 34*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 46*I*sq
rt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 174*sqrt(a)*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 - 54*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 394*s
qrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 22*I*sqrt(a)*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 - 550*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 550*
sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 22*I*sqrt(a)*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 394*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 +
 54*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 174*sqrt(a)*sin(d*x +
 c)^13/(cos(d*x + c) + 1)^13 + 46*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) +
 1)^14 - 34*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 13*I*sqrt(a)*si
n(d*x + c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 - 8*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 56*
a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)^8/(cos(d*x +
c) + 1)^8 - 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^3*sin(d*x +
 c)^12/(cos(d*x + c) + 1)^12 - 8*a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14
+ a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d*x + c)/(cos(d*x
+ c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2))
```

Fricas [B] time = 2.06984, size = 373, normalized size = 5.11

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(704i\,e^{(2i\,dx+2i\,c)}+128i\right)e^{(i\,dx+i\,c)}}{99\left(a^{3}de^{(11i\,dx+11i\,c)}+5\,a^{3}de^{(9i\,dx+9i\,c)}+10\,a^{3}de^{(7i\,dx+7i\,c)}+10\,a^{3}de^{(5i\,dx+5i\,c)}+5\,a^{3}de^{(3i\,dx+3i\,c)}+a^{3}de^{(i\,dx+i\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/99*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(704*I*e^(2*I*d*x + 2*I*c) +
 128*I)*e^(I*d*x + I*c)/(a^3*d*e^(11*I*d*x + 11*I*c) + 5*a^3*d*e^(9*I*d*x +
 9*I*c) + 10*a^3*d*e^(7*I*d*x + 7*I*c) + 10*a^3*d*e^(5*I*d*x + 5*I*c) + 5*a
^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^9}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.372 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

```
[Out] (((2*I)/7)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rubi [A] time = 0.0629306, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3493}

$$\frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/7)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

Mathematica [A] time = 0.255662, size = 57, normalized size = 1.63

$$-\frac{2(\tan(c+dx)+i)\sec^5(c+dx)}{7a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sec[c + d*x]^5*(I + Tan[c + d*x]))/(7*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[
a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.284, size = 100, normalized size = 2.9

$$\frac{16\,i\,(\cos(dx+c))^4+16\,(\cos(dx+c))^3\sin(dx+c)-16\,i\,(\cos(dx+c))^2-8\,\cos(dx+c)\sin(dx+c)+2\,i}{7\,da^3\,(\cos(dx+c))^3}\sqrt{\frac{a\,(i\sin(dx+\ldots}{\cos\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/7/d/a^3*(8*I*cos(d*x+c)^4+8*cos(d*x+c)^3*sin(d*x+c)-8*I*cos(d*x+c)^2-4*co
s(d*x+c)*sin(d*x+c)+I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d
*x+c)^3
```

Maxima [B] time = 1.7146, size = 659, normalized size = 18.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/7*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sqrt(a)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 10*sqrt(a)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 5*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*sqrt(a)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 - 20*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 5*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 10*sqrt(a)*sin(
d*x + c)^9/(cos(d*x + c) + 1)^9 + 4*I*sqrt(a)*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 - 2*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + I*sqrt(a)*sin(
d*x + c)^12/(cos(d*x + c) + 1)^12)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5
/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 - 6*a^3*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^
3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c)
 + 1)^8 - 6*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12
/(cos(d*x + c) + 1)^12)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2))
```

Fricas [B] time = 2.07692, size = 240, normalized size = 6.86

$$\frac{16i\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{(idx+ic)}}{7\left(a^3de^{(7idx+7ic)}+3a^3de^{(5idx+5ic)}+3a^3de^{(3idx+3ic)}+a^3de^{(idx+ic)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 16/7*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(a^3*d*e^(
7*I*d*x + 7*I*c) + 3*a^3*d*e^(5*I*d*x + 5*I*c) + 3*a^3*d*e^(3*I*d*x + 3*I*c
) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(5/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^7}{(ia\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.373 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{4i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] ((4*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(5/2)*d) - (((2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*
x])^(3/2)) - ((4*I)*Sec[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.169913, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3491, 3489, 206}

$$-\frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{4i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((4*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(5/2)*d) - (((2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*
x])^(3/2)) - ((4*I)*Sec[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3491

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e
+ f*x])^(n + 1))/(b*f*(m - 2)), x] + Dist[(2*d^2)/a, Int[(d*Sec[e + f*x])^(
m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}} + \frac{2\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx}{a}$$

$$= -\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{4\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx}{a^2}$$

$$= -\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{(8i)\,\mathrm{Subst}\left(\int \frac{1}{2-ax^2}dx, x, \frac{\sec(c}{\sqrt{a+ia\,ta}}\right.}{a^2d}$$

$$= \frac{4i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} - \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 0.728286, size = 82, normalized size = 0.67

$$-\frac{2\sec(c+dx)\left(\tan(c+dx) - 6i\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right) + 7i\right)}{3a^2d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sec[c + d*x]*(7*I - (6*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
+ E^((2*I)*(c + d*x))]] + Tan[c + d*x]))/(3*a^2*d*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [B] time = 0.285, size = 281, normalized size = 2.3

$$\frac{2}{3\,da^3\left(i\sin(dx+c)+\cos(dx+c)-1\right)\cos(dx+c)}\left(3\,\arctan\left(1/2\,\frac{\sqrt{2}\left(i\cos(dx+c)-i+\sin(dx+c)\right)}{\sin(dx+c)}\frac{1}{\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d/a^3*(3*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(3/2)*sin(
d*x+c)*cos(d*x+c)*2^(1/2)+3*2^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(
d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(3/2)*sin(d*x+c)+8*I*cos(d*x+c)^2-7*I*cos(d*x+c)+8*cos(d*x+c)*
sin(d*x+c)-I-sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*
sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)
```

Maxima [B] time = 2.17721, size = 1445, normalized size = 11.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*((12*I*sqrt(2)*cos(2*d*x + 2*c) - 12*sqrt(2)*sin(2*d*x + 2*c) + 16*I*sq
rt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 4*(3*sqrt
(2)*cos(2*d*x + 2*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2))*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + (6*(sqrt(2)*cos(2
*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 6*(sqrt(2)*cos(2
*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) + (-3*I*sqrt(2)*co
s(2*d*x + 2*c)^2 - 3*I*sqrt(2)*sin(2*d*x + 2*c)^2 - 6*I*sqrt(2)*cos(2*d*x +
 2*c) - 3*I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (3*I*sqrt(2)*cos(2*d*x
 + 2*c)^2 + 3*I*sqrt(2)*sin(2*d*x + 2*c)^2 + 6*I*sqrt(2)*cos(2*d*x + 2*c) +
 3*I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 +
sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/((a^3*cos(2*d*x + 2*c
)^2 + a^3*sin(2*d*x + 2*c)^2 + 2*a^3*cos(2*d*x + 2*c) + a^3)*d)
```

Fricas [B] time = 2.16131, size = 872, normalized size = 7.09

$$\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-12i\,e^{(2i\,dx+2i\,c)}-16i\right)e^{(i\,dx+i\,c)}+\sqrt{2}\left(6i\,a^3de^{(3i\,dx+3i\,c)}+6i\,a^3de^{(i\,dx+i\,c)}\right)\sqrt{\frac{1}{a^5d^2}}\log\left(\left(\sqrt{2}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(i\,dx+i\,c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-12*I*e^(2*I*d*x + 2*I*c) -
 16*I)*e^(I*d*x + I*c) + sqrt(2)*(6*I*a^3*d*e^(3*I*d*x + 3*I*c) + 6*I*a^3*d
*e^(I*d*x + I*c))*sqrt(1/(a^5*d^2))*log((sqrt(2)*a^3*d*sqrt(1/(a^5*d^2))*e^
(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I
*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*(-6*I*a^3*d*e^(3*I*d*
x + 3*I*c) - 6*I*a^3*d*e^(I*d*x + I*c))*sqrt(1/(a^5*d^2))*log(-(sqrt(2)*a^3
*d*sqrt(1/(a^5*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)))/(a^3*d*
e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{5}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.374 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=86

$$\frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}$$

```
[Out] ((-I)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])
/(Sqrt[2]*a^(5/2)*d) + (I*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.143849, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3501, 3502, 3489, 206}

$$\frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])
/(Sqrt[2]*a^(5/2)*d) + (I*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{a}$$
$$= \frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2a^2}$$
$$= \frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{i\,\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^2d}$$
$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] time = 0.975878, size = 149, normalized size = 1.73

$$\frac{ie^{-\frac{1}{2}i(2c+dx)}\sec^3(c+dx)\left(-e^{2i(c+dx)}+e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)-1\right)\left(\cos\left(c+\frac{dx}{2}\right)+i\sin\left(c+\frac{dx}{2}\right)\right)}{2a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/2)*(-1 - E^((2*I)*(c + d*x)) + E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^3*(Cos[c + (
d*x)/2] + I*Sin[c + (d*x)/2]))/(a^2*d*E^((I/2)*(2*c + d*x))*(-I + Tan[c + d
*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.282, size = 443, normalized size = 5.2

$$\frac{\sin(dx+c)}{4\,da^3\left(-1+(\cos(dx+c))^2\right)}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(i\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}\,(i\cos(dx+c)-i+\sin}{2\,\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/4/d/a^3*sin(d*x+c)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(I*(-2*
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x
+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)*cos(d*x+c)
*2^(1/2)+I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d
*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*
x+c)*2^(1/2)-(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos
(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(
d*x+c)^2*2^(1/2)-8*I*sin(d*x+c)*cos(d*x+c)^3+8*cos(d*x+c)^4+(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*
x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)+4*I*cos(d*x+c)*sin(d*x+c
)-8*cos(d*x+c)^2)/(-1+cos(d*x+c)^2)
```

Maxima [B] time = 1.99748, size = 1115, normalized size = 12.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/8*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*((4*I*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*(sqrt(2)*cos(2*d*x + 2*c)
 - I*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))*sqrt(a) + (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*s
qrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1))*sqrt(a))/(a^3*d)
```

Fricas [B] time = 1.99746, size = 761, normalized size = 8.85

$$\left(-i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(3idx+3ic)}\log\left(\left(2\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(idx+ic)}+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)+i\sqrt{\frac{1}{2}}a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(3*I*d*x + 3*I*c)*log((2*sqrt(1
/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
 I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(3*I*d*x + 3*I*c)*log(-(2*sqrt(1/2)*
a^3*d*sqrt(1/(a^5*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqr
t(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I)*e^(I*d*x
 + I*c))*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(i a \tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.375 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] (((3*I)/16)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/4)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])
^(5/2)) + (((3*I)/16)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.115477, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3502, 3489, 206}

$$\frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}}+\frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((3*I)/16)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/4)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])
^(5/2)) + (((3*I)/16)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{8a}$$
$$= \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}} + \frac{3\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{32a^2}$$
$$= \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}} + \frac{(3i)\,\mathrm{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec}{\sqrt{a+ia}}\right.}{16a^2d}$$
$$= \frac{3i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] time = 0.751814, size = 121, normalized size = 0.99

$$-\frac{i\sec^3(c+dx)\left(3i\sin(2(c+dx)) + 7\cos(2(c+dx)) + 3e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right) + 7\right)}{32a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I/32)*Sec[c + d*x]^3*(7 + 3*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d
*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] + 7*Cos[2*(c + d*x)] + (3*I)*S
in[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.283, size = 346, normalized size = 2.8

$$\frac{1}{64\,da^3}\left(64\,i\left(\cos(dx+c)\right)^5 + 3\,i\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}\left(i\cos(dx+c) - i + \sin(dx+c)\right)}{2\,\sin(dx+c)}\frac{1}{\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/64/d/a^3*(64*I*cos(d*x+c)^5+3*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
an(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*2^(1/2)*cos(d*x+c)+64*sin(d*x+c)*cos(d*x+c)^4+3*I*(-2*cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))
/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)+3*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*
x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)-24*I*cos(d*x+
c)^3+8*cos(d*x+c)^2*sin(d*x+c)-12*I*cos(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c)
)/cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 2.11828, size = 807, normalized size = 6.61

$$\left(3i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(5idx+5ic)}\log\left(\left(2\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(idx+ic)}+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-3i\sqrt{\frac{1}{2}}a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log((2*sqrt
(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
 - 3*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(5*I*d*x + 5*I*c)*log(-(2*sqrt(1
/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
 sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(5*I*e^(4*I*d*x + 4*I*c) + 7*I*e
^(2*I*d*x + 2*I*c) + 2*I)*e^(I*d*x + I*c))*e^(-5*I*d*x - 5*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(i a \tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.376 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=192

$$-\frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128a^3d}+\frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{35i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d}+\frac{7i\cos(c+a}{48ad(a+ia\tan(c}$$

```
[Out] (((35*I)/128)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/6)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x
])^(5/2)) + (((7*I)/48)*Cos[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) +
(((35*I)/192)*Cos[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((35*I)/1
28)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.25852, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3502, 3490, 3489, 206}

$$-\frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128a^3d}+\frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{35i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d}+\frac{7i\cos(c+a}{48ad(a+ia\tan(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```



```
[Out] (((35*I)/128)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/6)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x
])^(5/2)) + (((7*I)/48)*Cos[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) +
(((35*I)/192)*Cos[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((35*I)/1
28)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{12a}$$

$$= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{96a^2}$$

$$= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{35i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d} + \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3}}$$

Mathematica [A] time = 1.16669, size = 143, normalized size = 0.74

$$\frac{i\sec^3(c+dx)\left(7i\sin(2(c+dx)) + 56i\sin(4(c+dx)) - 85\cos(2(c+dx)) + 40\cos(4(c+dx)) - 105e^{2i(c+dx)}\sqrt{1+e^{2i(c+d}}\right.}{768a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/768)*Sec[c + d*x]^3*(-125 - 105*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(
c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 85*Cos[2*(c + d*x)] + 4
0*Cos[4*(c + d*x)] + (7*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)]))/(a^
2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.276, size = 373, normalized size = 1.9

$$\frac{1}{1536\,da^3}\sqrt{\frac{a\left(i\sin(dx+c)+\cos(dx+c)\right)}{\cos(dx+c)}}\left(1024\,i\left(\cos(dx+c)\right)^7 + 1024\,\left(\cos(dx+c)\right)^6\sin(dx+c) - 320\,i\left(\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/1536/d/a^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(1024*I*cos(d*x
+c)^7+1024*cos(d*x+c)^6*sin(d*x+c)-320*I*cos(d*x+c)^5+105*I*2^(1/2)*(-2*cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c)
)/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+192*sin(d*x+c
```

```
)*cos(d*x+c)^4+105*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/
2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))+56*I*cos(d*x+c)^3+105*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
an(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)+280*cos(d*x+c)^2*sin(d*x+c)-420*I*cos(d
*x+c))
```

Maxima [B] time = 2.22416, size = 3101, normalized size = 16.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3072*((cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*
arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*
x + 6*c), cos(6*d*x + 6*c))) + 1)^(3/4)*((544*I*sqrt(2)*cos(6*d*x + 6*c) +
544*sqrt(2)*sin(6*d*x + 6*c))*cos(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6
*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)
)) + 1)) - 544*(sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*sin(
3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*a
rctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + (cos(1/3*arcta
n2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c
), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6
*c))) + 1)^(1/4)*(((-348*I*sqrt(2)*cos(6*d*x + 6*c) - 348*sqrt(2)*sin(6*d*x
 + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (-348*I*s
qrt(2)*cos(6*d*x + 6*c) - 348*sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin
(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (-696*I*sqrt(2)*cos(6*d*x + 6*c) - 69
6*sqrt(2)*sin(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6
*c))) - 348*I*sqrt(2)*cos(6*d*x + 6*c) - 348*sqrt(2)*sin(6*d*x + 6*c))*cos(
5/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*a
rctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + (-228*I*sqrt(2)*cos(6*d
*x + 6*c) - 228*sqrt(2)*sin(6*d*x + 6*c) + 192*I*sqrt(2))*cos(1/2*arctan2(s
in(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*
d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + 348*((sqrt(2)*cos(6*d*x + 6*c) - I*s
qrt(2)*sin(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)
))^2 + (sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arct
an2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*(sqrt(2)*cos(6*d*x + 6*c) -
I*sqrt(2)*sin(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6
*c))) + sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*sin(5/2*arct
an2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(s
in(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + 12*(19*sqrt(2)*cos(6*d*x + 6*c)
 - 19*I*sqrt(2)*sin(6*d*x + 6*c) - 16*sqrt(2))*sin(1/2*arctan2(sin(1/3*arct
an2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c),
 cos(6*d*x + 6*c))) + 1)))*sqrt(a) + (210*sqrt(2)*arctan2((cos(1/3*arctan2(
sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c),
cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)
)) + 1)^(1/4)*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)), (cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d
*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6
*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c),
cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) +
1)) + 1) - 210*sqrt(2)*arctan2((cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
 + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*co
s(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*sin(1/2*arcta
```

```
n2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)), (cos(1/3*arctan2(sin(6*d*x + 6*c)
, cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)
))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*co
s(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3
*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) - 1) - 105*I*sqrt(2)*lo
g(sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arc
tan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x +
 6*c), cos(6*d*x + 6*c))) + 1)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x +
6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c
))) + 1))^2 + sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 +
 sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2
(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*sin(1/2*arctan2(sin(1/3*arctan2(
sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos
(6*d*x + 6*c))) + 1))^2 + 2*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(
sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + 1) + 105*I*sqrt(2)*log(sqrt(cos(1/3
*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x
 + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d
*x + 6*c))) + 1)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*
x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1))^2 +
sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arcta
n2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6
*c), cos(6*d*x + 6*c))) + 1)*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*
c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))
) + 1))^2 - 2*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin
(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/3*arctan
2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), c
os(6*d*x + 6*c))) + 1)) + 1))*sqrt(a))/(a^3*d)
```

Fricas [B] time = 2.11261, size = 892, normalized size = 4.65

$$\frac{\left(105 i \sqrt{\frac{1}{2}} a^{3} d \sqrt{\frac{1}{a^{5} d^{2}}} e^{(7 i dx+7 i c)} \log\left(\left(2 \sqrt{\frac{1}{2}} a^{3} d \sqrt{\frac{1}{a^{5} d^{2}}} e^{(i dx+i c)} + \sqrt{2} \sqrt{\frac{a}{e^{(2 i dx+2 i c)}+1}} \left(e^{(2 i dx+2 i c)}+1\right) e^{(i dx+i c)}\right) e^{(-i dx-i c)}\right) - 105 i \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/768*(105*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(7*I*d*x + 7*I*c)*log((2*s
qrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*
c)) - 105*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(7*I*d*x + 7*I*c)*log(-(2*s
qrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*
c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-48*I*e^(8*I*d*x + 8*I*c)
+ 39*I*e^(6*I*d*x + 6*I*c) + 125*I*e^(4*I*d*x + 4*I*c) + 46*I*e^(2*I*d*x +
2*I*c) + 8*I)*e^(I*d*x + I*c))*e^(-7*I*d*x - 7*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.377 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{77i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d}+\frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{1155i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4096a^3d}+\frac{\ }{2048a}$$

```
[Out] (((1155*I)/4096)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
 + d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/8)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c
+ d*x])^(5/2)) + (((11*I)/96)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^(
3/2)) + (((385*I)/2048)*Cos[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) +
(((33*I)/256)*Cos[c + d*x]^3)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((1155*
I)/4096)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d) - (((77*I)/512)*C
os[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.422262, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3502, 3497, 3490, 3489, 206}

$$-\frac{77i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d}+\frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{1155i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4096a^3d}+\frac{\ }{2048a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```


```
[Out] (((1155*I)/4096)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
 + d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/8)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c
+ d*x])^(5/2)) + (((11*I)/96)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^(
3/2)) + (((385*I)/2048)*Cos[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) +
(((33*I)/256)*Cos[c + d*x]^3)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((1155*
I)/4096)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d) - (((77*I)/512)*C
os[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
```

```
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{16a}$$

$$= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{33\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{64a^2}$$

$$= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}} + \cdots$$

$$= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}} - \cdots$$

$$= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{385i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}} + \cdots$$

$$= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{385i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}} + \cdots$$

$$= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{385i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}} + \cdots$$

$$= \frac{1155i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d} + \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} \cdots$$

Mathematica [A] time = 1.19126, size = 165, normalized size = 0.61

$$\frac{i\sec^3(c+dx)\left(1111i\sin(2(c+dx))+2552i\sin(4(c+dx))+176i\sin(6(c+dx))-1605\cos(2(c+dx))+1800\cos(4(c+d\cdots\right.}{24576a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/24576)*Sec[c + d*x]^3*(-3325 - 3465*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*
I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 1605*Cos[2*(c + d*x
)] + 1800*Cos[4*(c + d*x)] + 80*Cos[6*(c + d*x)] + (1111*I)*Sin[2*(c + d*x)
] + (2552*I)*Sin[4*(c + d*x)] + (176*I)*Sin[6*(c + d*x)]))/(a^2*d*(-I + Tan
[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.343, size = 400, normalized size = 1.5

$$\frac{1}{49152\,da^3}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(24576\,i\,(\cos(dx+c))^9+24576\,\sin(dx+c)\,(\cos(dx+c))^8-7168\,i\,(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/49152/d/a^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(24576*I*cos(d
*x+c)^9+24576*sin(d*x+c)*cos(d*x+c)^8-7168*I*cos(d*x+c)^7+5120*cos(d*x+c)^6
*sin(d*x+c)+704*I*cos(d*x+c)^5+3465*I*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c
)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+6336*sin(d*x+c)*cos(d*x+c)^4+3465*I
*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x
+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+1848*I*c
os(d*x+c)^3+3465*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I
*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
2^(1/2)*sin(d*x+c)+9240*cos(d*x+c)^2*sin(d*x+c)-13860*I*cos(d*x+c))
```

Maxima [B] time = 2.62346, size = 5107, normalized size = 18.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/98304*((cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*
arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*
x + 8*c), cos(8*d*x + 8*c))) + 1)^(3/4)*(((60*I*sqrt(2)*cos(8*d*x + 8*c) +
60*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
8*c)))^2 + (60*I*sqrt(2)*cos(8*d*x + 8*c) + 60*sqrt(2)*sin(8*d*x + 8*c))*si
n(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (120*I*sqrt(2)*cos(8
*d*x + 8*c) + 120*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c
), cos(8*d*x + 8*c))) + 60*I*sqrt(2)*cos(8*d*x + 8*c) + 60*sqrt(2)*sin(8*d*
x + 8*c))*cos(7/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c
))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (-220*I*s
qrt(2)*cos(8*d*x + 8*c) - 3840*I*sqrt(2)*cos(3/4*arctan2(sin(8*d*x + 8*c),
cos(8*d*x + 8*c))) + 5184*I*sqrt(2)*cos(1/2*arctan2(sin(8*d*x + 8*c), cos(8
*d*x + 8*c))) - 220*sqrt(2)*sin(8*d*x + 8*c) - 3840*sqrt(2)*sin(3/4*arctan2
(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 5184*sqrt(2)*sin(1/2*arctan2(sin(8*
d*x + 8*c), cos(8*d*x + 8*c))) - 512*I*sqrt(2))*cos(3/2*arctan2(sin(1/4*arc
tan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c)
, cos(8*d*x + 8*c))) + 1)) - (60*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(
8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 60*(
sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(
8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 120*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt
(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
+ 60*sqrt(2)*cos(8*d*x + 8*c) - 60*I*sqrt(2)*sin(8*d*x + 8*c))*sin(7/2*arct
an2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(s
in(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (220*sqrt(2)*cos(8*d*x + 8*c) +
 3840*sqrt(2)*cos(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 5184*s
```

qrt(2)*cos(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 220*I*sqrt(2)
*sin(8*d*x + 8*c) - 3840*I*sqrt(2)*sin(3/4*arctan2(sin(8*d*x + 8*c), cos(8*
d*x + 8*c))) + 5184*I*sqrt(2)*sin(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
 8*c))) + 512*sqrt(2))*sin(3/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)
))*sqrt(a) + (cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(
1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(
8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*(((292*I*sqrt(2)*cos(8*d*x + 8*
c) + 292*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*
d*x + 8*c)))^2 + (292*I*sqrt(2)*cos(8*d*x + 8*c) + 292*sqrt(2)*sin(8*d*x +
8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (3168*I*sqrt
(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 3168*I*sqrt(2)
*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 6336*I*sqrt(2)*co
s(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 3168*I*sqrt(2))*cos(3/
4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + (584*I*sqrt(2)*cos(8*d*x +
 8*c) + 584*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos
(8*d*x + 8*c))) + 3168*(sqrt(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x
 + 8*c)))^2 + sqrt(2)*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + sqrt(2
))*sin(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 292*I*sqrt(2)*cos
(8*d*x + 8*c) + 292*sqrt(2)*sin(8*d*x + 8*c))*cos(5/2*arctan2(sin(1/4*arcta
n2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c),
cos(8*d*x + 8*c))) + 1)) + (60*I*sqrt(2)*cos(8*d*x + 8*c) + 1440*I*sqrt(2)*
cos(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 4032*I*sqrt(2)*cos(1
/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 768*I*sqrt(2)*cos(1/4*arc
tan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 60*sqrt(2)*sin(8*d*x + 8*c) + 1
440*sqrt(2)*sin(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 4032*sqr
t(2)*sin(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 768*sqrt(2)*sin
(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 7680*I*sqrt(2))*cos(1/2
*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arct
an2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) - (292*(sqrt(2)*cos(8*d*x +
8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*
d*x + 8*c)))^2 + 292*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c)
)*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 3168*(sqrt(2)*co
s(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sqrt(2)*sin(1/4*arct
an2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(
8*d*x + 8*c), cos(8*d*x + 8*c))) + sqrt(2))*cos(3/4*arctan2(sin(8*d*x + 8*c
), cos(8*d*x + 8*c))) + 584*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x
 + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - (3168*I*sqr
t(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 3168*I*sqrt(2
)*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 6336*I*sqrt(2)*c
os(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 3168*I*sqrt(2))*sin(3
/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 292*sqrt(2)*cos(8*d*x + 8
*c) - 292*I*sqrt(2)*sin(8*d*x + 8*c))*sin(5/2*arctan2(sin(1/4*arctan2(sin(8
*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*
x + 8*c))) + 1)) - (60*sqrt(2)*cos(8*d*x + 8*c) + 1440*sqrt(2)*cos(3/4*arct
an2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 4032*sqrt(2)*cos(1/2*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) + 768*sqrt(2)*cos(1/4*arctan2(sin(8*d*x +
 8*c), cos(8*d*x + 8*c))) - 60*I*sqrt(2)*sin(8*d*x + 8*c) - 1440*I*sqrt(2)*
sin(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 4032*I*sqrt(2)*sin(1
/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 768*I*sqrt(2)*sin(1/4*arc
tan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 7680*sqrt(2))*sin(1/2*arctan2(s
in(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*
d*x + 8*c), cos(8*d*x + 8*c))) + 1)))*sqrt(a) - (6930*sqrt(2)*arctan2((cos(
1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*
d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(
8*d*x + 8*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c),
 cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) +
 1)), (cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arc

```
tan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x +
 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*
d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x
 + 8*c))) + 1)) + 1) - 6930*sqrt(2)*arctan2((cos(1/4*arctan2(sin(8*d*x + 8*
c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*
c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*
sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1
/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)), (cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos
(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
+ 1)^(1/4)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*
c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) - 1) - 346
5*I*sqrt(2)*log(sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2
 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arcta
n2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)*cos(1/2*arctan2(sin(1/4*arctan
2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), c
os(8*d*x + 8*c))) + 1))^2 + sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*
x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*c
os(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)*sin(1/2*arctan2(si
n(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d
*x + 8*c), cos(8*d*x + 8*c))) + 1))^2 + 2*(cos(1/4*arctan2(sin(8*d*x + 8*c)
, cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)
))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*co
s(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4
*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 1) + 3465*I*sqrt(2)*l
og(sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*ar
ctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x
+ 8*c), cos(8*d*x + 8*c))) + 1)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x +
 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*
c))) + 1))^2 + sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2
+ sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan
2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)*sin(1/2*arctan2(sin(1/4*arctan2
(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c))) + 1))^2 - 2*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(
1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*cos(1/2*arctan2
(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(
8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 1))*sqrt(a))/(a^3*d)
```

Fricas [A] time = 2.1253, size = 988, normalized size = 3.66

$$\left(3465i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(9idx+9ic)}\log\left(\left(2\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(idx+ic)}+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-346$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24576*(3465*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(9*I*d*x + 9*I*c)*log((
2*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x -
 I*c)) - 3465*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(9*I*d*x + 9*I*c)*log(-
(2*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x
- I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-128*I*e^(12*I*d*x + 1
2*I*c) - 2176*I*e^(10*I*d*x + 10*I*c) + 247*I*e^(8*I*d*x + 8*I*c) + 3325*I*
e^(6*I*d*x + 6*I*c) + 1358*I*e^(4*I*d*x + 4*I*c) + 376*I*e^(2*I*d*x + 2*I*c
```

```
) + 48*I)*e^(I*d*x + I*c))*e^(-9*I*d*x - 9*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.378 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^9d}+\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^8d}-\frac{48i(a+ia\tan(c+dx))^{7/2}}{7a^7d}+\frac{64i(a+ia\tan(c+dx))^{5/2}}{5a^6d}-32$$

```
[Out] (((-32*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^5*d) + (((64*I)/5)*(a + I*a*T
an[c + d*x])^(5/2))/(a^6*d) - (((48*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^
7*d) + (((16*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^8*d) - (((2*I)/11)*(a +
 I*a*Tan[c + d*x])^(11/2))/(a^9*d)
```

Rubi [A] time = 0.0931584, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$-\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^9d}+\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^8d}-\frac{48i(a+ia\tan(c+dx))^{7/2}}{7a^7d}+\frac{64i(a+ia\tan(c+dx))^{5/2}}{5a^6d}-32$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-32*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^5*d) + (((64*I)/5)*(a + I*a*T
an[c + d*x])^(5/2))/(a^6*d) - (((48*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^
7*d) + (((16*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^8*d) - (((2*I)/11)*(a +
 I*a*Tan[c + d*x])^(11/2))/(a^9*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^4\sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\
&= -\frac{i\,\text{Subst}\left(\int \left(16a^4\sqrt{a+x}-32a^3(a+x)^{3/2}+24a^2(a+x)^{5/2}-8a(a+x)^{7/2}+(a+x)^{9/2}\right)\right.}{a^9d}\\
&= -\frac{32i(a+ia\tan(c+dx))^{3/2}}{3a^5d}+\frac{64i(a+ia\tan(c+dx))^{5/2}}{5a^6d}-\frac{48i(a+ia\tan(c+dx))^{7/2}}{7a^7d}+
\end{aligned}$$

Mathematica [A] time = 0.852603, size = 114, normalized size = 0.78

$$\frac{2\sec^9(c+dx)(-1144i\sin(2(c+dx))-1027i\sin(4(c+dx))+2552\cos(2(c+dx))+1283\cos(4(c+dx))+1584)(\cos(}{3465a^3d(\tan(c+dx)-i)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*Sec[c + d*x]^9*(1584 + 2552*Cos[2*(c + d*x)] + 1283*Cos[4*(c + d*x)] - (
1144*I)*Sin[2*(c + d*x)] - (1027*I)*Sin[4*(c + d*x)])*(Cos[5*(c + d*x)] + I
*Sin[5*(c + d*x)]))/(3465*a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c +
d*x]])
```

Maple [A] time = 0.317, size = 117, normalized size = 0.8

$$-\frac{4096\,i\,(\cos(dx+c))^5-4096\,\sin(dx+c)\,(\cos(dx+c))^4+9752\,i\,(\cos(dx+c))^3+6680\,(\cos(dx+c))^2\sin(dx+c)-30}{3465\,a^4d\,(\cos(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -2/3465/d/a^4*(2048*I*cos(d*x+c)^5-2048*sin(d*x+c)*cos(d*x+c)^4+4876*I*cos(
d*x+c)^3+3340*cos(d*x+c)^2*sin(d*x+c)-1505*I*cos(d*x+c)-315*sin(d*x+c))*(a*
(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5
```

Maxima [A] time = 0.993697, size = 127, normalized size = 0.87

$$-\frac{2i\left(315\,(i\,a\tan(dx+c)+a)^{\frac{11}{2}}-3080\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a+11880\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^2-22176\,(i\,a\tan(dx+c)\right.}{3465\,a^9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3465*I*(315*(I*a*tan(d*x + c) + a)^(11/2) - 3080*(I*a*tan(d*x + c) + a)^
(9/2)*a + 11880*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 22176*(I*a*tan(d*x + c)
+ a)^(5/2)*a^3 + 18480*(I*a*tan(d*x + c) + a)^(3/2)*a^4)/(a^9*d)
```

Fricas [A] time = 2.15303, size = 513, normalized size = 3.51

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-8192i\,e^{(10i\,dx+10i\,c)}-45056i\,e^{(8i\,dx+8i\,c)}-101376i\,e^{(6i\,dx+6i\,c)}-118272i\,e^{(4i\,dx+4i\,c)}-73920i\,e^{(2i\,dx+2i\,c)}\right)e^{(i\,dx}}{3465\left(a^4de^{(10i\,dx+10i\,c)}+5\,a^4de^{(8i\,dx+8i\,c)}+10\,a^4de^{(6i\,dx+6i\,c)}+10\,a^4de^{(4i\,dx+4i\,c)}+5\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3465*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8192*I*e^(10*I*d*x + 10*
I*c) - 45056*I*e^(8*I*d*x + 8*I*c) - 101376*I*e^(6*I*d*x + 6*I*c) - 118272*
I*e^(4*I*d*x + 4*I*c) - 73920*I*e^(2*I*d*x + 2*I*c))*e^(I*d*x + I*c)/(a^4*d
*e^(10*I*d*x + 10*I*c) + 5*a^4*d*e^(8*I*d*x + 8*I*c) + 10*a^4*d*e^(6*I*d*x
+ 6*I*c) + 10*a^4*d*e^(4*I*d*x + 4*I*c) + 5*a^4*d*e^(2*I*d*x + 2*I*c) + a^4
```

```
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{10}}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^10/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.379 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=113

$$\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^7d}-\frac{12i(a+ia\tan(c+dx))^{5/2}}{5a^6d}+\frac{8i(a+ia\tan(c+dx))^{3/2}}{a^5d}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{a^4d}$$

```
[Out] ((-16*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d) + ((8*I)*(a + I*a*Tan[c + d*x]
)^(3/2))/(a^5*d) - (((12*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^6*d) + (((2
*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^7*d)
```

Rubi [A] time = 0.0863583, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^7d}-\frac{12i(a+ia\tan(c+dx))^{5/2}}{5a^6d}+\frac{8i(a+ia\tan(c+dx))^{3/2}}{a^5d}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-16*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d) + ((8*I)*(a + I*a*Tan[c + d*x]
)^(3/2))/(a^5*d) - (((12*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^6*d) + (((2
*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^7*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{(a-x)^3}{\sqrt{a+x}}\,dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(\frac{8a^3}{\sqrt{a+x}}-12a^2\sqrt{a+x}+6a(a+x)^{3/2}-(a+x)^{5/2}\right)dx,x,ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{16i\sqrt{a+ia\tan(c+dx)}}{a^4d}+\frac{8i(a+ia\tan(c+dx))^{3/2}}{a^5d}-\frac{12i(a+ia\tan(c+dx))^{5/2}}{5a^6d}+\frac{2i(a+\cdots}{}\end{aligned}$$

Mathematica [A] time = 0.440104, size = 110, normalized size = 0.97

$$\frac{2\sec^7(c+dx)(-i(14\sin(c+dx)+19\sin(3(c+dx)))+126\cos(c+dx)+51\cos(3(c+dx)))(\cos(4(c+dx))+i\sin(4(c+\cdots}{35a^3d(\tan(c+dx)-i)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*Sec[c + d*x]^7*(126*Cos[c + d*x] + 51*Cos[3*(c + d*x)] - I*(14*Sin[c + d
*x] + 19*Sin[3*(c + d*x)]))*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]))/(35*a^
3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.282, size = 90, normalized size = 0.8

$$-\frac{408\,i\,(\cos(dx+c))^3+152\,(\cos(dx+c))^2\sin(dx+c)-54\,i\cos(dx+c)-10\,\sin(dx+c)}{35\,a^4d\,(\cos(dx+c))^3}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(d}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -2/35/d/a^4*(204*I*cos(d*x+c)^3+76*cos(d*x+c)^2*sin(d*x+c)-27*I*cos(d*x+c)-
5*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3
```

Maxima [A] time = 0.980628, size = 103, normalized size = 0.91

$$\frac{2i\left(5\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}-42\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a+140\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^2-280\,\sqrt{i\,a\tan(dx+c)+a}a^3\right)}{35\,a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/35*I*(5*(I*a*tan(d*x + c) + a)^(7/2) - 42*(I*a*tan(d*x + c) + a)^(5/2)*a
+ 140*(I*a*tan(d*x + c) + a)^(3/2)*a^2 - 280*sqrt(I*a*tan(d*x + c) + a)*a^3
)/(a^7*d)
```

Fricas [A] time = 2.15842, size = 343, normalized size = 3.04

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-256i\,e^{(6i\,dx+6i\,c)}-896i\,e^{(4i\,dx+4i\,c)}-1120i\,e^{(2i\,dx+2i\,c)}-560i\right)e^{(i\,dx+i\,c)}}{35\left(a^4de^{(6i\,dx+6i\,c)}+3\,a^4de^{(4i\,dx+4i\,c)}+3\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-256*I*e^(6*I*d*x + 6*I*c)
- 896*I*e^(4*I*d*x + 4*I*c) - 1120*I*e^(2*I*d*x + 2*I*c) - 560*I)*e^(I*d*x
+ I*c)/(a^4*d*e^(6*I*d*x + 6*I*c) + 3*a^4*d*e^(4*I*d*x + 4*I*c) + 3*a^4*d*e
^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^8}{(i\,a\tan (dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.380 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^5d}+\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^4d}+\frac{8i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (8*I)/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) + ((8*I)*Sqrt[a + I*a*Tan[c + d*x]
])/(a^4*d) - (((2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^5*d)
```

Rubi [A] time = 0.0795285, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^5d}+\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^4d}+\frac{8i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (8*I)/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) + ((8*I)*Sqrt[a + I*a*Tan[c + d*x]
])/(a^4*d) - (((2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^5*d)
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= -\frac{i\,\mathrm{Subst}\left(\int \frac{(a-x)^2}{(a+x)^{3/2}}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{i\,\mathrm{Subst}\left(\int \left(\frac{4a^2}{(a+x)^{3/2}}-\frac{4a}{\sqrt{a+x}}+\sqrt{a+x}\right)dx,x,ia\tan(c+dx)\right)}{a^5d}\\
&= \frac{8i}{a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^4d}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^5d}
\end{aligned}$$

Mathematica [A] time = 0.295884, size = 61, normalized size = 0.73

$$\frac{2i\sec^2(c+dx)(5i\sin(2(c+dx))+11\cos(2(c+dx))+12)}{3a^3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((2*I)/3)*Sec[c + d*x]^2*(12 + 11*Cos[2*(c + d*x)] + (5*I)*Sin[2*(c + d*x)
]))/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.28, size = 88, normalized size = 1.1

$$\frac{24\, i\left(\cos\left(dx+c\right)\right)^{3}+24\,\left(\cos\left(dx+c\right)\right)^{2}\sin\left(dx+c\right)+22\, i\cos\left(dx+c\right)+2\,\sin\left(dx+c\right)}{3\, a^{4} d\cos\left(dx+c\right)}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 2/3/d/a^4*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(12*I*cos(d*x+c)^3
+12*cos(d*x+c)^2*sin(d*x+c)+11*I*cos(d*x+c)+sin(d*x+c))/cos(d*x+c)
```

Maxima [A] time = 0.978093, size = 84, normalized size = 1.

$$\frac{2i\left(\frac{12}{\sqrt{i\, a\tan\left(dx+c\right)+a}a^{2}}-\frac{\left(i\, a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}-12\,\sqrt{i\, a\tan\left(dx+c\right)+a}a}{a^{4}}\right)}{3\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*I*(12/(sqrt(I*a*tan(d*x + c) + a)*a^2) - ((I*a*tan(d*x + c) + a)^(3/2)
- 12*sqrt(I*a*tan(d*x + c) + a)*a)/a^4)/(a*d)
```

Fricas [A] time = 2.16177, size = 243, normalized size = 2.89

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{\left(2i\, dx+2i\, c\right)}+1}}\left(32i\, e^{\left(4i\, dx+4i\, c\right)}+48i\, e^{\left(2i\, dx+2i\, c\right)}+12i\right)e^{\left(i\, dx+i\, c\right)}}{3\left(a^{4} de^{\left(4i\, dx+4i\, c\right)}+a^{4} de^{\left(2i\, dx+2i\, c\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(32*I*e^(4*I*d*x + 4*I*c) + 4
8*I*e^(2*I*d*x + 2*I*c) + 12*I)*e^(I*d*x + I*c)/(a^4*d*e^(4*I*d*x + 4*I*c)
+ a^4*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^6}{(i a \tan (dx+c)+a)^{\frac{7}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.381 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=57

$$\frac{4i}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{2i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] ((4*I)/3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) - (2*I)/(a^3*d*Sqrt[a + I*a*
Tan[c + d*x]])
```

Rubi [A] time = 0.0736954, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 43}

$$\frac{4i}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{2i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((4*I)/3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) - (2*I)/(a^3*d*Sqrt[a + I*a*
Tan[c + d*x]])
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{a-x}{(a+x)^{5/2}}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(\frac{2a}{(a+x)^{5/2}}-\frac{1}{(a+x)^{3/2}}\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= \frac{4i}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{2i}{a^3d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.205322, size = 80, normalized size = 1.4

$$\frac{2(1+3i\tan(c+dx))\sec^2(c+dx)(\cos(2(c+dx))+i\sin(2(c+dx)))}{3a^3d(\tan(c+dx)-i)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*Sec[c + d*x]^2*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*(1 + (3*I)*Tan[c
+ d*x]))/(3*a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.271, size = 88, normalized size = 1.5

$$\frac{2\cos(dx+c)\left(4i(\cos(dx+c))^3+4(\cos(dx+c))^2\sin(dx+c)-5i\cos(dx+c)-3\sin(dx+c)\right)}{3a^4d}\sqrt{\frac{a(i\sin(dx+c)\cdots}{\cos(dx\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 2/3/d/a^4*cos(d*x+c)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(4*I*co
s(d*x+c)^3+4*cos(d*x+c)^2*sin(d*x+c)-5*I*cos(d*x+c)-3*sin(d*x+c))
```

Maxima [A] time = 0.977343, size = 43, normalized size = 0.75

$$-\frac{2i(3ia\tan(dx+c)+a)}{3(ia\tan(dx+c)+a)^{\frac{3}{2}}a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3*I*(3*I*a*tan(d*x + c) + a)/((I*a*tan(d*x + c) + a)^(3/2)*a^3*d)
```

Fricas [A] time = 2.04964, size = 177, normalized size = 3.11

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(-2ie^{(4idx+4ic)}-ie^{(2idx+2ic)}+i\right)e^{(-3idx-3ic)}}{3a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(4*I*d*x + 4*I*c) - I
*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^4}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.382 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] ((2*I)/5)/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] time = 0.0641361, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3487, 32}

$$\frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((2*I)/5)/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= -\frac{i\,\text{Subst}\left(\int \frac{1}{(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.252618, size = 39, normalized size = 1.34

$$-\frac{2\sqrt{a+ia\tan(c+dx)}}{5a^4d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-2*Sqrt[a + I*a*Tan[c + d*x]])/(5*a^4*d*(-I + Tan[c + d*x])^3)
```

Maple [A] time = 0.034, size = 24, normalized size = 0.8

$$\frac{\frac{2i}{5}}{ad}(a + ia\tan(dx + c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 2/5*I/a/d/(a+I*a*tan(d*x+c))^(5/2)
```

Maxima [A] time = 0.974485, size = 28, normalized size = 0.97

$$\frac{2i}{5(ia\tan(dx + c) + a)^{\frac{5}{2}}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*I/((I*a*tan(d*x + c) + a)^(5/2)*a*d)
```

Fricas [B] time = 2.15347, size = 212, normalized size = 7.31

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(6i\,dx+6i\,c)} + 3i\,e^{(4i\,dx+4i\,c)} + 3i\,e^{(2i\,dx+2i\,c)} + i\right)e^{(-5i\,dx-5i\,c)}}{20\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(6*I*d*x + 6*I*c) + 3*I
*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a
^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(ia\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.383 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}+\frac{11i}{96a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{11i}{64a^3d\sqrt{a+ia\tan(c+dx)}}-\frac{11i\tanh^{-1}(}{64}$$

```
[Out] (((-11*I)/64)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[
2]*a^(7/2)*d) + (((11*I)/36)*a)/(d*(a + I*a*Tan[c + d*x])^(9/2)) - ((I/2)*a
^2)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + ((11*I)/56)/(
d*(a + I*a*Tan[c + d*x])^(7/2)) + ((11*I)/80)/(a*d*(a + I*a*Tan[c + d*x])^(
5/2)) + ((11*I)/96)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((11*I)/64)/(a^3
*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.142192, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}+\frac{11i}{96a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{11i}{64a^3d\sqrt{a+ia\tan(c+dx)}}-\frac{11i\tanh^{-1}(}{64}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```


```
[Out] (((-11*I)/64)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[
2]*a^(7/2)*d) + (((11*I)/36)*a)/(d*(a + I*a*Tan[c + d*x])^(9/2)) - ((I/2)*a
^2)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + ((11*I)/56)/(
d*(a + I*a*Tan[c + d*x])^(7/2)) + ((11*I)/80)/(a*d*(a + I*a*Tan[c + d*x])^(
5/2)) + ((11*I)/96)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((11*I)/64)/(a^3
*d*Sqrt[a + I*a*Tan[c + d*x]])
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\left(ia^3\right)\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} - \frac{\left(11ia^2\right)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{11/2}}\,dx, x, ia\right.}{4d}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} - \frac{(11ia)\,\text{Sul}}{}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{}{56d(a+i}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{}{56d(a+i}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{}{56d(a+i}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{}{56d(a+i}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{}{56d(a+i}$$

$$= -\frac{11i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a-}$$

Mathematica [A] time = 1.51676, size = 176, normalized size = 0.76

$$-\frac{ie^{-11i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\sec^3(c+dx)\left(\sqrt{1+e^{2i(c+dx)}}\left(-460e^{2i(c+dx)}-1338e^{4i(c+dx)}-2416e^{6i(c+dx)}-4618e^{8i(c+dx)}+\right.\right.}{161280a^3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I/161280)*(1 + E^((2*I)*(c + d*x)))^(5/2)*(Sqrt[1 + E^((2*I)*(c + d*x))]
*(-70 - 460*E^((2*I)*(c + d*x)) - 1338*E^((4*I)*(c + d*x)) - 2416*E^((6*I)*
(c + d*x)) - 4618*E^((8*I)*(c + d*x)) + 315*E^((10*I)*(c + d*x))) + 3465*E^
((9*I)*(c + d*x))*ArcSinh[E^(I*(c + d*x))])*Sec[c + d*x]^3)/(a^3*d*E^((11*I
)*(c + d*x))*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.365, size = 422, normalized size = 1.8

$$\frac{1}{80640\,a^4 d}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(71680\,i\,(\cos(dx+c))^{10}+71680\,\sin(dx+c)\,(\cos(dx+c))^{9}-43520\,i\,(\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 1/80640/d/a^4*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(71680*I*cos(d
*x+c)^10+71680*sin(d*x+c)*cos(d*x+c)^9-43520*I*cos(d*x+c)^8-7680*sin(d*x+c)
*cos(d*x+c)^7+512*I*cos(d*x+c)^6+5632*cos(d*x+c)^5*sin(d*x+c)+3465*I*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c
))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*2^(1/2)+1056
*I*cos(d*x+c)^4+3465*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctan(
1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+
c)+1))^(1/2))+3465*2^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/s
in(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*sin(d*x+c)+7392*cos(d*x+c)^3*sin(d*x+c)+4620*I*cos(d*x+c)^2+13860
*cos(d*x+c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.74869, size = 1010, normalized size = 4.33

$$\left(-3465i\sqrt{\frac{1}{2}}a^4 d\sqrt{\frac{1}{a^7d^2}}e^{(10i\,dx+10i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}a^4 d\sqrt{\frac{1}{a^7d^2}}e^{(2i\,dx+2i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+34\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/40320*(-3465*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(10*I*d*x + 10*I*c)*lo
g((2*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(2*I*d*x + 2*I*c) + sqrt(2)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-
I*d*x - I*c)) + 3465*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(10*I*d*x + 10*I
*c)*log(-(2*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(2*I*d*x + 2*I*c) - sqrt(2)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c
))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-315*I*e^
(12*I*d*x + 12*I*c) + 4303*I*e^(10*I*d*x + 10*I*c) + 7034*I*e^(8*I*d*x + 8*
I*c) + 3754*I*e^(6*I*d*x + 6*I*c) + 1798*I*e^(4*I*d*x + 4*I*c) + 530*I*e^(2
*I*d*x + 2*I*c) + 70*I)*e^(I*d*x + I*c))*e^(-10*I*d*x - 10*I*c)/(a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.384 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=306

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}-\frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}+\frac{195ia^2}{352d(a+ia\tan(c+$$

```
[Out] (((-195*I)/1024)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*a^(7/2)*d) + (((195*I)/352)*a^2)/(d*(a + I*a*Tan[c + d*x])^(11/2)) -
((I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(11/2)) - ((
(15*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(11/2)) +
(((65*I)/192)*a)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + ((195*I)/896)/(d*(a + I
*a*Tan[c + d*x])^(7/2)) + ((39*I)/256)/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) +
 ((65*I)/512)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((195*I)/1024)/(a^3*d*
Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.176843, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3487, 51, 63, 206}

$$-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}-\frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}+\frac{195ia^2}{352d(a+ia\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```


```
[Out] (((-195*I)/1024)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*a^(7/2)*d) + (((195*I)/352)*a^2)/(d*(a + I*a*Tan[c + d*x])^(11/2)) -
((I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(11/2)) - ((
(15*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(11/2)) +
(((65*I)/192)*a)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + ((195*I)/896)/(d*(a + I
*a*Tan[c + d*x])^(7/2)) + ((39*I)/256)/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) +
 ((65*I)/512)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((195*I)/1024)/(a^3*d*
Sqrt[a + I*a*Tan[c + d*x]])
```


Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\left(ia^5\right)\operatorname{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{13/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{\left(15ia^4\right)\operatorname{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{13/2}}\,dx, x\right.}{8d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{}{16d(a}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{}{16d(a}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{}{16d(a}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{}{16d(a}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{}{16d(a}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{}{16d(a}$$

$$= \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{}{16d(a}$$

$$= -\frac{195i\tanh^{-1}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}a^{7/2}d} + \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{}{4d(a-ia\tan(c+dx))}$$

Mathematica [A] time = 2.14763, size = 202, normalized size = 0.66

$$-\frac{ie^{-13i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\sec^3(c+dx)\left(\sqrt{1+e^{2i(c+dx)}}\left(-1456e^{2i(c+dx)}-5728e^{4i(c+dx)}-13824e^{6i(c+dx)}-24688e^{8i(c+dx}\right.\right.}{1892352a^3d\sqrt{a+ia\tan(c+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I/1892352)*(1 + E^((2*I)*(c + d*x)))^(5/2)*(Sqrt[1 + E^((2*I)*(c + d*x))
]*(-168 - 1456*E^((2*I)*(c + d*x)) - 5728*E^((4*I)*(c + d*x)) - 13824*E^((6
*I)*(c + d*x)) - 24688*E^((8*I)*(c + d*x)) - 54112*E^((10*I)*(c + d*x)) + 6
699*E^((12*I)*(c + d*x)) + 462*E^((14*I)*(c + d*x))) + 45045*E^((11*I)*(c +
 d*x))*ArcSinh[E^(I*(c + d*x))])*Sec[c + d*x]^3)/(a^3*d*E^((13*I)*(c + d*x)
)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.706, size = 449, normalized size = 1.5

$$\frac{1}{946176\,a^4d}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(688128\,i\,(\cos(dx+c))^{12}+688128\,\sin(dx+c)\,(\cos(dx+c))^{11}-401408\,i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 1/946176/d/a^4*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(688128*I*cos
(d*x+c)^12+688128*sin(d*x+c)*cos(d*x+c)^11-401408*I*cos(d*x+c)^10-57344*sin
(d*x+c)*cos(d*x+c)^9+4096*I*cos(d*x+c)^8+61440*sin(d*x+c)*cos(d*x+c)^7+6656
*I*cos(d*x+c)^6+73216*cos(d*x+c)^5*sin(d*x+c)+45045*I*2^(1/2)*cos(d*x+c)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d
*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+13728*I*cos(d*x+c)^
4+45045*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*2^(1/2)*arctan(1/2*2^(1/2)*(
I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
+45045*2^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)/(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*si
n(d*x+c)+96096*cos(d*x+c)^3*sin(d*x+c)+60060*I*cos(d*x+c)^2+180180*cos(d*x+
c)*sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.85438, size = 1106, normalized size = 3.61

$$\left(-45045i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(12i\,dx+12i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(2i\,dx+2i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/473088*(-45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(12*I*d*x + 12*I*c)*
log((2*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(2*I*d*x + 2*I*c) + sqrt(2)*sqrt
```

```
(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^
(-I*d*x - I*c)) + 45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(12*I*d*x + 1
2*I*c)*log(-(2*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(2*I*d*x + 2*I*c) - sqrt
(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x +
I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-462*I
*e^(16*I*d*x + 16*I*c) - 7161*I*e^(14*I*d*x + 14*I*c) + 47413*I*e^(12*I*d*x
 + 12*I*c) + 78800*I*e^(10*I*d*x + 10*I*c) + 38512*I*e^(8*I*d*x + 8*I*c) +
19552*I*e^(6*I*d*x + 6*I*c) + 7184*I*e^(4*I*d*x + 4*I*c) + 1624*I*e^(2*I*d*
x + 2*I*c) + 168*I)*e^(I*d*x + I*c))*e^(-12*I*d*x - 12*I*c)/(a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.385 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}} dx$

Optimal. Leaf size=110

$$\frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}}+\frac{64ia^3\sec^{13}(c+dx)}{3315d(a+ia\tan(c+dx))^{13/2}}+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

```
[Out] (((64*I)/3315)*a^3*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (((
16*I)/255)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*I)
/17)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Rubi [A] time = 0.194414, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}}+\frac{64ia^3\sec^{13}(c+dx)}{3315d(a+ia\tan(c+dx))^{13/2}}+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((64*I)/3315)*a^3*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (((
16*I)/255)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*I)
/17)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}dx &= \frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}+\frac{1}{17}(8a)\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{9/2}}dx\\ &= \frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}+\frac{1}{255}\left(32a^2\right)\int \frac{\sec^{13}(c+}{(a+ia\tan(c+}\\ &= \frac{64ia^3\sec^{13}(c+dx)}{3315d(a+ia\tan(c+dx))^{13/2}}+\frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.659712, size = 92, normalized size = 0.84

$$-\frac{2\sec^{12}(c+dx)(247i\sin(2(c+dx))+263\cos(2(c+dx))+68)(\cos(3(c+dx))-i\sin(3(c+dx)))}{3315a^3d(\tan(c+dx)-i)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-2*Sec[c + d*x]^12*(68 + 263*Cos[2*(c + d*x)] + (247*I)*Sin[2*(c + d*x)])*
(Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]))/(3315*a^3*d*(-I + Tan[c + d*x])^3*
Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.629, size = 171, normalized size = 1.6

$$\frac{4096\, i\, (\cos (dx+c))^9 + 4096\, \sin (dx+c) (\cos (dx+c))^8 - 512\, i\, (\cos (dx+c))^7 + 1536\, (\cos (dx+c))^6 \sin (dx+c) - 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 2/3315/d/a^4*(2048*I*cos(d*x+c)^9+2048*sin(d*x+c)*cos(d*x+c)^8-256*I*cos(d*
x+c)^7+768*cos(d*x+c)^6*sin(d*x+c)-80*I*cos(d*x+c)^5+560*sin(d*x+c)*cos(d*x
+c)^4-2252*I*cos(d*x+c)^3-1748*cos(d*x+c)^2*sin(d*x+c)+871*I*cos(d*x+c)+195
*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^8
```

Maxima [B] time = 2.55558, size = 1218, normalized size = 11.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3315*(-331*I*sqrt(a) - 998*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 183
8*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 7522*sqrt(a)*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 - 4836*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 - 27882*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8954*I*sqrt(a)*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 68926*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 - 12631*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 125052*sqr
t(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 10540*I*sqrt(a)*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 168980*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^1
1 - 168980*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 10540*I*sqrt(a)*
sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 125052*sqrt(a)*sin(d*x + c)^15/(cos
(d*x + c) + 1)^15 + 12631*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 -
 68926*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 8954*I*sqrt(a)*sin(d
*x + c)^18/(cos(d*x + c) + 1)^18 - 27882*sqrt(a)*sin(d*x + c)^19/(cos(d*x +
 c) + 1)^19 + 4836*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20 - 7522*s
qrt(a)*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 + 1838*I*sqrt(a)*sin(d*x + c)^
22/(cos(d*x + c) + 1)^22 - 998*sqrt(a)*sin(d*x + c)^23/(cos(d*x + c) + 1)^2
3 + 331*I*sqrt(a)*sin(d*x + c)^24/(cos(d*x + c) + 1)^24)*(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4
 - 12*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 66*a^4*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 220*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 495*a^4*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 - 792*a^4*sin(d*x + c)^10/(cos(d*x + c) +
1)^10 + 924*a^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 792*a^4*sin(d*x + c
)^14/(cos(d*x + c) + 1)^14 + 495*a^4*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
- 220*a^4*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 66*a^4*sin(d*x + c)^20/(c
```

```
os(d*x + c) + 1)^20 - 12*a^4*sin(d*x + c)^22/(cos(d*x + c) + 1)^22 + a^4*si
n(d*x + c)^24/(cos(d*x + c) + 1)^24)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2))
```

Fricas [B] time = 2.34912, size = 555, normalized size = 5.05

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(130560i\,e^{(4i\,dx+4i\,c)}+34816i\,e^{(2i\,dx+2i\,c)}+4096i\right)e^{(i\,dx+i\,c)}}{3315\left(a^4de^{(17i\,dx+17i\,c)}+8\,a^4de^{(15i\,dx+15i\,c)}+28\,a^4de^{(13i\,dx+13i\,c)}+56\,a^4de^{(11i\,dx+11i\,c)}+70\,a^4de^{(9i\,dx+9i\,c)}+56\,a^4de^{(7i\,dx+7i\,c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(130560*I*e^(4*I*d*x + 4*I
*c) + 34816*I*e^(2*I*d*x + 2*I*c) + 4096*I)*e^(I*d*x + I*c)/(a^4*d*e^(17*I*
d*x + 17*I*c) + 8*a^4*d*e^(15*I*d*x + 15*I*c) + 28*a^4*d*e^(13*I*d*x + 13*I
*c) + 56*a^4*d*e^(11*I*d*x + 11*I*c) + 70*a^4*d*e^(9*I*d*x + 9*I*c) + 56*a^
4*d*e^(7*I*d*x + 7*I*c) + 28*a^4*d*e^(5*I*d*x + 5*I*c) + 8*a^4*d*e^(3*I*d*x
 + 3*I*c) + a^4*d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{13}}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^13/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.386 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=73

$$\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

```
[Out] (((8*I)/143)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*
I)/13)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Rubi [A] time = 0.127292, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3494, 3493}

$$\frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(7/2),x]
```


```
[Out] (((8*I)/143)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*
I)/13)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```


Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= \frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}+\frac{1}{13}(4a)\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{9/2}}\,dx\\ &= \frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\end{aligned}$$

Mathematica [A] time = 0.418194, size = 82, normalized size = 1.12

$$-\frac{2i(11\tan(c+dx)-15i)\sec^9(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))}{143a^3d(\tan(c+dx)-i)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/143)*Sec[c + d*x]^9*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-15*I
 + 11*Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [B] time = 0.37, size = 144, normalized size = 2.

$$\frac{256\,i\,(\cos(dx+c))^7 + 256\,(\cos(dx+c))^6\sin(dx+c) - 32\,i\,(\cos(dx+c))^5 + 96\,\sin(dx+c)\,(\cos(dx+c))^4 - 296\,i\,(\cos(}{143\,a^4 d\,(\cos(dx+c))^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 2/143/d/a^4*(128*I*cos(d*x+c)^7+128*cos(d*x+c)^6*sin(d*x+c)-16*I*cos(d*x+c)
^5+48*sin(d*x+c)*cos(d*x+c)^4-148*I*cos(d*x+c)^3-108*cos(d*x+c)^2*sin(d*x+c
)+51*I*cos(d*x+c)+11*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(
1/2)/cos(d*x+c)^6
```

Maxima [B] time = 2.1739, size = 1031, normalized size = 14.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/143*(-15*I*sqrt(a) - 38*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 88*I*s
qrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 278*sqrt(a)*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 - 213*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 920
*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 272*I*sqrt(a)*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 - 1848*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
182*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2548*sqrt(a)*sin(d*x +
c)^9/(cos(d*x + c) + 1)^9 - 2548*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11 + 182*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 1848*sqrt(a)*si
n(d*x + c)^13/(cos(d*x + c) + 1)^13 + 272*I*sqrt(a)*sin(d*x + c)^14/(cos(d*
x + c) + 1)^14 - 920*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 213*I*
sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 278*sqrt(a)*sin(d*x + c)^17
/(cos(d*x + c) + 1)^17 + 88*I*sqrt(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18
 - 38*sqrt(a)*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 + 15*I*sqrt(a)*sin(d*x
+ c)^20/(cos(d*x + c) + 1)^20)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4 - 10*a^4*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 45*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 120*a^4*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 210*a^4*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 - 252*a^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 210*a^4*sin(d*x +
c)^12/(cos(d*x + c) + 1)^12 - 120*a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14
 + 45*a^4*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 10*a^4*sin(d*x + c)^18/(c
os(d*x + c) + 1)^18 + a^4*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*d*(-2*I*si
n(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7
/2))
```

Fricas [B] time = 2.08415, size = 420, normalized size = 5.75

$$\frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(1664i\,e^{(2i\,dx+2i\,c)}+256i\right)e^{(i\,dx+i\,c)}}{143\left(a^4de^{(13i\,dx+13i\,c)}+6\,a^4de^{(11i\,dx+11i\,c)}+15\,a^4de^{(9i\,dx+9i\,c)}+20\,a^4de^{(7i\,dx+7i\,c)}+15\,a^4de^{(5i\,dx+5i\,c)}+6\,a^4de^{(3i\,dx+3i\,c)}+a^4d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/143*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(1664*I*e^(2*I*d*x + 2*I*c)
 + 256*I)*e^(I*d*x + I*c)/(a^4*d*e^(13*I*d*x + 13*I*c) + 6*a^4*d*e^(11*I*d*
x + 11*I*c) + 15*a^4*d*e^(9*I*d*x + 9*I*c) + 20*a^4*d*e^(7*I*d*x + 7*I*c) +
 15*a^4*d*e^(5*I*d*x + 5*I*c) + 6*a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*
x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec{(dx+c)}^{11}}{(i\,a\tan{(dx+c)}+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.387 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=35

$$\frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

```
[Out] (((2*I)/9)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Rubi [A] time = 0.0621879, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3493}

$$\frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((2*I)/9)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

Mathematica [A] time = 0.291017, size = 59, normalized size = 1.69

$$\frac{2i(\tan(c+dx)+i)\sec^7(c+dx)}{9a^3d(\tan(c+dx)-i)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((2*I)/9)*Sec[c + d*x]^7*(I + Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^3*
Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.303, size = 115, normalized size = 3.3

$$\frac{32\,i(\cos(dx+c))^5+32\,\sin(dx+c)(\cos(dx+c))^4-40\,i(\cos(dx+c))^3-24\,(\cos(dx+c))^2\sin(dx+c)+10\,i\cos(dx+}{9\,a^4d(\cos(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 2/9/d/a^4*(16*I*cos(d*x+c)^5+16*sin(d*x+c)*cos(d*x+c)^4-20*I*cos(d*x+c)^3-1
2*cos(d*x+c)^2*sin(d*x+c)+5*I*cos(d*x+c)+sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d
*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4
```

Maxima [B] time = 1.92944, size = 845, normalized size = 24.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/9*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*I*sqrt(a)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 14*sqrt(a)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 14*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 42*sqrt(a)*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*I*sqrt(a)*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 - 70*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 70*sqrt(a)*si
n(d*x + c)^9/(cos(d*x + c) + 1)^9 + 14*I*sqrt(a)*sin(d*x + c)^10/(cos(d*x +
 c) + 1)^10 - 42*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 14*I*sqrt(
a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 14*sqrt(a)*sin(d*x + c)^13/(cos(
d*x + c) + 1)^13 + 6*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 2*sq
rt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + I*sqrt(a)*sin(d*x + c)^16/(co
s(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x +
c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4 - 8*a^4*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 28*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 56*a^4*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 70*a^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 56*a^
4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^4*sin(d*x + c)^12/(cos(d*x +
 c) + 1)^12 - 8*a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a^4*sin(d*x + c
)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2))
```

Fricas [B] time = 2.09952, size = 281, normalized size = 8.03

$$\frac{32i\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{(idx+ic)}}{9\left(a^4de^{(9idx+9ic)}+4a^4de^{(7idx+7ic)}+6a^4de^{(5idx+5ic)}+4a^4de^{(3idx+3ic)}+a^4de^{(idx+ic)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 32/9*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(a^4*d*e^(
9*I*d*x + 9*I*c) + 4*a^4*d*e^(7*I*d*x + 7*I*c) + 6*a^4*d*e^(5*I*d*x + 5*I*c
) + 4*a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^9}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.388 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=160

$$-\frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{8i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] ((8*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(7/2)*d) - (((2*I)/5)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*
x])^(5/2)) - (((4*I)/3)*Sec[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)
) - ((8*I)*Sec[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.251263, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3491, 3489, 206}

$$-\frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{8i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(7/2),x]
```


```
[Out] ((8*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(7/2)*d) - (((2*I)/5)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*
x])^(5/2)) - (((4*I)/3)*Sec[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)
) - ((8*I)*Sec[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]])
```


Rule 3491

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e
+ f*x])^(n + 1))/(b*f*(m - 2)), x] + Dist[(2*d^2)/a, Int[(d*Sec[e + f*x])^(
m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} + \frac{2\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx}{a}$$

$$= -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} - \frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{4\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx}{a^2}$$

$$= -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} - \frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}} + \frac{8}{}$$

$$= -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} - \frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}} + \frac{(1}{}$$

$$= \frac{8i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} - \frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} - \frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/}}$$

Mathematica [A] time = 1.11821, size = 130, normalized size = 0.81

$$-\frac{128e^{7i(c+dx)}\left(-35e^{2i(c+dx)}-15e^{4i(c+dx)}+15\left(1+e^{2i(c+dx)}\right)^{5/2}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)-23\right)}{15a^3d\left(1+e^{2i(c+dx)}\right)^6(\tan(c+dx)-i)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-128*E^((7*I)*(c + d*x))*(-23 - 35*E^((2*I)*(c + d*x)) - 15*E^((4*I)*(c +
d*x)) + 15*(1 + E^((2*I)*(c + d*x)))^(5/2)*ArcTanh[Sqrt[1 + E^((2*I)*(c + d
*x))]]))/(15*a^3*d*(1 + E^((2*I)*(c + d*x)))^6*(-I + Tan[c + d*x])^3*Sqrt[a
 + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.302, size = 399, normalized size = 2.5

$$\frac{2}{15\,a^4d\,(i\sin(dx+c)+\cos(dx+c)-1)\,(\cos(dx+c))^2}\left(-15\sqrt{2}\sin(dx+c)\,(\cos(dx+c))^2\arctan\left(1/2\,\frac{\sqrt{2}\,(i\cos(dx+c)}{\sin(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 2/15/d/a^4*(-15*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*2^(1/2)*(I*cos(d
*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-2*co
s(d*x+c)/(cos(d*x+c)+1))^(5/2)-30*2^(1/2)*sin(d*x+c)*cos(d*x+c)*arctan(1/2*
2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)-15*2^(1/2)*arctan(1/2*2^(1/2
)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(5/2)*sin(d*x+c)+92*I*cos(d*x+c)^3-76*I*
cos(d*x+c)^2+92*cos(d*x+c)^2*sin(d*x+c)-19*I*cos(d*x+c)-16*cos(d*x+c)*sin(d
*x+c)+3*I-3*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*s
in(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)^2
```

Maxima [B] time = 2.35029, size = 1574, normalized size = 9.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/15*((60*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 1) - 60*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sq
rt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) - 1) - (30*I*sqrt(2)*cos(2*d*x + 2*c)^2 + 30*I*sqrt(2)*sin(2*d*x + 2*c)^
2 + 60*I*sqrt(2)*cos(2*d*x + 2*c) + 30*I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) - (-30*I*sqrt(2)*cos(2*d*x + 2*c)^2 - 30*I*sqrt(2)*sin(2*d*x + 2*c)^2
- 60*I*sqrt(2)*cos(2*d*x + 2*c) - 30*I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*sqrt(a) - ((-120*I*sqrt(2)*cos(4*d*x + 4*c) - 280*I*sqrt(2)*cos(2*d*x + 2
*c) + 120*sqrt(2)*sin(4*d*x + 4*c) + 280*sqrt(2)*sin(2*d*x + 2*c) - 184*I*s
qrt(2))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (120*sqr
t(2)*cos(4*d*x + 4*c) + 280*sqrt(2)*cos(2*d*x + 2*c) + 120*I*sqrt(2)*sin(4*
d*x + 4*c) + 280*I*sqrt(2)*sin(2*d*x + 2*c) + 184*sqrt(2))*sin(5/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))/((a^4*cos(2*d*x + 2*c)^2
 + a^4*sin(2*d*x + 2*c)^2 + 2*a^4*cos(2*d*x + 2*c) + a^4)*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*d)
```

Fricas [B] time = 2.25258, size = 1052, normalized size = 6.58

$$\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-120i\,e^{(4i\,dx+4i\,c)}-280i\,e^{(2i\,dx+2i\,c)}-184i\right)e^{(i\,dx+i\,c)}+\sqrt{2}\left(60i\,a^4de^{(5i\,dx+5i\,c)}+120i\,a^4de^{(3i\,dx+3i\,c)}+60i\,a^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-120*I*e^(4*I*d*x + 4*I*c)
 - 280*I*e^(2*I*d*x + 2*I*c) - 184*I)*e^(I*d*x + I*c) + sqrt(2)*(60*I*a^4*d
*e^(5*I*d*x + 5*I*c) + 120*I*a^4*d*e^(3*I*d*x + 3*I*c) + 60*I*a^4*d*e^(I*d*
x + I*c))*sqrt(1/(a^7*d^2))*log((sqrt(2)*a^4*d*sqrt(1/(a^7*d^2))*e^(I*d*x +
 I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)
```

```
*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*(-60*I*a^4*d*e^(5*I*d*x + 5*I
*c) - 120*I*a^4*d*e^(3*I*d*x + 3*I*c) - 60*I*a^4*d*e^(I*d*x + I*c))*sqrt(1/
(a^7*d^2))*log(-(sqrt(2)*a^4*d*sqrt(1/(a^7*d^2))*e^(I*d*x + I*c) - sqrt(2)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c)
)*e^(-I*d*x - I*c)))/(a^4*d*e^(5*I*d*x + 5*I*c) + 2*a^4*d*e^(3*I*d*x + 3*I*
c) + a^4*d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^7}{(i\,a\tan (dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.389 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=121

$$\frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{3i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] ((-3*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
+ d*x]])])/(a^(7/2)*d) - ((2*I)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])
^(5/2)) + ((6*I)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.21665, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3501, 3502, 3489, 206}

$$\frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{3i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d}-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-3*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
+ d*x]])])/(a^(7/2)*d) - ((2*I)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])
^(5/2)) + ((6*I)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}} + \frac{6\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{a}$$

$$= -\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}} + \frac{12i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{12\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{a^2}$$

$$= -\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}} + \frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{3\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a^3}$$

$$= -\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}} + \frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{(6i)\,\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec}{\sqrt{a+ia}}\right.}{a^3d}$$

$$= -\frac{3i\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} - \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}} + \frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] time = 0.766839, size = 126, normalized size = 1.04

$$\frac{16e^{5i(c+dx)}\left(-3e^{2i(c+dx)}+3e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)-1\right)}{a^3d\left(1+e^{2i(c+dx)}\right)^4(\tan(c+dx)-i)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (16*E^((5*I)*(c + d*x))*(-1 - 3*E^((2*I)*(c + d*x)) + 3*E^((2*I)*(c + d*x))
*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(a^
3*d*(1 + E^((2*I)*(c + d*x)))^4*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c +
d*x]])
```

Maple [B] time = 0.279, size = 318, normalized size = 2.6

$$-\frac{1}{2a^4d}\left(3i\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}(i\cos(dx+c)-i+\sin(dx+c))}{2\sin(dx+c)}\frac{1}{\sqrt{-2\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{2}\cos(dx+c)+3i\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -1/2/d/a^4*(3*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*
cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2
^(1/2)*cos(d*x+c)+3*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/
2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*2^(1/2)+3*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*c
os(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^
(1/2)*sin(d*x+c)-8*I*cos(d*x+c)^3-8*cos(d*x+c)^2*sin(d*x+c)-4*sin(d*x+c))*(
a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.11668, size = 756, normalized size = 6.25

$$\left(-3i\sqrt{2}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(3i\,dx+3i\,c)}\log\left(\left(\sqrt{2}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(i\,dx+i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)+3i\sqrt{2}a^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-3*I*sqrt(2)*a^4*d*sqrt(1/(a^7*d^2))*e^(3*I*d*x + 3*I*c)*log((sqrt(2)*
a^4*d*sqrt(1/(a^7*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 3*I
*sqrt(2)*a^4*d*sqrt(1/(a^7*d^2))*e^(3*I*d*x + 3*I*c)*log(-(sqrt(2)*a^4*d*sq
rt(1/(a^7*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*(6*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(I*d*x + I
*c))*e^(-3*I*d*x - 3*I*c)/(a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^5}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.390 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}a^{7/2}d}+\frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] ((-I/8)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])
])/(Sqrt[2]*a^(7/2)*d) + ((I/2)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(
5/2)) - ((I/8)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.187784, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3501, 3502, 3489, 206}

$$-\frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}a^{7/2}d}+\frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```


```
[Out] ((-I/8)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])
])/(Sqrt[2]*a^(7/2)*d) + ((I/2)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(
5/2)) - ((I/8)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```


Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}} - \frac{2\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{3a}$$
$$= \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}} - \frac{\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{4a^2}$$
$$= \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}} - \frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{16a^3}$$
$$= \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}} - \frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{i\,\mathrm{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\mathrm{se}}{\sqrt{a+i}}\right.}{8a^3d}$$
$$= -\frac{i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}a^{7/2}d} + \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}} - \frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3}}$$

Mathematica [A] time = 0.881641, size = 120, normalized size = 0.96

$$\frac{i\sec^3(c+dx)\left(i\sin(2(c+dx)) - 3\cos(2(c+dx)) + e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right) - 3\right)}{16a^3d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((I/16)*Sec[c + d*x]^3*(-3 + E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x
))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 3*Cos[2*(c + d*x)] + I*Sin[2*(
c + d*x)]))/(a^3*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.313, size = 345, normalized size = 2.8

$$-\frac{1}{32\,a^4d}\left(-64\,i\,(\cos(dx+c))^5 + i\cos(dx+c)\sqrt{2}\sqrt{-2\,\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\frac{\sqrt{2}\,(i\cos(dx+c) - i + \sin(dx+c))}{2\,\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] -1/32/d/a^4*(-64*I*cos(d*x+c)^5+I*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-64*sin(d*x+c)*cos(d*x+c)^4+I*2^(1/2)*(-
2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d
*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+(-2*cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)
/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)+56*I*cos(d*x+c)^3
+24*cos(d*x+c)^2*sin(d*x+c)-4*I*cos(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/co
s(d*x+c))^(1/2)
```

Maxima [B] time = 2.05719, size = 1318, normalized size = 10.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/64*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3
/4)*((4*I*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*sin(4*d*x + 4*c))*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*(sqrt(2)*cos(4*d*x + 4*c
) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*sqrt(a) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*((4*I*sqrt(2)*cos(4*d*x + 4*c) + 4*sqrt(2)*sin(4*d*x
 + 4*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 4*(sqrt
(2)*cos(4*d*x + 4*c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + (2*sqrt(2)*arctan2((cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sq
rt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + 1))*sqrt(a))/(a^4*d)
```

Fricas [B] time = 2.08509, size = 801, normalized size = 6.41

$$\left(-i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(5idx+5ic)}\log\left(\left(2\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(idx+ic)}+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)+i\sqrt{\frac{1}{2}}a^4d\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/16*(-I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(5*I*d*x + 5*I*c)*log((2*sqrt(
1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
+ I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(5*I*d*x + 5*I*c)*log(-(2*sqrt(1/2)
*a^4*d*sqrt(1/(a^7*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sq
rt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I
*d*x + 2*I*c) + 2*I)*e^(I*d*x + I*c))*e^(-5*I*d*x - 5*I*c)/(a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.391 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=157

$$\frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{5i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

```
[Out] (((5*I)/64)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*a^(7/2)*d) + ((I/6)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])
^(7/2)) + (((5*I)/48)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((
(5*I)/64)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.164806, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3502, 3489, 206}

$$\frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{5i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}}+\frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```


```
[Out] (((5*I)/64)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*a^(7/2)*d) + ((I/6)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])
^(7/2)) + (((5*I)/48)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((
(5*I)/64)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx}{12a}$$

$$= \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}} + \frac{5\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx}{32a^2}$$

$$= \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}} + \frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}}$$

$$= \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}} + \frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}}$$

$$= \frac{5i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A] time = 1.36328, size = 119, normalized size = 0.76

$$-\frac{\sec^3(c+dx)\left(50i\sin(2(c+dx))+82\cos(2(c+dx))+\frac{30e^{4i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+52\right)}{384a^3d(\tan(c+dx)-i)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] -(Sec[c + d*x]^3*(52 + (30*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c
 + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))] + 82*Cos[2*(c + d*x)] + (50*I)*Si
n[2*(c + d*x)]))/(384*a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [B] time = 0.236, size = 373, normalized size = 2.4

$$\frac{1}{768\,a^4d}\left(1024\,i\,(\cos(dx+c))^7+1024\,(\cos(dx+c))^6\sin(dx+c)-704\,i\,(\cos(dx+c))^5+15\,i\cos(dx+c)\sqrt{2}\sqrt{-2\,\frac{}{\mathrm{co}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 1/768/d/a^4*(1024*I*cos(d*x+c)^7+1024*cos(d*x+c)^6*sin(d*x+c)-704*I*cos(d*x
+c)^5+15*I*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1
/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))-192*sin(d*x+c)*cos(d*x+c)^4+15*I*2^(1/2)*(-2*cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(
-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+8*I*cos(d*x+c)^3+15*(-2*cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c
)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)+40*cos(d*x+c)^2*
sin(d*x+c)-60*I*cos(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(ia\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Fricas [B] time = 1.99066, size = 851, normalized size = 5.42

$$\left(15i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(7idx+7ic)}\log\left(\left(2\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(idx+ic)}+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-15i\sqrt{\frac{1}{2}}a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(15*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(7*I*d*x + 7*I*c)*log((2*sq
rt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c
)) - 15*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(7*I*d*x + 7*I*c)*log(-(2*sqr
t(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(33*I*e^(6*I*d*x + 6*I*c) + 5
9*I*e^(4*I*d*x + 4*I*c) + 34*I*e^(2*I*d*x + 2*I*c) + 8*I)*e^(I*d*x + I*c))*
e^(-7*I*d*x - 7*I*c)/(a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(ia\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.392 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=227

$$-\frac{315i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2048a^4d}+\frac{105i\cos(c+dx)}{1024a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{315i\tanh^{-1}}{204}$$

```
[Out] (((315*I)/2048)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
+ d*x]])])/(Sqrt[2]*a^(7/2)*d) + ((I/8)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d
*x])^(7/2)) + (((3*I)/32)*Cos[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
+ (((21*I)/256)*Cos[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((105
*I)/1024)*Cos[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((315*I)/2048
)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d)
```

Rubi [A] time = 0.375353, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3502, 3490, 3489, 206}

$$-\frac{315i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2048a^4d}+\frac{105i\cos(c+dx)}{1024a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{315i\tanh^{-1}}{204}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```


```
[Out] (((315*I)/2048)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
+ d*x]])])/(Sqrt[2]*a^(7/2)*d) + ((I/8)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d
*x])^(7/2)) + (((3*I)/32)*Cos[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
+ (((21*I)/256)*Cos[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((105
*I)/1024)*Cos[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((315*I)/2048
)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d)
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{9\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{16a}$$

$$= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{21\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{64a^2}$$

$$= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}$$

$$= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}$$

$$= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}$$

$$= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}$$

$$= \frac{315i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2048\sqrt{2}a^{7/2}d} + \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A] time = 1.9443, size = 141, normalized size = 0.62

$$-\frac{\sec^3(c+dx)\left(474i\sin(2(c+dx)) - 288i\sin(4(c+dx)) + 826\cos(2(c+dx)) - 224\cos(4(c+dx)) + \frac{630e^{4i(c+dx)}\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right.}{4096a^3d(\tan(c+dx)-i)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] -(Sec[c + d*x]^3*(420 + (630*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*
(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))] + 826*Cos[2*(c + d*x)] - 224*Co
s[4*(c + d*x)] + (474*I)*Sin[2*(c + d*x)] - (288*I)*Sin[4*(c + d*x)]))/(409
6*a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.303, size = 400, normalized size = 1.8

$$\frac{1}{8192\,a^4d}\sqrt{\frac{a\left(i\sin(dx+c)+\cos(dx+c)\right)}{\cos(dx+c)}}\left(8192\,i\left(\cos(dx+c)\right)^9 + 8192\,\sin(dx+c)\left(\cos(dx+c)\right)^8 - 5120\,i\left(\cos(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 1/8192/d/a^4*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(8192*I*cos(d*x
+c)^9+8192*sin(d*x+c)*cos(d*x+c)^8-5120*I*cos(d*x+c)^7-1024*cos(d*x+c)^6*si
n(d*x+c)+64*I*cos(d*x+c)^5+315*I*cos(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2
*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+576*sin(d*x+c)*cos(d*x+c)^4+315*I*2^(1/2
)*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+s
in(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+168*I*cos(d*x+c
)^3+315*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+
c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*s
in(d*x+c)+840*cos(d*x+c)^2*sin(d*x+c)-1260*I*cos(d*x+c))
```

Maxima [B] time = 2.25113, size = 3754, normalized size = 16.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/16384*((cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*
arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*
x + 8*c), cos(8*d*x + 8*c))) + 1)^(3/4)*(((1300*I*sqrt(2)*cos(8*d*x + 8*c)
+ 1300*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*
x + 8*c)))^2 + (1300*I*sqrt(2)*cos(8*d*x + 8*c) + 1300*sqrt(2)*sin(8*d*x +
8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (2600*I*sqrt
(2)*cos(8*d*x + 8*c) + 2600*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8
*d*x + 8*c), cos(8*d*x + 8*c))) + 1300*I*sqrt(2)*cos(8*d*x + 8*c) + 1300*sq
rt(2)*sin(8*d*x + 8*c))*cos(7/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), c
os(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1
)) + (2572*I*sqrt(2)*cos(8*d*x + 8*c) + 2572*sqrt(2)*sin(8*d*x + 8*c))*cos(
3/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*a
rctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) - (1300*(sqrt(2)*cos(8*d*
x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c)))^2 + 1300*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x +
 8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2600*(sqrt(
2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x
 + 8*c), cos(8*d*x + 8*c))) + 1300*sqrt(2)*cos(8*d*x + 8*c) - 1300*I*sqrt(2
)*sin(8*d*x + 8*c))*sin(7/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8
*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) -
 2572*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(3/2*arcta
n2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(si
n(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)))*sqrt(a) + (cos(1/4*arctan2(sin(8*
d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*
d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1
)^(1/4)*(((-3060*I*sqrt(2)*cos(8*d*x + 8*c) - 3060*sqrt(2)*sin(8*d*x + 8*c)
)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (-3060*I*sqrt(2)
*cos(8*d*x + 8*c) - 3060*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*
x + 8*c), cos(8*d*x + 8*c)))^2 + (-6120*I*sqrt(2)*cos(8*d*x + 8*c) - 6120*s
qrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)
)) - 3060*I*sqrt(2)*cos(8*d*x + 8*c) - 3060*sqrt(2)*sin(8*d*x + 8*c))*cos(5
/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*ar
ctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (-748*I*sqrt(2)*cos(8*d*
x + 8*c) - 748*sqrt(2)*sin(8*d*x + 8*c) - 512*I*sqrt(2))*cos(1/2*arctan2(si
n(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d
*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (3060*(sqrt(2)*cos(8*d*x + 8*c) - I*s
qrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)
))^2 + 3060*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4
```

```
*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 6120*(sqrt(2)*cos(8*d*x +
 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8
*d*x + 8*c))) + 3060*sqrt(2)*cos(8*d*x + 8*c) - 3060*I*sqrt(2)*sin(8*d*x +
8*c))*sin(5/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))),
 cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 4*(187*sqrt(2
)*cos(8*d*x + 8*c) - 187*I*sqrt(2)*sin(8*d*x + 8*c) + 128*sqrt(2))*sin(1/2*
arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arcta
n2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)))*sqrt(a) - (630*sqrt(2)*arcta
n2((cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan
2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*
c), cos(8*d*x + 8*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x
 + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
8*c))) + 1)), (cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin
(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/4*arctan
2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), c
os(8*d*x + 8*c))) + 1)) + 1) - 630*sqrt(2)*arctan2((cos(1/4*arctan2(sin(8*d
*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d
*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)
^(1/4)*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
, cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)), (cos(1/4*arct
an2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*
c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
8*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d
*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) - 1
) - 315*I*sqrt(2)*log(sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*
c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4
*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)*cos(1/2*arctan2(sin(1/4*
arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8
*c), cos(8*d*x + 8*c))) + 1))^2 + sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2
 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)*sin(1/2*arct
an2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(s
in(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1))^2 + 2*(cos(1/4*arctan2(sin(8*d*x
+ 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x
+ 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1
/4)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), c
os(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 1) + 315*I*sqrt
(2)*log(sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1
/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8
*d*x + 8*c), cos(8*d*x + 8*c))) + 1)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*
d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x
 + 8*c))) + 1))^2 + sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)
))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*a
rctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)*sin(1/2*arctan2(sin(1/4*ar
ctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c
), cos(8*d*x + 8*c))) + 1))^2 - 2*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*
d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2
*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2
(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 1))*sqrt(a))/(a^4*d)
```

Fricas [A] time = 2.12115, size = 940, normalized size = 4.14

$$\left(315i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(9idx+9ic)}\log\left(\left(2\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(idx+ic)}+\sqrt{2}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{(idx+ic)}\right)e^{(-idx-ic)}\right)-315i\sqrt{\frac{1}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4096*(315*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(9*I*d*x + 9*I*c)*log((2*
sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)) - 315*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(9*I*d*x + 9*I*c)*log(-(2*
sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d*x - I
*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-128*I*e^(10*I*d*x + 10*I
*c) + 197*I*e^(8*I*d*x + 8*I*c) + 535*I*e^(6*I*d*x + 6*I*c) + 298*I*e^(4*I*
d*x + 4*I*c) + 104*I*e^(2*I*d*x + 2*I*c) + 16*I)*e^(I*d*x + I*c))*e^(-9*I*d
*x - 9*I*c)/(a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.393 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d}+\frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{3003i\cos(c+$$

```
[Out] (((3003*I)/16384)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[
c + d*x]])])/(Sqrt[2]*a^(7/2)*d) + ((I/10)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[
c + d*x])^(7/2)) + (((13*I)/160)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x]
)^(5/2)) + (((143*I)/1920)*Cos[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3
/2)) + (((1001*I)/8192)*Cos[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) +
(((429*I)/5120)*Cos[c + d*x]^3)/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((300
3*I)/16384)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d) - (((1001*I)/1
0240)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d)
```

Rubi [A] time = 0.520107, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3502, 3497, 3490, 3489, 206}

$$-\frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d}+\frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{3003i\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```


```
[Out] (((3003*I)/16384)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[
c + d*x]])])/(Sqrt[2]*a^(7/2)*d) + ((I/10)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[
c + d*x])^(7/2)) + (((13*I)/160)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x]
)^(5/2)) + (((143*I)/1920)*Cos[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3
/2)) + (((1001*I)/8192)*Cos[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) +
(((429*I)/5120)*Cos[c + d*x]^3)/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((300
3*I)/16384)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d) - (((1001*I)/1
0240)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d)
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3490

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3489

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[(-2*a)/(b*f), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{20a}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{320a^2}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+d}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+d}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+d}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+d}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+d}$$

$$= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+d}$$

$$= \frac{3003i\tanh^{-1}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16384\sqrt{2}a^{7/2}d} + \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+}$$

Mathematica [A] time = 2.15792, size = 175, normalized size = 0.57

$$-\frac{\sec^3(c+dx)\left(20048e^{-2i(c+dx)} + 71190e^{2i(c+dx)} + 5856e^{-4i(c+dx)} - 48640e^{4i(c+dx)} + 768e^{-6i(c+dx)} - 2560e^{6i(c+dx)} + \frac{90090e}{\cdots}\right.}{491520a^3d(\tan(c+dx)-i)^3\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] -((42140 + 20048/E^((2*I)*(c + d*x)) + 71190*E^((2*I)*(c + d*x)) + 5856/E^(
(4*I)*(c + d*x)) - 48640*E^((4*I)*(c + d*x)) + 768/E^((6*I)*(c + d*x)) - 25
60*E^((6*I)*(c + d*x)) + (90090*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*
I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(491520*a^3
*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.452, size = 427, normalized size = 1.4

$$\frac{1}{983040\,a^4d}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}\left(786432\,i\left(\cos\left(dx+c\right)\right)^{11}+786432\,\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{10}-466944\,i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 1/983040/d/a^4*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(786432*I*cos
(d*x+c)^11+786432*sin(d*x+c)*cos(d*x+c)^10-466944*I*cos(d*x+c)^9-73728*sin(
d*x+c)*cos(d*x+c)^8+5120*I*cos(d*x+c)^7+66560*cos(d*x+c)^6*sin(d*x+c)+9152*
I*cos(d*x+c)^5+45045*I*(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1
/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*2^(1/2)*cos(d*x+c)+82368*sin(d*x+c)*cos(d*x+c)^4+45045*I*(-2*cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin
(d*x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)+45045*(-2*cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*arctan(1/2*2^(1/2)*(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*
x+c)/(-2*cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*2^(1/2)*sin(d*x+c)+24024*I*cos(d
*x+c)^3+120120*cos(d*x+c)^2*sin(d*x+c)-180180*I*cos(d*x+c))
```

Maxima [B] time = 2.80765, size = 7834, normalized size = 25.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/1966080*((cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + si
n(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan
2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)^(3/4)*(((3160*I*sqrt(2)*cos
(10*d*x + 10*c) + 3160*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c)))^2 + (3160*I*sqrt(2)*cos(10*d*x + 10*c) + 3
160*sqrt(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*
d*x + 10*c)))^2 + (33480*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(
10*d*x + 10*c)))^2 + 33480*I*sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), co
s(10*d*x + 10*c)))^2 + 66960*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c),
cos(10*d*x + 10*c))) + 33480*I*sqrt(2))*cos(4/5*arctan2(sin(10*d*x + 10*c),
 cos(10*d*x + 10*c))) + (6320*I*sqrt(2)*cos(10*d*x + 10*c) + 6320*sqrt(2)*s
in(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))
 + 33480*(sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^
2 + sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*
sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2))
*sin(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 3160*I*sqrt(2)*
cos(10*d*x + 10*c) + 3160*sqrt(2)*sin(10*d*x + 10*c))*cos(7/2*arctan2(sin(1
```

/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10
*d*x + 10*c), cos(10*d*x + 10*c))) + 1)) + (1960*I*sqrt(2)*cos(10*d*x + 10*
c) + 46200*I*sqrt(2)*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)
)) - 130560*I*sqrt(2)*cos(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c
))) + 24960*I*sqrt(2)*cos(2/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c
))) + 1960*sqrt(2)*sin(10*d*x + 10*c) + 46200*sqrt(2)*sin(4/5*arctan2(sin(1
0*d*x + 10*c), cos(10*d*x + 10*c))) - 130560*sqrt(2)*sin(3/5*arctan2(sin(10
*d*x + 10*c), cos(10*d*x + 10*c))) + 24960*sqrt(2)*sin(2/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c))) - 5120*I*sqrt(2))*cos(3/2*arctan2(sin(1/5*
arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*
x + 10*c), cos(10*d*x + 10*c))) + 1)) - (3160*(sqrt(2)*cos(10*d*x + 10*c) -
I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d
*x + 10*c)))^2 + 3160*(sqrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x +
10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 33480*(
sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sqrt(2
)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*sqrt(2)*co
s(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2))*cos(4/5*a
rctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 6320*(sqrt(2)*cos(10*d*x
+ 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c),
cos(10*d*x + 10*c))) - (33480*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c)
, cos(10*d*x + 10*c)))^2 + 33480*I*sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*
c), cos(10*d*x + 10*c)))^2 + 66960*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 1
0*c), cos(10*d*x + 10*c))) + 33480*I*sqrt(2))*sin(4/5*arctan2(sin(10*d*x +
10*c), cos(10*d*x + 10*c))) + 3160*sqrt(2)*cos(10*d*x + 10*c) - 3160*I*sqrt
(2)*sin(10*d*x + 10*c))*sin(7/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c),
cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*
c))) + 1)) - (1960*sqrt(2)*cos(10*d*x + 10*c) + 46200*sqrt(2)*cos(4/5*arcta
n2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 130560*sqrt(2)*cos(3/5*arctan
2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 24960*sqrt(2)*cos(2/5*arctan2(
sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 1960*I*sqrt(2)*sin(10*d*x + 10*c
) - 46200*I*sqrt(2)*sin(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))
) + 130560*I*sqrt(2)*sin(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)
)) - 24960*I*sqrt(2)*sin(2/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)
)) - 5120*sqrt(2))*sin(3/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(
10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))
+ 1)))*sqrt(a) + (cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^
2 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*
arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)^(1/4)*(((420*I*sqrt(2
)*cos(10*d*x + 10*c) + 420*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(
10*d*x + 10*c), cos(10*d*x + 10*c)))^4 + (420*I*sqrt(2)*cos(10*d*x + 10*c)
+ 420*sqrt(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(1
0*d*x + 10*c)))^4 + (1680*I*sqrt(2)*cos(10*d*x + 10*c) + 1680*sqrt(2)*sin(1
0*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^3 +
(2520*I*sqrt(2)*cos(10*d*x + 10*c) + 2520*sqrt(2)*sin(10*d*x + 10*c))*cos(
1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + ((840*I*sqrt(2)*co
s(10*d*x + 10*c) + 840*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c)))^2 + (1680*I*sqrt(2)*cos(10*d*x + 10*c) + 1
680*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*
d*x + 10*c))) + 840*I*sqrt(2)*cos(10*d*x + 10*c) + 840*sqrt(2)*sin(10*d*x +
10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + (1680*
I*sqrt(2)*cos(10*d*x + 10*c) + 1680*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arc
tan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 420*I*sqrt(2)*cos(10*d*x +
10*c) + 420*sqrt(2)*sin(10*d*x + 10*c))*cos(9/2*arctan2(sin(1/5*arctan2(sin
(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c),
cos(10*d*x + 10*c))) + 1)) + ((-3584*I*sqrt(2)*cos(10*d*x + 10*c) - 3584*sq
rt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
10*c)))^2 + (-3584*I*sqrt(2)*cos(10*d*x + 10*c) - 3584*sqrt(2)*sin(10*d*x +
10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + (-6132
0*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 - 61

320*I*sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 -
122640*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) -
61320*I*sqrt(2))*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))
+ (83520*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))
^2 + 83520*I*sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)
))^2 + 167040*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10
*c))) + 83520*I*sqrt(2))*cos(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 1
0*c))) + (-7168*I*sqrt(2)*cos(10*d*x + 10*c) - 7168*sqrt(2)*sin(10*d*x + 10
*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 61320*(sqrt
(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sqrt(2)*si
n(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*sqrt(2)*cos(1/
5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2))*sin(4/5*arcta
n2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 83520*(sqrt(2)*cos(1/5*arctan
2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sqrt(2)*sin(1/5*arctan2(sin(
10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*sqrt(2)*cos(1/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2))*sin(3/5*arctan2(sin(10*d*x + 10
*c), cos(10*d*x + 10*c))) - 3584*I*sqrt(2)*cos(10*d*x + 10*c) - 3584*sqrt(2
)*sin(10*d*x + 10*c))*cos(5/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), c
os(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)
)) + 1)) + (-420*I*sqrt(2)*cos(10*d*x + 10*c) - 12600*I*sqrt(2)*cos(4/5*arc
tan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 54720*I*sqrt(2)*cos(3/5*arc
tan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 21120*I*sqrt(2)*cos(2/5*arc
tan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 420*sqrt(2)*sin(10*d*x + 10
*c) - 12600*sqrt(2)*sin(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))
) + 54720*sqrt(2)*sin(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))
- 21120*sqrt(2)*sin(2/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) -
92160*I*sqrt(2))*cos(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10
*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) +
1)) - 420*((sqrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(
1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^4 + (sqrt(2)*cos(10*d*
x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c
), cos(10*d*x + 10*c)))^4 + 4*(sqrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(1
0*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^3 +
6*(sqrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arct
an2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*((sqrt(2)*cos(10*d*x + 1
0*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), co
s(10*d*x + 10*c)))^2 + 2*(sqrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x
+ 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2
)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10
*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 4*(sqrt(2)*cos(10*d*x + 10*c) - I*sq
rt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
10*c))) + sqrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*sin(9/
2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5
*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)) + (3584*(sqrt(2)*co
s(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x
+ 10*c), cos(10*d*x + 10*c)))^2 + 3584*(sqrt(2)*cos(10*d*x + 10*c) - I*sqr
t(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 1
0*c)))^2 + 61320*(sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
10*c)))^2 + sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))
)^2 + 2*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) +
sqrt(2))*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 83520*(
sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sqrt(2
)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*sqrt(2)*co
s(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2))*cos(3/5*a
rctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 7168*(sqrt(2)*cos(10*d*x
+ 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c),
cos(10*d*x + 10*c))) + (-61320*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c
), cos(10*d*x + 10*c)))^2 - 61320*I*sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10
*c), cos(10*d*x + 10*c)))^2 - 122640*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x +

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10*c), cos(10*d*x + 10*c))) - 61320*I*sqrt(2))*sin(4/5*arctan2(sin(10*d*x
+ 10*c), cos(10*d*x + 10*c))) + (83520*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x
 + 10*c), cos(10*d*x + 10*c)))^2 + 83520*I*sqrt(2)*sin(1/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c)))^2 + 167040*I*sqrt(2)*cos(1/5*arctan2(sin(1
0*d*x + 10*c), cos(10*d*x + 10*c))) + 83520*I*sqrt(2))*sin(3/5*arctan2(sin(
10*d*x + 10*c), cos(10*d*x + 10*c))) + 3584*sqrt(2)*cos(10*d*x + 10*c) - 35
84*I*sqrt(2)*sin(10*d*x + 10*c))*sin(5/2*arctan2(sin(1/5*arctan2(sin(10*d*x
 + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*
d*x + 10*c))) + 1)) + (420*sqrt(2)*cos(10*d*x + 10*c) + 12600*sqrt(2)*cos(4
/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 54720*sqrt(2)*cos(3/5
*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 21120*sqrt(2)*cos(2/5*a
rctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 420*I*sqrt(2)*sin(10*d*x
+ 10*c) - 12600*I*sqrt(2)*sin(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
10*c))) + 54720*I*sqrt(2)*sin(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
10*c))) - 21120*I*sqrt(2)*sin(2/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
10*c))) + 92160*sqrt(2))*sin(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c)
, cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10
*c))) + 1)))*sqrt(a) - (90090*sqrt(2)*arctan2((cos(1/5*arctan2(sin(10*d*x +
 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10
*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)
)) + 1)^(1/4)*sin(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*
x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1))
, (cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*arc
tan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*
d*x + 10*c), cos(10*d*x + 10*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/5*arctan
2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10
*c), cos(10*d*x + 10*c))) + 1)) + 1) - 90090*sqrt(2)*arctan2((cos(1/5*arcta
n2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*arctan2(sin(10*d*x
+ 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*d*x + 10*c), cos
(10*d*x + 10*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 1
0*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x
+ 10*c))) + 1)), (cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^
2 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*
arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)^(1/4)*cos(1/2*arctan2
(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(
sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)) - 1) - 45045*I*sqrt(2)*log(s
qrt(cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*ar
ctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10
*d*x + 10*c), cos(10*d*x + 10*c))) + 1)*cos(1/2*arctan2(sin(1/5*arctan2(sin
(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c),
cos(10*d*x + 10*c))) + 1))^2 + sqrt(cos(1/5*arctan2(sin(10*d*x + 10*c), cos
(10*d*x + 10*c)))^2 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c
)))^2 + 2*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)*sin
(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(
1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1))^2 + 2*(cos(1/5*a
rctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*arctan2(sin(10*
d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*d*x + 10*c),
 cos(10*d*x + 10*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x
 + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*
d*x + 10*c))) + 1)) + 1) + 45045*I*sqrt(2)*log(sqrt(cos(1/5*arctan2(sin(10*
d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*arctan2(sin(10*d*x + 10*c), c
os(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
10*c))) + 1)*cos(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x
 + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1))^
2 + sqrt(cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1
/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(s
in(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)*sin(1/2*arctan2(sin(1/5*arctan
2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10
*c), cos(10*d*x + 10*c))) + 1))^2 - 2*(cos(1/5*arctan2(sin(10*d*x + 10*c),
```

```
cos(10*d*x + 10*c)))^2 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 1
0*c)))^2 + 2*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)^
(1/4)*cos(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c
))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)) + 1))*s
qrt(a))/(a^4*d)
```

Fricas [A] time = 2.16289, size = 1054, normalized size = 3.43

$$\left(45045i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(11i\,dx+11i\,c)}\log\left(\left(2\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(i\,dx+i\,c)}+\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)-450$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/491520*(45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(11*I*d*x + 11*I*c)*l
og((2*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(I*d*x + I*c) + sqrt(2)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(-I*d
*x - I*c)) - 45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(11*I*d*x + 11*I*c
)*log(-(2*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(I*d*x + I*c) - sqrt(2)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(I*d*x + I*c))*e^(
-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1280*I*e^(14*I
*d*x + 14*I*c) - 25600*I*e^(12*I*d*x + 12*I*c) + 11275*I*e^(10*I*d*x + 10*I
*c) + 56665*I*e^(8*I*d*x + 8*I*c) + 31094*I*e^(6*I*d*x + 6*I*c) + 12952*I*e
^(4*I*d*x + 4*I*c) + 3312*I*e^(2*I*d*x + 2*I*c) + 384*I)*e^(I*d*x + I*c))*e
^(-11*I*d*x - 11*I*c)/(a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(7/2), x)
```

3.394 $\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=524

$$-\frac{ia^{3/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia^{3/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia^{3/2}e^{3/2}\,\mathrm{s}}{}$$

```
[Out] (I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (I*a^(3/2)*e^
(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt
[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + (I*a^(3/2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sq
rt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sq
rt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + ((I/2)*a^(
3/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/S
qrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(S
qrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^
(3/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(
Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.445326, antiderivative size = 524, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3498, 3499, 3495, 297, 1162, 617, 204, 1165, 628}

$$-\frac{ia^{3/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia^{3/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia^{3/2}e^{3/2}\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (I*a^(3/2)*e^
(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt
[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + (I*a^(3/2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sq
rt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sq
rt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + ((I/2)*a^(
3/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/S
qrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(S
qrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^
(3/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(
Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3499

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[(d*Sec[e + f*x])/(Sqrt[a - b*Tan[e + f*x]]*Sqr
t[a + b*Tan[e + f*x]]), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{1}{2}a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{(ae\sec(c+dx))\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+}}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(2ia^2e^3\sec(c+dx)\right)\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia}}{\sqrt{e s}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx}}$$
$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{\left(ia^2e^2\sec(c+dx)\right)\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\,t}}{\sqrt{e\,se}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx}}$$
$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(ia^2e\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a}}{\cdots}\right.}{2d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+}}$$
$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{3/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+\right.}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a}}$$
$$= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+d}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 1.79836, size = 373, normalized size = 0.71

$$\frac{e(\cos(c)-i\sin(c))\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}\left(\sqrt{\sin(c)+i\cos(c)-1}\left(\sqrt{-\sin(c)+i\cos(c)-1}\sqrt{\tan\left(\frac{dx}{2}\right)+i\,c}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (e*Sqrt[e*Sec[c + d*x]]*(Cos[c] - I*Sin[c])*(-(ArcTan[(Sqrt[-1 - I*Cos[c] +
 Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan
[(d*x)/2]])]*Cos[c + d*x]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1 - I*Cos[c] +
 Sin[c]]*Sqrt[I + Tan[(d*x)/2]]) + Sqrt[-1 + I*Cos[c] + Sin[c]]*(Sqrt[-1 -
I*Cos[c] - Sin[c]]*(I*Cos[d*x] + Sin[d*x])*Sqrt[I - Tan[(d*x)/2]] + ArcTan[
(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] -
 Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[-1 + I*Cos[c] - Sin[c]]
*Sqrt[I + Tan[(d*x)/2]]))*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[-1 - I*Cos[c]
 - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A] time = 0.391, size = 309, normalized size = 0.6

$$\frac{\cos(dx+c)(\cos(dx+c)-1)^2}{2d(\sin(dx+c))^3(i\sin(dx+c)+\cos(dx+c)-1)}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(i\text{Artanh}\left(\frac{\cos}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*c
os(d*x+c)*(cos(d*x+c)-1)^2*(I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1+sin(d*x+c)))*cos(d*x+c)-I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1-sin(d*x+c)))*cos(d*x+c)+2*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+arc
tanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+arc
tanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)-2*c
os(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-2*(1/(cos(d*x+c)+1))^(1/2))/sin(d*x+c)^3
/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(cos(d*x+c)+1))^(3/2)
```

Maxima [B] time = 2.30355, size = 2531, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -((16*sqrt(2)*e*cos(2*d*x + 2*c) + 16*I*sqrt(2)*e*sin(2*d*x + 2*c) + 16*sqr
t(2)*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
(16*sqrt(2)*e*cos(2*d*x + 2*c) + 16*I*sqrt(2)*e*sin(2*d*x + 2*c) + 16*sqrt(
2)*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (
16*sqrt(2)*e*cos(2*d*x + 2*c) + 16*I*sqrt(2)*e*sin(2*d*x + 2*c) + 16*sqrt(2
)*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (16
*sqrt(2)*e*cos(2*d*x + 2*c) + 16*I*sqrt(2)*e*sin(2*d*x + 2*c) + 16*sqrt(2)*
e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (16*
I*sqrt(2)*e*cos(2*d*x + 2*c) - 16*sqrt(2)*e*sin(2*d*x + 2*c) + 16*I*sqrt(2)
*e)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1) + (-16*I*sqrt(2)*e*cos(2*d*x + 2*c) + 16*sqrt(2)*e
*sin(2*d*x + 2*c) - 16*I*sqrt(2)*e)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 128*e*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (8*sqrt(2)*e*cos(2*d*x + 2*c
) + 8*I*sqrt(2)*e*sin(2*d*x + 2*c) + 8*sqrt(2)*e)*log(2*sqrt(2)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (8
*sqrt(2)*e*cos(2*d*x + 2*c) + 8*I*sqrt(2)*e*sin(2*d*x + 2*c) + 8*sqrt(2)*e)
*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) + (8*I*sqrt(2)*e*cos(2*d*x + 2*c) - 8*sqrt(2)*e*sin(
```

```
2*d*x + 2*c) + 8*I*sqrt(2)*e)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + (-8*I*sqrt(2)
*e*cos(2*d*x + 2*c) + 8*sqrt(2)*e*sin(2*d*x + 2*c) - 8*I*sqrt(2)*e)*log(2*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2) + (8*I*sqrt(2)*e*cos(2*d*x + 2*c) - 8*sqrt(2)*e*sin(
2*d*x + 2*c) + 8*I*sqrt(2)*e)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
- 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + (-8*I*sqrt(2)
*e*cos(2*d*x + 2*c) + 8*sqrt(2)*e*sin(2*d*x + 2*c) - 8*I*sqrt(2)*e)*log(2*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2) - 128*I*e*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*sqrt(a)*sqrt(e)/(d*(-64*I*cos(2*d*x + 2*c) + 64*sin(2*d*x + 2*c
) - 64*I))
```

Fricas [A] time = 2.21191, size = 1427, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(sqrt(I*a*e^3/d^2)*d*e^(I*d*x + I*c)*log(2*((e*e^(2*I*d*x + 2*I*c) + e
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/
2*I*d*x + 3/2*I*c) + sqrt(I*a*e^3/d^2)*d*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x -
 2*I*c)/e) - sqrt(I*a*e^3/d^2)*d*e^(I*d*x + I*c)*log(2*((e*e^(2*I*d*x + 2*I
*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(3/2*I*d*x + 3/2*I*c) - sqrt(I*a*e^3/d^2)*d*e^(2*I*d*x + 2*I*c))*e^(-2*
I*d*x - 2*I*c)/e) - sqrt(-I*a*e^3/d^2)*d*e^(I*d*x + I*c)*log(2*((e*e^(2*I*d
*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I
*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) + sqrt(-I*a*e^3/d^2)*d*e^(2*I*d*x + 2*I*c
))*e^(-2*I*d*x - 2*I*c)/e) + sqrt(-I*a*e^3/d^2)*d*e^(I*d*x + I*c)*log(2*((e
*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*
d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - sqrt(-I*a*e^3/d^2)*d*e^(2*I*d*
x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/e) - 4*I*e*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-I*d*x - I
*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{3}{2}}\sqrt{ia\tan(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*sqrt(I*a*tan(d*x + c) + a), x)
```

3.395 $\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=323

$$\frac{i\sqrt{2}\sqrt{a}\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d}-\frac{i\sqrt{2}\sqrt{a}\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d}-\frac{i\sqrt{a}\sqrt{e}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c-}}{\sqrt{e\sec(c+dx)}}\right.}{}$$

```
[Out] (I*Sqrt[2]*Sqrt[a]*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/d - (I*Sqrt[2]*Sqrt[a]*Sqrt[e]*Arc
Tan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c
+ d*x]])])/d - (I*Sqrt[a]*Sqrt[e]*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
 I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*
x])])/(Sqrt[2]*d) + (I*Sqrt[a]*Sqrt[e]*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqr
t[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
 + d*x])])/(Sqrt[2]*d)
```

Rubi [A] time = 0.190769, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3495, 297, 1162, 617, 204, 1165, 628}

$$\frac{i\sqrt{2}\sqrt{a}\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d}-\frac{i\sqrt{2}\sqrt{a}\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d}-\frac{i\sqrt{a}\sqrt{e}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c-}}{\sqrt{e\sec(c+dx)}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*Sqrt[a]*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/d - (I*Sqrt[2]*Sqrt[a]*Sqrt[e]*Arc
Tan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c
+ d*x]])])/d - (I*Sqrt[a]*Sqrt[e]*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
 I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*
x])])/(Sqrt[2]*d) + (I*Sqrt[a]*Sqrt[e]*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqr
t[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
 + d*x])])/(Sqrt[2]*d)
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{\left(4iae^2\right)\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d}$$

$$= \frac{(2iae)\,\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{(2iae)\,\text{Subst}\left(\int \frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \ldots\right.}{d}$$

$$= -\frac{(ia)\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{(ia)\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,d\ldots\right.}{d}$$

$$= -\frac{i\sqrt{a}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}d} + \ldots$$

$$= \frac{i\sqrt{2}\sqrt{a}\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{i\sqrt{2}\sqrt{a}\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan\ldots}}{\sqrt{a}\sqrt{e\sec(c+\ldots}}\right.}{d}$$

Mathematica [A] time = 0.930876, size = 289, normalized size = 0.89

$$\frac{2e\sqrt{\tan\left(\frac{dx}{2}\right)+i}\sqrt{a+ia\tan(c+dx)}\left(\sqrt{-\sin(c)+i\cos(c)-1}\sqrt{\sin(c)+i\cos(c)-1}\tan^{-1}\left(\frac{\sqrt{\sin(c)-i\cos(c)-1}\sqrt{-\tan\left(\frac{dx}{2}\right)+i}}{\sqrt{\sin(c)+i\cos(c)-1}\sqrt{\tan\left(\frac{dx}{2}\right)+i}}\right)-\ldots\right.}{d\sqrt{-\sin(c)+i\cos(c)-1}\sqrt{\sin(c)-i\cos(c)-1}\sqrt{-\tan\left(\frac{d\ldots}{2}\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*e*(-(ArcTan[(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[
-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c
]]*Sqrt[-1 - I*Cos[c] + Sin[c]]) + ArcTan[(Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqr
t[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]
*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*Sqrt[I + Tan[(d
*x)/2]]*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[-1 + I*Cos
[c] - Sin[c]]*Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A] time = 0.369, size = 225, normalized size = 0.7

$$\frac{\cos(dx+c)(\cos(dx+c)-1)}{d\sin(dx+c)(i\sin(dx+c)+\cos(dx+c)-1)}\sqrt{\frac{e}{\cos(dx+c)}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(i\mathrm{Artanh}\left(\frac{\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos
(d*x+c)*(cos(d*x+c)-1)*(I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1-sin(d*x+c)))-I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x
+c)))+arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+arcta
nh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))))/sin(d*x+c)/(I*s
in(d*x+c)+cos(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 2.20927, size = 1890, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/4*(-2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt
(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 1) - 2*sqrt(2)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)
 + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + I*sqr
t(2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
```

```
c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt
(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(2
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(
2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + sqrt(2)*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + sqrt(2)*log(2*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*sqrt(a)*sqrt(e)/d
```

Fricas [A] time = 2.24456, size = 1172, normalized size = 3.63

$$-\frac{1}{2}\sqrt{\frac{4iae}{d^2}}\log\left(\left(2\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{e}{e^{(2idx+2ic)}+1}}\left(e^{(2idx+2ic)}+1\right)e^{\left(\frac{3}{2}idx+\frac{3}{2}ic\right)}+id\sqrt{\frac{4iae}{d^2}}e^{(2idx+2ic)}\right)e^{(-2idx-2ic)}\right)+\frac{1}{2}\sqrt{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(4*I*a*e/d^2)*log((2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(
2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(3/2*I*d*x + 3/2*I*c) +
I*d*sqrt(4*I*a*e/d^2)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)) + 1/2*sqrt
(4*I*a*e/d^2)*log((2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x +
 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(3/2*I*d*x + 3/2*I*c) - I*d*sqrt(
4*I*a*e/d^2)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)) - 1/2*sqrt(-4*I*a*e
/d^2)*log((2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
+ 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(3/2*I*d*x + 3/2*I*c) + I*d*sqrt(-4*I*a*e
/d^2)*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)) + 1/2*sqrt(-4*I*a*e/d^2)*l
og((2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
e^(2*I*d*x + 2*I*c) + 1)*e^(3/2*I*d*x + 3/2*I*c) - I*d*sqrt(-4*I*a*e/d^2)*e
^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(i\tan\left(c+dx\right)+1\right)}\sqrt{e\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(I*tan(c + d*x) + 1))*sqrt(e*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\sec\left(dx+c\right)}\sqrt{i\,a\tan\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*sqrt(I*a*tan(d*x + c) + a), x)
```

3.396 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=36

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

```
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.0608407, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3488}

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

Mathematica [A] time = 0.0437845, size = 36, normalized size = 1.

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Maple [A] time = 0.322, size = 56, normalized size = 1.6

$$\frac{-2\,i\cos(dx+c)}{de}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\sqrt{\frac{e}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x)
```

```
[Out] -2*I/d*cos(d*x+c)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x
+c))^(1/2)/e
```

Maxima [B] time = 1.61616, size = 103, normalized size = 2.86

$$-\frac{2i\sqrt{a}\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{d\sqrt{e}\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2*I*sqrt(a)*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 1)/(d*sqrt(e)*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 - 1))
```

Fricas [B] time = 1.86377, size = 176, normalized size = 4.89

$$\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e
^(2*I*d*x + 2*I*c) - I)*e^(1/2*I*d*x + 1/2*I*c)/(d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\,(i\tan{(c+dx)}+1)}}{\sqrt{e\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(I*tan(c + d*x) + 1))/sqrt(e*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i a \tan(dx+c)+a}}{\sqrt{e \sec(dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/sqrt(e*sec(d*x + c)), x)
```

3.397 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] (((4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((
2*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.132141, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3497, 3488}

$$\frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((
2*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx &= -\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}+\frac{(2a)\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3e^2}\\ &= \frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.167859, size = 48, normalized size = 0.59

$$\frac{2(2\tan(c+dx)+i)\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(I + 2*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*(e*Sec[c + d*x])^(
3/2))
```

Maple [A] time = 0.333, size = 75, normalized size = 0.9

$$\frac{\left(2\,i\cos\left(dx+c\right)+4\,\sin\left(dx+c\right)\right)\left(\cos\left(dx+c\right)\right)^{2}}{3\,d e^{3}}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}\left(\frac{e}{\cos\left(dx+c\right)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x)
```

```
[Out] 2/3/d*(I*cos(d*x+c)+2*sin(d*x+c))*cos(d*x+c)^2*(a*(I*sin(d*x+c)+cos(d*x+c))
/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(3/2)/e^3
```

Maxima [A] time = 1.88359, size = 73, normalized size = 0.9

$$\frac{\sqrt{a}\left(-i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3i\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{3\,d e^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*sqrt(a)*(-I*cos(3/2*d*x + 3/2*c) + 3*I*cos(1/2*d*x + 1/2*c) + sin(3/2*d
*x + 3/2*c) + 3*sin(1/2*d*x + 1/2*c))/(d*e^(3/2))
```

Fricas [A] time = 1.93669, size = 220, normalized size = 2.72

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,d e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-I
*e^(4*I*d*x + 4*I*c) + 2*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-1/2*I*d*x - 1/2*I
*c)/(d*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\left(i\tan\left(c+dx\right)+1\right)}}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(I*tan(c + d*x) + 1))/(e*sec(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(3/2), x)
```

3.398 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{8ia}{15de^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] (((8*I)/15)*a)/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((
(2*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) - (((16*I)/
15)*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.196973, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3497, 3502, 3488}

$$\frac{8ia}{15de^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((8*I)/15)*a)/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((
(2*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) - (((16*I)/
15)*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}} + \frac{(4a)\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}dx}{5e^2}$$

$$= \frac{8ia}{15de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}} + \frac{8\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}dx}{15e^2}$$

$$= \frac{8ia}{15de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}} - \frac{16i\sqrt{a+ia\tan(c+}}{15de^2\sqrt{e\sec(c+d}}$$

Mathematica [A] time = 0.185985, size = 63, normalized size = 0.52

$$\frac{i\sqrt{a+ia\tan(c+dx)}(-4i\sin(2(c+dx))+\cos(2(c+dx))-15)}{15de^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((I/15)*(-15 + Cos[2*(c + d*x)] - (4*I)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[
c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [A] time = 0.35, size = 85, normalized size = 0.7

$$\frac{\left(2\,i\left(\cos\left(dx+c\right)\right)^{2}+8\,\cos\left(dx+c\right)\sin\left(dx+c\right)-16\,i\right)\left(\cos\left(dx+c\right)\right)^{3}}{15\,de^{5}}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}\left(\frac{e}{\cos\left(dx+c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x)
```

```
[Out] 2/15/d*(I*cos(d*x+c)^2+4*cos(d*x+c)*sin(d*x+c)-8*I)*cos(d*x+c)^3*(a*(I*sin(
d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(5/2)/e^5
```

Maxima [A] time = 1.94218, size = 176, normalized size = 1.44

$$\sqrt{a}\left(5i\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3i\cos\left(\frac{5}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)-30i\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/30*sqrt(a)*(5*I*cos(3/2*d*x + 3/2*c) - 3*I*cos(5/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 30*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 5*sin(3/2*d*x + 3/2*c) + 3*sin(5/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 30*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))))/(d*e^(5/2))
```

Fricas [A] time = 2.0929, size = 262, normalized size = 2.15

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(6i\,dx+6i\,c)}-33i\,e^{(4i\,dx+4i\,c)}-25i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{30\,de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/30*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-
3*I*e^(6*I*d*x + 6*I*c) - 33*I*e^(4*I*d*x + 4*I*c) - 25*I*e^(2*I*d*x + 2*I*
c) + 5*I)*e^(-3/2*I*d*x - 3/2*I*c)/(d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(5/2), x)
```

3.399 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=164

$$\frac{32ia\sqrt{e\sec(c+dx)}}{35de^4\sqrt{a+ia\tan(c+dx)}}+\frac{12ia}{35de^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{35de^2(e\sec(c+dx))^{3/2}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] (((12*I)/35)*a)/(d*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) +
 (((32*I)/35)*a*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) -
(((2*I)/7)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(7/2)) - (((16*I
)/35)*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.282532, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3497, 3502, 3488}

$$\frac{32ia\sqrt{e\sec(c+dx)}}{35de^4\sqrt{a+ia\tan(c+dx)}}+\frac{12ia}{35de^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{35de^2(e\sec(c+dx))^{3/2}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((12*I)/35)*a)/(d*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) +
 (((32*I)/35)*a*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) -
(((2*I)/7)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(7/2)) - (((16*I
)/35)*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(3/2))
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}} + \frac{(6a)\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx}{7e^2}$$

$$= \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}} + \frac{24\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{35e^2}$$

$$= \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{35de^2(e\sec(c+dx))^{3/2}}$$

$$= \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32ia\sqrt{e\sec(c+dx)}}{35de^4\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

Mathematica [A] time = 0.218766, size = 80, normalized size = 0.49

$$\frac{\sqrt{a+ia\tan(c+dx)}(70\sin(c+dx)+6\sin(3(c+dx))+35i\cos(c+dx)+i\cos(3(c+dx)))}{70de^3\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((35*I)*Cos[c + d*x] + I*Cos[3*(c + d*x)] + 70*Sin[c + d*x] + 6*Sin[3*(c +
 d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(70*d*e^3*Sqrt[e*Sec[c + d*x]])
```

Maple [A] time = 0.378, size = 102, normalized size = 0.6

$$\frac{\left(2i(\cos(dx+c))^3+12(\cos(dx+c))^2\sin(dx+c)+16i\cos(dx+c)+32\sin(dx+c)\right)(\cos(dx+c))^4}{35de^7}\sqrt{\frac{a(i\sin(dx+c)}{\cos(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x)
```

```
[Out] 2/35/d*(I*cos(d*x+c)^3+6*cos(d*x+c)^2*sin(d*x+c)+8*I*cos(d*x+c)+16*sin(d*x+
c))*cos(d*x+c)^4*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+
c))^(7/2)/e^7
```

Maxima [A] time = 1.94961, size = 240, normalized size = 1.46

$$\sqrt{a}\Big(7i\cos\left(\tfrac{5}{2}dx+\tfrac{5}{2}c\right)-5i\cos\left(\tfrac{7}{5}\arctan\left(\sin\left(\tfrac{5}{2}dx+\tfrac{5}{2}c\right),\cos\left(\tfrac{5}{2}dx+\tfrac{5}{2}c\right)\right)\right)-35i\cos\left(\tfrac{3}{5}\arctan\left(\sin\left(\tfrac{5}{2}dx+\tfrac{5}{2}c\right),c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] 1/140*sqrt(a)*(7*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(7/5*arctan2(sin(5/2*d*x +
 5/2*c), cos(5/2*d*x + 5/2*c))) - 35*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c)
```

```
, cos(5/2*d*x + 5/2*c))) + 105*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c))) + 7*sin(5/2*d*x + 5/2*c) + 5*sin(7/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c))) + 35*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c))) + 105*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))))/(d*e^(7/2))
```

Fricas [A] time = 2.09596, size = 301, normalized size = 1.84

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(8i\,dx+8i\,c)}-40i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+112i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{140\,de^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/140*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-5*I*e^(8*I*d*x + 8*I*c) - 40*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I
*c) + 112*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-5/2*I*d*x - 5/2*I*c)/(d*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(7/2), x)
```

3.400 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=453

$$\frac{7ia^{3/2}e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}-\frac{7ia^{3/2}e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}-\frac{7ia^{3/2}e^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right.}{1}$$

```
[Out] (((7*I)/8)*a^(3/2)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((7*I)/8)*a^(3/2)*e
^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqr
t[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((7*I)/16)*a^(3/2)*e^(5/2)*Log[a - (Sqr
t[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos
[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((7*I)/16)*a^(3/2)*e^(5/2
)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c
 + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((7*I)/12)*
a^2*(e*Sec[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((7*I)/8)*a*e
^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((I/3)*a*(e*Sec[c +
 d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.505928, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3498, 3501, 3495, 297, 1162, 617, 204, 1165, 628}

$$\frac{7ia^{3/2}e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}-\frac{7ia^{3/2}e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}-\frac{7ia^{3/2}e^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] (((7*I)/8)*a^(3/2)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((7*I)/8)*a^(3/2)*e
^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqr
t[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((7*I)/16)*a^(3/2)*e^(5/2)*Log[a - (Sqr
t[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos
[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((7*I)/16)*a^(3/2)*e^(5/2
)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c
 + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((7*I)/12)*
a^2*(e*Sec[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((7*I)/8)*a*e
^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((I/3)*a*(e*Sec[c +
 d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
```


Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
```

```
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{1}{6}(7a)\int (e\sec(c+dx))^{5/2}\sqrt{a+}$$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{1}{8}(7$$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} + \frac{ia(e}{}$$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} + \frac{ia(e}{}$$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} + \frac{ia(e}{}$$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} + \frac{ia(e}{}$$

$$= -\frac{7ia^{3/2}e^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right.}{16\sqrt{2}d}$$

$$= \frac{7ia^{3/2}e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d} - \frac{7ia^{3/2}e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia}}{\sqrt{a}\sqrt{e\sec}}\right.}{8\sqrt{2}d}$$

Mathematica [A] time = 3.33488, size = 358, normalized size = 0.79

$$\frac{(\cos(c)-i\sin(c))\cos^4(c+dx)(\cos(dx)-i\sin(dx))(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}\left(\frac{42\sqrt{\sin(c)+i\cos(c)-1}\sqrt{\tan\left(\frac{dx}{2}\right)+i\tan}}{\sqrt{\sin(c)-i\cos(c)-1}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^4*(e*Sec[c + d*x])^(5/2)*(Cos[c] - I*Sin[c])*(Cos[d*x] - I*Si
n[d*x])*(Sec[c + d*x]^3*(9*I - (7*I)*Cos[2*(c + d*x)] + 14*Sin[2*(c + d*x)]
) - (42*ArcTan[(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[
-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c
]]*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)
/2]]) + (42*ArcTan[(Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(S
qrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 + I*Cos[c] + S
in[c]]*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(
d*x)/2]]))*(a + I*a*Tan[c + d*x])^(3/2))/(48*d)
```

Maple [A] time = 0.325, size = 414, normalized size = 0.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 1/48/d*a*(e/cos(d*x+c))^(5/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)*(cos(d*x+c)-1)^3*(21*I*cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)+1-sin(d*x+c)))-21*I*cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-42*I*cos(d*x+c)^2*sin(d*x+c)*(1/(cos(d*x+
c)+1))^(1/2)+21*cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)+1-sin(d*x+c)))+21*cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1+sin(d*x+c)))-28*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-
42*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)+16*I*sin(d*x+c)*(1/(cos(d*x+c)+1))
^(1/2)-14*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)+44*cos(d*x+c)*(1/(cos(d*x+c
)+1))^(1/2)+16*(1/(cos(d*x+c)+1))^(1/2))/sin(d*x+c)^5/(I*sin(d*x+c)+cos(d*x
+c)-1)/(1/(cos(d*x+c)+1))^(5/2)
```

Maxima [B] time = 2.98898, size = 4070, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -(64512*a*e^2*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 55296
*a*e^2*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 21504*a*e^2*c
os(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 64512*I*a*e^2*sin(11/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 55296*I*a*e^2*sin(7/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 21504*I*a*e^2*sin(3/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + (8064*sqrt(2)*a*e^2*cos(6*d*x + 6*c) +
 24192*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 24192*sqrt(2)*a*e^2*cos(2*d*x + 2*c
) + 8064*I*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 24192*I*sqrt(2)*a*e^2*sin(4*d*x
 + 4*c) + 24192*I*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + 8064*sqrt(2)*a*e^2)*arct
an2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(
2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (8064*sqrt(2
)*a*e^2*cos(6*d*x + 6*c) + 24192*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 24192*sqr
t(2)*a*e^2*cos(2*d*x + 2*c) + 8064*I*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 24192
*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 24192*I*sqrt(2)*a*e^2*sin(2*d*x + 2*c)
+ 8064*sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 1) + (8064*sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 24192*sqrt(2)*a*e^2*c
os(4*d*x + 4*c) + 24192*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + 8064*I*sqrt(2)*a*e
^2*sin(6*d*x + 6*c) + 24192*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 24192*I*sqrt
(2)*a*e^2*sin(2*d*x + 2*c) + 8064*sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (8064*sqrt(2)*a*e^2*cos(6*d*x + 6*
c) + 24192*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 24192*sqrt(2)*a*e^2*cos(2*d*x +
 2*c) + 8064*I*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 24192*I*sqrt(2)*a*e^2*sin(4
*d*x + 4*c) + 24192*I*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + 8064*sqrt(2)*a*e^2)*
arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -
sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (8064*I
*sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 24192*I*sqrt(2)*a*e^2*cos(4*d*x + 4*c) +
24192*I*sqrt(2)*a*e^2*cos(2*d*x + 2*c) - 8064*sqrt(2)*a*e^2*sin(6*d*x + 6*c
) - 24192*sqrt(2)*a*e^2*sin(4*d*x + 4*c) - 24192*sqrt(2)*a*e^2*sin(2*d*x +
2*c) + 8064*I*sqrt(2)*a*e^2)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (-8064*I*sqrt(2)*a*e^2*
cos(6*d*x + 6*c) - 24192*I*sqrt(2)*a*e^2*cos(4*d*x + 4*c) - 24192*I*sqrt(2)
*a*e^2*cos(2*d*x + 2*c) + 8064*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 24192*sqrt(
```

2)*a*e^2*sin(4*d*x + 4*c) + 24192*sqrt(2)*a*e^2*sin(2*d*x + 2*c) - 8064*I*s
qrt(2)*a*e^2)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (4032*sqrt(2)*a*e^2*cos(6*d*x + 6*c)
 + 12096*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 12096*sqrt(2)*a*e^2*cos(2*d*x + 2
*c) + 4032*I*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 12096*I*sqrt(2)*a*e^2*sin(4*d
*x + 4*c) + 12096*I*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + 4032*sqrt(2)*a*e^2)*lo
g(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) + (4032*sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 12096*sqrt(2)*a
*e^2*cos(4*d*x + 4*c) + 12096*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + 4032*I*sqrt(
2)*a*e^2*sin(6*d*x + 6*c) + 12096*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 12096*
I*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + 4032*sqrt(2)*a*e^2)*log(-2*sqrt(2)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
) - (-4032*I*sqrt(2)*a*e^2*cos(6*d*x + 6*c) - 12096*I*sqrt(2)*a*e^2*cos(4*d
*x + 4*c) - 12096*I*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + 4032*sqrt(2)*a*e^2*sin
(6*d*x + 6*c) + 12096*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 12096*sqrt(2)*a*e^2*
sin(2*d*x + 2*c) - 4032*I*sqrt(2)*a*e^2)*log(2*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (4
032*I*sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 12096*I*sqrt(2)*a*e^2*cos(4*d*x + 4*
c) + 12096*I*sqrt(2)*a*e^2*cos(2*d*x + 2*c) - 4032*sqrt(2)*a*e^2*sin(6*d*x
+ 6*c) - 12096*sqrt(2)*a*e^2*sin(4*d*x + 4*c) - 12096*sqrt(2)*a*e^2*sin(2*d
*x + 2*c) + 4032*I*sqrt(2)*a*e^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (-4032*I*
sqrt(2)*a*e^2*cos(6*d*x + 6*c) - 12096*I*sqrt(2)*a*e^2*cos(4*d*x + 4*c) - 1
2096*I*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + 4032*sqrt(2)*a*e^2*sin(6*d*x + 6*c)
 + 12096*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 12096*sqrt(2)*a*e^2*sin(2*d*x + 2
*c) - 4032*I*sqrt(2)*a*e^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 -
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (4032*I*sqrt(2)
*a*e^2*cos(6*d*x + 6*c) + 12096*I*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 12096*I*
sqrt(2)*a*e^2*cos(2*d*x + 2*c) - 4032*sqrt(2)*a*e^2*sin(6*d*x + 6*c) - 1209
6*sqrt(2)*a*e^2*sin(4*d*x + 4*c) - 12096*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + 4
032*I*sqrt(2)*a*e^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2))*sqrt(a)*sqrt(e)/(d*(-36
864*I*cos(6*d*x + 6*c) - 110592*I*cos(4*d*x + 4*c) - 110592*I*cos(2*d*x + 2
*c) + 36864*sin(6*d*x + 6*c) + 110592*sin(4*d*x + 4*c) + 110592*sin(2*d*x +
 2*c) - 36864*I))

Fricas [B] time = 2.27307, size = 1989, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/12*((-21*I*a*e^2*e^(4*I*d*x + 4*I*c) + 18*I*a*e^2*e^(2*I*d*x + 2*I*c) + 7
*I*a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1
))*e^(3/2*I*d*x + 3/2*I*c) - 6*sqrt(-49/64*I*a^3*e^5/d^2)*(d*e^(4*I*d*x + 4
*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/7*(14*(a*e^2*e^(2*I*d*x + 2*I*c)
 + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1
))*e^(3/2*I*d*x + 3/2*I*c) + 16*I*sqrt(-49/64*I*a^3*e^5/d^2)*d*e^(2*I*d*x +
 2*I*c))*e^(-2*I*d*x - 2*I*c)/(a*e^2)) + 6*sqrt(-49/64*I*a^3*e^5/d^2)*(d*e^
(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(1/7*(14*(a*e^2*e^(2*I*
d*x + 2*I*c) + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - 16*I*sqrt(-49/64*I*a^3*e^5/d^2)*d*
e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(a*e^2)) - 6*sqrt(49/64*I*a^3*e^5
/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e
^2*e^(2*I*d*x + 2*I*c) + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e
^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) + 8*I*sqrt(49/64*I*a^3*e^5
/d^2)*d*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(a*e^2)) + 6*sqrt(49/64*I
*a^3*e^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/7
*(7*(a*e^2*e^(2*I*d*x + 2*I*c) + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - 8*I*sqrt(49/64*I
*a^3*e^5/d^2)*d*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(a*e^2)))/(d*e^(4
*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{5}{2}}(ia\tan(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.401 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=571

$$-\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia^{5/2}e^{3/2}\,\mathrm{s}}{}$$

```
[Out] (((5*I)/4)*a^2*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((
5*I)/4)*a^(5/2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*
x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*
Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/4)*a^(5/2)*e^(3/2)*ArcT
an[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c +
 d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*T
an[c + d*x]]) + (((5*I)/8)*a^(5/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*T
an[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a +
I*a*Tan[c + d*x]]) - (((5*I)/8)*a^(5/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a -
I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + ((I/2)*a*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[
c + d*x]])/d
```

Rubi [A] time = 0.542818, antiderivative size = 571, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3498, 3499, 3495, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia^{5/2}e^{3/2}\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((5*I)/4)*a^2*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((
5*I)/4)*a^(5/2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*
x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*
Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/4)*a^(5/2)*e^(3/2)*ArcT
an[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c +
 d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*T
an[c + d*x]]) + (((5*I)/8)*a^(5/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*T
an[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a +
I*a*Tan[c + d*x]]) - (((5*I)/8)*a^(5/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a -
I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + ((I/2)*a*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[
c + d*x]])/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
```

```
2*m, 2*n]
```

Rule 3499

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[(d*Sec[e + f*x])/(Sqrt[a - b*Tan[e + f*x]]*Sqr
t[a + b*Tan[e + f*x]]), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule Rubi steps

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d}+\frac{1}{4}(5a)\int (e\sec(c+dx))^{3/2}\sqrt{a+}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d}+\frac{1}{8}\left(5a\right.$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d}+\frac{\left(5a^2e\right.}{}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d}+\frac{\left(5ia^3e\right.}{}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{\left(5ia^3e\right.}{}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d}+\frac{\left(5ia^3e\right.}{}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}}+\frac{5ia^{5/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c\right.}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a}}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^{5/2}e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 21.0766, size = 395, normalized size = 0.69

$$\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}\left(\sqrt{-\sin(c)+i\cos(c)+1}\left(\sqrt{-\sin(c)+i\cos(c)-1}\sqrt{-\tan\left(\frac{dx}{2}\right)+i}(-5i\sin(2(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/8)*a*(e*Sec[c + d*x])^(3/2)*(-10*ArcTan[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sq
rt[I - Tan[(d*x)/2]])/(Sqrt[1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]
*Cos[c + d*x]^2*Sqrt[1 - I*Cos[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sq
rt[I + Tan[(d*x)/2]] + Sqrt[1 + I*Cos[c] - Sin[c]]*(Sqrt[-1 + I*Cos[c] - Si
n[c]]*(9 + 5*Cos[2*(c + d*x)] - (5*I)*Sin[2*(c + d*x)])*Sqrt[I - Tan[(d*x)/
2]] - 10*ArcTan[(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt
[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]^2*Sqrt[-1 -
I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]]))*Sqrt[a + I*a*Tan[c + d*x]])/(d*
Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)
/2]])
```

Maple [A] time = 0.337, size = 363, normalized size = 0.6

$$-\frac{a\left(\cos(dx+c)-1\right)^2}{8d\left(\sin(dx+c)\right)^3\left(i\sin(dx+c)+\cos(dx+c)-1\right)}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a\left(i\sin(dx+c)+\cos(dx+c)\right)}{\cos(dx+c)}}\left(5\,i\mathrm{Artanh}\left(\frac{\cos}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -1/8/d*a*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)*(cos(d*x+c)-1)^2*(5*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-
sin(d*x+c)))*cos(d*x+c)^2-5*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1+sin(d*x+c)))*cos(d*x+c)^2-10*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*si
n(d*x+c)+4*I*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-5*arctanh(1/2*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2-5*arctanh(1/2*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+10*cos(d*x+c)^2*(1
/(cos(d*x+c)+1))^(1/2)+14*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+4*(1/(cos(d*x
+c)+1))^(1/2))/sin(d*x+c)^3/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(cos(d*x+c)+1))^
(3/2)
```

Maxima [B] time = 2.55426, size = 3213, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] (4608*a*e*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2560*a*e*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4608*I*a*e*sin(5/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2560*I*a*e*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - (320*sqrt(2)*a*e*cos(4*d*x + 4*c) + 640
*sqrt(2)*a*e*cos(2*d*x + 2*c) + 320*I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 640*I*
sqrt(2)*a*e*sin(2*d*x + 2*c) + 320*sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (320*sqrt(2)*a*e*cos(4*d*x + 4*c) +
 640*sqrt(2)*a*e*cos(2*d*x + 2*c) + 320*I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 64
0*I*sqrt(2)*a*e*sin(2*d*x + 2*c) + 320*sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (320*sqrt(2)*a*e*cos(4*d*x + 4
*c) + 640*sqrt(2)*a*e*cos(2*d*x + 2*c) + 320*I*sqrt(2)*a*e*sin(4*d*x + 4*c)
 + 640*I*sqrt(2)*a*e*sin(2*d*x + 2*c) + 320*sqrt(2)*a*e)*arctan2(sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (320*sqrt(2)*a*e*cos(4*d*x
 + 4*c) + 640*sqrt(2)*a*e*cos(2*d*x + 2*c) + 320*I*sqrt(2)*a*e*sin(4*d*x +
4*c) + 640*I*sqrt(2)*a*e*sin(2*d*x + 2*c) + 320*sqrt(2)*a*e)*arctan2(sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (320*I*sqrt(2)*a*e*co
s(4*d*x + 4*c) + 640*I*sqrt(2)*a*e*cos(2*d*x + 2*c) - 320*sqrt(2)*a*e*sin(4
*d*x + 4*c) - 640*sqrt(2)*a*e*sin(2*d*x + 2*c) + 320*I*sqrt(2)*a*e)*arctan2
(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - (-320*I*sqrt(2)*a*e*cos(4*d*x + 4*c) - 640*I*sqrt(2)*a*e*co
s(2*d*x + 2*c) + 320*sqrt(2)*a*e*sin(4*d*x + 4*c) + 640*sqrt(2)*a*e*sin(2*d
*x + 2*c) - 320*I*sqrt(2)*a*e)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (160*sqrt(2)*a*e*co
s(4*d*x + 4*c) + 320*sqrt(2)*a*e*cos(2*d*x + 2*c) + 160*I*sqrt(2)*a*e*sin(4
*d*x + 4*c) + 320*I*sqrt(2)*a*e*sin(2*d*x + 2*c) + 160*sqrt(2)*a*e)*log(2*s
qrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*
```

```
x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) + (160*sqrt(2)*a*e*cos(4*d*x + 4*c) + 320*sqrt(2)*a*e*cos(2*d
*x + 2*c) + 160*I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 320*I*sqrt(2)*a*e*sin(2*d*
x + 2*c) + 160*sqrt(2)*a*e)*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (160*I*sqrt(2)*a*e*cos(
4*d*x + 4*c) + 320*I*sqrt(2)*a*e*cos(2*d*x + 2*c) - 160*sqrt(2)*a*e*sin(4*d
*x + 4*c) - 320*sqrt(2)*a*e*sin(2*d*x + 2*c) + 160*I*sqrt(2)*a*e)*log(2*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 2) - (-160*I*sqrt(2)*a*e*cos(4*d*x + 4*c) - 320*I*sqrt(2)
*a*e*cos(2*d*x + 2*c) + 160*sqrt(2)*a*e*sin(4*d*x + 4*c) + 320*sqrt(2)*a*e*
sin(2*d*x + 2*c) - 160*I*sqrt(2)*a*e)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (160*
I*sqrt(2)*a*e*cos(4*d*x + 4*c) + 320*I*sqrt(2)*a*e*cos(2*d*x + 2*c) - 160*s
qrt(2)*a*e*sin(4*d*x + 4*c) - 320*sqrt(2)*a*e*sin(2*d*x + 2*c) + 160*I*sqrt
(2)*a*e)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (-160*I*sqrt(2)*a*e*cos(4*d*x + 4*
c) - 320*I*sqrt(2)*a*e*cos(2*d*x + 2*c) + 160*sqrt(2)*a*e*sin(4*d*x + 4*c)
+ 320*sqrt(2)*a*e*sin(2*d*x + 2*c) - 160*I*sqrt(2)*a*e)*log(2*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 2))*sqrt(a)*sqrt(e)/(d*(-1024*I*cos(4*d*x + 4*c) - 2048*I*cos(2*d*x
 + 2*c) + 1024*sin(4*d*x + 4*c) + 2048*sin(2*d*x + 2*c) - 1024*I))
```

Fricas [A] time = 2.3083, size = 1808, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*((9*I*a*e*e^(2*I*d*x + 2*I*c) + 5*I*a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - sqrt(25/16*
I*a^3*e^3/d^2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*log(2/5*(5*(a*e*
e^(2*I*d*x + 2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I
*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) + 4*sqrt(25/16*I*a^3*e^3/d^2)*d
*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(a*e)) + sqrt(25/16*I*a^3*e^3/d^
2)*(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*log(2/5*(5*(a*e*e^(2*I*d*x +
 2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c
```

```
) + 1))*e^(3/2*I*d*x + 3/2*I*c) - 4*sqrt(25/16*I*a^3*e^3/d^2)*d*e^(2*I*d*x
+ 2*I*c))*e^(-2*I*d*x - 2*I*c)/(a*e)) + sqrt(-25/16*I*a^3*e^3/d^2)*(d*e^(3*
I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*log(2/5*(5*(a*e*e^(2*I*d*x + 2*I*c) + a
*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(
3/2*I*d*x + 3/2*I*c) + 4*sqrt(-25/16*I*a^3*e^3/d^2)*d*e^(2*I*d*x + 2*I*c))*
e^(-2*I*d*x - 2*I*c)/(a*e)) - sqrt(-25/16*I*a^3*e^3/d^2)*(d*e^(3*I*d*x + 3*
I*c) + d*e^(I*d*x + I*c))*log(2/5*(5*(a*e*e^(2*I*d*x + 2*I*c) + a*e)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x
+ 3/2*I*c) - 4*sqrt(-25/16*I*a^3*e^3/d^2)*d*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*
x - 2*I*c)/(a*e)))/(d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{3}{2}} (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.402 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=364

$$\frac{3ia^{3/2}\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d}-\frac{3ia^{3/2}\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d}-\frac{3ia^{3/2}\sqrt{e}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right.}{2}$$

```
[Out] ((3*I)*a^(3/2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x
]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - ((3*I)*a^(3/2)*Sqrt[e]*A
rcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[
c + d*x]])])/(Sqrt[2]*d) - (((3*I)/2)*a^(3/2)*Sqrt[e]*Log[a - (Sqrt[2]*Sqrt
[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]
*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((3*I)/2)*a^(3/2)*Sqrt[e]*Log[a +
(Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] +
 Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (I*a*Sqrt[e*Sec[c + d*
x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.315121, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3498, 3495, 297, 1162, 617, 204, 1165, 628}

$$\frac{3ia^{3/2}\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d}-\frac{3ia^{3/2}\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d}-\frac{3ia^{3/2}\sqrt{e}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] ((3*I)*a^(3/2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x
]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - ((3*I)*a^(3/2)*Sqrt[e]*A
rcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[
c + d*x]])])/(Sqrt[2]*d) - (((3*I)/2)*a^(3/2)*Sqrt[e]*Log[a - (Sqrt[2]*Sqrt
[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]
*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((3*I)/2)*a^(3/2)*Sqrt[e]*Log[a +
(Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] +
 Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (I*a*Sqrt[e*Sec[c + d*
x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```


Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}+\frac{1}{2}(3a)\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan$$

$$=\frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{(6ia^2e^2)\operatorname{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a+i}}{\sqrt{e}}\right.}{d}$$

$$=\frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}+\frac{(3ia^2e)\operatorname{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a+ia}}{\sqrt{es}}\right.}{d}$$

$$=\frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}-\frac{(3ia^2)\operatorname{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx,x,\frac{\sqrt{}}{}\right.}{2d}$$

$$=-\frac{3ia^{3/2}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}d}$$

$$=\frac{3ia^{3/2}\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d}-\frac{3ia^{3/2}\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan}}{\sqrt{a}\sqrt{e\sec(c+}}\right.}{\sqrt{2}d}$$

Mathematica [A] time = 1.74517, size = 348, normalized size = 0.96

$$\frac{ae\sqrt{a+ia\tan(c+dx)}\left(\sqrt{-\sin(c)+i\cos(c)-1}\left(3\sqrt{\sin(c)+i\cos(c)-1}\sqrt{\tan\left(\frac{dx}{2}\right)+i}\tan^{-1}\left(\frac{\sqrt{\sin(c)-i\cos(c)-1}\sqrt{-\tan\left(\frac{dx}{2}\right)+i}}{\sqrt{\sin(c)+i\cos(c)-1}\sqrt{\tan\left(\frac{dx}{2}\right)+i}}\right.\right.\right.}{d\sqrt{-\sin(c)+i\cos(c)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*e*(-3*ArcTan[(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt
[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[
c]]*Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-1 + I*Cos[c
] - Sin[c]]*(I*Sec[c + d*x]*Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)
/2]] + 3*ArcTan[(Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt
[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 + I*Cos[c] + Sin[
c]]*Sqrt[I + Tan[(d*x)/2]]))*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c +
d*x]]*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I - Ta
n[(d*x)/2]])
```

Maple [A] time = 0.336, size = 304, normalized size = 0.8

$$-\frac{a(\cos(dx+c)-1)}{2d\sin(dx+c)(i\sin(dx+c)+\cos(dx+c)-1)}\sqrt{\frac{e}{\cos(dx+c)}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(3i\operatorname{Artanh}\left(\frac{\cos(dx-}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d*a*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)*(cos(d*x+c)-1)*(3*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+si
```

```
n(d*x+c)))*cos(d*x+c)-3*I*cos(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))-2*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-3*arctanh
(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)-2*cos(d
*x+c)*(1/(cos(d*x+c)+1))^(1/2)-3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)+1-sin(d*x+c)))*cos(d*x+c)-2*(1/(cos(d*x+c)+1))^(1/2))/sin(d*x+c)/(1/
(cos(d*x+c)+1))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)
```

Maxima [B] time = 2.34972, size = 2539, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -((48*sqrt(2)*a*cos(2*d*x + 2*c) + 48*I*sqrt(2)*a*sin(2*d*x + 2*c) + 48*sqr
t(2)*a)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
(48*sqrt(2)*a*cos(2*d*x + 2*c) + 48*I*sqrt(2)*a*sin(2*d*x + 2*c) + 48*sqrt(
2)*a)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (
48*sqrt(2)*a*cos(2*d*x + 2*c) + 48*I*sqrt(2)*a*sin(2*d*x + 2*c) + 48*sqrt(2
)*a)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (48
*sqrt(2)*a*cos(2*d*x + 2*c) + 48*I*sqrt(2)*a*sin(2*d*x + 2*c) + 48*sqrt(2)*
a)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1
, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (48*
I*sqrt(2)*a*cos(2*d*x + 2*c) - 48*sqrt(2)*a*sin(2*d*x + 2*c) + 48*I*sqrt(2)
*a)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1) - (-48*I*sqrt(2)*a*cos(2*d*x + 2*c) + 48*sqrt(2)*a
*sin(2*d*x + 2*c) - 48*I*sqrt(2)*a)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 128*a*cos(3/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (24*sqrt(2)*a*cos(2*d*x + 2*
c) + 24*I*sqrt(2)*a*sin(2*d*x + 2*c) + 24*sqrt(2)*a)*log(2*sqrt(2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
 (24*sqrt(2)*a*cos(2*d*x + 2*c) + 24*I*sqrt(2)*a*sin(2*d*x + 2*c) + 24*sqrt
(2)*a)*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - (-24*I*sqrt(2)*a*cos(2*d*x + 2*c) + 24*sqrt(
2)*a*sin(2*d*x + 2*c) - 24*I*sqrt(2)*a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
```

```
 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (24
*I*sqrt(2)*a*cos(2*d*x + 2*c) - 24*sqrt(2)*a*sin(2*d*x + 2*c) + 24*I*sqrt(2
)*a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2) - (-24*I*sqrt(2)*a*cos(2*d*x + 2*c) + 24
*sqrt(2)*a*sin(2*d*x + 2*c) - 24*I*sqrt(2)*a)*log(2*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2)
 - (24*I*sqrt(2)*a*cos(2*d*x + 2*c) - 24*sqrt(2)*a*sin(2*d*x + 2*c) + 24*I*
sqrt(2)*a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 128*I*a*sin(3/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*sqrt(e)/(d*(-64*I*cos(2*d*x + 2*c) +
64*sin(2*d*x + 2*c) - 64*I))
```

Fricas [A] time = 2.13648, size = 1376, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(4*I*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) +
1))*e^(3/2*I*d*x + 3/2*I*c) - sqrt(9*I*a^3*e/d^2)*d*log(1/3*(6*(a*e^(2*I*d*
x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*
c) + 1))*e^(3/2*I*d*x + 3/2*I*c) + 2*I*sqrt(9*I*a^3*e/d^2)*d*e^(2*I*d*x + 2
*I*c))*e^(-2*I*d*x - 2*I*c)/a) + sqrt(9*I*a^3*e/d^2)*d*log(1/3*(6*(a*e^(2*I
*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2
*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - 2*I*sqrt(9*I*a^3*e/d^2)*d*e^(2*I*d*x
+ 2*I*c))*e^(-2*I*d*x - 2*I*c)/a) - sqrt(-9*I*a^3*e/d^2)*d*log(2/3*(3*(a*e^
(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x
 + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) + I*sqrt(-9*I*a^3*e/d^2)*d*e^(2*I*d
*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/a) + sqrt(-9*I*a^3*e/d^2)*d*log(2/3*(3*(a
*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*
d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - I*sqrt(-9*I*a^3*e/d^2)*d*e^(2*
I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/a))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e \sec (dx+c)}(i a \tan (dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.403 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=520

$$\frac{i\sqrt{2}a^{5/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{2}a^{5/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{ia^{5/2}\sec(c+dx}{}$$

```
[Out] (I*Sqrt[2]*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(d*Sqrt[e]*Sqrt[a - I*a*Tan[c
 + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (I*Sqrt[2]*a^(5/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[
c + d*x])/(d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
 - (I*a^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(
Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) +
(I*a^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqr
t[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqr
t[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((4
*I)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.433801, antiderivative size = 520, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3496, 3499, 3495, 297, 1162, 617, 204, 1165, 628}

$$\frac{i\sqrt{2}a^{5/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{2}a^{5/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{ia^{5/2}\sec(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[e*Sec[c + d*x]],x]
```


```
[Out] (I*Sqrt[2]*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(d*Sqrt[e]*Sqrt[a - I*a*Tan[c
 + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (I*Sqrt[2]*a^(5/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[
c + d*x])/(d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
 - (I*a^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(
Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) +
(I*a^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqr
t[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqr
t[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((4
*I)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```


Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3499

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[(d*Sec[e + f*x])/(Sqrt[a - b*Tan[e + f*x]]*Sqr
t[a + b*Tan[e + f*x]]), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{a^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{e^2}$$

$$= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{\left(a^2\sec(c+dx)\right)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{\left(4ia^3e\sec(c+dx)\right)\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{\left(2ia^3\sec(c+dx)\right)\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{(2 \cdots}{\cdots}$$

$$= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{\left(ia^3\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \cdots$$

$$= -\frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{5/2}\cdots}{\cdots}$$

$$= \frac{i\sqrt{2}a^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{2}a^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\cdots}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan\cdots}}$$

Mathematica [A] time = 0.668993, size = 354, normalized size = 0.68

$$-\frac{2ia\sqrt{a+ia\tan(c+dx)}\left(\sqrt{-\sin(c)+i\cos(c)+1}\left(2\sqrt{-\sin(c)+i\cos(c)-1}\sqrt{-\tan\left(\frac{dx}{2}\right)+i}-\sqrt{-\sin(c)-i\cos(c)-1}\sqrt{\cdots}\right.\right.}{d\sqrt{-\sin(c)+i\cos(c)-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*a*(-(ArcTan[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(S
qrt[1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] - Sin
[c]]*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]]) + Sqrt[1 + I*Cos[
c] - Sin[c]]*(2*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]] - ArcTa
n[(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c]
 - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + T
an[(d*x)/2]]))*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[-1
+ I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A] time = 0.293, size = 286, normalized size = 0.6

$$-\frac{a}{d\left(i\sin(dx+c)+\cos(dx+c)-1\right)}\left(-i\text{Artanh}\left(\frac{\cos(dx+c)+1-\sin(dx+c)}{2}\sqrt{(\cos(dx+c)+1)^{-1}}\right)\sqrt{(\cos(dx+c)+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x)
```

```
[Out] -1/d*a*(-I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*
(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)+1+sin(d*x+c)))*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+arctanh(1/2*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*(1/(cos(d*x+c)+1))^(1/2
)*sin(d*x+c)+arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))
)*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+4*I*cos(d*x+c)-4*I-4*sin(d*x+c))*(a*(
I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/(e/c
os(d*x+c))^(1/2)
```

Maxima [B] time = 2.15231, size = 1974, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/4*(2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 2*sqrt(2)*a*arctan2(sqrt(2)*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*sqrt(2)
*a*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) + I*sqrt(2)*a*log(2*sqrt(2)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - I*sqrt(2)*a*
log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) - sqrt(2)*a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + sqrt(2)*a
*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 2) - sqrt(2)*a*log(2*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
```

```
x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) +
sqrt(2)*a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*I*a*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 16*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*sqrt(a)/(d*sqrt(e))
```

Fricas [A] time = 2.18559, size = 1543, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(d*e*sqrt(4*I*a^3/(d^2*e))*e^(I*d*x + I*c)*log((d*e*sqrt(4*I*a^3/(d^2*e
))*e^(2*I*d*x + 2*I*c) + 2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^
(-2*I*d*x - 2*I*c)/a) - d*e*sqrt(4*I*a^3/(d^2*e))*e^(I*d*x + I*c)*log(-(d*e
*sqrt(4*I*a^3/(d^2*e))*e^(2*I*d*x + 2*I*c) - 2*(a*e^(2*I*d*x + 2*I*c) + a)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*
I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/a) - d*e*sqrt(-4*I*a^3/(d^2*e))*e^(I
*d*x + I*c)*log((d*e*sqrt(-4*I*a^3/(d^2*e))*e^(2*I*d*x + 2*I*c) + 2*(a*e^(2
*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x +
 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/a) + d*e*sqrt(-
4*I*a^3/(d^2*e))*e^(I*d*x + I*c)*log(-(d*e*sqrt(-4*I*a^3/(d^2*e))*e^(2*I*d*
x + 2*I*c) - 2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x -
2*I*c)/a) + 2*(-4*I*a*e^(2*I*d*x + 2*I*c) - 4*I*a)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-I*
d*x - I*c)/(d*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}{\sqrt{e\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/sqrt(e*sec(d*x + c)), x)
```

3.404 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.0914253, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3488}

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(3/2),x]

[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0623623, size = 38, normalized size = 1.

$$-\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(3/2),x]

[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

Maple [B] time = 0.288, size = 76, normalized size = 2.

$$-\frac{2a(i\cos(dx+c)-\sin(dx+c))(\cos(dx+c))^2}{3de^3}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d*a*(I*cos(d*x+c)-sin(d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))
^(1/2)*(e/cos(d*x+c))^(3/2)*cos(d*x+c)^2/e^3
```

Maxima [B] time = 1.66155, size = 103, normalized size = 2.71

$$-\frac{2i\,a^{\frac{3}{2}}\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}{3\,de^{\frac{3}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -2/3*I*a^(3/2)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1)^(3/2)/(d*e^(3/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 - 1)^(3/2))
```

Fricas [B] time = 2.02829, size = 186, normalized size = 4.89

$$\frac{2\left(-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}}{3\,de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/3*(-I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c)/(d*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^{\frac{3}{2}}}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(3/2), x)
```

3.405 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] (((-4*I)/5)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]]) - ((
(2*I)/5)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Rubi [A] time = 0.158046, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3497, 3488}

$$-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/5)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]]) - ((
(2*I)/5)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx &= -\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}+\frac{(2a)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}\\ &= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.365108, size = 84, normalized size = 1.04

$$-\frac{2a(2\tan(c+dx)+3i)(\cos(dx)-i\sin(dx))\sqrt{a+ia\tan(c+dx)}(\cos(c+2dx)+i\sin(c+2dx))}{5de(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*a*(Cos[d*x] - I*Sin[d*x])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])*(3*I + 2*
Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*e*(e*Sec[c + d*x])^(3/2))
```

Maple [A] time = 0.296, size = 86, normalized size = 1.1

$$-\frac{2a\left(i(\cos(dx+c))^2-\cos(dx+c)\sin(dx+c)+2i\right)(\cos(dx+c))^3}{5de^5}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/5/d*a*(I*cos(d*x+c)^2-cos(d*x+c)*sin(d*x+c)+2*I)*(a*(I*sin(d*x+c)+cos(d*
x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^3/e^5
```

Maxima [A] time = 1.88918, size = 80, normalized size = 0.99

$$\frac{\left(-ia\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5ia\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{5de^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/5*(-I*a*cos(5/2*d*x + 5/2*c) - 5*I*a*cos(1/2*d*x + 1/2*c) + a*sin(5/2*d*x
 + 5/2*c) + 5*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(5/2))
```

Fricas [A] time = 2.11201, size = 227, normalized size = 2.8

$$\frac{\left(-iae^{(4idx+4ic)}-6iae^{(2idx+2ic)}-5ia\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{e}{e^{(2idx+2ic)}+1}}e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{5de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/5*(-I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^(2*I*d*x + 2*I*c) - 5*I*a)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x +
1/2*I*c)/(d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan(dx+c)+a)^{\frac{3}{2}}}{(e \sec(dx+c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(5/2), x)
```

3.406 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=125

$$\frac{16ia^2\sqrt{e\sec(c+dx)}}{21de^4\sqrt{a+ia\tan(c+dx)}}-\frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] (((16*I)/21)*a^2*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((8*I)/21)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(3/2)) -
 (((2*I)/7)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(7/2))
```

Rubi [A] time = 0.229669, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3497, 3488}

$$\frac{16ia^2\sqrt{e\sec(c+dx)}}{21de^4\sqrt{a+ia\tan(c+dx)}}-\frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((16*I)/21)*a^2*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((8*I)/21)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(3/2)) -
 (((2*I)/7)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(7/2))
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx &= -\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}+\frac{(4a)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}\\ &= -\frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}+\frac{\left(8a^2\right)\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{21e^4}\\ &= \frac{16ia^2\sqrt{e\sec(c+dx)}}{21de^4\sqrt{a+ia\tan(c+dx)}}-\frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.426534, size = 98, normalized size = 0.78

$$\frac{a(\cos(dx)-i\sin(dx))\sqrt{a+ia\tan(c+dx)}(12\sin(2(c+dx))+9i\cos(2(c+dx))-7i)(\cos(c+2dx)+i\sin(c+2dx))}{21de^3\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (a*(Cos[d*x] - I*Sin[d*x])*(-7*I + (9*I)*Cos[2*(c + d*x)] + 12*Sin[2*(c + d
*x)])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(21*d
*e^3*Sqrt[e*Sec[c + d*x]])
```

Maple [A] time = 0.303, size = 103, normalized size = 0.8

$$-\frac{2a\left(3i(\cos(dx+c))^3-3(\cos(dx+c))^2\sin(dx+c)-4i\cos(dx+c)-8\sin(dx+c)\right)(\cos(dx+c))^4}{21\,de^7}\sqrt{\frac{a(i\sin(dx}{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x)
```

```
[Out] -2/21/d*a*(3*I*cos(d*x+c)^3-3*cos(d*x+c)^2*sin(d*x+c)-4*I*cos(d*x+c)-8*sin(
d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(7/2)
*cos(d*x+c)^4/e^7
```

Maxima [A] time = 1.92488, size = 113, normalized size = 0.9

$$\frac{\left(-3ia\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)-14ia\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)+21ia\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+14a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+}{42\,de^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] 1/42*(-3*I*a*cos(7/2*d*x + 7/2*c) - 14*I*a*cos(3/2*d*x + 3/2*c) + 21*I*a*co
s(1/2*d*x + 1/2*c) + 3*a*sin(7/2*d*x + 7/2*c) + 14*a*sin(3/2*d*x + 3/2*c) +
 21*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(7/2))
```

Fricas [A] time = 1.98865, size = 273, normalized size = 2.18

$$\frac{\left(-3i\,ae^{(6i\,dx+6i\,c)}-17i\,ae^{(4i\,dx+4i\,c)}+7i\,ae^{(2i\,dx+2i\,c)}+21i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{42\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/42*(-3*I*a*e^(6*I*d*x + 6*I*c) - 17*I*a*e^(4*I*d*x + 4*I*c) + 7*I*a*e^(2*
I*d*x + 2*I*c) + 21*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d
*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^{\frac{3}{2}}}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(7/2), x)
```

3.407 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=167

$$\frac{16ia^2}{45de^4\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{32ia\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx)}{9d(e\sec(c+dx))^{9/}}$$

```
[Out] (((16*I)/45)*a^2)/(d*e^4*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((4*I)/15)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(5/2)) -
 (((32*I)/45)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*Sqrt[e*Sec[c + d*x]]) -
(((2*I)/9)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(9/2))
```

Rubi [A] time = 0.285546, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3497, 3502, 3488}

$$\frac{16ia^2}{45de^4\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{32ia\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx)}{9d(e\sec(c+dx))^{9/}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (((16*I)/45)*a^2)/(d*e^4*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((4*I)/15)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(5/2)) -
 (((32*I)/45)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*Sqrt[e*Sec[c + d*x]]) -
(((2*I)/9)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(9/2))
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}} + \frac{(2a)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx}{3e^2}$$

$$= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}} + \frac{(8a^2)\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx}{15e^4}$$

$$= \frac{16ia^2}{45de^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx)}{9d(e\sec(c+dx))^{9}}$$

$$= \frac{16ia^2}{45de^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}} - \frac{32ia\sqrt{a+ia\tan(c+}}{45de^4\sqrt{e\sec(c+}}$$

Mathematica [A] time = 0.552318, size = 113, normalized size = 0.68

$$\frac{a(\cos(dx)-i\sin(dx))\sqrt{a+ia\tan(c+dx)}(-54\sin(c+dx)+10\sin(3(c+dx))-81i\cos(c+dx)+5i\cos(3(c+dx)))(\cos(}{90de^4\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (a*(Cos[d*x] - I*Sin[d*x])*((-81*I)*Cos[c + d*x] + (5*I)*Cos[3*(c + d*x)] -
 54*Sin[c + d*x] + 10*Sin[3*(c + d*x)])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])
*Sqrt[a + I*a*Tan[c + d*x]])/(90*d*e^4*Sqrt[e*Sec[c + d*x]])
```

Maple [A] time = 0.372, size = 113, normalized size = 0.7

$$-\frac{2a\left(5i(\cos(dx+c))^4-5(\cos(dx+c))^3\sin(dx+c)-2i(\cos(dx+c))^2-8\cos(dx+c)\sin(dx+c)+16i\right)(\cos(dx+}{45de^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x)
```

```
[Out] -2/45/d*a*(5*I*cos(d*x+c)^4-5*cos(d*x+c)^3*sin(d*x+c)-2*I*cos(d*x+c)^2-8*co
s(d*x+c)*sin(d*x+c)+16*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e
/cos(d*x+c))^(9/2)*cos(d*x+c)^5/e^9
```

Maxima [A] time = 1.949, size = 216, normalized size = 1.29

$$\left(-5ia\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)+15ia\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)-27ia\cos\left(\frac{5}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)-135ia\cos\left(\frac{1}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="maxim
a")
```

```
[Out] 1/180*(-5*I*a*cos(9/2*d*x + 9/2*c) + 15*I*a*cos(3/2*d*x + 3/2*c) - 27*I*a*c
os(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 135*I*a*cos(1
```

```
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*a*sin(9/2*d*x +
 9/2*c) + 15*a*sin(3/2*d*x + 3/2*c) + 27*a*sin(5/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 135*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))))*sqrt(a)/(d*e^(9/2))
```

Fricas [A] time = 2.16664, size = 317, normalized size = 1.9

$$\frac{\left(-5i\,ae^{(8i\,dx+8i\,c)}-32i\,ae^{(6i\,dx+6i\,c)}-162i\,ae^{(4i\,dx+4i\,c)}-120i\,ae^{(2i\,dx+2i\,c)}+15i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{180\,de^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/180*(-5*I*a*e^(8*I*d*x + 8*I*c) - 32*I*a*e^(6*I*d*x + 6*I*c) - 162*I*a*e^
(4*I*d*x + 4*I*c) - 120*I*a*e^(2*I*d*x + 2*I*c) + 15*I*a)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-3/2*I*d*x - 3/2*I*c)
/(d*e^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}{(e\sec(dx+c))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(9/2), x)
```

3.408 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=612

$$-\frac{15ia^{7/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{15ia^{7/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{15ia^{7/2}e\cdots}{\cdots}$$

```
[Out] (((15*I)/8)*a^3*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((
(15*I)/8)*a^(7/2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c +
d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*
a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((15*I)/8)*a^(7/2)*e^(3/2)*A
rcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[
c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*
a*Tan[c + d*x]]) + (((15*I)/16)*a^(7/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a -
I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) - (((15*I)/16)*a^(7/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqr
t[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x
]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x
]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((3*I)/4)*a^2*(e*Sec[c + d*x])^(3/2)*Sqrt
[a + I*a*Tan[c + d*x]])/d + ((I/3)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c
+ d*x])^(3/2))/d
```

Rubi [A] time = 0.689787, antiderivative size = 612, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3498, 3499, 3495, 297, 1162, 617, 204, 1165, 628}

$$-\frac{15ia^{7/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{15ia^{7/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{15ia^{7/2}e\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((15*I)/8)*a^3*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((
(15*I)/8)*a^(7/2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c +
d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*
a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((15*I)/8)*a^(7/2)*e^(3/2)*A
rcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[
c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*
a*Tan[c + d*x]]) + (((15*I)/16)*a^(7/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a -
I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) - (((15*I)/16)*a^(7/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqr
t[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x
]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x
]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((3*I)/4)*a^2*(e*Sec[c + d*x])^(3/2)*Sqrt
[a + I*a*Tan[c + d*x]])/d + ((I/3)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c
+ d*x])^(3/2))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
```

```
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3499

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[(d*Sec[e + f*x])/(Sqrt[a - b*Tan[e + f*x]]*Sqr
t[a + b*Tan[e + f*x]]), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{1}{2}(3a)\int (e\sec(c+dx))^{3/2}(a+$$

$$= \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan}{3d}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia(}{}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia(e}{}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia(e}{}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia(e}{}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia(e}{}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{15ia^{7/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c\right.}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{}}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} - \frac{15ia^{7/2}e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx}}$$

Mathematica [A] time = 20.5643, size = 417, normalized size = 0.68

$$\frac{ia^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}\left(\sqrt{-\sin(c)+i\cos(c)+1}\left(\sqrt{-\sin(c)+i\cos(c)-1}\sqrt{-\tan\left(\frac{dx}{2}\right)+i(-13i\sin(c+}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/96)*a^2*(e*Sec[c + d*x])^(5/2)*(-180*ArcTan[(Sqrt[1 + I*Cos[c] - Sin[c]
]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2
]])]*Cos[c + d*x]^3*Sqrt[1 - I*Cos[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] - Sin[c]
]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[1 + I*Cos[c] - Sin[c]]*(Sqrt[-1 + I*Cos[c]
- Sin[c]]*(239*Cos[c + d*x] + 45*Cos[3*(c + d*x)] - (13*I)*Sin[c + d*x] - (
45*I)*Sin[3*(c + d*x)])*Sqrt[I - Tan[(d*x)/2]] - 180*ArcTan[(Sqrt[-1 + I*Co
s[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I
 + Tan[(d*x)/2]])]*Cos[c + d*x]^3*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan
[(d*x)/2]]))*Sqrt[a + I*a*Tan[c + d*x]])/(d*e*Sqrt[-1 + I*Cos[c] - Sin[c]]*
Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A] time = 0.294, size = 424, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -1/48/d*a^2*(cos(d*x+c)-1)^2*(45*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3-45*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3-90*I*(1/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)^2*sin(d*x+c)-45*cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)+1-sin(d*x+c)))-45*cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^
(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+68*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*s
in(d*x+c)+90*cos(d*x+c)^3*(1/(cos(d*x+c)+1))^(1/2)+16*I*sin(d*x+c)*(1/(cos(
d*x+c)+1))^(1/2)+158*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)+52*cos(d*x+c)*(1
/(cos(d*x+c)+1))^(1/2)-16*(1/(cos(d*x+c)+1))^(1/2))*(a*(I*sin(d*x+c)+cos(d*
x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(3/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/(1
/(cos(d*x+c)+1))^(3/2)/cos(d*x+c)/sin(d*x+c)^3
```

Maxima [B] time = 3.39107, size = 4074, normalized size = 6.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] (347136*a^2*e*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 387072
*a^2*e*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 138240*a^2*e*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 347136*I*a^2*e*sin(9
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 387072*I*a^2*e*sin(5/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 138240*I*a^2*e*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (17280*sqrt(2)*a^2*e*cos(6*d*x + 6*
c) + 51840*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 51840*sqrt(2)*a^2*e*cos(2*d*x +
 2*c) + 17280*I*sqrt(2)*a^2*e*sin(6*d*x + 6*c) + 51840*I*sqrt(2)*a^2*e*sin(
4*d*x + 4*c) + 51840*I*sqrt(2)*a^2*e*sin(2*d*x + 2*c) + 17280*sqrt(2)*a^2*e
)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1,
 sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (17280
*sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 51840*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 51
840*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + 17280*I*sqrt(2)*a^2*e*sin(6*d*x + 6*c)
 + 51840*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 51840*I*sqrt(2)*a^2*e*sin(2*d*x
 + 2*c) + 17280*sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) - (17280*sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 51840*sqrt(
2)*a^2*e*cos(4*d*x + 4*c) + 51840*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + 17280*I*
sqrt(2)*a^2*e*sin(6*d*x + 6*c) + 51840*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 5
1840*I*sqrt(2)*a^2*e*sin(2*d*x + 2*c) + 17280*sqrt(2)*a^2*e)*arctan2(sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (17280*sqrt(2)*a^2*e*c
os(6*d*x + 6*c) + 51840*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 51840*sqrt(2)*a^2*
e*cos(2*d*x + 2*c) + 17280*I*sqrt(2)*a^2*e*sin(6*d*x + 6*c) + 51840*I*sqrt(
2)*a^2*e*sin(4*d*x + 4*c) + 51840*I*sqrt(2)*a^2*e*sin(2*d*x + 2*c) + 17280*
sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) - (17280*I*sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 51840*I*sqrt(2)*a^2*e*cos
(4*d*x + 4*c) + 51840*I*sqrt(2)*a^2*e*cos(2*d*x + 2*c) - 17280*sqrt(2)*a^2*
e*sin(6*d*x + 6*c) - 51840*sqrt(2)*a^2*e*sin(4*d*x + 4*c) - 51840*sqrt(2)*a
^2*e*sin(2*d*x + 2*c) + 17280*I*sqrt(2)*a^2*e)*arctan2(sqrt(2)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
```

```
, cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (-172
80*I*sqrt(2)*a^2*e*cos(6*d*x + 6*c) - 51840*I*sqrt(2)*a^2*e*cos(4*d*x + 4*c
) - 51840*I*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + 17280*sqrt(2)*a^2*e*sin(6*d*x
+ 6*c) + 51840*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 51840*sqrt(2)*a^2*e*sin(2*d
*x + 2*c) - 17280*I*sqrt(2)*a^2*e)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (8640*sqrt(2)*a
^2*e*cos(6*d*x + 6*c) + 25920*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 25920*sqrt(2
)*a^2*e*cos(2*d*x + 2*c) + 8640*I*sqrt(2)*a^2*e*sin(6*d*x + 6*c) + 25920*I*
sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 25920*I*sqrt(2)*a^2*e*sin(2*d*x + 2*c) + 8
640*sqrt(2)*a^2*e)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (8640*sqrt(2)*a^2*e*cos(6*d*x + 6
*c) + 25920*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 25920*sqrt(2)*a^2*e*cos(2*d*x
+ 2*c) + 8640*I*sqrt(2)*a^2*e*sin(6*d*x + 6*c) + 25920*I*sqrt(2)*a^2*e*sin(
4*d*x + 4*c) + 25920*I*sqrt(2)*a^2*e*sin(2*d*x + 2*c) + 8640*sqrt(2)*a^2*e)
*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) - (8640*I*sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 25920*I*s
qrt(2)*a^2*e*cos(4*d*x + 4*c) + 25920*I*sqrt(2)*a^2*e*cos(2*d*x + 2*c) - 86
40*sqrt(2)*a^2*e*sin(6*d*x + 6*c) - 25920*sqrt(2)*a^2*e*sin(4*d*x + 4*c) -
25920*sqrt(2)*a^2*e*sin(2*d*x + 2*c) + 8640*I*sqrt(2)*a^2*e)*log(2*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 2) - (-8640*I*sqrt(2)*a^2*e*cos(6*d*x + 6*c) - 25920*I*sqrt(2)
*a^2*e*cos(4*d*x + 4*c) - 25920*I*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + 8640*sqr
t(2)*a^2*e*sin(6*d*x + 6*c) + 25920*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 25920*
sqrt(2)*a^2*e*sin(2*d*x + 2*c) - 8640*I*sqrt(2)*a^2*e)*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) - (8640*I*sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 25920*I*sqrt(2)*a^2*e*
cos(4*d*x + 4*c) + 25920*I*sqrt(2)*a^2*e*cos(2*d*x + 2*c) - 8640*sqrt(2)*a^
2*e*sin(6*d*x + 6*c) - 25920*sqrt(2)*a^2*e*sin(4*d*x + 4*c) - 25920*sqrt(2)
*a^2*e*sin(2*d*x + 2*c) + 8640*I*sqrt(2)*a^2*e)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) - (-8640*I*sqrt(2)*a^2*e*cos(6*d*x + 6*c) - 25920*I*sqrt(2)*a^2*e*cos(4*
d*x + 4*c) - 25920*I*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + 8640*sqrt(2)*a^2*e*si
n(6*d*x + 6*c) + 25920*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 25920*sqrt(2)*a^2*e
*sin(2*d*x + 2*c) - 8640*I*sqrt(2)*a^2*e)*log(2*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2))*sq
rt(a)*sqrt(e)/(d*(-36864*I*cos(6*d*x + 6*c) - 110592*I*cos(4*d*x + 4*c) - 1
```

```
10592*I*cos(2*d*x + 2*c) + 36864*sin(6*d*x + 6*c) + 110592*sin(4*d*x + 4*c)
 + 110592*sin(2*d*x + 2*c) - 36864*I))
```

Fricas [A] time = 2.29891, size = 2105, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/12*((113*I*a^2*e*e^(4*I*d*x + 4*I*c) + 126*I*a^2*e*e^(2*I*d*x + 2*I*c) +
45*I*a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(3/2*I*d*x + 3/2*I*c) - 6*sqrt(225/64*I*a^5*e^3/d^2)*(d*e^(5*I*d*x +
 5*I*c) + 2*d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*log(2/15*(15*(a^2*e*
e^(2*I*d*x + 2*I*c) + a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) + 8*sqrt(225/64*I*a^5*e^3/d^2
)*d*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^2*e)) + 6*sqrt(225/64*I*a^
5*e^3/d^2)*(d*e^(5*I*d*x + 5*I*c) + 2*d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x +
I*c))*log(2/15*(15*(a^2*e*e^(2*I*d*x + 2*I*c) + a^2*e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - 8
*sqrt(225/64*I*a^5*e^3/d^2)*d*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^
2*e)) + 6*sqrt(-225/64*I*a^5*e^3/d^2)*(d*e^(5*I*d*x + 5*I*c) + 2*d*e^(3*I*d
*x + 3*I*c) + d*e^(I*d*x + I*c))*log(2/15*(15*(a^2*e*e^(2*I*d*x + 2*I*c) +
a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
e^(3/2*I*d*x + 3/2*I*c) + 8*sqrt(-225/64*I*a^5*e^3/d^2)*d*e^(2*I*d*x + 2*I*
c))*e^(-2*I*d*x - 2*I*c)/(a^2*e)) - 6*sqrt(-225/64*I*a^5*e^3/d^2)*(d*e^(5*I
*d*x + 5*I*c) + 2*d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))*log(2/15*(15*(
a^2*e*e^(2*I*d*x + 2*I*c) + a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - 8*sqrt(-225/64*I*a^5*
e^3/d^2)*d*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^2*e)))/(d*e^(5*I*d*
x + 5*I*c) + 2*d*e^(3*I*d*x + 3*I*c) + d*e^(I*d*x + I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e \sec (dx + c))^{\frac{3}{2}} (i\,a \tan (dx + c) + a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.409 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=411

$$\frac{21ia^{5/2}\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d}-\frac{21ia^{5/2}\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d}+\frac{7ia^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec}}{4d}$$

```
[Out] (((21*I)/4)*a^(5/2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c
+ d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((21*I)/4)*a^(5/2)
*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*S
qrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((21*I)/8)*a^(5/2)*Sqrt[e]*Log[a - (S
qrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + C
os[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((21*I)/8)*a^(5/2)*Sqrt
[e]*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec
[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((7*I)/4)
*a^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((I/2)*a*Sqrt[e*S
ec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.453239, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3498, 3495, 297, 1162, 617, 204, 1165, 628}

$$\frac{21ia^{5/2}\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d}-\frac{21ia^{5/2}\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d}+\frac{7ia^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((21*I)/4)*a^(5/2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c
+ d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((21*I)/4)*a^(5/2)
*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*S
qrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((21*I)/8)*a^(5/2)*Sqrt[e]*Log[a - (S
qrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + C
os[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((21*I)/8)*a^(5/2)*Sqrt
[e]*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec
[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((7*I)/4)
*a^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((I/2)*a*Sqrt[e*S
ec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d} + \frac{1}{4}(7a)\int \sqrt{e\sec(c+dx)}(a+ia$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c}{2d}$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c}{2d}$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c}{2d}$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c}{2d}$$

$$= -\frac{21ia^{5/2}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+d\right.}{8\sqrt{2}d}$$

$$= \frac{21ia^{5/2}\sqrt{e}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d} - \frac{21ia^{5/2}\sqrt{e}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a}}{\sqrt{a}\sqrt{e}}\right.}{4\sqrt{2}d}$$

Mathematica [A] time = 2.56917, size = 397, normalized size = 0.97

$$\frac{a^2(\cos(2dx)+i\sin(2dx))\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}\left(\sqrt{-\sin(c)+i\cos(c)-1}\left(21\sqrt{\sin(c)+i\cos(c)-1}\sqrt{\tan\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[e*Sec[c + d*x]]*(Cos[2*d*x] + I*Sin[2*d*x])*(-21*ArcTan[(Sqrt[-1
+ I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*
Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1
- I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-1 + I*Cos[c] - Sin[c]]*
(21*ArcTan[(Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 +
 I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[-1 + I*Cos[c
] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I -
Tan[(d*x)/2]]*(9*I - 2*Tan[c + d*x])))*Sqrt[a + I*a*Tan[c + d*x]])/(4*d*Sqr
t[-1 + I*Cos[c] - Sin[c]]*Sqrt[-1 - I*Cos[c] + Sin[c]]*(Cos[d*x] + I*Sin[d*
x])^2*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A] time = 0.359, size = 371, normalized size = 0.9

$$\frac{a^2(\cos(dx+c)-1)}{8d(i\sin(dx+c)+\cos(dx+c)-1)\cos(dx+c)\sin(dx+c)}\left(21\,i\mathrm{Artanh}\left(\frac{\cos(dx+c)+1-\sin(dx+c)}{2}\sqrt{(\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/8/d*a^2*(cos(d*x+c)-1)*(21*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1-sin(d*x+c)))*cos(d*x+c)^2-21*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+22*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)*sin(d*x+c)+4*I*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+21*arctanh(1/2*(1/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+21*arctanh(1/
2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+22*cos(d
*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)+18*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-4*(
1/(cos(d*x+c)+1))^(1/2))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/
cos(d*x+c))^(1/2)/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2)/cos(
d*x+c)/sin(d*x+c)
```

Maxima [B] time = 2.75543, size = 3291, normalized size = 8.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] (5632*a^2*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3584*a^2*c
os(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 5632*I*a^2*sin(7/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 3584*I*a^2*sin(3/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - (1344*sqrt(2)*a^2*cos(4*d*x + 4*c) + 26
88*sqrt(2)*a^2*cos(2*d*x + 2*c) + 1344*I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 268
8*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + 1344*sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (1344*sqrt(2)*a^2*cos(4*d*x +
4*c) + 2688*sqrt(2)*a^2*cos(2*d*x + 2*c) + 1344*I*sqrt(2)*a^2*sin(4*d*x + 4
*c) + 2688*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + 1344*sqrt(2)*a^2)*arctan2(sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (1344*sqrt(2)*a^2*co
s(4*d*x + 4*c) + 2688*sqrt(2)*a^2*cos(2*d*x + 2*c) + 1344*I*sqrt(2)*a^2*sin
(4*d*x + 4*c) + 2688*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + 1344*sqrt(2)*a^2)*arc
tan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (1344*sqrt(
2)*a^2*cos(4*d*x + 4*c) + 2688*sqrt(2)*a^2*cos(2*d*x + 2*c) + 1344*I*sqrt(2
)*a^2*sin(4*d*x + 4*c) + 2688*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + 1344*sqrt(2)
*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (
1344*I*sqrt(2)*a^2*cos(4*d*x + 4*c) + 2688*I*sqrt(2)*a^2*cos(2*d*x + 2*c) -
 1344*sqrt(2)*a^2*sin(4*d*x + 4*c) - 2688*sqrt(2)*a^2*sin(2*d*x + 2*c) + 13
44*I*sqrt(2)*a^2)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (-1344*I*sqrt(2)*a^2*cos(4*d*x + 4
*c) - 2688*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + 1344*sqrt(2)*a^2*sin(4*d*x + 4*
c) + 2688*sqrt(2)*a^2*sin(2*d*x + 2*c) - 1344*I*sqrt(2)*a^2)*arctan2(-sqrt(
2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) + (672*sqrt(2)*a^2*cos(4*d*x + 4*c) + 1344*sqrt(2)*a^2*cos(2*d*x +
 2*c) + 672*I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 1344*I*sqrt(2)*a^2*sin(2*d*x +
 2*c) + 672*sqrt(2)*a^2)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(
sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2
```

```
*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (672*sqrt(2)*a^2*cos(4*d*x
+ 4*c) + 1344*sqrt(2)*a^2*cos(2*d*x + 2*c) + 672*I*sqrt(2)*a^2*sin(4*d*x +
4*c) + 1344*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + 672*sqrt(2)*a^2)*log(-2*sqrt(2
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) + (-672*I*sqrt(2)*a^2*cos(4*d*x + 4*c) - 1344*I*sqrt(2)*a^2*cos(2*
d*x + 2*c) + 672*sqrt(2)*a^2*sin(4*d*x + 4*c) + 1344*sqrt(2)*a^2*sin(2*d*x
+ 2*c) - 672*I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + (672*I*sqrt(2)*
a^2*cos(4*d*x + 4*c) + 1344*I*sqrt(2)*a^2*cos(2*d*x + 2*c) - 672*sqrt(2)*a^
2*sin(4*d*x + 4*c) - 1344*sqrt(2)*a^2*sin(2*d*x + 2*c) + 672*I*sqrt(2)*a^2)
*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 2) + (-672*I*sqrt(2)*a^2*cos(4*d*x + 4*c) - 134
4*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + 672*sqrt(2)*a^2*sin(4*d*x + 4*c) + 1344*
sqrt(2)*a^2*sin(2*d*x + 2*c) - 672*I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2) + (672*I*sqrt(2)*a^2*cos(4*d*x + 4*c) + 1344*I*sqrt(2)*a^2*cos(2*d*x +
 2*c) - 672*sqrt(2)*a^2*sin(4*d*x + 4*c) - 1344*sqrt(2)*a^2*sin(2*d*x + 2*c
) + 672*I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqr
t(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2))*sqrt(a)*sqrt(e)/(d*(-
1024*I*cos(4*d*x + 4*c) - 2048*I*cos(2*d*x + 2*c) + 1024*sin(4*d*x + 4*c) +
 2048*sin(2*d*x + 2*c) - 1024*I))
```

Fricas [A] time = 2.3617, size = 1701, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/2*((11*I*a^2*e^(2*I*d*x + 2*I*c) + 7*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - sqrt(441/1
6*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(1/21*(42*(a^2*e^(2*I*d*x + 2
*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(3/2*I*d*x + 3/2*I*c) + 8*I*sqrt(441/16*I*a^5*e/d^2)*d*e^(2*I*d*x +
 2*I*c))*e^(-2*I*d*x - 2*I*c)/a^2) + sqrt(441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x
 + 2*I*c) + d)*log(1/21*(42*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c
) - 8*I*sqrt(441/16*I*a^5*e/d^2)*d*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c
```

```
)/a^2) - sqrt(-441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/21*(21
*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) + 4*I*sqrt(-441/16*I*a^5*
e/d^2)*d*e^(2*I*d*x + 2*I*c))*e^(-2*I*d*x - 2*I*c)/a^2) + sqrt(-441/16*I*a^
5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/21*(21*(a^2*e^(2*I*d*x + 2*I*c)
+ a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
e^(3/2*I*d*x + 3/2*I*c) - 4*I*sqrt(-441/16*I*a^5*e/d^2)*d*e^(2*I*d*x + 2*I*
c))*e^(-2*I*d*x - 2*I*c)/a^2))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e \sec(dx+c)}(i a \tan(dx+c) + a)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.410 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=563

$$\frac{5ia^{7/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^{7/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{10ia^2\sqrt{a+ia\,\text{ta}}}{d\sqrt{e\sec(c}}$$

```
[Out] ((5*I)*a^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqr
t[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*T
an[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((5*I)*a^(7/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[
c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((5*I)/2)*a^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a
*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]
*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*T
an[c + d*x]]) + (((5*I)/2)*a^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a
- I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d
*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a +
I*a*Tan[c + d*x]]) - ((10*I)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[
c + d*x]]) + (I*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.575009, antiderivative size = 563, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3498, 3496, 3499, 3495, 297, 1162, 617, 204, 1165, 628}

$$\frac{5ia^{7/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^{7/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{10ia^2\sqrt{a+ia\,\text{ta}}}{d\sqrt{e\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((5*I)*a^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqr
t[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*T
an[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((5*I)*a^(7/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[
c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((5*I)/2)*a^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a
*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]
*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*T
an[c + d*x]]) + (((5*I)/2)*a^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a
- I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d
*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a +
I*a*Tan[c + d*x]]) - ((10*I)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[
c + d*x]]) + (I*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3499

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[(d*Sec[e + f*x])/(Sqrt[a - b*Tan[e + f*x]]*Sqr
t[a + b*Tan[e + f*x]]), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}+\frac{1}{2}(5a)\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx$$

$$= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}-\frac{\left(5a^3\right)\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2e^2}$$

$$= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}-\frac{\left(5a^3\sec(c+dx)\right)\int \sqrt{e\sec(c}{2e\sqrt{a-ia\tan(c+dx}}$$

$$= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}-\frac{\left(10ia^4e\sec(c+dx)\right)\text{Subst}\Big(}{d\sqrt{a-ia\tan(c+dx}}$$

$$= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}+\frac{\left(5ia^4\sec(c+dx)\right)\text{Subst}\Big(\int}{d\sqrt{a-ia\tan(c+dx)}}$$

$$= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}-\frac{\left(5ia^4\sec(c+dx)\right)\text{Subst}\Big(\int}{2de\sqrt{a-ia\tan(c+}}$$

$$= -\frac{5ia^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+$$

$$= \frac{5ia^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{5ia^{7/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia}}$$

Mathematica [A] time = 1.93167, size = 367, normalized size = 0.65

$$-\frac{a^2\sqrt{a+ia\tan(c+dx)}\left(\sqrt{-\sin(c)+i\cos(c)+1}\left(\sqrt{-\sin(c)+i\cos(c)-1}\sqrt{-\tan\left(\frac{dx}{2}\right)+i}(\tan(c+dx)+9i)-5i\sqrt{-s}\right.\right.}{d\sqrt{-\sin(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] -((a^2*Sqrt[a + I*a*Tan[c + d*x]]*((-5*I)*ArcTan[(Sqrt[1 + I*Cos[c] - Sin[c
]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/
2]])]*Sqrt[1 - I*Cos[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan
[(d*x)/2]] + Sqrt[1 + I*Cos[c] - Sin[c]]*((-5*I)*ArcTan[(Sqrt[-1 + I*Cos[c]
 - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + T
an[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-
1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]]*(9*I + Tan[c + d*x]))))/(d*Sq
rt[e*Sec[c + d*x]]*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]
*Sqrt[I - Tan[(d*x)/2]]))
```

Maple [A] time = 0.345, size = 347, normalized size = 0.6

$$\frac{a^2}{2\,d\,(i\sin(dx+c)+\cos(dx+c)-1)\cos(dx+c)}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\Bigg(5\,i\cos(dx+c)\sin(dx+c)\,\mathrm{Artanh}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*a^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(5*I*cos(d*x+c)*si
n(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*(1
/(cos(d*x+c)+1))^(1/2)-5*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*a
rctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-5*cos(d*x+c)
*sin(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
*(1/(cos(d*x+c)+1))^(1/2)-5*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*
arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-16*I*cos(d*
x+c)^2+18*I*cos(d*x+c)+16*cos(d*x+c)*sin(d*x+c)-2*I+2*sin(d*x+c))/(I*sin(d*
x+c)+cos(d*x+c)-1)/cos(d*x+c)/(e/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.47983, size = 2720, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] ((80*sqrt(2)*a^2*cos(2*d*x + 2*c) + 80*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + 80*
sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
1) + (80*sqrt(2)*a^2*cos(2*d*x + 2*c) + 80*I*sqrt(2)*a^2*sin(2*d*x + 2*c) +
 80*sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 1) + (80*sqrt(2)*a^2*cos(2*d*x + 2*c) + 80*I*sqrt(2)*a^2*sin(2*d*x + 2
*c) + 80*sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + (80*sqrt(2)*a^2*cos(2*d*x + 2*c) + 80*I*sqrt(2)*a^2*sin(2*d*x
 + 2*c) + 80*sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1) + (80*I*sqrt(2)*a^2*cos(2*d*x + 2*c) - 80*sqrt(2)*a^2*sin(
2*d*x + 2*c) + 80*I*sqrt(2)*a^2)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (-80*I*sqrt(2)*a^2*
cos(2*d*x + 2*c) + 80*sqrt(2)*a^2*sin(2*d*x + 2*c) - 80*I*sqrt(2)*a^2)*arct
an2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) - (512*a^2*cos(2*d*x + 2*c) + 512*I*a^2*sin(2*d*x + 2*c)
 + 640*a^2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (40*sqrt
(2)*a^2*cos(2*d*x + 2*c) + 40*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + 40*sqrt(2)*a
^2)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
```

```
))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1) - (40*sqrt(2)*a^2*cos(2*d*x + 2*c) + 40*I*sqrt(2)*
a^2*sin(2*d*x + 2*c) + 40*sqrt(2)*a^2)*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (40*I*sqrt(2
)*a^2*cos(2*d*x + 2*c) - 40*sqrt(2)*a^2*sin(2*d*x + 2*c) + 40*I*sqrt(2)*a^2
)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2) + (-40*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + 40*
sqrt(2)*a^2*sin(2*d*x + 2*c) - 40*I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2) + (40*I*sqrt(2)*a^2*cos(2*d*x + 2*c) - 40*sqrt(2)*a^2*sin(2*d*x + 2*c)
+ 40*I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + (-40*I*sqrt(2)*a^2*cos(
2*d*x + 2*c) + 40*sqrt(2)*a^2*sin(2*d*x + 2*c) - 40*I*sqrt(2)*a^2)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) + (-512*I*a^2*cos(2*d*x + 2*c) + 512*a^2*sin(2*d*x +
2*c) - 640*I*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqr
t(a)*sqrt(e)/((-64*I*e*cos(2*d*x + 2*c) + 64*e*sin(2*d*x + 2*c) - 64*I*e)*d
)
```

Fricas [A] time = 2.35764, size = 1615, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(sqrt(25*I*a^5/(d^2*e))*d*e*e^(I*d*x + I*c)*log(2/5*(sqrt(25*I*a^5/(d^2
*e))*d*e*e^(2*I*d*x + 2*I*c) + 5*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/
2*I*c))*e^(-2*I*d*x - 2*I*c)/a^2) - sqrt(25*I*a^5/(d^2*e))*d*e*e^(I*d*x + I
*c)*log(-2/5*(sqrt(25*I*a^5/(d^2*e))*d*e*e^(2*I*d*x + 2*I*c) - 5*(a^2*e^(2*
I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/a^2) - sqrt(-2
5*I*a^5/(d^2*e))*d*e*e^(I*d*x + I*c)*log(2/5*(sqrt(-25*I*a^5/(d^2*e))*d*e*e
^(2*I*d*x + 2*I*c) + 5*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^
(-2*I*d*x - 2*I*c)/a^2) + sqrt(-25*I*a^5/(d^2*e))*d*e*e^(I*d*x + I*c)*log(-
2/5*(sqrt(-25*I*a^5/(d^2*e))*d*e*e^(2*I*d*x + 2*I*c) - 5*(a^2*e^(2*I*d*x +
2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
```

```
+ 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/a^2) + 2*(-8*I*a^2*e^(
2*I*d*x + 2*I*c) - 10*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-I*d*x - I*c)/(d*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{\sqrt{e\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/sqrt(e*sec(d*x + c)), x)
```

3.411 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=362

$$-\frac{i\sqrt{2}a^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}+\frac{i\sqrt{2}a^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}+\frac{ia^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\right.}{\sqrt{}}$$

```
[Out] ((-I)*Sqrt[2]*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]
])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(d*e^(3/2)) + (I*Sqrt[2]*a^(5/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])])/(d*e^(3/2)) + (I*a^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x
])])/(Sqrt[2]*d*e^(3/2)) - (I*a^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
+ d*x])])/(Sqrt[2]*d*e^(3/2)) - (((4*I)/3)*a*(a + I*a*Tan[c + d*x])^(3/2))/
(d*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.329807, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3496, 3495, 297, 1162, 617, 204, 1165, 628}

$$-\frac{i\sqrt{2}a^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}+\frac{i\sqrt{2}a^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}+\frac{ia^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\right.}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*Sqrt[2]*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]
])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(d*e^(3/2)) + (I*Sqrt[2]*a^(5/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])])/(d*e^(3/2)) + (I*a^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x
])])/(Sqrt[2]*d*e^(3/2)) - (I*a^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
+ d*x])])/(Sqrt[2]*d*e^(3/2)) - (((4*I)/3)*a*(a + I*a*Tan[c + d*x])^(3/2))/
(d*(e*Sec[c + d*x])^(3/2))
```

Rule 3496

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(
n - 1))/(f*m), x] - Dist[(b^2*(m + 2*n - 2))/(d^2*m), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
```

```
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} - \frac{a^2\int\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx}{e^2}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} + \frac{(4ia^3)\operatorname{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} - \frac{(2ia^3)\operatorname{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{de} + \frac{(2ia^3)\,\mathrm{Su}}{}$$

$$= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} + \frac{(ia^3)\operatorname{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{de^2} + \frac{(ia^3)}{}$$

$$= \frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}de^{3/2}} - \frac{ia^{5/2}\log\left(a+\right.}{}$$

$$= -\frac{i\sqrt{2}a^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}} + \frac{i\sqrt{2}a^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}} + \frac{ia}{}$$

Mathematica [A] time = 2.41451, size = 355, normalized size = 0.98

$$\frac{e(a+ia\tan(c+dx))^{5/2}\left(-\frac{4}{3}i(\cos(c)-i\sin(c))\cos(dx)+\frac{4}{3}(\cos(c)-i\sin(c))\sin(dx)+\frac{2(\cos(2c)-i\sin(2c))\sqrt{\tan\left(\frac{dx}{2}\right)+i}\left(\sqrt{-\sin\right.}{}\right.}{d(\cos(dx)+i\sin(d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (e*(((-4*I)/3)*Cos[d*x]*(Cos[c] - I*Sin[c]) + (4*(Cos[c] - I*Sin[c])*Sin[d*
x])/3 + (2*(ArcTan[(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(S
qrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - S
in[c]]*Sqrt[-1 - I*Cos[c] + Sin[c]] - ArcTan[(Sqrt[-1 - I*Cos[c] + Sin[c]]*
Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]
])]*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] -
I*Sin[2*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[-1 -
 I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]))*(a + I*a*Tan[c + d*x])^(5/2))/
(d*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.331, size = 323, normalized size = 0.9

$$-\frac{a^2}{3d(i\sin(dx+c)+\cos(dx+c)-1)\cos(dx+c)}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(-3i\operatorname{Artanh}\left(\frac{\cos(dx+c)+1-}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x)
```

```
[Out] -1/3/d*a^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(-3*I*arctanh(1/2
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*(1/(cos(d*x+c)+1))^(1/
```

```
2)*sin(d*x+c)+3*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*
x+c)))*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+8*I*cos(d*x+c)^2-3*arctanh(1/2*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*(1/(cos(d*x+c)+1))^(1/2)
*sin(d*x+c)-3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)
))*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-4*I*cos(d*x+c)-8*cos(d*x+c)*sin(d*x+
c)-4*I+4*sin(d*x+c))/(I*sin(d*x+c)+cos(d*x+c)-1)/cos(d*x+c)/(e/cos(d*x+c))^
(3/2)
```

Maxima [B] time = 2.43425, size = 2014, normalized size = 5.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/12*(-6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*sqrt(2)*a^2*arctan2(sqrt(2)*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
) + 6*sqrt(2)*a^2*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*I*sqrt(2)*a^2*log(2*sqrt(2)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) - 3*I*sqrt(2)*a^2*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(s
qrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*sqrt(2)*a^2*log(2*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*sqrt(2)*a
^2*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2
```

```
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2) + 16*I*a^2*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*a^
2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)/(d*e^(3/2))
```

Fricas [B] time = 2.27049, size = 1539, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(3*d*e^2*sqrt(4*I*a^5/(d^2*e^3))*log((I*d*e^2*sqrt(4*I*a^5/(d^2*e^3))*e
^(2*I*d*x + 2*I*c) + 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^
(-2*I*d*x - 2*I*c)/a^2) - 3*d*e^2*sqrt(4*I*a^5/(d^2*e^3))*log((-I*d*e^2*sqr
t(4*I*a^5/(d^2*e^3))*e^(2*I*d*x + 2*I*c) + 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/
2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/a^2) + 3*d*e^2*sqrt(-4*I*a^5/(d^2*
e^3))*log((I*d*e^2*sqrt(-4*I*a^5/(d^2*e^3))*e^(2*I*d*x + 2*I*c) + 2*(a^2*e^
(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d
*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/a^2) - 3*d*
e^2*sqrt(-4*I*a^5/(d^2*e^3))*log((-I*d*e^2*sqrt(-4*I*a^5/(d^2*e^3))*e^(2*I*
d*x + 2*I*c) + 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*
d*x - 2*I*c)/a^2) + 2*(-4*I*a^2*e^(2*I*d*x + 2*I*c) - 4*I*a^2)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*
I*c))/(d*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan(dx + c) + a)^{\frac{5}{2}}}{(e \sec(dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(3/2), x)
```

3.412 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Rubi [A] time = 0.0747706, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3488}

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx=-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

Mathematica [A] time = 0.0812213, size = 38, normalized size = 1.

$$-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Maple [B] time = 0.294, size = 88, normalized size = 2.3

$$-\frac{2a^2\left(2i(\cos(dx+c))^2-2\cos(dx+c)\sin(dx+c)-i\right)(\cos(dx+c))^3}{5de^5}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{e}{\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x)
```

```
[Out] -2/5/d*a^2*(2*I*cos(d*x+c)^2-2*cos(d*x+c)*sin(d*x+c)-I)*(a*(I*sin(d*x+c)+co
s(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^3/e^5
```

Maxima [B] time = 1.80131, size = 103, normalized size = 2.71

$$-\frac{2i\,a^{\frac{5}{2}}\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1\right)^{\frac{5}{2}}}{5\,de^{\frac{5}{2}}\left(-\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -2/5*I*a^(5/2)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1)^(5/2)/(d*e^(5/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 - 1)^(5/2))
```

Fricas [B] time = 2.46897, size = 213, normalized size = 5.61

$$\frac{2\left(-i\,a^{2}e^{(3i\,dx+3i\,c)}-i\,a^{2}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}}{5\,de^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/5*(-I*a^2*e^(3*I*d*x + 3*I*c) - I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c)/(
d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(5/2), x)
```

3.413 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] (((-4*I)/21)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(3/2))
 - (((2*I)/7)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(7/2))
```

Rubi [A] time = 0.153757, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3497, 3488}

$$-\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((-4*I)/21)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(3/2))
 - (((2*I)/7)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(7/2))
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx &= -\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}+\frac{(2a)\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}\\ &= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.39799, size = 92, normalized size = 1.14

$$-\frac{2a^2(2\tan(c+dx)+5i)\sqrt{a+ia\tan(c+dx)}(\cos(2(c+2dx))+i\sin(2(c+2dx)))}{21de^2(\cos(dx)+i\sin(dx))^2(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^2*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)])*(5*I + 2*Tan[c + d*x])*
Sqrt[a + I*a*Tan[c + d*x]])/(21*d*e^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*
Sin[d*x])^2)
```

Maple [A] time = 0.305, size = 105, normalized size = 1.3

$$-\frac{2\,a^2\left(6\,i\,(\cos(dx+c))^3-6\,(\cos(dx+c))^2\sin(dx+c)-i\cos(dx+c)-2\,\sin(dx+c)\right)(\cos(dx+c))^4}{21\,de^7}\sqrt{\frac{a\,(i\sin(dx\cdots}{\mathrm{co}\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x)
```

```
[Out] -2/21/d*a^2*(6*I*cos(d*x+c)^3-6*cos(d*x+c)^2*sin(d*x+c)-I*cos(d*x+c)-2*sin(
d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(7/2)
*cos(d*x+c)^4/e^7
```

Maxima [A] time = 2.07442, size = 127, normalized size = 1.57

$$\frac{\left(-7i\,a^2\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\,a^2\cos\left(\frac{7}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+7\,a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,a^2\sin\left(\cdots\right.}{21\,de^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] 1/21*(-7*I*a^2*cos(3/2*d*x + 3/2*c) - 3*I*a^2*cos(7/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 7*a^2*sin(3/2*d*x + 3/2*c) + 3*a^2*sin(7/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/(d*e^(7/2))
```

Fricas [A] time = 2.32796, size = 240, normalized size = 2.96

$$\frac{\left(-3i\,a^2e^{(4i\,dx+4i\,c)}-10i\,a^2e^{(2i\,dx+2i\,c)}-7i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}}{21\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/21*(-3*I*a^2*e^(4*I*d*x + 4*I*c) - 10*I*a^2*e^(2*I*d*x + 2*I*c) - 7*I*a^2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/
2*I*d*x + 3/2*I*c)/(d*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}}{(e\sec{(dx+c)})^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(7/2), x)
```

3.414 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}}-\frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

```
[Out] (((-16*I)/45)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*Sqrt[e*Sec[c + d*x]])
- (((8*I)/45)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(5/2)
) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(9/2))
```

Rubi [A] time = 0.221543, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3497, 3488}

$$-\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}}-\frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (((-16*I)/45)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*Sqrt[e*Sec[c + d*x]])
- (((8*I)/45)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(5/2)
) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(9/2))
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx &= -\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}+\frac{(4a)\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx}{9e^2}\\
&= -\frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}+\frac{\left(8a^2\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{45e^4}\\
&= -\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}}-\frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}
\end{aligned}$$

Mathematica [A] time = 0.459157, size = 104, normalized size = 0.83

$$\frac{a^2\sqrt{a+ia\tan(c+dx)}(-20i\sin(2(c+dx))+25\cos(2(c+dx))+9)(\sin(2(c+2dx))-i\cos(2(c+2dx)))}{45de^4(\cos(dx)+i\sin(dx))^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (a^2*(9 + 25*Cos[2*(c + d*x)] - (20*I)*Sin[2*(c + d*x)])*((-I)*Cos[2*(c + 2
*d*x)] + Sin[2*(c + 2*d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(45*d*e^4*Sqrt[e*S
ec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.329, size = 115, normalized size = 0.9

$$-\frac{2a^2\left(10i(\cos(dx+c))^4-10(\cos(dx+c))^3\sin(dx+c)-i(\cos(dx+c))^2-4\cos(dx+c)\sin(dx+c)+8i\right)(\cos(dx+}{45de^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x)
```

```
[Out] -2/45/d*a^2*(10*I*cos(d*x+c)^4-10*cos(d*x+c)^3*sin(d*x+c)-I*cos(d*x+c)^2-4*
cos(d*x+c)*sin(d*x+c)+8*I)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(
e/cos(d*x+c))^(9/2)*cos(d*x+c)^5/e^9
```

Maxima [A] time = 2.19263, size = 130, normalized size = 1.04

$$\frac{\left(-5ia^2\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)-18ia^2\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)-45ia^2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5a^2\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+18a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)}{90de^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="maxim
a")
```

```
[Out] 1/90*(-5*I*a^2*cos(9/2*d*x + 9/2*c) - 18*I*a^2*cos(5/2*d*x + 5/2*c) - 45*I*
a^2*cos(1/2*d*x + 1/2*c) + 5*a^2*sin(9/2*d*x + 9/2*c) + 18*a^2*sin(5/2*d*x
+ 5/2*c) + 45*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(9/2))
```

Fricas [A] time = 2.30069, size = 284, normalized size = 2.27

$$\frac{\left(-5ia^2e^{(6idx+6ic)}-23ia^2e^{(4idx+4ic)}-63ia^2e^{(2idx+2ic)}-45ia^2\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{e}{e^{(2idx+2ic)}+1}}e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}}{90de^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="frica
s")
```

```
[Out] 1/90*(-5*I*a^2*e^(6*I*d*x + 6*I*c) - 23*I*a^2*e^(4*I*d*x + 4*I*c) - 63*I*a^
2*e^(2*I*d*x + 2*I*c) - 45*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/
(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}}{(e\sec{(dx+c)})^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(9/2), x)
```

3.415 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal. Leaf size=169

$$\frac{32ia^3\sqrt{e\sec(c+dx)}}{77de^6\sqrt{a+ia\tan(c+dx)}}-\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}}-\frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

```
[Out] (((32*I)/77)*a^3*Sqrt[e*Sec[c + d*x]])/(d*e^6*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((16*I)/77)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*(e*Sec[c + d*x])^(3/2)
) - (((12*I)/77)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(7
/2)) - (((2*I)/11)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(11/2)
)
```

Rubi [A] time = 0.311941, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3497, 3488}

$$\frac{32ia^3\sqrt{e\sec(c+dx)}}{77de^6\sqrt{a+ia\tan(c+dx)}}-\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}}-\frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(11/2),x]
```


```
[Out] (((32*I)/77)*a^3*Sqrt[e*Sec[c + d*x]])/(d*e^6*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((16*I)/77)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*(e*Sec[c + d*x])^(3/2)
) - (((12*I)/77)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(7
/2)) - (((2*I)/11)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(11/2)
)
```


Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}} + \frac{(6a)\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}dx}{11e^2}$$

$$= -\frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}} + \frac{(24a^2)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}dx}{77e^4}$$

$$= -\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}} - \frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}} +$$

$$= \frac{32ia^3\sqrt{e\sec(c+dx)}}{77de^6\sqrt{a+ia\tan(c+dx)}} - \frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}} - \frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}} -$$

Mathematica [A] time = 0.645988, size = 121, normalized size = 0.72

$$\frac{a^2\sqrt{a+ia\tan(c+dx)}(-22\sin(c+dx)+42\sin(3(c+dx))-55i\cos(c+dx)+35i\cos(3(c+dx)))(\cos(2(c+2dx))+i}{154de^5(\cos(dx)+i\sin(dx))^2\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (a^2*((-55*I)*Cos[c + d*x] + (35*I)*Cos[3*(c + d*x)] - 22*Sin[c + d*x] + 42
*Sin[3*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)])*Sqrt[a + I*a
*Tan[c + d*x]])/(154*d*e^5*Sqrt[e*Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] time = 0.477, size = 132, normalized size = 0.8

$$-\frac{2a^2\left(14i(\cos(dx+c))^5-14\sin(dx+c)(\cos(dx+c))^4-i(\cos(dx+c))^3-6(\cos(dx+c))^2\sin(dx+c)-8i\cos(}{77de^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x)
```

```
[Out] -2/77/d*a^2*(14*I*cos(d*x+c)^5-14*sin(d*x+c)*cos(d*x+c)^4-I*cos(d*x+c)^3-6*
cos(d*x+c)^2*sin(d*x+c)-8*I*cos(d*x+c)-16*sin(d*x+c))*(a*(I*sin(d*x+c)+cos(
d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(11/2)*cos(d*x+c)^6/e^11
```

Maxima [A] time = 1.91745, size = 167, normalized size = 0.99

$$\frac{\left(-7ia^2\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)-33ia^2\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)-77ia^2\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)+77ia^2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7a^2\sin\left(\frac{11}{2}dx\right.\right.}{308de^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="maxi
ma")
```

```
[Out] 1/308*(-7*I*a^2*cos(11/2*d*x + 11/2*c) - 33*I*a^2*cos(7/2*d*x + 7/2*c) - 77
*I*a^2*cos(3/2*d*x + 3/2*c) + 77*I*a^2*cos(1/2*d*x + 1/2*c) + 7*a^2*sin(11/
```

```
2*d*x + 11/2*c) + 33*a^2*sin(7/2*d*x + 7/2*c) + 77*a^2*sin(3/2*d*x + 3/2*c)
 + 77*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(11/2))
```

Fricas [A] time = 2.34185, size = 288, normalized size = 1.7

$$\frac{\left(-7i\,a^2e^{(8i\,dx+8i\,c)} - 40i\,a^2e^{(6i\,dx+6i\,c)} - 110i\,a^2e^{(4i\,dx+4i\,c)} + 77i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{308\,de^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="fric
as")
```

```
[Out] 1/308*(-7*I*a^2*e^(8*I*d*x + 8*I*c) - 40*I*a^2*e^(6*I*d*x + 6*I*c) - 110*I*
a^2*e^(4*I*d*x + 4*I*c) + 77*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{(e\sec(dx+c))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="giac
")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(11/2), x)
```

3.416 $\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=369

$$\frac{ie^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d}-\frac{ie^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}-\frac{ie}{}$$

```
[Out] (I*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]
*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]*d) - (I*e^(5/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sq
rt[2]*Sqrt[a]*d) - ((I/2)*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
 I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*
x])])/(Sqrt[2]*Sqrt[a]*d) + ((I/2)*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*T
an[c + d*x])])/(Sqrt[2]*Sqrt[a]*d) - (I*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I
*a*Tan[c + d*x]])/(a*d)
```

Rubi [A] time = 0.307687, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3501, 3495, 297, 1162, 617, 204, 1165, 628}

$$\frac{ie^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d}-\frac{ie^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d}-\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}-\frac{ie}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```



```
[Out] (I*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]
*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]*d) - (I*e^(5/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sq
rt[2]*Sqrt[a]*d) - ((I/2)*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
 I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*
x])])/(Sqrt[2]*Sqrt[a]*d) + ((I/2)*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*T
an[c + d*x])])/(Sqrt[2]*Sqrt[a]*d) - (I*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I
*a*Tan[c + d*x]])/(a*d)
```


Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{e^2\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx}{2a}$$

$$= -\frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{(2ie^4)\,\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d}$$

$$= -\frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{(ie^3)\,\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d} - \ldots$$

$$= -\frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{(ie^2)\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d} \ldots$$

$$= -\frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d} + \frac{ie^{5/2}\log\left(a+\ldots\right.}{\ldots}$$

$$= \frac{ie^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{ie^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{ie^{5/2}\log\left(a-\ldots\right.}{\ldots}$$

Mathematica [A] time = 1.85106, size = 366, normalized size = 0.99

$$\frac{e^2(\tan(c+dx)-i)\sqrt{e\sec(c+dx)}\left(\sqrt{-\sin(c)+i\cos(c)-1}\left(i\sqrt{\sin(c)+i\cos(c)-1}\sqrt{\tan\left(\frac{dx}{2}\right)+i}\cos(c+dx)\tan^{-1}\left(\ldots\right.\right.\right.}{d\sqrt{-\sin(\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (e^2*Sqrt[e*Sec[c + d*x]]*((-I)*ArcTan[(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I
 - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Co
s[c + d*x]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I
 + Tan[(d*x)/2]] + Sqrt[-1 + I*Cos[c] - Sin[c]]*(Sqrt[-1 - I*Cos[c] + Sin[c
]]*Sqrt[I - Tan[(d*x)/2]] + I*ArcTan[(Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I -
 Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[
c + d*x]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]]))*(-I + Tan[c
+ d*x]))/(d*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[
I - Tan[(d*x)/2]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.35, size = 316, normalized size = 0.9

$$-\frac{(\cos(dx+c))^2(\cos(dx+c)-1)^3}{2\,ad\,(\sin(dx+c))^5\,(i\sin(dx+c)+\cos(dx+c)-1)}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{5}{2}}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(i\text{Artanh}\left(\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d/a*(e/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(cos(d*x+c)-1)^3*(a*(I*sin(d*x+c
)+cos(d*x+c))/cos(d*x+c))^(1/2)*(I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(co
```

```
s(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)-I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+2*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
)-arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c
)-arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c
)+2*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+2*(1/(cos(d*x+c)+1))^(1/2))/sin(d*x
+c)^5/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(cos(d*x+c)+1))^(5/2)
```

Maxima [B] time = 2.26565, size = 3069, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -(128*e^2*cos(3/2*d*x + 3/2*c) + 128*I*e^2*sin(3/2*d*x + 3/2*c) + (16*sqrt(
2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*I*
sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
16*sqrt(2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 1) + (16*sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 16*I*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*e^2)*arctan2(sqrt(2)*cos(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + (16*sqrt(2)*e^2*c
os(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*I*sqrt(2)*
e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(
2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) - 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 1) + (16*sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 16*I*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (16*I*sqrt(2)*e^2*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 16*sqrt(2)*e^2*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*I*sqrt(2)*e^2
)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (-16*I*sqrt(2)*e^2
*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*
e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 16*I*sqr
t(2)*e^2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
, -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + c
os(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (8*sqrt(
2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*I*s
qrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8
*sqrt(2)*e^2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*
```

```
c), cos(3/2*d*x + 3/2*c))) + 1) + (8*sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*I*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*sqrt(2)*e^2)*log(-2*sqrt(2)*sin(
2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)
- (-8*I*sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 8*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) - 8*I*sqrt(2)*e^2)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 2) - (8*I*sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) - 8*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 8*I*sqrt(2)*e^2)*log(2*cos(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 2) - (-8*I*sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*I*sqrt(2)*e^2)*log(2*cos(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - (8*I*sqrt(2)*e^2*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*sqrt(2)*e^2*sin(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*I*sqrt(2)*e^2)*log(2*co
s(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*sqrt(a)*sqrt(e)/((-6
4*I*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 64*a*s
in(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 64*I*a)*d)
```

Fricas [A] time = 2.29068, size = 1420, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(-4*I*e^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(3/2*I*d*x + 3/2*I*c) + sqrt(I*e^5/(a*d^2))*a*d*log(-2*(I*sqrt(I*e
^5/(a*d^2))*a*d*e^(2*I*d*x + 2*I*c) - (e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x
 + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^2) - sqrt(I*e^5/(a*d^2))*a*d*log(-2*(-I
*sqrt(I*e^5/(a*d^2))*a*d*e^(2*I*d*x + 2*I*c) - (e^2*e^(2*I*d*x + 2*I*c) + e
^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(
3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^2) + sqrt(-I*e^5/(a*d^2))*a*d*
log(-2*(I*sqrt(-I*e^5/(a*d^2))*a*d*e^(2*I*d*x + 2*I*c) - (e^2*e^(2*I*d*x +
```

```
2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^2) - sqrt(-I*e^5/(a*
d^2))*a*d*log(-2*(-I*sqrt(-I*e^5/(a*d^2))*a*d*e^(2*I*d*x + 2*I*c) - (e^2*e^
(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d
*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^2))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.417 $\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=483

$$-\frac{i\sqrt{2}\sqrt{a}e^{3/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{2}\sqrt{a}e^{3/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{a}\ldots}{\ldots}$$

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[
c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(d*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (I*Sqrt[2]*Sqrt[a]*e^(3/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])]*Sec[c + d*x])/(d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]]) + (I*Sqrt[a]*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[
c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[
c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
 - (I*Sqrt[a]*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c +
 d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c +
 d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.352719, antiderivative size = 483, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3499, 3495, 297, 1162, 617, 204, 1165, 628}

$$-\frac{i\sqrt{2}\sqrt{a}e^{3/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{2}\sqrt{a}e^{3/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{a}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[
c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(d*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (I*Sqrt[2]*Sqrt[a]*e^(3/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])]*Sec[c + d*x])/(d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]]) + (I*Sqrt[a]*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[
c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[
c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
 - (I*Sqrt[a]*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c +
 d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c +
 d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3499

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[(d*Sec[e + f*x])/(Sqrt[a - b*Tan[e + f*x]]*Sqr
t[a + b*Tan[e + f*x]]), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
```

```
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(e\sec(c+dx))\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{\left(4iae^3\sec(c+dx)\right)\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{\left(2iae^2\sec(c+dx)\right)\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(2iae^2\sec(c+dx)\right)\text{Subst}\left(\right.}{d\sqrt{a-ia\tan(c+d}}$$

$$= \frac{(iae\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(iae\sec(c+dx))\,\text{Subst}\left(\int\right.}{d\sqrt{a-ia\tan(c+}}$$

$$= \frac{i\sqrt{a}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{}}{}$$

$$= -\frac{i\sqrt{2}\sqrt{a}e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{i\sqrt{2}\sqrt{a}e^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-i}}{\sqrt{a}\sqrt{e\,\text{se}}}\right.}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a}}$$

Mathematica [A] time = 1.10315, size = 302, normalized size = 0.63

$$-\frac{2e\sqrt{\tan\left(\frac{dx}{2}\right)+i}(\cos(dx)+i\sin(dx))\sqrt{e\sec(c+dx)}\left(\sqrt{-\sin(c)-i\cos(c)-1}\sqrt{\sin(c)-i\cos(c)-1}\tan^{-1}\left(\frac{\sqrt{\sin(c)-i\cos}}{\sqrt{\sin(c)+i\cos}}\right.\right.}{d\sqrt{-\sin(c)-i\cos(c)-1}\sqrt{\sin(c)+i\cos(c)-1}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*e*Sqrt[e*Sec[c + d*x]]*(ArcTan[(Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I - T
an[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-
1 - I*Cos[c] - Sin[c]]*Sqrt[-1 - I*Cos[c] + Sin[c]] - ArcTan[(Sqrt[-1 + I*C
os[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[
I + Tan[(d*x)/2]])]*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c
]])*(Cos[d*x] + I*Sin[d*x])*Sqrt[I + Tan[(d*x)/2]])/(d*Sqrt[-1 - I*Cos[c] -
 Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]*Sqrt[a + I*a*T
an[c + d*x]])
```

Maple [A] time = 0.333, size = 237, normalized size = 0.5

$$-\frac{(\cos(dx+c))^2(\cos(dx+c)-1)^2}{ad(\sin(dx+c))^3(i\sin(dx+c)+\cos(dx+c)-1)}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(i\text{Artanh}\left(\text{cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/d/a*(e/cos(d*x+c))^(3/2)*cos(d*x+c)^2*(cos(d*x+c)-1)^2*(a*(I*sin(d*x+c)+
cos(d*x+c))/cos(d*x+c))^(1/2)*(I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(
```

```
d*x+c)+1-sin(d*x+c)))-I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+
sin(d*x+c)))-arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c))
)-arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))))/sin(d*x+
c)^3/(1/(cos(d*x+c)+1))^(3/2)/(I*sin(d*x+c)+cos(d*x+c)-1)
```

Maxima [A] time = 2.04854, size = 980, normalized size = 2.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/4*(2*I*sqrt(2)*e*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1
/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) +
 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e*arctan2(sqrt(2)*cos(
1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e*arc
tan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) -
 2*sqrt(2)*e*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*c
os(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + 2*sqrt(2)*e*arctan2(-sqrt(2)*sin(
1/2*d*x + 1/2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x +
c) + 1) + I*sqrt(2)*e*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(
sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2
*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 1) - I*sqrt(2)*e*log(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*
x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d*x +
c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - sqrt(2)*e*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*e*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - sqrt(2)*e*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + sqrt(2)*e*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sqrt
(e)/(sqrt(a)*d)
```

Fricas [A] time = 2.16165, size = 1239, normalized size = 2.57

$$-\frac{1}{2}\sqrt{\frac{4ie^3}{ad^2}}\log\left(\frac{\left(ad\sqrt{\frac{4ie^3}{ad^2}}e^{(2idx+2ic)}+2\left(ee^{(2idx+2ic)}+e\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{e}{e^{(2idx+2ic)}+1}}e^{\left(\frac{3}{2}idx+\frac{3}{2}ic\right)}\right)e^{(-2idx-2ic)}}{e}\right)+\frac{1}{2}\sqrt{\frac{4ie^3}{ad^2}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(4*I*e^3/(a*d^2))*log((a*d*sqrt(4*I*e^3/(a*d^2))*e^(2*I*d*x + 2*I*
c) + 2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e
) + 1/2*sqrt(4*I*e^3/(a*d^2))*log(-(a*d*sqrt(4*I*e^3/(a*d^2))*e^(2*I*d*x +
2*I*c) - 2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
```

```
rt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*
c)/e) + 1/2*sqrt(-4*I*e^3/(a*d^2))*log((a*d*sqrt(-4*I*e^3/(a*d^2))*e^(2*I*d
*x + 2*I*c) + 2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x -
 2*I*c)/e) - 1/2*sqrt(-4*I*e^3/(a*d^2))*log(-(a*d*sqrt(-4*I*e^3/(a*d^2))*e^
(2*I*d*x + 2*I*c) - 2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I
*d*x - 2*I*c)/e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{\sqrt{ia\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.418 $\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=36

$$\frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] ((2*I)*Sqrt[e*Sec[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.068347, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3488}

$$\frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[e*Sec[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 0.0504171, size = 36, normalized size = 1.

$$\frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[e*Sec[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.308, size = 74, normalized size = 2.1

$$\frac{2\,i\cos(dx+c)}{ad\,(i\sin(dx+c)+\cos(dx+c))}\sqrt{\frac{e}{\cos(dx+c)}}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2*I/d/a*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
*cos(d*x+c)/(I*sin(d*x+c)+cos(d*x+c))
```

Maxima [B] time = 1.6033, size = 103, normalized size = 2.86

$$\frac{2i\sqrt{e}\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{\sqrt{a}d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 2*I*sqrt(e)*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)/(sqrt(a)*d*sqrt(
-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 1))
```

Fricas [B] time = 1.96547, size = 176, normalized size = 4.89

$$\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^
(2*I*d*x + 2*I*c) + I)*e^(-1/2*I*d*x - 1/2*I*c)/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a(i\tan(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e \sec (dx + c)}}{\sqrt{i\,a \tan (dx + c) + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*sec(d*x + c))/sqrt(I*a*tan(d*x + c) + a), x)
```

3.419 $\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}$$

```
[Out] ((2*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.138318, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3502, 3488}

$$\frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((2*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx&=\frac{2i}{3d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{2\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{3a}\\&=\frac{2i}{3d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.101304, size = 48, normalized size = 0.6

$$\frac{4\tan(c+dx)-2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (-2*I + 4*Tan[c + d*x])/(3*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [A] time = 0.319, size = 85, normalized size = 1.1

$$-\frac{2\,i\cos(dx+c)-4\,\sin(dx+c)}{3\,ad\,(i\sin(dx+c)+\cos(dx+c))}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\frac{1}{\sqrt{\frac{e}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/a*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(I*cos(d*x+c)-2*sin
(d*x+c))/(I*sin(d*x+c)+cos(d*x+c))/(e/cos(d*x+c))^(1/2)
```

Maxima [A] time = 1.87588, size = 108, normalized size = 1.35

$$\frac{i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\cos\left(\frac{1}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,c\right.\right.\right.}{3\,\sqrt{a}d\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/3*(I*cos(3/2*d*x + 3/2*c) - 3*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d*sqrt(e))
```

Fricas [A] time = 2.02834, size = 220, normalized size = 2.75

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(4i\,dx+4i\,c)}-2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-3
*I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3/2*I*d*x - 3/2*I
*c)/(a*d*e)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(i\tan\left(c+dx\right)+1\right)}\sqrt{e\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(I*tan(c + d*x) + 1))*sqrt(e*sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\sec\left(dx+c\right)}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(e*sec(d*x + c))*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.420 $\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=121

$$\frac{16i\sqrt{e\sec(c+dx)}}{15de^2\sqrt{a+ia\tan(c+dx)}}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{15ad(e\sec(c+dx))^{3/2}}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

```
[Out] ((2*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I)/
15)*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((8*I)/15)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.220198, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3502, 3497, 3488}

$$\frac{16i\sqrt{e\sec(c+dx)}}{15de^2\sqrt{a+ia\tan(c+dx)}}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{15ad(e\sec(c+dx))^{3/2}}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((2*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I)/
15)*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((8*I)/15)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(3/2))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{4\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{5a}$$

$$= \frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{8i\sqrt{a+ia\tan(c+dx)}}{15ad(e\sec(c+dx))^{3/2}} + \frac{8\int}{}$$

$$= \frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{16i\sqrt{e\sec(c+dx)}}{15de^2\sqrt{a+ia\tan(c+dx)}} -$$

Mathematica [A] time = 0.250152, size = 68, normalized size = 0.56

$$-\frac{i\sec^2(c+dx)(4i\sin(2(c+dx))+\cos(2(c+dx))-15)}{15d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-I/15)*Sec[c + d*x]^2*(-15 + Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)]))/
(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.319, size = 105, normalized size = 0.9

$$\frac{2\left(\cos(dx+c)\right)^2\left(3i\left(\cos(dx+c)\right)^3+3\left(\cos(dx+c)\right)^2\sin(dx+c)+4i\cos(dx+c)+8\sin(dx+c)\right)}{15ade^3}\left(\frac{e}{\cos(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d/a*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)*cos(d*x+c)^2*(3*I*cos(d*x+c)^3+3*cos(d*x+c)^2*sin(d*x+c)+4*I*cos(d*x+c)+8
*sin(d*x+c))/e^3
```

Maxima [A] time = 1.93319, size = 176, normalized size = 1.45

$$\frac{3i\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5i\cos\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+30i\cos\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/30*(3*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c),
cos(5/2*d*x + 5/2*c))) + 30*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c))) + 3*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x +
5/2*c), cos(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
```

```
os(5/2*d*x + 5/2*c))))/(sqrt(a)*d*e^(3/2))
```

Fricas [A] time = 2.06385, size = 265, normalized size = 2.19

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(6i\,dx+6i\,c)}+25i\,e^{(4i\,dx+4i\,c)}+33i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{30\,ade^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/30*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-
5*I*e^(6*I*d*x + 6*I*c) + 25*I*e^(4*I*d*x + 4*I*c) + 33*I*e^(2*I*d*x + 2*I*
c) + 3*I)*e^(-5/2*I*d*x - 5/2*I*c)/(a*d*e^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(i\tan\left(c+dx\right)+1\right)}\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(I*tan(c + d*x) + 1))*(e*sec(c + d*x))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.421 $\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=165

$$-\frac{32i\sqrt{a+ia\tan(c+dx)}}{35ade^2\sqrt{e\sec(c+dx)}}+\frac{16i}{35de^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{12i\sqrt{a+ia\tan(c+dx)}}{35ad(e\sec(c+dx))^{5/2}}+\frac{2\cdots}{7d\sqrt{a+ia\tan(c+\cdots}}$$

```
[Out] ((2*I)/7)/(d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((16*I)/3
5)/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((12*I)/35)*S
qrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(5/2)) - (((32*I)/35)*Sqrt
[a + I*a*Tan[c + d*x]])/(a*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.290697, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3502, 3497, 3488}

$$-\frac{32i\sqrt{a+ia\tan(c+dx)}}{35ade^2\sqrt{e\sec(c+dx)}}+\frac{16i}{35de^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{12i\sqrt{a+ia\tan(c+dx)}}{35ad(e\sec(c+dx))^{5/2}}+\frac{2\cdots}{7d\sqrt{a+ia\tan(c+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((2*I)/7)/(d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((16*I)/3
5)/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((12*I)/35)*S
qrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(5/2)) - (((32*I)/35)*Sqrt
[a + I*a*Tan[c + d*x]])/(a*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{6\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx}{7a}$$

$$= \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{12i\sqrt{a+ia\tan(c+dx)}}{35ad(e\sec(c+dx))^{5/2}} + \frac{24\int \cdots}{\cdots}$$

$$= \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{16i}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan\cdots}}$$

$$= \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{16i}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan\cdots}}$$

Mathematica [A] time = 0.339108, size = 79, normalized size = 0.48

$$-\frac{i(\cos(2(c+dx))+35i\tan(c+dx)+3i\sin(3(c+dx))\sec(c+dx)+17)}{35de^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-I/35)*(17 + Cos[2*(c + d*x)] + (3*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (35
*I)*Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.332, size = 115, normalized size = 0.7

$$\frac{2(\cos(dx+c))^3\left(5i(\cos(dx+c))^4+5(\cos(dx+c))^3\sin(dx+c)+2i(\cos(dx+c))^2+8\cos(dx+c)\sin(dx+c)-16\cdots\right.}{35ade^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/35/d/a*cos(d*x+c)^3*(e/cos(d*x+c))^(5/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos
(d*x+c))^(1/2)*(5*I*cos(d*x+c)^4+5*cos(d*x+c)^3*sin(d*x+c)+2*I*cos(d*x+c)^2
+8*cos(d*x+c)*sin(d*x+c)-16*I)/e^5
```

Maxima [A] time = 1.9702, size = 240, normalized size = 1.45

$$\frac{5i\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)-7i\cos\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+35i\cos\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/140*(5*I*cos(7/2*d*x + 7/2*c) - 7*I*cos(5/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c))) + 35*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
```

```
2*d*x + 7/2*c))) - 105*I*cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
+ 7/2*c))) + 5*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))))/(sqrt(a)*d*e^(5/2))
```

Fricas [A] time = 2.08237, size = 304, normalized size = 1.84

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(8i\,dx+8i\,c)}-112i\,e^{(6i\,dx+6i\,c)}-70i\,e^{(4i\,dx+4i\,c)}+40i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{\left(-\frac{7}{2}i\,dx-\frac{7}{2}i\,c\right)}}{140\,ade^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/140*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-7*I*e^(8*I*d*x + 8*I*c) - 112*I*e^(6*I*d*x + 6*I*c) - 70*I*e^(4*I*d*x + 4*
I*c) + 40*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7/2*I*d*x - 7/2*I*c)/(a*d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(5/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.422 $\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=206

$$\frac{256i\sqrt{e\sec(c+dx)}}{315de^4\sqrt{a+ia\tan(c+dx)}}-\frac{128i\sqrt{a+ia\tan(c+dx)}}{315ade^2(e\sec(c+dx))^{3/2}}+\frac{32i}{105de^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}}$$

```
[Out] ((2*I)/9)/(d*(e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((32*I)/1
05)/(d*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((256*I)/3
15)*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/63)
*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(7/2)) - (((128*I)/315)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*e^2*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.387039, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3502, 3497, 3488}

$$\frac{256i\sqrt{e\sec(c+dx)}}{315de^4\sqrt{a+ia\tan(c+dx)}}-\frac{128i\sqrt{a+ia\tan(c+dx)}}{315ade^2(e\sec(c+dx))^{3/2}}+\frac{32i}{105de^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```


```
[Out] ((2*I)/9)/(d*(e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((32*I)/1
05)/(d*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((256*I)/3
15)*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/63)
*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(7/2)) - (((128*I)/315)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*e^2*(e*Sec[c + d*x])^(3/2))
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{8\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx}{9a}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}} + \frac{16}{\ }$$

$$= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32i}{105de^2(e\sec(c+dx))^{3/2}\sqrt{a+}}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32i}{105de^2(e\sec(c+dx))^{3/2}\sqrt{a+}}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32i}{105de^2(e\sec(c+dx))^{3/2}\sqrt{a+}}$$

Mathematica [A] time = 0.367011, size = 87, normalized size = 0.42

$$\frac{\sqrt{e\sec(c+dx)}(336\sin(2(c+dx)) + 40\sin(4(c+dx)) - 84i\cos(2(c+dx)) - 5i\cos(4(c+dx)) + 945i)}{1260de^4\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (Sqrt[e*Sec[c + d*x]]*(945*I - (84*I)*Cos[2*(c + d*x)] - (5*I)*Cos[4*(c + d
*x)] + 336*Sin[2*(c + d*x)] + 40*Sin[4*(c + d*x)]))/(1260*d*e^4*Sqrt[a + I*
a*Tan[c + d*x]])
```

Maple [A] time = 0.352, size = 132, normalized size = 0.6

$$\frac{2\,(\cos(dx+c))^4\left(35\,i\,(\cos(dx+c))^5 + 35\,\sin(dx+c)\,(\cos(dx+c))^4 + 8\,i\,(\cos(dx+c))^3 + 48\,(\cos(dx+c))^2\sin(dx\right.}{315\,ade^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/315/d/a*cos(d*x+c)^4*(e/cos(d*x+c))^(7/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/co
s(d*x+c))^(1/2)*(35*I*cos(d*x+c)^5+35*sin(d*x+c)*cos(d*x+c)^4+8*I*cos(d*x+c
)^3+48*cos(d*x+c)^2*sin(d*x+c)+64*I*cos(d*x+c)+128*sin(d*x+c))/e^7
```

Maxima [A] time = 2.02318, size = 305, normalized size = 1.48

$$\frac{35i\cos\left(\frac{9}{2}dx + \frac{9}{2}c\right) - 45i\cos\left(\frac{7}{9}\arctan\left(\sin\left(\frac{9}{2}dx + \frac{9}{2}c\right), \cos\left(\frac{9}{2}dx + \frac{9}{2}c\right)\right)\right) + 252i\cos\left(\frac{5}{9}\arctan\left(\sin\left(\frac{9}{2}dx + \frac{9}{2}c\right.\right.\right.}{\ }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/2520*(35*I*cos(9/2*d*x + 9/2*c) - 45*I*cos(7/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 252*I*cos(5/9*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c))) - 420*I*cos(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*
d*x + 9/2*c))) + 1890*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c))) + 35*sin(9/2*d*x + 9/2*c) + 45*sin(7/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 252*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) + 420*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))) + 1890*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))))/(sqrt(a)*d*e^(7/2))
```

Fricas [A] time = 2.19531, size = 354, normalized size = 1.72

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-45i\,e^{(10i\,dx+10i\,c)}-465i\,e^{(8i\,dx+8i\,c)}+1470i\,e^{(6i\,dx+6i\,c)}+2142i\,e^{(4i\,dx+4i\,c)}+287i\,e^{(2i\,dx+2i\,c)}+35i\right)e}{2520\,ade^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2520*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-45*I*e^(10*I*d*x + 10*I*c) - 465*I*e^(8*I*d*x + 8*I*c) + 1470*I*e^(6*I*d*
x + 6*I*c) + 2142*I*e^(4*I*d*x + 4*I*c) + 287*I*e^(2*I*d*x + 2*I*c) + 35*I)
*e^(-9/2*I*d*x - 9/2*I*c)/(a*d*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{7}{2}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(7/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.423 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=529

$$-\frac{3ie^{7/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{3ie^{7/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{ie^2(e\sec(c+}{ad\sqrt{a+ia\tan}}$$

```
[Out] ((-I)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - ((3*I)
*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*S
qrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + ((3*I)*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqr
t[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*
x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]
]) + (((3*I)/2)*e^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c
 + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c
 + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c +
 d*x]]) - (((3*I)/2)*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*
Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*
Sec[c + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Ta
n[c + d*x]])
```

Rubi [A] time = 0.56732, antiderivative size = 529, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3500, 3498, 3499, 3495, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3ie^{7/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{3ie^{7/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{ie^2(e\sec(c+}{ad\sqrt{a+ia\tan}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] ((-I)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - ((3*I)
*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*S
qrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + ((3*I)*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqr
t[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*
x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]
]) + (((3*I)/2)*e^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c
 + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c
 + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c +
 d*x]]) - (((3*I)/2)*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*
Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*
Sec[c + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Ta
n[c + d*x]])
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
```

```
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3499

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[(d*Sec[e + f*x])/(Sqrt[a - b*Tan[e + f*x]]*Sqr
t[a + b*Tan[e + f*x]]), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}dx = -\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3e^2)\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx}{a^2}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3e^2)\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2a}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3e^3\sec(c+dx))\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{2a\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(6ie^5\sec(c+dx))\,\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{(3ie^4\sec(c+dx))\,\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \cdots$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3ie^3\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}} \cdots\right.}{2d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{3ie^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan \cdots\right.}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c \cdots}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{3ie^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{3ie^{7/2}\,\text{ta}\cdots}{\sqrt{2}\sqrt{a}d\cdots}$$

Mathematica [A] time = 15.9922, size = 405, normalized size = 0.77

$$\frac{ie^2(e\sec(c+dx))^{3/2}(\cos(2(c+dx))+i\sin(2(c+dx)))\left(3i\sqrt{-\sin(c)-i\cos(c)+1}\sqrt{-\sin(c)+i\cos(c)-1}\sqrt{\tan\left(\frac{dx}{2}\right)+\cdots}\right.}{ad\sqrt{-\text{si}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*e^2*(e*Sec[c + d*x])^(3/2)*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*((3*I
)*ArcTan[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[1 - I*C
os[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] - Sin[c]]*Sqrt[-
1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[1 + I*Cos[c] - Sin[c]]
*((3*I)*ArcTan[(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[
-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c
]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)
/2]]*(I + Tan[c + d*x]))))/(a*d*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos
```

```
[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]]*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c
 + d*x]])
```

Maple [B] time = 0.311, size = 1022, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -1/4/d/a^2*(cos(d*x+c)-1)^3*(-3*I*cos(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-6*I*cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-6*I*cos(d*x+c)^2*sin(d*x+c)*arctanh
(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-6*I*cos(d*x+c)^2*s
in(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-3
*I*cos(d*x+c)^2*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+
c)))-4*I*cos(d*x+c)*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+3*I*cos(d*x+c)^2*ar
ctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+3*I*cos(d*x+c
)*sin(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))
)+3*I*cos(d*x+c)*sin(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)+1-sin(d*x+c)))-6*cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1-sin(d*x+c)))-6*cos(d*x+c)^2*sin(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-6*cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+6*cos(d*x+c)^2*sin(d*x+c)*arctanh(1
/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+6*I*cos(d*x+c)^3*arc
tanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+3*I*arctanh(1/
2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+3*arctanh(
1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+3*cos(
d*x+c)*sin(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*
x+c)))+3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*co
s(d*x+c)^2-3*cos(d*x+c)*sin(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1+sin(d*x+c)))-4*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)+3*arctanh(1
/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+3*arctanh
(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+4*(1/(c
os(d*x+c)+1))^(1/2))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x
+c)^3*(e/cos(d*x+c))^(7/2)/(2*I*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-1)/(1/
(cos(d*x+c)+1))^(7/2)/sin(d*x+c)^7
```

Maxima [B] time = 2.2251, size = 2456, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -(48*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*
d*x + 1/2*c) + 1) + 48*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1
, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 48*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1
/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 48*sqrt(2)*e^3*arc
tan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) +
 24*I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
```

```
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 24*
I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 24*I*sq
rt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 24*I*sqrt(2
)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 128*e^3*cos(1/2
*d*x + 1/2*c) + 128*I*e^3*sin(1/2*d*x + 1/2*c) + (48*I*sqrt(2)*e^3*cos(2*d*
x + 2*c) - 48*sqrt(2)*e^3*sin(2*d*x + 2*c) + 48*I*sqrt(2)*e^3)*arctan2(sqrt
(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x + 1/2*c) + cos
(d*x + c) + 1) + (-48*I*sqrt(2)*e^3*cos(2*d*x + 2*c) + 48*sqrt(2)*e^3*sin(2
*d*x + 2*c) - 48*I*sqrt(2)*e^3)*arctan2(-sqrt(2)*sin(1/2*d*x + 1/2*c) + sin
(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + (48*sqrt(2)*
e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 1) + 48*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*si
n(1/2*d*x + 1/2*c) + 1) + 48*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*
c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 48*sqrt(2)*e^3*arctan2(sqrt(2)*
cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 24*I*sqrt(2)
*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 24*I*sqrt(2)*e^3
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 24*I*sqrt(2)*e^3*log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 24*I*sqrt(2)*e^3*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + (24*sqrt(2
)*e^3*cos(2*d*x + 2*c) + 24*I*sqrt(2)*e^3*sin(2*d*x + 2*c) + 24*sqrt(2)*e^3
)*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(sqrt(2)*cos(1/2*d*x
+ 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + si
n(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
1) - (24*sqrt(2)*e^3*cos(2*d*x + 2*c) + 24*I*sqrt(2)*e^3*sin(2*d*x + 2*c) +
 24*sqrt(2)*e^3)*log(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) - 2*(sqrt
(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x
 + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 1) + (48*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 48*I*sqrt(2)*e^3*arctan2(sqrt(2)*
cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 48*I*sqrt(2)
*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 1) + 48*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)
*sin(1/2*d*x + 1/2*c) + 1) - 24*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) + 24*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) - 24*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + 24*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2))*sin(2*d*x + 2*c))*sqrt(a)*sqrt(e)/((-64*I*a^2*cos(2*d*x + 2*c) + 64*a^2
*sin(2*d*x + 2*c) - 64*I*a^2)*d)
```

Fricas [A] time = 2.28874, size = 1602, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(sqrt(9*I*e^7/(a^3*d^2))*a^2*d*e^(I*d*x + I*c)*log(2/3*(sqrt(9*I*e^7/(
a^3*d^2))*a^2*d*e^(2*I*d*x + 2*I*c) + 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d
*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^3) - sqrt(9*I*e^7/(a^3*d^2))*a^2*d*e^
(I*d*x + I*c)*log(-2/3*(sqrt(9*I*e^7/(a^3*d^2))*a^2*d*e^(2*I*d*x + 2*I*c) -
 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e
^3) - sqrt(-9*I*e^7/(a^3*d^2))*a^2*d*e^(I*d*x + I*c)*log(2/3*(sqrt(-9*I*e^7
/(a^3*d^2))*a^2*d*e^(2*I*d*x + 2*I*c) + 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I
*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^3) + sqrt(-9*I*e^7/(a^3*d^2))*a^2*d
*e^(I*d*x + I*c)*log(-2/3*(sqrt(-9*I*e^7/(a^3*d^2))*a^2*d*e^(2*I*d*x + 2*I*
c) - 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*
c)/e^3) + 4*I*e^3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*
I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-I*d*x - I*c)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec (dx + c))^{\frac{7}{2}}}{(i a \tan (dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.424 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=365

$$-\frac{i\sqrt{2}e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{i\sqrt{2}e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{ie^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\right.}{\sqrt{2}}$$

```
[Out] ((-I)*Sqrt[2]*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]
])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(a^(3/2)*d) + (I*Sqrt[2]*e^(5/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])])/(a^(3/2)*d) + (I*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x
])])/(Sqrt[2]*a^(3/2)*d) - (I*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
+ d*x])])/(Sqrt[2]*a^(3/2)*d) + ((4*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[
a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.315878, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3500, 3495, 297, 1162, 617, 204, 1165, 628}

$$-\frac{i\sqrt{2}e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{i\sqrt{2}e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{ie^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\right.}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```


```
[Out] ((-I)*Sqrt[2]*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]
])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(a^(3/2)*d) + (I*Sqrt[2]*e^(5/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])])/(a^(3/2)*d) + (I*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x
])])/(Sqrt[2]*a^(3/2)*d) - (I*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
+ d*x])])/(Sqrt[2]*a^(3/2)*d) + ((4*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[
a + I*a*Tan[c + d*x]])
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx}{a^2}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(4ie^4)\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{(2ie^3)\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad} + \frac{(2ie^3)\,\text{Subst}}{}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(ie^2)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad} + \frac{(ie^2)\,\text{Subs}}{}$$

$$= \frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}a^{3/2}d} - \frac{ie^{5/2}\log\left(a+\right.}{}$$

$$= -\frac{i\sqrt{2}e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{i\sqrt{2}e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{ie}{}$$

Mathematica [A] time = 2.68322, size = 350, normalized size = 0.96

$$\frac{e(\cos(dx)+i\sin(dx))^2(e\sec(c+dx))^{3/2}\left((-4\sin(c)+4i\cos(c))\cos(dx)+4(\cos(c)+i\sin(c))\sin(dx)+\frac{2(\cos(2c)+i\sin(2c)}{}\right.}{d(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (e*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])^2*(Cos[d*x]*((4*I)*Cos[c]
 - 4*Sin[c]) + 4*(Cos[c] + I*Sin[c])*Sin[d*x] + (2*(ArcTan[(Sqrt[-1 + I*Cos
[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I
+ Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1 - I*Cos[c] + Sin[c]]
 - ArcTan[(Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 +
I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 + I*Cos[c] - Sin[c]]*Sq
rt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] + I*Sin[2*c])*Sqrt[I + Tan[(d*x)/2]])
/(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d
*x)/2]])))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Maple [B] time = 0.337, size = 957, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -1/2/d/a^2*(cos(d*x+c)-1)^2*(-I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1-sin(d*x+c)))*cos(d*x+c)-8*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin
```

```
(d*x+c)+2*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))
*cos(d*x+c)^2-I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+
c)))*sin(d*x+c)-I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*
x+c)))-2*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*
cos(d*x+c)^2+2*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x
+c)))*cos(d*x+c)*sin(d*x+c)+2*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)+1-sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)+I*arctanh(1/2*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+2*arctanh(1/2*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+2*arctanh(1/2*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-8*cos(d*x+c)^2*(1/(cos
(d*x+c)+1))^(1/2)-2*cos(d*x+c)*sin(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))+2*cos(d*x+c)*sin(d*x+c)*arctanh(1/2*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-I*arctanh(1/2*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)+I*arctanh(1/2*(1/(cos(d*x+c)+
1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)-arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))*sin(d*x+c)-arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1+sin(d*x+c)))*sin(d*x+c)-arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(
cos(d*x+c)+1-sin(d*x+c)))-arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+
1+sin(d*x+c)))+8*(1/(cos(d*x+c)+1))^(1/2))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos
(d*x+c))^(1/2)*cos(d*x+c)^3*(e/cos(d*x+c))^(5/2)/(2*I*cos(d*x+c)*sin(d*x+c)
+2*cos(d*x+c)^2-1)/(1/(cos(d*x+c)+1))^(5/2)/sin(d*x+c)^5
```

Maxima [B] time = 2.11249, size = 1050, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/4*(2*I*sqrt(2)*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin
(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*
c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e^2*arctan2(sqrt(2
)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)
*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c
) + 1) + 2*sqrt(2)*e^2*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c),
 sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) - 2*sqrt(2)*e^2*arctan2(-
sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c)
+ cos(d*x + c) + 1) + I*sqrt(2)*e^2*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x
+ 1/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)
^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - I*sqrt(2)*e^2*log(-2*sqrt(2)*sin(d*x
 + c)*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x +
 c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) + sqrt(2)*e^2*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*e^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*e^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*e^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 16*I*e^2*cos(1/2*d*x + 1/2*c) - 16*e^2*sin(1/2*d*x + 1
/2*c))*sqrt(e)/(a^(3/2)*d)
```

Fricas [B] time = 2.29494, size = 1663, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(a^2*d*sqrt(4*I*e^5/(a^3*d^2))*e^(2*I*d*x + 2*I*c)*log((I*a^2*d*sqrt(4*
I*e^5/(a^3*d^2))*e^(2*I*d*x + 2*I*c) + 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*
d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^2) - a^2*d*sqrt(4*I*e^5/(a^3*d^2))*e
^(2*I*d*x + 2*I*c)*log((-I*a^2*d*sqrt(4*I*e^5/(a^3*d^2))*e^(2*I*d*x + 2*I*c
) + 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c
)/e^2) + a^2*d*sqrt(-4*I*e^5/(a^3*d^2))*e^(2*I*d*x + 2*I*c)*log((I*a^2*d*sq
rt(-4*I*e^5/(a^3*d^2))*e^(2*I*d*x + 2*I*c) + 2*(e^2*e^(2*I*d*x + 2*I*c) + e
^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(
3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^2) - a^2*d*sqrt(-4*I*e^5/(a^3*
d^2))*e^(2*I*d*x + 2*I*c)*log((-I*a^2*d*sqrt(-4*I*e^5/(a^3*d^2))*e^(2*I*d*x
 + 2*I*c) + 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x
 - 2*I*c)/e^2) + 2*(4*I*e^2*e^(2*I*d*x + 2*I*c) + 4*I*e^2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c)
)*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec(dx+c))^{\frac{5}{2}}}{(i a \tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.425 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] (((2*I)/3)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.0758921, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3488}

$$\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]

[Out] (((2*I)/3)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0685318, size = 38, normalized size = 1.

$$\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]

[Out] (((2*I)/3)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Maple [B] time = 0.275, size = 87, normalized size = 2.3

$$\frac{\frac{2i}{3}(\cos(dx+c))^2}{a^2d\left(2i\cos(dx+c)\sin(dx+c)+2(\cos(dx+c))^2-1\right)}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(\frac{e}{\cos(dx+c)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2/3*I/d/a^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e/cos(d*x+c))^(
3/2)*cos(d*x+c)^2/(2*I*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-1)
```

Maxima [B] time = 1.63574, size = 103, normalized size = 2.71

$$\frac{2 i e^{\frac{3}{2}} \left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1\right)^{\frac{3}{2}}}{3 a^{\frac{3}{2}} d \left(-\frac{2 i \sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*I*e^(3/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)/(a^(3/2)*d*(
-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 1)^(3/2))
```

Fricas [B] time = 2.04543, size = 186, normalized size = 4.89

$$\frac{2 \left(i e e^{(2 i dx+2 i c)} + i e\right) \sqrt{\frac{a}{e^{(2 i dx+2 i c)}+1}} \sqrt{\frac{e}{e^{(2 i dx+2 i c)}+1}} e^{\left(-\frac{3}{2} i dx - \frac{3}{2} i c\right)}}{3 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 2/3*(I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-3/2*I*d*x - 3/2*I*c)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec (dx + c))^{\frac{3}{2}}}{(i\, a \tan (dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.426 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=80

$$\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] (((2*I)/5)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((4*I)
/5)*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.144943, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3502, 3488}

$$\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/5)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((4*I)
/5)*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}+\frac{2\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{5a}\\ &= \frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}+\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.120254, size = 63, normalized size = 0.79

$$\frac{2(3+2i\tan(c+dx))\sqrt{e\sec(c+dx)}}{5ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[e*Sec[c + d*x]]*(3 + (2*I)*Tan[c + d*x]))/(5*a*d*(-I + Tan[c + d*x]
)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.308, size = 101, normalized size = 1.3

$$\frac{-\frac{2i}{5}\cos(dx+c)\left(2i(\cos(dx+c))^2\sin(dx+c)-2(\cos(dx+c))^3+2i\sin(dx+c)-\cos(dx+c)\right)}{a^2d}\sqrt{\frac{e}{\cos(dx+c)}}\sqrt{\frac{a(i\text{s}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -2/5*I/d/a^2*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^
(1/2)*cos(d*x+c)*(2*I*cos(d*x+c)^2*sin(d*x+c)-2*cos(d*x+c)^3+2*I*sin(d*x+c)
-cos(d*x+c))
```

Maxima [A] time = 1.91847, size = 108, normalized size = 1.35

$$\frac{\sqrt{e}\left(i\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5i\cos\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sin\left(\frac{1}{5}\arctan\left(\sin\right.\right.\right.}{5a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/5*sqrt(e)*(I*cos(5/2*d*x + 5/2*c) + 5*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))) + sin(5/2*d*x + 5/2*c) + 5*sin(1/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))/(a^(3/2)*d)
```

Fricas [A] time = 2.10288, size = 219, normalized size = 2.74

$$\frac{\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{e}{e^{(2idx+2ic)}+1}}\left(5ie^{(4idx+4ic)}+6ie^{(2idx+2ic)}+i\right)e^{\left(-\frac{5}{2}idx-\frac{5}{2}ic\right)}}{5a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/5*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(5*
I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-5/2*I*d*x - 5/2*I*
c)/(a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a(i\tan(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))/(a*(I*tan(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sec(dx+c)}}{(ia\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.427 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=121

$$-\frac{16i\sqrt{a+ia\tan(c+dx)}}{21a^2d\sqrt{e\sec(c+dx)}}+\frac{8i}{21ad\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

```
[Out] ((2*I)/7)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + ((8*I)/21
)/(a*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/21)*Sqrt
[a + I*a*Tan[c + d*x]])/(a^2*d*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.209232, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3502, 3488}

$$-\frac{16i\sqrt{a+ia\tan(c+dx)}}{21a^2d\sqrt{e\sec(c+dx)}}+\frac{8i}{21ad\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((2*I)/7)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + ((8*I)/21
)/(a*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/21)*Sqrt
[a + I*a*Tan[c + d*x]])/(a^2*d*Sqrt[e*Sec[c + d*x]])
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx &= \frac{2i}{7d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}+\frac{4\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx}{7a}\\ &= \frac{2i}{7d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}+\frac{8i}{21ad\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(}\\ &= \frac{2i}{7d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}+\frac{8i}{21ad\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(}\end{aligned}$$

Mathematica [A] time = 0.287931, size = 83, normalized size = 0.69

$$-\frac{\sec^2(c+dx)(12i\sin(2(c+dx))+9\cos(2(c+dx))-7)}{21ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -(Sec[c + d*x]^2*(-7 + 9*Cos[2*(c + d*x)] + (12*I)*Sin[2*(c + d*x)]))/(21*a
*d*Sqrt[e*Sec[c + d*x]]*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.308, size = 106, normalized size = 0.9

$$-\frac{18\,i\left(\cos\left(dx+c\right)\right)^{2}-24\,\cos\left(dx+c\right)\sin\left(dx+c\right)-16\,i}{21\,a^{2}d\left(2\,i\cos\left(dx+c\right)\sin\left(dx+c\right)+2\,\left(\cos\left(dx+c\right)\right)^{2}-1\right)}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}\frac{1}{\sqrt{\frac{e}{\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] -2/21/d/a^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(9*I*cos(d*x+c)^
2-12*cos(d*x+c)*sin(d*x+c)-8*I)/(2*I*cos(d*x+c)*sin(d*x+c)+2*cos(d*x+c)^2-1
)/(e/cos(d*x+c))^(1/2)
```

Maxima [A] time = 1.93716, size = 176, normalized size = 1.45

$$3i\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)+14i\cos\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)-21i\cos\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="max
ima")
```

```
[Out] 1/42*(3*I*cos(7/2*d*x + 7/2*c) + 14*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c))) - 21*I*cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 3*sin(7/2*d*x + 7/2*c) + 14*sin(3/7*arctan2(sin(7/2*d*x
+ 7/2*c), cos(7/2*d*x + 7/2*c))) + 21*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c))))/(a^(3/2)*d*sqrt(e))
```

Fricas [A] time = 2.04918, size = 265, normalized size = 2.19

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-21i\,e^{(6i\,dx+6i\,c)}-7i\,e^{(4i\,dx+4i\,c)}+17i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(-\frac{7}{2}i\,dx-\frac{7}{2}i\,c\right)}}{42\,a^{2}de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/42*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-
21*I*e^(6*I*d*x + 6*I*c) - 7*I*e^(4*I*d*x + 4*I*c) + 17*I*e^(2*I*d*x + 2*I*
c) + 3*I)*e^(-7/2*I*d*x - 7/2*I*c)/(a^2*d*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e \sec(dx+c)}(i a \tan(dx+c)+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(3/2)), x)
```

3.428 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{16i\sqrt{a+ia\tan(c+dx)}}{45a^2d(e\sec(c+dx))^{3/2}}+\frac{32i\sqrt{e\sec(c+dx)}}{45ade^2\sqrt{a+ia\tan(c+dx)}}+\frac{4i}{15ad\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}+\frac{}{9d(a+ia\tan(c}$$

```
[Out] ((2*I)/9)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((4*I)/
15)/(a*d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((32*I)/45)*
Sqrt[e*Sec[c + d*x]])/(a*d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/45)*S
qrt[a + I*a*Tan[c + d*x]])/(a^2*d*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.305762, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3502, 3497, 3488}

$$-\frac{16i\sqrt{a+ia\tan(c+dx)}}{45a^2d(e\sec(c+dx))^{3/2}}+\frac{32i\sqrt{e\sec(c+dx)}}{45ade^2\sqrt{a+ia\tan(c+dx)}}+\frac{4i}{15ad\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}+\frac{}{9d(a+ia\tan(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```


```
[Out] ((2*I)/9)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((4*I)/
15)/(a*d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((32*I)/45)*
Sqrt[e*Sec[c + d*x]])/(a*d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/45)*S
qrt[a + I*a*Tan[c + d*x]])/(a^2*d*(e*Sec[c + d*x])^(3/2))
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{2\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+d}}}{3a}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{4i}{15ad(e\sec(c+dx))^{3/2}\sqrt{a+}}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{4i}{15ad(e\sec(c+dx))^{3/2}\sqrt{a+}}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{4i}{15ad(e\sec(c+dx))^{3/2}\sqrt{a+}}$$

Mathematica [A] time = 0.438154, size = 100, normalized size = 0.61

$$-\frac{\sec^3(c+dx)(-54i\sin(c+dx)+10i\sin(3(c+dx))-81\cos(c+dx)+5\cos(3(c+dx)))}{90ad(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -(Sec[c + d*x]^3*(-81*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - (54*I)*Sin[c + d*
x] + (10*I)*Sin[3*(c + d*x)]))/(90*a*d*(e*Sec[c + d*x])^(3/2)*(-I + Tan[c +
 d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.304, size = 132, normalized size = 0.8

$$\frac{2\left(\cos(dx+c)\right)^2\left(10\,i\left(\cos(dx+c)\right)^5+10\,\sin(dx+c)\left(\cos(dx+c)\right)^4+i\left(\cos(dx+c)\right)^3+6\left(\cos(dx+c)\right)^2\sin(dx+c)+\right.}{45\,a^2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2/45/d/a^2*cos(d*x+c)^2*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/c
os(d*x+c))^(1/2)*(10*I*cos(d*x+c)^5+10*sin(d*x+c)*cos(d*x+c)^4+I*cos(d*x+c)
^3+6*cos(d*x+c)^2*sin(d*x+c)+8*I*cos(d*x+c)+16*sin(d*x+c))/e^3
```

Maxima [A] time = 1.97288, size = 240, normalized size = 1.45

$$5i\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)+27i\cos\left(\frac{5}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)-15i\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="max
ima")
```

```
[Out] 1/180*(5*I*cos(9/2*d*x + 9/2*c) + 27*I*cos(5/9*arctan2(sin(9/2*d*x + 9/2*c)
, cos(9/2*d*x + 9/2*c))) - 15*I*cos(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9
```

```
/2*d*x + 9/2*c))) + 135*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c))) + 5*sin(9/2*d*x + 9/2*c) + 27*sin(5/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) + 15*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) + 135*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))))/(a^(3/2)*d*e^(3/2))
```

Fricas [A] time = 1.98072, size = 309, normalized size = 1.87

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-15i\,e^{(8i\,dx+8i\,c)}+120i\,e^{(6i\,dx+6i\,c)}+162i\,e^{(4i\,dx+4i\,c)}+32i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{\left(-\frac{9}{2}i\,dx-\frac{9}{2}i\,c\right)}}{180\,a^2de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/180*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-15*I*e^(8*I*d*x + 8*I*c) + 120*I*e^(6*I*d*x + 6*I*c) + 162*I*e^(4*I*d*x +
4*I*c) + 32*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-9/2*I*d*x - 9/2*I*c)/(a^2*d*e^
2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(3/2)), x)
```

3.429 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{256i\sqrt{a+ia\tan(c+dx)}}{385a^2de^2\sqrt{e\sec(c+dx)}}-\frac{96i\sqrt{a+ia\tan(c+dx)}}{385a^2d(e\sec(c+dx))^{5/2}}+\frac{128i}{385ade^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{}{77ad\sqrt{a+ia\tan(c}}$$

```
[Out] ((2*I)/11)/(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((16*I
)/77)/(a*d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((128*I)/38
5)/(a*d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((96*I)/385
)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d*(e*Sec[c + d*x])^(5/2)) - (((256*I)/38
5)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.392568, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3502, 3497, 3488}

$$-\frac{256i\sqrt{a+ia\tan(c+dx)}}{385a^2de^2\sqrt{e\sec(c+dx)}}-\frac{96i\sqrt{a+ia\tan(c+dx)}}{385a^2d(e\sec(c+dx))^{5/2}}+\frac{128i}{385ade^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{}{77ad\sqrt{a+ia\tan(c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```


```
[Out] ((2*I)/11)/(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((16*I
)/77)/(a*d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((128*I)/38
5)/(a*d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((96*I)/385
)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d*(e*Sec[c + d*x])^(5/2)) - (((256*I)/38
5)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d*e^2*Sqrt[e*Sec[c + d*x]])
```


Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{8\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan}}}{11a}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{}}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{}}$$

Mathematica [A] time = 0.471438, size = 100, normalized size = 0.48

$$-\frac{(e\sec(c+dx))^{3/2}(880i\sin(2(c+dx))+56i\sin(4(c+dx))+660\cos(2(c+dx))+21\cos(4(c+dx))-385)}{1540ade^4(\tan(c+dx)-i)\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -((e*Sec[c + d*x])^(3/2)*(-385 + 660*Cos[2*(c + d*x)] + 21*Cos[4*(c + d*x)]
 + (880*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)]))/(1540*a*d*e^4*(-I +
 Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.312, size = 142, normalized size = 0.7

$$\frac{2\,(\cos(dx+c))^3\left(70\,i\,(\cos(dx+c))^6+70\,(\cos(dx+c))^5\sin(dx+c)+5\,i\,(\cos(dx+c))^4+40\,(\cos(dx+c))^3\sin(dx\right.}{385\,a^2de^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] 2/385/d/a^2*(e/cos(d*x+c))^(5/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(
1/2)*cos(d*x+c)^3*(70*I*cos(d*x+c)^6+70*cos(d*x+c)^5*sin(d*x+c)+5*I*cos(d*x
+c)^4+40*cos(d*x+c)^3*sin(d*x+c)+16*I*cos(d*x+c)^2+64*cos(d*x+c)*sin(d*x+c)
-128*I)/e^5
```

Maxima [A] time = 2.03733, size = 305, normalized size = 1.46

$$35i\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)+220i\cos\left(\frac{7}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right),\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)\right)\right)-77i\cos\left(\frac{5}{11}\arctan\left(\sin\left(\frac{11}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="max
ima")
```

```
[Out] 1/3080*(35*I*cos(11/2*d*x + 11/2*c) + 220*I*cos(7/11*arctan2(sin(11/2*d*x +
 11/2*c), cos(11/2*d*x + 11/2*c))) - 77*I*cos(5/11*arctan2(sin(11/2*d*x + 1
1/2*c), cos(11/2*d*x + 11/2*c))) + 770*I*cos(3/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) - 1540*I*cos(1/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) + 35*sin(11/2*d*x + 11/2*c) + 220*sin(7/11*
arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(5/11*arct
an2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 770*sin(3/11*arctan2
(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 1540*sin(1/11*arctan2(s
in(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))/(a^(3/2)*d*e^(5/2))
```

Fricas [A] time = 2.05023, size = 358, normalized size = 1.71

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-77i\,e^{(10i\,dx+10i\,c)}-1617i\,e^{(8i\,dx+8i\,c)}-770i\,e^{(6i\,dx+6i\,c)}+990i\,e^{(4i\,dx+4i\,c)}+255i\,e^{(2i\,dx+2i\,c)}+35i\right)e^{(}}{3080\,a^2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/3080*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-77*I*e^(10*I*d*x + 10*I*c) - 1617*I*e^(8*I*d*x + 8*I*c) - 770*I*e^(6*I*d*
x + 6*I*c) + 990*I*e^(4*I*d*x + 4*I*c) + 255*I*e^(2*I*d*x + 2*I*c) + 35*I)*
e^(-11/2*I*d*x - 11/2*I*c)/(a^2*d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^(3/2)), x)
```

3.430 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=411

$$-\frac{5ie^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}+\frac{5ie^{9/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}+\frac{5ie^4\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx}}{a^3d}$$

```
[Out] ((-5*I)*e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((5*I)*e^(9/2)*ArcTan[1
 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x
]])])/(Sqrt[2]*a^(5/2)*d) + (((5*I)/2)*e^(9/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqr
t[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I
*a*Tan[c + d*x])])/(Sqrt[2]*a^(5/2)*d) - (((5*I)/2)*e^(9/2)*Log[a + (Sqrt[2
]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c
+ d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*a^(5/2)*d) + ((4*I)*e^2*(e*Sec[c +
 d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((5*I)*e^4*Sqrt[e*Sec[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.433811, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3500, 3501, 3495, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5ie^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}+\frac{5ie^{9/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}+\frac{5ie^4\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx}}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```


```
[Out] ((-5*I)*e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((5*I)*e^(9/2)*ArcTan[1
 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x
]])])/(Sqrt[2]*a^(5/2)*d) + (((5*I)/2)*e^(9/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqr
t[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I
*a*Tan[c + d*x])])/(Sqrt[2]*a^(5/2)*d) - (((5*I)/2)*e^(9/2)*Log[a + (Sqrt[2
]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c
+ d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*a^(5/2)*d) + ((4*I)*e^2*(e*Sec[c +
 d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((5*I)*e^4*Sqrt[e*Sec[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```


Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
```

```
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{\left(5e^2\right)\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}dx}{a^2}$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} - \frac{\left(5e^4\right)\int \sqrt{e\sec(c}$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} + \frac{\left(10ie^6\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} - \frac{\left(5ie^5\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} + \frac{\left(5ie^4\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{5ie^{9/2}\log\left(a - \frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}} + \cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}a^{5/2}d} - \frac{5ie^{9/2}\log\left(a+\right.}{}$$

$$= -\frac{5ie^{9/2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{5ie^{9/2}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{5ie^{9/2}}{}$$

Mathematica [A] time = 5.43652, size = 369, normalized size = 0.9

$$\frac{e^2(\cos(dx)+i\sin(dx))^3(e\sec(c+dx))^{5/2}\left(-\frac{5\sqrt{\sin(c)+i\cos(c)-1}(\cos(3c)+i\sin(3c))\sqrt{\tan\left(\frac{dx}{2}\right)+i}\tan^{-1}\left(\frac{\sqrt{\sin(c)-i\cos(c)-1}\sqrt{-\tan\left(\frac{dx}{2}\right)+i}}{\sqrt{\sin(c)+i\cos(c)-1}\sqrt{\tan\left(\frac{dx}{2}\right)+i}}\right)}{\sqrt{\sin(c)-i\cos(c)-1}\sqrt{-\tan\left(\frac{dx}{2}\right)+i}} + \ldots\right.}{d(a+ia\,\mathrm{t}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (e^2*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^3*((5*ArcTan[(Sqrt[-1 +
 I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*S
qrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*(Cos[3*c] + I*Sin[3*c]
)*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/
2]]) - (5*ArcTan[(Sqrt[-1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqr
t[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 + I*Cos[c] + Sin
[c]]*(Cos[3*c] + I*Sin[3*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] +
Sin[c]]*Sqrt[I - Tan[(d*x)/2]]) - (Cos[2*c - d*x] + I*Sin[2*c - d*x])*(-9*I
 + Tan[c + d*x])))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Maple [B] time = 0.346, size = 1439, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -1/4/d/a^3*(cos(d*x+c)-1)^4*(10*cos(d*x+c)^2*sin(d*x+c)*arctanh(1/2*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-10*cos(d*x+c)^2*sin(d*x+c)*arc
tanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+5*cos(d*x+c)*s
in(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-5
*cos(d*x+c)*sin(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+s
in(d*x+c)))+20*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c
)))*cos(d*x+c)^4+20*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(
d*x+c)))*cos(d*x+c)^4-80*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4-10*cos(d*x+c
)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-10*cos(
d*x+c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+84
*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)+5*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+5*arctanh(1/2*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)-20*arctanh(1/2*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3*sin(d*x+c)+20*I*arctanh(1/2
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^3*sin(d*x+c
)+20*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(
d*x+c)^3*sin(d*x+c)-80*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3*sin(d*x+c)-1
0*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x
+c)^2*sin(d*x+c)-10*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+si
n(d*x+c)))*cos(d*x+c)^2*sin(d*x+c)-5*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)-5*I*arctanh(1/2*(1/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)+44*I*(1/(c
os(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-4*(1/(cos(d*x+c)+1))^(1/2)+20*arc
tanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3*s
in(d*x+c)+20*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c
)))*cos(d*x+c)^4-20*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+si
n(d*x+c)))*cos(d*x+c)^4-10*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)+1-sin(d*x+c)))*cos(d*x+c)^3+10*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^3-15*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+15*I*arctanh(1/2*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2+5*I*arctanh(1/2*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)-5*I*arctanh(1/2*(1/
(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)-15*arctanh(1/2*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2-15*arctanh
(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2)*(a*(
I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(e/cos(d*x+c))^(9/2
)/(4*I*sin(d*x+c)*cos(d*x+c)^2+4*cos(d*x+c)^3-I*sin(d*x+c)-3*cos(d*x+c))/(1
/(cos(d*x+c)+1))^(9/2)/sin(d*x+c)^9
```

Maxima [B] time = 2.25316, size = 3312, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -(64*e^4*cos(1/2*d*x + 1/2*c)^2 + 64*e^4*sin(1/2*d*x + 1/2*c)^2 + 16*e^4 +
(10*I*sqrt(2)*e^4*cos(3/2*d*x + 3/2*c) + 10*I*sqrt(2)*e^4*cos(1/2*d*x + 1/2
*c) + 10*sqrt(2)*e^4*sin(3/2*d*x + 3/2*c) - 10*sqrt(2)*e^4*sin(1/2*d*x + 1/
2*c))*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*
d*x + 1/2*c) + cos(d*x + c) + 1) + (-10*I*sqrt(2)*e^4*cos(3/2*d*x + 3/2*c)
- 10*I*sqrt(2)*e^4*cos(1/2*d*x + 1/2*c) - 10*sqrt(2)*e^4*sin(3/2*d*x + 3/2*
c) + 10*sqrt(2)*e^4*sin(1/2*d*x + 1/2*c))*arctan2(-sqrt(2)*sin(1/2*d*x + 1/
2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) - (1
0*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x
```

+ 1/2*c) + 1) + 10*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -
sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*
d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*sqrt(2)*e^4*arctan
2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 5*
I*sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*I*sqr
t(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*I*sqrt(2)*
e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*I*sqrt(2)*e^4*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 64*e^4*cos(1/2*d*x + 1
/2*c) + 64*I*e^4*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - (10*sqrt(2)*e
^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) +
1) + 10*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin
(1/2*d*x + 1/2*c) + 1) + 10*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c
) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*sqrt(2)*e^4*arctan2(sqrt(2)*c
os(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 5*I*sqrt(2)*e
^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*I*sqrt(2)*e^4*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*I*sqrt(2)*e^4*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*I*sqrt(2)*e^4*log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c) - (5*sqrt(2)*
e^4*cos(3/2*d*x + 3/2*c) + 5*sqrt(2)*e^4*cos(1/2*d*x + 1/2*c) - 5*I*sqrt(2)
*e^4*sin(3/2*d*x + 3/2*c) + 5*I*sqrt(2)*e^4*sin(1/2*d*x + 1/2*c))*log(2*sqr
t(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) +
1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) + (5*sqr
t(2)*e^4*cos(3/2*d*x + 3/2*c) + 5*sqrt(2)*e^4*cos(1/2*d*x + 1/2*c) - 5*I*sq
rt(2)*e^4*sin(3/2*d*x + 3/2*c) + 5*I*sqrt(2)*e^4*sin(1/2*d*x + 1/2*c))*log(
-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2
*c) - 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x
+ c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) +
(10*I*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2
*d*x + 1/2*c) + 1) + 10*I*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*I*sqrt(2)*e^4*arctan2(sqrt(2)*
cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*I*sqrt(2)*
e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 1) + 5*sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
5*sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*sqrt
(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*e^4
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 64*I*e^4*cos(1/2*d*x
+ 1/2*c) - 64*e^4*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c) + (-10*I*sqrt
(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2
*c) + 1) - 10*I*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt
(2)*sin(1/2*d*x + 1/2*c) + 1) - 10*I*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*
x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 10*I*sqrt(2)*e^4*arctan
2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 5*
sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*sqrt(2)
*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*e^4*lo

```
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*sqrt(2)*e^4*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(1/2*d*x + 1/2*c))*sqrt(a)*s
qrt(e)/((8*I*a^3*cos(3/2*d*x + 3/2*c) + 8*I*a^3*cos(1/2*d*x + 1/2*c) + 8*a^
3*sin(3/2*d*x + 3/2*c) - 8*a^3*sin(1/2*d*x + 1/2*c))*d)
```

Fricas [B] time = 2.51361, size = 1704, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(sqrt(25*I*e^9/(a^5*d^2))*a^3*d*e^(2*I*d*x + 2*I*c)*log(-2/5*(I*sqrt(2
5*I*e^9/(a^5*d^2))*a^3*d*e^(2*I*d*x + 2*I*c) - 5*(e^4*e^(2*I*d*x + 2*I*c) +
 e^4)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e
^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^4) - sqrt(25*I*e^9/(a^5*d^2)
)*a^3*d*e^(2*I*d*x + 2*I*c)*log(-2/5*(-I*sqrt(25*I*e^9/(a^5*d^2))*a^3*d*e^(
2*I*d*x + 2*I*c) - 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-
2*I*d*x - 2*I*c)/e^4) + sqrt(-25*I*e^9/(a^5*d^2))*a^3*d*e^(2*I*d*x + 2*I*c)
*log(-2/5*(I*sqrt(-25*I*e^9/(a^5*d^2))*a^3*d*e^(2*I*d*x + 2*I*c) - 5*(e^4*e
^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*
d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^4) - sqr
t(-25*I*e^9/(a^5*d^2))*a^3*d*e^(2*I*d*x + 2*I*c)*log(-2/5*(-I*sqrt(-25*I*e^
9/(a^5*d^2))*a^3*d*e^(2*I*d*x + 2*I*c) - 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*
I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^4) - 2*(10*I*e^4*e^(2*I*d*x + 2*I*
c) + 8*I*e^4)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec(dx+c))^{\frac{9}{2}}}{(i a \tan(dx+c)+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.431 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=527

$$\frac{i\sqrt{2}e^{7/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{2}e^{7/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{ie^{7/2}\sec(c+dx}{}$$

```
[Out] (((4*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) +
 (I*Sqrt[2]*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])
/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(a^(3/2)*d*Sqrt[a - I*a*Tan[
c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (I*Sqrt[2]*e^(7/2)*ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec
[c + d*x])/(a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]
) - (I*e^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])
/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/
(Sqrt[2]*a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) +
 (I*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sq
rt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sq
rt[2]*a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.45357, antiderivative size = 527, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3500, 3499, 3495, 297, 1162, 617, 204, 1165, 628}

$$\frac{i\sqrt{2}e^{7/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{2}e^{7/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{ie^{7/2}\sec(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((4*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) +
 (I*Sqrt[2]*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])
/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(a^(3/2)*d*Sqrt[a - I*a*Tan[
c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (I*Sqrt[2]*e^(7/2)*ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec
[c + d*x])/(a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]
) - (I*e^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])
/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/
(Sqrt[2]*a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) +
 (I*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sq
rt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sq
rt[2]*a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3499

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[(d*Sec[e + f*x])/(Sqrt[a - b*Tan[e + f*x]]*Sqr
t[a + b*Tan[e + f*x]]), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{e^2\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a^2}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{\left(e^3\sec(c+dx)\right)\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{a^2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{\left(4ie^5\sec(c+dx)\right)\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} + \frac{\left(2ie^4\sec(c+dx)\right)\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{(2}{}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{\left(ie^3\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} -$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{ie^{7/2}\log\left(a - \frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}} + \cos(c+dx)(a-ia\tan(c\right.}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+}}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} + \frac{i\sqrt{2}e^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{2}e^{7/2}}{a^{3/2}d}$$

Mathematica [A] time = 11.6297, size = 422, normalized size = 0.8

$$-\frac{2e^2(e\sec(c+dx))^{3/2}(\cos(2(c+dx))+i\sin(2(c+dx)))\left(\sqrt{-\sin(c)+i\cos(c)+1}\left(2\sqrt{-\sin(c)+i\cos(c)-1}\sqrt{-\tan\left(\frac{dx}{2}\right)}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*e^2*(e*Sec[c + d*x])^(3/2)*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])*(-3*
ArcTan[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[1 - I*Cos
[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] - Sin[c]]*Sqrt[-1
+ I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]]*(-I + Tan[c + d*x]) + Sqrt[1 +
I*Cos[c] - Sin[c]]*(-3*ArcTan[(Sqrt[-1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d
*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I
*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]]*(-I + Tan[c + d*x]) + 2*Sqrt[-1 +
I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]]*(I + Tan[c + d*x]))))/(3*a^2*d*Sq
rt[-1 + I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2
]]*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.303, size = 1324, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/6/d/a^3*(cos(d*x+c)-1)^3*(-3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)+1-sin(d*x+c)))+3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-si
n(d*x+c)))*sin(d*x+c)-3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+
sin(d*x+c)))*sin(d*x+c)-12*cos(d*x+c)^2*sin(d*x+c)*arctanh(1/2*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))+12*cos(d*x+c)^2*sin(d*x+c)*arctanh(
1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+6*cos(d*x+c)*sin(d*
x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-6*cos(
d*x+c)*sin(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*
x+c)))-12*cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-s
in(d*x+c)))-12*cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c
)+1+sin(d*x+c)))-8*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)+9*arctanh(1/2*(1/(
cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+9*arctanh(1/2*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)-3*arctanh(1/2*
(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+8*(1/(cos(d*x+c)+1))^(1
/2)+3*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*sin
(d*x+c)+3*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))
*sin(d*x+c)+12*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x
+c)))*cos(d*x+c)^3-12*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+
sin(d*x+c)))*cos(d*x+c)^3-6*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x
+c)+1-sin(d*x+c)))*cos(d*x+c)^2+6*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-9*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)+9*I*arctanh(1/2*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+3*I*arctanh(1/2*(1/(cos(d*x+c
)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))-3*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)+1+sin(d*x+c)))+6*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)^2+6*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-8*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*
x+c)*sin(d*x+c)-12*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin
(d*x+c)))*cos(d*x+c)^2*sin(d*x+c)-12*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2*sin(d*x+c)+6*I*arctanh(1/2*(1/(cos
(d*x+c)+1))^(1/2)*(cos(d*x+c)+1-sin(d*x+c)))*cos(d*x+c)*sin(d*x+c)+6*I*arct
anh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)*sin(
d*x+c))*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^4*(e/cos(
d*x+c))^(7/2)/(4*I*sin(d*x+c)*cos(d*x+c)^2+4*cos(d*x+c)^3-I*sin(d*x+c)-3*co
s(d*x+c))/(1/(cos(d*x+c)+1))^(7/2)/sin(d*x+c)^7
```

Maxima [B] time = 2.26944, size = 1979, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/12*(6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 1) + 6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 6*I*sqrt(2)*e^3*arct
an2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) -
1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1
) + 6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 1) - 6*sqrt(2)*e^3*arctan2(sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2
```

```
*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) + 6*sqrt(2)*e^3*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 1) + 3*I*sqrt(2)*e^3*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 3*I*sqrt(2)*e^3*log(
-2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 1) - 3*sqrt(2)*e^3*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 2) + 3*sqrt(2)*e^3*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 2) - 3*sqrt(2)*e^3*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*e^3*log(2*cos(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 2) + 16*I*e^3*cos(3/2*d*x + 3/2*c) + 16*e^3*sin(3/2
*d*x + 3/2*c))*sqrt(e)/(a^(5/2)*d)
```

Fricas [A] time = 2.40564, size = 1663, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(3*a^3*d*sqrt(4*I*e^7/(a^5*d^2))*e^(3*I*d*x + 3*I*c)*log((a^3*d*sqrt(4*
I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*
d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^3) - 3*a^3*d*sqrt(4*I*e^7/(a^5*d^2))
*e^(3*I*d*x + 3*I*c)*log(-(a^3*d*sqrt(4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c
) - 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c
```

```
)/e^3) - 3*a^3*d*sqrt(-4*I*e^7/(a^5*d^2))*e^(3*I*d*x + 3*I*c)*log((a^3*d*sq
rt(-4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e
^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(
3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x - 2*I*c)/e^3) + 3*a^3*d*sqrt(-4*I*e^7/(a^
5*d^2))*e^(3*I*d*x + 3*I*c)*log(-(a^3*d*sqrt(-4*I*e^7/(a^5*d^2))*e^(2*I*d*x
 + 2*I*c) - 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c))*e^(-2*I*d*x
 - 2*I*c)/e^3) + 2*(4*I*e^3*e^(2*I*d*x + 2*I*c) + 4*I*e^3)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c)
)*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sec(dx+c))^{\frac{7}{2}}}{(i a \tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.432 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=38

$$\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] (((2*I)/5)*(e*Sec[c + d*x])^(5/2))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rubi [A] time = 0.0783137, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3488}

$$\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/5)*(e*Sec[c + d*x])^(5/2))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A] time = 0.220448, size = 38, normalized size = 1.

$$\frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/5)*(e*Sec[c + d*x])^(5/2))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Maple [B] time = 0.281, size = 105, normalized size = 2.8

$$\frac{\frac{2i}{5}(\cos(dx+c))^3}{da^3\left(4i\sin(dx+c)(\cos(dx+c))^2+4(\cos(dx+c))^3-i\sin(dx+c)-3\cos(dx+c)\right)}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/5*I/d/a^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(e/
cos(d*x+c))^(5/2)/(4*I*sin(d*x+c)*cos(d*x+c)^2+4*cos(d*x+c)^3-I*sin(d*x+c)-
3*cos(d*x+c))
```

Maxima [B] time = 1.61859, size = 103, normalized size = 2.71

$$\frac{2i\,e^{\frac{5}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}{5\,a^{\frac{5}{2}}d\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 2/5*I*e^(5/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2)/(a^(5/2)*d*(
-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 1)^(5/2))
```

Fricas [B] time = 2.08998, size = 192, normalized size = 5.05

$$\frac{2\left(i\,e^2e^{(2i\,dx+2i\,c)}+i\,e^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 2/5*(I*e^2*e^(2*I*d*x + 2*I*c) + I*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2*I*d*x - 5/2*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.433 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=80

$$\frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}+\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] (((2*I)/7)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((4*
I)/21)*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rubi [A] time = 0.165989, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3502, 3488}

$$\frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}+\frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/7)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((4*
I)/21)*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}+\frac{2\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx}{7a}\\ &= \frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}+\frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.199161, size = 63, normalized size = 0.79

$$\frac{2(2\tan(c+dx)-5i)(e\sec(c+dx))^{3/2}}{21a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*(e*Sec[c + d*x])^(3/2)*(-5*I + 2*Tan[c + d*x]))/(21*a^2*d*(-I + Tan[c +
d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.291, size = 112, normalized size = 1.4

$$\frac{-\frac{2i}{21}(\cos(dx+c))^2\left(12i(\cos(dx+c))^3\sin(dx+c)-12(\cos(dx+c))^4+i\sin(dx+c)\cos(dx+c)+5(\cos(dx+c))^2+\right.}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -2/21*I/d/a^3*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))
^(1/2)*cos(d*x+c)^2*(12*I*cos(d*x+c)^3*sin(d*x+c)-12*cos(d*x+c)^4+I*sin(d*x
+c)*cos(d*x+c)+5*cos(d*x+c)^2+2)
```

Maxima [A] time = 1.9025, size = 116, normalized size = 1.45

$$\frac{\left(3ie\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7ie\cos\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+3e\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7e\sin\left(\frac{3}{7}\arctan\right.\right.}{21a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/21*(3*I*e*cos(7/2*d*x + 7/2*c) + 7*I*e*cos(3/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c))) + 3*e*sin(7/2*d*x + 7/2*c) + 7*e*sin(3/7*arctan2
(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(e)/(a^(5/2)*d)
```

Fricas [A] time = 2.01614, size = 232, normalized size = 2.9

$$\frac{\left(7iee^{(4idx+4ic)}+10iee^{(2idx+2ic)}+3ie\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{e}{e^{(2idx+2ic)}+1}}e^{\left(-\frac{7}{2}idx-\frac{7}{2}ic\right)}}{21a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/21*(7*I*e*e^(4*I*d*x + 4*I*c) + 10*I*e*e^(2*I*d*x + 2*I*c) + 3*I*e)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2*I*d*
x - 7/2*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.434 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=121

$$\frac{16i\sqrt{e\sec(c+dx)}}{45a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] (((2*I)/9)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((8*I)
/45)*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((16*I)/45
)*Sqrt[e*Sec[c + d*x]])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.219756, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3502, 3488}

$$\frac{16i\sqrt{e\sec(c+dx)}}{45a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}}+\frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/9)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((8*I)
/45)*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((16*I)/45
)*Sqrt[e*Sec[c + d*x]])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx &= \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}+\frac{4\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx}{9a}\\ &= \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}+\frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}}+\frac{8\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{45a^2}\\ &= \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}+\frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}}+\frac{16i\sqrt{e\sec(c+dx)}}{45a^2d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.305609, size = 85, normalized size = 0.7

$$-\frac{i\sec^2(c+dx)\sqrt{e\sec(c+dx)}(20i\sin(2(c+dx))+25\cos(2(c+dx))+9)}{45a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I/45)*Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(9 + 25*Cos[2*(c + d*x)] + (20
*I)*Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*
x]])
```

Maple [A] time = 0.313, size = 128, normalized size = 1.1

$$\frac{-\frac{2i}{45}\cos(dx+c)\left(20\,i(\cos(dx+c))^4\sin(dx+c)-20\,(\cos(dx+c))^5+3\,i(\cos(dx+c))^2\sin(dx+c)+7\,(\cos(dx+c}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] -2/45*I/d/a^3*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))
^(1/2)*cos(d*x+c)*(20*I*cos(d*x+c)^4*sin(d*x+c)-20*cos(d*x+c)^5+3*I*cos(d*x
+c)^2*sin(d*x+c)+7*cos(d*x+c)^3+8*I*sin(d*x+c)-4*cos(d*x+c))
```

Maxima [A] time = 1.88968, size = 176, normalized size = 1.45

$$\sqrt{e}\left(5i\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)+18i\cos\left(\frac{5}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right),\cos\left(\frac{9}{2}dx+\frac{9}{2}c\right)\right)\right)+45i\cos\left(\frac{1}{9}\arctan\left(\sin\left(\frac{9}{2}dx+\frac{9}{2}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/90*sqrt(e)*(5*I*cos(9/2*d*x + 9/2*c) + 18*I*cos(5/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) + 45*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c)
, cos(9/2*d*x + 9/2*c))) + 5*sin(9/2*d*x + 9/2*c) + 18*sin(5/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 45*sin(1/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))))/(a^(5/2)*d)
```

Fricas [A] time = 1.87818, size = 262, normalized size = 2.17

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(45i\,e^{(6i\,dx+6i\,c)}+63i\,e^{(4i\,dx+4i\,c)}+23i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{\left(-\frac{9}{2}i\,dx-\frac{9}{2}i\,c\right)}}{90\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/90*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(4
5*I*e^(6*I*d*x + 6*I*c) + 63*I*e^(4*I*d*x + 4*I*c) + 23*I*e^(2*I*d*x + 2*I*
c) + 5*I)*e^(-9/2*I*d*x - 9/2*I*c)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e \sec(dx+c)}}{(i a \tan(dx+c) + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.435 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=162

$$-\frac{32i\sqrt{a+ia\tan(c+dx)}}{77a^3d\sqrt{e\sec(c+dx)}}+\frac{16i}{77a^2d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{12i}{77ad(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}+\frac{}{1\cdots}$$

```
[Out] ((2*I)/11)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)) + ((12*I)/
77)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + ((16*I)/77)/(
a^2*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((32*I)/77)*Sqrt[
a + I*a*Tan[c + d*x]])/(a^3*d*Sqrt[e*Sec[c + d*x]])
```

Rubi [A] time = 0.301009, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3502, 3488}

$$-\frac{32i\sqrt{a+ia\tan(c+dx)}}{77a^3d\sqrt{e\sec(c+dx)}}+\frac{16i}{77a^2d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{12i}{77ad(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}+\frac{}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((2*I)/11)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)) + ((12*I)/
77)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + ((16*I)/77)/(
a^2*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((32*I)/77)*Sqrt[
a + I*a*Tan[c + d*x]])/(a^3*d*Sqrt[e*Sec[c + d*x]])
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{11d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}} + \frac{6\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,d}{11a}$$

$$= \frac{2i}{11d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}} + \frac{12i}{77ad\sqrt{e\sec(c+dx)}(a+ia\tan(}$$

$$= \frac{2i}{11d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}} + \frac{12i}{77ad\sqrt{e\sec(c+dx)}(a+ia\tan(}$$

$$= \frac{2i}{11d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}} + \frac{12i}{77ad\sqrt{e\sec(c+dx)}(a+ia\tan(}$$

Mathematica [A] time = 0.400181, size = 102, normalized size = 0.63

$$\frac{i\sec^3(c+dx)(-22i\sin(c+dx)+42i\sin(3(c+dx))-55\cos(c+dx)+35\cos(3(c+dx)))}{154a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((I/154)*Sec[c + d*x]^3*(-55*Cos[c + d*x] + 35*Cos[3*(c + d*x)] - (22*I)*Si
n[c + d*x] + (42*I)*Sin[3*(c + d*x)]))/(a^2*d*Sqrt[e*Sec[c + d*x]]*(-I + Ta
n[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.32, size = 140, normalized size = 0.9

$$\frac{2\cos(dx+c)\left(28\,i(\cos(dx+c))^6+28\,(\cos(dx+c))^5\sin(dx+c)-9\,i(\cos(dx+c))^4+5\,(\cos(dx+c))^3\sin(dx+c)+\right.}{77\,da^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/77/d/a^3*(e/cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1
/2)*cos(d*x+c)*(28*I*cos(d*x+c)^6+28*cos(d*x+c)^5*sin(d*x+c)-9*I*cos(d*x+c)
^4+5*cos(d*x+c)^3*sin(d*x+c)+2*I*cos(d*x+c)^2+8*cos(d*x+c)*sin(d*x+c)-16*I)
/e
```

Maxima [A] time = 1.9415, size = 240, normalized size = 1.48

$$\frac{7i\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)+33i\cos\left(\frac{7}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right),\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)\right)\right)+77i\cos\left(\frac{3}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="max
ima")
```

```
[Out] 1/308*(7*I*cos(11/2*d*x + 11/2*c) + 33*I*cos(7/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) + 77*I*cos(3/11*arctan2(sin(11/2*d*x + 11/2
*c), cos(11/2*d*x + 11/2*c))) - 77*I*cos(1/11*arctan2(sin(11/2*d*x + 11/2*c
), cos(11/2*d*x + 11/2*c))) + 7*sin(11/2*d*x + 11/2*c) + 33*sin(7/11*arctan
2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(3/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(1/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))/(a^(5/2)*d*sqrt(e))
```

Fricas [A] time = 2.12603, size = 271, normalized size = 1.67

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-77i\,e^{(8i\,dx+8i\,c)}+110i\,e^{(4i\,dx+4i\,c)}+40i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{\left(-\frac{11}{2}i\,dx-\frac{11}{2}i\,c\right)}}{308\,a^3de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/308*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-77*I*e^(8*I*d*x + 8*I*c) + 110*I*e^(4*I*d*x + 4*I*c) + 40*I*e^(2*I*d*x + 2
*I*c) + 7*I)*e^(-11/2*I*d*x - 11/2*I*c)/(a^3*d*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(5/2)), x)
```

3.436 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=206

$$\frac{256i\sqrt{e\sec(c+dx)}}{585a^2de^2\sqrt{a+ia\tan(c+dx)}}-\frac{128i\sqrt{a+ia\tan(c+dx)}}{585a^3d(e\sec(c+dx))^{3/2}}+\frac{32i}{195a^2d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}+\frac{}{117ad(a+ia\text{t}}$$

```
[Out] ((2*I)/13)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)) + ((16*I
)/117)/(a*d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((32*I)/
195)/(a^2*d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((256*I)/
585)*Sqrt[e*Sec[c + d*x]])/(a^2*d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((128*
I)/585)*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d*(e*Sec[c + d*x])^(3/2))
```

Rubi [A] time = 0.398962, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3502, 3497, 3488}

$$\frac{256i\sqrt{e\sec(c+dx)}}{585a^2de^2\sqrt{a+ia\tan(c+dx)}}-\frac{128i\sqrt{a+ia\tan(c+dx)}}{585a^3d(e\sec(c+dx))^{3/2}}+\frac{32i}{195a^2d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}+\frac{}{117ad(a+ia\text{t}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((2*I)/13)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)) + ((16*I
)/117)/(a*d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((32*I)/
195)/(a^2*d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((256*I)/
585)*Sqrt[e*Sec[c + d*x]])/(a^2*d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((128*
I)/585)*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d*(e*Sec[c + d*x])^(3/2))
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{8\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan\ldots}}{13a}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{16\ldots}{117ad(e\sec(c+dx))^{3/2}(\ldots}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{16\ldots}{117ad(e\sec(c+dx))^{3/2}(\ldots}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{16\ldots}{117ad(e\sec(c+dx))^{3/2}(\ldots}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{16\ldots}{117ad(e\sec(c+dx))^{3/2}(\ldots}$$

Mathematica [A] time = 0.548076, size = 107, normalized size = 0.52

$$\frac{\sec^4(c+dx)(1040\sin(2(c+dx)) - 120\sin(4(c+dx)) - 1300i\cos(2(c+dx)) + 75i\cos(4(c+dx)) - 351i)}{2340a^2d(\tan(c+dx)-i)^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (Sec[c + d*x]^4*(-351*I - (1300*I)*Cos[2*(c + d*x)] + (75*I)*Cos[4*(c + d*x
)] + 1040*Sin[2*(c + d*x)] - 120*Sin[4*(c + d*x)]))/(2340*a^2*d*(e*Sec[c +
d*x])^(3/2)*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.315, size = 159, normalized size = 0.8

$$\frac{2(\cos(dx+c))^2\left(180i(\cos(dx+c))^7 + 180(\cos(dx+c))^6\sin(dx+c) - 55i(\cos(dx+c))^5 + 35\sin(dx+c)(\cos(d\ldots\right.}{585\,da^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2/585/d/a^3*(e/cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(
1/2)*cos(d*x+c)^2*(180*I*cos(d*x+c)^7+180*cos(d*x+c)^6*sin(d*x+c)-55*I*cos(
d*x+c)^5+35*sin(d*x+c)*cos(d*x+c)^4+8*I*cos(d*x+c)^3+48*cos(d*x+c)^2*sin(d*
x+c)+64*I*cos(d*x+c)+128*sin(d*x+c))/e^3
```

Maxima [A] time = 2.05163, size = 305, normalized size = 1.48

$$45i\cos\left(\frac{13}{2}dx + \frac{13}{2}c\right) + 260i\cos\left(\frac{9}{13}\arctan\left(\sin\left(\frac{13}{2}dx + \frac{13}{2}c\right), \cos\left(\frac{13}{2}dx + \frac{13}{2}c\right)\right)\right) + 702i\cos\left(\frac{5}{13}\arctan\left(\sin\left(\frac{13}{2}\ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="max
ima")
```

```
[Out] 1/4680*(45*I*cos(13/2*d*x + 13/2*c) + 260*I*cos(9/13*arctan2(sin(13/2*d*x +
 13/2*c), cos(13/2*d*x + 13/2*c))) + 702*I*cos(5/13*arctan2(sin(13/2*d*x +
13/2*c), cos(13/2*d*x + 13/2*c))) - 195*I*cos(3/13*arctan2(sin(13/2*d*x + 1
3/2*c), cos(13/2*d*x + 13/2*c))) + 2340*I*cos(1/13*arctan2(sin(13/2*d*x + 1
3/2*c), cos(13/2*d*x + 13/2*c))) + 45*sin(13/2*d*x + 13/2*c) + 260*sin(9/13
*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 702*sin(5/13*ar
ctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 195*sin(3/13*arcta
n2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 2340*sin(1/13*arctan2
(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))))/(a^(5/2)*d*e^(3/2))
```

Fricas [A] time = 2.16607, size = 360, normalized size = 1.75

$$\frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-195i\,e^{(10i\,dx+10i\,c)}+2145i\,e^{(8i\,dx+8i\,c)}+3042i\,e^{(6i\,dx+6i\,c)}+962i\,e^{(4i\,dx+4i\,c)}+305i\,e^{(2i\,dx+2i\,c)}+45i\right.}{4680\,a^3de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/4680*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-195*I*e^(10*I*d*x + 10*I*c) + 2145*I*e^(8*I*d*x + 8*I*c) + 3042*I*e^(6*I*
d*x + 6*I*c) + 962*I*e^(4*I*d*x + 4*I*c) + 305*I*e^(2*I*d*x + 2*I*c) + 45*I
)*e^(-13/2*I*d*x - 13/2*I*c)/(a^3*d*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(5/2)), x)
```

3.437 $\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=86

$$\frac{3i2^{2/3}a\sqrt[3]{1+i\tan(c+dx)}(e\sec(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{13}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{7d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] (((3*I)/7)*2^(2/3)*a*Hypergeometric2F1[1/3, 7/6, 13/6, (1 - I*Tan[c + d*x])
/2]*(e*Sec[c + d*x])^(7/3)*(1 + I*Tan[c + d*x])^(1/3))/(d*(a + I*a*Tan[c +
d*x])^(3/2))
```

Rubi [A] time = 0.210934, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3505, 3523, 70, 69}

$$\frac{3i2^{2/3}a\sqrt[3]{1+i\tan(c+dx)}(e\sec(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{13}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{7d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(7/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/7)*2^(2/3)*a*Hypergeometric2F1[1/3, 7/6, 13/6, (1 - I*Tan[c + d*x])
/2]*(e*Sec[c + d*x])^(7/3)*(1 + I*Tan[c + d*x])^(1/3))/(d*(a + I*a*Tan[c +
d*x])^(3/2))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{(e\sec(c+dx))^{7/3}\int (a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{2/3}\,dx}{(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}$$
$$= \frac{\left(a^2(e\sec(c+dx))^{7/3}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a-iax}}{\sqrt[3]{a+iax}}\,dx, x, \tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}$$
$$= \frac{\left(a^2(e\sec(c+dx))^{7/3}\sqrt[3]{\frac{a+ia\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a-iax}}{\sqrt[3]{\frac{1}{2}+\frac{ix}{2}}}\,dx, x, \tan(c+dx)\right)}{\sqrt[3]{2}d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{3/2}}$$
$$= \frac{3i2^{2/3}a\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{13}{6};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{7/3}\sqrt[3]{1+i\tan(c+dx)}}{7d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] time = 0.662903, size = 118, normalized size = 1.37

$$-\frac{3i\sqrt[3]{2}ee^{i(c+dx)}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{4/3}\left(4+\left(1+e^{2i(c+dx)}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{5}{3},-e^{2i(c+dx)}\right)\right)}{5d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(7/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-3*I)/5)*2^(1/3)*e*E^(I*(c + d*x))*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c
 + d*x))))^(4/3)*(4 + (1 + E^((2*I)*(c + d*x)))^(5/6)*Hypergeometric2F1[2/3
, 5/6, 5/3, -E^((2*I)*(c + d*x))]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{7}{3}}\frac{1}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((e*sec(d*x + c))^(7/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{-6i\cdot 2^{\frac{5}{6}}e^{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(\frac{e}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{4}{3}i\,dx+\frac{4}{3}i\,c\right)}+5\,ad\mathrm{integral}\left(-\frac{2i\cdot 2^{\frac{5}{6}}e^{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(\frac{e}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{1}{3}i\,dx+\frac{1}{3}i\,c\right)}}{5\,ad},x\right)}{5\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/5*(-6*I*2^(5/6)*e^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*
I*c) + 1))^(1/3)*e^(4/3*I*d*x + 4/3*I*c) + 5*a*d*integral(-2/5*I*2^(5/6)*e^
2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(
1/3*I*d*x + 1/3*I*c)/(a*d), x))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(7/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(7/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.438 $\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=86

$$\frac{3i\sqrt[3]{2}a(1+i\tan(c+dx))^{2/3}(e\sec(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{5d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] (((3*I)/5)*2^(1/3)*a*Hypergeometric2F1[2/3, 5/6, 11/6, (1 - I*Tan[c + d*x])
/2]*(e*Sec[c + d*x])^(5/3)*(1 + I*Tan[c + d*x])^(2/3))/(d*(a + I*a*Tan[c +
d*x])^(3/2))
```

Rubi [A] time = 0.198889, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3505, 3523, 70, 69}

$$\frac{3i\sqrt[3]{2}a(1+i\tan(c+dx))^{2/3}(e\sec(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{5d(a+ia\tan(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/5)*2^(1/3)*a*Hypergeometric2F1[2/3, 5/6, 11/6, (1 - I*Tan[c + d*x])
/2]*(e*Sec[c + d*x])^(5/3)*(1 + I*Tan[c + d*x])^(2/3))/(d*(a + I*a*Tan[c +
d*x])^(3/2))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{(e\sec(c+dx))^{5/3}\int (a-ia\tan(c+dx))^{5/6}\sqrt[3]{a+ia\tan(c+dx)}\,dx}{(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}\\ &= \frac{\left(a^2(e\sec(c+dx))^{5/3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{a-iax}(a+iax)^{2/3}}\,dx,x,\tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}\\ &= \frac{\left(a^2(e\sec(c+dx))^{5/3}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{2/3}\sqrt[6]{a-iax}}\,dx,x,\tan(c+dx)\right)}{2^{2/3}d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{3/2}}\\ &= \frac{3i\sqrt[3]{2}a\,{}_2F_1\left(\frac{2}{3},\frac{5}{6};\frac{11}{6};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/3}(1+i\tan(c+dx))^{2/3}}{5d(a+ia\tan(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.550094, size = 116, normalized size = 1.35

$$\frac{3i2^{2/3}ee^{i(c+dx)}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(-2+\sqrt[6]{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{4}{3},-e^{2i(c+dx)}\right)\right)}{d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5/3)/Sqrt[a + I*a*Tan[c + d*x]],x]

[Out] ((3*I)*2^(2/3)*e*E^(I*(c + d*x))*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*
x))))^(2/3)*(-2 + (1 + E^((2*I)*(c + d*x)))^(1/6)*Hypergeometric2F1[1/6, 1/
3, 4/3, -E^((2*I)*(c + d*x))]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F] time = 0.33, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{5}{3}}\frac{1}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x)

[Out] int((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{5}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

[Out] integrate((e*sec(d*x + c))^(5/3)/sqrt(I*a*tan(d*x + c) + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(ade^{(idx+ic)}\text{integral}\left(\frac{2^{\frac{1}{6}}\left(iee^{(2idx+2ic)}+ie\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(\frac{e}{e^{(2idx+2ic)}+1}\right)^{\frac{2}{3}}e^{\left(-\frac{4}{3}idx-\frac{4}{3}ic\right)}}{ad},x\right)+2\cdot 2^{\frac{1}{6}}\left(-3iee^{(2idx+2ic)}-3ie\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\left(\cdots\right.}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")

[Out] (a*d*e^(I*d*x + I*c)*integral(2^(1/6)*(I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(-4/3*I*d*x - 4/3*I*c)/(a*d), x) + 2*2^(1/6)*(-3*I*e*e^(2*I*d*x + 2*I*c) - 3*I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(5/3*I*d*x + 5/3*I*c))*e^(-I*d*x - I*c)/(a*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sec(d*x+c))**(5/3)/(a+I*a*tan(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec{(dx+c)})^{\frac{5}{3}}}{\sqrt{ia\tan{(dx+c)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((e*sec(d*x + c))^(5/3)/sqrt(I*a*tan(d*x + c) + a), x)

3.439 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{3i\sqrt[6]{1+i\tan(c+dx)}(e\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{4}{3},\frac{1}{2}(1-i\tan(c+dx))\right)}{2\sqrt[6]{2}d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (((3*I)/2)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - I*Tan[c + d*x])/2]*(e*Sec[
c + d*x])^(2/3)*(1 + I*Tan[c + d*x])^(1/6))/(2^(1/6)*d*Sqrt[a + I*a*Tan[c +
 d*x]])
```

Rubi [A] time = 0.190387, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3505, 3523, 70, 69}

$$\frac{3i\sqrt[6]{1+i\tan(c+dx)}(e\sec(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{4}{3},\frac{1}{2}(1-i\tan(c+dx))\right)}{2\sqrt[6]{2}d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/2)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - I*Tan[c + d*x])/2]*(e*Sec[
c + d*x])^(2/3)*(1 + I*Tan[c + d*x])^(1/6))/(2^(1/6)*d*Sqrt[a + I*a*Tan[c +
 d*x]])
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{(e\sec(c+dx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(c+dx)}}{\sqrt[6]{a+ia\tan(c+dx)}}\,dx}{\sqrt[3]{a-ia\tan(c+dx)}\sqrt[3]{a+ia\tan(c+dx)}}\\
&= \frac{\left(a^2(e\sec(c+dx))^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{(a-iax)^{2/3}(a+iax)^{7/6}}\,dx,x,\tan(c+dx)\right)}{d\sqrt[3]{a-ia\tan(c+dx)}\sqrt[3]{a+ia\tan(c+dx)}}\\
&= \frac{\left(a(e\sec(c+dx))^{2/3}\sqrt[6]{\frac{a+ia\tan(c+dx)}{a}}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{7/6}(a-iax)^{2/3}}\,dx,x,\tan(c+dx)\right)}{2\sqrt[6]{2}d\sqrt[3]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&= \frac{3i\,{}_2F_1\left(\frac{1}{3},\frac{7}{6};\frac{4}{3};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2/3}\sqrt[6]{1+i\tan(c+dx)}}{2\sqrt[6]{2}d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.428968, size = 116, normalized size = 1.36

$$\frac{3i\sqrt[6]{2}\sqrt[6]{1+e^{2i(c+dx)}}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},-e^{2i(c+dx)}\right)}{d\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]

[Out] ((3*I)*2^(1/6)*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(1 + E
^((2*I)*(c + d*x)))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -E^((2*I)*(c +
d*x))])/(d*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])
```

Maple [F] time = 0.448, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{2}{3}}\frac{1}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)

[Out] int((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((e*sec(d*x + c))^(2/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2^{\frac{1}{6}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(\frac{e}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(3i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(\frac{5}{3}i\,dx+\frac{5}{3}i\,c\right)}+\left(ade^{(4i\,dx+4i\,c)}-2\,ade^{(3i\,dx+3i\,c)}+ade^{(2i\,dx+\cdots}\right.}{ade^{(4i\,dx+4i\,c)}-2\,ade^{(3i\,dx+3i\,c)}+a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] (2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2
/3)*(3*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(5/3*I*d*x
+ 5/3*I*c) + (a*d*e^(4*I*d*x + 4*I*c) - 2*a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(
2*I*d*x + 2*I*c))*integral(2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^
(2*I*d*x + 2*I*c) + 1))^(2/3)*(I*e^(4*I*d*x + 4*I*c) + 7*I*e^(3*I*d*x + 3*I
*c) + 5*I*e^(2*I*d*x + 2*I*c) + 7*I*e^(I*d*x + I*c) + 4*I)*e^(5/3*I*d*x + 5
/3*I*c)/(a*d*e^(5*I*d*x + 5*I*c) - 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(3*I
*d*x + 3*I*c) - a*d*e^(2*I*d*x + 2*I*c)), x))/(a*d*e^(4*I*d*x + 4*I*c) - 2*
a*d*e^(3*I*d*x + 3*I*c) + a*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(c+dx))^{\frac{2}{3}}}{\sqrt{a\,(i\tan(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(2/3)/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(2/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.440 $\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=83

$$\frac{3i\sqrt[3]{1+i\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{4}{3},\frac{7}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{\sqrt[3]{2}d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] ((3*I)*Hypergeometric2F1[1/6, 4/3, 7/6, (1 - I*Tan[c + d*x])/2]*(e*Sec[c +
d*x])^(1/3)*(1 + I*Tan[c + d*x])^(1/3))/(2^(1/3)*d*Sqrt[a + I*a*Tan[c + d*x
]])
```

Rubi [A] time = 0.182116, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3505, 3523, 70, 69}

$$\frac{3i\sqrt[3]{1+i\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{4}{3},\frac{7}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{\sqrt[3]{2}d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((3*I)*Hypergeometric2F1[1/6, 4/3, 7/6, (1 - I*Tan[c + d*x])/2]*(e*Sec[c +
d*x])^(1/3)*(1 + I*Tan[c + d*x])^(1/3))/(2^(1/3)*d*Sqrt[a + I*a*Tan[c + d*x
]])
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt[3]{e\sec(c+dx)}\int \frac{\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx}{\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}}$$
$$= \frac{\left(a^2\sqrt[3]{e\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a-iax)^{5/6}(a+iax)^{4/3}}\,dx, x, \tan(c+dx)\right)}{d\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}}$$
$$= \frac{\left(a\sqrt[3]{e\sec(c+dx)}\sqrt[3]{\frac{a+ia\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{4/3}(a-iax)^{5/6}}\,dx, x, \tan(c+dx)\right)}{2\sqrt[3]{2}d\sqrt[6]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$= \frac{3i\,{}_2F_1\left(\frac{1}{6},\frac{4}{3};\frac{7}{6};\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[3]{e\sec(c+dx)}\sqrt[3]{1+i\tan(c+dx)}}{\sqrt[3]{2}d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 0.519591, size = 95, normalized size = 1.14

$$\frac{3\left(8i-\frac{2ie^{2i(c+dx)}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{5}{3},-e^{2i(c+dx)}\right)}{\sqrt[6]{1+e^{2i(c+dx)}}}\right)\sqrt[3]{e\sec(c+dx)}}{16d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (3*(8*I - ((2*I)*E^((2*I)*(c + d*x))*Hypergeometric2F1[2/3, 5/6, 5/3, -E^((
2*I)*(c + d*x))])/(1 + E^((2*I)*(c + d*x)))^(1/6))*(e*Sec[c + d*x])^(1/3))/
(16*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F] time = 0.408, size = 0, normalized size = 0.

$$\int \sqrt[3]{e\sec(dx+c)}\frac{1}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{\frac{1}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((e*sec(d*x + c))^(1/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(4\,ade^{(2i\,dx+2i\,c)}\text{integral}\left(-\frac{i\cdot 2^{\frac{5}{6}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(\frac{e}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}e^{\left(\frac{1}{3}i\,dx+\frac{1}{3}i\,c\right)}}{4\,ad},x\right)+2^{\frac{5}{6}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(\frac{e}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{1}{3}}\left(3i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(\frac{4}{3}\right.}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a*d*e^(2*I*d*x + 2*I*c)*integral(-1/4*I*2^(5/6)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(1/3*I*d*x + 1/3*I*c)/(
a*d), x) + 2^(5/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c
) + 1))^(1/3)*(3*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(4/3*I*d*x + 4/3*I*c))*e^(-
2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{e\sec\left(c+dx\right)}}{\sqrt{a\left(i\tan\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(1/3)/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(1/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.441 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=83

$$-\frac{3i(1+i\tan(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{3},\frac{5}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{2^{2/3}d\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}$$

```
[Out] ((-3*I)*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - I*Tan[c + d*x])/2]*(1 + I*Ta
n[c + d*x])^(2/3))/(2^(2/3)*d*(e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d
*x]])
```

Rubi [A] time = 0.196309, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3505, 3523, 70, 69}

$$-\frac{3i(1+i\tan(c+dx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{3},\frac{5}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{2^{2/3}d\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-3*I)*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - I*Tan[c + d*x])/2]*(1 + I*Ta
n[c + d*x])^(2/3))/(2^(2/3)*d*(e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d
*x]])
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}\right)\int \frac{1}{\sqrt[6]{a-ia\tan(c+dx)}(a+ia\tan(c+dx))^{2/3}}\,dx}{\sqrt[3]{e\sec(c+dx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{(a-iax)^{7/6}(a+iax)^{5/3}}\,dx, x, t\right.}{d\sqrt[3]{e\sec(c+dx)}}$$

$$= \frac{\left(a\sqrt[6]{a-ia\tan(c+dx)}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{5/3}(a-iax)^{7/6}}\,dx, x, \tan\right.}{2\,2^{2/3}d\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{3i\,{}_2F_1\left(-\frac{1}{6},\frac{5}{3};\frac{5}{6};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{2/3}}{2^{2/3}d\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 0.738996, size = 95, normalized size = 1.14

$$\frac{12i-\frac{30ie^{2i(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{4}{3},-e^{2i(c+dx)}\right)}{\left(1+e^{2i(c+dx)}\right)^{5/6}}}{16d\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (12*I - ((30*I)*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/6, 1/3, 4/3, -E^((2
*I)*(c + d*x))])/(1 + E^((2*I)*(c + d*x)))^(5/6))/(16*d*(e*Sec[c + d*x])^(1
/3)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F] time = 0.359, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{e\sec(dx+c)}}\frac{1}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\sec(dx+c))^{\frac{1}{3}}\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(1/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{2^{\frac{1}{6}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(\frac{e}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(-12i\,e^{(6i\,dx+6i\,c)}-27i\,e^{(4i\,dx+4i\,c)}-18i\,e^{(2i\,dx+2i\,c)}-3i\right)e^{\left(\frac{5}{3}i\,dx+\frac{5}{3}i\,c\right)}+8\left(adee^{(5i\,dx+5i\,c)}-ade\right.}{8\left(adee^{(5i\,dx+5i\,c)}-adee^{(3i\,dx+3i\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/8*(2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1)
)^(2/3)*(-12*I*e^(6*I*d*x + 6*I*c) - 27*I*e^(4*I*d*x + 4*I*c) - 18*I*e^(2*I
*d*x + 2*I*c) - 3*I)*e^(5/3*I*d*x + 5/3*I*c) + 8*(a*d*e*e^(5*I*d*x + 5*I*c)
 - a*d*e*e^(3*I*d*x + 3*I*c))*integral(1/16*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-45*I*e^(4*I*d*x + 4*I*c) -
 60*I*e^(2*I*d*x + 2*I*c) - 15*I)*e^(5/3*I*d*x + 5/3*I*c)/(a*d*e*e^(7*I*d*x
 + 7*I*c) - 2*a*d*e*e^(5*I*d*x + 5*I*c) + a*d*e*e^(3*I*d*x + 3*I*c)), x))/(
a*d*e*e^(5*I*d*x + 5*I*c) - a*d*e*e^(3*I*d*x + 3*I*c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\,(i\tan{(c+dx)}+1)}\,\sqrt[3]{e\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(1/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(I*tan(c + d*x) + 1))*(e*sec(c + d*x))**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\sec(dx+c))^{\frac{1}{3}}\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(1/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.442 $\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=88

$$-\frac{3i\sqrt[6]{1+i\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{13}{6},\frac{1}{3},\frac{1}{2}(1-i\tan(c+dx))\right)}{8\sqrt[6]{2}ad(e\sec(c+dx))^{4/3}}$$

```
[Out] (((-3*I)/8)*Hypergeometric2F1[-2/3, 13/6, 1/3, (1 - I*Tan[c + d*x])/2]*(1 +
 I*Tan[c + d*x])^(1/6)*Sqrt[a + I*a*Tan[c + d*x]])/(2^(1/6)*a*d*(e*Sec[c +
d*x])^(4/3))
```

Rubi [A] time = 0.209879, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3505, 3523, 70, 69}

$$-\frac{3i\sqrt[6]{1+i\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{13}{6},\frac{1}{3},\frac{1}{2}(1-i\tan(c+dx))\right)}{8\sqrt[6]{2}ad(e\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (((-3*I)/8)*Hypergeometric2F1[-2/3, 13/6, 1/3, (1 - I*Tan[c + d*x])/2]*(1 +
 I*Tan[c + d*x])^(1/6)*Sqrt[a + I*a*Tan[c + d*x]])/(2^(1/6)*a*d*(e*Sec[c +
d*x])^(4/3))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left((a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3}\right)\int \frac{1}{(a-ia\tan(c+dx))^{2/3}(a+ia\tan(}}{(e\sec(c+dx))^{4/3}}$$

$$= \frac{\left(a^2(a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{1}{(a-iax)^{5/3}(a+iax}\right.}{d(e\sec(c+dx))^{4/3}}$$

$$= \frac{\left((a-ia\tan(c+dx))^{2/3}\sqrt{a+ia\tan(c+dx)}\sqrt[6]{\frac{a+ia\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{}{\left(\frac{1}{2}\right.}\right.}{4\sqrt[6]{2}d(e\sec(c+dx))^{4/3}}$$

$$= -\frac{3i\,{}_2F_1\left(-\frac{2}{3},\frac{13}{6};\frac{1}{3};\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[6]{1+i\tan(c+dx)}\sqrt{a+ia\tan(}}{8\sqrt[6]{2}ad(e\sec(c+dx))^{4/3}}$$

Mathematica [A] time = 0.655092, size = 112, normalized size = 1.27

$$-\frac{3i\sec^2(c+dx)\left(-55\sqrt[6]{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},-e^{2i(c+dx)}\right)+11i\sin(2(c+dx))+3\cos(2(c+dx))\right.}{112d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (((-3*I)/112)*Sec[c + d*x]^2*(3 + 3*Cos[2*(c + d*x)] - 55*(1 + E^((2*I)*(c
+ d*x)))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -E^((2*I)*(c + d*x))] + (1
1*I)*Sin[2*(c + d*x)]))/(d*(e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-\frac{4}{3}}\frac{1}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\sec(dx+c))^{\frac{4}{3}}\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(4/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$2^{\frac{1}{6}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(\frac{e}{e^{(2i\,dx+2i\,c)}+1}\right)^{\frac{2}{3}}\left(-21i\,e^{(8i\,dx+8i\,c)}+42i\,e^{(7i\,dx+7i\,c)}+114i\,e^{(6i\,dx+6i\,c)}+60i\,e^{(5i\,dx+5i\,c)}+303i\,e^{(4i\,dx+4i\,c)}-6i\,e^{(3i\,dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/112*(2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) +
1))^(2/3)*(-21*I*e^(8*I*d*x + 8*I*c) + 42*I*e^(7*I*d*x + 7*I*c) + 114*I*e^(
6*I*d*x + 6*I*c) + 60*I*e^(5*I*d*x + 5*I*c) + 303*I*e^(4*I*d*x + 4*I*c) - 6
*I*e^(3*I*d*x + 3*I*c) + 180*I*e^(2*I*d*x + 2*I*c) - 24*I*e^(I*d*x + I*c) +
 12*I)*e^(5/3*I*d*x + 5/3*I*c) + 112*(a*d*e^2*e^(6*I*d*x + 6*I*c) - 2*a*d*e
^2*e^(5*I*d*x + 5*I*c) + a*d*e^2*e^(4*I*d*x + 4*I*c))*integral(1/112*2^(1/6
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(55
*I*e^(4*I*d*x + 4*I*c) + 385*I*e^(3*I*d*x + 3*I*c) + 275*I*e^(2*I*d*x + 2*I
*c) + 385*I*e^(I*d*x + I*c) + 220*I)*e^(5/3*I*d*x + 5/3*I*c)/(a*d*e^2*e^(5*
I*d*x + 5*I*c) - 3*a*d*e^2*e^(4*I*d*x + 4*I*c) + 3*a*d*e^2*e^(3*I*d*x + 3*I
*c) - a*d*e^2*e^(2*I*d*x + 2*I*c)), x))/(a*d*e^2*e^(6*I*d*x + 6*I*c) - 2*a*
d*e^2*e^(5*I*d*x + 5*I*c) + a*d*e^2*e^(4*I*d*x + 4*I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))**(4/3)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{4}{3}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(4/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.443 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$

Optimal. Leaf size=437

$$\frac{5i\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{12\ 2^{2/3}\sqrt{3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}-\frac{5x(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}+\frac{}{24f\sqrt[3]{}}$$

```
[Out] ((I/4)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(7/3)) - (5*x*(d*S
ec[e + f*x])^(2/3))/(72*2^(2/3)*a^(5/3)*(a - I*a*Tan[e + f*x])^(1/3)*(a + I
*a*Tan[e + f*x])^(1/3)) + (((5*I)/12)*ArcTan[(a^(1/3) + 2^(2/3)*(a - I*a*Ta
n[e + f*x])^(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*Sqrt
[3]*a^(5/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) -
(((5*I)/72)*Log[Cos[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(5/3)*f*(a
 - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - (((5*I)/24)*Log[
2^(1/3)*a^(1/3) - (a - I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(2^
(2/3)*a^(5/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
+ (((5*I)/24)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)*(a^2
+ I*a^2*Tan[e + f*x]))
```

Rubi [A] time = 0.397968, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3505, 3522, 3487, 51, 57, 617, 204, 31}

$$\frac{5i\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{12\ 2^{2/3}\sqrt{3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}-\frac{5x(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}+\frac{}{24f\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(7/3),x]
```

```
[Out] ((I/4)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(7/3)) - (5*x*(d*S
ec[e + f*x])^(2/3))/(72*2^(2/3)*a^(5/3)*(a - I*a*Tan[e + f*x])^(1/3)*(a + I
*a*Tan[e + f*x])^(1/3)) + (((5*I)/12)*ArcTan[(a^(1/3) + 2^(2/3)*(a - I*a*Ta
n[e + f*x])^(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*Sqrt
[3]*a^(5/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) -
(((5*I)/72)*Log[Cos[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(5/3)*f*(a
 - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - (((5*I)/24)*Log[
2^(1/3)*a^(1/3) - (a - I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(2^
(2/3)*a^(5/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
+ (((5*I)/24)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)*(a^2
+ I*a^2*Tan[e + f*x]))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3522

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
```

```
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{(d\sec(e+fx))^{2/3}\int \cos^4(e+fx)(a-ia\tan(e+fx))^{7/3}\,dx}{a^4\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(ia(d\sec(e+fx))^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{2/3}}\,dx, x, -ia\tan(e+fx)\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{\left(5i(d\sec(e+fx))^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{2/3}}\,dx, x, -ia\tan(\right.}{12f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}} + \frac{\left(5i(d\sec(e+fx))^{2/3}\right)\operatorname{Subs}}{36af\sqrt[3]{a-ia\tan}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}} - \frac{5x(d\sec(e+}{72\,2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}} - \frac{5x(d\sec(e+}{72\,2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}} - \frac{5x(d\sec(e+}{72\,2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+}}$$

Mathematica [A] time = 1.79464, size = 240, normalized size = 0.55

$$\frac{e^{-2i(e+fx)}\sec^2(e+fx)(d\sec(e+fx))^{2/3}\left(-10fxe^{4i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}+33ie^{2i(e+fx)}+24ie^{4i(e+fx)}-15ie^{4i(e+fx)}\sqrt[3]{1+e^{2i(e}}\right.}{144f(a+ia\tan(e+fx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(7/3),x]
```

```
[Out] ((9*I + (33*I)*E^((2*I)*(e + f*x)) + (24*I)*E^((4*I)*(e + f*x)) - 10*E^((4*
I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*f*x - (10*I)*Sqrt[3]*E^((4*I)
*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*ArcTan[(1 + 2*(1 + E^((2*I)*(e
+ f*x)))^(1/3))/Sqrt[3]] - (15*I)*E^((4*I)*(e + f*x))*(1 + E^((2*I)*(e + f*
x)))^(1/3)*Log[1 - (1 + E^((2*I)*(e + f*x)))^(1/3)])*Sec[e + f*x]^2*(d*Sec[
e + f*x])^(2/3))/(144*E^((2*I)*(e + f*x))*f*(a + I*a*Tan[e + f*x])^(7/3))
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(a+ia\tan\left(fx+e\right)\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x)
```

Maxima [B] time = 2.82842, size = 5268, normalized size = 12.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="maxim
a")
```

```
[Out] -1/288*((cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*a
rctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x
 + 4*e), cos(4*f*x + 4*e))) + 1)^(5/6)*((-48*I*2^(1/3)*cos(4*f*x + 4*e) - 4
8*2^(1/3)*sin(4*f*x + 4*e))*cos(5/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e
), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))
 + 1)) + 48*(2^(1/3)*cos(4*f*x + 4*e) - I*2^(1/3)*sin(4*f*x + 4*e))*sin(5/3
*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arct
an2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)))*d^(2/3) + (cos(1/2*arctan2(
sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e),
cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)
)) + 1)^(1/3)*((30*I*2^(1/3)*cos(4*f*x + 4*e) + 30*2^(1/3)*sin(4*f*x + 4*e)
)*cos(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos
(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) - 30*(2^(1/3)*cos(4
*f*x + 4*e) - I*2^(1/3)*sin(4*f*x + 4*e))*sin(2/3*arctan2(sin(1/2*arctan2(s
in(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(
4*f*x + 4*e))) + 1)))*d^(2/3) + (10*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(
cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(si
n(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e),
cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4
*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)
)) + 1)) + 1/3*sqrt(3), 1/3*sqrt(3)*(2*(cos(1/2*arctan2(sin(4*f*x + 4*e), c
os(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^
2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*sin(1
/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*ar
ctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + sqrt(3))) + 10*I*sqrt(3)
*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x +
 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(
1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3*arctan2
(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(
4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + 1/3*sqrt(3), -1/3*sqrt(3)*(2*(cos(
1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*
f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(
4*f*x + 4*e))) + 1)^(1/6)*sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e),
 cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) +
 1)) - sqrt(3))) - 5*sqrt(3)*2^(1/3)*log(4/3*(cos(1/2*arctan2(sin(4*f*x + 4
*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4
*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/3)
*(cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos
(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2 + sin(1/3*arctan2
(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(
4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2) + 4/3*(cos(1/2*arctan2(sin(4*f*x
+ 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x
+ 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1
/6)*(sqrt(3)*sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x +
4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + cos(1/
3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arc
```

```
tan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))) + 4/3) + 5*sqrt(3)*2^(1/3)
*log(4/3*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*
arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*
x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/3)*(cos(1/3*arctan2(sin(1/2*arctan2(si
n(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4
*f*x + 4*e))) + 1))^2 + sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), c
os(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1
))^2) - 4/3*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1
/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(1/2
*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x +
4*e), cos(4*f*x + 4*e))) + 1)) - cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x
+ 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4
*e))) + 1))) + 4/3) + 10*2^(1/3)*arctan2((cos(1/2*arctan2(sin(4*f*x + 4*e),
 cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))
)^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/3)*sin
(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*
arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + (cos(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4
*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) +
1)^(1/6)*sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)
)), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)), (cos(1/2*ar
ctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x +
4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x
+ 4*e))) + 1)^(1/3)*cos(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4
*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) +
 (cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(
sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e)
, cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x +
 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*
e))) + 1)) + 1) - 20*2^(1/3)*arctan2((cos(1/2*arctan2(sin(4*f*x + 4*e), cos
(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2
+ 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*sin(1/3
*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arct
an2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)), (cos(1/2*arctan2(sin(4*f*x
+ 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x
+ 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1
/6)*cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), c
os(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) - 1) + 10*I*2^(1/
3)*log((cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*ar
ctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x
+ 4*e), cos(4*f*x + 4*e))) + 1)^(1/3)*cos(1/3*arctan2(sin(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*
x + 4*e))) + 1))^2 + (cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^
2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arct
an2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/3)*sin(1/3*arctan2(sin(1/2
*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x +
4*e), cos(4*f*x + 4*e))) + 1))^2 - 2*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos
(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2
+ 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3
*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arct
an2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + 1) - 5*I*2^(1/3)*log((cos(
1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*
f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(
4*f*x + 4*e))) + 1)^(2/3)*(cos(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e)
, cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))
+ 1))^2 + sin(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e
))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2) + (cos(1
/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f
```

```
*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4
*f*x + 4*e))) + 1)^(1/3)*(cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e),
 cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) +
 1))^2 + sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)
)), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2) + 2*(cos(
1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*
f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(
4*f*x + 4*e))) + 1)^(1/3)*((cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4
*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/
2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*(cos(2/3*arctan2(
sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e))) + 1))*cos(1/3*arctan2(sin(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*
x + 4*e))) + 1)) + sin(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*
f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))*si
n(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2
*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))) + cos(2/3*arctan2(sin(
1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x
 + 4*e), cos(4*f*x + 4*e))) + 1))) + 2*(cos(1/2*arctan2(sin(4*f*x + 4*e), c
os(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^
2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1
/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*ar
ctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + 1))*d^(2/3))/(a^(7/3)*f)
```

Fricas [A] time = 2.33395, size = 1608, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="frica
s")
```

```
[Out] 1/96*(96*a^3*f*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6*I*e)*log(-
2/5*(72*I*a^3*f*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I*e) - 5*
2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(
2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) +
 2*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1)
)^(2/3)*(8*I*e^(6*I*f*x + 6*I*e) + 19*I*e^(4*I*f*x + 4*I*e) + 14*I*e^(2*I*f
*x + 2*I*e) + 3*I)*e^(2*I*f*x + 2*I*e) + (48*I*sqrt(3)*a^3*f - 48*a^3*f)*(1
25/186624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6*I*e)*log(2/5*(5*2^(1/3)*(a/
(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*
I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) + 36*(sqrt(3)*a^3*f + I*a^3*f)*(125
/186624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) +
 (-48*I*sqrt(3)*a^3*f - 48*a^3*f)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I
*f*x + 6*I*e)*log(2/5*(5*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^
(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e)
 - 36*(sqrt(3)*a^3*f - I*a^3*f)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f
*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(7/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}{\left(i\,a\tan\left(fx+e\right)+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(7/3), x)
```

3.444 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx$

Optimal. Leaf size=378

$$\frac{i\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}\sqrt{3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}-\frac{x(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}-\frac{i(d\sec(e+f\ldots}{2\ 2^{2/3}a^{2/3}f\ldots}$$

```
[Out] ((I/2)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(4/3)) - (x*(d*Sec
[e + f*x])^(2/3))/(6*2^(2/3)*a^(2/3)*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*
Tan[e + f*x])^(1/3)) + (I*ArcTan[(a^(1/3) + 2^(2/3)*(a - I*a*Tan[e + f*x])^
(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*Sqrt[3]*a^(2/3)*
f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - ((I/6)*Log[C
os[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(2/3)*f*(a - I*a*Tan[e + f*
x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - ((I/2)*Log[2^(1/3)*a^(1/3) - (a -
 I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(2/3)*f*(a - I
*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

Rubi [A] time = 0.342637, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3505, 3522, 3487, 51, 57, 617, 204, 31}

$$\frac{i\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}\sqrt{3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}-\frac{x(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}-\frac{i(d\sec(e+f\ldots}{2\ 2^{2/3}a^{2/3}f\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(4/3),x]
```



```
[Out] ((I/2)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(4/3)) - (x*(d*Sec
[e + f*x])^(2/3))/(6*2^(2/3)*a^(2/3)*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*
Tan[e + f*x])^(1/3)) + (I*ArcTan[(a^(1/3) + 2^(2/3)*(a - I*a*Tan[e + f*x])^
(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*Sqrt[3]*a^(2/3)*
f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - ((I/6)*Log[C
os[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(2/3)*f*(a - I*a*Tan[e + f*
x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - ((I/2)*Log[2^(1/3)*a^(1/3) - (a -
 I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(2/3)*f*(a - I
*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```


Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3522

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \frac{(d\sec(e+fx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$= \frac{(d\sec(e+fx))^{2/3}\int \cos^2(e+fx)(a-ia\tan(e+fx))^{4/3}\,dx}{a^2\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$= \frac{\left(ia(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{2/3}}\,dx, x, -ia\tan(e+fx)\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} + \frac{\left(i(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{2/3}}\,dx, x, -ia\tan(e+f\right.}{3f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} - \frac{x(d\sec(e+fx))^{2/3}}{6\,2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i}{6\,2^{2/3}a^{2/}}$$
$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} - \frac{x(d\sec(e+fx))^{2/3}}{6\,2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i}{6\,2^{2/3}a^{2/}}$$
$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} - \frac{x(d\sec(e+fx))^{2/3}}{6\,2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} + \frac{i\tan}{2^{2/3}\sqrt{3}a}$$

Mathematica [A] time = 1.23268, size = 220, normalized size = 0.58

$$\frac{e^{-i(e+fx)}(d\sec(e+fx))^{5/3}\left(-2fxe^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}} + 3ie^{2i(e+fx)} - 3ie^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\log\left(1-\sqrt[3]{1+e^{2i(e+fx)}}\right) - 2i\sqrt{}\right.}{12df(a+ia\tan(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(4/3),x]
```

```
[Out] ((3*I + (3*I)*E^((2*I)*(e + f*x)) - 2*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e
+ f*x)))^(1/3)*f*x - (2*I)*Sqrt[3]*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f
*x)))^(1/3)*ArcTan[(1 + 2*(1 + E^((2*I)*(e + f*x)))^(1/3))/Sqrt[3]] - (3*I)
*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Log[1 - (1 + E^((2*I)*
(e + f*x)))^(1/3)])*(d*Sec[e + f*x])^(5/3))/(12*d*E^(I*(e + f*x))*f*(a + I*
a*Tan[e + f*x])^(4/3))
```

Maple [F] time = 0.157, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(a+ia\tan\left(fx+e\right)\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x)
```

Maxima [B] time = 2.32492, size = 2572, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="maxim
a")
```

```
[Out] 1/24*((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1
/3)*((6*I*2^(1/3)*cos(2*f*x + 2*e) + 6*2^(1/3)*sin(2*f*x + 2*e))*cos(2/3*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 6*(2^(1/3)*cos(2*f*x + 2*e
) - I*2^(1/3)*sin(2*f*x + 2*e))*sin(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)))*d^(2/3) + (-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), 1/3*sqrt(3)*(
2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*
sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sqrt(3))) - 2*I*
sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)) + 1/3*sqrt(3), -1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1)) - sqrt(3))) + sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e) + 1))^2) + 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1)) + cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1))) + 4/3) - sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1))^2) - 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1)) - cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) +
4/3) - 2*2^(1/3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/3)*sin(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)) + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*cos(2/3*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) + 1) + 4*2^(1/3)*arctan2((cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)) - 1) - 2*I*2^(1/3)*log((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)^(1/3)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2
- 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6
)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) + I*2^(1/3)
*log((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(2/
3)*(cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(2/3*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2) + (cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1))^2) + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/3)*((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)^(1/6)*(cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
```

```
+ 1))*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(2/3*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/3*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1))) + cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1))) + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) +
 1))*d^(2/3))/(a^(4/3)*f)
```

Fricas [A] time = 2.07599, size = 1496, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="frica
s")
```

```
[Out] 1/8*(8*a^2*f*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x + 4*I*e)*log(-2*(6*I*
a^2*f*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e) - 2^(1/3)*(a/(e^(2*
I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x
+ 2*I*e) + 1)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 2*2^(1/3)*(a/(e^
(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(I*e^(4*I
*f*x + 4*I*e) + 2*I*e^(2*I*f*x + 2*I*e) + I)*e^(2*I*f*x + 2*I*e) + (4*I*sqr
t(3)*a^2*f - 4*a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x + 4*I*e)*log
(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1
))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) + 3*(sqrt(3)*a^2*f +
 I*a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x -
2*I*e)) + (-4*I*sqrt(3)*a^2*f - 4*a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4
*I*f*x + 4*I*e)*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2
*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) -
 3*(sqrt(3)*a^2*f - I*a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I
*e))*e^(-2*I*f*x - 2*I*e)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec\left(fx+e\right)\right)^{\frac{2}{3}}}{\left(i\,a \tan\left(fx+e\right)+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(4/3), x)
```

3.445 $\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx$

Optimal. Leaf size=340

$$\frac{i\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}-\frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}-\frac{3i\sqrt[3]{a}(d\sec(e+}{2\,2^{2/3}f\sqrt[3]{}}$$

```
[Out] -(a^(1/3)*x*(d*Sec[e + f*x])^(2/3))/(2*2^(2/3)*(a - I*a*Tan[e + f*x])^(1/3)
*(a + I*a*Tan[e + f*x])^(1/3)) + (I*Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/
3)*(a - I*a*Tan[e + f*x])^(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3))
/(2^(2/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - ((
I/2)*a^(1/3)*Log[Cos[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*f*(a - I*a*
Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - (((3*I)/2)*a^(1/3)*Log[
2^(1/3)*a^(1/3) - (a - I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(2^
(2/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

Rubi [A] time = 0.171269, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3492, 3481, 57, 617, 204, 31}

$$\frac{i\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}-\frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}-\frac{3i\sqrt[3]{a}(d\sec(e+}{2\,2^{2/3}f\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(1/3),x]
```

```
[Out] -(a^(1/3)*x*(d*Sec[e + f*x])^(2/3))/(2*2^(2/3)*(a - I*a*Tan[e + f*x])^(1/3)
*(a + I*a*Tan[e + f*x])^(1/3)) + (I*Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/
3)*(a - I*a*Tan[e + f*x])^(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3))
/(2^(2/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - ((
I/2)*a^(1/3)*Log[Cos[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*f*(a - I*a*
Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - (((3*I)/2)*a^(1/3)*Log[
2^(1/3)*a^(1/3) - (a - I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(2^
(2/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

Rule 3492

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Dist[((a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Fra
cPart[n]*(a - b*Tan[e + f*x])^FracPart[n])/(d*Sec[e + f*x])^(2*FracPart[n])
, Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Rule 3481

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \frac{(d\sec(e+fx))^{2/3}\int \sqrt[3]{a-ia\tan(e+fx)}\,dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(ia(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{2/3}}\,dx, x, -ia\tan(e+fx)\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= -\frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i\sqrt[3]{a}\log(\cos(e+fx))(d\sec(e+f\ldots}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan\ldots}$$

$$= -\frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i\sqrt[3]{a}\log(\cos(e+fx))(d\sec(e+f\ldots}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan\ldots}$$

$$= -\frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} + \frac{i\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)(d\,\text{s}\ldots}{2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\,\text{t}\ldots}$$

Mathematica [A] time = 0.636429, size = 161, normalized size = 0.47

$$-\frac{\sqrt[3]{1+e^{2i(e+fx)}}\left(\frac{de^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{2/3}\left(3i\log\left(1-\sqrt[3]{1+e^{2i(e+fx)}}\right)+2i\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+e^{2i(e+fx)}}}{\sqrt{3}}\right)+2fx\right)}{2\,2^{2/3}f\sqrt[3]{\frac{ae^{2i(e+fx)}}{1+e^{2i(e+fx)}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(1/3),x]
```

```
[Out] -(((d*E^(I*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(2/3)*(1 + E^((2*I)*(e +
f*x)))^(1/3)*(2*f*x + (2*I)*Sqrt[3]*ArcTan[(1 + 2*(1 + E^((2*I)*(e + f*x)))
^(1/3))/Sqrt[3]] + (3*I)*Log[1 - (1 + E^((2*I)*(e + f*x)))^(1/3)]))/(2*2^(2
```

```
/3)*((a*E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(1/3)*f)
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}} \frac{1}{\sqrt[3]{a+ia\tan\left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x)

[Out] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x)
```

Maxima [B] time = 2.25678, size = 2367, normalized size = 6.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="maxim
a")

[Out] 1/8*(-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), 1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sqrt(3))) - 2*I*sqrt(3)*2^(1/3)*arcta
n2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1/3
*sqrt(3), -1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) - sqrt(3))) + sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1))^2) + 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) + cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) + 4/3)
 - sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2) - 4
/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)
*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/
3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) + 4/3) - 2*2^(1/3)*arct
an2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3
)*sin(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*cos(2/3*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1)) + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) + 1) + 4*2^(1/3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*I
```

```
*2^(1/3)*log((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/3)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*sin(1
/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 - 2*(cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) + I*2^(1/3)*log((cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(2/3)*(cos(2/3*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(2/3*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1))^2) + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)
 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/
3)*((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6
)*(cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/3*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(2/3*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1))*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))) + cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) + 2*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(
1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1))*d^(2/3)/(a^(1/3)
*f)
```

Fricas [A] time = 2.14621, size = 1064, normalized size = 3.13

$$\frac{1}{2}\left(i\sqrt{3}-1\right)\left(\frac{id^2}{4af^3}\right)^{\frac{1}{3}}\log\Bigg(2\Bigg(2^{\frac{1}{3}}\left(\frac{a}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}\left(e^{(2ifx+2ie)}+1\right)e^{(2ifx+2ie)}+\left(\sqrt{3}af+iaf\right)\Bigg(\frac{id^2}{4af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="frica
s")
```

```
[Out] 1/2*(I*sqrt(3) - 1)*(1/4*I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I
*e) + 1)*e^(2*I*f*x + 2*I*e) + (sqrt(3)*a*f + I*a*f)*(1/4*I*d^2/(a*f^3))^(1
/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 1/2*(-I*sqrt(3) - 1)*(1/4*
I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/
(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I
*e) - (sqrt(3)*a*f - I*a*f)*(1/4*I*d^2/(a*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*
e^(-2*I*f*x - 2*I*e)) + (1/4*I*d^2/(a*f^3))^(1/3)*log((2*2^(1/3)*(a/(e^(2*I
*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x +
 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) - 4*I*a*f*(1/4*I*d^2/(a*f^3))^(1/3)*e^(2*I
*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^{\frac{2}{3}}}{\sqrt[3]{a\left(i\tan\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(1/3),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2/3)/(a*(I*tan(e + f*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}{\left(ia\tan\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(1/3), x)
```

3.446 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx$

Optimal. Leaf size=37

$$\frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

```
[Out] ((3*I)*a*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
```

Rubi [A] time = 0.0772428, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3493}

$$\frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3),x]
```

```
[Out] ((3*I)*a*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx=\frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

Mathematica [A] time = 0.324315, size = 47, normalized size = 1.27

$$\frac{3d^2(\tan(e+fx)+i)(a+ia\tan(e+fx))^{2/3}}{f(d\sec(e+fx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3),x]
```

```
[Out] (3*d^2*(I + Tan[e + f*x])*(a + I*a*Tan[e + f*x])^(2/3))/(f*(d*Sec[e + f*x])
^(4/3))
```

Maple [F] time = 0.129, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(a+ia\tan\left(fx+e\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x)
```

Maxima [B] time = 1.99047, size = 144, normalized size = 3.89

$$-\frac{3\left(-i\cdot 2^{\frac{1}{3}}\cos\left(\frac{1}{3}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)\right)-2^{\frac{1}{3}}\sin\left(\frac{1}{3}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+\right.\right.\right.}{\left(\cos\left(2fx+2e\right)^2+\sin\left(2fx+2e\right)^2+2\cos\left(2fx+2e\right)+1\right)^{\frac{1}{6}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="maxim
a")
```

```
[Out] -3*(-I*2^(1/3)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 2
^(1/3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*d^
(2/3)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/6)*f)
```

Fricas [A] time = 2.08586, size = 155, normalized size = 4.19

$$\frac{2^{\frac{1}{3}}\left(\frac{a}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2ifx+2ie)}+1}\right)^{\frac{2}{3}}\left(3ie^{(2ifx+2ie)}+3i\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="frica
s")
```

```
[Out] 2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(
2/3)*(3*I*e^(2*I*f*x + 2*I*e) + 3*I)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(ia\tan\left(fx+e\right)+a\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(2/3), x)
```

3.447 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx$

Optimal. Leaf size=81

$$\frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}$$

```
[Out] (((9*I)/2)*a^2*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)) + (
((3*I)/4)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f
```

Rubi [A] time = 0.154354, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3494, 3493}

$$\frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3),x]
```

```
[Out] (((9*I)/2)*a^2*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)) + (
((3*I)/4)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx &= \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{4f}+\frac{1}{2}(3a)\int (d\sec(e+fx))^{2/3}(\\ &= \frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{4f}\end{aligned}$$

Mathematica [A] time = 0.514008, size = 70, normalized size = 0.86

$$-\frac{3ad(\cos(e)-i\sin(e))(\tan(e+fx)-7i)(\cos(fx)-i\sin(fx))(a+ia\tan(e+fx))^{2/3}}{4f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3),x]
```

```
[Out] (-3*a*d*(Cos[e] - I*Sin[e])*(Cos[f*x] - I*Sin[f*x])*(-7*I + Tan[e + f*x])*(
a + I*a*Tan[e + f*x])^(2/3))/(4*f*(d*Sec[e + f*x])^(1/3))
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int \left(d \sec \left(fx+e\right)\right)^{\frac{2}{3}} \left(a+ia \tan \left(fx+e\right)\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x)
```

Maxima [B] time = 2.00544, size = 429, normalized size = 5.3

$$\frac{3\left(-i \cdot 2^{\frac{1}{3}} a \cos\left(\frac{4}{3} \arctan\left(\sin\left(2fx+2e\right), \cos\left(2fx+2e\right)+1\right)\right) - 2^{\frac{1}{3}} a \sin\left(\frac{4}{3} \arctan\left(\sin\left(2fx+2e\right), \cos\left(2fx+2\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="maxim
a")
```

```
[Out] 1/2*(3*(-I*2^(1/3)*a*cos(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)) - 2^(1/3)*a*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sq
rt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a^(2/3
)*d^(2/3) - ((-12*I*2^(1/3)*a*cos(2*f*x + 2*e)^2 - 12*I*2^(1/3)*a*sin(2*f*x
 + 2*e)^2 - 24*I*2^(1/3)*a*cos(2*f*x + 2*e) - 12*I*2^(1/3)*a)*cos(1/3*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 12*(2^(1/3)*a*cos(2*f*x + 2*e
)^2 + 2^(1/3)*a*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a*cos(2*f*x + 2*e) + 2^(1/3)
*a)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*d^(2/
3))/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(7/
6)*f)
```

Fricas [A] time = 2.0753, size = 167, normalized size = 2.06

$$\frac{2^{\frac{1}{3}} \left(12 i a e^{\left(2 i fx+2 i e\right)} + 9 i a\right) \left(\frac{a}{e^{\left(2 i fx+2 i e\right)}+1}\right)^{\frac{2}{3}} \left(\frac{d}{e^{\left(2 i fx+2 i e\right)}+1}\right)^{\frac{2}{3}}}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="frica
s")
```

```
[Out] 1/2*2^(1/3)*(12*I*a*e^(2*I*f*x + 2*I*e) + 9*I*a)*(a/(e^(2*I*f*x + 2*I*e) +
1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)/f
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(ia\tan\left(fx+e\right)+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(5/3), x)
```

3.448 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx$

Optimal. Leaf size=122

$$\frac{54ia^3(d\sec(e+fx))^{2/3}}{7f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{9ia^2(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{7f}+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

```
[Out] (((54*I)/7)*a^3*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)) +
(((9*I)/7)*a^2*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f + (((
3*I)/7)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f
```

Rubi [A] time = 0.23504, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3494, 3493}

$$\frac{54ia^3(d\sec(e+fx))^{2/3}}{7f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{9ia^2(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{7f}+\frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3),x]
```

```
[Out] (((54*I)/7)*a^3*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)) +
(((9*I)/7)*a^2*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f + (((
3*I)/7)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx &= \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{7f}+\frac{1}{7}(12a)\int (d\sec(e+fx)\\ &= \frac{9ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{7f}+\frac{3ia(d\sec(e+fx))^{2/3}(a}{7f}\\ &= \frac{54ia^3(d\sec(e+fx))^{2/3}}{7f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{9ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{7f}\end{aligned}$$

Mathematica [A] time = 0.595814, size = 100, normalized size = 0.82

$$\frac{3a^2(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{5/3}(\sin(e-fx)+i\cos(e-fx))(5i\sin(2(e+fx))+23\cos(2(e+fx))+21)}{14df(\cos(fx)+i\sin(fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3),x]
```

```
[Out] (3*a^2*(d*Sec[e + f*x])^(5/3)*(I*Cos[e - f*x] + Sin[e - f*x])*(21 + 23*Cos[
2*(e + f*x)] + (5*I)*Sin[2*(e + f*x)])*(a + I*a*Tan[e + f*x])^(2/3))/(14*d*
f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [F] time = 0.14, size = 0, normalized size = 0.

$$\int \left(d \sec \left(f x+e\right)\right)^{\frac{2}{3}} \left(a+i a \tan \left(f x+e\right)\right)^{\frac{8}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x)
```

Maxima [B] time = 2.20946, size = 543, normalized size = 4.45

$$42\left(-i \cdot 2^{\frac{1}{3}} a^2 \cos\left(\frac{4}{3} \arctan\left(\sin\left(2 f x+2 e\right), \cos\left(2 f x+2 e\right)+1\right)\right) - 2^{\frac{1}{3}} a^2 \sin\left(\frac{4}{3} \arctan\left(\sin\left(2 f x+2 e\right), \cos\left(2 f x+2\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="maxim
a")
```

```
[Out] 1/7*(42*(-I*2^(1/3)*a^2*cos(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) - 2^(1/3)*a^2*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
))*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a
^(2/3)*d^(2/3) - (-12*I*2^(1/3)*a^2*cos(7/3*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e) + 1)) - 12*2^(1/3)*a^2*sin(7/3*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1)) + (-84*I*2^(1/3)*a^2*cos(2*f*x + 2*e)^2 - 84*I*2^(1/3)*a^2
*sin(2*f*x + 2*e)^2 - 168*I*2^(1/3)*a^2*cos(2*f*x + 2*e) - 84*I*2^(1/3)*a^2
)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 84*(2^(1/3)*a^
2*cos(2*f*x + 2*e)^2 + 2^(1/3)*a^2*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^2*cos(2
*f*x + 2*e) + 2^(1/3)*a^2)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)))*a^(2/3)*d^(2/3))/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)^(7/6)*f)
```

Fricas [A] time = 2.06202, size = 305, normalized size = 2.5

$$\frac{2 \cdot 2^{\frac{1}{3}} \left(42 i a^2 e^{\left(4 i f x+4 i e\right)} + 63 i a^2 e^{\left(2 i f x+2 i e\right)} + 27 i a^2\right) \left(\frac{a}{e^{\left(2 i f x+2 i e\right)}+1}\right)^{\frac{2}{3}} \left(\frac{d}{e^{\left(2 i f x+2 i e\right)}+1}\right)^{\frac{2}{3}} e^{\left(2 i f x+2 i e\right)}}{7 \left(f e^{\left(4 i f x+4 i e\right)} + f e^{\left(2 i f x+2 i e\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="frica
s")
```

```
[Out] 2/7*2^(1/3)*(42*I*a^2*e^(4*I*f*x + 4*I*e) + 63*I*a^2*e^(2*I*f*x + 2*I*e) +
27*I*a^2)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))
^(2/3)*e^(2*I*f*x + 2*I*e)/(f*e^(4*I*f*x + 4*I*e) + f*e^(2*I*f*x + 2*I*e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(8/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \sec \left(fx+e\right)\right)^{\frac{2}{3}} \left(i\,a \tan \left(fx+e\right)+a\right)^{\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(8/3), x)
```

3.449 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx$

Optimal. Leaf size=163

$$\frac{486ia^4(d\sec(e+fx))^{2/3}}{35f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{81ia^3(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{35f}+\frac{27ia^2(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{35f}$$

```
[Out] (((486*I)/35)*a^4*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
+ (((81*I)/35)*a^3*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f +
 (((27*I)/35)*a^2*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f +
(((3*I)/10)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3))/f
```

Rubi [A] time = 0.320744, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3494, 3493}

$$\frac{486ia^4(d\sec(e+fx))^{2/3}}{35f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{81ia^3(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{35f}+\frac{27ia^2(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{35f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(11/3),x]
```

```
[Out] (((486*I)/35)*a^4*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
+ (((81*I)/35)*a^3*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f +
 (((27*I)/35)*a^2*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f +
(((3*I)/10)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3))/f
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}
\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx &= \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}}{10f}+\frac{1}{5}(9a)\int (d\sec(e+fx))^{2/3}\\
&= \frac{27ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{35f}+\frac{3ia(d\sec(e+fx))^{2/3}(a}{10f}\\
&= \frac{81ia^3(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{35f}+\frac{27ia^2(d\sec(e+fx))^{2/3}(}{35}\\
&= \frac{486ia^4(d\sec(e+fx))^{2/3}}{35f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{81ia^3(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{35f}
\end{aligned}$$

Mathematica [A] time = 1.11004, size = 116, normalized size = 0.71

$$\frac{3a^3(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{5/3}(\sin(e-2fx)+i\cos(e-2fx))(442\cos(2(e+fx))+45i\tan(e+fx)+59i}{140df(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(11/3),x]
```

```
[Out] (3*a^3*(d*Sec[e + f*x])^(5/3)*(I*Cos[e - 2*f*x] + Sin[e - 2*f*x])*(364 + 44
2*Cos[2*(e + f*x)] + (59*I)*Sec[e + f*x]*Sin[3*(e + f*x)] + (45*I)*Tan[e +
f*x])*(a + I*a*Tan[e + f*x])^(2/3))/(140*d*f*(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(a+ia\tan\left(fx+e\right)\right)^{\frac{11}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x)
```

Maxima [B] time = 2.26319, size = 1319, normalized size = 8.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="maxi
ma")
```

```
[Out] -1/35*((84*I*2^(1/3)*a^3*cos(10/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) + 84*2^(1/3)*a^3*sin(10/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) + (630*I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 630*I*2^(1/3)*a^3*sin(2*f
*x + 2*e)^2 + 1260*I*2^(1/3)*a^3*cos(2*f*x + 2*e) + 630*I*2^(1/3)*a^3)*cos(
4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 630*(2^(1/3)*a^3*cos
(2*f*x + 2*e)^2 + 2^(1/3)*a^3*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f*x
+ 2*e) + 2^(1/3)*a^3)*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)))*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*a^(2/3)*d^(2/3) + ((-360*I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 - 360*I*2^(1/3)*
a^3*sin(2*f*x + 2*e)^2 - 720*I*2^(1/3)*a^3*cos(2*f*x + 2*e) - 360*I*2^(1/3)
*a^3)*cos(7/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (-840*I*2^
(1/3)*a^3*cos(2*f*x + 2*e)^4 - 840*I*2^(1/3)*a^3*sin(2*f*x + 2*e)^4 - 3360*
I*2^(1/3)*a^3*cos(2*f*x + 2*e)^3 - 5040*I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 -
3360*I*2^(1/3)*a^3*cos(2*f*x + 2*e) - 840*I*2^(1/3)*a^3 + (-1680*I*2^(1/3)*
a^3*cos(2*f*x + 2*e)^2 - 3360*I*2^(1/3)*a^3*cos(2*f*x + 2*e) - 1680*I*2^(1/
3)*a^3)*sin(2*f*x + 2*e)^2)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1)) - 360*(2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 2^(1/3)*a^3*sin(2*f*x + 2
*e)^2 + 2*2^(1/3)*a^3*cos(2*f*x + 2*e) + 2^(1/3)*a^3)*sin(7/3*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 840*(2^(1/3)*a^3*cos(2*f*x + 2*e)^4 +
 2^(1/3)*a^3*sin(2*f*x + 2*e)^4 + 4*2^(1/3)*a^3*cos(2*f*x + 2*e)^3 + 6*2^(1
/3)*a^3*cos(2*f*x + 2*e)^2 + 4*2^(1/3)*a^3*cos(2*f*x + 2*e) + 2^(1/3)*a^3 +
```

```
 2*(2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f*x + 2*e) + 2^(1/
3)*a^3)*sin(2*f*x + 2*e)^2)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1)))*a^(2/3)*d^(2/3))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*c
os(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f
*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*(cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*f)
```

Fricas [A] time = 2.19718, size = 389, normalized size = 2.39

$$\frac{2 \cdot 2^{\frac{1}{3}} \left(420 i a^3 e^{\left(6 i f x + 6 i e\right)} + 945 i a^3 e^{\left(4 i f x + 4 i e\right)} + 810 i a^3 e^{\left(2 i f x + 2 i e\right)} + 243 i a^3\right) \left(\frac{a}{e^{\left(2 i f x + 2 i e\right)} + 1}\right)^{\frac{2}{3}} \left(\frac{d}{e^{\left(2 i f x + 2 i e\right)} + 1}\right)^{\frac{2}{3}} e^{\left(2 i f x + 2 i e\right)}}{35 \left(f e^{\left(6 i f x + 6 i e\right)} + 2 f e^{\left(4 i f x + 4 i e\right)} + f e^{\left(2 i f x + 2 i e\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="fric
as")
```

```
[Out] 2/35*2^(1/3)*(420*I*a^3*e^(6*I*f*x + 6*I*e) + 945*I*a^3*e^(4*I*f*x + 4*I*e)
 + 810*I*a^3*e^(2*I*f*x + 2*I*e) + 243*I*a^3)*(a/(e^(2*I*f*x + 2*I*e) + 1))
^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2*I*f*x + 2*I*e)/(f*e^(6*I*f*
x + 6*I*e) + 2*f*e^(4*I*f*x + 4*I*e) + f*e^(2*I*f*x + 2*I*e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(11/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \sec\left(f x + e\right)\right)^{\frac{2}{3}} \left(i a \tan\left(f x + e\right) + a\right)^{\frac{11}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="giac
")
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(11/3), x)
```

3.450 $\int (e\sec(c+dx))^m (a+ia\tan(c+dx))^5\,dx$

Optimal. Leaf size=86

$$\frac{ia^5 2^{\frac{m}{2}+5}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2}-4,\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

```
[Out] (I*2^(5 + m/2)*a^5*Hypergeometric2F1[-4 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rubi [A] time = 0.149067, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3505, 3523, 70, 69}

$$\frac{ia^5 2^{\frac{m}{2}+5}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2}-4,\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (I*2^(5 + m/2)*a^5*Hypergeometric2F1[-4 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^m (a+ia\tan(c+dx))^5\,dx = \left((e\sec(c+dx))^m (a-ia\tan(c+dx))^{-m/2} (a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\,\mathrm{t}$$

$$= \frac{\left(a^2 (e\sec(c+dx))^m (a-ia\tan(c+dx))^{-m/2} (a+ia\tan(c+dx))^{-m/2}\right)\mathrm{Subst}\,(}{d}$$

$$= \frac{\left(2^{4+\frac{m}{2}} a^6 (e\sec(c+dx))^m (a-ia\tan(c+dx))^{-m/2} \left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\mathrm{Subst}\,(}{d}$$

$$= \frac{i2^{5+\frac{m}{2}} a^5\,{}_2F_1\left(-4-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m (1+i\tan}{dm}$$

Mathematica [A] time = 3.20352, size = 151, normalized size = 1.76

$$\frac{a^5 2^{m+5} e^{5i(c+2dx)} \left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m (\tan(c+dx)-i)^5 \sec^{-m-5}(c+dx)\mathrm{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m}{2}+6,-e^{2i(c+dx)}\right)(e\sec(c}{d(m+10)\left(1+e^{2i(c+dx)}\right)^4 (\cos(dx)+i\sin(dx))^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (2^(5 + m)*a^5*E^((5*I)*(c + 2*d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*
x))))^m*Hypergeometric2F1[1, 1 - m/2, 6 + m/2, -E^((2*I)*(c + d*x))]*Sec[c
+ d*x]^(-5 - m)*(e*Sec[c + d*x])^m*(-I + Tan[c + d*x])^5)/(d*(1 + E^((2*I)*
(c + d*x)))^4*(10 + m)*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [F] time = 0.474, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^5 (e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^5*(e*sec(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{32\,a^5\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^m e^{(10i\,dx+10i\,c)}}{e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] integral(32*a^5*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(10*I*d
*x + 10*I*c)/(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x
 + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^5\,(e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^5*(e*sec(d*x + c))^m, x)
```

3.451 $\int (e\sec(c+dx))^m (a+ia\tan(c+dx))^3\,dx$

Optimal. Leaf size=86

$$\frac{ia^3 2^{\frac{m}{2}+3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2}-2,\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

```
[Out] (I*2^(3 + m/2)*a^3*Hypergeometric2F1[-2 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rubi [A] time = 0.147617, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3505, 3523, 70, 69}

$$\frac{ia^3 2^{\frac{m}{2}+3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2}-2,\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (I*2^(3 + m/2)*a^3*Hypergeometric2F1[-2 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subs}}{d}$$

$$= \frac{\left(2^{2+\frac{m}{2}}a^4(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\text{Sub}}{d}$$

$$= \frac{i2^{3+\frac{m}{2}}a^3\,{}_2F_1\left(-2-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i}{dm}$$

Mathematica [A] time = 1.35644, size = 154, normalized size = 1.79

$$-\frac{i2^{m+3}e^{3i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m(a+ia\tan(c+dx))^3\sec^{-m-3}(c+dx)\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m+8}{2},-e^{2i(c+dx)}\right)(e\,\text{s}}{d(m+6)\left(1+e^{2i(c+dx)}\right)^2(\cos(dx)+i\sin(dx))^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-I)*2^(3 + m)*E^((3*I)*(c + 2*d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d
*x))))^m*Hypergeometric2F1[1, 1 - m/2, (8 + m)/2, -E^((2*I)*(c + d*x))]*Sec
[c + d*x]^(-3 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^3)/(d*(1 + E^(
(2*I)*(c + d*x)))^2*(6 + m)*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [F] time = 0.497, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m(a+ia\tan(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ia\tan(dx+c)+a)^3(e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*(e*sec(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{8\,a^3 \left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^m e^{(6i\,dx+6i\,c)}}{e^{(6i\,dx+6i\,c)} + 3\,e^{(4i\,dx+4i\,c)} + 3\,e^{(2i\,dx+2i\,c)} + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(8*a^3*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(6*I*d*x
 + 6*I*c)/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I
*c) + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^3\,(e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3*(e*sec(d*x + c))^m, x)
```

3.452 $\int (e\sec(c+dx))^m (a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=86

$$\frac{ia^2 2^{\frac{m}{2}+2}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2}-1,\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

```
[Out] (I*2^(2 + m/2)*a^2*Hypergeometric2F1[-1 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rubi [A] time = 0.149719, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3505, 3523, 70, 69}

$$\frac{ia^2 2^{\frac{m}{2}+2}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2}-1,\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```


```
[Out] (I*2^(2 + m/2)*a^2*Hypergeometric2F1[-1 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```


Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\,\mathrm{t}$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\mathrm{Subst}\big(}{d}$$

$$= \frac{\left(2^{1+\frac{m}{2}}a^3(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\mathrm{Subst}\big(}{d}$$

$$= \frac{i2^{2+\frac{m}{2}}a^2\,{}_2F_1\left(-1-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan}{dm}$$

Mathematica [A] time = 0.956044, size = 141, normalized size = 1.64

$$-\frac{i2^{m+2}e^{i(c+3dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+1}(a+ia\tan(c+dx))^2\sec^{-m-2}(c+dx)\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m+6}{2},-e^{2i(c+dx)}\right)(e\sec}{d(m+4)(\cos(dx)+i\sin(dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(2 + m)*E^(I*(c + 3*d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
))^(1 + m)*Hypergeometric2F1[1, 1 - m/2, (6 + m)/2, -E^((2*I)*(c + d*x))]*S
ec[c + d*x]^(-2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^2)/(d*(4 + m
)*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [F] time = 0.364, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m(a+ia\tan(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ia\tan(dx+c)+a)^2(e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*(e*sec(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{4\,a^{2}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{m}e^{(4i\,dx+4i\,c)}}{e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(4*a^2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(4*I*d*x
 + 4*I*c)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int(e\sec(c+dx))^{m}\,dx+\int-(e\sec(c+dx))^{m}\tan^{2}(c+dx)\,dx+\int 2i\,(e\sec(c+dx))^{m}\tan(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*(Integral((e*sec(c + d*x))**m, x) + Integral(-(e*sec(c + d*x))**m*tan(
c + d*x)**2, x) + Integral(2*I*(e*sec(c + d*x))**m*tan(c + d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(i\,a\tan(dx+c)+a)^{2}(e\sec(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*(e*sec(d*x + c))^m, x)
```

3.453 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=82

$$\frac{ia2^{\frac{m}{2}+1}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

```
[Out] (I*2^(1 + m/2)*a*Hypergeometric2F1[-m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d*x
])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rubi [A] time = 0.125838, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3505, 3523, 70, 69}

$$\frac{ia2^{\frac{m}{2}+1}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I*2^(1 + m/2)*a*Hypergeometric2F1[-m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d*x
])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-i$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subs}}{d}$$

$$= \frac{\left(2^{m/2}a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\text{Subst}}{d}$$

$$= \frac{i2^{1+\frac{m}{2}}a\,{}_2F_1\left(-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c}{dm}$$

Mathematica [A] time = 0.60924, size = 130, normalized size = 1.59

$$\frac{a2^{m+1}e^{i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m(\tan(c+dx)-i)(\cos(dx)-i\sin(dx))\sec^{-m-1}(c+dx)\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m+4}{2}\right.}{d(m+2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2^(1 + m)*a*E^(I*(c + 2*d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
m*Hypergeometric2F1[1, 1 - m/2, (4 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*
x]^(-1 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] - I*Sin[d*x])*(-I + Tan[c + d*x]))
/(d*(2 + m))
```

Maple [F] time = 0.477, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m\,(a+ia\tan(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ia\tan(dx+c)+a)\,(e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\,a\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{m}e^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(2*a*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(2*I*d*x +
 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\left(e\sec\left(c+dx\right)\right)^{m}\,dx+\int i\left(e\sec\left(c+dx\right)\right)^{m}\tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*(Integral((e*sec(c + d*x))**m, x) + Integral(I*(e*sec(c + d*x))**m*tan(c
+ d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(i\,a\tan\left(dx+c\right)+a\right)\left(e\sec\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

3.454 $\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{i2^{\frac{m}{2}-1}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(2-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{adm}$$

```
[Out] (I*2^(-1 + m/2)*Hypergeometric2F1[2 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rubi [A] time = 0.153605, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3505, 3523, 70, 69}

$$\frac{i2^{\frac{m}{2}-1}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(2-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{adm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I*2^(-1 + m/2)*Hypergeometric2F1[2 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a\right.}{d}$$

$$= \frac{\left(2^{-2+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-2+\frac{m}{2}}(a\right.}{d}$$

$$= \frac{i2^{-1+\frac{m}{2}}\,{}_2F_1\left(2-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{adm}$$

Mathematica [A] time = 0.608412, size = 150, normalized size = 1.74

$$-\frac{i2^{m-1}e^{-i(c+2dx)}\left(1+e^{2i(c+dx)}\right)^2\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m(\cos(dx)+i\sin(dx))\sec^{1-m}(c+dx)\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m}{2},-e^{2i(c\right.}{d(m-2)(a+ia\tan(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*2^(-1 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I
)*(c + d*x)))^2*Hypergeometric2F1[1, 1 - m/2, m/2, -E^((2*I)*(c + d*x))]*Se
c[c + d*x]^(1 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x]))/(d*E^(I*(c +
 2*d*x))*(-2 + m)*(a + I*a*Tan[c + d*x]))
```

Maple [F] time = 0.79, size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^m}{a+ia\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{m}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(-2i\,dx-2i\,c)}}{2\,a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(2*I*d*x
+ 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c)/a, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{m}}{i\,a\tan(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a), x)
```

3.455 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{i2^{\frac{m}{2}-2}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(3-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm}$$

```
[Out] (I*2^(-2 + m/2)*Hypergeometric2F1[3 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a^2*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rubi [A] time = 0.158533, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3505, 3523, 70, 69}

$$\frac{i2^{\frac{m}{2}-2}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(3-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```


```
[Out] (I*2^(-2 + m/2)*Hypergeometric2F1[3 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a^2*d*m*(1 + I*Tan[c + d*x])^(m/2))
```


Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\tan(c+dx))$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int (a-iax)^{-}\right.}{d}$$

$$= \frac{\left(2^{-3+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\text{Subst}\left(\int \left(\frac{1}{2}+\frac{ix}{2}\right)^{-3}\right.}{ad}$$

$$= \frac{i2^{-2+\frac{m}{2}}\,{}_2F_1\left(3-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{a^2dm}$$

Mathematica [A] time = 18.1943, size = 154, normalized size = 1.79

$$-\frac{i2^{m-2}e^{-2i(c+2dx)}\left(1+e^{2i(c+dx)}\right)^3\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m(\cos(dx)+i\sin(dx))^2\sec^{2-m}(c+dx)\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m-2}{2}\right.}{d(m-4)(a+ia\tan(c+dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(-2 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I
)*(c + d*x)))^3*Hypergeometric2F1[1, 1 - m/2, (-2 + m)/2, -E^((2*I)*(c + d*
x))]*Sec[c + d*x]^(2 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x])^2)/(d*
E^((2*I)*(c + 2*d*x))*(-4 + m)*(a + I*a*Tan[c + d*x])^2)
```

Maple [F] time = 1.02, size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{m}\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-4i\,dx-4i\,c)}}{4\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(4*I*d*x
+ 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/a^2, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{m}}{(i\,a\tan(dx+c)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^2, x)
```

3.456 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx$

Optimal. Leaf size=86

$$\frac{i2^{\frac{m}{2}-3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(4-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^3dm}$$

```
[Out] (I*2^(-3 + m/2)*Hypergeometric2F1[4 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a^3*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rubi [A] time = 0.157505, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3505, 3523, 70, 69}

$$\frac{i2^{\frac{m}{2}-3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(4-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^3dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^3,x]
```


```
[Out] (I*2^(-3 + m/2)*Hypergeometric2F1[4 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a^3*d*m*(1 + I*Tan[c + d*x])^(m/2))
```


Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\tan(c+dx))^{m/2}($$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\mathrm{Subst}\left(\int (a-iax)^{-1+\frac{m}{2}}\right.}{d}$$

$$= \frac{\left(2^{-4+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\mathrm{Subst}\left(\int \left(\frac{1}{2}+\frac{ix}{2}\right)^{-4+\frac{m}{2}}\right.}{a^2d}$$

$$= \frac{i2^{-3+\frac{m}{2}}\,{}_2F_1\left(4-\frac{m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{a^3dm}$$

Mathematica [A] time = 11.8327, size = 151, normalized size = 1.76

$$\frac{2^{m-3}e^{-3i(c+2dx)}\left(1+e^{2i(c+dx)}\right)^4\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m(\cos(dx)+i\sin(dx))^3\sec^{3-m}(c+dx)\mathrm{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m-4}{2},-e^{2i}\right.}{a^3d(m-6)(\tan(c+dx)-i)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (2^(-3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c
+ d*x)))^4*Hypergeometric2F1[1, 1 - m/2, (-4 + m)/2, -E^((2*I)*(c + d*x))]*
Sec[c + d*x]^(3 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x])^3)/(a^3*d*E
^((3*I)*(c + 2*d*x))*(-6 + m)*(-I + Tan[c + d*x])^3)
```

Maple [F] time = 1.264, size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{m}\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-6i\,dx-6i\,c)}}{8\,a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/8*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(6*I*d*x
+ 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-6*I*d*x -
 6*I*c)/a^3, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{m}}{(i\,a\tan(dx+c)+a)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^3, x)
```

3.457 $\int (e\sec(c+dx))^m (a+ia\tan(c+dx))^{7/2}\,dx$

Optimal. Leaf size=109

$$\frac{ia^3 2^{\frac{m+7}{2}}\sqrt{a+ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{1}{2}(-m-5),\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan\right.}{dm}$$

```
[Out] (I*2^((7 + m)/2)*a^3*Hypergeometric2F1[(-5 - m)/2, m/2, (2 + m)/2, (1 - I*T
an[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*m)
```

Rubi [A] time = 0.206088, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{ia^3 2^{\frac{m+7}{2}}\sqrt{a+ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{1}{2}(-m-5),\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan\right.}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (I*2^((7 + m)/2)*a^3*Hypergeometric2F1[(-5 - m)/2, m/2, (2 + m)/2, (1 - I*T
an[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*m)
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Sul}}{d}$$

$$= \frac{\left(2^{\frac{5}{2}+\frac{m}{2}}a^4(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\sqrt{a+ia\tan(c+dx)}\left(\frac{a}{}\right.\right.}{}$$

$$= \frac{i2^{\frac{7+m}{2}}a^3\,{}_2F_1\left(\frac{1}{2}(-5-m),\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1}{dm}$$

Mathematica [A] time = 2.40871, size = 178, normalized size = 1.63

$$-\frac{i2^{m+\frac{7}{2}}\sqrt{e^{idx}}e^{3i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{1}{2}}(a+ia\tan(c+dx))^{7/2}\sec^{-m-\frac{7}{2}}(c+dx)\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m+9}{2},-e^{2i}\right.}{d(m+7)\left(1+e^{2i(c+dx)}\right)^2(\cos(dx)+i\sin(dx))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I)*2^(7/2 + m)*E^((3*I)*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1
 + E^((2*I)*(c + d*x))))^(1/2 + m)*Hypergeometric2F1[1, 1 - m/2, (9 + m)/2,
 -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-7/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*
Tan[c + d*x])^(7/2))/(d*(1 + E^((2*I)*(c + d*x)))^2*(7 + m)*(Cos[d*x] + I*S
in[d*x])^(7/2))
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m(a+ia\tan(dx+c))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}(e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*(e*sec(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{8\sqrt{2}a^3\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^m\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(7i\,dx+7i\,c)}}{e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(8*sqrt(2)*a^3*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(7*I*d*x + 7*I*c)/(e^(6*I*d*x + 6*I*c) +
3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}(e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*(e*sec(d*x + c))^m, x)
```

3.458 $\int (e\sec(c+dx))^m (a+ia\tan(c+dx))^{5/2}\,dx$

Optimal. Leaf size=109

$$\frac{ia^2 2^{\frac{m+5}{2}}\sqrt{a+ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(\frac{1}{2}(-m-3),\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-\right.}{dm}$$

```
[Out] (I*2^((5 + m)/2)*a^2*Hypergeometric2F1[(-3 - m)/2, m/2, (2 + m)/2, (1 - I*T
an[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*m)
```

Rubi [A] time = 0.198288, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{ia^2 2^{\frac{m+5}{2}}\sqrt{a+ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m \text{Hypergeometric2F1}\left(\frac{1}{2}(-m-3),\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-\right.}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*2^((5 + m)/2)*a^2*Hypergeometric2F1[(-3 - m)/2, m/2, (2 + m)/2, (1 - I*T
an[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*m)
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}}{d}$$

$$= \frac{\left(2^{\frac{3}{2}+\frac{m}{2}}a^3(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\sqrt{a+ia\tan(c+dx)}\left(\frac{a+ia\text{t}}{}\right.\right.}{d}$$

$$= \frac{i2^{\frac{5+m}{2}}a^2\,{}_2F_1\left(\frac{1}{2}(-3-m),\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+}{dm}$$

Mathematica [A] time = 1.6516, size = 163, normalized size = 1.5

$$-\frac{i2^{m+\frac{5}{2}}\sqrt{e^{idx}}e^{i(c+3dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{3}{2}}(a+ia\tan(c+dx))^{5/2}\sec^{-m-\frac{5}{2}}(c+dx)\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m+7}{2},-e^{2i(c+dx}\right.}{d(m+5)(\cos(dx)+i\sin(dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*2^(5/2 + m)*E^(I*(c + 3*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x))))^(3/2 + m)*Hypergeometric2F1[1, 1 - m/2, (7 + m)/2, -E^
((2*I)*(c + d*x))]*Sec[c + d*x]^(-5/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[
c + d*x])^(5/2))/(d*(5 + m)*(Cos[d*x] + I*Sin[d*x])^(5/2))
```

Maple [F] time = 0.316, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}(e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*(e*sec(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{4\sqrt{2}a^2\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^m\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(5i\,dx+5i\,c)}}{e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(4*sqrt(2)*a^2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(5*I*d*x + 5*I*c)/(e^(4*I*d*x + 4*I*c) +
2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}(e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*(e*sec(d*x + c))^m, x)
```

3.459 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal. Leaf size=107

$$\frac{ia2^{\frac{m+3}{2}}\sqrt{a+ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{1}{2}(-m-1),\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan\right.}{dm}$$

```
[Out] (I*2^((3 + m)/2)*a*Hypergeometric2F1[(-1 - m)/2, m/2, (2 + m)/2, (1 - I*Tan
[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a +
 I*a*Tan[c + d*x]])/(d*m)
```

Rubi [A] time = 0.196241, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{ia2^{\frac{m+3}{2}}\sqrt{a+ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{1}{2}(-m-1),\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan\right.}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*2^((3 + m)/2)*a*Hypergeometric2F1[(-1 - m)/2, m/2, (2 + m)/2, (1 - I*Tan
[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a +
 I*a*Tan[c + d*x]])/(d*m)
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Su}}{d}$$

$$= \frac{\left(2^{\frac{1}{2}+\frac{m}{2}}a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\sqrt{a+ia\tan(c+dx)}\left(\frac{a}{}\right.\right.}{}$$

$$= \frac{i2^{\frac{3+m}{2}}a\,{}_2F_1\left(\frac{1}{2}(-1-m),\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1}{dm}$$

Mathematica [A] time = 1.02954, size = 163, normalized size = 1.52

$$-\frac{i2^{m+\frac{3}{2}}\sqrt{e^{idx}}e^{i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{1}{2}}(a+ia\tan(c+dx))^{3/2}\sec^{-m-\frac{3}{2}}(c+dx)\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m+5}{2},-e^{2i(}\right.}{d(m+3)(\cos(dx)+i\sin(dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*2^(3/2 + m)*E^(I*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x))))^(1/2 + m)*Hypergeometric2F1[1, 1 - m/2, (5 + m)/2, -E^
((2*I)*(c + d*x))]*Sec[c + d*x]^(-3/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[
c + d*x])^(3/2))/(d*(3 + m)*(Cos[d*x] + I*Sin[d*x])^(3/2))
```

Maple [F] time = 0.324, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m(a+ia\tan(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}(e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*(e*sec(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\sqrt{2}a\left(\frac{2ee^{(idx+ic)}}{e^{(2idx+2ic)}+1}\right)^m\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{(3idx+3ic)}}{e^{(2idx+2ic)}+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(2*sqrt(2)*a*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3*I*d*x + 3*I*c)/(e^(2*I*d*x + 2*I*c) + 1)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan(dx + c) + a)^{\frac{3}{2}} (e \sec(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*(e*sec(d*x + c))^m, x)
```

3.460 $\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{ia2^{\frac{m+1}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (I*2^((1 + m)/2)*a*Hypergeometric2F1[(1 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[
c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(d*m*Sqrt
[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.178756, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{ia2^{\frac{m+1}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*2^((1 + m)/2)*a*Hypergeometric2F1[(1 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[
c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(d*m*Sqrt
[a + I*a*Tan[c + d*x]])
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\,\mathrm{ta}$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\mathrm{Subst}\left(\right.}{d}$$

$$= \frac{\left(2^{-\frac{1}{2}+\frac{m}{2}}a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-\frac{m}{2}}\right)\mathrm{Subst}}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{1+m}{2}}a\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 0.657975, size = 162, normalized size = 1.51

$$-\frac{i2^{m+\frac{1}{2}}\sqrt{e^{idx}}e^{i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m-\frac{1}{2}}\sqrt{a+ia\tan(c+dx)}\sec^{-m-\frac{1}{2}}(c+dx)\mathrm{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m+3}{2},-e^{2i(c+dx)}\right)(}{d(m+1)\sqrt{\cos(dx)+i\sin(dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*2^(1/2 + m)*E^(I*(c + d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x))))^(-1/2 + m)*Hypergeometric2F1[1, 1 - m/2, (3 + m)/2, -E^(
(2*I)*(c + d*x))]*Sec[c + d*x]^(-1/2 - m)*(e*Sec[c + d*x])^m*Sqrt[a + I*a*T
an[c + d*x]])/(d*(1 + m)*Sqrt[Cos[d*x] + I*Sin[d*x]])
```

Maple [F] time = 0.403, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m\sqrt{a+ia\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\tan(dx+c)+a}\,(e\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{2}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{m}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\,(i\tan{(c+dx)}+1)}\,(e\sec{(c+dx)})^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(I*tan(c + d*x) + 1))*(e*sec(c + d*x))**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{i\,a\tan{(dx+c)}+a}\,(e\sec{(dx+c)})^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

3.461 $\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{i2^{\frac{m-1}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{3-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (I*2^((-1 + m)/2)*Hypergeometric2F1[(3 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(d*m*Sqrt[
a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.193542, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{i2^{\frac{m-1}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{3-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*2^((-1 + m)/2)*Hypergeometric2F1[(3 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(d*m*Sqrt[
a + I*a*Tan[c + d*x]])
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\tan(c+dx))^{m}$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int (a-iax)^{-1}\right.}{d}$$

$$= \frac{\left(2^{-\frac{3}{2}+\frac{m}{2}}a(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int \left(\frac{1}{2}+\frac{ix}{2}\right)^{-}\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{1}{2}(-1+m)}\,{}_2F_1\left(\frac{3-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 1.127, size = 162, normalized size = 1.53

$$-\frac{i2^{m-\frac{1}{2}}e^{i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m-\frac{3}{2}}\sqrt{\cos(dx)+i\sin(dx)}\sec^{\frac{1}{2}-m}(c+dx)\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{m+1}{2},-e^{2i(c+dx)}\right)(e\,s}{d(m-1)\sqrt{e^{idx}}\sqrt{a+ia\tan(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*2^(-1/2 + m)*E^(I*(c + d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
)))^(-3/2 + m)*Hypergeometric2F1[1, 1 - m/2, (1 + m)/2, -E^((2*I)*(c + d*x)
)]*Sec[c + d*x]^(1/2 - m)*(e*Sec[c + d*x])^m*Sqrt[Cos[d*x] + I*Sin[d*x]])/(
d*Sqrt[E^(I*d*x)]*(-1 + m)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F] time = 0.357, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m \frac{1}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2}\left(\frac{2\,ee^{\left(i\,dx+i\,c\right)}}{e^{\left(2i\,dx+2i\,c\right)}+1}\right)^{m}\sqrt{\frac{a}{e^{\left(2i\,dx+2i\,c\right)}+1}}\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)e^{\left(-i\,dx-i\,c\right)}}{2\,a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-I*d*x - I*c)/a,
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(e\sec\left(c+dx\right)\right)^{m}}{\sqrt{a\left(i\tan\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**m/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(e\sec\left(dx+c\right)\right)^{m}}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

3.462 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=109

$$\frac{i2^{\frac{m-3}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{5-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{adm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (I*2^((-3 + m)/2)*Hypergeometric2F1[(5 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(a*d*m*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.208719, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{i2^{\frac{m-3}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{5-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{adm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*2^((-3 + m)/2)*Hypergeometric2F1[(5 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(a*d*m*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\tan(c+dx))^{m/}$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int (a-iax)^{-1+}\right.}{d}$$

$$= \frac{\left(2^{-\frac{5}{2}+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int \left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{5}{2}+}\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{1}{2}(-3+m)}\,{}_2F_1\left(\frac{5-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{adm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 1.58322, size = 178, normalized size = 1.63

$$-\frac{i2^{m-\frac{3}{2}}\sqrt{e^{idx}}e^{-2i(c+2dx)}\left(1+e^{2i(c+dx)}\right)^3\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{1}{2}}(\cos(dx)+i\sin(dx))^{3/2}\sec^{\frac{3}{2}-m}(c+dx)\text{Hypergeometric2F1}\left(1,1-\right.}{d(m-3)(a+ia\tan(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*2^(-3/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
)))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^3*Hypergeometric2F1[1, 1 - m/2, (-1
 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(3/2 - m)*(e*Sec[c + d*x])^m*(C
os[d*x] + I*Sin[d*x])^(3/2))/(d*E^((2*I)*(c + 2*d*x))*(-3 + m)*(a + I*a*Tan
[c + d*x])^(3/2))
```

Maple [F] time = 0.337, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m(a+ia\tan(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^m}{(ia\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^m\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-3i\,dx-3i\,c)}}{4\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/4*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c)
+ 1)*e^(-3*I*d*x - 3*I*c)/a^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(e\sec(c+dx))^m}{(a(i\tan(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**m/(a*(I*tan(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(3/2), x)
```

3.463 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{i2^{\frac{m-5}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{7-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (I*2^((-5 + m)/2)*Hypergeometric2F1[(7 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(a^2*d*m*S
qrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.210445, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{i2^{\frac{m-5}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\text{Hypergeometric2F1}\left(\frac{7-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*2^((-5 + m)/2)*Hypergeometric2F1[(7 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(a^2*d*m*S
qrt[a + I*a*Tan[c + d*x]])
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\tan(c+dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int (a-iax)\right.}{d}$$

$$= \frac{\left(2^{-\frac{7}{2}+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int \left(\frac{1}{2}+\frac{ix}{2}\right)\right.}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{1}{2}(-5+m)}\,{}_2F_1\left(\frac{7-m}{2},\frac{m}{2};\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{a^2dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 3.14941, size = 178, normalized size = 1.63

$$-\frac{i2^{m-\frac{5}{2}}\sqrt{e^{idx}}e^{-3i(c+2dx)}\left(1+e^{2i(c+dx)}\right)^4\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+\frac{1}{2}}(\cos(dx)+i\sin(dx))^{5/2}\sec^{\frac{5}{2}-m}(c+dx)\text{Hypergeometric2F1}\left(1,\right.}{d(m-5)(a+ia\tan(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*2^(-5/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
)))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^4*Hypergeometric2F1[1, 1 - m/2, (-3
 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(5/2 - m)*(e*Sec[c + d*x])^m*(C
os[d*x] + I*Sin[d*x])^(5/2))/(d*E^((3*I)*(c + 2*d*x))*(-5 + m)*(a + I*a*Tan
[c + d*x])^(5/2))
```

Maple [F] time = 0.398, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m(a+ia\tan(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{m}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-5i\,dx-5i\,c)}}{8\,a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/8*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c)
+ 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-5*I*d*x - 5*I*c)/a^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sec(dx+c))^{m}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(5/2), x)
```

3.464 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=105

$$\frac{i2^{\frac{m}{2}+n}(a+ia\tan(c+dx))^n(e\sec(c+dx))^m(1+i\tan(c+dx))^{-\frac{m}{2}-n}\text{Hypergeometric2F1}\left(\frac{m}{2},-\frac{m}{2}-n+1,\frac{m+2}{2},\frac{1}{2}(1-i\right.}{dm}$$

```
[Out] (I*2^(m/2 + n)*Hypergeometric2F1[m/2, 1 - m/2 - n, (2 + m)/2, (1 - I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^(-m/2 - n)*(a + I*a*Tan[
c + d*x])^n)/(d*m)
```

Rubi [A] time = 0.145732, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3505, 3523, 70, 69}

$$\frac{i2^{\frac{m}{2}+n}(a+ia\tan(c+dx))^n(e\sec(c+dx))^m(1+i\tan(c+dx))^{-\frac{m}{2}-n}\text{Hypergeometric2F1}\left(\frac{m}{2},-\frac{m}{2}-n+1,\frac{m+2}{2},\frac{1}{2}(1-i\right.}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(m/2 + n)*Hypergeometric2F1[m/2, 1 - m/2 - n, (2 + m)/2, (1 - I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^(-m/2 - n)*(a + I*a*Tan[
c + d*x])^n)/(d*m)
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\,\mathrm{t}$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\mathrm{Subst}\,\Big(}{d}$$

$$= \frac{\Big(2^{-1+\frac{m}{2}+n}a(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^n\left(\frac{a+i}{}\right.}{a}$$

$$= \frac{i2^{\frac{m}{2}+n}\,{}_2F_1\left(\frac{m}{2},1-\frac{m}{2}-n;\frac{2+m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\,\mathrm{ta}}{dm}$$

Mathematica [A] time = 8.78721, size = 159, normalized size = 1.51

$$-\frac{i2^{m+n}\left(1+e^{2i(c+dx)}\right)\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+n}(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n(e\sec(c+dx))^m\sec^{-m-n}(c+dx)\mathrm{H}}{d(m+2n)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(m + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(
m + n)*(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1, 1 - m/2, 1 + m/2 + n,
 -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-m - n)*(e*Sec[c + d*x])^m*(a + I*a*Ta
n[c + d*x])^n)/(d*(m + 2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.73, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^m(i\,a\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*
x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{m}(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

3.465 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=97

$$-\frac{4i(a+ia\tan(c+dx))^{n+3}}{a^3d(n+3)}+\frac{4i(a+ia\tan(c+dx))^{n+4}}{a^4d(n+4)}-\frac{i(a+ia\tan(c+dx))^{n+5}}{a^5d(n+5)}$$

```
[Out] ((-4*I)*(a + I*a*Tan[c + d*x])^(3 + n))/(a^3*d*(3 + n)) + ((4*I)*(a + I*a*T
an[c + d*x])^(4 + n))/(a^4*d*(4 + n)) - (I*(a + I*a*Tan[c + d*x])^(5 + n))/
(a^5*d*(5 + n))
```

Rubi [A] time = 0.0696611, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$-\frac{4i(a+ia\tan(c+dx))^{n+3}}{a^3d(n+3)}+\frac{4i(a+ia\tan(c+dx))^{n+4}}{a^4d(n+4)}-\frac{i(a+ia\tan(c+dx))^{n+5}}{a^5d(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-4*I)*(a + I*a*Tan[c + d*x])^(3 + n))/(a^3*d*(3 + n)) + ((4*I)*(a + I*a*T
an[c + d*x])^(4 + n))/(a^4*d*(4 + n)) - (I*(a + I*a*Tan[c + d*x])^(5 + n))/
(a^5*d*(5 + n))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)^2(a+x)^{2+n}\,dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(4a^2(a+x)^{2+n}-4a(a+x)^{3+n}+(a+x)^{4+n}\right)dx,x,ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)}+\frac{4i(a+ia\tan(c+dx))^{4+n}}{a^4d(4+n)}-\frac{i(a+ia\tan(c+dx))^5}{a^5d(5+n)}\end{aligned}$$

Mathematica [A] time = 14.1055, size = 171, normalized size = 1.76

$$-\frac{i2^{n+5}e^{6i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(2(n+5)e^{2i(c+dx)}+2e^{4i(c+dx)}+n^2+9n+20\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+}{d(n+3)(n+4)(n+5)\left(1+e^{2i(c+dx)}\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(5 + n)*E^((6*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x))))^n*(20 + 2*E^((4*I)*(c + d*x)) + 9*n + n^2 + 2*E^((2*I)*(
c + d*x))*(5 + n))*(a + I*a*Tan[c + d*x])^n)/(d*(1 + E^((2*I)*(c + d*x)))^5
*(3 + n)*(4 + n)*(5 + n)*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [C] time = 0.577, size = 3316, normalized size = 34.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] -32*I/(3+n)/d/(5+n)/(4+n)/(exp(2*I*(d*x+c))+1)^5*(2*a^n*2^n*(exp(I*(Re(d*x)
+Re(c)))^n)^2/((exp(2*I*(d*x+c))+1)^n)*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*
I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))
^2*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/
(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n)*exp(1/2*I*Pi*csgn(I*exp
(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(1
/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*
x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*
exp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp(-1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1
)*exp(2*I*(d*x+c)))^3*n)*exp(10*I*d*x)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)
))*csgn(I*exp(I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(
I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))*n)*ex
p(10*I*c)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))^3*n)*exp(-1/2*I*Pi*csgn(I*
exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c)
))^2*csgn(I*exp(I*(d*x+c)))*n)+2*a^n*2^n*(exp(I*(Re(d*x)+Re(c)))^n)^2/((exp
(2*I*(d*x+c))+1)^n)*n*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*I*Pi*csgn(I*exp(2
*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*n)*exp(-1/2*I*
Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))
+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(ex
p(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(1/2*I*Pi*csgn(I*ex
p(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*
(d*x+c)))^2*n)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))
^2*csgn(I*a)*n)*exp(-1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)
))^3*n)*exp(8*I*d*x)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d
*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))
/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(8*I*c)*exp(-1/2*
I*Pi*csgn(I*exp(2*I*(d*x+c)))^3*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))^3*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(
d*x+c)))*n)+a^n*2^n*(exp(I*(Re(d*x)+Re(c)))^n)^2/((exp(2*I*(d*x+c))+1)^n)*n
^2*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I
*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(
d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)
))*csgn(I*a)*n)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^
2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp
(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(
1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp
(-1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(6*I*d*x
)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n)*exp(-1
/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+
1))*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(6*I*c)*exp(-1/2*I*Pi*csgn(I*exp(2*I
```

```
*(d*x+c)))^3*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))
^3*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n)+10*a^n*
2^n*(exp(I*(Re(d*x)+Re(c)))^n)^2/((exp(2*I*(d*x+c))+1)^n)*exp(-2*n*Im(d*x)-
2*n*Im(c))*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))^2*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*
x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n)*exp(
1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d
*x+c))+1))*n)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*cs
gn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(1/2*I*Pi*csgn(I*a/(e
xp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp(-1/2*I*Pi*csgn(I*a/
(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(8*I*d*x)*exp(-1/2*I*Pi*csgn
(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I*exp(2
*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*
(d*x+c))+1))*n)*exp(8*I*c)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))^3*n)*exp(
-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*n)*exp(I*Pi*csgn(
I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n)+9*a^n*2^n*(exp(I*(Re(d*x)+R
e(c)))^n)^2/((exp(2*I*(d*x+c))+1)^n)*n*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*
I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))
^2*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/
(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n)*exp(1/2*I*Pi*csgn(I*exp
(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(1
/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*
x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*
exp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp(-1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1
)*exp(2*I*(d*x+c)))^3*n)*exp(6*I*d*x)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))
)*csgn(I*exp(I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I
*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp
(6*I*c)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))^3*n)*exp(-1/2*I*Pi*csgn(I*ex
p(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c)))
^2*csgn(I*exp(I*(d*x+c)))*n)+20*a^n*2^n*(exp(I*(Re(d*x)+Re(c)))^n)^2/((exp(
2*I*(d*x+c))+1)^n)*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*I*Pi*csgn(I*exp(2*I*
(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*n)*exp(-1/2*I*Pi*
csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)
*exp(2*I*(d*x+c)))*csgn(I*a)*n)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2
*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(1/2*I*Pi*csgn(I*exp(2
*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*
x+c)))^2*n)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*
csgn(I*a)*n)*exp(-1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^
3*n)*exp(6*I*d*x)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+
c)))^2*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(6*I*c)*exp(-1/2*I*P
i*csgn(I*exp(2*I*(d*x+c)))^3*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(
2*I*(d*x+c))+1))^3*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x
+c)))*n))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.20734, size = 686, normalized size = 7.07

$$\frac{\left((-64 i n - 320 i) e^{(8 i dx + 8 i c)} + \left(-32 i n^2 - 2\right.\right.}{dn^3 + 12\,dn^2 + 47\,dn + \left(dn^3 + 12\,dn^2 + 47\,dn + 60\,d\right)e^{(10 i dx + 10 i c)} + 5\left(dn^3 + 12\,dn^2 + 47\,dn + 60\,d\right)e^{(8 i dx + 8 i c)} + 10\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] ((-64*I*n - 320*I)*e^(8*I*d*x + 8*I*c) + (-32*I*n^2 - 288*I*n - 640*I)*e^(6
*I*d*x + 6*I*c) - 64*I*e^(10*I*d*x + 10*I*c))*(2*a*e^(2*I*d*x + 2*I*c)/(e^(
2*I*d*x + 2*I*c) + 1))^n/(d*n^3 + 12*d*n^2 + 47*d*n + (d*n^3 + 12*d*n^2 + 4
7*d*n + 60*d)*e^(10*I*d*x + 10*I*c) + 5*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*
e^(8*I*d*x + 8*I*c) + 10*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(6*I*d*x + 6*
I*c) + 10*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(4*I*d*x + 4*I*c) + 5*(d*n^3
 + 12*d*n^2 + 47*d*n + 60*d)*e^(2*I*d*x + 2*I*c) + 60*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan(dx + c) + a)^n \sec(dx + c)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^6, x)
```

3.466 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=65

$$\frac{i(a+ia\tan(c+dx))^{n+3}}{a^3d(n+3)}-\frac{2i(a+ia\tan(c+dx))^{n+2}}{a^2d(n+2)}$$

```
[Out] ((-2*I)*(a + I*a*Tan[c + d*x])^(2 + n))/(a^2*d*(2 + n)) + (I*(a + I*a*Tan[c
 + d*x])^(3 + n))/(a^3*d*(3 + n))
```

Rubi [A] time = 0.0555908, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 43}

$$\frac{i(a+ia\tan(c+dx))^{n+3}}{a^3d(n+3)}-\frac{2i(a+ia\tan(c+dx))^{n+2}}{a^2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-2*I)*(a + I*a*Tan[c + d*x])^(2 + n))/(a^2*d*(2 + n)) + (I*(a + I*a*Tan[c
 + d*x])^(3 + n))/(a^3*d*(3 + n))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx &= -\frac{i\,\text{Subst}\left(\int (a-x)(a+x)^{1+n}\,dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\,\text{Subst}\left(\int \left(2a(a+x)^{1+n}-(a+x)^{2+n}\right)dx,x,ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{2i(a+ia\tan(c+dx))^{2+n}}{a^2d(2+n)}+\frac{i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)}\end{aligned}$$

Mathematica [B] time = 13.4347, size = 143, normalized size = 2.2

$$-\frac{i2^{n+3}e^{4i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(e^{2i(c+dx)}+n+3\right)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(n+2)(n+3)\left(1+e^{2i(c+dx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(3 + n)*E^((4*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x))))^n*(3 + E^((2*I)*(c + d*x)) + n)*(a + I*a*Tan[c + d*x])^n
)/(d*(1 + E^((2*I)*(c + d*x)))^3*(2 + n)*(3 + n)*Sec[c + d*x]^n*(Cos[d*x] +
 I*Sin[d*x])^n)
```

Maple [C] time = 0.236, size = 1668, normalized size = 25.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] -8*I/(2+n)/d/(3+n)/(exp(2*I*(d*x+c))+1)^3*(a^n*2^n*(exp(I*(Re(d*x)+Re(c)))^
n)^2/((exp(2*I*(d*x+c))+1)^n)*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*I*Pi*csgn
(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*n)*exp
(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*
(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x
+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(1/2*I*Pi*c
sgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*
exp(2*I*(d*x+c)))^2*n)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(
d*x+c)))^2*csgn(I*a)*n)*exp(-1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I
*(d*x+c)))^3*n)*exp(6*I*d*x)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*
exp(I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*
(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(6*I*c)*e
xp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))^3*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*
x+c))/(exp(2*I*(d*x+c))+1))^3*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I
*exp(I*(d*x+c)))*n)+(exp(I*(Re(d*x)+Re(c)))^n)^2/((exp(2*I*(d*x+c))+1)^n)*a
^n*2^n*n*exp(-2*n*Im(d*x)-2*n*Im(c))*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(
exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n)*e
xp(-1/2*I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(-1/2*
I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*n)*exp(-1/2*I*Pi*csgn(
I*exp(2*I*(d*x+c)))^3*n)*exp(4*I*d*x)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/
(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(1/2*I*Pi*csgn(I
*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*n)*exp(-
1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n)*exp(1/2*I*Pi*
csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp(4*I*c)*e
xp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*
I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c
)))^2*csgn(I*exp(I*(d*x+c)))*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn
(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))*n)+3
*(exp(I*(Re(d*x)+Re(c)))^n)^2/((exp(2*I*(d*x+c))+1)^n)*a^n*2^n*exp(-2*n*Im(
d*x)-2*n*Im(c))*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*
csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n)*exp(-1/2*I*Pi*csgn(I*a
/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(
d*x+c))/(exp(2*I*(d*x+c))+1))^3*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))^3
*n)*exp(4*I*d*x)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))
^2*csgn(I/(exp(2*I*(d*x+c))+1))*n)*exp(1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*cs
gn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*n)*exp(-1/2*I*Pi*csgn(I*exp(2
*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n)*exp(1/2*I*Pi*csgn(I*a/(exp(2*I*(d*
x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n)*exp(4*I*c)*exp(-1/2*I*Pi*csgn(I*e
xp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I
*(d*x+c)))*csgn(I*a)*n)*exp(I*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d
*x+c)))*n)*exp(-1/2*I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(
```

```
exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))*n))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.17595, size = 374, normalized size = 5.75

$$\frac{\left((-8i\,n-24i)e^{(4i\,dx+4i\,c)}-8i\,e^{(6i\,dx+6i\,c)}\right)\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n}{dn^2+5\,dn+\left(dn^2+5\,dn+6\,d\right)e^{(6i\,dx+6i\,c)}+3\left(dn^2+5\,dn+6\,d\right)e^{(4i\,dx+4i\,c)}+3\left(dn^2+5\,dn+6\,d\right)e^{(2i\,dx+2i\,c)}+6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] ((-8*I*n - 24*I)*e^(4*I*d*x + 4*I*c) - 8*I*e^(6*I*d*x + 6*I*c))*(2*a*e^(2*I
*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n/(d*n^2 + 5*d*n + (d*n^2 + 5*d*n
+ 6*d)*e^(6*I*d*x + 6*I*c) + 3*(d*n^2 + 5*d*n + 6*d)*e^(4*I*d*x + 4*I*c) +
3*(d*n^2 + 5*d*n + 6*d)*e^(2*I*d*x + 2*I*c) + 6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^4, x)
```

3.467 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=32

$$-\frac{i(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

```
[Out] ((-I)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Rubi [A] time = 0.0462205, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 32}

$$-\frac{i(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx &= -\frac{i\,\text{Subst}\left(\int (a+x)^n\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{i(a+ia\tan(c+dx))^{1+n}}{ad(1+n)}\end{aligned}$$

Mathematica [B] time = 12.9838, size = 111, normalized size = 3.47

$$-\frac{i2^{n+1}e^{i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+1}\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a+ia\tan(c+dx))^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(1 + n)*E^(I*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I
)*(c + d*x))))^(1 + n)*(a + I*a*Tan[c + d*x])^n)/(d*(1 + n)*Sec[c + d*x]^n*
```

```
(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [A] time = 0.019, size = 31, normalized size = 1.

$$-\frac{i(a+ia\tan(dx+c))^{1+n}}{ad(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] -I*(a+I*a*tan(d*x+c))^(1+n)/a/d/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.00667, size = 165, normalized size = 5.16

$$-\frac{2i\left(\frac{2ae^{(2idx+2ic)}}{e^{(2idx+2ic)}+1}\right)^{n}e^{(2idx+2ic)}}{dn+(dn+d)e^{(2idx+2ic)}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -2*I*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(2*I*d*x + 2*I
*c)/(d*n + (d*n + d)*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a(i\tan(c+dx)+1))^{n}\sec^{2}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((a*(I*tan(c + d*x) + 1))**n*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan{(dx+c)}+a)^n\sec{(dx+c)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^2, x)
```

3.468 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=56

$$\frac{ia(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(2,n-1,n,\frac{1}{2}(1+i\tan(c+dx))\right)}{4d(1-n)}$$

```
[Out] ((I/4)*a*Hypergeometric2F1[2, -1 + n, n, (1 + I*Tan[c + d*x])/2]*(a + I*a*T
an[c + d*x])^(-1 + n))/(d*(1 - n))
```

Rubi [A] time = 0.0572094, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 68}

$$\frac{ia(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(2,n-1,n,\frac{1}{2}(1+i\tan(c+dx))\right)}{4d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/4)*a*Hypergeometric2F1[2, -1 + n, n, (1 + I*Tan[c + d*x])/2]*(a + I*a*T
an[c + d*x])^(-1 + n))/(d*(1 - n))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx &= -\frac{\left(ia^3\right)\text{Subst}\left(\int \frac{(a+x)^{-2+n}}{(a-x)^2}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= \frac{ia\,{}_2F_1\left(2,-1+n;n;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-1+n}}{4d(1-n)}\end{aligned}$$

Mathematica [B] time = 13.1871, size = 141, normalized size = 2.52

$$-\frac{i2^{n-3}e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^3\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(1,2,n,-e^{2i}\right.}{d(n-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-3 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*(1 + E^((2*I)*(c + d*x)))^3*Hypergeometric2F1[1, 2, n, -E^((2*I)*(c + d*x
))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((2*I)*(c + d*x))*(-1 + n)*Sec[c + d*x]^
n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 1.096, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^2 (a+ia\tan (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan (dx+c)+a)^n \cos (dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{4}\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n \left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-2i\,dx-2i\,c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/4*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(4*I*
d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

3.469 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=60

$$\frac{ia^2(a+ia\tan(c+dx))^{n-2}\text{Hypergeometric2F1}\left(3,n-2,n-1,\frac{1}{2}(1+i\tan(c+dx))\right)}{8d(2-n)}$$

```
[Out] ((I/8)*a^2*Hypergeometric2F1[3, -2 + n, -1 + n, (1 + I*Tan[c + d*x])/2]*(a
+ I*a*Tan[c + d*x])^(-2 + n))/(d*(2 - n))
```

Rubi [A] time = 0.0550382, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 68}

$$\frac{ia^2(a+ia\tan(c+dx))^{n-2}\text{Hypergeometric2F1}\left(3,n-2,n-1,\frac{1}{2}(1+i\tan(c+dx))\right)}{8d(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/8)*a^2*Hypergeometric2F1[3, -2 + n, -1 + n, (1 + I*Tan[c + d*x])/2]*(a
+ I*a*Tan[c + d*x])^(-2 + n))/(d*(2 - n))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx &= -\frac{\left(ia^5\right)\text{Subst}\left(\int \frac{(a+x)^{-3+n}}{(a-x)^3}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= \frac{ia^2\,{}_2F_1\left(3,-2+n;-1+n;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-2+n}}{8d(2-n)}\end{aligned}$$

Mathematica [B] time = 4.13387, size = 143, normalized size = 2.38

$$-\frac{i2^{n-5}e^{-4i(c+dx)}\left(1+e^{2i(c+dx)}\right)^5\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(1,3,n-\right.}{d(n-2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-5 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*(1 + E^((2*I)*(c + d*x)))^5*Hypergeometric2F1[1, 3, -1 + n, -E^((2*I)*(c
+ d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((4*I)*(c + d*x))*(-2 + n)*Sec[c +
d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.743, size = 0, normalized size = 0.

$$\int (\cos (dx+c))^4 (a+ia\tan (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan (dx+c)+a)^n \cos (dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{16}\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n \left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-4i\,dx-4i\,c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/16*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(8*I
*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*
x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ia\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

3.470 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=60

$$\frac{ia^3(a+ia\tan(c+dx))^{n-3}\text{Hypergeometric2F1}\left(4,n-3,n-2,\frac{1}{2}(1+i\tan(c+dx))\right)}{16d(3-n)}$$

```
[Out] ((I/16)*a^3*Hypergeometric2F1[4, -3 + n, -2 + n, (1 + I*Tan[c + d*x])/2]*(a
 + I*a*Tan[c + d*x])^(-3 + n))/(d*(3 - n))
```

Rubi [A] time = 0.0539792, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3487, 68}

$$\frac{ia^3(a+ia\tan(c+dx))^{n-3}\text{Hypergeometric2F1}\left(4,n-3,n-2,\frac{1}{2}(1+i\tan(c+dx))\right)}{16d(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/16)*a^3*Hypergeometric2F1[4, -3 + n, -2 + n, (1 + I*Tan[c + d*x])/2]*(a
 + I*a*Tan[c + d*x])^(-3 + n))/(d*(3 - n))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx &= -\frac{\left(ia^7\right)\text{Subst}\left(\int \frac{(a+x)^{-4+n}}{(a-x)^4}\,dx,x,ia\tan(c+dx)\right)}{d}\\ &= \frac{ia^3\,{}_2F_1\left(4,-3+n;-2+n;\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-3+n}}{16d(3-n)}\end{aligned}$$

Mathematica [B] time = 5.48584, size = 143, normalized size = 2.38

$$-\frac{i2^{n-7}e^{-6i(c+dx)}\left(1+e^{2i(c+dx)}\right)^7\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(1,4,n-2,\right.}{d(n-3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-7 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*(1 + E^((2*I)*(c + d*x)))^7*Hypergeometric2F1[1, 4, -2 + n, -E^((2*I)*(c
+ d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((6*I)*(c + d*x))*(-3 + n)*Sec[c +
d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.801, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^6 (a+ia\tan(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^n \cos(dx+c)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{64}\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n \left(e^{(12i\,dx+12i\,c)}+6\,e^{(10i\,dx+10i\,c)}+15\,e^{(8i\,dx+8i\,c)}+20\,e^{(6i\,dx+6i\,c)}+15\,e^{(4i\,dx+4i\,c)}+6\,e^{(2i\,dx+2i\,c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/64*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(12*
I*d*x + 12*I*c) + 6*e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*x + 8*I*c) + 20*e^(
6*I*d*x + 6*I*c) + 15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x + 2*I*c) + 1)*e^(-
6*I*d*x - 6*I*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i a \tan(dx + c) + a)^n \cos(dx + c)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^6, x)
```

3.471 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=94

$$\frac{ia^2 2^{n+\frac{5}{2}}\sec^5(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-2}\text{Hypergeometric2F1}\left(\frac{5}{2},-n-\frac{3}{2},\frac{7}{2},\frac{1}{2}(1-i\tan(c+\right.}{5d}$$

```
[Out] ((I/5)*2^(5/2 + n)*a^2*Hypergeometric2F1[5/2, -3/2 - n, 7/2, (1 - I*Tan[c +
 d*x])/2]*Sec[c + d*x]^5*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c + d
*x])^(-2 + n))/d
```

Rubi [A] time = 0.187376, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3505, 3523, 70, 69}

$$\frac{ia^2 2^{n+\frac{5}{2}}\sec^5(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-2}\text{Hypergeometric2F1}\left(\frac{5}{2},-n-\frac{3}{2},\frac{7}{2},\frac{1}{2}(1-i\tan(c+\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/5)*2^(5/2 + n)*a^2*Hypergeometric2F1[5/2, -3/2 - n, 7/2, (1 - I*Tan[c +
 d*x])/2]*Sec[c + d*x]^5*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c + d
*x])^(-2 + n))/d
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{\sec^5(c+dx)\int(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{\frac{5}{2}+n}\,dx}{(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$

$$= \frac{\left(a^2\sec^5(c+dx)\right)\text{Subst}\left(\int(a-iax)^{3/2}(a+iax)^{\frac{3}{2}+n}\,dx,x,\tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$

$$= \frac{\left(2^{\frac{3}{2}+n}a^3\sec^5(c+dx)(a+ia\tan(c+dx))^{-2+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}-n}\right)\text{Subst}\left(\int\left(\frac{1}{2}\right.\right.}{d(a-ia\tan(c+dx))^{5/2}}$$

$$= \frac{i2^{\frac{5}{2}+n}a^2\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-n;\frac{7}{2};\frac{1}{2}(1-i\tan(c+dx))\right)\sec^5(c+dx)(1+i\tan(c+dx))^{-}}{5d}$$

Mathematica [A] time = 13.9095, size = 149, normalized size = 1.59

$$-\frac{i2^{n+5}e^{5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},1,n+\frac{7}{2},-e^{2i(c+dx)}\right)(a\,-}{d(2n+5)\left(1+e^{2i(c+dx)}\right)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(5 + n)*E^((5*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x))))^n*Hypergeometric2F1[-3/2, 1, 7/2 + n, -E^((2*I)*(c + d*x
))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + E^((2*I)*(c + d*x)))^4*(5 + 2*n)*Sec[
c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.345, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^5(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{32 \left(\frac{2 \, a e^{\left(2 i \, d x + 2 i \, c\right)}}{e^{\left(2 i \, d x + 2 i \, c\right)} + 1}\right)^{n} e^{\left(5 i \, d x + 5 i \, c\right)}}{e^{\left(10 i \, d x + 10 i \, c\right)} + 5 \, e^{\left(8 i \, d x + 8 i \, c\right)} + 10 \, e^{\left(6 i \, d x + 6 i \, c\right)} + 10 \, e^{\left(4 i \, d x + 4 i \, c\right)} + 5 \, e^{\left(2 i \, d x + 2 i \, c\right)} + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(32*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(5*I*d*
x + 5*I*c)/(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i \, a \tan \left(dx + c\right) + a)^{n} \sec \left(dx + c\right)^{5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^5, x)
```

3.472 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=92

$$\frac{ia2^{n+\frac{3}{2}}\sec^3(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{3}{2},-n-\frac{1}{2},\frac{5}{2},\frac{1}{2}(1-i\tan(c+dx\right.}{3d}$$

```
[Out] ((I/3)*2^(3/2 + n)*a*Hypergeometric2F1[3/2, -1/2 - n, 5/2, (1 - I*Tan[c + d
*x])/2]*Sec[c + d*x]^3*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c + d*x
])^(-1 + n))/d
```

Rubi [A] time = 0.182036, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3505, 3523, 70, 69}

$$\frac{ia2^{n+\frac{3}{2}}\sec^3(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{3}{2},-n-\frac{1}{2},\frac{5}{2},\frac{1}{2}(1-i\tan(c+dx\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/3)*2^(3/2 + n)*a*Hypergeometric2F1[3/2, -1/2 - n, 5/2, (1 - I*Tan[c + d
*x])/2]*Sec[c + d*x]^3*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c + d*x
])^(-1 + n))/d
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{\sec^3(c+dx)\int(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{\frac{3}{2}+n}\,dx}{(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$
$$= \frac{\left(a^2\sec^3(c+dx)\right)\text{Subst}\left(\int\sqrt{a-iax}(a+iax)^{\frac{1}{2}+n}\,dx,x,\tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$
$$= \frac{\left(2^{\frac{1}{2}+n}a^2\sec^3(c+dx)(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}-n}\right)\text{Subst}\left(\int\right.}{d(a-ia\tan(c+dx))^{3/2}}$$
$$= \frac{i2^{\frac{3}{2}+n}a\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-n;\frac{5}{2};\frac{1}{2}(1-i\tan(c+dx))\right)\sec^3(c+dx)(1+i\tan(c+dx)}{3d}$$

Mathematica [A] time = 13.2185, size = 149, normalized size = 1.62

$$-\frac{i2^{n+3}e^{3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},1,n+\frac{5}{2},-e^{2i(c+dx)}\right)}{d(2n+3)\left(1+e^{2i(c+dx)}\right)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(3 + n)*E^((3*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x))))^n*Hypergeometric2F1[-1/2, 1, 5/2 + n, -E^((2*I)*(c + d*x
))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + E^((2*I)*(c + d*x)))^2*(3 + 2*n)*Sec[
c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.607, size = 0, normalized size = 0.

$$\int(\sec(dx+c))^3(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(ia\tan(dx+c)+a)^n\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{8\left(\frac{2ae^{(2idx+2ic)}}{e^{(2idx+2ic)}+1}\right)^{n}e^{(3idx+3ic)}}{e^{(6idx+6ic)}+3e^{(4idx+4ic)}+3e^{(2idx+2ic)}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(8*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(3*I*d*x
 + 3*I*c)/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I
*c) + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{n}\sec(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

3.473 $\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=88

$$\frac{ia2^{n+\frac{1}{2}}\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx)\right)}{d}$$

```
[Out] (I*2^(1/2 + n)*a*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - I*Tan[c + d*x])/
2]*Sec[c + d*x]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^(-1 +
 n))/d
```

Rubi [A] time = 0.149262, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3505, 3523, 70, 69}

$$\frac{ia2^{n+\frac{1}{2}}\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(1/2 + n)*a*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - I*Tan[c + d*x])/
2]*Sec[c + d*x]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^(-1 +
 n))/d
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{\sec(c+dx)\int \sqrt{a-ia\tan(c+dx)}(a+ia\tan(c+dx))^{\frac{1}{2}+n}\,dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{\left(a^2\sec(c+dx)\right)\operatorname{Subst}\left(\int \frac{(a+iax)^{-\frac{1}{2}+n}}{\sqrt{a-iax}}\,dx, x, \tan(c+dx)\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{\left(2^{-\frac{1}{2}+n}a^2\sec(c+dx)(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-n}\right)\operatorname{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right.}{\sqrt{a-}}\right.}{d\sqrt{a-ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{1}{2}+n}a\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-n;\frac{3}{2};\frac{1}{2}(1-i\tan(c+dx))\right)\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a}{d}$$

Mathematica [A] time = 8.20648, size = 134, normalized size = 1.52

$$-\frac{i2^{n+1}e^{i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},1,n+\frac{3}{2},-e^{2i(c+dx)}\right)(a+i}{d(2n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(1 + n)*E^(I*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I
)*(c + d*x))))^n*Hypergeometric2F1[1/2, 1, 3/2 + n, -E^((2*I)*(c + d*x))]*(
a + I*a*Tan[c + d*x])^n)/(d*(1 + 2*n)*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x]
)^n)
```

Maple [F] time = 0.444, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(a+ia\tan(dx+c))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^n\sec(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}e^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(I*d*x +
 I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\,(i\tan{(c+dx)}+1))^{n}\sec{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((a*(I*tan(c + d*x) + 1))**n*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan{(dx+c)}+a)^{n}\sec{(dx+c)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

3.474 $\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=85

$$-\frac{i2^{n-\frac{1}{2}}\cos(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^n\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-n,\frac{1}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{d}$$

```
[Out] ((-I)*2^(-1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[-1/2, 3/2 - n, 1/2, (1 -
I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^n)
/d
```

Rubi [A] time = 0.166354, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3505, 3523, 70, 69}

$$-\frac{i2^{n-\frac{1}{2}}\cos(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^n\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-n,\frac{1}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[-1/2, 3/2 - n, 1/2, (1 -
I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^n)
/d
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \left(\cos(c+dx)\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))}{\sqrt{a-ia\tan(c+d}}$$

$$= \frac{\left(a^2\cos(c+dx)\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+iax)^{-\frac{3}{2}+n}}{(a-iax)^{3/2}}\right.}{d}$$

$$= \frac{\left(2^{-\frac{3}{2}+n}a\cos(c+dx)\sqrt{a-ia\tan(c+dx)}(a+ia\tan(c+dx))^n\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}}\right.}{d}$$

$$= -\frac{i2^{-\frac{1}{2}+n}\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-n;\frac{1}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx}{d}$$

Mathematica [A] time = 12.8407, size = 136, normalized size = 1.6

$$-\frac{i2^{n-1}e^{i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n-2}\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(1,\frac{3}{2},n+\frac{1}{2},-e^{2i(c+dx)}\right)(}{d(2n-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-1 + n)*E^(I*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*
I)*(c + d*x))))^(-2 + n)*Hypergeometric2F1[1, 3/2, 1/2 + n, -E^((2*I)*(c +
d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(-1 + 2*n)*Sec[c + d*x]^n*(Cos[d*x] + I
*Sin[d*x])^n)
```

Maple [F] time = 0.858, size = 0, normalized size = 0.

$$\int \cos(dx+c)\left(a+ia\tan(dx+c)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^n\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{2}\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(-i\,dx-i\,c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(2*I*
d*x + 2*I*c) + 1)*e^(-I*d*x - I*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan\left(dx+c\right)+a)^{n}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

3.475 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=94

$$-\frac{i2^{n-\frac{3}{2}}\cos^3(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{1}{2}(1-i\tan(c+\right.}{3ad}$$

```
[Out] ((-I/3)*2^(-3/2 + n)*Cos[c + d*x]^3*Hypergeometric2F1[-3/2, 5/2 - n, -1/2,
(1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x
])^(1 + n))/(a*d)
```

Rubi [A] time = 0.189261, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3505, 3523, 70, 69}

$$-\frac{i2^{n-\frac{3}{2}}\cos^3(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{1}{2}(1-i\tan(c+\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I/3)*2^(-3/2 + n)*Cos[c + d*x]^3*Hypergeometric2F1[-3/2, 5/2 - n, -1/2,
(1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x
])^(1 + n))/(a*d)
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \left(\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\right)\int \frac{(a+ia\tan(c+d}{(a-ia\tan(c+}$$

$$= \frac{\left(a^2\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+iax)^{-}}{(a-iax)}\right.}{d}$$

$$= \frac{\left(2^{-\frac{5}{2}+n}\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right.\right.}{d}$$

$$= -\frac{i2^{-\frac{3}{2}+n}\cos^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-n;-\frac{1}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx}{3ad}$$

Mathematica [A] time = 13.4614, size = 149, normalized size = 1.59

$$-\frac{i2^{n-3}e^{-3i(c+dx)}\left(1+e^{2i(c+dx)}\right)^4\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(1,\frac{5}{2},n-\frac{1}{2},\right.}{d(2n-3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-3 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*(1 + E^((2*I)*(c + d*x)))^4*Hypergeometric2F1[1, 5/2, -1/2 + n, -E^((2*I)
*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((3*I)*(c + d*x))*(-3 + 2*n)*Se
c[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 1.84, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^3(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ia\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{8}\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-3i\,dx-3i\,c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/8*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(6*I*
d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-3*I*d
*x - 3*I*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{n}\cos(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

3.476 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=94

$$-\frac{i2^{n-\frac{5}{2}}\cos^5(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+2}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx\right.}{5a^2d}$$

```
[Out] ((-I/5)*2^(-5/2 + n)*Cos[c + d*x]^5*Hypergeometric2F1[-5/2, 7/2 - n, -3/2,
(1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x
])^(2 + n))/(a^2*d)
```

Rubi [A] time = 0.193491, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3505, 3523, 70, 69}

$$-\frac{i2^{n-\frac{5}{2}}\cos^5(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+2}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx\right.}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I/5)*2^(-5/2 + n)*Cos[c + d*x]^5*Hypergeometric2F1[-5/2, 7/2 - n, -3/2,
(1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x
])^(2 + n))/(a^2*d)
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \left(\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\right)\int \frac{(a+ia\tan(c}{(a-ia\tan(c}$$

$$= \frac{\left(a^2\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\right)\text{Subst}\left(\int \frac{(a+i}{(a-}\right.}{d}$$

$$= \frac{\left(2^{-\frac{7}{2}+n}\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{2+n}\left(\frac{a+ia\tan(c+}{a}\right.\right.}{ad}$$

$$= -\frac{i2^{-\frac{5}{2}+n}\cos^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-n;-\frac{3}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+}{5a^2d}$$

Mathematica [A] time = 6.01809, size = 149, normalized size = 1.59

$$-\frac{i2^{n-5}e^{-5i(c+dx)}\left(1+e^{2i(c+dx)}\right)^6\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(1,\frac{7}{2},n-\right.}{d(2n-5)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-5 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*(1 + E^((2*I)*(c + d*x)))^6*Hypergeometric2F1[1, 7/2, -3/2 + n, -E^((2*I)
*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((5*I)*(c + d*x))*(-5 + 2*n)*Se
c[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.734, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^5\,(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{32}\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-5i\,dx-5i\,c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/32*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(10*
I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6*I*c) + 10*e^(4*
I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*e^(-5*I*d*x - 5*I*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{n}\cos(dx+c)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^5, x)
```

3.477 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=96

$$\frac{ia2^{n+\frac{9}{4}}(e\sec(c+dx))^{5/2}(1+i\tan(c+dx))^{-n-\frac{1}{4}}(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{5}{4},-n-\frac{1}{4},\frac{9}{4},\frac{1}{2}(1-i\tan\right.}{5d}$$

```
[Out] ((I/5)*2^(9/4 + n)*a*Hypergeometric2F1[5/4, -1/4 - n, 9/4, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(5/2)*(1 + I*Tan[c + d*x])^(-1/4 - n)*(a + I*a*Tan
[c + d*x])^(-1 + n))/d
```

Rubi [A] time = 0.201624, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{ia2^{n+\frac{9}{4}}(e\sec(c+dx))^{5/2}(1+i\tan(c+dx))^{-n-\frac{1}{4}}(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{5}{4},-n-\frac{1}{4},\frac{9}{4},\frac{1}{2}(1-i\tan\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/5)*2^(9/4 + n)*a*Hypergeometric2F1[5/4, -1/4 - n, 9/4, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(5/2)*(1 + I*Tan[c + d*x])^(-1/4 - n)*(a + I*a*Tan
[c + d*x])^(-1 + n))/d
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \frac{(e\sec(c+dx))^{5/2}\int (a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{\frac{5}{4}+n}\,dx}{(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}}$$
$$= \frac{\left(a^2(e\sec(c+dx))^{5/2}\right)\text{Subst}\left(\int \sqrt[4]{a-iax}(a+iax)^{\frac{1}{4}+n}\,dx,x,\tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}}$$
$$= \frac{\left(2^{\frac{1}{4}+n}a^2(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{1}{4}-n}\right)\text{Subs}}{d(a-ia\tan(c+dx))^{5/4}}$$
$$= \frac{i2^{\frac{9}{4}+n}a\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-n;\frac{9}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/2}(1+i\tan(c}{5d}$$

Mathematica [A] time = 8.86914, size = 156, normalized size = 1.62

$$-\frac{i2^{n+\frac{7}{2}}e^{i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{3}{2}}(e\sec(c+dx))^{5/2}\sec^{-n-\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(-\frac{1}{4},1\right.}{d(4n+5)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(7/2 + n)*E^(I*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2
*I)*(c + d*x))))^(3/2 + n)*Hypergeometric2F1[-1/4, 1, 9/4 + n, -E^((2*I)*(c
 + d*x))]*Sec[c + d*x]^(-5/2 - n)*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d
*x])^n)/(d*(5 + 4*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.198, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{5}{2}}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{5}{2}}(ia\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{4\sqrt{2}e^2\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{5}{2}i\,dx+\frac{5}{2}i\,c\right)}}{e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(4*sqrt(2)*e^2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^
n*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(5/2*I*d*x + 5/2*I*c)/(e^(4*I*d*x + 4
*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.478 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=96

$$\frac{ia2^{n+\frac{7}{4}}(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}-n,\frac{7}{4},\frac{1}{2}(1-i\tan(c+d\right.}{3d}$$

```
[Out] ((I/3)*2^(7/4 + n)*a*Hypergeometric2F1[3/4, 1/4 - n, 7/4, (1 - I*Tan[c + d*
x])/2]*(e*Sec[c + d*x])^(3/2)*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c
 + d*x])^(-1 + n))/d
```

Rubi [A] time = 0.198503, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{ia2^{n+\frac{7}{4}}(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}-n,\frac{7}{4},\frac{1}{2}(1-i\tan(c+d\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/3)*2^(7/4 + n)*a*Hypergeometric2F1[3/4, 1/4 - n, 7/4, (1 - I*Tan[c + d*
x])/2]*(e*Sec[c + d*x])^(3/2)*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c
 + d*x])^(-1 + n))/d
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \frac{(e\sec(c+dx))^{3/2}\int (a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{\frac{3}{4}+n}\,dx}{(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+iax)^{-\frac{1}{4}+n}}{\sqrt[4]{a-iax}}\,dx, x, \tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$= \frac{\left(2^{-\frac{1}{4}+n}a^2(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{4}-n}\text{S}\right.}{d(a-ia\tan(c+dx))^{3/4}}$$

$$= \frac{i2^{\frac{7}{4}+n}a\,{}_2F_1\left(\frac{3}{4},\frac{1}{4}-n;\frac{7}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3/2}(1+i\tan(}{3d}$$

Mathematica [A] time = 8.7107, size = 156, normalized size = 1.62

$$-\frac{i2^{n+\frac{5}{2}}e^{i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{1}{2}}(e\sec(c+dx))^{3/2}\sec^{-n-\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{4}\right.}{d(4n+3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(5/2 + n)*E^(I*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2
*I)*(c + d*x))))^(1/2 + n)*Hypergeometric2F1[1/4, 1, 7/4 + n, -E^((2*I)*(c
+ d*x))]*Sec[c + d*x]^(-3/2 - n)*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*
x])^n)/(d*(3 + 4*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{3}{2}}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\,\sqrt{2}e\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}}{e^{(2i\,dx+2i\,c)}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(2*sqrt(2)*e*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c)/(e^(2*I*d*x + 2*I
*c) + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.479 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=94

$$\frac{ia2^{n+\frac{5}{4}}\sqrt{e\sec(c+dx)}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4}-n,\frac{5}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{d}$$

```
[Out] (I*2^(5/4 + n)*a*Hypergeometric2F1[1/4, 3/4 - n, 5/4, (1 - I*Tan[c + d*x])/
2]*Sqrt[e*Sec[c + d*x]]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d*x
])^(-1 + n))/d
```

Rubi [A] time = 0.174919, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$\frac{ia2^{n+\frac{5}{4}}\sqrt{e\sec(c+dx)}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^{n-1}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4}-n,\frac{5}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(5/4 + n)*a*Hypergeometric2F1[1/4, 3/4 - n, 5/4, (1 - I*Tan[c + d*x])/
2]*Sqrt[e*Sec[c + d*x]]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d*x
])^(-1 + n))/d
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \frac{\sqrt{e\sec(c+dx)}\int \sqrt[4]{a-ia\tan(c+dx)}(a+ia\tan(c+dx))^{\frac{1}{4}+n}\,dx}{\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}}$$

$$= \frac{\left(a^2\sqrt{e\sec(c+dx)}\right)\text{Subst}\left(\int \frac{(a+iax)^{-\frac{3}{4}+n}}{(a-iax)^{3/4}}\,dx, x, \tan(c+dx)\right)}{d\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}}$$

$$= \frac{\left(2^{-\frac{3}{4}+n}a^2\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{3}{4}-n}\right)\text{Subst}\left(\int \right.}{d\sqrt[4]{a-ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{5}{4}+n}a\,{}_2F_1\left(\frac{1}{4},\frac{3}{4}-n;\frac{5}{4};\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt{e\sec(c+dx)}(1+i\tan(c+dx)}{d}$$

Mathematica [A] time = 8.34891, size = 156, normalized size = 1.66

$$-\frac{i2^{n+\frac{3}{2}}e^{i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n-\frac{1}{2}}\sqrt{e\sec(c+dx)}\sec^{-n-\frac{1}{2}}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(\frac{3}{4},1,n\right.}{d(4n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(3/2 + n)*E^(I*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2
*I)*(c + d*x))))^(-1/2 + n)*Hypergeometric2F1[3/4, 1, 5/4 + n, -E^((2*I)*(c
 + d*x))]*Sec[c + d*x]^(-1/2 - n)*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x
])^n)/(d*(1 + 4*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.194, size = 0, normalized size = 0.

$$\int \sqrt{e\sec(dx+c)}\,(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{2}\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^n, x)
```

3.480 $\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal. Leaf size=91

$$-\frac{i2^{n+\frac{3}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^n\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4}-n,\frac{3}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{d\sqrt{e\sec(c+dx)}}$$

```
[Out] ((-I)*2^(3/4 + n)*Hypergeometric2F1[-1/4, 5/4 - n, 3/4, (1 - I*Tan[c + d*x]
)/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*Sqrt[e*Sec
[c + d*x]])
```

Rubi [A] time = 0.178157, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$-\frac{i2^{n+\frac{3}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^n\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4}-n,\frac{3}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{d\sqrt{e\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-I)*2^(3/4 + n)*Hypergeometric2F1[-1/4, 5/4 - n, 3/4, (1 - I*Tan[c + d*x]
)/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*Sqrt[e*Sec
[c + d*x]])
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = \frac{\left(\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}\right)\int \frac{(a+ia\tan(c+dx))^{-\frac{1}{4}+n}}{\sqrt[4]{a-ia\tan(c+dx)}}\,dx}{\sqrt{e\sec(c+dx)}}$$

$$= \frac{\left(a^2\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}\right)\operatorname{Subst}\left(\int \frac{(a+iax)^{-\frac{5}{4}+n}}{(a-iax)^{5/4}}\,dx,x,\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}$$

$$= \frac{\left(2^{-\frac{5}{4}+n}a\sqrt[4]{a-ia\tan(c+dx)}(a+ia\tan(c+dx))^n\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{4}-n}\right)\operatorname{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{5}{4}+\ldots}}{(a-iax)^{5/4}}\ldots\right.}{d\sqrt{e\sec(c+dx)}}$$

$$= -\frac{i2^{\frac{3}{4}+n}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4}-n;\frac{3}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx)\ldots}{d\sqrt{e\sec(c+dx)}}$$

Mathematica [A] time = 9.53452, size = 129, normalized size = 1.42

$$-\frac{i2^{n+\frac{1}{2}}e^{i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n-\frac{3}{2}}\sec^{\frac{1}{2}-n}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{5}{4},n+\frac{3}{4},-e^{2i(c+dx)}\right)(a+ia\tan(c+dx))^n}{d(4n-1)\sqrt{e\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-I)*2^(1/2 + n)*E^(I*(c + d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
))^(-3/2 + n)*Hypergeometric2F1[1, 5/4, 3/4 + n, -E^((2*I)*(c + d*x))]*Sec[
c + d*x]^(1/2 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(-1 + 4*n)*Sqrt[e*Sec[c + d
*x]])
```

Maple [F] time = 0.199, size = 0, normalized size = 0.

$$\int (a+ia\tan(dx+c))^n\frac{1}{\sqrt{e\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^n}{\sqrt{e\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/sqrt(e*sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2}\left(\frac{2\,ae^{\left(2i\,dx+2i\,c\right)}}{e^{\left(2i\,dx+2i\,c\right)}+1}\right)^{n}\sqrt{\frac{e}{e^{\left(2i\,dx+2i\,c\right)}+1}}\left(e^{\left(2i\,dx+2i\,c\right)}+1\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{2\,e},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x -
 1/2*I*c)/e, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\left(i\tan\left(c+dx\right)+1\right)\right)^{n}}{\sqrt{e\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((a*(I*tan(c + d*x) + 1))**n/sqrt(e*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(i\,a\tan\left(dx+c\right)+a\right)^{n}}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/sqrt(e*sec(d*x + c)), x)
```

3.481 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{i2^{n+\frac{1}{4}}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^n\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4}-n,\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] ((-I/3)*2^(1/4 + n)*Hypergeometric2F1[-3/4, 7/4 - n, 1/4, (1 - I*Tan[c + d*
x])/2]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(e*Sec[c
 + d*x])^(3/2))
```

Rubi [A] time = 0.208344, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$-\frac{i2^{n+\frac{1}{4}}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^n\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4}-n,\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{3d(e\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-I/3)*2^(1/4 + n)*Hypergeometric2F1[-3/4, 7/4 - n, 1/4, (1 - I*Tan[c + d*
x])/2]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(e*Sec[c
 + d*x])^(3/2))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\left((a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\right)\int \frac{(a+ia\tan(c+dx))^{-\frac{3}{4}+n}}{(a-ia\tan(c+dx))^{3/4}}\,dx}{(e\sec(c+dx))^{3/2}}$$

$$= \frac{\left(a^2(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\right)\operatorname{Subst}\left(\int \frac{(a+iax)^{-\frac{7}{4}+n}}{(a-iax)^{7/4}}\,dx,x,\tan(c+dx)\right)}{d(e\sec(c+dx))^{3/2}}$$

$$= \frac{\left(2^{-\frac{7}{4}+n}a(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^n\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{3}{4}-n}\right)\operatorname{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{7}{4}+n}}{(a-iax)^{7/4}}\right.}{d(e\sec(c+dx))^{3/2}}$$

$$= -\frac{i2^{\frac{1}{4}+n}\,{}_2F_1\left(-\frac{3}{4},\frac{7}{4}-n;\frac{1}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^n}{3d(e\sec(c+dx))^{3/2}}$$

Mathematica [A] time = 10.1949, size = 129, normalized size = 1.39

$$-\frac{i2^{n-\frac{1}{2}}e^{i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n-\frac{5}{2}}\sec^{\frac{3}{2}-n}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{7}{4},n+\frac{1}{4},-e^{2i(c+dx)}\right)(a+ia\tan(c+dx))^n}{d(4n-3)(e\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*2^(-1/2 + n)*E^(I*(c + d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
)))^(-5/2 + n)*Hypergeometric2F1[1, 7/4, 1/4 + n, -E^((2*I)*(c + d*x))]*Sec
[c + d*x]^(3/2 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(-3 + 4*n)*(e*Sec[c + d*x]
)^(3/2))
```

Maple [F] time = 0.19, size = 0, normalized size = 0.

$$\int (a+ia\tan(dx+c))^n (e\sec(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2}\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{4\,e^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/4*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I
*c) + 1)*e^(-3/2*I*d*x - 3/2*I*c)/e^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^{n}}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(3/2), x)
```

3.482 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{i2^{n-\frac{1}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4}-n,-\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{5ad(e\sec(c+dx))^{5/2}}$$

```
[Out] ((-I/5)*2^(-1/4 + n)*Hypergeometric2F1[-5/4, 9/4 - n, -1/4, (1 - I*Tan[c +
d*x])/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*
d*(e*Sec[c + d*x])^(5/2))
```

Rubi [A] time = 0.210164, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3505, 3523, 70, 69}

$$-\frac{i2^{n-\frac{1}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4}-n,-\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{5ad(e\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-I/5)*2^(-1/4 + n)*Hypergeometric2F1[-5/4, 9/4 - n, -1/4, (1 - I*Tan[c +
d*x])/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*
d*(e*Sec[c + d*x])^(5/2))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\left((a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\right)\int \frac{(a+ia\tan(c+dx))^{-\frac{5}{4}+n}}{(a-ia\tan(c+dx))^{5/4}}\,dx}{(e\sec(c+dx))^{5/2}}$$

$$= \frac{\left(a^2(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\right)\text{Subst}\left(\int \frac{(a+iax)^{-\frac{9}{4}+n}}{(a-iax)^{9/4}}\,dx, x, \tan(c+dx)\right)}{d(e\sec(c+dx))^{5/2}}$$

$$= \frac{\left(2^{-\frac{9}{4}+n}(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{4}-n}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right.}{(a-ia}\right.}{d(e\sec(c+dx))^{5/2}}$$

$$= -\frac{i2^{-\frac{1}{4}+n}\,{}_2F_1\left(-\frac{5}{4},\frac{9}{4}-n;-\frac{1}{4};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+a}{5ad(e\sec(c+dx))^{5/2}}$$

Mathematica [A] time = 10.4131, size = 147, normalized size = 1.5

$$-\frac{i2^{n-\frac{3}{2}}e^{-3i(c+dx)}\left(1+e^{2i(c+dx)}\right)^4\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{n+\frac{1}{2}}\sec^{\frac{1}{2}-n}(c+dx)\text{Hypergeometric2F1}\left(1,\frac{9}{4},n-\frac{1}{4},-e^{2i(c+dx)}\right)(a+ia\tan(}{de^2(4n-5)\sqrt{e\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*2^(-3/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/2 + n)*(1
 + E^((2*I)*(c + d*x)))^4*Hypergeometric2F1[1, 9/4, -1/4 + n, -E^((2*I)*(c
+ d*x))]*Sec[c + d*x]^(1/2 - n)*(a + I*a*Tan[c + d*x])^n)/(d*e^2*E^((3*I)*(
c + d*x))*(-5 + 4*n)*Sqrt[e*Sec[c + d*x]])
```

Maple [F] time = 0.181, size = 0, normalized size = 0.

$$\int (a+ia\tan(dx+c))^n (e\sec(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2}\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{8\,e^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/8*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I
*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-5/2*I*d*x - 5/2*I*c)/e^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(5/2), x)
```

3.483 $\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=269

$$\frac{24i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-4}}{a^3d(4-n)n\left(4-n^2\right)}-\frac{24i(a+ia\tan(c+dx))^{n+4}(e\sec(c+dx))^{-n-4}}{a^4dn\left(n^4-20n^2+64\right)}-\frac{12i(a+ia\tan(c+dx)}{a^2d(2-n}$$

```
[Out] (I*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(4 - n)) + ((4*I)
*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(8 - 6*n +
n^2)) - ((12*I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(
a^2*d*(2 - n)*(4 - n)*n) + ((24*I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c
 + d*x])^(3 + n))/(a^3*d*(4 - n)*n*(4 - n^2)) - ((24*I)*(e*Sec[c + d*x])^(-
4 - n)*(a + I*a*Tan[c + d*x])^(4 + n))/(a^4*d*n*(64 - 20*n^2 + n^4))
```

Rubi [A] time = 0.406169, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3504, 3488}

$$\frac{24i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-4}}{a^3d(4-n)n\left(4-n^2\right)}-\frac{24i(a+ia\tan(c+dx))^{n+4}(e\sec(c+dx))^{-n-4}}{a^4dn\left(n^4-20n^2+64\right)}-\frac{12i(a+ia\tan(c+dx)}{a^2d(2-n}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(4 - n)) + ((4*I)
*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(8 - 6*n +
n^2)) - ((12*I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(
a^2*d*(2 - n)*(4 - n)*n) + ((24*I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c
 + d*x])^(3 + n))/(a^3*d*(4 - n)*n*(4 - n^2)) - ((24*I)*(e*Sec[c + d*x])^(-
4 - n)*(a + I*a*Tan[c + d*x])^(4 + n))/(a^4*d*n*(64 - 20*n^2 + n^4))
```

Rule 3504

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx = \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4\int (e\sec(c+dx))^{-4-n}(a+ia\,\mathrm{t}}{a(4-n)}$$
$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+dx))^{-4-n}(a+ia\,\mathrm{ta}}{ad\left(8-6n+n^2\right)}$$
$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+dx))^{-4-n}(a+ia\,\mathrm{ta}}{ad\left(8-6n+n^2\right)}$$
$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+dx))^{-4-n}(a+ia\,\mathrm{ta}}{ad\left(8-6n+n^2\right)}$$
$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+dx))^{-4-n}(a+ia\,\mathrm{ta}}{ad\left(8-6n+n^2\right)}$$

Mathematica [A] time = 0.622421, size = 165, normalized size = 0.61

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}\left(-8in^3\sin(2(c+dx))-4in^3\sin(4(c+dx))+4\left(n^2-16\right)n^2\cos(2(c+dx))+\left(n^2\right.\right.}{8de^4(n-4)(n-2)n(n+2)(n+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I/8)*(192 - 60*n^2 + 3*n^4 + 4*n^2*(-16 + n^2)*Cos[2*(c + d*x)] + n^2*(-
4 + n^2)*Cos[4*(c + d*x)] + (128*I)*n*Sin[2*(c + d*x)] - (8*I)*n^3*Sin[2*(c
 + d*x)] + (16*I)*n*Sin[4*(c + d*x)] - (4*I)*n^3*Sin[4*(c + d*x)])*(a + I*a
*Tan[c + d*x])^n)/(d*e^4*(-4 + n)*(-2 + n)*n*(2 + n)*(4 + n)*(e*Sec[c + d*x
])^n)
```

Maple [C] time = 1.361, size = 5866, normalized size = 21.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 2.0563, size = 583, normalized size = 2.17

$$\frac{\left(-i\,a^n n^4+4i\,a^n n^3+4i\,a^n n^2-16i\,a^n n\right)\cos\left((dx+c)(n+4)\right)+\left(-4i\,a^n n^4+8i\,a^n n^3+64i\,a^n n^2-128i\,a^n n\right)\cos\left((dx+c)(n\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 1/16*((-I*a^n*n^4 + 4*I*a^n*n^3 + 4*I*a^n*n^2 - 16*I*a^n*n)*cos((d*x + c)*(
n + 4)) + (-4*I*a^n*n^4 + 8*I*a^n*n^3 + 64*I*a^n*n^2 - 128*I*a^n*n)*cos((d*
```

```
x + c)*(n + 2)) + (-4*I*a^n*n^4 - 8*I*a^n*n^3 + 64*I*a^n*n^2 + 128*I*a^n*n)
*cos((d*x + c)*(n - 2)) + (-I*a^n*n^4 - 4*I*a^n*n^3 + 4*I*a^n*n^2 + 16*I*a^
n*n)*cos((d*x + c)*(n - 4)) + (-6*I*a^n*n^4 + 120*I*a^n*n^2 - 384*I*a^n)*co
s((d*x + c)*n) + (a^n*n^4 - 4*a^n*n^3 - 4*a^n*n^2 + 16*a^n*n)*sin((d*x + c)
*(n + 4)) + 4*(a^n*n^4 - 2*a^n*n^3 - 16*a^n*n^2 + 32*a^n*n)*sin((d*x + c)*(
n + 2)) + 4*(a^n*n^4 + 2*a^n*n^3 - 16*a^n*n^2 - 32*a^n*n)*sin((d*x + c)*(n
- 2)) + (a^n*n^4 + 4*a^n*n^3 - 4*a^n*n^2 - 16*a^n*n)*sin((d*x + c)*(n - 4))
 + 6*(a^n*n^4 - 20*a^n*n^2 + 64*a^n)*sin((d*x + c)*n))/((e^(n + 4)*n^5 - 20
*e^(n + 4)*n^3 + 64*e^(n + 4)*n)*d)
```

Fricas [A] time = 2.20184, size = 844, normalized size = 3.14

$$\frac{\left(-i n^4 - 4i n^3 + 4i n^2 + \left(-i n^4 + 4i n^3 + 4i n^2 - 16i n\right)e^{(8i dx+8i c)} + \left(-4i n^4 + 8i n^3 + 64i n^2 - 128i n\right)e^{(6i dx+6i c)} + \left(-6i n \right.\right.}{dn^5 - 20 dn^3 + 64 dn + \left(dn^5 - 20 dn^3 + 64 dn\right)e^{(8i dx+8i c)} + 4\left(dn^5 - 20 dn^3 + 64 dn\right)e^{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (-I*n^4 - 4*I*n^3 + 4*I*n^2 + (-I*n^4 + 4*I*n^3 + 4*I*n^2 - 16*I*n)*e^(8*I*
d*x + 8*I*c) + (-4*I*n^4 + 8*I*n^3 + 64*I*n^2 - 128*I*n)*e^(6*I*d*x + 6*I*c
) + (-6*I*n^4 + 120*I*n^2 - 384*I)*e^(4*I*d*x + 4*I*c) + (-4*I*n^4 - 8*I*n^
3 + 64*I*n^2 + 128*I*n)*e^(2*I*d*x + 2*I*c) + 16*I*n)*(2*a*e^(2*I*d*x + 2*I
*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c)
+ 1))^(-n - 4)/(d*n^5 - 20*d*n^3 + 64*d*n + (d*n^5 - 20*d*n^3 + 64*d*n)*e^(
8*I*d*x + 8*I*c) + 4*(d*n^5 - 20*d*n^3 + 64*d*n)*e^(6*I*d*x + 6*I*c) + 6*(d
*n^5 - 20*d*n^3 + 64*d*n)*e^(4*I*d*x + 4*I*c) + 4*(d*n^5 - 20*d*n^3 + 64*d*
n)*e^(2*I*d*x + 2*I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-4-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-n-4} (i a \tan(dx+c) + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n - 4)*(I*a*tan(d*x + c) + a)^n, x)
```

3.484 $\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=205

$$-\frac{6i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-3}}{a^2d(3-n)\left(1-n^2\right)}+\frac{6i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-3}}{a^3d\left(n^4-10n^2+9\right)}+\frac{3i(a+ia\tan(c+dx))^{n+1}(}{ad\left(n^2-4n\right.}$$

```
[Out] (I*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(3 - n)) + ((3*I)
*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(3 - 4*n +
n^2)) - ((6*I)*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(a
^2*d*(3 - n)*(1 - n^2)) + ((6*I)*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c +
 d*x])^(3 + n))/(a^3*d*(9 - 10*n^2 + n^4))
```

Rubi [A] time = 0.304566, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3504, 3488}

$$-\frac{6i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-3}}{a^2d(3-n)\left(1-n^2\right)}+\frac{6i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-3}}{a^3d\left(n^4-10n^2+9\right)}+\frac{3i(a+ia\tan(c+dx))^{n+1}(}{ad\left(n^2-4n\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```


```
[Out] (I*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(3 - n)) + ((3*I)
*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(3 - 4*n +
n^2)) - ((6*I)*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(a
^2*d*(3 - n)*(1 - n^2)) + ((6*I)*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c +
 d*x])^(3 + n))/(a^3*d*(9 - 10*n^2 + n^4))
```


Rule 3504

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx = \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3\int (e\sec(c+dx))^{-3-n}(a+}{a(3-}$$
$$= \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3i(e\sec(c+dx))^{-3-n}(a+i}{ad\left(3-4n+\right.}$$
$$= \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3i(e\sec(c+dx))^{-3-n}(a+i}{ad\left(3-4n+\right.}$$
$$= \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3i(e\sec(c+dx))^{-3-n}(a+i}{ad\left(3-4n+\right.}$$

Mathematica [A] time = 0.615162, size = 119, normalized size = 0.58

$$\frac{(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}\left(-3in\left(n^2-9\right)\cos(c+dx)-in\left(n^2-1\right)\cos(3(c+dx))-6\sin(c+dx)\left(\left(n^2-1\right)\right.\right.}{4de^3(n-3)(n-1)(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (((-3*I)*n*(-9 + n^2)*Cos[c + d*x] - I*n*(-1 + n^2)*Cos[3*(c + d*x)] - 6*(-
5 + n^2 + (-1 + n^2)*Cos[2*(c + d*x)])*Sin[c + d*x])*(a + I*a*Tan[c + d*x])
^n)/(4*d*e^3*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(e*Sec[c + d*x])^n)
```

Maple [C] time = 1.178, size = 4990, normalized size = 24.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] 1/8/(-3*I*d+I*n*d)*a^n*e^(-n)/e^3*exp(I*(d*x+c))^n*exp(-1/2*I*(6*c+Pi*csgn(
I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*n+6*d*x+3*Pi*csgn(I/(exp(2*I*(d*
x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+3*Pi*csgn(I*exp(I*(
d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))
^2+Pi*csgn(I*exp(2*I*(d*x+c)))^3*n-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x
+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))
*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*cs
gn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+
c))+1)*exp(I*(d*x+c)))+3*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))^2+Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*
n+3*Pi*csgn(I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2-n*Pi*csgn(
I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3-n*Pi*csgn(I*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))^3-3*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3-3*P
i*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-3*Pi*csgn(I*e)*csgn(I*exp(I
*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)
))-3*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d
*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+
c)))^2*csgn(I*a)*n-2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n
+Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c)
)+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2
*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))*n+n*Pi*csgn(
```

```
I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+n*Pi*csgn(I/(exp(2*I*(
d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I
*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*
(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c))
)^2+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n-Pi*csgn(I*exp(2*
I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*n-Pi*csgn(I*exp
(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(
d*x+c)))^2*n))+1/8/(I*n*d+3*I*d)*a^n*e^(-n)/e^3*exp(I*(d*x+c))^n*exp(-1/2*I
*(-6*c+Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*n-6*d*x+3*Pi*csgn
(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+3*Pi
*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*
exp(I*(d*x+c)))^2+Pi*csgn(I*exp(2*I*(d*x+c)))^3*n-Pi*csgn(I*exp(2*I*(d*x+c)
)/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n-n*Pi*csgn(I/(exp(2
*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+
c))+1))-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e
/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))+3*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*ex
p(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2
*I*(d*x+c)))^3*n+3*Pi*csgn(I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)
))^2-n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3-n*Pi*csgn(I*exp(I
*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-3*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*
(d*x+c)))^3-3*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-3*Pi*csgn(I*
e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1
)*exp(I*(d*x+c)))-3*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*
csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I*a/(exp(2*I*(d*x+c))+1
)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n-2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*ex
p(I*(d*x+c)))*n+Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(
exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n+Pi*csgn(I*exp(2*I*(d*x+c)
))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1
))*n+n*Pi*csgn(I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+n*Pi*cs
gn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*
Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*P
i*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)
*exp(I*(d*x+c)))^2+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n-P
i*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*
n-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c
))+1)*exp(2*I*(d*x+c)))^2*n))+3/8/(-I*d+I*n*d)*a^n*e^(-n)/e^3*exp(I*(d*x+c)
)^n*exp(-1/2*I*(2*c+Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*n+2*
d*x+3*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c
))+1))^2+3*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I
*(d*x+c))+1)*exp(I*(d*x+c)))^2+Pi*csgn(I*exp(2*I*(d*x+c)))^3*n-Pi*csgn(I*ex
p(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n-n*Pi*
csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(
exp(2*I*(d*x+c))+1))-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))
+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))+3*Pi*csgn(I*exp(I*(d*x+c
)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I*a/(exp(2*I*(d*x
+c))+1)*exp(2*I*(d*x+c)))^3*n+3*Pi*csgn(I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*
exp(I*(d*x+c)))^2-n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3-n*Pi
*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-3*Pi*csgn(I*e/(exp(2*I*(d*x+
c))+1)*exp(I*(d*x+c)))^3-3*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3
-3*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2
*I*(d*x+c))+1)*exp(I*(d*x+c)))-3*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp
(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I*a/(exp(2
*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n-2*Pi*csgn(I*exp(2*I*(d*x+c))
)^2*csgn(I*exp(I*(d*x+c)))*n+Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1
))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n+Pi*csgn(I*ex
p(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2
*I*(d*x+c))+1))*n+n*Pi*csgn(I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c
)))^2+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x
+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+
```

```
c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*
I*(d*x+c))+1)*exp(I*(d*x+c)))^2+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d
*x+c)))^2*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d
*x+c))+1))^2*n-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(e
xp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n))+3/8/(I*d+I*n*d)*a^n*e^(-n)/e^3*e
xp(I*(d*x+c))^n*exp(-1/2*I*(-2*c+Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c
))+1))^3*n-2*d*x+3*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))^2+3*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn
(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+Pi*csgn(I*exp(2*I*(d*x+c)))^3*n
-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c)
)+1))*n-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp
(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp
(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))+3*Pi*csgn(I
*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I*a/
(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n+3*Pi*csgn(I*e)*csgn(I*e/(exp(2*I
*(d*x+c))+1)*exp(I*(d*x+c)))^2-n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*
x+c)))^3-n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-3*Pi*csgn(I*e/(
exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3-3*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(
d*x+c))+1))^3-3*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*cs
gn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))-3*Pi*csgn(I/(exp(2*I*(d*x+c))+1
))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*cs
gn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n-2*Pi*csgn(I*exp
(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n+Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2
*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n
+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*
csgn(I/(exp(2*I*(d*x+c))+1))*n+n*Pi*csgn(I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)
*exp(I*(d*x+c)))^2+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/
(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(
exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csg
n(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+Pi*csgn(I*exp(2*I*(d*x+c)))*cs
gn(I*exp(I*(d*x+c)))^2*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c)
)/(exp(2*I*(d*x+c))+1))^2*n-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1)
)*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n))
```

Maxima [A] time = 1.9043, size = 464, normalized size = 2.26

$$\frac{\left(-i\,a^n n^3+3i\,a^n n^2+i\,a^n n-3i\,a^n\right)\cos\left((dx+c)(n+3)\right)+\left(-3i\,a^n n^3+3i\,a^n n^2+27i\,a^n n-27i\,a^n\right)\cos\left((dx+c)(n+1)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 1/8*((-I*a^n*n^3 + 3*I*a^n*n^2 + I*a^n*n - 3*I*a^n)*cos((d*x + c)*(n + 3))
+ (-3*I*a^n*n^3 + 3*I*a^n*n^2 + 27*I*a^n*n - 27*I*a^n)*cos((d*x + c)*(n + 1
)) + (-3*I*a^n*n^3 - 3*I*a^n*n^2 + 27*I*a^n*n + 27*I*a^n)*cos((d*x + c)*(n
- 1)) + (-I*a^n*n^3 - 3*I*a^n*n^2 + I*a^n*n + 3*I*a^n)*cos((d*x + c)*(n - 3
)) + (a^n*n^3 - 3*a^n*n^2 - a^n*n + 3*a^n)*sin((d*x + c)*(n + 3)) + 3*(a^n*
n^3 - a^n*n^2 - 9*a^n*n + 9*a^n)*sin((d*x + c)*(n + 1)) + 3*(a^n*n^3 + a^n*
n^2 - 9*a^n*n - 9*a^n)*sin((d*x + c)*(n - 1)) + (a^n*n^3 + 3*a^n*n^2 - a^n*
n - 3*a^n)*sin((d*x + c)*(n - 3)))/((e^(n + 3)*n^4 - 10*e^(n + 3)*n^2 + 9*e
^(n + 3))*d)
```

Fricas [A] time = 2.13417, size = 653, normalized size = 3.19

$$\frac{\left(-i\,n^{3}-3i\,n^{2}+\left(-i\,n^{3}+3i\,n^{2}+i\,n-3i\right)e^{(6i\,dx+6i\,c)}+\left(-3i\,n^{3}+3i\,n^{2}+27i\,n-27i\right)e^{(4i\,dx+4i\,c)}+\left(-3i\,n^{3}-3i\,n^{2}+27i\,n+2\right.}{dn^{4}-10\,dn^{2}+\left(dn^{4}-10\,dn^{2}+9\,d\right)e^{(6i\,dx+6i\,c)}+3\left(dn^{4}-10\,dn^{2}+9\,d\right)e^{(4i\,dx+4i\,c)}+3\left(dn^{4}-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (-I*n^3 - 3*I*n^2 + (-I*n^3 + 3*I*n^2 + I*n - 3*I)*e^(6*I*d*x + 6*I*c) + (-
3*I*n^3 + 3*I*n^2 + 27*I*n - 27*I)*e^(4*I*d*x + 4*I*c) + (-3*I*n^3 - 3*I*n^
2 + 27*I*n + 27*I)*e^(2*I*d*x + 2*I*c) + I*n + 3*I)*(2*a*e^(2*I*d*x + 2*I*c
)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) +
1))^(-n - 3)/(d*n^4 - 10*d*n^2 + (d*n^4 - 10*d*n^2 + 9*d)*e^(6*I*d*x + 6*I*
c) + 3*(d*n^4 - 10*d*n^2 + 9*d)*e^(4*I*d*x + 4*I*c) + 3*(d*n^4 - 10*d*n^2 +
 9*d)*e^(2*I*d*x + 2*I*c) + 9*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-3-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-n-3}\,(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

3.485 $\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=148

$$\frac{2i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-2}}{a^2dn\left(4-n^2\right)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}-\frac{2i(a+ia\tan(c+dx))^{n+1}(e\,\mathrm{s}}{ad(2-n)n}$$

```
[Out] (I*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(2 - n)) - ((2*I)
*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(2 - n)*n)
+ ((2*I)*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(a^2*d*n
*(4 - n^2))
```

Rubi [A] time = 0.197928, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3504, 3488}

$$\frac{2i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-2}}{a^2dn\left(4-n^2\right)}+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)}-\frac{2i(a+ia\tan(c+dx))^{n+1}(e\,\mathrm{s}}{ad(2-n)n}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(2 - n)) - ((2*I)
*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(2 - n)*n)
+ ((2*I)*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(a^2*d*n
*(4 - n^2))
```

Rule 3504

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx &= \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)}+\frac{2\int (e\sec(c+dx))^{-2-n}(a+}{a(2-}\\ &= \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)}-\frac{2i(e\sec(c+dx))^{-2-n}(a+i}{ad(2-n)}\\ &= \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)}-\frac{2i(e\sec(c+dx))^{-2-n}(a+i}{ad(2-n)}\end{aligned}$$

Mathematica [A] time = 0.219606, size = 82, normalized size = 0.55

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}\left(n^2\cos(2(c+dx))-2in\sin(2(c+dx))+n^2-4\right)}{2de^2(n-2)n(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I/2)*(-4 + n^2 + n^2*Cos[2*(c + d*x)] - (2*I)*n*Sin[2*(c + d*x)])*(a + I
*a*Tan[c + d*x])^n)/(d*e^2*(-2 + n)*n*(2 + n)*(e*Sec[c + d*x])^n)
```

Maple [C] time = 0.823, size = 3376, normalized size = 22.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] 1/4/(-2*I*d+I*n*d)*a^n*e^(-n)/e^2*exp(I*(d*x+c))^n*exp(1/2*I*(-4*c-Pi*csgn(
I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*n-4*d*x-2*Pi*csgn(I/(exp(2*I*(d*
x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-2*Pi*csgn(I*exp(I*(
d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))
^2-Pi*csgn(I*exp(2*I*(d*x+c)))^3*n+Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x
+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))
*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*cs
gn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+
c))+1)*exp(I*(d*x+c)))-2*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))^2-Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*
n-2*Pi*csgn(I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+n*Pi*csgn(
I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))^3+2*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3+2*P
i*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+2*Pi*csgn(I*e)*csgn(I*exp(I
*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)
))+2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d
*x+c))/(exp(2*I*(d*x+c))+1))+Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+
c)))^2*csgn(I*a)*n+2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n
-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c)
)+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2
*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))*n-n*Pi*csgn(
I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2-n*Pi*csgn(I/(exp(2*I*(
d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*exp(I
*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*exp(I*
(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c))
)^2-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+Pi*csgn(I*exp(2*
I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*n+Pi*csgn(I*exp
(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(
d*x+c)))^2*n))+1/4/(2*I*d+I*n*d)*a^n*e^(-n)/e^2*exp(I*(d*x+c))^n*exp(1/2*I*
(4*c-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*n+4*d*x-2*Pi*csgn(I
/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-2*Pi*c
sgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*ex
p(I*(d*x+c)))^2-Pi*csgn(I*exp(2*I*(d*x+c)))^3*n+Pi*csgn(I*exp(2*I*(d*x+c))/
(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))*n+n*Pi*csgn(I/(exp(2*I
*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c)
)+1))+n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(
```

```
exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))-2*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(
I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I
*(d*x+c)))^3*n-2*Pi*csgn(I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))
^2+n*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3+n*Pi*csgn(I*exp(I*(
d*x+c))/(exp(2*I*(d*x+c))+1))^3+2*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d
*x+c)))^3+2*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+2*Pi*csgn(I*e)
*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*
exp(I*(d*x+c)))+2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*cs
gn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*
exp(2*I*(d*x+c)))^2*csgn(I*a)*n+2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(
I*(d*x+c)))*n-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(ex
p(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a)*n-Pi*csgn(I*exp(2*I*(d*x+c)))
*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))
*n-n*Pi*csgn(I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2-n*Pi*csgn
(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi
*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*
csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*e
xp(I*(d*x+c)))^2-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+Pi*
csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*n+
Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))
+1)*exp(2*I*(d*x+c)))^2*n))-1/2*I*a^n/e^2/(e^n)/n/d*exp(-1/2*n*(-I*Pi*csgn(
I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-I*Pi*cs
gn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I/(exp(2*I*(d*x+c))+1))-
2*I*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))+I*Pi*csgn(I*exp(2*
I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x
+c)))*csgn(I*a)-I*Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3-I*Pi*c
sgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x
+c))+1)*exp(I*(d*x+c)))+I*Pi*csgn(I*exp(2*I*(d*x+c)))^3+I*Pi*csgn(I*a/(exp(
2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3+I*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(
I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+I*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I
*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-I*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*c
sgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-I*Pi*csgn
(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)-I*Pi*csgn(I*exp(I*(
d*x+c))/(exp(2*I*(d*x+c))+1))^3+I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*
I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))+I*Pi*csgn(I*e
)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+I*Pi*csgn(I*exp(2*I*(d*x+
c))/(exp(2*I*(d*x+c))+1))^3+I*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1)
)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2-I*Pi*csgn(I*exp(2*I*(d*x+
c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2
+I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2-2*ln(exp(I*(d*x+c))
)))
```

Maxima [A] time = 2.08959, size = 235, normalized size = 1.59

$$\frac{\left(-i a^n n^2 + 2 i a^n n\right)\cos\left((dx+c)(n+2)\right) + \left(-i a^n n^2 - 2 i a^n n\right)\cos\left((dx+c)(n-2)\right) + \left(-2 i a^n n^2 + 8 i a^n\right)\cos\left((dx+c)n\right)}{4\left(e^{n+2} n^3 - 4 e^{r}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 1/4*((-I*a^n*n^2 + 2*I*a^n*n)*cos((d*x + c)*(n + 2)) + (-I*a^n*n^2 - 2*I*a^
n*n)*cos((d*x + c)*(n - 2)) + (-2*I*a^n*n^2 + 8*I*a^n)*cos((d*x + c)*n) + (
a^n*n^2 - 2*a^n*n)*sin((d*x + c)*(n + 2)) + (a^n*n^2 + 2*a^n*n)*sin((d*x +
c)*(n - 2)) + 2*(a^n*n^2 - 4*a^n)*sin((d*x + c)*n))/((e^(n + 2)*n^3 - 4*e^(
n + 2)*n)*d)
```

Fricas [A] time = 2.39426, size = 406, normalized size = 2.74

$$\frac{\left(-i\,n^2+\left(-i\,n^2+2i\,n\right)e^{(4i\,dx+4i\,c)}+\left(-2i\,n^2+8i\right)e^{(2i\,dx+2i\,c)}-2i\,n\right)\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n-2}}{dn^3-4\,dn+\left(dn^3-4\,dn\right)e^{(4i\,dx+4i\,c)}+2\left(dn^3-4\,dn\right)e^{(2i\,dx+2i\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (-I*n^2 + (-I*n^2 + 2*I*n)*e^(4*I*d*x + 4*I*c) + (-2*I*n^2 + 8*I)*e^(2*I*d*
x + 2*I*c) - 2*I*n)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(
2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n - 2)/(d*n^3 - 4*d*n + (d
*n^3 - 4*d*n)*e^(4*I*d*x + 4*I*c) + 2*(d*n^3 - 4*d*n)*e^(2*I*d*x + 2*I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-2-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-n-2}\,(i\,a\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.486 $\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=94

$$\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}}{d(1-n)}-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-1}}{ad\left(1-n^2\right)}$$

```
[Out] (I*(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(1 - n)) - (I*(e*
Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 - n^2))
```

Rubi [A] time = 0.116357, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3504, 3488}

$$\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}}{d(1-n)}-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-1}}{ad\left(1-n^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(1 - n)) - (I*(e*
Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 - n^2))
```

Rule 3504

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx &= \frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n}{d(1-n)}+\frac{\int (e\sec(c+dx))^{-1-n}(a+ia}{a(1-n}\\ &= \frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n}{d(1-n)}-\frac{i(e\sec(c+dx))^{-1-n}(a+ia}{ad\left(1-n^2\right)}\end{aligned}$$

Mathematica [A] time = 0.188712, size = 58, normalized size = 0.62

$$-\frac{i(n-i\tan(c+dx))(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}}{d(n-1)(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*(e*Sec[c + d*x])^(-1 - n)*(n - I*Tan[c + d*x])*(a + I*a*Tan[c + d*x])
^n)/(d*(-1 + n)*(1 + n))
```

Maple [C] time = 0.87, size = 2488, normalized size = 26.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] 1/2/(I*d+I*n*d)*a^n*e^(-n)/e*exp(I*(d*x+c))^n*exp(-1/2*I*(-2*c+Pi*csgn(I*ex
p(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*n-2*d*x+Pi*csgn(I/(exp(2*I*(d*x+c))+
1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I*exp(I*(d*x+c))/
(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+Pi*cs
gn(I*exp(2*I*(d*x+c)))^3*n-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))
^2*csgn(I/(exp(2*I*(d*x+c))+1))*n-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*
exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I*e)*
csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*e
xp(I*(d*x+c)))+Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*
x+c))+1))^2+Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n+Pi*csgn(
I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2-n*Pi*csgn(I*e/(exp(2*I
*(d*x+c))+1)*exp(I*(d*x+c)))^3-n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))
+1))^3-Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3-Pi*csgn(I*exp(I*(
d*x+c))/(exp(2*I*(d*x+c))+1))^3-Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I
*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))-Pi*csgn(I/(exp(
2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x
+c))+1))-Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n-2
*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n+Pi*csgn(I*exp(2*I*(
d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)
))*csgn(I*a)*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I
*(d*x+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))*n+n*Pi*csgn(I*e)*csgn(I*e/(exp(2
*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*e
xp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*ex
p(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d
*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+Pi*csgn(I*exp(2*
I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*e
xp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*n-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2
*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n))+1/2/(
-I*d+I*n*d)*a^n*e^(-n)/e*exp(I*(d*x+c))^n*exp(-1/2*I*(2*c+Pi*csgn(I*exp(2*I
*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*n+2*d*x+Pi*csgn(I/(exp(2*I*(d*x+c))+1))*c
sgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+Pi*csgn(I*
exp(2*I*(d*x+c)))^3*n-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*cs
gn(I/(exp(2*I*(d*x+c))+1))*n-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I
*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I*e)*csgn(
I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*
(d*x+c)))+Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))
+1))^2+Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*n+Pi*csgn(I*e)*
csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2-n*Pi*csgn(I*e/(exp(2*I*(d*x
+c))+1)*exp(I*(d*x+c)))^3-n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^
3-Pi*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^3-Pi*csgn(I*exp(I*(d*x+c
))/(exp(2*I*(d*x+c))+1))^3-Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x
+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))-Pi*csgn(I/(exp(2*I*(
d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+
```

```
1))-Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*a)*n-2*Pi*c
sgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n+Pi*csgn(I*exp(2*I*(d*x+c
))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*cs
gn(I*a)*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x
+c))+1))*csgn(I/(exp(2*I*(d*x+c))+1))*n+n*Pi*csgn(I*e)*csgn(I*e/(exp(2*I*(d
*x+c))+1)*exp(I*(d*x+c)))^2+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*
(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(
d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c)
)+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+Pi*csgn(I*exp(2*I*(d*
x+c)))*csgn(I*exp(I*(d*x+c)))^2*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*
I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*n-Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d
*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*n))
```

Maxima [A] time = 1.98974, size = 153, normalized size = 1.63

$$\frac{(-i a^n n + i a^n)\cos((dx+c)(n+1)) + (-i a^n n - i a^n)\cos((dx+c)(n-1)) + (a^n n - a^n)\sin((dx+c)(n+1)) + (a^n n + a}{2\left(e^{n+1} n^2 - e^{n+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 1/2*((-I*a^n*n + I*a^n)*cos((d*x + c)*(n + 1)) + (-I*a^n*n - I*a^n)*cos((d*
x + c)*(n - 1)) + (a^n*n - a^n)*sin((d*x + c)*(n + 1)) + (a^n*n + a^n)*sin(
(d*x + c)*(n - 1)))/((e^(n + 1)*n^2 - e^(n + 1))*d)
```

Fricas [A] time = 2.41416, size = 271, normalized size = 2.88

$$\frac{\left((-i n + i)e^{(2i dx + 2i c)} - i n - i\right)\left(\frac{2 a e^{(2i dx+2i c)}}{e^{(2i dx+2i c)}+1}\right)^n \left(\frac{2 e e^{(i dx+i c)}}{e^{(2i dx+2i c)}+1}\right)^{-n-1}}{dn^2 + \left(dn^2 - d\right)e^{(2i dx+2i c)} - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] ((-I*n + I)*e^(2*I*d*x + 2*I*c) - I*n - I)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I
*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n -
 1)/(d*n^2 + (d*n^2 - d)*e^(2*I*d*x + 2*I*c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-1-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-n-1}(ia\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

3.487 $\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=37

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}}{dn}$$

```
[Out] ((-I)*(a + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n)
```

Rubi [A] time = 0.0477303, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {3488}

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^n,x]
```

```
[Out] ((-I)*(a + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n)
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx=-\frac{i(e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n}{dn}$$

Mathematica [A] time = 0.0365944, size = 37, normalized size = 1.

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^n,x]
```

```
[Out] ((-I)*(a + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n)
```

Maple [C] time = 0.387, size = 874, normalized size = 23.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x)
```

```
[Out] -I/n/d*exp(-1/2*n*(I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3+I
*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2-I*Pi*csgn(I*e/(exp(2*
I*(d*x+c))+1)*exp(I*(d*x+c)))^3+I*Pi*csgn(I*exp(2*I*(d*x+c)))^3+I*Pi*csgn(I
*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-I*Pi*csgn(I*
e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1
)*exp(I*(d*x+c)))-I*Pi*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn
(I/(exp(2*I*(d*x+c))+1))+I*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*
x+c))/(exp(2*I*(d*x+c))+1))^2-I*Pi*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d
*x+c)))^2*csgn(I*a)-I*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))
)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-2*I*Pi*csgn(I*exp(2*I*(d*x+c)
))^2*csgn(I*exp(I*(d*x+c)))-I*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1)
)^3+I*Pi*csgn(I*e)*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2+I*Pi*csg
n(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+I*Pi*csgn(I*exp(2*I*(d*x+c))/(
exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I
*a)+I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))
+1))*csgn(I/(exp(2*I*(d*x+c))+1))+I*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+
c))+1))*csgn(I*e/(exp(2*I*(d*x+c))+1)*exp(I*(d*x+c)))^2-I*Pi*csgn(I*exp(2*I
*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+
c)))^2-I*Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(2*I*(d*x+c))/(exp(2*I*(d*x+
c))+1))^2-2*ln(exp(I*(d*x+c)))+2*ln(e)-2*ln(a)))
```

Maxima [B] time = 1.78293, size = 116, normalized size = 3.14

$$-\frac{i\,a^{n}e^{\left(n\log\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1\right)-n\log\left(-\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1\right)\right)}}{de^{n}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="maxima")
```

```
[Out] -I*a^n*e^(n*log(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1) - n*log(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))/(d*
e^n*n)
```

Fricas [A] time = 2.19733, size = 154, normalized size = 4.16

$$-\frac{i\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}}{dn\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="fricas")
```

```
[Out] -I*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n/(d*n*(2*e*e^(I*d*x
 + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/((e*sec(d*x+c))**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \tan(dx + c) + a)^n}{(e \sec(dx + c))^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^n, x)
```

3.488 $\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=118

$$\frac{i2^{\frac{n+1}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-n-1)}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-n}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-i\tan(c+\right.}{d(1-n)}$$

```
[Out] (I*2^((1 + n)/2)*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - I*
Tan[c + d*x])/2]*(e*Sec[c + d*x])^(1 - n)*(1 + I*Tan[c + d*x])^((-1 - n)/2)
*(a + I*a*Tan[c + d*x])^n)/(d*(1 - n))
```

Rubi [A] time = 0.217809, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3505, 3523, 70, 69}

$$\frac{i2^{\frac{n+1}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-n-1)}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-n}\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-i\tan(c+\right.}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^((1 + n)/2)*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - I*
Tan[c + d*x])/2]*(e*Sec[c + d*x])^(1 - n)*(1 + I*Tan[c + d*x])^((-1 - n)/2)
*(a + I*a*Tan[c + d*x])^n)/(d*(1 - n))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{1-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-1+n)}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{1-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-1}}{d}$$

$$= \frac{\Big(2^{-\frac{1}{2}+\frac{n}{2}}a(e\sec(c+dx))^{1-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+n)}(a+ia\tan(c+dx}{}$$

$$= \frac{i2^{\frac{1+n}{2}}\,{}_2F_1\left(\frac{1-n}{2},\frac{1-n}{2};\frac{3-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{1-n}(1+i\tan}{d(1-n)}$$

Mathematica [A] time = 4.4395, size = 87, normalized size = 0.74

$$-\frac{e(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}(\text{Hypergeometric2F1}(1,n,n+1,-\sin(c+dx)+i\cos(c+dx))-\text{Hypergeo}}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] -((e*(Hypergeometric2F1[1, n, 1 + n, I*Cos[c + d*x] - Sin[c + d*x]] - Hyper
geometric2F1[1, n, 1 + n, (-I)*Cos[c + d*x] + Sin[c + d*x]])*(a + I*a*Tan[c
 + d*x])^n)/(d*n*(e*Sec[c + d*x])^n))
```

Maple [F] time = 0.971, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{1-n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*
x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-n+1}(i\,a\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

3.489 $\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=113

$$\frac{ia2^{\frac{n}{2}+1}(1+i\tan(c+dx))^{-n/2}(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{2-n}\text{Hypergeometric2F1}\left(\frac{2-n}{2},-\frac{n}{2},\frac{4-n}{2},\frac{1}{2}(1-i\tan\right.}{d(2-n)}$$

```
[Out] (I*2^(1 + n/2)*a*Hypergeometric2F1[(2 - n)/2, -n/2, (4 - n)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(2
 - n)*(1 + I*Tan[c + d*x])^(n/2))
```

Rubi [A] time = 0.183201, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3505, 3523, 70, 69}

$$\frac{ia2^{\frac{n}{2}+1}(1+i\tan(c+dx))^{-n/2}(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{2-n}\text{Hypergeometric2F1}\left(\frac{2-n}{2},-\frac{n}{2},\frac{4-n}{2},\frac{1}{2}(1-i\tan\right.}{d(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(1 + n/2)*a*Hypergeometric2F1[(2 - n)/2, -n/2, (4 - n)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(2
 - n)*(1 + I*Tan[c + d*x])^(n/2))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \left((e\sec(c+dx))^{2-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-2+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-2+n)}\right)\int$$

$$= \frac{\left(a^2(e\sec(c+dx))^{2-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-2+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-2+n)}\right.}{d}$$

$$= \frac{\left(2^{n/2}a^2(e\sec(c+dx))^{2-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-2+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-2}\right.}{}$$

$$= \frac{i2^{1+\frac{n}{2}}a\,{}_2F_1\left(\frac{2-n}{2},-\frac{n}{2};\frac{4-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2-n}(1+i\tan(}{d(2-n)}$$

Mathematica [A] time = 12.5509, size = 112, normalized size = 0.99

$$\frac{4e^2(\cos(2c)-i\sin(2c))(\tan(dx)+i)(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},-\cos(2(\right.}{d(n-2)(-1-i\tan(dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (4*e^2*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, -Cos[2*(c + d*x)] + I*Sin[2*(
c + d*x)]]*(Cos[2*c] - I*Sin[2*c])*(I + Tan[d*x])*(a + I*a*Tan[c + d*x])^n)
/(d*(-2 + n)*(e*Sec[c + d*x])^n*(-1 - I*Tan[d*x]))
```

Maple [F] time = 0.967, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{2-n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*
x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-n+2}(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n + 2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.490 $\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=121

$$\frac{ia2^{\frac{n+3}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-n-1)}(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{3-n}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-\right.}{d(3-n)}$$

```
[Out] (I*2^((3 + n)/2)*a*Hypergeometric2F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 -
 I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(3 - n)*(1 + I*Tan[c + d*x])^((-1 - n)
/2)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(3 - n))
```

Rubi [A] time = 0.235446, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3505, 3523, 70, 69}

$$\frac{ia2^{\frac{n+3}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-n-1)}(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{3-n}\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-1),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-\right.}{d(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```


```
[Out] (I*2^((3 + n)/2)*a*Hypergeometric2F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 -
 I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(3 - n)*(1 + I*Tan[c + d*x])^((-1 - n)
/2)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(3 - n))
```


Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx = \left((e\sec(c+dx))^{3-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-3+n)}\right.$$

$$= \frac{\left(a^2(e\sec(c+dx))^{3-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-3\right.}{d}$$

$$= \frac{\left(2^{\frac{1}{2}+\frac{n}{2}}a^2(e\sec(c+dx))^{3-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+n)}(a+ia\tan(c+dx\right.}{}$$

$$= \frac{i2^{\frac{3+n}{2}}a\,{}_2F_1\left(\frac{1}{2}(-1-n),\frac{3-n}{2};\frac{5-n}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3-n}(}{d(3-n)}$$

Mathematica [A] time = 11.0644, size = 116, normalized size = 0.96

$$\frac{8e^3(\tan(dx)+i)\sec(dx)(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}\text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},-\cos(2(c+dx))+i\right.}{d(n-3)(\cos(c)+i\sin(c))^3(\tan(dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (8*e^3*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, -Cos[2*(c + d*x)] + I*Sin
[2*(c + d*x)]]*Sec[d*x]*(I + Tan[d*x])*(a + I*a*Tan[c + d*x])^n)/(d*(-3 + n
)*(e*Sec[c + d*x])^n*(Cos[c] + I*Sin[c])^3*(-I + Tan[d*x])^2)
```

Maple [F] time = 0.886, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{3-n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*
x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-n+3}\,(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

3.491 $\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=156

$$\frac{8ia^3(a+ia\tan(c+dx))^{n-3}(e\sec(c+dx))^{6-2n}}{d(5-n)\left(n^2-7n+12\right)}+\frac{4ia^2(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{6-2n}}{d\left(n^2-9n+20\right)}+\frac{ia(a+ia\tan(c+dx)}{d(5}$$

```
[Out] ((8*I)*a^3*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^(-3 + n))/(d*(
5 - n)*(12 - 7*n + n^2)) + ((4*I)*a^2*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*T
an[c + d*x])^(-2 + n))/(d*(20 - 9*n + n^2)) + (I*a*(e*Sec[c + d*x])^(6 - 2*
n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(5 - n))
```

Rubi [A] time = 0.225592, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3494, 3493}

$$\frac{8ia^3(a+ia\tan(c+dx))^{n-3}(e\sec(c+dx))^{6-2n}}{d(5-n)\left(n^2-7n+12\right)}+\frac{4ia^2(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{6-2n}}{d\left(n^2-9n+20\right)}+\frac{ia(a+ia\tan(c+dx)}{d(5}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((8*I)*a^3*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^(-3 + n))/(d*(
5 - n)*(12 - 7*n + n^2)) + ((4*I)*a^2*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*T
an[c + d*x])^(-2 + n))/(d*(20 - 9*n + n^2)) + (I*a*(e*Sec[c + d*x])^(6 - 2*
n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(5 - n))
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx &= \frac{ia(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-1+n}}{d(5-n)}+\frac{(4a)\int (e\sec(c+dx))^{6-}}{}\\ &= \frac{4ia^2(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-2+n}}{d\left(20-9n+n^2\right)}+\frac{ia(e\sec(c+dx))^{6-2n}}{d(}\\ &= \frac{8ia^3(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-3+n}}{d(3-n)\left(20-9n+n^2\right)}+\frac{4ia^2(e\sec(c+dx))^{6-}}{d\left(20\right.}\end{aligned}$$

Mathematica [A] time = 2.12678, size = 122, normalized size = 0.78

$$-\frac{e^6 \sec^5(c+dx)(\sin(3(c+dx))+i\cos(3(c+dx)))(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\left(i\left(n^2-9n+18\right)\sin(2(c+dx))\right)}{d(n-5)(n-4)(n-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] -((e^6*Sec[c + d*x]^5*(-2*(-5 + n) + (22 - 9*n + n^2)*Cos[2*(c + d*x)] + I*
(18 - 9*n + n^2)*Sin[2*(c + d*x)])*(I*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*
(a + I*a*Tan[c + d*x])^n)/(d*(-5 + n)*(-4 + n)*(-3 + n)*(e*Sec[c + d*x])^(2
*n)))
```

Maple [F] time = 0.758, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{6-2n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [B] time = 12.563, size = 1434, normalized size = 9.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] -(64*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
2*n)*a^n*e^6*cos(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 64*I*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*
a^n*e^6*sin(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 32*(a^n*e^
6*n^2 - 9*a^n*e^6*n + 20*a^n*e^6)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*cos(4*d*x + n*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1) + 4*c) - 64*(a^n*e^6*n - 5*a^n*e^6)*(cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*cos(2*d*x + n*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*c) + (32*I*a^n*e^6*n^2 - 28
8*I*a^n*e^6*n + 640*I*a^n*e^6)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(4*d*x + n*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1) + 4*c) + (-64*I*a^n*e^6*n + 320*I*a^n*e^6)*(cos(2*d*x + 2
*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(2*d*x + n*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*c))/(((-I*e^(2*n)*n^3 +
 12*I*e^(2*n)*n^2 - 47*I*e^(2*n)*n + 60*I*e^(2*n))*2^n*cos(10*d*x + 10*c) +
 (-5*I*e^(2*n)*n^3 + 60*I*e^(2*n)*n^2 - 235*I*e^(2*n)*n + 300*I*e^(2*n))*2^
n*cos(8*d*x + 8*c) + (-10*I*e^(2*n)*n^3 + 120*I*e^(2*n)*n^2 - 470*I*e^(2*n)
*n + 600*I*e^(2*n))*2^n*cos(6*d*x + 6*c) + (-10*I*e^(2*n)*n^3 + 120*I*e^(2*
n)*n^2 - 470*I*e^(2*n)*n + 600*I*e^(2*n))*2^n*cos(4*d*x + 4*c) + (-5*I*e^(2
*n)*n^3 + 60*I*e^(2*n)*n^2 - 235*I*e^(2*n)*n + 300*I*e^(2*n))*2^n*cos(2*d*x
```

```
 + 2*c) + (e^(2*n)*n^3 - 12*e^(2*n)*n^2 + 47*e^(2*n)*n - 60*e^(2*n))*2^n*si
n(10*d*x + 10*c) + 5*(e^(2*n)*n^3 - 12*e^(2*n)*n^2 + 47*e^(2*n)*n - 60*e^(2
*n))*2^n*sin(8*d*x + 8*c) + 10*(e^(2*n)*n^3 - 12*e^(2*n)*n^2 + 47*e^(2*n)*n
 - 60*e^(2*n))*2^n*sin(6*d*x + 6*c) + 10*(e^(2*n)*n^3 - 12*e^(2*n)*n^2 + 47
*e^(2*n)*n - 60*e^(2*n))*2^n*sin(4*d*x + 4*c) + 5*(e^(2*n)*n^3 - 12*e^(2*n)
*n^2 + 47*e^(2*n)*n - 60*e^(2*n))*2^n*sin(2*d*x + 2*c) + (-I*e^(2*n)*n^3 +
12*I*e^(2*n)*n^2 - 47*I*e^(2*n)*n + 60*I*e^(2*n))*2^n)*d)
```

Fricas [A] time = 2.07946, size = 419, normalized size = 2.69

$$\frac{\left(\left(-i\,n^2+9i\,n-20i\right)e^{(6i\,dx+6i\,c)}+\left(-i\,n^2+11i\,n-30i\right)e^{(4i\,dx+4i\,c)}+(2i\,n-12i)e^{(2i\,dx+2i\,c)}-2i\right)\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right.}{2\left(dn^3-12\,dn^2+47\,dn-60\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] 1/2*((-I*n^2 + 9*I*n - 20*I)*e^(6*I*d*x + 6*I*c) + (-I*n^2 + 11*I*n - 30*I)
*e^(4*I*d*x + 4*I*c) + (2*I*n - 12*I)*e^(2*I*d*x + 2*I*c) - 2*I)*(2*a*e^(2*
I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*
x + 2*I*c) + 1))^(-2*n + 6)*e^(-6*I*d*x - 6*I*c)/(d*n^3 - 12*d*n^2 + 47*d*n
 - 60*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(6-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2n+6}(i\,a\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 6)*(I*a*tan(d*x + c) + a)^n, x)
```

3.492 $\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=97

$$-\frac{i2^{\frac{5}{2}-n}(1-i\tan(c+dx))^{n-\frac{5}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{5-2n}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}(2n-3),\frac{7}{2},\frac{1}{2}(1+i\tan(c\right.}{5d}$$

```
[Out] ((-I/5)*2^(5/2 - n)*Hypergeometric2F1[5/2, (-3 + 2*n)/2, 7/2, (1 + I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^(5 - 2*n)*(1 - I*Tan[c + d*x])^(-5/2 + n)*(a +
I*a*Tan[c + d*x])^n)/d
```

Rubi [A] time = 0.208412, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3505, 3523, 7, 70, 69}

$$-\frac{i2^{\frac{5}{2}-n}(1-i\tan(c+dx))^{n-\frac{5}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{5-2n}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}(2n-3),\frac{7}{2},\frac{1}{2}(1+i\tan(c\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I/5)*2^(5/2 - n)*Hypergeometric2F1[5/2, (-3 + 2*n)/2, 7/2, (1 + I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^(5 - 2*n)*(1 - I*Tan[c + d*x])^(-5/2 + n)*(a +
I*a*Tan[c + d*x])^n)/d
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \left((e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-5}\right.$$

$$= \frac{\left(a^2(e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}}\right.}{}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}}\right.}{d}$$

$$= \frac{\left(2^{\frac{3}{2}-n}a^3(e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}-n+\frac{1}{2}(-5+2n)}\left(\frac{a-ia\tan(c+d}{a}\right.\right.}{}$$

$$= -\frac{i2^{\frac{5}{2}-n}\,{}_2F_1\left(\frac{5}{2},\frac{1}{2}(-3+2n);\frac{7}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{5-2n}(}{5d}$$

Mathematica [A] time = 13.2251, size = 166, normalized size = 1.71

$$-\frac{i2^{5-n}e^{5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\sec^{n-5}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(\frac{5}{2},5-\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I/5)*2^(5 - n)*E^((5*I)*(c + d*x))*(E^(I*d*x))^n*Hypergeometric2F1[5/2,
5 - n, 7/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-5 + n)*(e*Sec[c + d*x])^(5
 - 2*n)*(a + I*a*Tan[c + d*x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*
x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.781, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{5-2n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2n+5}(ia\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 5)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*
x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(5-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2\,n+5}\,(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 5)*(I*a*tan(d*x + c) + a)^n, x)
```

3.493 $\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=98

$$\frac{2ia^2(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{4-2n}}{d\left(n^2-5n+6\right)}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{4-2n}}{d(3-n)}$$

```
[Out] ((2*I)*a^2*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^(-2 + n))/(d*(
6 - 5*n + n^2)) + (I*a*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^(-
1 + n))/(d*(3 - n))
```

Rubi [A] time = 0.131831, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3494, 3493}

$$\frac{2ia^2(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{4-2n}}{d\left(n^2-5n+6\right)}+\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{4-2n}}{d(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((2*I)*a^2*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^(-2 + n))/(d*(
6 - 5*n + n^2)) + (I*a*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^(-
1 + n))/(d*(3 - n))
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx=\frac{ia(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-1+n}}{d(3-n)}+\frac{(2a)\int (e\sec(c+dx))^{4-}}{}$$

$$=\frac{2ia^2(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-2+n}}{d\left(6-5n+n^2\right)}+\frac{ia(e\sec(c+dx))^{4-2n}}{d(}$$

Mathematica [A] time = 1.1796, size = 91, normalized size = 0.93

$$\frac{e^4\sec^2(c+dx)((n-2)\tan(c+dx)-i(n-4))(\cos(2(c+dx))-i\sin(2(c+dx)))(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}}{d(n-3)(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (e^4*Sec[c + d*x]^2*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(a + I*a*Tan[c
+ d*x])^n*((-I)*(-4 + n) + (-2 + n)*Tan[c + d*x]))/(d*(-3 + n)*(-2 + n)*(e*
Sec[c + d*x])^(2*n))
```

Maple [F] time = 0.746, size = 0, normalized size = 0.

$$\int (e\sec (dx+c))^{4-2n} (a+ia\tan (dx+c))^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [B] time = 3.26385, size = 802, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] (8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*
n)*a^n*e^4*cos(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 8*I*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*a^n
*e^4*sin(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 8*(a^n*e^4*n
- 3*a^n*e^4)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/2*n)*cos(2*d*x + n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
+ 2*c) - (8*I*a^n*e^4*n - 24*I*a^n*e^4)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(2*d*x + n*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1) + 2*c))/(((-I*e^(2*n)*n^2 + 5*I*e^(2*n)*n - 6*I*
e^(2*n))*2^n*cos(6*d*x + 6*c) + (-3*I*e^(2*n)*n^2 + 15*I*e^(2*n)*n - 18*I*e
^(2*n))*2^n*cos(4*d*x + 4*c) + (-3*I*e^(2*n)*n^2 + 15*I*e^(2*n)*n - 18*I*e^
(2*n))*2^n*cos(2*d*x + 2*c) + (e^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*s
in(6*d*x + 6*c) + 3*(e^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*sin(4*d*x +
 4*c) + 3*(e^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*sin(2*d*x + 2*c) + (-
I*e^(2*n)*n^2 + 5*I*e^(2*n)*n - 6*I*e^(2*n))*2^n)*d)
```

Fricas [A] time = 1.96662, size = 317, normalized size = 3.23

$$\frac{\left((-i\,n+3i)e^{(4i\,dx+4i\,c)}+(-i\,n+4i)e^{(2i\,dx+2i\,c)}+i\right)\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+4}e^{(-4i\,dx-4i\,c)}}{2\left(dn^{2}-5\,dn+6\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] 1/2*((-I*n + 3*I)*e^(4*I*d*x + 4*I*c) + (-I*n + 4*I)*e^(2*I*d*x + 2*I*c) +
I)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*x + I*
c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 4)*e^(-4*I*d*x - 4*I*c)/(d*n^2 - 5*d*
n + 6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(4-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e \sec (dx + c))^{-2n+4} (i a \tan (dx + c) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 4)*(I*a*tan(d*x + c) + a)^n, x)
```

3.494 $\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=97

$$-\frac{i2^{\frac{3}{2}-n}(1-i\tan(c+dx))^{n-\frac{3}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-2n}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(2n-1),\frac{5}{2},\frac{1}{2}(1+i\tan(c\right.}{3d}$$

```
[Out] ((-I/3)*2^(3/2 - n)*Hypergeometric2F1[3/2, (-1 + 2*n)/2, 5/2, (1 + I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^(3 - 2*n)*(1 - I*Tan[c + d*x])^(-3/2 + n)*(a +
I*a*Tan[c + d*x])^n)/d
```

Rubi [A] time = 0.208014, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3505, 3523, 7, 70, 69}

$$-\frac{i2^{\frac{3}{2}-n}(1-i\tan(c+dx))^{n-\frac{3}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-2n}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(2n-1),\frac{5}{2},\frac{1}{2}(1+i\tan(c\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```


```
[Out] ((-I/3)*2^(3/2 - n)*Hypergeometric2F1[3/2, (-1 + 2*n)/2, 5/2, (1 + I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^(3 - 2*n)*(1 - I*Tan[c + d*x])^(-3/2 + n)*(a +
I*a*Tan[c + d*x])^n)/d
```


Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-3$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}}}{}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}}}{d}$$

$$= \frac{\Big(2^{\frac{1}{2}-n}a^2(e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}-n+\frac{1}{2}(-3+2n)}\Big(\frac{a-ia\tan(c+d}{a}}{}$$

$$= -\frac{i2^{\frac{3}{2}-n}\,{}_2F_1\left(\frac{3}{2},\frac{1}{2}(-1+2n);\frac{5}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{3-2n}(}{3d}$$

Mathematica [A] time = 11.8003, size = 166, normalized size = 1.71

$$-\frac{i2^{3-n}e^{3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\sec^{n-3}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(\frac{3}{2},3-\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I/3)*2^(3 - n)*E^((3*I)*(c + d*x))*(E^(I*d*x))^n*Hypergeometric2F1[3/2,
3 - n, 5/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-3 + n)*(e*Sec[c + d*x])^(3
 - 2*n)*(a + I*a*Tan[c + d*x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*
x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.782, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{3-2n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2n+3}(i\,a\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*
x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(3-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2\,n+3}\,(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

3.495 $\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=46

$$\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{2-2n}}{d(1-n)}$$

```
[Out] (I*a*(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(1 - n)
)
```

Rubi [A] time = 0.0563509, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3493}

$$\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{2-2n}}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*a*(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(1 - n)
)
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx=\frac{ia(e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^{-1+n}}{d(1-n)}$$

Mathematica [A] time = 0.603248, size = 59, normalized size = 1.28

$$-\frac{e^2(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}(\sec(c)\sin(dx)\sec(c+dx)+\tan(c)+i)}{d(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] -((e^2*(I + Sec[c]*Sec[c + d*x]*Sin[d*x] + Tan[c])*(a + I*a*Tan[c + d*x])^n
)/(d*(-1 + n)*(e*Sec[c + d*x])^(2*n)))
```

Maple [F] time = 0.782, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{2-2n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [B] time = 2.26599, size = 293, normalized size = 6.37

$$\frac{\left(-i\,a^{n}e^{2}-\frac{2\,a^{n}e^{2}\sin(dx+c)}{\cos(dx+c)+1}+\frac{i\,a^{n}e^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)e^{\left(n\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)+n\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)+n\log\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1\right)-2\,n\log\left(-\frac{\sin(dx}{(\cos(dx+}\right.\right.}}{\left(e^{2\,n}(n-1)-\frac{e^{2\,n}(n-1)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] (-I*a^n*e^2 - 2*a^n*e^2*sin(d*x + c)/(cos(d*x + c) + 1) + I*a^n*e^2*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2)*e^(n*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
 + n*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1) + n*log(-2*I*sin(d*x + c)/(co
s(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1) - 2*n*log(-sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 1))/((e^(2*n)*(n - 1) - e^(2*n)*(n - 1)*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2)*d)
```

Fricas [B] time = 1.88639, size = 240, normalized size = 5.22

$$\frac{\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+2}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)e^{(-2i\,dx-2i\,c)}}{2\,(dn-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*x + I
*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 2)*(-I*e^(2*I*d*x + 2*I*c) - I)*e^(-
2*I*d*x - 2*I*c)/(d*n - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(2-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e \sec (dx+c))^{-2n+2} (ia \tan (dx+c)+a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")

[Out] integrate((e*sec(d*x + c))^(-2*n + 2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.496 $\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=95

$$-\frac{i2^{\frac{1}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-2n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2n+1),\frac{3}{2},\frac{1}{2}(1+i\tan(c\right.}{d}$$

```
[Out] ((-I)*2^(1/2 - n)*Hypergeometric2F1[1/2, (1 + 2*n)/2, 3/2, (1 + I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(1 - 2*n)*(1 - I*Tan[c + d*x])^(-1/2 + n)*(a + I*a
*Tan[c + d*x])^n)/d
```

Rubi [A] time = 0.184963, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3505, 3523, 7, 70, 69}

$$-\frac{i2^{\frac{1}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-2n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2n+1),\frac{3}{2},\frac{1}{2}(1+i\tan(c\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(1/2 - n)*Hypergeometric2F1[1/2, (1 + 2*n)/2, 3/2, (1 + I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(1 - 2*n)*(1 - I*Tan[c + d*x])^(-1/2 + n)*(a + I*a
*Tan[c + d*x])^n)/d
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-1}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}}}{}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}}}{d}$$

$$= \frac{\Big(2^{-\frac{1}{2}-n}a(e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}-n+\frac{1}{2}(-1+2n)}\Big(\frac{a-ia\tan(c+a}{a}}{}$$

$$= -\frac{i2^{\frac{1}{2}-n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+2n);\frac{3}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{1-2n}(1}{d}$$

Mathematica [A] time = 8.10103, size = 154, normalized size = 1.62

$$-\frac{ie2^{1-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{1-n}\left(1+e^{2i(c+dx)}\right)^{1-n}\sec^n(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(1 - n)*e*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^(1 - n)*(1 + E^((2*I)*(c + d*x)))^(1 - n)*Hypergeometric2F1[1/2, 1 - n, 3/
2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c + d*x])^n)/(d*(e*Sec
[c + d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.741, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{1-2n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2n+1}(ia\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*
x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(1-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2\,n+1}\,(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

3.497 $\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=65

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{1}{2}(1-i\tan(c+dx))\right)}{2dn}$$

```
[Out] ((-I/2)*Hypergeometric2F1[1, -n, 1 - n, (1 - I*Tan[c + d*x])/2]*(a + I*a*Ta
n[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^(2*n))
```

Rubi [A] time = 0.0781432, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3492, 3481, 68}

$$-\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{1}{2}(1-i\tan(c+dx))\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(2*n),x]
```

```
[Out] ((-I/2)*Hypergeometric2F1[1, -n, 1 - n, (1 - I*Tan[c + d*x])/2]*(a + I*a*Ta
n[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^(2*n))
```

Rule 3492

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Dist[((a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Fra
cPart[n]*(a - b*Tan[e + f*x])^FracPart[n])/(d*Sec[e + f*x])^(2*FracPart[n])
, Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Rule 3481

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \left((e\sec(c+dx))^{-2n}(a-ia\tan(c+dx))^n(a+ia\tan(c+dx))^n\right)\int (a-ia$$

$$= \frac{\left(ia(e\sec(c+dx))^{-2n}(a-ia\tan(c+dx))^n(a+ia\tan(c+dx))^n\right)\text{Subst}\big(}{d}$$

$$= -\frac{i\,{}_2F_1\left(1,-n;1-n;\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2n}(a+ia\tan(c}{2dn}$$

Mathematica [B] time = 1.65388, size = 146, normalized size = 2.25

$$\frac{i2^{-n-1}\left(1+e^{2i(c+dx)}\right)\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\sec^n(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(1,n+1,n+2,1+e^{2}\right.}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(2*n),x]
```

```
[Out] (I*2^(-1 - n)*(E^(I*d*x))^n*(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1,
1 + n, 2 + n, 1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c + d*x]
)^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(1 + n)*(e*Sec[c + d*
x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.487, size = 0, normalized size = 0.

$$\int \frac{(a+ia\tan(dx+c))^n}{(e\sec(dx+c))^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^n}{(e\sec(dx+c))^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="maxima"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(2*n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n}{\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{2\,n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="fricas"
)
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n/(2*e*e^(I*d*
x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(2*n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**n/((e*sec(d*x+c))**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia\tan(dx+c)+a)^n}{(e\sec(dx+c))^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(2*n), x)
```

3.498 $\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=95

$$\frac{i2^{-n-\frac{1}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(2n+3),\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{d}$$

```
[Out] (I*2^(-1/2 - n)*Hypergeometric2F1[-1/2, (3 + 2*n)/2, 1/2, (1 + I*Tan[c + d*
x])/2]*(e*Sec[c + d*x])^(-1 - 2*n)*(1 - I*Tan[c + d*x])^(1/2 + n)*(a + I*a*
Tan[c + d*x])^n)/d
```

Rubi [A] time = 0.19187, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3505, 3523, 7, 70, 69}

$$\frac{i2^{-n-\frac{1}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(2n+3),\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(-1/2 - n)*Hypergeometric2F1[-1/2, (3 + 2*n)/2, 1/2, (1 + I*Tan[c + d*
x])/2]*(e*Sec[c + d*x])^(-1 - 2*n)*(1 - I*Tan[c + d*x])^(1/2 + n)*(a + I*a*
Tan[c + d*x])^n)/d
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \left((e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(1+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(1\cdots}\right.$$

$$= \frac{\left(a^2(e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(1+2n)}(a+ia\tan(c+dx))\cdots\right.}{\cdots}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(1+2n)}(a+ia\tan(c+dx))\cdots\right.}{\cdots d \cdots}$$

$$= \frac{\left(2^{-\frac{3}{2}-n}a(e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{-\frac{1}{2}-n+\frac{1}{2}(1+2n)}\left(\frac{a-ia\tan(\cdots}{a}\right.\right.}{\cdots}$$

$$= \frac{i2^{-\frac{1}{2}-n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(3+2n);\frac{1}{2};\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{-1-2\cdots}}{d}$$

Mathematica [A] time = 12.8261, size = 157, normalized size = 1.65

$$\frac{i2^{-n-1}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n-1}\left(1+e^{2i(c+dx)}\right)^{-n-1}\sec^{n+1}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},-n-\cdots\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(-1 - n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1
 - n)*(1 + E^((2*I)*(c + d*x)))^(-1 - n)*Hypergeometric2F1[-1/2, -1 - n, 1/
2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1 + n)*(e*Sec[c + d*x])^(-1 - 2*n)*(
a + I*a*Tan[c + d*x])^n)/(d*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 1.25, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-1-2n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2n-1}(ia\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*
x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-1-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2\,n-1}\,(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

3.499 $\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=74

$$-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\text{Hypergeometric2F1}\left(2,-n-1,-n,\frac{1}{2}(1-i\tan(c+dx))\right)}{4ad(n+1)}$$

```
[Out] ((-I/4)*Hypergeometric2F1[2, -1 - n, -n, (1 - I*Tan[c + d*x])/2]*(a + I*a*T
an[c + d*x])^(1 + n))/(a*d*(1 + n)*(e*Sec[c + d*x])^(2*(1 + n)))
```

Rubi [A] time = 0.148941, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3505, 3523, 7, 68}

$$-\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\text{Hypergeometric2F1}\left(2,-n-1,-n,\frac{1}{2}(1-i\tan(c+dx))\right)}{4ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I/4)*Hypergeometric2F1[2, -1 - n, -n, (1 - I*Tan[c + d*x])/2]*(a + I*a*T
an[c + d*x])^(1 + n))/(a*d*(1 + n)*(e*Sec[c + d*x])^(2*(1 + n)))
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{-2-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(2+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(2+2n}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{-2-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(2+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(2-}}{}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{-2-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(2+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(2-}}{d}$$

$$= -\frac{i\,{}_2F_1\left(2,-1-n;-n;\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2(1+n)}(a+ia\,\mathrm{ta}}{4ad(1+n)}$$

Mathematica [B] time = 13.0512, size = 151, normalized size = 2.04

$$-\frac{i2^{-n-3}\left(1+e^{2i(c+dx)}\right)^3\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\sec^n(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(2,n+3,n+4,1+\right.}{de^2(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-3 - n)*(E^(I*d*x))^n*(1 + E^((2*I)*(c + d*x)))^3*Hypergeometric2F
1[2, 3 + n, 4 + n, 1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c +
 d*x])^n)/(d*e^2*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(3 + n)*(e*S
ec[c + d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 1.239, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2n-2}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-2*n-2)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(-2*n-2)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2n-2}(ia\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*
x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-2-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2\,n-2}(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

3.500 $\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=97

$$\frac{i2^{-n-\frac{3}{2}}(1-i\tan(c+dx))^{n+\frac{3}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-3}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(2n+5),-\frac{1}{2},\frac{1}{2}(1+i\,\text{ta}\right.}{3d}$$

```
[Out] ((I/3)*2^(-3/2 - n)*Hypergeometric2F1[-3/2, (5 + 2*n)/2, -1/2, (1 + I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^(-3 - 2*n)*(1 - I*Tan[c + d*x])^(3/2 + n)*(a +
 I*a*Tan[c + d*x])^n)/d
```

Rubi [A] time = 0.214257, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3505, 3523, 7, 70, 69}

$$\frac{i2^{-n-\frac{3}{2}}(1-i\tan(c+dx))^{n+\frac{3}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-3}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(2n+5),-\frac{1}{2},\frac{1}{2}(1+i\,\text{ta}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```


```
[Out] ((I/3)*2^(-3/2 - n)*Hypergeometric2F1[-3/2, (5 + 2*n)/2, -1/2, (1 + I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^(-3 - 2*n)*(1 - I*Tan[c + d*x])^(3/2 + n)*(a +
 I*a*Tan[c + d*x])^n)/d
```


Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \Big((e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(3+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(3}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(3+2n)}(a+ia\tan(c+dx))}{}$$

$$= \frac{\Big(a^2(e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(3+2n)}(a+ia\tan(c+dx))}{d}$$

$$= \frac{\Big(2^{-\frac{5}{2}-n}(e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{-\frac{1}{2}-n+\frac{1}{2}(3+2n)}\Big(\frac{a-ia\tan(c+}{a}}{}$$

$$= \frac{i2^{-\frac{3}{2}-n}\,{}_2F_1\Big(-\frac{3}{2},\frac{1}{2}(5+2n);-\frac{1}{2};\frac{1}{2}(1+i\tan(c+dx))\Big)(e\sec(c+dx))^{-3-}}{3d}$$

Mathematica [A] time = 13.3671, size = 166, normalized size = 1.71

$$\frac{i2^{-n-3}e^{-3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\sec^{n+3}(c+dx)(\cos(dx)+i\sin(dx))^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},\cdot\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/3)*2^(-3 - n)*(E^(I*d*x))^n*Hypergeometric2F1[-3/2, -3 - n, -1/2, -E^((
2*I)*(c + d*x))]*Sec[c + d*x]^(3 + n)*(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*
Tan[c + d*x])^n)/(d*E^((3*I)*(c + d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 1.461, size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-3-2n}(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2n-3}(ia\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n}\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(2*e*e^(I*d*
x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))**(-3-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sec(dx+c))^{-2\,n-3}\,(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

3.501 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx$

Optimal. Leaf size=66

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\text{Hypergeometric2F1}\left(3,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{8a^2fn}$$

```
[Out] ((I/8)*Hypergeometric2F1[3, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(a^2*f*n*(a + I*a*Tan[e + f*x])^n)
```

Rubi [A] time = 0.189005, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3505, 3522, 3487, 68}

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\text{Hypergeometric2F1}\left(3,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{8a^2fn}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-2 - n),x]
```

```
[Out] ((I/8)*Hypergeometric2F1[3, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(a^2*f*n*(a + I*a*Tan[e + f*x])^n)
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3522

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \left((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\int \frac{(a-i}{(a+i}$$

$$= \frac{\left((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\int \cos^4(e}{a^4}$$

$$= \frac{\left(ia(d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\text{Subst}}{f}$$

$$= \frac{i\,{}_2F_1\left(3,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+f}{8a^2fn}$$

Mathematica [B] time = 118.315, size = 165, normalized size = 2.5

$$-\frac{ie^{2ie}2^{n-3}\left(1+e^{2i(e+fx)}\right)^3\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n\sec^{2-n}(e+fx)(\cos(fx)+i\sin(fx))^{n+2}\text{Hypergeometric2F1}\left(3,3-n,4-\right.}{f(n-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-2 - n),x]
```

```
[Out] ((-I)*2^(-3 + n)*E^((2*I)*e)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*
(1 + E^((2*I)*(e + f*x)))^3*Hypergeometric2F1[3, 3 - n, 4 - n, 1 + E^((2*I)
*(e + f*x))]*Sec[e + f*x]^(2 - n)*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[
f*x])^(2 + n)*(a + I*a*Tan[e + f*x])^(-2 - n))/((E^(I*f*x))^n*f*(-3 + n))
```

Maple [F] time = 1.473, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{2n}\left(a+ia\tan\left(fx+e\right)\right)^{-2-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{\left(2i\,fx+2i\,e\right)}}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{-n-2}\left(\frac{2\,de^{\left(i\,fx+i\,e\right)}}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{2\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="fric
as")
```

```
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(-n - 2)*(2*d*
e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(-2-n),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{2\,n}\left(i\,a\tan\left(fx+e\right)+a\right)^{-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="giac
")
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n - 2), x)
```

3.502 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$

Optimal. Leaf size=66

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\text{Hypergeometric2F1}\left(2,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{4afn}$$

```
[Out] ((I/4)*Hypergeometric2F1[2, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(a*f*n*(a + I*a*Tan[e + f*x])^n)
```

Rubi [A] time = 0.182586, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3505, 3522, 3487, 68}

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\text{Hypergeometric2F1}\left(2,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{4afn}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-1 - n),x]
```

```
[Out] ((I/4)*Hypergeometric2F1[2, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(a*f*n*(a + I*a*Tan[e + f*x])^n)
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3522

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rule 3487

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \left((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\int \frac{(a}{a}$$

$$= \frac{\left((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\int \cos}{a^2}$$

$$= \frac{\left(ia(d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\text{Su}}{f}$$

$$= \frac{i\,{}_2F_1\left(2,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e}{4afn}$$

Mathematica [B] time = 13.9642, size = 165, normalized size = 2.5

$$\frac{ie^{ie}2^{n-2}\left(1+e^{2i(e+fx)}\right)^2\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n\sec^{1-n}(e+fx)(\cos(fx)+i\sin(fx))^{n+1}\text{Hypergeometric2F1}\left(2,2-n,3-\right.}{f(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-1 - n),x]
```

```
[Out] (I*2^(-2 + n)*E^(I*e)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + E^
((2*I)*(e + f*x)))^2*Hypergeometric2F1[2, 2 - n, 3 - n, 1 + E^((2*I)*(e + f
*x))]*Sec[e + f*x]^(1 - n)*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[f*x])^(
1 + n)*(a + I*a*Tan[e + f*x])^(-1 - n))/((E^(I*f*x))^n*f*(-2 + n))
```

Maple [F] time = 1.344, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{2n}\left(a+ia\tan\left(fx+e\right)\right)^{-1-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{2\,ae^{\left(2i\,fx+2i\,e\right)}}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{-n-1}\left(\frac{2\,de^{\left(i\,fx+i\,e\right)}}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{2\,n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="fric
as")
```

```
[Out] integral((2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(-n - 1)*(2*d*
e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(-1-n),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{2\,n}\left(i\,a\tan\left(fx+e\right)+a\right)^{-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="giac
")
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n - 1), x)
```

3.503 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx$

Optimal. Leaf size=63

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{2fn}$$

```
[Out] ((I/2)*Hypergeometric2F1[1, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Rubi [A] time = 0.0744578, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3492, 3481, 68}

$$\frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{2fn}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2*n)/(a + I*a*Tan[e + f*x])^n,x]
```

```
[Out] ((I/2)*Hypergeometric2F1[1, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Rule 3492

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Dist[((a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Fra
cPart[n]*(a - b*Tan[e + f*x])^FracPart[n])/(d*Sec[e + f*x])^(2*FracPart[n])
, Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Rule 3481

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \left((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\int (a-$$

$$= \frac{\left(ia(d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\text{Subs}}{f}$$

$$= \frac{i\,{}_2F_1\left(1,n;1+n;\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+}{2fn}$$

Mathematica [B] time = 1.05455, size = 150, normalized size = 2.38

$$-\frac{i2^{n-1}\left(1+e^{2i(e+fx)}\right)\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n}\sec^{-n}(e+fx)(\cos(fx)+i\sin(fx))^{n}\text{Hypergeometric2F1}\left(1,1-n,2-n,1+e\right.}{f(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)/(a + I*a*Tan[e + f*x])^n,x]
```

```
[Out] ((-I)*2^(-1 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + E^((2*I
)*(e + f*x)))*Hypergeometric2F1[1, 1 - n, 2 - n, 1 + E^((2*I)*(e + f*x))]*(
d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[f*x])^n)/((E^(I*f*x))^n*f*(-1 + n)*
Sec[e + f*x]^n*(a + I*a*Tan[e + f*x])^n)
```

Maple [F] time = 0.486, size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{2n}}{\left(a+ia\tan\left(fx+e\right)\right)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{2n}}{\left(ia\tan\left(fx+e\right)+a\right)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="maxima"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)/(I*a*tan(f*x + e) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{2\,de^{\left(ifx+ie\right)}}{e^{\left(2ifx+2ie\right)}+1}\right)^{2n}}{\left(\frac{2\,ae^{\left(2ifx+2ie\right)}}{e^{\left(2ifx+2ie\right)}+1}\right)^{n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="fricas"
)
```

```
[Out] integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)/(2*a*e^(2*I*
f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)/((a+I*a*tan(f*x+e))**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{2n}}{\left(i\,a\tan\left(fx+e\right)+a\right)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)/(I*a*tan(f*x + e) + a)^n, x)
```

3.504 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$

Optimal. Leaf size=40

$$\frac{ia(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn}$$

```
[Out] (I*a*(d*Sec[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Rubi [A] time = 0.0574708, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {3493}

$$\frac{ia(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n),x]
```

```
[Out] (I*a*(d*Sec[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx=\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn}$$

Mathematica [A] time = 0.411444, size = 40, normalized size = 1.

$$\frac{ia(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n),x]
```

```
[Out] (I*a*(d*Sec[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Maple [C] time = 0.596, size = 1294, normalized size = 32.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x)
```

```
[Out] I/f*a^(-n)*2^n*a*d^(2*n)*exp(1/2*I*Pi*(-n*csgn(I*a)*csgn(I*a/(exp(2*I*(f*x+
e))+1)*exp(2*I*(f*x+e)))^2-n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(2*I*(f
*x+e))/(exp(2*I*(f*x+e))+1))^2-csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(2*I*
(f*x+e)))*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))-2*n*csgn(I*exp(I*(f
*x+e)))*csgn(I*exp(2*I*(f*x+e)))^2-n*csgn(I*exp(2*I*(f*x+e)))*csgn(I*exp(2*
I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2+n*csgn(I*exp(I*(f*x+e)))^2*csgn(I*exp(2*
I*(f*x+e)))-n*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*a/(exp(2
*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2-csgn(I*a)*csgn(I*exp(2*I*(f*x+e))/(exp(2
*I*(f*x+e))+1))*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))+csgn(I*exp(
2*I*(f*x+e)))*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2+csgn(I*exp(2*
I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x
+e)))^2+csgn(I*a)*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2+csgn(I/
(exp(2*I*(f*x+e))+1))*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-csgn(
I*exp(I*(f*x+e)))^2*csgn(I*exp(2*I*(f*x+e)))+n*csgn(I*exp(2*I*(f*x+e)))^3+n
*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3+n*csgn(I*a/(exp(2*I*(f*x+e
))+1)*exp(2*I*(f*x+e)))^3-2*n*csgn(I*d/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))
^3-2*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3-csgn(I*exp(2*I*(f*x+e)
)/(exp(2*I*(f*x+e))+1))^3-csgn(I*exp(2*I*(f*x+e)))^3-csgn(I*a/(exp(2*I*(f*x
+e))+1)*exp(2*I*(f*x+e)))^3+2*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(2*I*(f*x+e)
))^2+n*csgn(I*a)*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*a/(ex
p(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))-2*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I
*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))+n*csgn(I/(exp(
2*I*(f*x+e))+1))*csgn(I*exp(2*I*(f*x+e)))*csgn(I*exp(2*I*(f*x+e))/(exp(2*I*
(f*x+e))+1))-2*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*d)*csgn
(I*d/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))+2*n*csgn(I*d)*csgn(I*d/(exp(2*I*(
f*x+e))+1)*exp(I*(f*x+e)))^2+2*n*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e
))/(exp(2*I*(f*x+e))+1))^2+2*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*
csgn(I*d/(exp(2*I*(f*x+e))+1)*exp(I*(f*x+e)))^2+2*n*csgn(I/(exp(2*I*(f*x+e)
)+1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2))/n*(exp(2*I*(f*x+e))+1
)^(-n)
```

Maxima [B] time = 2.47156, size = 185, normalized size = 4.62

$$\frac{i\,a^{-n+1}d^{2\,n}e^{\left(-n\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right)-n\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)-n\log\left(-\frac{2i\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-1\right)+2\,n\log\left(-\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-1\right)\right)}}{fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="maxim
a")
```

```
[Out] I*a^(-n + 1)*d^(2*n)*e^(-n*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - n*log
(sin(f*x + e)/(cos(f*x + e) + 1) - 1) - n*log(-2*I*sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1) + 2*n*log(-sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 1))/(f*n)
```

Fricas [B] time = 1.90458, size = 236, normalized size = 5.9

$$\frac{\left(\frac{2\,ae^{\left(2i\,fx+2i\,e\right)}}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{-n+1}\left(\frac{2\,de^{\left(i\,fx+i\,e\right)}}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{2\,n}\left(i\,e^{\left(2i\,fx+2i\,e\right)}+i\right)e^{\left(-2i\,fx-2i\,e\right)}}{2\,fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(-n + 1)*(2*d*e^(I*
f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*(I*e^(2*I*f*x + 2*I*e) + I)*e^(
-2*I*f*x - 2*I*e)/(f*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(1-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \sec\left(fx+e\right)\right)^{2n} \left(i a \tan\left(fx+e\right)+a\right)^{-n+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 1), x)
```

3.505 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$

Optimal. Leaf size=92

$$\frac{2ia^2(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn(n+1)}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}$$

```
[Out] (I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(1 + n)) + (
(2*I)*a^2*(d*Sec[e + f*x])^(2*n))/(f*n*(1 + n)*(a + I*a*Tan[e + f*x])^n)
```

Rubi [A] time = 0.127938, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3494, 3493}

$$\frac{2ia^2(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn(n+1)}+\frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n),x]
```

```
[Out] (I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(1 + n)) + (
(2*I)*a^2*(d*Sec[e + f*x])^(2*n))/(f*n*(1 + n)*(a + I*a*Tan[e + f*x])^n)
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx=\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(1+n)}+\frac{(2a)\int (d\sec(e+fx))^{2n}(a}{1+}$$
$$=\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(1+n)}+\frac{2ia^2(d\sec(e+fx))^{2n}(a+}{fn(1+}$$

Mathematica [A] time = 1.07498, size = 61, normalized size = 0.66

$$-\frac{a^2(n\tan(e+fx)-i(n+2))(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n),x]
```

```
[Out] -((a^2*(d*Sec[e + f*x])^(2*n)*((-I)*(2 + n) + n*Tan[e + f*x]))/(f*n*(1 + n)
*(a + I*a*Tan[e + f*x])^n))
```

Maple [F] time = 0.809, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{2n}\left(a+ia\tan\left(fx+e\right)\right)^{2-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x)
```

Maxima [B] time = 3.05998, size = 410, normalized size = 4.46

$$\frac{2^{n+1}a^2d^{2n}\cos\left(n\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)\right)-i\cdot 2^{n+1}a^2d^{2n}\sin\left(n\arctan\left(\sin\left(2fx+2e\right),\cos\left(2f\right.\right.\right.}{\left(-ia^nn^2-ia^nn+\left(-ia^nn^2-ia^nn\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="maxim
a")
```

```
[Out] (2^(n + 1)*a^2*d^(2*n)*cos(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)) - I*2^(n + 1)*a^2*d^(2*n)*sin(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) + 2*(a^2*d^(2*n)*n + a^2*d^(2*n))*2^n*cos(-2*f*x + n*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e) - (2*I*a^2*d^(2*n)*n + 2*I*a^2*d^
(2*n))*2^n*sin(-2*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) -
 2*e))/((-I*a^n*n^2 - I*a^n*n + (-I*a^n*n^2 - I*a^n*n)*cos(2*f*x + 2*e) + (
a^n*n^2 + a^n*n)*sin(2*f*x + 2*e))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*f)
```

Fricas [A] time = 1.90475, size = 304, normalized size = 3.3

$$\frac{\left((in+i)e^{(4ifx+4ie)}+(in+2i)e^{(2ifx+2ie)}+i\right)\left(\frac{2ae^{(2ifx+2ie)}}{e^{(2ifx+2ie)}+1}\right)^{-n+2}\left(\frac{2de^{(ifx+ie)}}{e^{(2ifx+2ie)}+1}\right)^{2n}e^{(-4ifx-4ie)}}{2\left(fn^2+fn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="frica
s")
```

```
[Out] 1/2*((I*n + I)*e^(4*I*f*x + 4*I*e) + (I*n + 2*I)*e^(2*I*f*x + 2*I*e) + I)*(
2*a*e^(2*I*f*x + 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(-n + 2)*(2*d*e^(I*f*x +
 I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-4*I*f*x - 4*I*e)/(f*n^2 + f*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(2-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{2n}\left(ia\tan\left(fx+e\right)+a\right)^{-n+2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 2), x)
```

3.506 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$

Optimal. Leaf size=148

$$\frac{4ia^2(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f\left(n^2+3n+2\right)}+\frac{8ia^3(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn\left(n^2+3n+2\right)}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d}{f(n+2)}$$

```
[Out] ((4*I)*a^2*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(2 + 3
*n + n^2)) + (I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n))/(f
*(2 + n)) + ((8*I)*a^3*(d*Sec[e + f*x])^(2*n))/(f*n*(2 + 3*n + n^2)*(a + I*
a*Tan[e + f*x])^n)
```

Rubi [A] time = 0.211018, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3494, 3493}

$$\frac{4ia^2(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f\left(n^2+3n+2\right)}+\frac{8ia^3(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn\left(n^2+3n+2\right)}+\frac{ia(a+ia\tan(e+fx))^{2-n}(d}{f(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(3 - n),x]
```

```
[Out] ((4*I)*a^2*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(2 + 3
*n + n^2)) + (I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n))/(f
*(2 + n)) + ((8*I)*a^3*(d*Sec[e + f*x])^(2*n))/(f*n*(2 + 3*n + n^2)*(a + I*
a*Tan[e + f*x])^n)
```

Rule 3494

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rule 3493

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(2*b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^
(n - 1))/(f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\begin{aligned}\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx &= \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}}{f(2+n)}+\frac{(4a)\int (d\sec(e+fx))^{2n}(a+}{2+n}\\ &= \frac{4ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f\left(2+3n+n^2\right)}+\frac{ia(d\sec(e+fx))^{2n}(a+ia}{f(2+n)}\\ &= \frac{4ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f\left(2+3n+n^2\right)}+\frac{ia(d\sec(e+fx))^{2n}(a+ia}{f(2+n)}\end{aligned}$$

Mathematica [A] time = 1.84091, size = 129, normalized size = 0.87

$$\frac{ia^3\sec^2(e+fx)(\cos(3fx)+i\sin(3fx))(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\left(\left(n^2+3n+4\right)\cos(2(e+fx))+in(n+\right.}{fn(n+1)(n+2)(\cos(fx)+i\sin(fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(3 - n),x]
```

```
[Out] (I*a^3*Sec[e + f*x]^2*(d*Sec[e + f*x])^(2*n)*(Cos[3*f*x] + I*Sin[3*f*x])*(2
*(2 + n) + (4 + 3*n + n^2)*Cos[2*(e + f*x)] + I*n*(3 + n)*Sin[2*(e + f*x)])
)/(f*n*(1 + n)*(2 + n)*(Cos[f*x] + I*Sin[f*x])^3*(a + I*a*Tan[e + f*x])^n)
```

Maple [F] time = 0.862, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{2n}\left(a+ia\tan\left(fx+e\right)\right)^{3-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x)
```

Maxima [B] time = 10.3702, size = 833, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="maxim
a")
```

```
[Out] (2^(n + 3)*a^3*d^(2*n)*cos(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)) - I*2^(n + 3)*a^3*d^(2*n)*sin(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) + 8*(a^3*d^(2*n)*n + 2*a^3*d^(2*n))*2^n*cos(-2*f*x + n*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e) + 4*(a^3*d^(2*n)*n^2 + 3*a^3*d^
(2*n)*n + 2*a^3*d^(2*n))*2^n*cos(-4*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e) + 1) - 4*e) - (8*I*a^3*d^(2*n)*n + 16*I*a^3*d^(2*n))*2^n*sin(-2
*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e) - (4*I*a^3*
d^(2*n)*n^2 + 12*I*a^3*d^(2*n)*n + 8*I*a^3*d^(2*n))*2^n*sin(-4*f*x + n*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 4*e))/(((-I*a^n*n^3 - 3*I*a^n
*n^2 - 2*I*a^n*n)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)^(1/2*n)*cos(4*f*x + 4*e) + (a^n*n^3 + 3*a^n*n^2 + 2*a^n*n)*(cos(2
*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*sin(4*
f*x + 4*e) + (-I*a^n*n^3 - 3*I*a^n*n^2 - 2*I*a^n*n + (-2*I*a^n*n^3 - 6*I*a^
n*n^2 - 4*I*a^n*n)*cos(2*f*x + 2*e) + 2*(a^n*n^3 + 3*a^n*n^2 + 2*a^n*n)*sin
(2*f*x + 2*e))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)^(1/2*n))*f)
```

Fricas [A] time = 1.93509, size = 401, normalized size = 2.71

$$\frac{\left(\left(i\,n^2+3i\,n+2i\right)e^{\left(6i\,fx+6i\,e\right)}+\left(i\,n^2+5i\,n+6i\right)e^{\left(4i\,fx+4i\,e\right)}+\left(2i\,n+6i\right)e^{\left(2i\,fx+2i\,e\right)}+2i\right)\left(\frac{2\,ae^{\left(2i\,fx+2i\,e\right)}}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{-n+3}\left(\frac{2\,de^{\left(i\,fx+i\,e\right)}}{e^{\left(2i\,fx+2i\,e\right)}+1}\right)^{2\,n}e^{(}}{2\left(fn^3+3\,fn^2+2\,fn\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="frica
s")
```

```
[Out] 1/2*((I*n^2 + 3*I*n + 2*I)*e^(6*I*f*x + 6*I*e) + (I*n^2 + 5*I*n + 6*I)*e^(4
*I*f*x + 4*I*e) + (2*I*n + 6*I)*e^(2*I*f*x + 2*I*e) + 2*I)*(2*a*e^(2*I*f*x
+ 2*I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(-n + 3)*(2*d*e^(I*f*x + I*e)/(e^(2*I*f
*x + 2*I*e) + 1))^(2*n)*e^(-6*I*f*x - 6*I*e)/(f*n^3 + 3*f*n^2 + 2*f*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(3-n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{2\,n}\left(i\,a\tan\left(fx+e\right)+a\right)^{-n+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 3), x)
```

3.507 $\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
+ (a*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0369258, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3486, 3767}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
+ (a*Tan[c + d*x]^5)/(5*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx&=\frac{b\sec^6(c+dx)}{6d}+a\int \sec^6(c+dx)\,dx\\&=\frac{b\sec^6(c+dx)}{6d}-\frac{a\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx,x,-\tan(c+dx)\right)}{d}\\&=\frac{b\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.160658, size = 53, normalized size = 0.88

$$\frac{a\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
+ d*x]^5/5))/d
```

Maple [A] time = 0.042, size = 48, normalized size = 0.8

$$\frac{1}{d}\left(\frac{b}{6\left(\cos\left(dx+c\right)\right)^{6}}-a\left(-\frac{8}{15}-\frac{\left(\sec\left(dx+c\right)\right)^{4}}{5}-\frac{4\left(\sec\left(dx+c\right)\right)^{2}}{15}\right)\tan\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/6*b/cos(d*x+c)^6-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*
x+c))
```

Maxima [A] time = 1.47607, size = 95, normalized size = 1.58

$$\frac{5\,b\tan\left(dx+c\right)^{6}+6\,a\tan\left(dx+c\right)^{5}+15\,b\tan\left(dx+c\right)^{4}+20\,a\tan\left(dx+c\right)^{3}+15\,b\tan\left(dx+c\right)^{2}+30\,a\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a*
tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Fricas [A] time = 1.8321, size = 147, normalized size = 2.45

$$\frac{2\left(8\,a\cos\left(dx+c\right)^{5}+4\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+5\,b}{30\,d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(2*(8*a*cos(d*x + c)^5 + 4*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*
x + c) + 5*b)/(d*cos(d*x + c)^6)
```

Sympy [A] time = 5.74066, size = 56, normalized size = 0.93

$$\begin{cases}\frac{a\left(\frac{\tan^{5}(c+dx)}{5}+\frac{2\tan^{3}(c+dx)}{3}+\tan(c+dx)\right)+\frac{b\sec^{6}(c+dx)}{6}}{d} & \text{for}\: d\neq 0\\ x\left(a+b\tan\left(c\right)\right)\sec^{6}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**5/5 + 2*tan(c + d*x)**3/3 + tan(c + d*x)) + b*
sec(c + d*x)**6/6)/d, Ne(d, 0)), (x*(a + b*tan(c))*sec(c)**6, True))
```

Giac [A] time = 1.26476, size = 95, normalized size = 1.58

$$\frac{5\,b\tan\left(dx+c\right)^{6}+6\,a\tan\left(dx+c\right)^{5}+15\,b\tan\left(dx+c\right)^{4}+20\,a\tan\left(dx+c\right)^{3}+15\,b\tan\left(dx+c\right)^{2}+30\,a\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a*
tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

3.508 $\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=74

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c +
 d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.0464041, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3486, 3768, 3770}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c +
 d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx &= \frac{b\sec^5(c+dx)}{5d}+a\int \sec^5(c+dx)\,dx\\ &= \frac{b\sec^5(c+dx)}{5d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}(3a)\int \sec^3(c+dx)\,dx\\ &= \frac{b\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{8}(3a\\ &= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c\end{aligned}$$

Mathematica [A] time = 0.199198, size = 68, normalized size = 0.92

$$\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(Ar
cTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A] time = 0.039, size = 74, normalized size = 1.

$$\frac{b}{5\,d\left(\cos\left(dx+c\right)\right)^5}+\frac{a\left(\sec\left(dx+c\right)\right)^3\tan\left(dx+c\right)}{4\,d}+\frac{3\,a\sec\left(dx+c\right)\tan\left(dx+c\right)}{8\,d}+\frac{3\,a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/5/d*b/cos(d*x+c)^5+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d
*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.52377, size = 116, normalized size = 1.57

$$-\frac{5\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{16\,b}{\cos(dx+c)^5}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*b/
cos(d*x + c)^5)/d
```

Fricas [A] time = 1.91627, size = 242, normalized size = 3.27

$$\frac{15\,a\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,a\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+10\left(3\,a\cos\left(dx+c\right)^3+2\,a\cos\left(dx\right.\right.}{80\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/80*(15*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a*cos(d*x + c)^5*log(-
sin(d*x + c) + 1) + 10*(3*a*cos(d*x + c)^3 + 2*a*cos(d*x + c))*sin(d*x + c)
 + 16*b)/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\tan(c + dx))\sec^5(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sec(c + d*x)**5, x)
```

Giac [B] time = 1.3023, size = 190, normalized size = 2.57

$$\frac{15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 - 10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 -}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*log(abs(tan(1/2*d*x +
1/2*c) - 1)) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*b*tan(1/2*d*x + 1/2*c)^8
 - 10*a*tan(1/2*d*x + 1/2*c)^7 - 80*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2
*d*x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*b)/(tan(1/2*d*x + 1/2*c)^2
 - 1)^5)/d
```

3.509 $\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=44

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0318142, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3486, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx &= \frac{b\sec^4(c+dx)}{4d}+a\int \sec^4(c+dx)\,dx\\ &= \frac{b\sec^4(c+dx)}{4d}-\frac{a\,\text{Subst}\left(\int \left(1+x^2\right)dx, x, -\tan(c+dx)\right)}{d}\\ &= \frac{b\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0820923, size = 41, normalized size = 0.93

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.037, size = 38, normalized size = 0.9

$$\frac{1}{d}\left(\frac{b}{4\left(\cos\left(dx+c\right)\right)^{4}}-a\left(-\frac{2}{3}-\frac{\left(\sec\left(dx+c\right)\right)^{2}}{3}\right)\tan\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/4*b/cos(d*x+c)^4-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.58285, size = 65, normalized size = 1.48

$$\frac{3\,b\tan\left(dx+c\right)^{4}+4\,a\tan\left(dx+c\right)^{3}+6\,b\tan\left(dx+c\right)^{2}+12\,a\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*t
an(d*x + c))/d
```

Fricas [A] time = 1.88274, size = 116, normalized size = 2.64

$$\frac{4\left(2\,a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,b}{12\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*(2*a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c) + 3*b)/(d*cos(d*
x + c)^4)
```

Sympy [A] time = 3.36366, size = 44, normalized size = 1.

$$\begin{cases}\frac{a\left(\frac{\tan^{3}\left(c+dx\right)}{3}+\tan\left(c+dx\right)\right)+\frac{b\sec^{4}\left(c+dx\right)}{4}}{d} & \text{for}\: d\neq 0\\x\left(a+b\tan\left(c\right)\right)\sec^{4}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**3/3 + tan(c + d*x)) + b*sec(c + d*x)**4/4)/d,
Ne(d, 0)), (x*(a + b*tan(c))*sec(c)**4, True))
```

Giac [A] time = 1.21218, size = 65, normalized size = 1.48

$$\frac{3\,b\tan\left(dx+c\right)^4+4\,a\tan\left(dx+c\right)^3+6\,b\tan\left(dx+c\right)^2+12\,a\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*t
an(d*x + c))/d
```

3.510 $\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0347972, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3486, 3768, 3770}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx &= \frac{b\sec^3(c+dx)}{3d}+a\int \sec^3(c+dx)\,dx\\ &= \frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a\int \sec(c+dx)\,dx\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0160117, size = 52, normalized size = 1.

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.039, size = 54, normalized size = 1.

$$\frac{b}{3d\left(\cos\left(dx+c\right)\right)^{3}}+\frac{a\sec\left(dx+c\right)\tan\left(dx+c\right)}{2d}+\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/3/d*b/cos(d*x+c)^3+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+ta
n(d*x+c))
```

Maxima [A] time = 1.52192, size = 82, normalized size = 1.58

$$-\frac{3\,a\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{4\,b}{\cos\left(dx+c\right)^{3}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*b/cos(d*x + c)^3)/d
```

Fricas [A] time = 1.96352, size = 203, normalized size = 3.9

$$\frac{3\,a\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+6\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+4\,b}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 6*a*cos(d*x + c)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] time = 1.31583, size = 134, normalized size = 2.58

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 - 3*
a*tan(1/2*d*x + 1/2*c) - 2*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.511 $\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=28

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

```
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0284686, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3486, 3767, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx &= \frac{b\sec^2(c+dx)}{2d}+a\int \sec^2(c+dx)\,dx\\ &= \frac{b\sec^2(c+dx)}{2d}-\frac{a\,\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0133805, size = 28, normalized size = 1.

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.035, size = 25, normalized size = 0.9

$$\frac{1}{d}\left(\frac{b}{2\left(\cos\left(dx+c\right)\right)^{2}}+a\tan\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/2*b/cos(d*x+c)^2+a*tan(d*x+c))
```

Maxima [A] time = 1.5035, size = 27, normalized size = 0.96

$$\frac{\left(b\tan\left(dx+c\right)+a\right)^{2}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*tan(d*x + c) + a)^2/(b*d)
```

Fricas [A] time = 1.76126, size = 81, normalized size = 2.89

$$\frac{2\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+b}{2\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cos(d*x + c)*sin(d*x + c) + b)/(d*cos(d*x + c)^2)
```

Sympy [A] time = 2.19389, size = 34, normalized size = 1.21

$$\begin{cases}\frac{a\tan\left(c+dx\right)+\frac{b\tan^{2}\left(c+dx\right)}{2}}{d} & \text{for}\: d\neq 0\\x\left(a+b\tan\left(c\right)\right)\sec^{2}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*tan(c + d*x) + b*tan(c + d*x)**2/2)/d, Ne(d, 0)), (x*(a + b*t
an(c))*sec(c)**2, True))
```

Giac [A] time = 1.2868, size = 34, normalized size = 1.21

$$\frac{b\tan\left(dx+c\right)^{2}+2\,a\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b*tan(d*x + c)^2 + 2*a*tan(d*x + c))/d
```

3.512 $\int \sec(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Rubi [A] time = 0.0151662, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3486, 3770}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\tan(c+dx))\,dx &= \frac{b\sec(c+dx)}{d}+a\int \sec(c+dx)\,dx\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0091772, size = 24, normalized size = 1.

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Maple [A] time = 0.011, size = 34, normalized size = 1.4

$$\frac{b}{d\cos(dx+c)}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*b/cos(d*x+c)+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.3044, size = 42, normalized size = 1.75

$$\frac{a\log(\sec(dx+c)+\tan(dx+c))+\frac{b}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + b/cos(d*x + c))/d
```

Fricas [B] time = 1.8153, size = 144, normalized size = 6.

$$\frac{a\cos(dx+c)\log(\sin(dx+c)+1)-a\cos(dx+c)\log(-\sin(dx+c)+1)+2\,b}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*b)/(d*cos(d*x + c))
```

Sympy [A] time = 3.8325, size = 37, normalized size = 1.54

$$\begin{cases}\frac{a\log(\tan(c+dx)+\sec(c+dx))+b\sec(c+dx)}{d} & \text{for } d\neq 0\\ x(a+b\tan(c))\sec(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + b*sec(c + d*x))/d, Ne(d, 0
)), (x*(a + b*tan(c))*sec(c), True))
```

Giac [B] time = 1.27402, size = 73, normalized size = 3.04

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*b/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.513 $\int \cos(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0196113, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3486, 2637}

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\tan(c+dx))\,dx &= -\frac{b\cos(c+dx)}{d}+a\int \cos(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0187987, size = 46, normalized size = 1.92

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}-\frac{b\cos(c)\cos(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
(b*Sin[c]*Sin[d*x])/d
```

Maple [A] time = 0.033, size = 23, normalized size = 1.

$$\frac{-b\cos(dx+c)+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-b*cos(d*x+c)+a*sin(d*x+c))
```

Maxima [A] time = 1.18442, size = 31, normalized size = 1.29

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d
```

Fricas [A] time = 1.84741, size = 51, normalized size = 2.12

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\tan(c+dx))\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x), x)
```

Giac [B] time = 1.24499, size = 174, normalized size = 7.25

$$-\frac{b\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2+2a\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)+2a\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx\right)^2-4b\tan\left(\frac{1}{2}dx\right)\tan\left(\frac{1}{2}c\right.}{d\tan\left(\frac{1}{2}dx\right)^2\tan\left(\frac{1}{2}c\right)^2+d\tan\left(\frac{1}{2}dx\right)^2+d\tan\left(\frac{1}{2}c\right)^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*tan(1
/2*d*x)*tan(1/2*c)^2 - b*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c) - b*t
an(1/2*c)^2 - 2*a*tan(1/2*d*x) - 2*a*tan(1/2*c) + b)/(d*tan(1/2*d*x)^2*tan(
1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

3.514 $\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\cos^2(c+dx)}{2d}$$

```
[Out] (a*x)/2 - (b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0281819, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3486, 2635, 8}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\cos^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*x)/2 - (b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx &= -\frac{b\cos^2(c+dx)}{2d}+a\int \cos^2(c+dx)\,dx\\ &= -\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\ &= \frac{ax}{2}-\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0538012, size = 46, normalized size = 1.07

$$\frac{a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}-\frac{b\cos^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^2)/(2*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.042, size = 41, normalized size = 1.

$$\frac{1}{d}\left(-\frac{b\left(\cos\left(dx+c\right)\right)^{2}}{2}+a\left(\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/2*b*cos(d*x+c)^2+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 2.34818, size = 51, normalized size = 1.19

$$\frac{(dx+c)a+\frac{a\tan(dx+c)-b}{\tan(dx+c)^{2}+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((d*x + c)*a + (a*tan(d*x + c) - b)/(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.84076, size = 86, normalized size = 2.

$$\frac{adx-b\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x - b*cos(d*x + c)^2 + a*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)\cos^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x)**2, x)
```

Giac [B] time = 1.35676, size = 197, normalized size = 4.58

$$\frac{2\,adx\tan(dx)^2\tan(c)^2 + 2\,adx\tan(dx)^2 + 2\,adx\tan(c)^2 - b\tan(dx)^2\tan(c)^2 - 2\,a\tan(dx)^2\tan(c) - 2\,a\tan(dx)\tan}{4\left(d\tan(dx)^2\tan(c)^2 + d\tan(dx)^2 + d\,t\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*d*x*tan(d*x)^2*tan(c)^2 + 2*a*d*x*tan(d*x)^2 + 2*a*d*x*tan(c)^2 -
b*tan(d*x)^2*tan(c)^2 - 2*a*tan(d*x)^2*tan(c) - 2*a*tan(d*x)*tan(c)^2 + 2*a
*d*x + b*tan(d*x)^2 + 4*b*tan(d*x)*tan(c) + b*tan(c)^2 + 2*a*tan(d*x) + 2*a
*tan(c) - b)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

3.515 $\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=44

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

```
[Out] -(b*Cos[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0315729, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3486, 2633}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx &= -\frac{b\cos^3(c+dx)}{3d}+a\int \cos^3(c+dx)\,dx\\
&= -\frac{b\cos^3(c+dx)}{3d}-\frac{a\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\
&= -\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.012072, size = 44, normalized size = 1.

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.046, size = 36, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{(\cos(dx+c))^3 b}{3}+\frac{a\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/3*cos(d*x+c)^3*b+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.75519, size = 47, normalized size = 1.07

$$-\frac{b\cos(dx+c)^3+\left(\sin(dx+c)^3-3\sin(dx+c)\right)a}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(b*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

Fricas [A] time = 1.7645, size = 90, normalized size = 2.05

$$-\frac{b\cos(dx+c)^3-\left(a\cos(dx+c)^2+2a\right)\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(b*cos(d*x + c)^3 - (a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\tan(c+dx))\cos^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.516 $\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}$$

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^4)/(4*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.040945, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3486, 2635, 8}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^4)/(4*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx &= -\frac{b\cos^4(c+dx)}{4d}+a\int \cos^4(c+dx)\,dx\\
&= -\frac{b\cos^4(c+dx)}{4d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3a)\int \cos^2(c+dx)\,dx\\
&= -\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{8}(3\\
&= \frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.0920675, size = 62, normalized size = 0.95

$$\frac{3a(c+dx)}{8d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^4)/(4*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.04, size = 52, normalized size = 0.8

$$\frac{1}{d}\left(-\frac{b(\cos(dx+c))^4}{4}+a\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-1/4*b*cos(d*x+c)^4+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/
8*d*x+3/8*c))
```

Maxima [A] time = 2.31804, size = 82, normalized size = 1.26

$$\frac{3(dx+c)a+\frac{3a\tan(dx+c)^3+5a\tan(dx+c)-2b}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(d*x + c)*a + (3*a*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 2*b)/(tan(d*x
 + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.81161, size = 127, normalized size = 1.95

$$-\frac{2b\cos(dx+c)^4-3adx-\left(2a\cos(dx+c)^3+3a\cos(dx+c)\right)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - (2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\tan(c + dx))\cos^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x)**4, x)
```

Giac [B] time = 1.52752, size = 575, normalized size = 8.85

$$\frac{12\,adx\tan(dx)^4\tan(c)^4 + 24\,adx\tan(dx)^4\tan(c)^2 + 24\,adx\tan(dx)^2\tan(c)^4 - 5\,b\tan(dx)^4\tan(c)^4 - 20\,a\tan(dx)^4\,\mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(12*a*d*x*tan(d*x)^4*tan(c)^4 + 24*a*d*x*tan(d*x)^4*tan(c)^2 + 24*a*d*
x*tan(d*x)^2*tan(c)^4 - 5*b*tan(d*x)^4*tan(c)^4 - 20*a*tan(d*x)^4*tan(c)^3
- 20*a*tan(d*x)^3*tan(c)^4 + 12*a*d*x*tan(d*x)^4 + 48*a*d*x*tan(d*x)^2*tan(
c)^2 + 6*b*tan(d*x)^4*tan(c)^2 + 32*b*tan(d*x)^3*tan(c)^3 + 12*a*d*x*tan(c)
^4 + 6*b*tan(d*x)^2*tan(c)^4 - 12*a*tan(d*x)^4*tan(c) + 24*a*tan(d*x)^3*tan
(c)^2 + 24*a*tan(d*x)^2*tan(c)^3 - 12*a*tan(d*x)*tan(c)^4 + 24*a*d*x*tan(d*
x)^2 + 3*b*tan(d*x)^4 + 24*a*d*x*tan(c)^2 - 36*b*tan(d*x)^2*tan(c)^2 + 3*b*
tan(c)^4 + 12*a*tan(d*x)^3 - 24*a*tan(d*x)^2*tan(c) - 24*a*tan(d*x)*tan(c)^
2 + 12*a*tan(c)^3 + 12*a*d*x + 6*b*tan(d*x)^2 + 32*b*tan(d*x)*tan(c) + 6*b*
tan(c)^2 + 20*a*tan(d*x) + 20*a*tan(c) - 5*b)/(d*tan(d*x)^4*tan(c)^4 + 2*d*
tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4 + 4*d*tan(d*x)
^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

3.517 $\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=119

$$\frac{(a^2+3b^2)\tan^7(c+dx)}{7d}+\frac{3(a^2+b^2)\tan^5(c+dx)}{5d}+\frac{(3a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec^8(c+dx)}{4d}+$$

```
[Out] (a*b*Sec[c + d*x]^8)/(4*d) + (a^2*Tan[c + d*x])/d + ((3*a^2 + b^2)*Tan[c +
d*x]^3)/(3*d) + (3*(a^2 + b^2)*Tan[c + d*x]^5)/(5*d) + ((a^2 + 3*b^2)*Tan[c
 + d*x]^7)/(7*d) + (b^2*Tan[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.106558, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3506, 696, 1810}

$$\frac{(a^2+3b^2)\tan^7(c+dx)}{7d}+\frac{3(a^2+b^2)\tan^5(c+dx)}{5d}+\frac{(3a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec^8(c+dx)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^8)/(4*d) + (a^2*Tan[c + d*x])/d + ((3*a^2 + b^2)*Tan[c +
d*x]^3)/(3*d) + (3*(a^2 + b^2)*Tan[c + d*x]^5)/(5*d) + ((a^2 + 3*b^2)*Tan[c
 + d*x]^7)/(7*d) + (b^2*Tan[c + d*x]^9)/(9*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 696

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*m*d^(m - 1)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(1+\frac{x^2}{b^2}\right)^3 dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{ab\sec^8(c+dx)}{4d} + \frac{\text{Subst}\left(\int \left(1+\frac{x^2}{b^2}\right)^3 \left(-2ax+(a+x)^2\right) dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{ab\sec^8(c+dx)}{4d} + \frac{\text{Subst}\left(\int \left(a^2+\frac{(3a^2+b^2)x^2}{b^2}+\frac{3(a^2+b^2)x^4}{b^4}+\frac{(a^2+3b^2)x^6}{b^6}+\frac{x^8}{b^6}\right) dx, x\right.}{bd}$$
$$= \frac{ab\sec^8(c+dx)}{4d} + \frac{a^2\tan(c+dx)}{d} + \frac{(3a^2+b^2)\tan^3(c+dx)}{3d} + \frac{3(a^2+b^2)\tan^5(}{5d}$$

Mathematica [A] time = 0.482637, size = 133, normalized size = 1.12

$$\frac{\tan(c+dx)\left(180\left(a^2+3b^2\right)\tan^6(c+dx)+756\left(a^2+b^2\right)\tan^4(c+dx)+420\left(3a^2+b^2\right)\tan^2(c+dx)+1260a^2+315ab\,\text{t}\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Tan[c + d*x]*(1260*a^2 + 1260*a*b*Tan[c + d*x] + 420*(3*a^2 + b^2)*Tan[c +
 d*x]^2 + 1890*a*b*Tan[c + d*x]^3 + 756*(a^2 + b^2)*Tan[c + d*x]^4 + 1260*a
*b*Tan[c + d*x]^5 + 180*(a^2 + 3*b^2)*Tan[c + d*x]^6 + 315*a*b*Tan[c + d*x]
^7 + 140*b^2*Tan[c + d*x]^8))/(1260*d)
```

Maple [A] time = 0.054, size = 138, normalized size = 1.2

$$\frac{1}{d}\left(b^2\left(\frac{(\sin(dx+c))^3}{9(\cos(dx+c))^9}+\frac{2(\sin(dx+c))^3}{21(\cos(dx+c))^7}+\frac{8(\sin(dx+c))^3}{105(\cos(dx+c))^5}+\frac{16(\sin(dx+c))^3}{315(\cos(dx+c))^3}\right)+\frac{ab}{4(\cos(dx+c))^8}-a^2\left(-\frac{16}{35}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/10
5*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+1/4*a*b/cos(d
*x+c)^8-a^2*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*t
an(d*x+c))
```

Maxima [A] time = 1.59889, size = 180, normalized size = 1.51

$$\frac{140\,b^2\tan(dx+c)^9+315\,ab\tan(dx+c)^8+1260\,ab\tan(dx+c)^6+180\left(a^2+3\,b^2\right)\tan(dx+c)^7+1890\,ab\tan(dx+c)}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(140*b^2*tan(d*x + c)^9 + 315*a*b*tan(d*x + c)^8 + 1260*a*b*tan(d*x
+ c)^6 + 180*(a^2 + 3*b^2)*tan(d*x + c)^7 + 1890*a*b*tan(d*x + c)^4 + 756*(
```

```
a^2 + b^2)*tan(d*x + c)^5 + 1260*a*b*tan(d*x + c)^2 + 420*(3*a^2 + b^2)*tan
(d*x + c)^3 + 1260*a^2*tan(d*x + c))/d
```

Fricas [A] time = 2.1421, size = 282, normalized size = 2.37

$$\frac{315\,ab\cos(dx+c)+4\left(16\left(9\,a^2-b^2\right)\cos(dx+c)^8+8\left(9\,a^2-b^2\right)\cos(dx+c)^6+6\left(9\,a^2-b^2\right)\cos(dx+c)^4+5\left(9\,a^2\right.\right.}{1260\,d\cos(dx+c)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(315*a*b*cos(d*x + c) + 4*(16*(9*a^2 - b^2)*cos(d*x + c)^8 + 8*(9*a^
2 - b^2)*cos(d*x + c)^6 + 6*(9*a^2 - b^2)*cos(d*x + c)^4 + 5*(9*a^2 - b^2)*
cos(d*x + c)^2 + 35*b^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^2\sec^8(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**8, x)
```

Giac [A] time = 1.33762, size = 211, normalized size = 1.77

$$\frac{140\,b^2\tan(dx+c)^9+315\,ab\tan(dx+c)^8+180\,a^2\tan(dx+c)^7+540\,b^2\tan(dx+c)^7+1260\,ab\tan(dx+c)^6+756}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1260*(140*b^2*tan(d*x + c)^9 + 315*a*b*tan(d*x + c)^8 + 180*a^2*tan(d*x +
 c)^7 + 540*b^2*tan(d*x + c)^7 + 1260*a*b*tan(d*x + c)^6 + 756*a^2*tan(d*x
+ c)^5 + 756*b^2*tan(d*x + c)^5 + 1890*a*b*tan(d*x + c)^4 + 1260*a^2*tan(d*
x + c)^3 + 420*b^2*tan(d*x + c)^3 + 1260*a*b*tan(d*x + c)^2 + 1260*a^2*tan(
d*x + c))/d
```

3.518 $\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=97

$$\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec^6(c+dx)}{3d}+\frac{b^2\tan^7(c+dx)}{7d}$$

```
[Out] (a*b*Sec[c + d*x]^6)/(3*d) + (a^2*Tan[c + d*x])/d + ((2*a^2 + b^2)*Tan[c +
d*x]^3)/(3*d) + ((a^2 + 2*b^2)*Tan[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)
/(7*d)
```

Rubi [A] time = 0.0825734, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3506, 696, 1810}

$$\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec^6(c+dx)}{3d}+\frac{b^2\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^6)/(3*d) + (a^2*Tan[c + d*x])/d + ((2*a^2 + b^2)*Tan[c +
d*x]^3)/(3*d) + ((a^2 + 2*b^2)*Tan[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)
/(7*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 696

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*m*d^(m - 1)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(1+\frac{x^2}{b^2}\right)^2 dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{ab\sec^6(c+dx)}{3d} + \frac{\text{Subst}\left(\int \left(1+\frac{x^2}{b^2}\right)^2 \left(-2ax+(a+x)^2\right) dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{ab\sec^6(c+dx)}{3d} + \frac{\text{Subst}\left(\int \left(a^2+\frac{(2a^2+b^2)x^2}{b^2}+\frac{(a^2+2b^2)x^4}{b^4}+\frac{x^6}{b^4}\right) dx, x, b\tan(c\right.}{bd}$$
$$= \frac{ab\sec^6(c+dx)}{3d} + \frac{a^2\tan(c+dx)}{d} + \frac{(2a^2+b^2)\tan^3(c+dx)}{3d} + \frac{(a^2+2b^2)\tan}{5d}$$

Mathematica [A] time = 0.638971, size = 104, normalized size = 1.07

$$\frac{\tan(c+dx)\left(21\left(a^2+2b^2\right)\tan^4(c+dx)+35\left(2a^2+b^2\right)\tan^2(c+dx)+105a^2+35ab\tan^5(c+dx)+105ab\tan^3(c+d\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Tan[c + d*x]*(105*a^2 + 105*a*b*Tan[c + d*x] + 35*(2*a^2 + b^2)*Tan[c + d*
x]^2 + 105*a*b*Tan[c + d*x]^3 + 21*(a^2 + 2*b^2)*Tan[c + d*x]^4 + 35*a*b*Ta
n[c + d*x]^5 + 15*b^2*Tan[c + d*x]^6))/(105*d)
```

Maple [A] time = 0.053, size = 110, normalized size = 1.1

$$\frac{1}{d}\left(b^2\left(\frac{(\sin(dx+c))^3}{7(\cos(dx+c))^7}+\frac{4(\sin(dx+c))^3}{35(\cos(dx+c))^5}+\frac{8(\sin(dx+c))^3}{105(\cos(dx+c))^3}\right)+\frac{ab}{3(\cos(dx+c))^6}-a^2\left(-\frac{8}{15}-\frac{(\sec(dx+c))^4}{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/10
5*sin(d*x+c)^3/cos(d*x+c)^3)+1/3*a*b/cos(d*x+c)^6-a^2*(-8/15-1/5*sec(d*x+c)
^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.35239, size = 140, normalized size = 1.44

$$\frac{15\,b^2\tan(dx+c)^7+35\,ab\tan(dx+c)^6+105\,ab\tan(dx+c)^4+21\left(a^2+2\,b^2\right)\tan(dx+c)^5+105\,ab\tan(dx+c)^2+}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 105*a*b*tan(d*x + c)
^4 + 21*(a^2 + 2*b^2)*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^2 + 35*(2*a^2 +
 b^2)*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.95376, size = 231, normalized size = 2.38

$$\frac{35\,ab\cos\left(dx+c\right)+\left(8\left(7\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{6}+4\left(7\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}+3\left(7\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+15\,b^{2}\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(35*a*b*cos(d*x + c) + (8*(7*a^2 - b^2)*cos(d*x + c)^6 + 4*(7*a^2 - b
^2)*cos(d*x + c)^4 + 3*(7*a^2 - b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c))
/(d*cos(d*x + c)^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)^{2}\sec^{6}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**6, x)
```

Giac [A] time = 1.39152, size = 159, normalized size = 1.64

$$\frac{15\,b^{2}\tan\left(dx+c\right)^{7}+35\,ab\tan\left(dx+c\right)^{6}+21\,a^{2}\tan\left(dx+c\right)^{5}+42\,b^{2}\tan\left(dx+c\right)^{5}+105\,ab\tan\left(dx+c\right)^{4}+70\,a^{2}\tan\left(dx+c\right)^{3}+35\,b^{2}\tan\left(dx+c\right)^{3}+105\,ab\tan\left(dx+c\right)^{2}+105\,a^{2}\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 21*a^2*tan(d*x + c)^
5 + 42*b^2*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^4 + 70*a^2*tan(d*x + c)^3
+ 35*b^2*tan(d*x + c)^3 + 105*a*b*tan(d*x + c)^2 + 105*a^2*tan(d*x + c))/d
```

3.519 $\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=75

$$\frac{(a^2+b^2)(a+b\tan(c+dx))^3}{3b^3d}+\frac{(a+b\tan(c+dx))^5}{5b^3d}-\frac{a(a+b\tan(c+dx))^4}{2b^3d}$$

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^3)/(3*b^3*d) - (a*(a + b*Tan[c + d*x])^4)
/(2*b^3*d) + (a + b*Tan[c + d*x])^5/(5*b^3*d)
```

Rubi [A] time = 0.0659744, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$\frac{(a^2+b^2)(a+b\tan(c+dx))^3}{3b^3d}+\frac{(a+b\tan(c+dx))^5}{5b^3d}-\frac{a(a+b\tan(c+dx))^4}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^3)/(3*b^3*d) - (a*(a + b*Tan[c + d*x])^4)
/(2*b^3*d) + (a + b*Tan[c + d*x])^5/(5*b^3*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx&=\frac{\text{Subst}\left(\int(a+x)^2\left(1+\frac{x^2}{b^2}\right)dx,x,b\tan(c+dx)\right)}{bd}\\&=\frac{\text{Subst}\left(\int\left(\frac{(a^2+b^2)(a+x)^2}{b^2}-\frac{2a(a+x)^3}{b^2}+\frac{(a+x)^4}{b^2}\right)dx,x,b\tan(c+dx)\right)}{bd}\\&=\frac{(a^2+b^2)(a+b\tan(c+dx))^3}{3b^3d}-\frac{a(a+b\tan(c+dx))^4}{2b^3d}+\frac{(a+b\tan(c+dx))^5}{5b^3d}\end{aligned}$$

Mathematica [A] time = 0.186638, size = 54, normalized size = 0.72

$$\frac{(a+b\tan(c+dx))^3\left(a^2-3ab\tan(c+dx)+6b^2\tan^2(c+dx)+10b^2\right)}{30b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a + b*Tan[c + d*x])^3*(a^2 + 10*b^2 - 3*a*b*Tan[c + d*x] + 6*b^2*Tan[c +
d*x]^2))/(30*b^3*d)
```

Maple [A] time = 0.052, size = 82, normalized size = 1.1

$$\frac{1}{d}\left(b^2\left(\frac{(\sin(dx+c))^3}{5(\cos(dx+c))^5}+\frac{2(\sin(dx+c))^3}{15(\cos(dx+c))^3}\right)+\frac{ab}{2(\cos(dx+c))^4}-a^2\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+1/2
*a*b/cos(d*x+c)^4-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.52541, size = 96, normalized size = 1.28

$$\frac{6\,b^2\tan(dx+c)^5+15\,ab\tan(dx+c)^4+30\,ab\tan(dx+c)^2+10\left(a^2+b^2\right)\tan(dx+c)^3+30\,a^2\tan(dx+c)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 30*a*b*tan(d*x + c)^2
+ 10*(a^2 + b^2)*tan(d*x + c)^3 + 30*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.88768, size = 184, normalized size = 2.45

$$\frac{15\,ab\cos(dx+c)+2\left(2\left(5\,a^2-b^2\right)\cos(dx+c)^4+\left(5\,a^2-b^2\right)\cos(dx+c)^2+3\,b^2\right)\sin(dx+c)}{30\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*b*cos(d*x + c) + 2*(2*(5*a^2 - b^2)*cos(d*x + c)^4 + (5*a^2 - b^
2)*cos(d*x + c)^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^2\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**4, x)
```

Giac [A] time = 1.32014, size = 108, normalized size = 1.44

$$\frac{6\,b^{2}\tan\left(dx+c\right)^{5}+15\,ab\tan\left(dx+c\right)^{4}+10\,a^{2}\tan\left(dx+c\right)^{3}+10\,b^{2}\tan\left(dx+c\right)^{3}+30\,ab\tan\left(dx+c\right)^{2}+30\,a^{2}\tan\left(d\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 10*a^2*tan(d*x + c)^3
+ 10*b^2*tan(d*x + c)^3 + 30*a*b*tan(d*x + c)^2 + 30*a^2*tan(d*x + c))/d
```

3.520 $\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\tan(c+dx))^3}{3bd}$$

```
[Out] (a + b*Tan[c + d*x])^3/(3*b*d)
```

Rubi [A] time = 0.0365484, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 32}

$$\frac{(a+b\tan(c+dx))^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a + b*Tan[c + d*x])^3/(3*b*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a+b\tan(c+dx))^3}{3bd}\end{aligned}$$

Mathematica [B] time = 0.0419874, size = 46, normalized size = 2.09

$$\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Maple [B] time = 0.048, size = 48, normalized size = 2.2

$$\frac{1}{d}\left(\frac{b^2 (\sin (dx+c))^3}{3 (\cos (dx+c))^3}+\frac{ab}{(\cos (dx+c))^2}+a^2\tan (dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3+a*b/cos(d*x+c)^2+a^2*tan(d*x+c))
```

Maxima [A] time = 1.36252, size = 27, normalized size = 1.23

$$\frac{(b\tan (dx+c)+a)^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*tan(d*x + c) + a)^3/(b*d)
```

Fricas [B] time = 1.71534, size = 131, normalized size = 5.95

$$\frac{3\,ab\cos (dx+c)+\left(\left(3\,a^2-b^2\right)\cos (dx+c)^2+b^2\right)\sin (dx+c)}{3\,d\cos (dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*b*cos(d*x + c) + ((3*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c)
)/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan (c+dx))^2\sec^2 (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [B] time = 1.36354, size = 55, normalized size = 2.5

$$\frac{b^2\tan (dx+c)^3+3\,ab\tan (dx+c)^2+3\,a^2\tan (dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^2 + 3*a^2*tan(d*x + c))/d
```

3.521 $\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d}$$

```
[Out] ((a^2 + b^2)*x)/2 - (Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x
]))/(2*d)
```

Rubi [A] time = 0.0528003, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3506, 723, 203}

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/2 - (Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x
]))/(2*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2}{\left(1+\frac{x^2}{b^2}\right)^2}dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d} + \frac{\left(a^2+b^2\right)\text{Subst}\left(\int \frac{1}{1+\frac{x^2}{b^2}}\right.}{2bd}$$

$$= \frac{1}{2}\left(a^2+b^2\right)x - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d}$$

Mathematica [A] time = 0.125098, size = 52, normalized size = 1.06

$$\frac{2\left(a^2+b^2\right)(c+dx)+\left(a^2-b^2\right)\sin(2(c+dx))-2ab\cos(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c + 
d*x)])/(4*d)
```

Maple [A] time = 0.046, size = 70, normalized size = 1.4

$$\frac{1}{d}\left(b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-(\cos(dx+c))^2ab+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-cos(d*x+c)^2*a*b+a^2*(1
/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 2.24816, size = 74, normalized size = 1.51

$$\frac{\left(a^2+b^2\right)(dx+c)-\frac{2\,ab-\left(a^2-b^2\right)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 + b^2)*(d*x + c) - (2*a*b - (a^2 - b^2)*tan(d*x + c))/(tan(d*x + 
c)^2 + 1))/d
```

Fricas [A] time = 1.77913, size = 120, normalized size = 2.45

$$-\frac{2\,ab\cos\left(dx+c\right)^{2}-\left(a^{2}+b^{2}\right)dx-\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*sin
(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(c+dx\right)\right)^{2}\cos^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + dx))**2*cos(c + d*x)**2, x)
```

Giac [B] time = 1.47207, size = 331, normalized size = 6.76

$$\frac{a^{2}dx\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+b^{2}dx\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+a^{2}dx\tan\left(dx\right)^{2}+b^{2}dx\tan\left(dx\right)^{2}+a^{2}dx\tan\left(c\right)^{2}+b^{2}dx\tan\left(c\right)^{2}-a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d*x*tan(d*x)^2*tan(c)^2 + b^2*d*x*tan(d*x)^2*tan(c)^2 + a^2*d*x*ta
n(d*x)^2 + b^2*d*x*tan(d*x)^2 + a^2*d*x*tan(c)^2 + b^2*d*x*tan(c)^2 - a*b*t
an(d*x)^2*tan(c)^2 - a^2*tan(d*x)^2*tan(c) + b^2*tan(d*x)^2*tan(c) - a^2*ta
n(d*x)*tan(c)^2 + b^2*tan(d*x)*tan(c)^2 + a^2*d*x + b^2*d*x + a*b*tan(d*x)^
2 + 4*a*b*tan(d*x)*tan(c) + a*b*tan(c)^2 + a^2*tan(d*x) - b^2*tan(d*x) + a^
2*tan(c) - b^2*tan(c) - a*b)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(
c)^2 + d)
```

3.522 $\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=88

$$-\frac{\cos^2(c+dx)\left(2ab-\left(3a^2+b^2\right)\tan(c+dx)\right)}{8d}+\frac{1}{8}x\left(3a^2+b^2\right)-\frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d}$$

```
[Out] ((3*a^2 + b^2)*x)/8 - (Cos[c + d*x]^4*(b - a*Tan[c + d*x])*(a + b*Tan[c + d
*x]))/(4*d) - (Cos[c + d*x]^2*(2*a*b - (3*a^2 + b^2)*Tan[c + d*x]))/(8*d)
```

Rubi [A] time = 0.0783186, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3506, 739, 639, 203}

$$-\frac{\cos^2(c+dx)\left(2ab-\left(3a^2+b^2\right)\tan(c+dx)\right)}{8d}+\frac{1}{8}x\left(3a^2+b^2\right)-\frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + b^2)*x)/8 - (Cos[c + d*x]^4*(b - a*Tan[c + d*x])*(a + b*Tan[c + d
*x]))/(4*d) - (Cos[c + d*x]^2*(2*a*b - (3*a^2 + b^2)*Tan[c + d*x]))/(8*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2}{\left(1+\frac{x^2}{b^2}\right)^3}dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d} + \frac{b\,\text{Subst}\left(\int \frac{1+\frac{3a^2}{b^2}+\frac{2ax}{b^2}}{\left(1+\frac{x^2}{b^2}\right)^2}\right.}{4d}$$

$$= -\frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d} - \frac{\cos^2(c+dx)\left(2ab-(\right.}{8}$$

$$= \frac{1}{8}\left(3a^2+b^2\right)x - \frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d} - \frac{\cos^2(c}{}$$

Mathematica [B] time = 2.91466, size = 216, normalized size = 2.45

$$\frac{4\left(a^2+b^2\right)\cos^4(c+dx)(a\tan(c+dx)+b)(a+b\tan(c+dx))^3 + \frac{\left(3a^2+b^2\right)\left(-\sqrt{-b^2}\left(b^4-a^4\right)\sin(2(c+dx))-2ab\sqrt{-b^2}\left(a^2+b^2\right)\cos(2(c+dx)\right.}{}}{16d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (((3*a^2 + b^2)*(2*a*b*Sqrt[-b^2]*(2*a^2 + b^2) - 2*a*b*Sqrt[-b^2]*(a^2 + b
^2)*Cos[2*(c + d*x)] + b*(a^2 + b^2)^2*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - b
*(a^2 + b^2)^2*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - Sqrt[-b^2]*(-a^4 + b^4)*S
in[2*(c + d*x)]))/Sqrt[-b^2] + 4*(a^2 + b^2)*Cos[c + d*x]^4*(b + a*Tan[c +
d*x])*(a + b*Tan[c + d*x])^3)/(16*(a^2 + b^2)^2*d)
```

Maple [A] time = 0.054, size = 97, normalized size = 1.1

$$\frac{1}{d}\left(b^2\left(-\frac{(\cos(dx+c))^3\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab(\cos(dx+c))^4}{2}+a^2\left(\frac{\sin(dx+c)}{4}\left((\text{co}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-1/2*a*b*cos(d*x+c)^4+a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+3/8*d*x+3/8*c))
```

Maple [A] time = 2.44639, size = 115, normalized size = 1.31

$$\frac{\left(3a^2+b^2\right)(dx+c)+\frac{\left(3a^2+b^2\right)\tan(dx+c)^3-4ab+\left(5a^2-b^2\right)\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + b^2)*(d*x + c) + ((3*a^2 + b^2)*tan(d*x + c)^3 - 4*a*b + (5*a
^2 - b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.80958, size = 170, normalized size = 1.93

$$-\frac{4\,ab\cos\left(dx+c\right)^{4}-\left(3\,a^{2}+b^{2}\right)dx-\left(2\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{3}+\left(3\,a^{2}+b^{2}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*cos(d*x + c)^4 - (3*a^2 + b^2)*d*x - (2*(a^2 - b^2)*cos(d*x + c
)^3 + (3*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)^{2}\cos^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**4, x)
```

Giac [B] time = 5.97267, size = 3086, normalized size = 35.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 24*a^2*d*x
*tan(d*x)^4*tan(c)^4 + 8*b^2*d*x*tan(d*x)^4*tan(c)^4 + 3*pi*b^2*sgn(-2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan
(c)^4 + 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 6*pi*b^2
*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c
)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 6*b^2*arctan((tan(d*x) +
 tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 6*b^2*arctan(-(tan(d*
x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 48*a^2*d*x*tan(d*
x)^4*tan(c)^2 + 16*b^2*d*x*tan(d*x)^4*tan(c)^2 + 6*pi*b^2*sgn(-2*tan(d*x)^2
*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2
+ 48*a^2*d*x*tan(d*x)^2*tan(c)^4 + 16*b^2*d*x*tan(d*x)^2*tan(c)^4 + 6*pi*b^
2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*t
```

```
an(d*x)^2*tan(c)^4 - 20*a*b*tan(d*x)^4*tan(c)^4 + 3*pi*b^2*sgn(2*tan(d*x)^2
*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
- 2*tan(c))*tan(d*x)^4 + 12*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*ta
n(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*t
an(c)^2 + 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)
^4*tan(c)^2 - 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan
(d*x)^4*tan(c)^2 - 40*a^2*tan(d*x)^4*tan(c)^3 + 8*b^2*tan(d*x)^4*tan(c)^3 +
 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d
*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 12*b^2*arctan((tan(d*x) +
tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 12*b^2*arctan(-(tan(d*
x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 40*a^2*tan(d*x)^3
*tan(c)^4 + 8*b^2*tan(d*x)^3*tan(c)^4 + 24*a^2*d*x*tan(d*x)^4 + 8*b^2*d*x*t
an(d*x)^4 + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan
(d*x) - 2*tan(c))*tan(d*x)^4 + 96*a^2*d*x*tan(d*x)^2*tan(c)^2 + 32*b^2*d*x*
tan(d*x)^2*tan(c)^2 + 12*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c
)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 24*a*b*tan(d*x)^4*tan(c)
^2 + 128*a*b*tan(d*x)^3*tan(c)^3 + 24*a^2*d*x*tan(c)^4 + 8*b^2*d*x*tan(c)^4
 + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2
*tan(c))*tan(c)^4 + 24*a*b*tan(d*x)^2*tan(c)^4 + 6*pi*b^2*sgn(2*tan(d*x)^2*
tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) -
 2*tan(c))*tan(d*x)^2 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -
 1))*tan(d*x)^4 - 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*
tan(d*x)^4 - 24*a^2*tan(d*x)^4*tan(c) - 8*b^2*tan(d*x)^4*tan(c) + 6*pi*b^2*
sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 24*b^2*arctan((tan(d*x) + tan(c))/(t
an(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 24*b^2*arctan(-(tan(d*x) - tan(c
))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 48*a^2*tan(d*x)^3*tan(c)^2
- 48*b^2*tan(d*x)^3*tan(c)^2 + 48*a^2*tan(d*x)^2*tan(c)^3 - 48*b^2*tan(d*x)
^2*tan(c)^3 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c
)^4 - 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^4 - 2
4*a^2*tan(d*x)*tan(c)^4 - 8*b^2*tan(d*x)*tan(c)^4 + 48*a^2*d*x*tan(d*x)^2 +
 16*b^2*d*x*tan(d*x)^2 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 12*a*b*tan(d*x)^4 + 48*a^2*d*x*
tan(c)^2 + 16*b^2*d*x*tan(c)^2 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(
d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 - 144*a*b*tan(d*x)^2*tan(c)
^2 + 12*a*b*tan(c)^4 + 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d
*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 12*b^2*arctan
((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2 - 12*b^2*arctan(-(ta
n(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 24*a^2*tan(d*x)^3 + 8*
b^2*tan(d*x)^3 - 48*a^2*tan(d*x)^2*tan(c) + 48*b^2*tan(d*x)^2*tan(c) + 12*b
^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^2 - 12*b^2*arct
an(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^2 - 48*a^2*tan(d*x)*t
an(c)^2 + 48*b^2*tan(d*x)*tan(c)^2 + 24*a^2*tan(c)^3 + 8*b^2*tan(c)^3 + 24*
a^2*d*x + 8*b^2*d*x + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c)) + 24*a*b*tan(d*x)^2 + 128*a*b*tan(d*x)*tan(c) +
 24*a*b*tan(c)^2 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))
- 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1)) + 40*a^2*tan(d*x
) - 8*b^2*tan(d*x) + 40*a^2*tan(c) - 8*b^2*tan(c) - 20*a*b)/(d*tan(d*x)^4*t
an(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4
+ 4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

3.523 $\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=163

$$\frac{5(8a^2-b^2)\tanh^{-1}(\sin(c+dx))}{128d}+\frac{(8a^2-b^2)\tan(c+dx)\sec^5(c+dx)}{48d}+\frac{5(8a^2-b^2)\tan(c+dx)\sec^3(c+dx)}{192d}+\frac{5(8a^2$$

```
[Out] (5*(8*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(128*d) + (9*a*b*Sec[c + d*x]^7)/(5
6*d) + (5*(8*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*(8*a^2 - b^
2)*Sec[c + d*x]^3*Tan[c + d*x])/(192*d) + ((8*a^2 - b^2)*Sec[c + d*x]^5*Tan
[c + d*x])/(48*d) + (b*Sec[c + d*x]^7*(a + b*Tan[c + d*x]))/(8*d)
```

Rubi [A] time = 0.131723, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3508, 3486, 3768, 3770}

$$\frac{5(8a^2-b^2)\tanh^{-1}(\sin(c+dx))}{128d}+\frac{(8a^2-b^2)\tan(c+dx)\sec^5(c+dx)}{48d}+\frac{5(8a^2-b^2)\tan(c+dx)\sec^3(c+dx)}{192d}+\frac{5(8a^2$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```


```
[Out] (5*(8*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(128*d) + (9*a*b*Sec[c + d*x]^7)/(5
6*d) + (5*(8*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*(8*a^2 - b^
2)*Sec[c + d*x]^3*Tan[c + d*x])/(192*d) + ((8*a^2 - b^2)*Sec[c + d*x]^5*Tan
[c + d*x])/(48*d) + (b*Sec[c + d*x]^7*(a + b*Tan[c + d*x]))/(8*d)
```


Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d} + \frac{1}{8}\int \sec^7(c+dx)\left(8a^2-b^2+9ab\tan(c\right.$$
$$= \frac{9ab\sec^7(c+dx)}{56d} + \frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d} + \frac{1}{8}\left(8a^2-b^2\right)\int \sec^7($$
$$= \frac{9ab\sec^7(c+dx)}{56d} + \frac{\left(8a^2-b^2\right)\sec^5(c+dx)\tan(c+dx)}{48d} + \frac{b\sec^7(c+dx)(a+}{8d}$$
$$= \frac{9ab\sec^7(c+dx)}{56d} + \frac{5\left(8a^2-b^2\right)\sec^3(c+dx)\tan(c+dx)}{192d} + \frac{\left(8a^2-b^2\right)\sec^5(c}{4}$$
$$= \frac{9ab\sec^7(c+dx)}{56d} + \frac{5\left(8a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{128d} + \frac{5\left(8a^2-b^2\right)\sec^3(}{19}$$
$$= \frac{5\left(8a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{128d} + \frac{9ab\sec^7(c+dx)}{56d} + \frac{5\left(8a^2-b^2\right)\sec(c+d}{128d}$$

Mathematica [A] time = 0.785572, size = 131, normalized size = 0.8

$$\frac{105\left(8a^2-b^2\right)\tanh^{-1}(\sin(c+dx)) + 56\left(8a^2-b^2\right)\tan(c+dx)\sec^5(c+dx) + 70\left(8a^2-b^2\right)\tan(c+dx)\sec^3(c+dx)}{2688d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (105*(8*a^2 - b^2)*ArcTanh[Sin[c + d*x]] + 105*(8*a^2 - b^2)*Sec[c + d*x]*T
an[c + d*x] + 70*(8*a^2 - b^2)*Sec[c + d*x]^3*Tan[c + d*x] + 56*(8*a^2 - b^
2)*Sec[c + d*x]^5*Tan[c + d*x] + 48*b*Sec[c + d*x]^7*(16*a + 7*b*Tan[c + d*
x]))/(2688*d)
```

Maple [A] time = 0.053, size = 235, normalized size = 1.4

$$\frac{b^2(\sin(dx+c))^3}{8d(\cos(dx+c))^8} + \frac{5b^2(\sin(dx+c))^3}{48d(\cos(dx+c))^6} + \frac{5b^2(\sin(dx+c))^3}{64d(\cos(dx+c))^4} + \frac{5b^2(\sin(dx+c))^3}{128d(\cos(dx+c))^2} + \frac{5\sin(dx+c)b^2}{128d} - \frac{5b^2\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/8/d*b^2*sin(d*x+c)^3/cos(d*x+c)^8+5/48/d*b^2*sin(d*x+c)^3/cos(d*x+c)^6+5/
64/d*b^2*sin(d*x+c)^3/cos(d*x+c)^4+5/128/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+5/
128/d*sin(d*x+c)*b^2-5/128/d*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/7/d*a*b/cos(d*
x+c)^7+1/6/d*a^2*tan(d*x+c)*sec(d*x+c)^5+5/24*a^2*sec(d*x+c)^3*tan(d*x+c)/d
+5/16*a^2*sec(d*x+c)*tan(d*x+c)/d+5/16/d*a^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.298, size = 297, normalized size = 1.82

$$\frac{7b^2\left(\frac{2\left(15\sin(dx+c)^7-55\sin(dx+c)^5+73\sin(dx+c)^3+15\sin(dx+c)\right)}{\sin(dx+c)^8-4\sin(dx+c)^6+6\sin(dx+c)^4-4\sin(dx+c)^2+1} - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1)\right) - 56a^2\left(\right.}{5376d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5376*(7*b^2*(2*(15*sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^3
 + 15*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 -
 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1
)) - 56*a^2*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x +
 c) + 1) + 15*log(sin(d*x + c) - 1)) + 1536*a*b/cos(d*x + c)^7)/d
```

Fricas [A] time = 2.04581, size = 397, normalized size = 2.44

$$\frac{105\left(8a^2-b^2\right)\cos(dx+c)^8\log(\sin(dx+c)+1)-105\left(8a^2-b^2\right)\cos(dx+c)^8\log(-\sin(dx+c)+1)+1536ab\cos(dx}{5376d\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5376*(105*(8*a^2 - b^2)*cos(d*x + c)^8*log(sin(d*x + c) + 1) - 105*(8*a^2
 - b^2)*cos(d*x + c)^8*log(-sin(d*x + c) + 1) + 1536*a*b*cos(d*x + c) + 14*
(15*(8*a^2 - b^2)*cos(d*x + c)^6 + 10*(8*a^2 - b^2)*cos(d*x + c)^4 + 8*(8*a
^2 - b^2)*cos(d*x + c)^2 + 48*b^2)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^2\sec^7(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**7, x)
```

Giac [B] time = 1.63664, size = 590, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2688*(105*(8*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(8*a^2 -
 b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(1848*a^2*tan(1/2*d*x + 1/2*c)
^15 + 105*b^2*tan(1/2*d*x + 1/2*c)^15 - 5376*a*b*tan(1/2*d*x + 1/2*c)^14 -
3416*a^2*tan(1/2*d*x + 1/2*c)^13 + 2779*b^2*tan(1/2*d*x + 1/2*c)^13 + 5376*
a*b*tan(1/2*d*x + 1/2*c)^12 + 6328*a^2*tan(1/2*d*x + 1/2*c)^11 + 6265*b^2*t
an(1/2*d*x + 1/2*c)^11 - 26880*a*b*tan(1/2*d*x + 1/2*c)^10 - 4760*a^2*tan(1
/2*d*x + 1/2*c)^9 + 12355*b^2*tan(1/2*d*x + 1/2*c)^9 + 26880*a*b*tan(1/2*d*
x + 1/2*c)^8 - 4760*a^2*tan(1/2*d*x + 1/2*c)^7 + 12355*b^2*tan(1/2*d*x + 1/
2*c)^7 - 16128*a*b*tan(1/2*d*x + 1/2*c)^6 + 6328*a^2*tan(1/2*d*x + 1/2*c)^5
 + 6265*b^2*tan(1/2*d*x + 1/2*c)^5 + 16128*a*b*tan(1/2*d*x + 1/2*c)^4 - 341
```

```
6*a^2*tan(1/2*d*x + 1/2*c)^3 + 2779*b^2*tan(1/2*d*x + 1/2*c)^3 - 768*a*b*ta
n(1/2*d*x + 1/2*c)^2 + 1848*a^2*tan(1/2*d*x + 1/2*c) + 105*b^2*tan(1/2*d*x
+ 1/2*c) + 768*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

3.524 $\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=131

$$\frac{(6a^2-b^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(6a^2-b^2)\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{(6a^2-b^2)\tan(c+dx)\sec(c+dx)}{16d}+\frac{7ab\sec^5(c}{30d}$$

```
[Out] ((6*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(16*d) + (7*a*b*Sec[c + d*x]^5)/(30*d
) + ((6*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((6*a^2 - b^2)*Sec[c
 + d*x]^3*Tan[c + d*x])/(24*d) + (b*Sec[c + d*x]^5*(a + b*Tan[c + d*x]))/(6
*d)
```

Rubi [A] time = 0.115188, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3508, 3486, 3768, 3770}

$$\frac{(6a^2-b^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(6a^2-b^2)\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{(6a^2-b^2)\tan(c+dx)\sec(c+dx)}{16d}+\frac{7ab\sec^5(c}{30d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```


```
[Out] ((6*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(16*d) + (7*a*b*Sec[c + d*x]^5)/(30*d
) + ((6*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((6*a^2 - b^2)*Sec[c
 + d*x]^3*Tan[c + d*x])/(24*d) + (b*Sec[c + d*x]^5*(a + b*Tan[c + d*x]))/(6
*d)
```


Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d} + \frac{1}{6}\int \sec^5(c+dx)\left(6a^2-b^2+7ab\tan(c\right.$$
$$= \frac{7ab\sec^5(c+dx)}{30d} + \frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d} + \frac{1}{6}\left(6a^2-b^2\right)\int \sec^5($$
$$= \frac{7ab\sec^5(c+dx)}{30d} + \frac{\left(6a^2-b^2\right)\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{b\sec^5(c+dx)(a+}{6d}$$
$$= \frac{7ab\sec^5(c+dx)}{30d} + \frac{\left(6a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{16d} + \frac{\left(6a^2-b^2\right)\sec^3(c+}{24d}$$
$$= \frac{\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{7ab\sec^5(c+dx)}{30d} + \frac{\left(6a^2-b^2\right)\sec(c+dx)}{16d}$$

Mathematica [A] time = 0.504837, size = 104, normalized size = 0.79

$$\frac{15\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))+10\left(6a^2-b^2\right)\tan(c+dx)\sec^3(c+dx)+15\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)+8}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (15*(6*a^2 - b^2)*ArcTanh[Sin[c + d*x]] + 15*(6*a^2 - b^2)*Sec[c + d*x]*Tan
[c + d*x] + 10*(6*a^2 - b^2)*Sec[c + d*x]^3*Tan[c + d*x] + 8*b*Sec[c + d*x]
^5*(12*a + 5*b*Tan[c + d*x]))/(240*d)
```

Maple [A] time = 0.052, size = 189, normalized size = 1.4

$$\frac{b^2(\sin(dx+c))^3}{6d(\cos(dx+c))^6} + \frac{b^2(\sin(dx+c))^3}{8d(\cos(dx+c))^4} + \frac{b^2(\sin(dx+c))^3}{16d(\cos(dx+c))^2} + \frac{\sin(dx+c)b^2}{16d} - \frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{16d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/6/d*b^2*sin(d*x+c)^3/cos(d*x+c)^6+1/8/d*b^2*sin(d*x+c)^3/cos(d*x+c)^4+1/1
6/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/16/d*sin(d*x+c)*b^2-1/16/d*b^2*ln(sec(d
*x+c)+tan(d*x+c))+2/5/d*a*b/cos(d*x+c)^5+1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+
3/8*a^2*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.41491, size = 243, normalized size = 1.85

$$\frac{5b^2\left(\frac{2\left(3\sin(dx+c)^5-8\sin(dx+c)^3-3\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right) - 30a^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(\right.}{\sin(dx+c)^4-2\sin(dx+}\right.}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/480*(5*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin
(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c)
```

```
 + 1) + 3*log(sin(d*x + c) - 1)) - 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x
+ c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3
*log(sin(d*x + c) - 1)) + 192*a*b/cos(d*x + c)^5)/d
```

Fricas [A] time = 1.93548, size = 343, normalized size = 2.62

$$\frac{15\left(6a^2-b^2\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\left(6a^2-b^2\right)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+192\,ab\cos\left(dx+c\right)\dots}{480\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(6*a^2 - b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(6*a^2 -
b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 192*a*b*cos(d*x + c) + 10*(3*(
6*a^2 - b^2)*cos(d*x + c)^4 + 2*(6*a^2 - b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d
*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)^2\sec^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**5, x)
```

Giac [B] time = 1.55437, size = 463, normalized size = 3.53

$$15\left(6a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(6a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(150a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+15b^2\tan\left(\frac{1}{2}dx\dots\right.}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(15*(6*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(6*a^2 - b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(150*a^2*tan(1/2*d*x + 1/2*c)^11
+ 15*b^2*tan(1/2*d*x + 1/2*c)^11 - 480*a*b*tan(1/2*d*x + 1/2*c)^10 - 210*a^
2*tan(1/2*d*x + 1/2*c)^9 + 235*b^2*tan(1/2*d*x + 1/2*c)^9 + 480*a*b*tan(1/2
*d*x + 1/2*c)^8 + 60*a^2*tan(1/2*d*x + 1/2*c)^7 + 390*b^2*tan(1/2*d*x + 1/2
*c)^7 - 960*a*b*tan(1/2*d*x + 1/2*c)^6 + 60*a^2*tan(1/2*d*x + 1/2*c)^5 + 39
0*b^2*tan(1/2*d*x + 1/2*c)^5 + 960*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^2*tan
(1/2*d*x + 1/2*c)^3 + 235*b^2*tan(1/2*d*x + 1/2*c)^3 - 96*a*b*tan(1/2*d*x +
 1/2*c)^2 + 150*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c) + 96
*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.525 $\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=99

$$\frac{(4a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2-b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{5ab\sec^3(c+dx)}{12d}+\frac{b\sec^3(c+dx)(a+b\tan(c$$

```
[Out] ((4*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a*b*Sec[c + d*x]^3)/(12*d)
 + ((4*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*(a +
 b*Tan[c + d*x]))/(4*d)
```

Rubi [A] time = 0.0988038, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3508, 3486, 3768, 3770}

$$\frac{(4a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2-b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{5ab\sec^3(c+dx)}{12d}+\frac{b\sec^3(c+dx)(a+b\tan(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((4*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a*b*Sec[c + d*x]^3)/(12*d)
 + ((4*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*(a +
 b*Tan[c + d*x]))/(4*d)
```

Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d} + \frac{1}{4}\int \sec^3(c+dx)\left(4a^2-b^2+5ab\tan(c+d\ldots\right.$$
$$= \frac{5ab\sec^3(c+dx)}{12d} + \frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d} + \frac{1}{4}\left(4a^2-b^2\right)\int \sec^3(c+\ldots$$
$$= \frac{5ab\sec^3(c+dx)}{12d} + \frac{\left(4a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)(a+b\,\mathrm{ta}\ldots}{4d}$$
$$= \frac{\left(4a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{5ab\sec^3(c+dx)}{12d} + \frac{\left(4a^2-b^2\right)\sec(c+dx)\tan\ldots}{8d}$$

Mathematica [A] time = 0.0594228, size = 120, normalized size = 1.21

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2\tan(c+dx)\sec(c+dx)}{2d} + \frac{2ab\sec^3(c+dx)}{3d} - \frac{b^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b^2\tan(c+dx)\sec^3(\ldots}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*
a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.054, size = 143, normalized size = 1.4

$$\frac{b^2(\sin(dx+c))^3}{4d(\cos(dx+c))^4} + \frac{b^2(\sin(dx+c))^3}{8d(\cos(dx+c))^2} + \frac{\sin(dx+c)b^2}{8d} - \frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{2ab}{3d(\cos(dx+c))^3} + \frac{a^2 \mathrm{s}\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/4/d*b^2*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/8
/d*sin(d*x+c)*b^2-1/8/d*b^2*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a*b/cos(d*x+c)^
3+1/2*a^2*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.42239, size = 174, normalized size = 1.76

$$\frac{3b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 12a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \ldots\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
+ c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^2*(2*si
n(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
- 1)) + 32*a*b/cos(d*x + c)^3)/d
```

Fricas [A] time = 2.04285, size = 289, normalized size = 2.92

$$\frac{3\left(4a^2-b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(4a^2-b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+32ab\cos\left(dx+c\right)}{48d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(4*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 - b^2
)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 32*a*b*cos(d*x + c) + 6*((4*a^2 -
 b^2)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)^2\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [B] time = 1.49328, size = 336, normalized size = 3.39

$$\frac{3\left(4a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(4a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+3b^2\tan\left(\frac{1}{2}dx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(4*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 - b^2)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3*b
^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*tan(1/2*
d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 + 48*a*b*tan(1/2*d*x + 1/2*c
)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*a*
b*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x
+ 1/2*c) + 16*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.526 $\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=65

$$\frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab\sec(c+dx)}{2d}+\frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}$$

```
[Out] ((2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b*Sec[c + d*x])/(2*d) +
(b*Sec[c + d*x]*(a + b*Tan[c + d*x]))/(2*d)
```

Rubi [A] time = 0.051964, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3508, 3486, 3770}

$$\frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab\sec(c+dx)}{2d}+\frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b*Sec[c + d*x])/(2*d) +
(b*Sec[c + d*x]*(a + b*Tan[c + d*x]))/(2*d)
```

Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}+\frac{1}{2}\int \sec(c+dx)\left(2a^2-b^2+3ab\tan(c+dx)\right) \\
&= \frac{3ab\sec(c+dx)}{2d}+\frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}+\frac{1}{2}\left(2a^2-b^2\right)\int \sec(c+dx) \\
&= \frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab\sec(c+dx)}{2d}+\frac{b\sec(c+dx)(a+b\tan(c+
\end{aligned}$$

Mathematica [A] time = 0.038608, size = 67, normalized size = 1.03

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*
Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.021, size = 98, normalized size = 1.5

$$\frac{b^2(\sin(dx+c))^3}{2d(\cos(dx+c))^2}+\frac{\sin(dx+c)b^2}{2d}-\frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{2d}+2\frac{ab}{d\cos(dx+c)}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/2/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/2/d*sin(d*x+c)*b^2-1/2/d*b^2*ln(sec(d
*x+c)+tan(d*x+c))+2/d*a*b/cos(d*x+c)+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.36742, size = 111, normalized size = 1.71

$$-\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-4a^2\log(\sec(dx+c)+\tan(dx+c))-\frac{8ab}{\cos(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a*b/cos(d
*x + c))/d
```

Fricas [A] time = 1.79038, size = 234, normalized size = 3.6

$$\frac{\left(2a^2-b^2\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-\left(2a^2-b^2\right)\cos(dx+c)^2\log(-\sin(dx+c)+1)+8ab\cos(dx+c)+2b^2\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 - b^2)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 8*a*b*cos(d*x + c) + 2*b^2*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^2 \sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] time = 1.48975, size = 165, normalized size = 2.54

$$\frac{(2a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(2a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+b^2\tan\ldots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 - b^2)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1
/2*d*x + 1/2*c)^2 + b^2*tan(1/2*d*x + 1/2*c) + 4*a*b)/(tan(1/2*d*x + 1/2*c)
^2 - 1)^2)/d
```

3.527 $\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=47

$$\frac{(a^2-b^2)\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Cos[c + d*x])/d + ((a^2 - b^2)*Sin[c
 + d*x])/d
```

Rubi [A] time = 0.0332454, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3507}

$$\frac{(a^2-b^2)\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Cos[c + d*x])/d + ((a^2 - b^2)*Sin[c
 + d*x])/d
```

Rule 3507

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2/sec[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Simp[(b^2*ArcTanh[Sin[e + f*x]])/f, x] + (-Simp[(2*a*b*Cos[e + f*x]
)/f, x] + Simp[((a^2 - b^2)*Sin[e + f*x])/f, x]) /; FreeQ[{a, b, e, f}, x]
&& NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{(a^2-b^2)\sin(c+dx)}{d}$$

Mathematica [A] time = 0.138971, size = 84, normalized size = 1.79

$$\frac{(a^2-b^2)\sin(c+dx)-2ab\cos(c+dx)+b^2\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x] + b^2*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a^2 - b^2)*Sin[c + d*x])/d
```

Maple [A] time = 0.046, size = 63, normalized size = 1.3

$$\frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\sin(dx+c)}{d}-\frac{\sin(dx+c)\,b^2}{d}-2\,\frac{ab\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*b^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*sin(d*x+c)/d-1/d*sin(d*x+c)*b^2-2*a*b
*cos(d*x+c)/d
```

Maxima [A] time = 1.38506, size = 81, normalized size = 1.72

$$\frac{b^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))-4ab\cos(dx+c)+2a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) -
 4*a*b*cos(d*x + c) + 2*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.80178, size = 155, normalized size = 3.3

$$-\frac{4ab\cos(dx+c)-b^2\log(\sin(dx+c)+1)+b^2\log(-\sin(dx+c)+1)-2\left(a^2-b^2\right)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b*cos(d*x + c) - b^2*log(sin(d*x + c) + 1) + b^2*log(-sin(d*x + c
) + 1) - 2*(a^2 - b^2)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^2\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x), x)
```

Giac [B] time = 1.94314, size = 1777, normalized size = 37.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(
1/2*d*x)^2*tan(1/2*c)^2 - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + ta
n(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*tan(1/2*d*x)^2*tan(1/2*
c)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan
(1/2*d*x)^2 - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
1))*tan(1/2*d*x)^2 + 4*a^2*tan(1/2*d*x)^2*tan(1/2*c) - 4*b^2*tan(1/2*d*x)^2
*tan(1/2*c) + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) +
1))*tan(1/2*c)^2 - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d
*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*
c) + 1))*tan(1/2*c)^2 + 4*a^2*tan(1/2*d*x)*tan(1/2*c)^2 - 4*b^2*tan(1/2*d*x
)*tan(1/2*c)^2 - 4*a*b*tan(1/2*d*x)^2 - 16*a*b*tan(1/2*d*x)*tan(1/2*c) - 4*
a*b*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d
*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*
c) + 1)) - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))
 - 4*a^2*tan(1/2*d*x) + 4*b^2*tan(1/2*d*x) - 4*a^2*tan(1/2*c) + 4*b^2*tan(1
/2*c) + 4*a*b)/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/
2*c)^2 + d)
```

3.528 $\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=90

$$-\frac{(2a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(2a^2+b^2)\sin(c+dx)}{2d}-\frac{ab\cos^3(c+dx)}{6d}-\frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d}$$

```
[Out] -(a*b*Cos[c + d*x]^3)/(6*d) + ((2*a^2 + b^2)*Sin[c + d*x])/(2*d) - ((2*a^2
+ b^2)*Sin[c + d*x]^3)/(6*d) - (b*Cos[c + d*x]^3*(a + b*Tan[c + d*x]))/(2*d
)
```

Rubi [A] time = 0.0945082, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3508, 3486, 2633}

$$-\frac{(2a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(2a^2+b^2)\sin(c+dx)}{2d}-\frac{ab\cos^3(c+dx)}{6d}-\frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(a*b*Cos[c + d*x]^3)/(6*d) + ((2*a^2 + b^2)*Sin[c + d*x])/(2*d) - ((2*a^2
+ b^2)*Sin[c + d*x]^3)/(6*d) - (b*Cos[c + d*x]^3*(a + b*Tan[c + d*x]))/(2*d
)
```

Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d} - \frac{1}{2}\int \cos^3(c+dx)\left(-2a^2-b^2-ab\tan(\right.$$
$$= -\frac{ab\cos^3(c+dx)}{6d} - \frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d} - \frac{1}{2}\left(-2a^2-b^2\right)\int \cos$$
$$= -\frac{ab\cos^3(c+dx)}{6d} - \frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d} - \frac{\left(2a^2+b^2\right)\text{Subst}\left(\int\right.}{}$$
$$= -\frac{ab\cos^3(c+dx)}{6d} + \frac{\left(2a^2+b^2\right)\sin(c+dx)}{2d} - \frac{\left(2a^2+b^2\right)\sin^3(c+dx)}{6d} - \frac{b\cos}{}$$

Mathematica [A] time = 0.456233, size = 64, normalized size = 0.71

$$\frac{\sin(c+dx)\left(\left(a^2-b^2\right)\cos(2(c+dx))+5a^2+b^2\right)-3ab\cos(c+dx)-ab\cos(3(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a*b*Cos[c + d*x] - a*b*Cos[3*(c + d*x)] + (5*a^2 + b^2 + (a^2 - b^2)*Co
s[2*(c + d*x)])*Sin[c + d*x])/(6*d)
```

Maple [A] time = 0.056, size = 52, normalized size = 0.6

$$\frac{1}{d}\left(\frac{b^2(\sin(dx+c))^3}{3} - \frac{2ab(\cos(dx+c))^3}{3} + \frac{a^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*b^2*sin(d*x+c)^3-2/3*a*b*cos(d*x+c)^3+1/3*a^2*(2+cos(d*x+c)^2)*sin
(d*x+c))
```

Maxima [A] time = 1.43123, size = 70, normalized size = 0.78

$$-\frac{2ab\cos(dx+c)^3 - b^2\sin(dx+c)^3 + \left(\sin(dx+c)^3 - 3\sin(dx+c)\right)a^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d
*x + c))*a^2)/d
```

Fricas [A] time = 1.86064, size = 120, normalized size = 1.33

$$-\frac{2ab\cos(dx+c)^3 - \left(\left(a^2-b^2\right)\cos(dx+c)^2 + 2a^2 + b^2\right)\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - ((a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 + b^2)*sin
(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\tan(c + dx))^2 \cos^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.529 $\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=114

$$\frac{(4a^2+b^2)\sin^5(c+dx)}{20d}-\frac{(4a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(4a^2+b^2)\sin(c+dx)}{4d}-\frac{3ab\cos^5(c+dx)}{20d}-\frac{b\cos^5(c+dx)(a+b}{4d}$$

```
[Out] (-3*a*b*Cos[c + d*x]^5)/(20*d) + ((4*a^2 + b^2)*Sin[c + d*x])/(4*d) - ((4*a
^2 + b^2)*Sin[c + d*x]^3)/(6*d) + ((4*a^2 + b^2)*Sin[c + d*x]^5)/(20*d) - (
b*Cos[c + d*x]^5*(a + b*Tan[c + d*x]))/(4*d)
```

Rubi [A] time = 0.102317, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3508, 3486, 2633}

$$\frac{(4a^2+b^2)\sin^5(c+dx)}{20d}-\frac{(4a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(4a^2+b^2)\sin(c+dx)}{4d}-\frac{3ab\cos^5(c+dx)}{20d}-\frac{b\cos^5(c+dx)(a+b}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a*b*Cos[c + d*x]^5)/(20*d) + ((4*a^2 + b^2)*Sin[c + d*x])/(4*d) - ((4*a
^2 + b^2)*Sin[c + d*x]^3)/(6*d) + ((4*a^2 + b^2)*Sin[c + d*x]^5)/(20*d) - (
b*Cos[c + d*x]^5*(a + b*Tan[c + d*x]))/(4*d)
```

Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d} - \frac{1}{4}\int \cos^5(c+dx)\left(-4a^2-b^2-3ab\tan(c\right.$$
$$= -\frac{3ab\cos^5(c+dx)}{20d} - \frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d} - \frac{1}{4}\left(-4a^2-b^2\right)\int \cos^5($$
$$= -\frac{3ab\cos^5(c+dx)}{20d} - \frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d} - \frac{\left(4a^2+b^2\right)\text{Subst}\left(\int (\right.}{}$$
$$= -\frac{3ab\cos^5(c+dx)}{20d} + \frac{\left(4a^2+b^2\right)\sin(c+dx)}{4d} - \frac{\left(4a^2+b^2\right)\sin^3(c+dx)}{6d} + \frac{(4a^2+}{}$$

Mathematica [A] time = 0.210422, size = 116, normalized size = 1.02

$$\frac{150a^2\sin(c+dx)+25a^2\sin(3(c+dx))+3a^2\sin(5(c+dx))-60ab\cos(c+dx)-30ab\cos(3(c+dx))-6ab\cos(5(c+dx}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-60*a*b*Cos[c + d*x] - 30*a*b*Cos[3*(c + d*x)] - 6*a*b*Cos[5*(c + d*x)] +
150*a^2*Sin[c + d*x] + 30*b^2*Sin[c + d*x] + 25*a^2*Sin[3*(c + d*x)] - 5*b^
2*Sin[3*(c + d*x)] + 3*a^2*Sin[5*(c + d*x)] - 3*b^2*Sin[5*(c + d*x)])/(240*
d)
```

Maple [A] time = 0.055, size = 88, normalized size = 0.8

$$\frac{1}{d}\left(b^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{15}\right)-\frac{2ab(\cos(dx+c))^5}{5}+\frac{a^2\sin(dx+c)}{5}\left(\frac{8}{3}+(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-2/
5*a*b*cos(d*x+c)^5+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.36548, size = 104, normalized size = 0.91

$$-\frac{6\,ab\cos(dx+c)^5-\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)a^2+\left(3\,\sin(dx+c)^5-5\,\sin(dx+c)^3\right)b^2}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*si
n(d*x + c))*a^2 + (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*b^2)/d
```

Fricas [A] time = 1.90536, size = 169, normalized size = 1.48

$$-\frac{6\,ab\cos\left(dx+c\right)^{5}-\left(3\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}+\left(4\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}+8\,a^{2}+2\,b^{2}\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*(a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 + b^2)
*cos(d*x + c)^2 + 8*a^2 + 2*b^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.530 $\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=138

$$-\frac{(6a^2+b^2)\sin^7(c+dx)}{42d}+\frac{(6a^2+b^2)\sin^5(c+dx)}{10d}-\frac{(6a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(6a^2+b^2)\sin(c+dx)}{6d}-\frac{5ab\cos^7(c+dx}{42d}$$

```
[Out] (-5*a*b*Cos[c + d*x]^7)/(42*d) + ((6*a^2 + b^2)*Sin[c + d*x])/(6*d) - ((6*a
^2 + b^2)*Sin[c + d*x]^3)/(6*d) + ((6*a^2 + b^2)*Sin[c + d*x]^5)/(10*d) - (
(6*a^2 + b^2)*Sin[c + d*x]^7)/(42*d) - (b*Cos[c + d*x]^7*(a + b*Tan[c + d*x
]))/(6*d)
```

Rubi [A] time = 0.107396, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3508, 3486, 2633}

$$-\frac{(6a^2+b^2)\sin^7(c+dx)}{42d}+\frac{(6a^2+b^2)\sin^5(c+dx)}{10d}-\frac{(6a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(6a^2+b^2)\sin(c+dx)}{6d}-\frac{5ab\cos^7(c+dx}{42d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-5*a*b*Cos[c + d*x]^7)/(42*d) + ((6*a^2 + b^2)*Sin[c + d*x])/(6*d) - ((6*a
^2 + b^2)*Sin[c + d*x]^3)/(6*d) + ((6*a^2 + b^2)*Sin[c + d*x]^5)/(10*d) - (
(6*a^2 + b^2)*Sin[c + d*x]^7)/(42*d) - (b*Cos[c + d*x]^7*(a + b*Tan[c + d*x
]))/(6*d)
```

Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d} - \frac{1}{6}\int \cos^7(c+dx)\left(-6a^2-b^2-5ab\tan\right.$$
$$= -\frac{5ab\cos^7(c+dx)}{42d} - \frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d} - \frac{1}{6}\left(-6a^2-b^2\right)\int \mathrm{co}$$
$$= -\frac{5ab\cos^7(c+dx)}{42d} - \frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d} - \frac{\left(6a^2+b^2\right)\mathrm{Subst}\left(\right.}{}$$
$$= -\frac{5ab\cos^7(c+dx)}{42d} + \frac{\left(6a^2+b^2\right)\sin(c+dx)}{6d} - \frac{\left(6a^2+b^2\right)\sin^3(c+dx)}{6d} + \frac{(6a}{}$$

Mathematica [A] time = 0.426256, size = 154, normalized size = 1.12

$$-\frac{-3675a^2\sin(c+dx) - 735a^2\sin(3(c+dx)) - 147a^2\sin(5(c+dx)) - 15a^2\sin(7(c+dx)) + 1050ab\cos(c+dx) + 630}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(1050*a*b*Cos[c + d*x] + 630*a*b*Cos[3*(c + d*x)] + 210*a*b*Cos[5*(c + d*x
)] + 30*a*b*Cos[7*(c + d*x)] - 3675*a^2*Sin[c + d*x] - 525*b^2*Sin[c + d*x]
 - 735*a^2*Sin[3*(c + d*x)] + 35*b^2*Sin[3*(c + d*x)] - 147*a^2*Sin[5*(c +
d*x)] + 63*b^2*Sin[5*(c + d*x)] - 15*a^2*Sin[7*(c + d*x)] + 15*b^2*Sin[7*(c
 + d*x)])/(6720*d)
```

Maple [A] time = 0.057, size = 108, normalized size = 0.8

$$\frac{1}{d}\left(b^2\left(-\frac{(\cos(dx+c))^6\sin(dx+c)}{7} + \frac{\sin(dx+c)}{35}\left(\frac{8}{3} + (\cos(dx+c))^4 + \frac{4(\cos(dx+c))^2}{3}\right)\right) - \frac{2ab(\cos(dx+c))^7}{7} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/7*cos(d*x+c)^6*sin(d*x+c)+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-2/7*a*b*cos(d*x+c)^7+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*cos(d*
x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.5216, size = 132, normalized size = 0.96

$$-\frac{30\,ab\cos(dx+c)^7 + 3\left(5\sin(dx+c)^7 - 21\sin(dx+c)^5 + 35\sin(dx+c)^3 - 35\sin(dx+c)\right)a^2 - \left(15\sin(dx+c)^7\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 3
5*sin(d*x + c)^3 - 35*sin(d*x + c))*a^2 - (15*sin(d*x + c)^7 - 42*sin(d*x +
 c)^5 + 35*sin(d*x + c)^3)*b^2)/d
```

Fricas [A] time = 1.93409, size = 221, normalized size = 1.6

$$-\frac{30\,ab\cos\left(dx+c\right)^{7}-\left(15\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{6}+3\left(6\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{4}+4\left(6\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}+48\,a^{2}+8\,b^{2}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 - (15*(a^2 - b^2)*cos(d*x + c)^6 + 3*(6*a^2 +
 b^2)*cos(d*x + c)^4 + 4*(6*a^2 + b^2)*cos(d*x + c)^2 + 48*a^2 + 8*b^2)*sin
(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.531 $\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=194

$$\frac{a\left(a^2+9b^2\right)\tan^7(c+dx)}{7d}+\frac{3a\left(a^2+3b^2\right)\tan^5(c+dx)}{5d}+\frac{a\left(a^2+b^2\right)\tan^3(c+dx)}{d}+\frac{3a^2b\sec^8(c+dx)}{8d}+\frac{a^3\tan(c+\ldots}{d}$$

```
[Out] (3*a^2*b*Sec[c + d*x]^8)/(8*d) + (a^3*Tan[c + d*x])/d + (a*(a^2 + b^2)*Tan[
c + d*x]^3)/d + (b^3*Tan[c + d*x]^4)/(4*d) + (3*a*(a^2 + 3*b^2)*Tan[c + d*x
]^5)/(5*d) + (b^3*Tan[c + d*x]^6)/(2*d) + (a*(a^2 + 9*b^2)*Tan[c + d*x]^7)/
(7*d) + (3*b^3*Tan[c + d*x]^8)/(8*d) + (a*b^2*Tan[c + d*x]^9)/(3*d) + (b^3*
Tan[c + d*x]^10)/(10*d)
```

Rubi [A] time = 0.147028, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3506, 696, 1810}

$$\frac{a\left(a^2+9b^2\right)\tan^7(c+dx)}{7d}+\frac{3a\left(a^2+3b^2\right)\tan^5(c+dx)}{5d}+\frac{a\left(a^2+b^2\right)\tan^3(c+dx)}{d}+\frac{3a^2b\sec^8(c+dx)}{8d}+\frac{a^3\tan(c+\ldots}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^3,x]
```


```
[Out] (3*a^2*b*Sec[c + d*x]^8)/(8*d) + (a^3*Tan[c + d*x])/d + (a*(a^2 + b^2)*Tan[
c + d*x]^3)/d + (b^3*Tan[c + d*x]^4)/(4*d) + (3*a*(a^2 + 3*b^2)*Tan[c + d*x
]^5)/(5*d) + (b^3*Tan[c + d*x]^6)/(2*d) + (a*(a^2 + 9*b^2)*Tan[c + d*x]^7)/
(7*d) + (3*b^3*Tan[c + d*x]^8)/(8*d) + (a*b^2*Tan[c + d*x]^9)/(3*d) + (b^3*
Tan[c + d*x]^10)/(10*d)
```


Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 696

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*m*d^(m - 1)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)^3\,dx,x,b\tan(c+dx)\right)}{bd}$$

$$= \frac{3a^2b\sec^8(c+dx)}{8d} + \frac{\text{Subst}\left(\int \left(1+\frac{x^2}{b^2}\right)^3\left(-3a^2x+(a+x)^3\right)\,dx,x,b\tan(c+dx)\right)}{bd}$$

$$= \frac{3a^2b\sec^8(c+dx)}{8d} + \frac{\text{Subst}\left(\int \left(a^3+\frac{3a(a^2+b^2)x^2}{b^2}+x^3+\frac{3a(a^2+3b^2)x^4}{b^4}+\frac{3x^5}{b^2}+\frac{a(a^2+9}{b}\right.}{bd}$$

$$= \frac{3a^2b\sec^8(c+dx)}{8d} + \frac{a^3\tan(c+dx)}{d} + \frac{a\left(a^2+b^2\right)\tan^3(c+dx)}{d} + \frac{b^3\tan^4(c+dx}{4d}$$

Mathematica [A] time = 1.95957, size = 177, normalized size = 0.91

$$\frac{\frac{3}{8}\left(5a^2+b^2\right)(a+b\tan(c+dx))^8-\frac{4}{7}a\left(5a^2+3b^2\right)(a+b\tan(c+dx))^7+\frac{1}{2}\left(a^2+b^2\right)\left(5a^2+b^2\right)(a+b\tan(c+dx))^6-\frac{6}{5}a}{b^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^3*(a + b*Tan[c + d*x])^4)/4 - (6*a*(a^2 + b^2)^2*(a + b*Tan[c
 + d*x])^5)/5 + ((a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/2 - (4*a
*(5*a^2 + 3*b^2)*(a + b*Tan[c + d*x])^7)/7 + (3*(5*a^2 + b^2)*(a + b*Tan[c
+ d*x])^8)/8 - (2*a*(a + b*Tan[c + d*x])^9)/3 + (a + b*Tan[c + d*x])^10/10)
/(b^7*d)
```

Maple [A] time = 0.072, size = 219, normalized size = 1.1

$$\frac{1}{d}\left(b^3\left(\frac{(\sin(dx+c))^4}{10\,(\cos(dx+c))^{10}}+\frac{3\,(\sin(dx+c))^4}{40\,(\cos(dx+c))^8}+\frac{(\sin(dx+c))^4}{20\,(\cos(dx+c))^6}+\frac{(\sin(dx+c))^4}{40\,(\cos(dx+c))^4}\right)+3\,ab^2\left(1/9\,\frac{(\sin(dx+c))^3}{(\cos(dx+c))^9}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/
20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4)+3*a*b^2*(1/9*s
in(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/
cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+3/8*b*a^2/cos(d*x+c)^8-a^3*(
-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.18849, size = 238, normalized size = 1.23

$$\frac{84\,b^3\tan(dx+c)^{10}+280\,ab^2\tan(dx+c)^9+315\left(a^2b+b^3\right)\tan(dx+c)^8+120\left(a^3+9\,ab^2\right)\tan(dx+c)^7+420\left(3\,a^2b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*(a^2*b + b^3
)*tan(d*x + c)^8 + 120*(a^3 + 9*a*b^2)*tan(d*x + c)^7 + 420*(3*a^2*b + b^3)
*tan(d*x + c)^6 + 504*(a^3 + 3*a*b^2)*tan(d*x + c)^5 + 1260*a^2*b*tan(d*x +
 c)^2 + 210*(9*a^2*b + b^3)*tan(d*x + c)^4 + 840*a^3*tan(d*x + c) + 840*(a^
3 + a*b^2)*tan(d*x + c)^3)/d
```

Fricas [A] time = 2.06488, size = 344, normalized size = 1.77

$$\frac{84\,b^3 + 105\left(3\,a^2b - b^3\right)\cos\left(dx + c\right)^2 + 8\left(16\left(3\,a^3 - ab^2\right)\cos\left(dx + c\right)^9 + 8\left(3\,a^3 - ab^2\right)\cos\left(dx + c\right)^7 + 6\left(3\,a^3 - ab^2\right)\cos\cdots}{840\,d\cos\left(dx + c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(84*b^3 + 105*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(16*(3*a^3 - a*b^2)*
cos(d*x + c)^9 + 8*(3*a^3 - a*b^2)*cos(d*x + c)^7 + 6*(3*a^3 - a*b^2)*cos(d
*x + c)^5 + 35*a*b^2*cos(d*x + c) + 5*(3*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d
*x + c))/(d*cos(d*x + c)^10)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan\left(c + dx\right)\right)^3 \sec^8\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**8, x)
```

Giac [A] time = 1.8627, size = 297, normalized size = 1.53

$$\frac{84\,b^3\tan\left(dx + c\right)^{10} + 280\,ab^2\tan\left(dx + c\right)^9 + 315\,a^2b\tan\left(dx + c\right)^8 + 315\,b^3\tan\left(dx + c\right)^8 + 120\,a^3\tan\left(dx + c\right)^7 + 10\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*a^2*b*tan(d*
x + c)^8 + 315*b^3*tan(d*x + c)^8 + 120*a^3*tan(d*x + c)^7 + 1080*a*b^2*tan
(d*x + c)^7 + 1260*a^2*b*tan(d*x + c)^6 + 420*b^3*tan(d*x + c)^6 + 504*a^3*
tan(d*x + c)^5 + 1512*a*b^2*tan(d*x + c)^5 + 1890*a^2*b*tan(d*x + c)^4 + 21
0*b^3*tan(d*x + c)^4 + 840*a^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 +
1260*a^2*b*tan(d*x + c)^2 + 840*a^3*tan(d*x + c))/d
```

3.532 $\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=138

$$\frac{(3a^2+b^2)(a+b\tan(c+dx))^6}{3b^5d}-\frac{4a(a^2+b^2)(a+b\tan(c+dx))^5}{5b^5d}+\frac{(a^2+b^2)^2(a+b\tan(c+dx))^4}{4b^5d}+\frac{(a+b\tan(c+dx}{8b^5d}$$

```
[Out] ((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/(4*b^5*d) - (4*a*(a^2 + b^2)*(a + b*
Tan[c + d*x])^5)/(5*b^5*d) + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/(3*b^5*
d) - (4*a*(a + b*Tan[c + d*x])^7)/(7*b^5*d) + (a + b*Tan[c + d*x])^8/(8*b^5
*d)
```

Rubi [A] time = 0.125019, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$\frac{(3a^2+b^2)(a+b\tan(c+dx))^6}{3b^5d}-\frac{4a(a^2+b^2)(a+b\tan(c+dx))^5}{5b^5d}+\frac{(a^2+b^2)^2(a+b\tan(c+dx))^4}{4b^5d}+\frac{(a+b\tan(c+dx}{8b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/(4*b^5*d) - (4*a*(a^2 + b^2)*(a + b*
Tan[c + d*x])^5)/(5*b^5*d) + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/(3*b^5*
d) - (4*a*(a + b*Tan[c + d*x])^7)/(7*b^5*d) + (a + b*Tan[c + d*x])^8/(8*b^5
*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)^2\,dx, x, b\tan(c+dx)\right)}{bd}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)^2(a+x)^3}{b^4}-\frac{4a(a^2+b^2)(a+x)^4}{b^4}+\frac{2(3a^2+b^2)(a+x)^5}{b^4}-\frac{4a(a+x)^6}{b^4}+\frac{(a+x)^7}{b^4}\right)dx,\right.}{bd}\\
&= \frac{(a^2+b^2)^2(a+b\tan(c+dx))^4}{4b^5d}-\frac{4a(a^2+b^2)(a+b\tan(c+dx))^5}{5b^5d}+\frac{(3a^2+b^2)}{}
\end{aligned}$$

Mathematica [A] time = 0.570698, size = 115, normalized size = 0.83

$$\frac{\frac{1}{3}\left(3a^2+b^2\right)(a+b\tan(c+dx))^6-\frac{4}{5}a\left(a^2+b^2\right)(a+b\tan(c+dx))^5+\frac{1}{4}\left(a^2+b^2\right)^2(a+b\tan(c+dx))^4+\frac{1}{8}(a+b\tan}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/4 - (4*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^5)/5 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/3 - (4*a*(a + b*Tan[c
+ d*x])^7)/7 + (a + b*Tan[c + d*x])^8/8)/(b^5*d)
```

Maple [A] time = 0.069, size = 173, normalized size = 1.3

$$\frac{1}{d}\left(b^3\left(\frac{(\sin(dx+c))^4}{8(\cos(dx+c))^8}+\frac{(\sin(dx+c))^4}{12(\cos(dx+c))^6}+\frac{(\sin(dx+c))^4}{24(\cos(dx+c))^4}\right)+3ab^2\left(1/7\frac{(\sin(dx+c))^3}{(\cos(dx+c))^7}+\frac{4(\sin(dx+c))^3}{35(\cos(dx+c))^5}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24
*sin(d*x+c)^4/cos(d*x+c)^4)+3*a*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin
(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+1/2*b*a^2/cos(d*x+c
)^6-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.16301, size = 192, normalized size = 1.39

$$\frac{105\,b^3\tan(dx+c)^8+360\,ab^2\tan(dx+c)^7+140\left(3\,a^2b+2\,b^3\right)\tan(dx+c)^6+168\left(a^3+6\,ab^2\right)\tan(dx+c)^5+1260}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 140*(3*a^2*b + 2
*b^3)*tan(d*x + c)^6 + 168*(a^3 + 6*a*b^2)*tan(d*x + c)^5 + 1260*a^2*b*tan(
d*x + c)^2 + 210*(6*a^2*b + b^3)*tan(d*x + c)^4 + 840*a^3*tan(d*x + c) + 28
0*(2*a^3 + 3*a*b^2)*tan(d*x + c)^3)/d
```

Fricas [A] time = 2.12018, size = 304, normalized size = 2.2

$$\frac{105\,b^3+140\left(3\,a^2b-b^3\right)\cos(dx+c)^2+8\left(8\left(7\,a^3-3\,ab^2\right)\cos(dx+c)^7+4\left(7\,a^3-3\,ab^2\right)\cos(dx+c)^5+45\,ab^2\cos(\right.}{840\,d\cos(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^3 + 140*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(8*(7*a^3 - 3*a*b^2
)*cos(d*x + c)^7 + 4*(7*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 45*a*b^2*cos(d*x +
c) + 3*(7*a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\tan(c + dx))^3 \sec^6(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**6, x)
```

Giac [A] time = 1.76878, size = 224, normalized size = 1.62

$$\frac{105\, b^3 \tan(dx+c)^8 + 360\, ab^2 \tan(dx+c)^7 + 420\, a^2 b \tan(dx+c)^6 + 280\, b^3 \tan(dx+c)^6 + 168\, a^3 \tan(dx+c)^5 + 1008}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 420*a^2*b*tan(d*
x + c)^6 + 280*b^3*tan(d*x + c)^6 + 168*a^3*tan(d*x + c)^5 + 1008*a*b^2*tan
(d*x + c)^5 + 1260*a^2*b*tan(d*x + c)^4 + 210*b^3*tan(d*x + c)^4 + 560*a^3*
tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 + 1260*a^2*b*tan(d*x + c)^2 + 840
*a^3*tan(d*x + c))/d
```

3.533 $\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=75

$$\frac{(a^2+b^2)(a+b\tan(c+dx))^4}{4b^3d}+\frac{(a+b\tan(c+dx))^6}{6b^3d}-\frac{2a(a+b\tan(c+dx))^5}{5b^3d}$$

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^4)/(4*b^3*d) - (2*a*(a + b*Tan[c + d*x])^
5)/(5*b^3*d) + (a + b*Tan[c + d*x])^6/(6*b^3*d)
```

Rubi [A] time = 0.0708607, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$\frac{(a^2+b^2)(a+b\tan(c+dx))^4}{4b^3d}+\frac{(a+b\tan(c+dx))^6}{6b^3d}-\frac{2a(a+b\tan(c+dx))^5}{5b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^4)/(4*b^3*d) - (2*a*(a + b*Tan[c + d*x])^
5)/(5*b^3*d) + (a + b*Tan[c + d*x])^6/(6*b^3*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)dx,x,b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)(a+x)^3}{b^2}-\frac{2a(a+x)^4}{b^2}+\frac{(a+x)^5}{b^2}\right)dx,x,b\tan(c+dx)\right)}{bd}\\ &= \frac{(a^2+b^2)(a+b\tan(c+dx))^4}{4b^3d}-\frac{2a(a+b\tan(c+dx))^5}{5b^3d}+\frac{(a+b\tan(c+dx)}{6b^3d}\end{aligned}$$

Mathematica [A] time = 0.346668, size = 54, normalized size = 0.72

$$\frac{(a+b\tan(c+dx))^4\left(a^2-4ab\tan(c+dx)+10b^2\tan^2(c+dx)+15b^2\right)}{60b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a + b*Tan[c + d*x])^4*(a^2 + 15*b^2 - 4*a*b*Tan[c + d*x] + 10*b^2*Tan[c +
 d*x]^2))/(60*b^3*d)
```

Maple [A] time = 0.063, size = 127, normalized size = 1.7

$$\frac{1}{d}\left(b^3\left(\frac{(\sin(dx+c))^4}{6(\cos(dx+c))^6}+\frac{(\sin(dx+c))^4}{12(\cos(dx+c))^4}\right)+3ab^2\left(1/5\frac{(\sin(dx+c))^3}{(\cos(dx+c))^5}+2/15\frac{(\sin(dx+c))^3}{(\cos(dx+c))^3}\right)+\frac{3ba^2}{4(\cos(dx+c))^4}-a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)+3*a
*b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+3/4*b*a
^2/cos(d*x+c)^4-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A] time = 1.15361, size = 132, normalized size = 1.76

$$\frac{10b^3\tan(dx+c)^6+36ab^2\tan(dx+c)^5+90a^2b\tan(dx+c)^2+15\left(3a^2b+b^3\right)\tan(dx+c)^4+60a^3\tan(dx+c)+20}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 90*a^2*b*tan(d*x +
c)^2 + 15*(3*a^2*b + b^3)*tan(d*x + c)^4 + 60*a^3*tan(d*x + c) + 20*(a^3 +
3*a*b^2)*tan(d*x + c)^3)/d
```

Fricas [A] time = 1.88605, size = 246, normalized size = 3.28

$$\frac{10b^3+15\left(3a^2b-b^3\right)\cos(dx+c)^2+4\left(2\left(5a^3-3ab^2\right)\cos(dx+c)^5+9ab^2\cos(dx+c)+\left(5a^3-3ab^2\right)\cos(dx+c)^3\right)}{60d\cos(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^3 + 15*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(2*(5*a^3 - 3*a*b^2)*c
os(d*x + c)^5 + 9*a*b^2*cos(d*x + c) + (5*a^3 - 3*a*b^2)*cos(d*x + c)^3)*si
n(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^3\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**4, x)
```

Giac [A] time = 1.75557, size = 151, normalized size = 2.01

$$\frac{10\,b^3\tan\left(dx+c\right)^6+36\,ab^2\tan\left(dx+c\right)^5+45\,a^2b\tan\left(dx+c\right)^4+15\,b^3\tan\left(dx+c\right)^4+20\,a^3\tan\left(dx+c\right)^3+60\,ab^2\,\mathrm{ta}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 45*a^2*b*tan(d*x +
c)^4 + 15*b^3*tan(d*x + c)^4 + 20*a^3*tan(d*x + c)^3 + 60*a*b^2*tan(d*x + c
)^3 + 90*a^2*b*tan(d*x + c)^2 + 60*a^3*tan(d*x + c))/d
```

3.534 $\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=22

$$\frac{(a+b\tan(c+dx))^4}{4bd}$$

```
[Out] (a + b*Tan[c + d*x])^4/(4*b*d)
```

Rubi [A] time = 0.0363167, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 32}

$$\frac{(a+b\tan(c+dx))^4}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a + b*Tan[c + d*x])^4/(4*b*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a+b\tan(c+dx))^4}{4bd}\end{aligned}$$

Mathematica [B] time = 0.154683, size = 57, normalized size = 2.59

$$\frac{\tan(c+dx)\left(6a^2b\tan(c+dx)+4a^3+4ab^2\tan^2(c+dx)+b^3\tan^3(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Tan[c + d*x]*(4*a^3 + 6*a^2*b*Tan[c + d*x] + 4*a*b^2*Tan[c + d*x]^2 + b^3*
Tan[c + d*x]^3))/(4*d)
```

Maple [B] time = 0.061, size = 72, normalized size = 3.3

$$\frac{1}{d}\left(\frac{b^3(\sin(dx+c))^4}{4(\cos(dx+c))^4}+\frac{ab^2(\sin(dx+c))^3}{(\cos(dx+c))^3}+\frac{3ba^2}{2(\cos(dx+c))^2}+a^3\tan(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(1/4*b^3*sin(d*x+c)^4/cos(d*x+c)^4+a*b^2*sin(d*x+c)^3/cos(d*x+c)^3+3/2*
b*a^2/cos(d*x+c)^2+a^3*tan(d*x+c))
```

Maxima [A] time = 1.18808, size = 27, normalized size = 1.23

$$\frac{(b\tan(dx+c)+a)^4}{4bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*tan(d*x + c) + a)^4/(b*d)
```

Fricas [B] time = 1.83864, size = 181, normalized size = 8.23

$$\frac{b^3+2\left(3a^2b-b^3\right)\cos(dx+c)^2+4\left(ab^2\cos(dx+c)+\left(a^3-ab^2\right)\cos(dx+c)^3\right)\sin(dx+c)}{4d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3 + 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b^2*cos(d*x + c) + (a^3
- a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^3\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**2, x)
```

Giac [B] time = 1.82681, size = 77, normalized size = 3.5

$$\frac{b^3 \tan{(dx+c)}^4 + 4\,ab^2 \tan{(dx+c)}^3 + 6\,a^2 b \tan{(dx+c)}^2 + 4\,a^3 \tan{(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*tan(d*x + c)^4 + 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)^2 +
 4*a^3*tan(d*x + c))/d
```

3.535 $\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}ax\left(a^2+3b^2\right)-\frac{ab^2\tan(c+dx)}{2d}-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d}-\frac{b^3\log(\cos(c+dx))}{d}$$

```
[Out] (a*(a^2 + 3*b^2)*x)/2 - (b^3*Log[Cos[c + d*x]])/d - (a*b^2*Tan[c + d*x])/(2
*d) - (Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.0932581, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3506, 739, 774, 635, 203, 260}

$$\frac{1}{2}ax\left(a^2+3b^2\right)-\frac{ab^2\tan(c+dx)}{2d}-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d}-\frac{b^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/2 - (b^3*Log[Cos[c + d*x]])/d - (a*b^2*Tan[c + d*x])/(2
*d) - (Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d} + \frac{b\,\text{Subst}\left(\int \frac{(a+x)\left(2+\frac{a^2}{b^2}-\frac{a}{b}\right.}{1+\frac{x^2}{b^2}}\right.}{2d}$$

$$= -\frac{ab^2\tan(c+dx)}{2d} - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d} + \frac{b^3\,\text{Su}}{}$$

$$= -\frac{ab^2\tan(c+dx)}{2d} - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d} + \frac{b\,\text{Su}}{}$$

$$= \frac{1}{2}a\left(a^2+3b^2\right)x - \frac{b^3\log(\cos(c+dx))}{d} - \frac{ab^2\tan(c+dx)}{2d} - \frac{\cos^2(c+dx)(b-a\,\text{ta}}{}$$

Mathematica [B] time = 0.746237, size = 401, normalized size = 4.66

$$\frac{ab\left(-2a^2b^2+a^4-3b^4\right)\sin(2(c+dx))+\left(-2a^2b^4-3a^4b^2+b^6\right)\cos(2(c+dx))+2a^2b^4\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2a^2b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (5*a^4*b^2 + 2*a^2*b^4 - b^6 + (-3*a^4*b^2 - 2*a^2*b^4 + b^6)*Cos[2*(c + d*
x)] + 2*a^2*b^4*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 2*b^6*Log[Sqrt[-b^2] - b
*Tan[c + d*x]] - a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*a^3*(-
b^2)^(3/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 3*a*(-b^2)^(5/2)*Log[Sqrt[-b^
2] - b*Tan[c + d*x]] + 2*a^2*b^4*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 2*b^6*L
og[Sqrt[-b^2] + b*Tan[c + d*x]] + a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]] + 3*a*b^4*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - 4*a^3*(-b^2)
^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a*b*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin
[2*(c + d*x)])/(4*b*(a^2 + b^2)*d)
```

Maple [A] time = 0.059, size = 123, normalized size = 1.4

$$-\frac{(\sin(dx+c))^2 b^3}{2d} - \frac{b^3\ln(\cos(dx+c))}{d} - \frac{3\cos(dx+c)\sin(dx+c)ab^2}{2d} + \frac{3ab^2x}{2} + \frac{3ab^2c}{2d} - \frac{3a^2(\cos(dx+c))^2 b}{2d} + \frac{a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/d*sin(d*x+c)^2*b^3-b^3*ln(cos(d*x+c))/d-3/2/d*sin(d*x+c)*cos(d*x+c)*a*
b^2+3/2*a*b^2*x+3/2/d*a*b^2*c-3/2/d*a^2*cos(d*x+c)^2*b+1/2/d*a^3*sin(d*x+c)
*cos(d*x+c)+1/2*a^3*x+1/2/d*a^3*c
```

Maxima [A] time = 1.73724, size = 109, normalized size = 1.27

$$\frac{b^3\log\left(\tan(dx+c)^2+1\right)+\left(a^3+3\,ab^2\right)(dx+c)-\frac{3\,a^2b-b^3-\left(a^3-3\,ab^2\right)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*log(tan(d*x + c)^2 + 1) + (a^3 + 3*a*b^2)*(d*x + c) - (3*a^2*b - b
^3 - (a^3 - 3*a*b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.98182, size = 181, normalized size = 2.1

$$-\frac{2\,b^3\log\left(-\cos(dx+c)\right)-\left(a^3+3\,ab^2\right)dx+\left(3\,a^2b-b^3\right)\cos(dx+c)^2-\left(a^3-3\,ab^2\right)\cos(dx+c)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*log(-cos(d*x + c)) - (a^3 + 3*a*b^2)*d*x + (3*a^2*b - b^3)*cos(
d*x + c)^2 - (a^3 - 3*a*b^2)*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\tan(c+dx))^3\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**2, x)
```

Giac [B] time = 1.91175, size = 811, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*a^3*d*x*tan(d*x)^2*tan(c)^2 + 6*a*b^2*d*x*tan(d*x)^2*tan(c)^2 - 2*b^
3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x
)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 2
*a^3*d*x*tan(d*x)^2 + 6*a*b^2*d*x*tan(d*x)^2 + 2*a^3*d*x*tan(c)^2 + 6*a*b^2
*d*x*tan(c)^2 - 3*a^2*b*tan(d*x)^2*tan(c)^2 + b^3*tan(d*x)^2*tan(c)^2 - 2*b
^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*
x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2 - 2*a^3*tan
(d*x)^2*tan(c) + 6*a*b^2*tan(d*x)^2*tan(c) - 2*b^3*log(4*(tan(c)^2 + 1)/(ta
n(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
- 2*tan(d*x)*tan(c) + 1))*tan(c)^2 - 2*a^3*tan(d*x)*tan(c)^2 + 6*a*b^2*tan(
d*x)*tan(c)^2 + 2*a^3*d*x + 6*a*b^2*d*x + 3*a^2*b*tan(d*x)^2 - b^3*tan(d*x)
^2 + 12*a^2*b*tan(d*x)*tan(c) - 4*b^3*tan(d*x)*tan(c) + 3*a^2*b*tan(c)^2 -
b^3*tan(c)^2 - 2*b^3*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)
^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 2*
a^3*tan(d*x) - 6*a*b^2*tan(d*x) + 2*a^3*tan(c) - 6*a*b^2*tan(c) - 3*a^2*b +
 b^3)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

3.536 $\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=84

$$\frac{3}{8}ax\left(a^2+b^2\right)-\frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d}+\frac{\sin(c+dx)\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d}$$

```
[Out] (3*a*(a^2 + b^2)*x)/8 - (3*a*Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan
[c + d*x]))/(8*d) + (Cos[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(4
*d)
```

Rubi [A] time = 0.0685598, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3506, 729, 723, 203}

$$\frac{3}{8}ax\left(a^2+b^2\right)-\frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d}+\frac{\sin(c+dx)\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 + b^2)*x)/8 - (3*a*Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan
[c + d*x]))/(8*d) + (Cos[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(4
*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 729

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^m*(2*c*x)*(a + c*x^2)^(p + 1))/(4*a*c*(p + 1)), x] - Dist[(m*(2
*c*d))/(4*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x] /
; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3,
 0] && LtQ[p, -1]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^3}dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{4d} + \frac{(3a)\,\text{Subst}\left(\int \frac{(a+x)^2}{\left(1+\frac{x^2}{b^2}\right)^2}dx, x, b\tan\right.}{4bd}$$

$$= -\frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(c+}{4}$$

$$= \frac{3}{8}a\left(a^2+b^2\right)x - \frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \cos^3(c$$

Mathematica [B] time = 3.51926, size = 257, normalized size = 3.06

$$2ab^5\left(b^2-3a^2\right)\tan^3(c+dx) - 24a^4b^4\tan^2(c+dx) - 6ab^3\left(3a^2b^2+6a^4+b^4\right)\tan(c+dx) - 3a\sqrt{-b^2}\left(a^2+b^2\right)^3\left(\log\left(\sqrt{-}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a*Sqrt[-b^2]*(a^2 + b^2)^3*(Log[Sqrt[-b^2] - b*Tan[c + d*x]] - Log[Sqrt
[-b^2] + b*Tan[c + d*x]]) - 6*a*b^3*(6*a^4 + 3*a^2*b^2 + b^4)*Tan[c + d*x]
- 24*a^4*b^4*Tan[c + d*x]^2 + 2*a*b^5*(-3*a^2 + b^2)*Tan[c + d*x]^3 + 4*b*(
a^2 + b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^4*(b + a*Tan[c + d*x]) + 8*a^2
*b^2*Cos[c + d*x]^2*(a + b*Tan[c + d*x])^4 + 2*a*b*(3*a^2 - b^2)*Cos[c + d*
x]*Sin[c + d*x]*(a + b*Tan[c + d*x])^4)/(16*b*(a^2 + b^2)^2*d)
```

Maple [A] time = 0.071, size = 114, normalized size = 1.4

$$\frac{1}{d}\left(\frac{b^3(\sin(dx+c))^4}{4} + 3ab^2\left(-1/4(\cos(dx+c))^3\sin(dx+c) + 1/8\cos(dx+c)\sin(dx+c) + 1/8dx + c/8\right) - \frac{3ba^2(\cos}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(1/4*b^3*sin(d*x+c)^4+3*a*b^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x
+c)*sin(d*x+c)+1/8*d*x+1/8*c)-3/4*b*a^2*cos(d*x+c)^4+a^3*(1/4*(cos(d*x+c)^3
+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.77752, size = 149, normalized size = 1.77

$$\frac{3\left(a^3+ab^2\right)(dx+c) - \frac{4b^3\tan(dx+c)^2 - 3\left(a^3+ab^2\right)\tan(dx+c)^3 + 6a^2b + 2b^3 - \left(5a^3 - 3ab^2\right)\tan(dx+c)}{\tan(dx+c)^4 + 2\tan(dx+c)^2 + 1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^3 + a*b^2)*(d*x + c) - (4*b^3*tan(d*x + c)^2 - 3*(a^3 + a*b^2)*ta
n(d*x + c)^3 + 6*a^2*b + 2*b^3 - (5*a^3 - 3*a*b^2)*tan(d*x + c))/(tan(d*x +
 c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Fricas [A] time = 1.86007, size = 228, normalized size = 2.71

$$-\frac{4\,b^3\cos\left(dx+c\right)^2+2\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^4-3\left(a^3+ab^2\right)dx-\left(2\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+3\left(a^3+ab^2\right)\cos\left(d\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*cos(d*x + c)^2 + 2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 3*(a^3 + a*
b^2)*d*x - (2*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(a^3 + a*b^2)*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 22.987, size = 3370, normalized size = 40.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(9*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 24*a^3*d
*x*tan(d*x)^4*tan(c)^4 + 24*a*b^2*d*x*tan(d*x)^4*tan(c)^4 + 9*pi*a*b^2*sgn(
-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x
)^4*tan(c)^4 + 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2
*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2
+ 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*t
an(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 18*a*b^2*ar
ctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 18*a*
b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4
+ 48*a^3*d*x*tan(d*x)^4*tan(c)^2 + 48*a*b^2*d*x*tan(d*x)^4*tan(c)^2 + 18*pi
*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(
```

c))*tan(d*x)^4*tan(c)^2 + 48*a^3*d*x*tan(d*x)^2*tan(c)^4 + 48*a*b^2*d*x*tan(d*x)^2*tan(c)^4 + 18*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 - 30*a^2*b*tan(d*x)^4*tan(c)^4 - 6*b^3*tan(d*x)^4*tan(c)^4 + 9*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 36*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 36*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^2 - 36*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^2 - 40*a^3*tan(d*x)^4*tan(c)^3 + 24*a*b^2*tan(d*x)^4*tan(c)^3 + 9*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 36*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 36*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 40*a^3*tan(d*x)^3*tan(c)^4 + 24*a*b^2*tan(d*x)^3*tan(c)^4 + 24*a^3*d*x*tan(d*x)^4 + 24*a*b^2*d*x*tan(d*x)^4 + 9*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 96*a^3*d*x*tan(d*x)^2*tan(c)^2 + 96*a*b^2*d*x*tan(d*x)^2*tan(c)^2 + 36*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 36*a^2*b*tan(d*x)^4*tan(c)^2 - 12*b^3*tan(d*x)^4*tan(c)^2 + 192*a^2*b*tan(d*x)^3*tan(c)^3 + 24*a^3*d*x*tan(c)^4 + 24*a*b^2*d*x*tan(c)^4 + 9*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 36*a^2*b*tan(d*x)^2*tan(c)^4 - 12*b^3*tan(d*x)^2*tan(c)^4 + 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 18*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4 - 18*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4 - 24*a^3*tan(d*x)^4*tan(c) - 24*a*b^2*tan(d*x)^4*tan(c) + 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 72*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 72*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 48*a^3*tan(d*x)^3*tan(c)^2 - 144*a*b^2*tan(d*x)^3*tan(c)^2 + 48*a^3*tan(d*x)^2*tan(c)^3 - 144*a*b^2*tan(d*x)^2*tan(c)^3 + 18*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^4 - 18*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^4 - 24*a^3*tan(d*x)*tan(c)^4 - 24*a*b^2*tan(d*x)*tan(c)^4 + 48*a^3*d*x*tan(d*x)^2 + 48*a*b^2*d*x*tan(d*x)^2 + 18*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 18*a^2*b*tan(d*x)^4 + 10*b^3*tan(d*x)^4 + 64*b^3*tan(d*x)^3*tan(c) + 48*a^3*d*x*tan(c)^2 + 48*a*b^2*d*x*tan(c)^2 + 18*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 - 216*a^2*b*tan(d*x)^2*tan(c)^2 + 72*b^3*tan(d*x)^2*tan(c)^2 + 64*b^3*tan(d*x)*tan(c)^3 + 18*a^2*b*tan(c)^4 + 10*b^3*tan(c)^4 + 9*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 36*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2 - 36*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 24*a^3*tan(d*x)^3 + 24*a*b^2*tan(d*x)^3 - 48*a^3*tan(d*x)^2*tan(c) + 144*a*b^2*tan(d*x)^2*tan(c) + 36*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^2 - 36*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^2 - 48*a^3*tan(d*x)*tan(c)^2 + 144*a*b^2*tan(d*x)*tan(c)^2 + 24*a^3*tan(c)^3 + 24*a*b^2*tan(c)^3 + 24*a^3*d*x + 24*a*b^2*d*x + 9*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 36*a^2*b*tan(d*x)^2 - 12*b^3*tan(d*x)^2 + 192*a^2*b*tan(d*x)*tan(c) + 36*a^2*b*tan(c)^2 - 12*b^3*tan(c)^2 + 18*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) - 18*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1)) + 40*a^3*tan(d*x) - 24*a*b^2*tan(d*x) + 40*a^3*tan(c) - 24*a*b^2*tan(c) - 30*a^2*b - 6*b^3)/(d*tan(d*x)^4*tan(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4 + 4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)

3.537 $\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=159

$$\frac{3a\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{b\sec^5(c+dx)\left(4\left(8a^2-b^2\right)+15ab\tan(c+dx)\right)}{70d}+\frac{a\left(2a^2-b^2\right)\tan(c+dx)\sec^3(\ldots}{8d}$$

```
[Out] (3*a*(2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(16*d) + (3*a*(2*a^2 - b^2)*Sec[c
 + d*x]*Tan[c + d*x])/(16*d) + (a*(2*a^2 - b^2)*Sec[c + d*x]^3*Tan[c + d*x]
)/(8*d) + (b*Sec[c + d*x]^5*(a + b*Tan[c + d*x])^2)/(7*d) + (b*Sec[c + d*x]
^5*(4*(8*a^2 - b^2) + 15*a*b*Tan[c + d*x]))/(70*d)
```

Rubi [A] time = 0.14451, antiderivative size = 177, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3512, 743, 780, 195, 215}

$$\frac{b\sec^5(c+dx)\left(4\left(8a^2-b^2\right)+15ab\tan(c+dx)\right)}{70d}+\frac{a\left(2a^2-b^2\right)\tan(c+dx)\sec^3(c+dx)}{8d}+\frac{3a\left(2a^2-b^2\right)\tan(c+dx)s\ldots}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(2*a^2 - b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(16*d*Sqrt[Sec[c + d
*x]^2]) + (3*a*(2*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a*(2*a^2
- b^2)*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^5*(a + b*Tan[c
+ d*x])^2)/(7*d) + (b*Sec[c + d*x]^5*(4*(8*a^2 - b^2) + 15*a*b*Tan[c + d*x]
))/(70*d)
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/2}dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d} + \frac{(b\sec(c+dx))\,\text{Subst}\left(\int (a+x)\left(-2+\frac{7a^2}{b^2}\right.\right.}{7d\sqrt{\sec^2(c}}$$

$$= \frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d} + \frac{b\sec^5(c+dx)\left(4\left(8a^2-b^2\right)+15ab\tan(c+\right.}{70d}$$

$$= \frac{a\left(2a^2-b^2\right)\sec^3(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d} + \frac{b\,\text{s}}{}$$

$$= \frac{3a\left(2a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{16d} + \frac{a\left(2a^2-b^2\right)\sec^3(c+dx)\tan(c+dx)}{8d} +$$

$$= \frac{3a\left(2a^2-b^2\right)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{16d\sqrt{\sec^2(c+dx)}} + \frac{3a\left(2a^2-b^2\right)\sec(c+dx)\tan(c}{16d}$$

Mathematica [B] time = 2.14269, size = 637, normalized size = 4.01

$$\frac{\sec^7(c+dx)\left(3584\left(3a^2b-b^3\right)\cos(2(c+dx))-3675a\left(2a^2-b^2\right)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\text{l}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^7*(10752*a^2*b + 1536*b^3 + 3584*(3*a^2*b - b^3)*Cos[2*(c + d
*x)] - 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 147
0*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 735*a*b^2
*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 210*a^3*Cos[7*
(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*a*b^2*Cos[7*(c +
d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3675*a*(2*a^2 - b^2)*Cos[c
 + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) + 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(
```

```
c + d*x)/2]] - 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] - 735*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
210*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 105*a*b
^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4340*a^3*Sin
[2*(c + d*x)] + 6790*a*b^2*Sin[2*(c + d*x)] + 2800*a^3*Sin[4*(c + d*x)] - 1
400*a*b^2*Sin[4*(c + d*x)] + 420*a^3*Sin[6*(c + d*x)] - 210*a*b^2*Sin[6*(c
+ d*x)]))/(35840*d)
```

Maple [B] time = 0.074, size = 328, normalized size = 2.1

$$\frac{b^3 (\sin(dx+c))^4}{7d(\cos(dx+c))^7} + \frac{3b^3 (\sin(dx+c))^4}{35d(\cos(dx+c))^5} + \frac{b^3 (\sin(dx+c))^4}{35d(\cos(dx+c))^3} - \frac{b^3 (\sin(dx+c))^4}{35d\cos(dx+c)} - \frac{b^3\cos(dx+c)(\sin(dx+c))^2}{35d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/7/d*b^3*sin(d*x+c)^4/cos(d*x+c)^7+3/35/d*b^3*sin(d*x+c)^4/cos(d*x+c)^5+1/
35/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3-1/35/d*b^3*sin(d*x+c)^4/cos(d*x+c)-1/35/
d*b^3*cos(d*x+c)*sin(d*x+c)^2-2/35/d*b^3*cos(d*x+c)+1/2/d*a*b^2*sin(d*x+c)^
3/cos(d*x+c)^6+3/8/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^4+3/16/d*a*b^2*sin(d*x+c
)^3/cos(d*x+c)^2+3/16/d*a*b^2*sin(d*x+c)-3/16/d*a*b^2*ln(sec(d*x+c)+tan(d*x
+c))+3/5/d*b*a^2/cos(d*x+c)^5+1/4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8*a^3*sec
(d*x+c)*tan(d*x+c)/d+3/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.17441, size = 281, normalized size = 1.77

$$\frac{35\,ab^2\left(\frac{2\left(3\sin(dx+c)^5-8\sin(dx+c)^3-3\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right) - 70\,a^3\left(\frac{2\left(3\sin(dx+c)^3-5\,\mathrm{s}\right.}{\sin(dx+c)^4-2\sin(}\right.}{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/1120*(35*a*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/
(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 70*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(
d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
 + 3*log(sin(d*x + c) - 1)) + 672*a^2*b/cos(d*x + c)^5 - 32*(7*cos(d*x + c)
^2 - 5)*b^3/cos(d*x + c)^7)/d
```

Fricas [A] time = 2.08916, size = 410, normalized size = 2.58

$$\frac{105\left(2a^3-ab^2\right)\cos(dx+c)^7\log(\sin(dx+c)+1) - 105\left(2a^3-ab^2\right)\cos(dx+c)^7\log(-\sin(dx+c)+1) + 160\,b^3 + }{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/1120*(105*(2*a^3 - a*b^2)*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(2*a
^3 - a*b^2)*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 160*b^3 + 224*(3*a^2*b
- b^3)*cos(d*x + c)^2 + 70*(3*(2*a^3 - a*b^2)*cos(d*x + c)^5 + 8*a*b^2*cos(
d*x + c) + 2*(2*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^
7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\tan(c + dx))^3 \sec^5(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**5, x)
```

Giac [B] time = 1.89232, size = 628, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/560*(105*(2*a^3 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(2*a^3
- a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(350*a^3*tan(1/2*d*x + 1/2*
c)^13 + 105*a*b^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)
^12 - 840*a^3*tan(1/2*d*x + 1/2*c)^11 + 1540*a*b^2*tan(1/2*d*x + 1/2*c)^11
+ 3360*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 1120*b^3*tan(1/2*d*x + 1/2*c)^10 + 6
30*a^3*tan(1/2*d*x + 1/2*c)^9 + 1085*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 5040*a^
2*b*tan(1/2*d*x + 1/2*c)^8 - 1120*b^3*tan(1/2*d*x + 1/2*c)^8 + 6720*a^2*b*t
an(1/2*d*x + 1/2*c)^6 - 2240*b^3*tan(1/2*d*x + 1/2*c)^6 - 630*a^3*tan(1/2*d
*x + 1/2*c)^5 - 1085*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3696*a^2*b*tan(1/2*d*x
+ 1/2*c)^4 - 448*b^3*tan(1/2*d*x + 1/2*c)^4 + 840*a^3*tan(1/2*d*x + 1/2*c)^
3 - 1540*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 672*a^2*b*tan(1/2*d*x + 1/2*c)^2 -
224*b^3*tan(1/2*d*x + 1/2*c)^2 - 350*a^3*tan(1/2*d*x + 1/2*c) - 105*a*b^2*t
an(1/2*d*x + 1/2*c) - 336*a^2*b + 32*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.538 $\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=126

$$\frac{a\left(4a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\sec^3(c+dx)\left(8\left(6a^2-b^2\right)+21ab\tan(c+dx)\right)}{60d}+\frac{a\left(4a^2-3b^2\right)\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (a*(4*a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(4*a^2 - 3*b^2)*Sec[c
+ d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2)/(5*d
) + (b*Sec[c + d*x]^3*(8*(6*a^2 - b^2) + 21*a*b*Tan[c + d*x]))/(60*d)
```

Rubi [A] time = 0.130012, antiderivative size = 144, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3512, 743, 780, 195, 215}

$$\frac{b\sec^3(c+dx)\left(8\left(6a^2-b^2\right)+21ab\tan(c+dx)\right)}{60d}+\frac{a\left(4a^2-3b^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a\left(4a^2-3b^2\right)\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{8d\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(4*a^2 - 3*b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(8*d*Sqrt[Sec[c + d*
x]^2]) + (a*(4*a^2 - 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d
*x]^3*(a + b*Tan[c + d*x])^2)/(5*d) + (b*Sec[c + d*x]^3*(8*(6*a^2 - b^2) +
21*a*b*Tan[c + d*x]))/(60*d)
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int (a+x)^3\sqrt{1+\frac{x^2}{b^2}}\,dx,x,b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{(b\sec(c+dx))\,\text{Subst}\left(\int (a+x)\left(-2+\frac{5a^2}{b^2}\right.\right.}{5d\sqrt{\sec^2(c}}$$

$$= \frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{b\sec^3(c+dx)\left(8\left(6a^2-b^2\right)+21ab\tan(c+\right.}{60d}$$

$$= \frac{a\left(4a^2-3b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{b\,\text{s}}{}$$

$$= \frac{a\left(4a^2-3b^2\right)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{8d\sqrt{\sec^2(c+dx)}} + \frac{a\left(4a^2-3b^2\right)\sec(c+dx)\tan(c}{8d}$$

Mathematica [B] time = 1.27308, size = 464, normalized size = 3.68

$$\frac{\sec^5(c+dx)\left(320\left(3a^2b-b^3\right)\cos(2(c+dx))-150a\left(4a^2-3b^2\right)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\text{lo}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^5*(960*a^2*b + 64*b^3 + 320*(3*a^2*b - b^3)*Cos[2*(c + d*x)]
- 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 225*a
*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 60*a^3*Cos
[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b^2*Cos[5*(c
+ d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 150*a*(4*a^2 - 3*b^2)*Co
s[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) + 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] - 225*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + 60*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 45*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24
0*a^3*Sin[2*(c + d*x)] + 540*a*b^2*Sin[2*(c + d*x)] + 120*a^3*Sin[4*(c + d*
x)] - 90*a*b^2*Sin[4*(c + d*x)]))/(1920*d)
```

Maple [B] time = 0.068, size = 256, normalized size = 2.

$$\frac{b^3(\sin(dx+c))^4}{5d(\cos(dx+c))^5} + \frac{b^3(\sin(dx+c))^4}{15d(\cos(dx+c))^3} - \frac{b^3(\sin(dx+c))^4}{15d\cos(dx+c)} - \frac{b^3\cos(dx+c)(\sin(dx+c))^2}{15d} - \frac{2b^3\cos(dx+c)}{15d} + \frac{3ab^2}{4d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/5/d*b^3*sin(d*x+c)^4/cos(d*x+c)^5+1/15/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3-1/
15/d*b^3*sin(d*x+c)^4/cos(d*x+c)-1/15/d*b^3*cos(d*x+c)*sin(d*x+c)^2-2/15/d*
b^3*cos(d*x+c)+3/4/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*a*b^2*sin(d*x+c)
^3/cos(d*x+c)^2+3/8/d*a*b^2*sin(d*x+c)-3/8/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c)
)+1/d*b*a^2/cos(d*x+c)^3+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^3*ln(sec(d
*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.06815, size = 212, normalized size = 1.68

$$\frac{45\,ab^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - 60\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/240*(45*a*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*a^3*(
2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) + 240*a^2*b/cos(d*x + c)^3 - 16*(5*cos(d*x + c)^2 - 3)*b^3/cos(d*
x + c)^5)/d
```

Fricas [A] time = 1.94477, size = 362, normalized size = 2.87

$$\frac{15\left(4\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right) - 15\left(4\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right) + 48\,b^3 + \cdots}{240\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(4*a^3 - 3*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a^
3 - 3*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^3 + 80*(3*a^2*b -
 b^3)*cos(d*x + c)^2 + 30*(6*a*b^2*cos(d*x + c) + (4*a^3 - 3*a*b^2)*cos(d*x
 + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\tan\left(c + dx\right)\right)^3 \sec^3\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + dx))**3*sec(c + d*x)**3, x)
```

Giac [B] time = 1.86256, size = 450, normalized size = 3.57

$$15\left(4a^3 - 3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 15\left(4a^3 - 3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{2\left(60a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9 + 45ab^2\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(4*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
- 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1/2
*c)^9 + 45*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^8
- 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 270*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*a
^2*b*tan(1/2*d*x + 1/2*c)^6 - 240*b^3*tan(1/2*d*x + 1/2*c)^6 - 480*a^2*b*ta
n(1/2*d*x + 1/2*c)^4 - 80*b^3*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 270*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b*tan(1/2*d*x + 1/2
*c)^2 - 80*b^3*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c) - 45*a*
b^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b + 16*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^5)/d
```

3.539 $\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{a\left(2a^2-3b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\sec(c+dx)\left(4\left(4a^2-b^2\right)+5ab\tan(c+dx)\right)}{6d}+\frac{b\sec(c+dx)(a+b\tan(c+dx))}{3d}$$

```
[Out] (a*(2*a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]*(a + b*Ta
n[c + d*x])^2)/(3*d) + (b*Sec[c + d*x]*(4*(4*a^2 - b^2) + 5*a*b*Tan[c + d*x
]))/(6*d)
```

Rubi [A] time = 0.0851151, antiderivative size = 109, normalized size of antiderivative = 1.2, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3512, 743, 780, 215}

$$\frac{b\sec(c+dx)\left(4\left(4a^2-b^2\right)+5ab\tan(c+dx)\right)}{6d}+\frac{a\left(2a^2-3b^2\right)\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{2d\sqrt{\sec^2(c+dx)}}+\frac{b\sec(c+dx)(a+b\tan}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 3*b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(2*d*Sqrt[Sec[c + d*
x]^2]) + (b*Sec[c + d*x]*(a + b*Tan[c + d*x])^2)/(3*d) + (b*Sec[c + d*x]*(4
*(4*a^2 - b^2) + 5*a*b*Tan[c + d*x]))/(6*d)
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{(a+x)^3}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d} + \frac{(b\sec(c+dx))\,\text{Subst}\left(\int \frac{(a+x)\left(-2+\frac{3a^2}{b^2}+\frac{5ax}{b^2}\right)}{\sqrt{1+\frac{x^2}{b^2}}}\,dx\right.}{3d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d} + \frac{b\sec(c+dx)\left(4\left(4a^2-b^2\right)+5ab\tan(c+dx)\right.}{6d}$$

$$= \frac{a\left(2a^2-3b^2\right)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2d\sqrt{\sec^2(c+dx)}} + \frac{b\sec(c+dx)(a+b\tan(c+dx))}{3d}$$

Mathematica [B] time = 1.55495, size = 293, normalized size = 3.22

$$\frac{-6a\left(2a^2-3b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b\sin^2\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\left(18a^2-5b^2\right)\cos(2(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (36*a^2*b - 10*b^3 - 6*a*(2*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 12*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 18*a*b^2*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] + (9*a*b^2)/(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2 + b^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 2*b*(18*a^2 - b^
2 + 2*b^2*Cos[c + d*x] + (18*a^2 - 5*b^2)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*
Sin[(c + d*x)/2]^2 - (9*a*b^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + b^
3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(12*d)
```

Maple [B] time = 0.027, size = 187, normalized size = 2.1

$$\frac{b^3(\sin(dx+c))^4}{3d(\cos(dx+c))^3} - \frac{b^3(\sin(dx+c))^4}{3d\cos(dx+c)} - \frac{b^3\cos(dx+c)(\sin(dx+c))^2}{3d} - \frac{2b^3\cos(dx+c)}{3d} + \frac{3ab^2(\sin(dx+c))^3}{2d(\cos(dx+c))^2} + \frac{3ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/3/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3-1/3/d*b^3*sin(d*x+c)^4/cos(d*x+c)-1/3/d
*b^3*cos(d*x+c)*sin(d*x+c)^2-2/3/d*b^3*cos(d*x+c)+3/2/d*a*b^2*sin(d*x+c)^3/
cos(d*x+c)^2+3/2/d*a*b^2*sin(d*x+c)-3/2/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+3
/d*b*a^2/cos(d*x+c)+1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.12219, size = 150, normalized size = 1.65

$$-\frac{9\,ab^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,a^3\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)-\frac{36\,a^2b}{\cos(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(9*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) - 12*a^3*log(sec(d*x + c) + tan(d*x + c)) - 36*a^
2*b/cos(d*x + c) + 4*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3)/d
```

Fricas [A] time = 1.94229, size = 304, normalized size = 3.34

$$\frac{3\left(2\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+18\,ab^2\cos}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(2*a^3 - 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3 -
 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 18*a*b^2*cos(d*x + c)*sin
(d*x + c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)^3\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x), x)
```

Giac [B] time = 1.83717, size = 231, normalized size = 2.54

$$\frac{3\left(2\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-18\,a^2b\tan}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 - 3*
a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)
^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 12
*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 4*b
^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.540 $\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=84

$$-\frac{b\sec(c+dx)\left(2\left(a^2-b^2\right)+ab\tan(c+dx)\right)}{d}+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+$$

```
[Out] (3*a*b^2*ArcTanh[Sin[c + d*x]])/d - (Cos[c + d*x]*(b - a*Tan[c + d*x])*(a +
 b*Tan[c + d*x])^2)/d - (b*Sec[c + d*x]*(2*(a^2 - b^2) + a*b*Tan[c + d*x]))
/d
```

Rubi [A] time = 0.0801492, antiderivative size = 102, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3512, 739, 780, 215}

$$-\frac{b\sec(c+dx)\left(2\left(a^2-b^2\right)+ab\tan(c+dx)\right)}{d}+\frac{3ab^2\cos(c+dx)\sqrt{\sec^2(c+dx)}\sinh^{-1}(\tan(c+dx))}{d}-\frac{\cos(c+dx)(b-a\,t}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*ArcSinh[Tan[c + d*x]]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/d - (Cos[
c + d*x]*(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/d - (b*Sec[c + d*x]*(
2*(a^2 - b^2) + a*b*Tan[c + d*x]))/d
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d} + \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c\right.}{}$$

$$= -\frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d} - \frac{b\sec(c+dx)\left(2\left(a^2-b\right.\right.}{d}$$

$$= \frac{3ab^2\sinh^{-1}(\tan(c+dx))\cos(c+dx)\sqrt{\sec^2(c+dx)}}{d} - \frac{\cos(c+dx)(b-a\tan(}{}$$

Mathematica [A] time = 1.06517, size = 131, normalized size = 1.56

$$\frac{\sec(c+dx)\left(\left(b^3-3a^2b\right)\cos(2(c+dx))-3a^2b+a^3\sin(2(c+dx))-3ab^2\sin(2(c+dx))-6ab^2\cos(c+dx)\left(\log\left(\cos\left(\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(-3*a^2*b + 3*b^3 + (-3*a^2*b + b^3)*Cos[2*(c + d*x)] - 6*a*b
^2*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[2*(c + d*x)] - 3*a*b^2*Sin[2*(c + d*x)
]))/(2*d)
```

Maple [A] time = 0.049, size = 126, normalized size = 1.5

$$\frac{b^3(\sin(dx+c))^4}{d\cos(dx+c)} + \frac{b^3\cos(dx+c)(\sin(dx+c))^2}{d} + 2\frac{b^3\cos(dx+c)}{d} - 3\frac{ab^2\sin(dx+c)}{d} + 3\frac{ab^2\ln(\sec(dx+c)+\text{ta}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*b^3*sin(d*x+c)^4/cos(d*x+c)+1/d*b^3*cos(d*x+c)*sin(d*x+c)^2+2/d*b^3*cos
(d*x+c)-3/d*a*b^2*sin(d*x+c)+3/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))-3/d*b*a^2*
cos(d*x+c)+a^3*sin(d*x+c)/d
```

Maxima [A] time = 1.1417, size = 113, normalized size = 1.35

$$\frac{2\,b^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+3\,ab^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))-6\,a^2b\cos(dx+c}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a*b^2*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 6*a^2*b*cos(d*x + c) + 2*a^3*s
in(d*x + c))/d
```

Fricas [A] time = 2.04388, size = 273, normalized size = 3.25

$$\frac{3\,ab^{2}\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab^{2}\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,b^{3}-2\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{2}+\ldots}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*a*b^2*cos(d*x + c)*log(
-sin(d*x + c) + 1) + 2*b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 2*(a^3 - 3*
a*b^2)*cos(d*x + c)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(c+dx\right)\right)^{3}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x), x)
```

Giac [B] time = 6.00224, size = 6400, normalized size = 76.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(3*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*pi*a
^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a^2*b*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 6*a*b^2*log(2*(t
an(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c)
 + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2
*c)^4 - 6*a*b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*
```

```
c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)
)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 12*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d
*x)^3*tan(1/2*c)^3 + 12*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 8*b^3*tan(1/2*d
*x)^4*tan(1/2*c)^4 + 12*pi*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + 24*a^2*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 +
 24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^3*t
an(1/2*c)^3 - 24*a*b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d
*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*
c) + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 24*a*b^2*log(2*(tan(1/2*c)^2 + 1)/(t
an(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 8*a^3*tan(1
/2*d*x)^4*tan(1/2*c)^3 - 24*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^3 + 8*a^3*tan(1
/2*d*x)^3*tan(1/2*c)^4 - 24*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - 3*pi*a^2*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4 - 12*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*ta
n(1/2*d*x)^3*tan(1/2*c) - 24*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^2 - 12*pi*a^2*
b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)*tan(1/2*c)^3 - 96*a^2*b*tan(1/2*d*x)^3
*tan(1/2*c)^3 + 32*b^3*tan(1/2*d*x)^3*tan(1/2*c)^3 - 3*pi*a^2*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*c)^2 + 1)*tan(1/2*c)^4 - 24*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^4 + 3*pi*a^2*b
*tan(1/2*d*x)^4 + 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*
tan(1/2*d*x)^4 + 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*t
an(1/2*d*x)^4 - 6*a*b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*
d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2
*c) + 1))*tan(1/2*d*x)^4 + 6*a*b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2
 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
+ 2*tan(1/2*c) + 1))*tan(1/2*d*x)^4 + 12*pi*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)
 + 24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^3
*tan(1/2*c) + 24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan
(1/2*d*x)^3*tan(1/2*c) - 24*a*b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2
+ tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 24*a*b^2*log(2*(tan(1/2*c)^
2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^3*tan(1/2*c) - 8*a^
3*tan(1/2*d*x)^4*tan(1/2*c) + 24*a*b^2*tan(1/2*d*x)^4*tan(1/2*c) - 48*a^3*t
an(1/2*d*x)^3*tan(1/2*c)^2 + 144*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^2 + 12*pi*
a^2*b*tan(1/2*d*x)*tan(1/2*c)^3 + 24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
```

```
tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 24*a^2*b*arctan((tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^3 - 24*a*b^2*log(2*(tan
(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c)^
3 + 24*a*b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*t
an(1/2*d*x)*tan(1/2*c)^3 - 48*a^3*tan(1/2*d*x)^2*tan(1/2*c)^3 + 144*a*b^2*t
an(1/2*d*x)^2*tan(1/2*c)^3 + 3*pi*a^2*b*tan(1/2*c)^4 + 6*a^2*b*arctan((tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^4 + 6*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 - 6*a*b^2*log(2*(tan(1/2*c)^
2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^4 + 6*a*b^2*log(2*(ta
n(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c)
 - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)^4 - 8*a^3*ta
n(1/2*d*x)*tan(1/2*c)^4 + 24*a*b^2*tan(1/2*d*x)*tan(1/2*c)^4 + 12*a^2*b*tan
(1/2*d*x)^4 - 8*b^3*tan(1/2*d*x)^4 - 12*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(
1/2*d*x)*tan(1/2*c) + 96*a^2*b*tan(1/2*d*x)^3*tan(1/2*c) - 32*b^3*tan(1/2*d
*x)^3*tan(1/2*c) + 240*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^2 - 96*b^3*tan(1/2*d
*x)^2*tan(1/2*c)^2 + 96*a^2*b*tan(1/2*d*x)*tan(1/2*c)^3 - 32*b^3*tan(1/2*d*
x)*tan(1/2*c)^3 + 12*a^2*b*tan(1/2*c)^4 - 8*b^3*tan(1/2*c)^4 + 8*a^3*tan(1/
2*d*x)^3 - 24*a*b^2*tan(1/2*d*x)^3 + 12*pi*a^2*b*tan(1/2*d*x)*tan(1/2*c) +
24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)*tan(
1/2*c) + 24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*
c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*
d*x)*tan(1/2*c) - 24*a*b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1
/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*
x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(
1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c) + 24*a*b^2*log(2*(tan(1/2*c)^2 + 1)/(t
an(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)*tan(1/2*c) + 48*a^3*tan(1/2*
d*x)^2*tan(1/2*c) - 144*a*b^2*tan(1/2*d*x)^2*tan(1/2*c) + 48*a^3*tan(1/2*d*
x)*tan(1/2*c)^2 - 144*a*b^2*tan(1/2*d*x)*tan(1/2*c)^2 + 8*a^3*tan(1/2*c)^3
 - 24*a*b^2*tan(1/2*c)^3 + 3*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(
1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1) - 24*a^2*b*tan(1
/2*d*x)^2 - 96*a^2*b*tan(1/2*d*x)*tan(1/2*c) + 32*b^3*tan(1/2*d*x)*tan(1/2*
c) - 24*a^2*b*tan(1/2*c)^2 - 3*pi*a^2*b - 6*a^2*b*arctan((tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) - tan(1/2*c) - 1)) - 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c
) - 1)) + 6*a*b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4
 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) +
```

```
1)) - 6*a*b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))
- 8*a^3*tan(1/2*d*x) + 24*a*b^2*tan(1/2*d*x) - 8*a^3*tan(1/2*c) + 24*a*b^2*
tan(1/2*c) + 12*a^2*b - 8*b^3)/(d*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(1/2
*d*x)^3*tan(1/2*c)^3 - d*tan(1/2*d*x)^4 - 4*d*tan(1/2*d*x)^3*tan(1/2*c) - 4
*d*tan(1/2*d*x)*tan(1/2*c)^3 - d*tan(1/2*c)^4 - 4*d*tan(1/2*d*x)*tan(1/2*c)
 + d)
```

3.541 $\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=70

$$-\frac{2\left(a^2+b^2\right)\cos(c+dx)(b-a\tan(c+dx))}{3d}-\frac{\cos^3(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{3d}$$

```
[Out] (-2*(a^2 + b^2)*Cos[c + d*x]*(b - a*Tan[c + d*x]))/(3*d) - (Cos[c + d*x]^3*
(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(3*d)
```

Rubi [A] time = 0.0687944, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3512, 723, 637}

$$-\frac{2\left(a^2+b^2\right)\cos(c+dx)(b-a\tan(c+dx))}{3d}-\frac{\cos^3(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```


```
[Out] (-2*(a^2 + b^2)*Cos[c + d*x]*(b - a*Tan[c + d*x]))/(3*d) - (Cos[c + d*x]^3*
(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(3*d)
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{\cos^3(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{3d} + \frac{\left(2\left(a^2+b^2\right)\cos(c+\right.}{}$$

$$= -\frac{2\left(a^2+b^2\right)\cos(c+dx)(b-a\tan(c+dx))}{3d} - \frac{\cos^3(c+dx)(b-a\tan(c+dx)}{3d}$$

Mathematica [A] time = 0.356454, size = 81, normalized size = 1.16

$$\frac{-9b\left(a^2+b^2\right)\cos(c+dx)+\left(b^3-3a^2b\right)\cos(3(c+dx))+2a\sin(c+dx)\left(\left(a^2-3b^2\right)\cos(2(c+dx))+5a^2+3b^2\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)
```

Maple [A] time = 0.055, size = 75, normalized size = 1.1

$$\frac{1}{d}\left(-\frac{b^3\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}+ab^2(\sin(dx+c))^3-ba^2(\cos(dx+c))^3+\frac{a^3\left(2+(\cos(dx+c))^2\right)\sin(dx+}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+a*b^2*sin(d*x+c)^3-b*a^2*cos(d*x+
c)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.1216, size = 104, normalized size = 1.49

$$-\frac{3\,a^2b\cos(dx+c)^3-3\,ab^2\sin(dx+c)^3+\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^3-\left(\cos(dx+c)^3-3\,\cos(dx+c)\right)b^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^2*b*cos(d*x + c)^3 - 3*a*b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3
*sin(d*x + c))*a^3 - (cos(d*x + c)^3 - 3*cos(d*x + c))*b^3)/d
```

Fricas [A] time = 1.75308, size = 173, normalized size = 2.47

$$-\frac{3\,b^3\cos\left(dx+c\right)+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^3-\left(2\,a^3+3\,ab^2+\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b^
2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\tan\left(c+dx\right)\right)^3\cos^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.542 $\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=105

$$-\frac{2\left(4a^2+b^2\right)\cos(c+dx)(b-a\tan(c+dx))}{15d}-\frac{\cos^3(c+dx)(b-4a\tan(c+dx))(a+b\tan(c+dx))^2}{15d}+\frac{\sin(c+dx)\cos}{}$$

```
[Out] (-2*(4*a^2 + b^2)*Cos[c + d*x]*(b - a*Tan[c + d*x]))/(15*d) - (Cos[c + d*x]
^3*(b - 4*a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(15*d) + (Cos[c + d*x]^4*
Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(5*d)
```

Rubi [A] time = 0.0946877, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3512, 737, 805, 637}

$$-\frac{2\left(4a^2+b^2\right)\cos(c+dx)(b-a\tan(c+dx))}{15d}-\frac{\cos^3(c+dx)(b-4a\tan(c+dx))(a+b\tan(c+dx))^2}{15d}+\frac{\sin(c+dx)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-2*(4*a^2 + b^2)*Cos[c + d*x]*(b - a*Tan[c + d*x]))/(15*d) - (Cos[c + d*x]
^3*(b - 4*a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(15*d) + (Cos[c + d*x]^4*
Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(5*d)
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{7/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{5d} - \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Sub}}{}$$

$$= -\frac{\cos^3(c+dx)(b-4a\tan(c+dx))(a+b\tan(c+dx))^2}{15d} + \frac{\cos^4(c+dx)\sin(c+d}{5}$$

$$= -\frac{2\left(4a^2+b^2\right)\cos(c+dx)(b-a\tan(c+dx))}{15d} - \frac{\cos^3(c+dx)(b-4a\tan(c+dx)}{15d}$$

Mathematica [A] time = 0.691361, size = 150, normalized size = 1.43

$$\frac{-30b\left(3a^2+b^2\right)\cos(c+dx)-5\left(9a^2b+b^3\right)\cos(3(c+dx))-9a^2b\cos(5(c+dx))+150a^3\sin(c+dx)+25a^3\sin(3(c+dx}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-30*b*(3*a^2 + b^2)*Cos[c + d*x] - 5*(9*a^2*b + b^3)*Cos[3*(c + d*x)] - 9*
a^2*b*Cos[5*(c + d*x)] + 3*b^3*Cos[5*(c + d*x)] + 150*a^3*Sin[c + d*x] + 90
*a*b^2*Sin[c + d*x] + 25*a^3*Sin[3*(c + d*x)] - 15*a*b^2*Sin[3*(c + d*x)] +
 3*a^3*Sin[5*(c + d*x)] - 9*a*b^2*Sin[5*(c + d*x)])/(240*d)
```

Maple [A] time = 0.065, size = 125, normalized size = 1.2

$$\frac{1}{d}\left(b^3\left(-\frac{(\cos(dx+c))^3(\sin(dx+c))^2}{5}-\frac{2(\cos(dx+c))^3}{15}\right)+3ab^2\left(-1/5\sin(dx+c)(\cos(dx+c))^4+1/15\left(2+(\cos(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(d*x+c)^3)+3*a*b^2*(-1/5*s
in(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/5*b*a^2*cos(d*x+
c)^5+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.13995, size = 144, normalized size = 1.37

$$-\frac{9a^2b\cos(dx+c)^5-\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^3+3\left(3\sin(dx+c)^5-5\sin(dx+c)^3\right)ab^2-}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(9*a^2*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*
sin(d*x + c))*a^3 + 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a*b^2 - (3*cos(
d*x + c)^5 - 5*cos(d*x + c)^3)*b^3)/d
```

Fricas [A] time = 1.77587, size = 230, normalized size = 2.19

$$-\frac{5\,b^3\cos\left(dx+c\right)^3+3\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^5-\left(3\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^4+8\,a^3+6\,ab^2+\left(4\,a^3+3\,ab^2\right)\cos\left(dx\right.\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^3*cos(d*x + c)^3 + 3*(3*a^2*b - b^3)*cos(d*x + c)^5 - (3*(a^3 -
3*a*b^2)*cos(d*x + c)^4 + 8*a^3 + 6*a*b^2 + (4*a^3 + 3*a*b^2)*cos(d*x + c)^
2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.543 $\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=142

$$\frac{8a\left(2a^2+b^2\right)\sin(c+dx)}{35d}-\frac{2\cos^3(c+dx)\left(b\left(6a^2+b^2\right)-a\left(4a^2-b^2\right)\tan(c+dx)\right)}{35d}-\frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(}{35d}$$

```
[Out] (8*a*(2*a^2 + b^2)*Sin[c + d*x])/(35*d) - (3*Cos[c + d*x]^5*(b - 2*a*Tan[c
+ d*x])*(a + b*Tan[c + d*x])^2)/(35*d) + (Cos[c + d*x]^6*Sin[c + d*x]*(a +
b*Tan[c + d*x])^3)/(7*d) - (2*Cos[c + d*x]^3*(b*(6*a^2 + b^2) - a*(4*a^2 -
b^2)*Tan[c + d*x]))/(35*d)
```

Rubi [A] time = 0.151976, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3512, 737, 821, 778, 191}

$$\frac{8a\left(2a^2+b^2\right)\sin(c+dx)}{35d}-\frac{2\cos^3(c+dx)\left(b\left(6a^2+b^2\right)-a\left(4a^2-b^2\right)\tan(c+dx)\right)}{35d}-\frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (8*a*(2*a^2 + b^2)*Sin[c + d*x])/(35*d) - (3*Cos[c + d*x]^5*(b - 2*a*Tan[c
+ d*x])*(a + b*Tan[c + d*x])^2)/(35*d) + (Cos[c + d*x]^6*Sin[c + d*x]*(a +
b*Tan[c + d*x])^3)/(7*d) - (2*Cos[c + d*x]^3*(b*(6*a^2 + b^2) - a*(4*a^2 -
b^2)*Tan[c + d*x]))/(35*d)
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[(x*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*a*(p + 1)), x] + Dist[1/(2*a*(p + 1
)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -
1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && In
tQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{9/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^6(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{7d} - \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{S}}{}$$

$$= -\frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d} + \frac{\cos^6(c+dx)\sin(}{}$$

$$= -\frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d} + \frac{\cos^6(c+dx)\sin(}{}$$

$$= \frac{8a\left(2a^2+b^2\right)\sin(c+dx)}{35d} - \frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+}{35d}$$

Mathematica [A] time = 1.05369, size = 204, normalized size = 1.44

$$\frac{-105b\left(5a^2+b^2\right)\cos(c+dx) - 35\left(9a^2b+b^3\right)\cos(3(c+dx)) - 105a^2b\cos(5(c+dx)) - 15a^2b\cos(7(c+dx)) + 1225a^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-105*b*(5*a^2 + b^2)*Cos[c + d*x] - 35*(9*a^2*b + b^3)*Cos[3*(c + d*x)] -
105*a^2*b*Cos[5*(c + d*x)] + 7*b^3*Cos[5*(c + d*x)] - 15*a^2*b*Cos[7*(c + d
*x)] + 5*b^3*Cos[7*(c + d*x)] + 1225*a^3*Sin[c + d*x] + 525*a*b^2*Sin[c + d
*x] + 245*a^3*Sin[3*(c + d*x)] - 35*a*b^2*Sin[3*(c + d*x)] + 49*a^3*Sin[5*(
c + d*x)] - 63*a*b^2*Sin[5*(c + d*x)] + 5*a^3*Sin[7*(c + d*x)] - 15*a*b^2*S
in[7*(c + d*x)])/(2240*d)
```

Maple [A] time = 0.069, size = 145, normalized size = 1.

$$\frac{1}{d}\left(b^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^5}{7} - \frac{2(\cos(dx+c))^5}{35}\right) + 3ab^2\left(-1/7(\cos(dx+c))^6\sin(dx+c) + 1/35\left(8/3+(\text{co}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a*b^2*(-1/7*c
os(d*x+c)^6*sin(d*x+c)+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
-3/7*b*a^2*cos(d*x+c)^7+1/7*a^3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos
(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.15782, size = 170, normalized size = 1.2

$$-\frac{15\,a^2b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^3-\left(15\,\sin\left(dx+c\right)^7-4\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35*(15*a^2*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35
*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3 - (15*sin(d*x + c)^7 - 42*sin(d*x +
c)^5 + 35*sin(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^3
)/d
```

Fricas [A] time = 1.98461, size = 278, normalized size = 1.96

$$-\frac{7\,b^3\cos\left(dx+c\right)^5+5\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^7-\left(5\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^6+3\left(2\,a^3+ab^2\right)\cos\left(dx+c\right)^4+16\,a^3+8\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(7*b^3*cos(d*x + c)^5 + 5*(3*a^2*b - b^3)*cos(d*x + c)^7 - (5*(a^3 -
3*a*b^2)*cos(d*x + c)^6 + 3*(2*a^3 + a*b^2)*cos(d*x + c)^4 + 16*a^3 + 8*a*b
^2 + 4*(2*a^3 + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.544 $\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{(a^2+2b^2)\tan^2(c+dx)}{2b^3d}-\frac{a(a^2+2b^2)\tan(c+dx)}{b^4d}+\frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^5d}-\frac{a\tan^3(c+dx)}{3b^2d}+\frac{\tan^4(c+dx)}{4bd}$$

```
[Out] ((a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(b^5*d) - (a*(a^2 + 2*b^2)*Tan[c +
d*x])/(b^4*d) + ((a^2 + 2*b^2)*Tan[c + d*x]^2)/(2*b^3*d) - (a*Tan[c + d*x]^
3)/(3*b^2*d) + Tan[c + d*x]^4/(4*b*d)
```

Rubi [A] time = 0.101835, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$\frac{(a^2+2b^2)\tan^2(c+dx)}{2b^3d}-\frac{a(a^2+2b^2)\tan(c+dx)}{b^4d}+\frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^5d}-\frac{a\tan^3(c+dx)}{3b^2d}+\frac{\tan^4(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(b^5*d) - (a*(a^2 + 2*b^2)*Tan[c +
d*x])/(b^4*d) + ((a^2 + 2*b^2)*Tan[c + d*x]^2)/(2*b^3*d) - (a*Tan[c + d*x]^
3)/(3*b^2*d) + Tan[c + d*x]^4/(4*b*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^2}{a+x}\,dx,x,b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a(-a^2-2b^2)}{b^4}+\frac{(a^2+2b^2)x}{b^4}-\frac{ax^2}{b^4}+\frac{x^3}{b^4}+\frac{(a^2+b^2)^2}{b^4(a+x)}\right)dx,x,b\tan(c+dx)\right)}{bd}\\ &= \frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^5d}-\frac{a(a^2+2b^2)\tan(c+dx)}{b^4d}+\frac{(a^2+2b^2)\tan^2(c+dx)}{2b^3d}-\frac{a\,\text{ta}}{}\end{aligned}$$

Mathematica [A] time = 1.21426, size = 99, normalized size = 0.85

$$\frac{6b^2(a^2+b^2)\tan^2(c+dx)-12ab(a^2+2b^2)\tan(c+dx)+12(a^2+b^2)^2\log(a+b\tan(c+dx))-4ab^3\tan^3(c+dx)+}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] (12*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] + 3*b^4*Sec[c + d*x]^4 - 12*a*b*(
a^2 + 2*b^2)*Tan[c + d*x] + 6*b^2*(a^2 + b^2)*Tan[c + d*x]^2 - 4*a*b^3*Tan[
c + d*x]^3)/(12*b^5*d)
```

Maple [A] time = 0.08, size = 162, normalized size = 1.4

$$\frac{(\tan(dx+c))^4}{4bd}-\frac{a(\tan(dx+c))^3}{3b^2d}+\frac{(\tan(dx+c))^2a^2}{2db^3}+\frac{(\tan(dx+c))^2}{bd}-\frac{a^3\tan(dx+c)}{db^4}-2\frac{a\tan(dx+c)}{b^2d}+\frac{\ln(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/4*tan(d*x+c)^4/b/d-1/3*a*tan(d*x+c)^3/b^2/d+1/2/d/b^3*tan(d*x+c)^2*a^2+ta
n(d*x+c)^2/b/d-1/d/b^4*a^3*tan(d*x+c)-2*a*tan(d*x+c)/b^2/d+1/d/b^5*ln(a+b*t
an(d*x+c))*a^4+2/d/b^3*ln(a+b*tan(d*x+c))*a^2+ln(a+b*tan(d*x+c))/b/d
```

Maxima [A] time = 1.14714, size = 146, normalized size = 1.26

$$\frac{\frac{3b^3\tan(dx+c)^4-4ab^2\tan(dx+c)^3+6(a^2b+2b^3)\tan(dx+c)^2-12(a^3+2ab^2)\tan(dx+c)}{b^4}+\frac{12(a^4+2a^2b^2+b^4)\log(b\tan(dx+c)+a)}{b^5}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*(a^2*b + 2*b^3)*ta
n(d*x + c)^2 - 12*(a^3 + 2*a*b^2)*tan(d*x + c))/b^4 + 12*(a^4 + 2*a^2*b^2 +
 b^4)*log(b*tan(d*x + c) + a)/b^5)/d
```

Fricas [A] time = 2.13068, size = 439, normalized size = 3.78

$$\frac{6(a^4+2a^2b^2+b^4)\cos(dx+c)^4\log(2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2)-6(a^4+2a^2b^2+b}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^4 + 2*a^2*b^2 + b^4)*cos(d*x + c)^4*log(2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^4 + 2*a^2*b^2 + b^4)*cos
```

```
(d*x + c)^4*log(cos(d*x + c)^2) + 3*b^4 + 6*(a^2*b^2 + b^4)*cos(d*x + c)^2
- 4*(a*b^3*cos(d*x + c) + (3*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))
/(b^5*d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)), x)
```

Giac [A] time = 1.73845, size = 162, normalized size = 1.4

$$\frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,a^2b\tan(dx+c)^2+12\,b^3\tan(dx+c)^2-12\,a^3\tan(dx+c)-24\,ab^2\tan(dx+c)}{b^4}+\frac{12\left(a^4+2\,a^2b^2+b^4\right)\log(|b\tan(dx+c)+a|)}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)
^2 + 12*b^3*tan(d*x + c)^2 - 12*a^3*tan(d*x + c) - 24*a*b^2*tan(d*x + c))/b
^4 + 12*(a^4 + 2*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^5)/d
```

3.545 $\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{(a^2+b^2)\log(a+b\tan(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}$$

```
[Out] ((a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^3*d) - (a*Tan[c + d*x])/(b^2*d) +
Tan[c + d*x]^2/(2*b*d)
```

Rubi [A] time = 0.0645737, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$\frac{(a^2+b^2)\log(a+b\tan(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^3*d) - (a*Tan[c + d*x])/(b^2*d) +
Tan[c + d*x]^2/(2*b*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1+\frac{x^2}{b^2}}{a+x}\,dx,x,b\tan(c+dx)\right)}{bd}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b^2}+\frac{a^2+b^2}{b^2(a+x)}\right)dx,x,b\tan(c+dx)\right)}{bd}\\
&= \frac{(a^2+b^2)\log(a+b\tan(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}
\end{aligned}$$

Mathematica [A] time = 0.126072, size = 52, normalized size = 0.88

$$\frac{(a^2+b^2)\log(a+b\tan(c+dx))-ab\tan(c+dx)+\frac{1}{2}b^2\tan^2(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Tan[c + d*x]] - a*b*Tan[c + d*x] + (b^2*Tan[c + d*x]
^2)/2)/(b^3*d)
```

Maple [A] time = 0.064, size = 72, normalized size = 1.2

$$\frac{(\tan (dx+c))^2}{2\,bd}-\frac{a\tan (dx+c)}{b^2d}+\frac{\ln (a+b\tan (dx+c))\,a^2}{db^3}+\frac{\ln (a+b\tan (dx+c))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/2*tan(d*x+c)^2/b/d-a*tan(d*x+c)/b^2/d+1/d/b^3*ln(a+b*tan(d*x+c))*a^2+ln(a
+b*tan(d*x+c))/b/d
```

Maxima [A] time = 1.10986, size = 72, normalized size = 1.22

$$\frac{\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}+\frac{2\,\left(a^2+b^2\right)\log(b\tan(dx+c)+a)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(b*tan(d*
x + c) + a)/b^3)/d
```

Fricas [B] time = 1.91328, size = 294, normalized size = 4.98

$$\frac{\left(a^2+b^2\right)\cos (dx+c)^2\log \left(2\,ab\cos (dx+c)\sin (dx+c)+\left(a^2-b^2\right)\cos (dx+c)^2+b^2\right)-\left(a^2+b^2\right)\cos (dx+c)^2\log \left(\cos\right.}{2\,b^3d\cos (dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)*cos(d*x + c)^2*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 + b^2) - (a^2 + b^2)*cos(d*x + c)^2*log(cos(d*x + c)^
2) - 2*a*b*cos(d*x + c)*sin(d*x + c) + b^2)/(b^3*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{a+b\tan (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

Giac [A] time = 1.50554, size = 73, normalized size = 1.24

$$\frac{\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}+\frac{2\left(a^2+b^2\right)\log(|b\tan(dx+c)+a|)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(abs(b*ta
n(d*x + c) + a))/b^3)/d
```

3.546 $\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(a+b\tan(c+dx))}{bd}$$

```
[Out] Log[a + b*Tan[c + d*x]]/(b*d)
```

Rubi [A] time = 0.0417421, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 31}

$$\frac{\log(a+b\tan(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] Log[a + b*Tan[c + d*x]]/(b*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\tan(c+dx))}{bd}\end{aligned}$$

Mathematica [A] time = 0.0137411, size = 18, normalized size = 1.

$$\frac{\log(a+b\tan(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] Log[a + b*Tan[c + d*x]]/(b*d)
```

Maple [A] time = 0.029, size = 19, normalized size = 1.1

$$\frac{\ln\left(a+b\tan\left(dx+c\right)\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)),x)
```

```
[Out] ln(a+b*tan(d*x+c))/b/d
```

Maxima [A] time = 1.13644, size = 24, normalized size = 1.33

$$\frac{\log\left(b\tan\left(dx+c\right)+a\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] log(b*tan(d*x + c) + a)/(b*d)
```

Fricas [B] time = 1.95648, size = 144, normalized size = 8.

$$\frac{\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-\log\left(\cos\left(dx+c\right)^2\right)}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
) - log(cos(d*x + c)^2))/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^2\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Giac [A] time = 1.82938, size = 26, normalized size = 1.44

$$\frac{\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*tan(d*x + c) + a))/(b*d)
```

3.547 $\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\left(a^2+b^2\right)}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{ax\left(a^2+3b^2\right)}{2\left(a^2+b^2\right)^2}$$

```
[Out] (a*(a^2 + 3*b^2)*x)/(2*(a^2 + b^2)^2) + (b^3*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/((a^2 + b^2)^2*d) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d)
```

Rubi [A] time = 0.136052, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3506, 741, 801, 635, 203, 260}

$$\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\left(a^2+b^2\right)}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{ax\left(a^2+3b^2\right)}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(2*(a^2 + b^2)^2) + (b^3*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/((a^2 + b^2)^2*d) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b\,\text{Subst}\left(\int \frac{-2-\frac{a^2}{b^2}-\frac{ax}{b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)}\,dx, x, b\tan(c+dx)\right)}{2\left(a^2+b^2\right)d}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b\,\text{Subst}\left(\int \left(-\frac{2b^2}{\left(a^2+b^2\right)(a+x)} + \frac{-a^3-3ab^2+2b^2x}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right) dx, x, b\tan(c+d$$

$$= \frac{b^3\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b\,\text{Subst}\left(\int \frac{-a^3-3ab^2+2b^2x}{b^2+x^2}\,dx, x, b$$

$$= \frac{b^3\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d} - \frac{b^3\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c$$

$$= \frac{a\left(a^2+3b^2\right)x}{2\left(a^2+b^2\right)^2} + \frac{b^3\log(\cos(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{b^3\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^2 d} + \frac{\cos^2(c+dx)(b+a\tan(c+$$

Mathematica [C] time = 0.22499, size = 143, normalized size = 1.54

$$\frac{b\left(a^2+b^2\right)\cos(2(c+dx)) + a^3\sin(2(c+dx)) + 2a^3c + 2a^3dx + ab^2\sin(2(c+dx)) + 2b^3\log\left((a\cos(c+dx)+b\sin(c+d\right.}{4d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*a^3*c + 6*a*b^2*c + (4*I)*b^3*c + 2*a^3*d*x + 6*a*b^2*d*x + (4*I)*b^3*d*
x - (4*I)*b^3*ArcTan[Tan[c + d*x]] + b*(a^2 + b^2)*Cos[2*(c + d*x)] + 2*b^3
*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2] + a^3*Sin[2*(c + d*x)] + a*b^2*Si
n[2*(c + d*x)])/(4*(a^2 + b^2)^2*d)
```

Maple [B] time = 0.098, size = 236, normalized size = 2.5

$$\frac{a^3\tan(dx+c)}{2d\left(a^2+b^2\right)^2\left(1+(\tan(dx+c))^2\right)} + \frac{a\tan(dx+c)\,b^2}{2d\left(a^2+b^2\right)^2\left(1+(\tan(dx+c))^2\right)} + \frac{ba^2}{2d\left(a^2+b^2\right)^2\left(1+(\tan(dx+c))^2\right)} + \frac{}{2d\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/2/d/(a^2+b^2)^2/(1+tan(d*x+c)^2)*tan(d*x+c)*a^3+1/2/d/(a^2+b^2)^2/(1+tan(
d*x+c)^2)*tan(d*x+c)*a*b^2+1/2/d/(a^2+b^2)^2/(1+tan(d*x+c)^2)*b*a^2+1/2/d/(
a^2+b^2)^2/(1+tan(d*x+c)^2)*b^3-1/2/d/(a^2+b^2)^2*b^3*ln(1+tan(d*x+c)^2)+3/
2/d/(a^2+b^2)^2*arctan(tan(d*x+c))*a*b^2+1/2/d/(a^2+b^2)^2*arctan(tan(d*x+c
))*a^3+1/d*b^3/(a^2+b^2)^2*ln(a+b*tan(d*x+c))
```

Maxima [A] time = 1.64093, size = 190, normalized size = 2.04

$$\frac{\frac{2\,b^3\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}-\frac{b^3\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3+3\,ab^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{a\tan(dx+c)+b}{\left(a^2+b^2\right)\tan(dx+c)^2+a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(tan(d*
x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/(a^4 + 2*
a^2*b^2 + b^4) + (a*tan(d*x + c) + b)/((a^2 + b^2)*tan(d*x + c)^2 + a^2 + b
^2))/d
```

Fricas [A] time = 1.93546, size = 278, normalized size = 2.99

$$\frac{b^3\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+\left(a^3+3\,ab^2\right)dx+\left(a^2b+b^3\right)\cos\left(dx+c\right)^2+\left(a^3\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2) + (a^3 + 3*a*b^2)*d*x + (a^2*b + b^3)*cos(d*x + c)^2 + (a^3 + a*b^2)*
cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.60913, size = 246, normalized size = 2.65

$$\frac{\frac{2\,b^4\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3+3\,ab^2\right)\left(dx+c\right)}{a^4+2\,a^2b^2+b^4}+\frac{b^3\tan\left(dx+c\right)^2+a^3\tan\left(dx+c\right)+ab^2\tan\left(dx+c\right)+a^2b+2\,b^3}{\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(dx+c\right)^2+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^4*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - b^3*log
(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/(a
^4 + 2*a^2*b^2 + b^4) + (b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*tan(
d*x + c) + a^2*b + 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d
```

3.548 $\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\left(a^2+b^2\right)}+\frac{\cos^2(c+dx)\left(a\left(3a^2+7b^2\right)\tan(c+dx)+4b^3\right)}{8d\left(a^2+b^2\right)^2}+\frac{b^5\log(a\cos(c+dx)+b\sin(c+d}{d\left(a^2+b^2\right)^3}$$

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*x)/(8*(a^2 + b^2)^3) + (b^5*Log[a*Cos[c +
d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(b + a*Tan[c +
d*x]))/(4*(a^2 + b^2)*d) + (Cos[c + d*x]^2*(4*b^3 + a*(3*a^2 + 7*b^2)*Tan[c
 + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rubi [A] time = 0.197114, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3506, 741, 823, 801, 635, 203, 260}

$$\frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\left(a^2+b^2\right)}+\frac{\cos^2(c+dx)\left(a\left(3a^2+7b^2\right)\tan(c+dx)+4b^3\right)}{8d\left(a^2+b^2\right)^2}+\frac{b^5\log(a\cos(c+dx)+b\sin(c+d}{d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```


```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*x)/(8*(a^2 + b^2)^3) + (b^5*Log[a*Cos[c +
d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(b + a*Tan[c +
d*x]))/(4*(a^2 + b^2)*d) + (Cos[c + d*x]^2*(4*b^3 + a*(3*a^2 + 7*b^2)*Tan[c
 + d*x]))/(8*(a^2 + b^2)^2*d)
```


Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} - \frac{b\,\text{Subst}\left(\int \frac{-4-\frac{3a^2}{b^2}-\frac{3ax}{b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{4\left(a^2+b^2\right)d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} + \frac{\cos^2(c+dx)\left(4b^3+a\left(3a^2+7b^2\right)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2 d} + \frac{b^5\,\text{Subst}}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} + \frac{\cos^2(c+dx)\left(4b^3+a\left(3a^2+7b^2\right)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2 d} + \frac{b^5\,\text{Subst}}{}$$

$$= \frac{b^5\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} + \frac{\cos^2(c+dx)\left(4b^3+a\left(3a^2+7b\right.\right.}{8\left(a^2+b^2\right)^2 d}$$

$$= \frac{b^5\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d} + \frac{\cos^2(c+dx)\left(4b^3+a\left(3a^2+7b\right.\right.}{8\left(a^2+b^2\right)^2 d}$$

$$= \frac{a\left(3a^4+10a^2b^2+15b^4\right)x}{8\left(a^2+b^2\right)^3} + \frac{b^5\log(\cos(c+dx))}{\left(a^2+b^2\right)^3 d} + \frac{b^5\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3 d} + \frac{\cos^4(c+dx)(b}{4\left(a^2\right.}$$

Mathematica [A] time = 0.395122, size = 218, normalized size = 1.43

$$\frac{24a^3b^2\sin(2(c+dx))+2a^3b^2\sin(4(c+dx))+4b\left(4a^2b^2+a^4+3b^4\right)\cos(2(c+dx))+b\left(a^2+b^2\right)^2\cos(4(c+dx))+40\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (12*a^5*c + 40*a^3*b^2*c + 60*a*b^4*c + 12*a^5*d*x + 40*a^3*b^2*d*x + 60*a*
b^4*d*x + 4*b*(a^4 + 4*a^2*b^2 + 3*b^4)*Cos[2*(c + d*x)] + b*(a^2 + b^2)^2*
Cos[4*(c + d*x)] + 32*b^5*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 8*a^5*Sin[
2*(c + d*x)] + 24*a^3*b^2*Sin[2*(c + d*x)] + 16*a*b^4*Sin[2*(c + d*x)] + a^
5*Sin[4*(c + d*x)] + 2*a^3*b^2*Sin[4*(c + d*x)] + a*b^4*Sin[4*(c + d*x)])/(
32*(a^2 + b^2)^3*d)
```

Maple [B] time = 0.086, size = 524, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c)),x)
```

```
[Out] 3/8/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^5+5/4/d/(a^2+b^2)^3/(1+
tan(d*x+c)^2)^2*tan(d*x+c)^3*a^3*b^2+7/8/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*t
an(d*x+c)^3*a*b^4+1/2/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^2*b^3
+1/2/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*b^5+7/4/d/(a^2+b^2)^3/(1
+tan(d*x+c)^2)^2*tan(d*x+c)*a^3*b^2+9/8/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*ta
n(d*x+c)*a*b^4+5/8/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^5+1/4/d/(a
^2+b^2)^3/(1+tan(d*x+c)^2)^2*a^4*b+1/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*a^2*b
^3+3/4/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)^2*b^5-1/2/d/(a^2+b^2)^3*b^5*ln(1+tan(
d*x+c)^2)+15/8/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a*b^4+3/8/d/(a^2+b^2)^3*arc
tan(tan(d*x+c))*a^5+5/4/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^3*b^2+1/d*b^5/(a
^2+b^2)^3*ln(a+b*tan(d*x+c))
```

Maxima [A] time = 1.80161, size = 366, normalized size = 2.41

$$\frac{\frac{8\,b^5\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,b^5\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{4\,b^3\tan(dx+c)^2+\left(3\,a^3+7\,ab^2\right)\tan(dx+c)^3+2\,a^2b+6\,b^3+\left(5\,a^3+9\,ab\cdots\right.}{\left(a^4+2\,a^2b^2+b^4\right)\tan(dx+c)^4+a^4+2\,a^2b^2+b^4+2\left(a^4+2\,a^2b^2+b^4\right)\mathrm{t}\cdots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(8*b^5*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4*
b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^5 +
10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (4*b
^3*tan(d*x + c)^2 + (3*a^3 + 7*a*b^2)*tan(d*x + c)^3 + 2*a^2*b + 6*b^3 + (5
*a^3 + 9*a*b^2)*tan(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4 + a^4
 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^2))/d
```

Fricas [A] time = 2.05428, size = 471, normalized size = 3.1

$$\frac{4\,b^5\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+\left(3\,a^5+10\,a^3\ldots\right.}{8\left(a^6+3\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^5*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 + 10*a^3*b^2
+ 15*a*b^4)*d*x + 4*(a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*a^3*b^2 +
a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.4601, size = 435, normalized size = 2.86

$$\frac{\frac{8\,b^6\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{4\,b^5\log\left(\tan\left(dx+c\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)\left(dx+c\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{6\,b^5\tan\left(dx+c\right)^4+3\,a^5\tan\left(dx+c\right)^3+10\,a^3b^2\tan\left(dx+c\right)^3+7\,ab^4\tan\left(dx\ldots\right.}{\ldots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^6*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^
7) - 4*b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3
*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 + (6*b^5*tan(d*x + c)^4 + 3*a^5*tan(d*x + c)^3 + 10*a^3*b^2*tan(d*x + c)^3
 + 7*a*b^4*tan(d*x + c)^3 + 4*a^2*b^3*tan(d*x + c)^2 + 16*b^5*tan(d*x + c)^
2 + 5*a^5*tan(d*x + c) + 14*a^3*b^2*tan(d*x + c) + 9*a*b^4*tan(d*x + c) + 2
*a^4*b + 8*a^2*b^3 + 12*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(d*x
+ c)^2 + 1)^2))/d
```

3.549 $\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{(a^2+b^2)\sec(c+dx)}{b^3d}-\frac{a(2a^2+3b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a\tan(c+d}{2}$$

```
[Out] -(a*(2*a^2 + 3*b^2)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - ((a^2 + b^2)^(3/2)*A
rcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/(b^4*d) + ((a^
2 + b^2)*Sec[c + d*x])/(b^3*d) + Sec[c + d*x]^3/(3*b*d) - (a*Sec[c + d*x]*T
an[c + d*x])/(2*b^2*d)
```

Rubi [A] time = 0.198018, antiderivative size = 152, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3510, 3486, 3768, 3770, 3509, 206}

$$\frac{(a^2+b^2)\sec(c+dx)}{b^3d}-\frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a\tanh^{-1}(\sin(}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(2*b^2*d) - (a*(a^2 + b^2)*ArcTanh[Sin[c + d*x]]
)/(b^4*d) - ((a^2 + b^2)^(3/2)*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/
Sqrt[a^2 + b^2]])/(b^4*d) + ((a^2 + b^2)*Sec[c + d*x])/(b^3*d) + Sec[c + d*
x]^3/(3*b*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d)
```

Rule 3510

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Dist[d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[(d^2*(a^2 + b^2))/b^2, Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\int \sec^3(c+dx)(a-b\tan(c+dx))\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx}{b^2}$$

$$= \frac{\sec^3(c+dx)}{3bd} - \frac{a\int \sec^3(c+dx)\,dx}{b^2} - \frac{\left(a^2+b^2\right)\int \sec(c+dx)(a-b\tan(c+dx))\,dx}{b^4} + \frac{\left(a^2+b^2\right.}{}$$

$$= \frac{\left(a^2+b^2\right)\sec(c+dx)}{b^3d} + \frac{\sec^3(c+dx)}{3bd} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} - \frac{a\int \sec(c+dx)\,dx}{2b^2} - \frac{\left(a\left(a\right.\right.}{}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{2b^2d} - \frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\,\mathrm{t}}{\sqrt{a^2+b}}\right.}{b^4d}$$

Mathematica [B] time = 1.90942, size = 321, normalized size = 2.29

$$48\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right) + \sec^3(c+dx)\left(12b\left(a^2+b^2\right)\cos(2(c+dx)) + 9a\left(2a^2+3b^2\right)\cos(c+dx)\left(\log\left(\mathrm{c}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] (48*(a^2 + b^2)^(3/2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
Sec[c + d*x]^3*(12*a^2*b + 20*b^3 + 12*b*(a^2 + b^2)*Cos[2*(c + d*x)] + 6*a
^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*b^2*Cos[
3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*(2*a^2 + 3*b^2)
*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]) - 6*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] - 9*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] - 6*a*b^2*Sin[2*(c + d*x)]))/(24*b^4*d)
```

Maple [B] time = 0.088, size = 488, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/3/d/b/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)^2*a-1/2/d
/b/(tan(1/2*d*x+1/2*c)+1)^2+1/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a^2-1/2/d/b^2/(t
an(1/2*d*x+1/2*c)+1)*a+3/2/d/b/(tan(1/2*d*x+1/2*c)+1)-1/d*a^3/b^4*ln(tan(1/
2*d*x+1/2*c)+1)-3/2/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2/d/b^4/(a^2+b^2)^(1/2
)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^4+4/d/b^2/(a^
2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^2+
2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
))-1/3/d/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2*a-1/
2/d/b/(tan(1/2*d*x+1/2*c)-1)^2-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b^2
/(tan(1/2*d*x+1/2*c)-1)*a-3/2/d/b/(tan(1/2*d*x+1/2*c)-1)+1/d*a^3/b^4*ln(tan
(1/2*d*x+1/2*c)-1)+3/2/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.95833, size = 633, normalized size = 4.52

$$\frac{6\left(a^2+b^2\right)^{\frac{3}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)-3\left(2\,a^3+3\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + b^2)^(3/2)*cos(d*x + c)^3*log(-(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d
*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 + b^2)) - 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c)
 + 1) + 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 6*a*b^2
*cos(d*x + c)*sin(d*x + c) + 4*b^3 + 12*(a^2*b + b^3)*cos(d*x + c)^2)/(b^4*
d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)), x)
```

Giac [B] time = 1.60799, size = 375, normalized size = 2.68

$$-\frac{\frac{3\left(2a^3+3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}-\frac{3\left(2a^3+3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}+\frac{6\left(a^4+2a^2b^2+b^4\right)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4}+\frac{2\left(3ab\tan\left(\frac{1}{2}\right.\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^3
 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^4 + 2*a^2*b^2 + b
^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(3*
a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 12*b^2*tan(1/2*
d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c
)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 + 8*b^2)/((tan(1/2*d*x + 1/2*c)^2
- 1)^3*b^3))/d
```

3.550 $\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}$$

```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(Cos[c + d*
x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)
```

Rubi [A] time = 0.0944538, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3510, 3486, 3770, 3509, 206}

$$-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(Cos[c + d*
x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)
```

Rule 3510

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Dist[d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[(d^2*(a^2 + b^2))/b^2, Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\int \sec(c+dx)(a-b\tan(c+dx))\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx}{b^2}$$
$$= \frac{\sec(c+dx)}{bd} - \frac{a\int \sec(c+dx)\,dx}{b^2} - \frac{\left(a^2+b^2\right)\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, \cos(c+dx)(b-a\tan(c-$$
$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d} + \frac{\sec(c+dx)}{bd}$$

Mathematica [A] time = 0.141189, size = 109, normalized size = 1.38

$$\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)+a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + a*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) + b*Sec[c + d*x])/(b^2*d)
```

Maple [B] time = 0.069, size = 174, normalized size = 2.2

$$\frac{1}{bd}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{a}{b^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) + 2\frac{a^2}{b^2d\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)-2\,b}{\sqrt{a^2+b^2}}\right) + 2\,\frac{}{d\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d/b/(tan(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2/d/b^2/(a^
2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^2+
2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
))-1/d/b/(tan(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10969, size = 471, normalized size = 5.96

$$-\frac{a\cos(dx+c)\log(\sin(dx+c)+1)-a\cos(dx+c)\log(-\sin(dx+c)+1)-\sqrt{a^2+b^2}\cos(dx+c)\log\left(-\frac{2\,ab\cos(dx+c)}{\ldots}\right.}{2\,b^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) - sqrt(a^2 + b^2)*cos(d*x + c)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x
 + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 + b^2)) - 2*b)/(b^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

Giac [A] time = 1.65507, size = 184, normalized size = 2.33

$$-\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/b^2 + sqrt(a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqr
t(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/b^2
+ 2/((tan(1/2*d*x + 1/2*c)^2 - 1)*b))/d
```

3.551 $\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=46

$$-\frac{\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

```
[Out] -(ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]]/(Sqrt[a^2 +
b^2]*d))
```

Rubi [A] time = 0.0307225, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3509, 206}

$$-\frac{\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]]/(Sqrt[a^2 +
b^2]*d))
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, \cos(c+dx)(b-a\tan(c+dx))\right)}{d}\\ &= -\frac{\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}\end{aligned}$$

Mathematica [A] time = 0.0450385, size = 45, normalized size = 0.98

$$\frac{2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Maple [A] time = 0.031, size = 43, normalized size = 0.9

$$2\,\frac{1}{d\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.92076, size = 312, normalized size = 6.78

$$\frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos
(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 + b^2
)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x)), x)
```

Giac [A] time = 1.58734, size = 100, normalized size = 2.17

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/
2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

3.552 $\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=90

$$\frac{a\sin(c+dx)}{d(a^2+b^2)}+\frac{b\cos(c+dx)}{d(a^2+b^2)}-\frac{b^2\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}$$

```
[Out] -((b^2*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(3/2)*d)) + (b*Cos[c + d*x])/((a^2 + b^2)*d) + (a*Sin[c + d*x])/((a^
2 + b^2)*d)
```

Rubi [A] time = 0.100533, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3511, 3486, 2637, 3509, 206}

$$\frac{a\sin(c+dx)}{d(a^2+b^2)}+\frac{b\cos(c+dx)}{d(a^2+b^2)}-\frac{b^2\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b^2*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(3/2)*d)) + (b*Cos[c + d*x])/((a^2 + b^2)*d) + (a*Sin[c + d*x])/((a^
2 + b^2)*d)
```

Rule 3511

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx &= \frac{\int \cos(c+dx)(a-b\tan(c+dx))\,dx}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}\\ &= \frac{b\cos(c+dx)}{(a^2+b^2)\,d}+\frac{a\int \cos(c+dx)\,dx}{a^2+b^2}-\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, \cos(c+dx)(b-a\tan(c+dx)\right)}{(a^2+b^2)\,d}\\ &= -\frac{b^2\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{b\cos(c+dx)}{(a^2+b^2)\,d}+\frac{a\sin(c+dx)}{(a^2+b^2)\,d}\end{aligned}$$

Mathematica [A] time = 0.305512, size = 79, normalized size = 0.88

$$\frac{\sqrt{a^2+b^2}(a\sin(c+dx)+b\cos(c+dx))+2b^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2]
*(b*Cos[c + d*x] + a*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

Maple [A] time = 0.08, size = 90, normalized size = 1.

$$\frac{1}{d}\left(-2\,\frac{-a\tan(1/2\,dx+c/2)-b}{\left(a^2+b^2\right)\left(1+(\tan(1/2\,dx+c/2))^2\right)}+2\,\frac{b^2}{\left(a^2+b^2\right)^{3/2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)-2\,b}{\sqrt{a^2+b^2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-2/(a^2+b^2)*(-a*tan(1/2*d*x+1/2*c)-b)/(1+tan(1/2*d*x+1/2*c)^2)+2*b^2/
(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.0526, size = 436, normalized size = 4.84

$$\frac{\sqrt{a^2+b^2}b^2\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)+2\left(a^2b+b^3\right)\cos\left(dx+c\right)-}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b^2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(
d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
b^2)) + 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^4
+ 2*a^2*b^2 + b^4)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x)), x)
```

Giac [A] time = 1.52948, size = 159, normalized size = 1.77

$$-\frac{\frac{b^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{\left(a^2+b^2\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*ta
n(1/2*d*x + 1/2*c) + b)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

3.553 $\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal. Leaf size=165

$$-\frac{a\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{b^3\cos(c+dx)}{d\left(a^2+b^2\right)^2}-\frac{b^4\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}$$

```
[Out] -((b^4*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(5/2)*d)) + (b^3*Cos[c + d*x])/((a^2 + b^2)^2*d) + (b*Cos[c + d*x]^3
)/(3*(a^2 + b^2)*d) + (a*b^2*Sin[c + d*x])/((a^2 + b^2)^2*d) + (a*Sin[c + d
*x])/((a^2 + b^2)*d) - (a*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rubi [A] time = 0.194423, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3511, 3486, 2633, 2637, 3509, 206}

$$-\frac{a\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{b^3\cos(c+dx)}{d\left(a^2+b^2\right)^2}-\frac{b^4\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```


```
[Out] -((b^4*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(5/2)*d)) + (b^3*Cos[c + d*x])/((a^2 + b^2)^2*d) + (b*Cos[c + d*x]^3
)/(3*(a^2 + b^2)*d) + (a*b^2*Sin[c + d*x])/((a^2 + b^2)^2*d) + (a*Sin[c + d
*x])/((a^2 + b^2)*d) - (a*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```


Rule 3511

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3509

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> -Dist[f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\int \cos^3(c+dx)(a-b\tan(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{b^2\int \cos(c+dx)(a-b\tan(c+dx))\,dx}{\left(a^2+b^2\right)^2} + \frac{b^4\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx}{\left(a^2+b^2\right)^2} + \frac{a\int \cos^3(c}{a^2+}$$

$$= \frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d} + \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{\left(ab^2\right)\int \cos(c+dx)\,dx}{\left(a^2+b^2\right)^2} - \frac{b^4\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, \cos(}{\left(a^2+b\right.}$$

$$= -\frac{b^4\tanh^{-1}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} + \frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d} + \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{ab^2\sin(c+dx)}{\left(a^2+b^2\right)^2 d} + $$

Mathematica [A] time = 1.34926, size = 137, normalized size = 0.83

$$\frac{\sqrt{a^2+b^2}\left(3b\left(a^2+5b^2\right)\cos(c+dx)+b\left(a^2+b^2\right)\cos(3(c+dx))+2a\sin(c+dx)\left(\left(a^2+b^2\right)\cos(2(c+dx))+5a^2+11\right.\right.}{12d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (24*b^4*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(3*b*(a^2 + 5*b^2)*Cos[c + d*x] + b*(a^2 + b^2)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 11*b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 + b^2
)^(5/2)*d)
```

Maple [A] time = 0.082, size = 221, normalized size = 1.3

$$\frac{1}{d}\left(-2\,\frac{\left(-a^3-2\,ab^2\right)(\tan(1/2\,dx+c/2))^5+\left(-ba^2-2\,b^3\right)(\tan(1/2\,dx+c/2))^4+\left(-2/3\,a^3-8/3\,ab^2\right)(\tan(1/2\,dx+c/2)}{\left(a^4+2\,a^2b^2+b^4\right)\left(1+(\tan(1/2\,dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tan(1/2*d*x+1/2*c)^5+(-a^2*b-2*
b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^3-8/3*a*b^2)*tan(1/2*d*x+1/2*c)^3-2*b^3*t
an(1/2*d*x+1/2*c)^2+(-a^3-2*a*b^2)*tan(1/2*d*x+1/2*c)-1/3*b*a^2-4/3*b^3)/(1
+tan(1/2*d*x+1/2*c)^2)^3+2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(
1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94192, size = 598, normalized size = 3.62

$$\frac{3\sqrt{a^2+b^2}b^4\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos(d}{6\left(a^6+3\,a^4b^2+3\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*b^4*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b
^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*si
n(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2)) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + 6*(a^2*b^3 + b^5)*co
s(d*x + c) + 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.54098, size = 386, normalized size = 2.34

$$-\frac{\frac{3\,b^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right.}{\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}\,d\right.\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b
^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a*b^2*tan(1/2*d*
x + 1/2*c)^5 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^
4 + 2*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*b^3*t
an(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*d*x +
1/2*c) + a^2*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 +
1)^3))/d
```

3.554 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=178

$$\frac{(a^2+b^2)\tan^3(c+dx)}{b^4d}-\frac{a(2a^2+3b^2)\tan^2(c+dx)}{b^5d}+\frac{(9a^2b^2+5a^4+3b^4)\tan(c+dx)}{b^6d}-\frac{(a^2+b^2)^3}{b^7d(a+b\tan(c+dx))}-\frac{6a(a}{}$$

```
[Out] (-6*a*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(b^7*d) + ((5*a^4 + 9*a^2*b^2
+ 3*b^4)*Tan[c + d*x])/(b^6*d) - (a*(2*a^2 + 3*b^2)*Tan[c + d*x]^2)/(b^5*d)
 + ((a^2 + b^2)*Tan[c + d*x]^3)/(b^4*d) - (a*Tan[c + d*x]^4)/(2*b^3*d) + Ta
n[c + d*x]^5/(5*b^2*d) - (a^2 + b^2)^3/(b^7*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.151808, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$\frac{(a^2+b^2)\tan^3(c+dx)}{b^4d}-\frac{a(2a^2+3b^2)\tan^2(c+dx)}{b^5d}+\frac{(9a^2b^2+5a^4+3b^4)\tan(c+dx)}{b^6d}-\frac{(a^2+b^2)^3}{b^7d(a+b\tan(c+dx))}-\frac{6a(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-6*a*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(b^7*d) + ((5*a^4 + 9*a^2*b^2
+ 3*b^4)*Tan[c + d*x])/(b^6*d) - (a*(2*a^2 + 3*b^2)*Tan[c + d*x]^2)/(b^5*d)
 + ((a^2 + b^2)*Tan[c + d*x]^3)/(b^4*d) - (a*Tan[c + d*x]^4)/(2*b^3*d) + Ta
n[c + d*x]^5/(5*b^2*d) - (a^2 + b^2)^3/(b^7*d*(a + b*Tan[c + d*x]))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^3}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{5a^4+9a^2b^2+3b^4}{b^6}-\frac{2a(2a^2+3b^2)x}{b^6}+\frac{3(a^2+b^2)x^2}{b^6}-\frac{2ax^3}{b^6}+\frac{x^4}{b^6}+\frac{(a^2+b^2)^3}{b^6(a+x)^2}-\frac{6a(a^2+b^2)^2}{b^6(a+x)}\right)dx, x, b\right.}{bd}\\ &= -\frac{6a(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^7d}+\frac{(5a^4+9a^2b^2+3b^4)\tan(c+dx)}{b^6d}-\frac{a(2a^2+3b^2)\tan}{b^5d}\end{aligned}$$

Mathematica [A] time = 2.41941, size = 229, normalized size = 1.29

$$-2\left(-2a^2b^4\tan^4(c+dx)+ab^3\left(5a^2+7b^2\right)\tan^3(c+dx)-b^2\left(29a^2b^2+15a^4+8b^4\right)\tan^2(c+dx)+2ab\tan(c+dx)\left(1\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*b^6*Sec[c + d*x]^6 + b^4*Sec[c + d*x]^4*(a^2 + 4*b^2 - 3*a*b*Tan[c + d*x
]) - 2*(8*(a^2 + b^2)^3 + 30*a^2*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] + 2*
a*b*(-11*a^4 - 18*a^2*b^2 - 4*b^4 + 15*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]
])*Tan[c + d*x] - b^2*(15*a^4 + 29*a^2*b^2 + 8*b^4)*Tan[c + d*x]^2 + a*b^3*
(5*a^2 + 7*b^2)*Tan[c + d*x]^3 - 2*a^2*b^4*Tan[c + d*x]^4))/(10*b^7*d*(a +
b*Tan[c + d*x]))
```

Maple [A] time = 0.106, size = 305, normalized size = 1.7

$$\frac{(\tan(dx+c))^5}{5b^2d}-\frac{a(\tan(dx+c))^4}{2b^3d}+\frac{(\tan(dx+c))^3a^2}{db^4}+\frac{(\tan(dx+c))^3}{b^2d}-2\frac{(\tan(dx+c))^2a^3}{db^5}-3\frac{a(\tan(dx+c))^2}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/5*tan(d*x+c)^5/b^2/d-1/2*a*tan(d*x+c)^4/b^3/d+1/d/b^4*tan(d*x+c)^3*a^2+ta
n(d*x+c)^3/b^2/d-2/d/b^5*tan(d*x+c)^2*a^3-3*a*tan(d*x+c)^2/b^3/d+5/d/b^6*a^
4*tan(d*x+c)+9/d/b^4*a^2*tan(d*x+c)+3*tan(d*x+c)/b^2/d-1/d/b^7/(a+b*tan(d*x
+c))*a^6-3/d/b^5/(a+b*tan(d*x+c))*a^4-3/d/b^3/(a+b*tan(d*x+c))*a^2-1/b/d/(a
+b*tan(d*x+c))-6/d*a^5/b^7*ln(a+b*tan(d*x+c))-12/d*a^3/b^5*ln(a+b*tan(d*x+c
))-6*a*ln(a+b*tan(d*x+c))/b^3/d
```

Maxima [A] time = 1.13519, size = 251, normalized size = 1.41

$$-\frac{\frac{10\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}{b^8\tan(dx+c)+ab^7}-\frac{2b^4\tan(dx+c)^5-5ab^3\tan(dx+c)^4+10\left(a^2b^2+b^4\right)\tan(dx+c)^3-10\left(2a^3b+3ab^3\right)\tan(dx+c)^2+10\left(5a^4+9a^2b^2+3b^4\right)\tan(dx+}{b^6}}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/(b^8*tan(d*x + c) + a*b^7) -
(2*b^4*tan(d*x + c)^5 - 5*a*b^3*tan(d*x + c)^4 + 10*(a^2*b^2 + b^4)*tan(d*x
 + c)^3 - 10*(2*a^3*b + 3*a*b^3)*tan(d*x + c)^2 + 10*(5*a^4 + 9*a^2*b^2 + 3
*b^4)*tan(d*x + c))/b^6 + 60*(a^5 + 2*a^3*b^2 + a*b^4)*log(b*tan(d*x + c) +
 a)/b^7)/d
```

Fricas [B] time = 2.55605, size = 890, normalized size = 5.

$$-\frac{4\left(15a^4b^2+25a^2b^4+8b^6\right)\cos(dx+c)^6-2b^6-2\left(15a^4b^2+25a^2b^4+8b^6\right)\cos(dx+c)^4-\left(5a^2b^4+4b^6\right)\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/10*(4*(15*a^4*b^2 + 25*a^2*b^4 + 8*b^6)*cos(d*x + c)^6 - 2*b^6 - 2*(15*a
^4*b^2 + 25*a^2*b^4 + 8*b^6)*cos(d*x + c)^4 - (5*a^2*b^4 + 4*b^6)*cos(d*x +
 c)^2 + 30*((a^6 + 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^6 + (a^5*b + 2*a^3*b^3
 + a*b^5)*cos(d*x + c)^5*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c)
+ (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*((a^6 + 2*a^4*b^2 + a^2*b^4)*cos(d
*x + c)^6 + (a^5*b + 2*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c))*log(co
s(d*x + c)^2) + (3*a*b^5*cos(d*x + c) - 4*(15*a^5*b + 25*a^3*b^3 + 8*a*b^5)
*cos(d*x + c)^5 + 2*(5*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^3)*sin(d*x + c))/(a*
b^7*d*cos(d*x + c)^6 + b^8*d*cos(d*x + c)^5*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31501, size = 342, normalized size = 1.92

$$-\frac{\frac{60\left(a^{5}+2a^{3}b^{2}+ab^{4}\right)\log\left(|b\tan(dx+c)+a|\right)}{b^{7}}-\frac{10\left(6a^{5}b\tan(dx+c)+12a^{3}b^{3}\tan(dx+c)+6ab^{5}\tan(dx+c)+5a^{6}+9a^{4}b^{2}+3a^{2}b^{4}-b^{6}\right)}{(b\tan(dx+c)+a)b^{7}}-\frac{2b^{8}\tan(dx+c)^{5}-5ab^{7}\tan\cdots}{\cdots}}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(60*(a^5 + 2*a^3*b^2 + a*b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 10*(
6*a^5*b*tan(d*x + c) + 12*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c) + 5*a
^6 + 9*a^4*b^2 + 3*a^2*b^4 - b^6)/((b*tan(d*x + c) + a)*b^7) - (2*b^8*tan(d
*x + c)^5 - 5*a*b^7*tan(d*x + c)^4 + 10*a^2*b^6*tan(d*x + c)^3 + 10*b^8*tan
(d*x + c)^3 - 20*a^3*b^5*tan(d*x + c)^2 - 30*a*b^7*tan(d*x + c)^2 + 50*a^4*
b^4*tan(d*x + c) + 90*a^2*b^6*tan(d*x + c) + 30*b^8*tan(d*x + c))/b^10)/d
```

3.555 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$\frac{(3a^2+2b^2)\tan(c+dx)}{b^4d}-\frac{(a^2+b^2)^2}{b^5d(a+b\tan(c+dx))}-\frac{4a(a^2+b^2)\log(a+b\tan(c+dx))}{b^5d}-\frac{a\tan^2(c+dx)}{b^3d}+\frac{\tan^3(c+\dots}{3b^2d}$$

```
[Out] (-4*a*(a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^5*d) + ((3*a^2 + 2*b^2)*Tan[c
 + d*x])/(b^4*d) - (a*Tan[c + d*x]^2)/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d) -
(a^2 + b^2)^2/(b^5*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.0968805, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$\frac{(3a^2+2b^2)\tan(c+dx)}{b^4d}-\frac{(a^2+b^2)^2}{b^5d(a+b\tan(c+dx))}-\frac{4a(a^2+b^2)\log(a+b\tan(c+dx))}{b^5d}-\frac{a\tan^2(c+dx)}{b^3d}+\frac{\tan^3(c+\dots}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-4*a*(a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^5*d) + ((3*a^2 + 2*b^2)*Tan[c
 + d*x])/(b^4*d) - (a*Tan[c + d*x]^2)/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d) -
(a^2 + b^2)^2/(b^5*d*(a + b*Tan[c + d*x]))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^2}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{3a^2+2b^2}{b^4}-\frac{2ax}{b^4}+\frac{x^2}{b^4}+\frac{(a^2+b^2)^2}{b^4(a+x)^2}-\frac{4a(a^2+b^2)}{b^4(a+x)}\right)dx, x, b\tan(c+dx)\right)}{bd}\\ &= -\frac{4a(a^2+b^2)\log(a+b\tan(c+dx))}{b^5d}+\frac{(3a^2+2b^2)\tan(c+dx)}{b^4d}-\frac{a\tan^2(c+dx)}{b^3d}+\tan\dots\end{aligned}$$

Mathematica [A] time = 2.77048, size = 122, normalized size = 1.05

$$\frac{4b\left(2a^2+b^2\right)\tan(c+dx)+\frac{b^4\sec^4(c+dx)-4\left(a^2+b^2\right)\left(3a^2\log(a+b\tan(c+dx))+a^2+3ab\tan(c+dx)\log(a+b\tan(c+dx))+b^2\right)}{a+b\tan(c+dx)}-2ab^2\tan^2(c+dx)}{3b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (4*b*(2*a^2 + b^2)*Tan[c + d*x] - 2*a*b^2*Tan[c + d*x]^2 + (b^4*Sec[c + d*x
]^4 - 4*(a^2 + b^2)*(a^2 + b^2 + 3*a^2*Log[a + b*Tan[c + d*x]] + 3*a*b*Log[
a + b*Tan[c + d*x]]*Tan[c + d*x]))/(a + b*Tan[c + d*x]))/(3*b^5*d)
```

Maple [A] time = 0.098, size = 174, normalized size = 1.5

$$\frac{(\tan(dx+c))^3}{3b^2d}-\frac{a(\tan(dx+c))^2}{b^3d}+3\frac{a^2\tan(dx+c)}{db^4}+2\frac{\tan(dx+c)}{b^2d}-\frac{a^4}{db^5(a+b\tan(dx+c))}-2\frac{a^2}{b^3d(a+b\tan(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/3*tan(d*x+c)^3/b^2/d-a*tan(d*x+c)^2/b^3/d+3/d/b^4*a^2*tan(d*x+c)+2*tan(d*
x+c)/b^2/d-1/d/b^5/(a+b*tan(d*x+c))*a^4-2/d/b^3/(a+b*tan(d*x+c))*a^2-1/b/d/
(a+b*tan(d*x+c))-4/d*a^3/b^5*ln(a+b*tan(d*x+c))-4*a*ln(a+b*tan(d*x+c))/b^3/
d
```

Maxima [A] time = 1.1341, size = 155, normalized size = 1.34

$$-\frac{\frac{3\left(a^4+2a^2b^2+b^4\right)}{b^6\tan(dx+c)+ab^5}-\frac{b^2\tan(dx+c)^3-3ab\tan(dx+c)^2+3\left(3a^2+2b^2\right)\tan(dx+c)}{b^4}+\frac{12\left(a^3+ab^2\right)\log(b\tan(dx+c)+a)}{b^5}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a^4 + 2*a^2*b^2 + b^4)/(b^6*tan(d*x + c) + a*b^5) - (b^2*tan(d*x +
 c)^3 - 3*a*b*tan(d*x + c)^2 + 3*(3*a^2 + 2*b^2)*tan(d*x + c))/b^4 + 12*(a^
3 + a*b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

Fricas [B] time = 2.29584, size = 656, normalized size = 5.66

$$-\frac{4\left(3a^2b^2+2b^4\right)\cos(dx+c)^4-b^4-2\left(3a^2b^2+2b^4\right)\cos(dx+c)^2+6\left(\left(a^4+a^2b^2\right)\cos(dx+c)^4+\left(a^3b+ab^3\right)\cos(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(3*a^2*b^2 + 2*b^4)*cos(d*x + c)^4 - b^4 - 2*(3*a^2*b^2 + 2*b^4)*co
s(d*x + c)^2 + 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(
d*x + c)^2 + b^2) - 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos
(d*x + c)^3*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(a*b^3*cos(d*x + c) - 2*(
3*a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4 +
b^6*d*cos(d*x + c)^3*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x))**2, x)
```

Giac [A] time = 1.4407, size = 201, normalized size = 1.73

$$-\frac{\frac{12\left(a^3+ab^2\right)\log(|b\tan(dx+c)+a|)}{b^5}-\frac{b^4\tan(dx+c)^3-3\,ab^3\tan(dx+c)^2+9\,a^2b^2\tan(dx+c)+6\,b^4\tan(dx+c)}{b^6}-\frac{3\left(4\,a^3b\tan(dx+c)+4\,ab^3\tan(dx+c)+3\,a^4+2\right.}{(b\tan(dx+c)+a)b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3 + a*b^2)*log(abs(b*tan(d*x + c) + a))/b^5 - (b^4*tan(d*x + c)
^3 - 3*a*b^3*tan(d*x + c)^2 + 9*a^2*b^2*tan(d*x + c) + 6*b^4*tan(d*x + c))/
b^6 - 3*(4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 3*a^4 + 2*a^2*b^2 -
b^4)/((b*tan(d*x + c) + a)*b^5))/d
```

3.556 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=61

$$-\frac{a^2+b^2}{b^3d(a+b\tan(c+dx))}-\frac{2a\log(a+b\tan(c+dx))}{b^3d}+\frac{\tan(c+dx)}{b^2d}$$

```
[Out] (-2*a*Log[a + b*Tan[c + d*x]])/(b^3*d) + Tan[c + d*x]/(b^2*d) - (a^2 + b^2)
/(b^3*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.0660522, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$-\frac{a^2+b^2}{b^3d(a+b\tan(c+dx))}-\frac{2a\log(a+b\tan(c+dx))}{b^3d}+\frac{\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-2*a*Log[a + b*Tan[c + d*x]])/(b^3*d) + Tan[c + d*x]/(b^2*d) - (a^2 + b^2)
/(b^3*d*(a + b*Tan[c + d*x]))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1+\frac{x^2}{b^2}}{(a+x)^2}\,dx,x,b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2}+\frac{a^2+b^2}{b^2(a+x)^2}-\frac{2a}{b^2(a+x)}\right)dx,x,b\tan(c+dx)\right)}{bd}\\ &= -\frac{2a\log(a+b\tan(c+dx))}{b^3d}+\frac{\tan(c+dx)}{b^2d}-\frac{a^2+b^2}{b^3d(a+b\tan(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0607277, size = 51, normalized size = 0.84

$$\frac{-\frac{a^2+b^2}{a+b\tan(c+dx)}-2a\log(a+b\tan(c+dx))+b\tan(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-2*a*Log[a + b*Tan[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c +
 d*x]))/(b^3*d)
```

Maple [A] time = 0.083, size = 78, normalized size = 1.3

$$\frac{\tan(dx+c)}{b^2d}-\frac{a^2}{db^3(a+b\tan(dx+c))}-\frac{1}{bd(a+b\tan(dx+c))}-2\frac{a\ln(a+b\tan(dx+c))}{db^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x)
```

```
[Out] tan(d*x+c)/b^2/d-1/d/b^3/(a+b*tan(d*x+c))*a^2-1/b/d/(a+b*tan(d*x+c))-2*a*ln
(a+b*tan(d*x+c))/b^3/d
```

Maxima [A] time = 1.06808, size = 81, normalized size = 1.33

$$-\frac{\frac{a^2+b^2}{b^4\tan(dx+c)+ab^3}+\frac{2a\log(b\tan(dx+c)+a)}{b^3}-\frac{\tan(dx+c)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((a^2 + b^2)/(b^4*tan(d*x + c) + a*b^3) + 2*a*log(b*tan(d*x + c) + a)/b^3
- tan(d*x + c)/b^2)/d
```

Fricas [B] time = 2.00202, size = 440, normalized size = 7.21

$$-\frac{2b^2\cos(dx+c)^2-2ab\cos(dx+c)\sin(dx+c)-b^2+\left(a^2\cos(dx+c)^2+ab\cos(dx+c)\sin(dx+c)\right)\log\left(2ab\cos\right.}{ab^3d\cos(dx+c)^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - b^2 + (a^2*cos(d
*x + c)^2 + a*b*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*cos(d*x + c)^2 + a*b*cos(d*x
 + c)*sin(d*x + c))*log(cos(d*x + c)^2))/(a*b^3*d*cos(d*x + c)^2 + b^4*d*co
s(d*x + c)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x))**2, x)
```

Giac [A] time = 1.35014, size = 96, normalized size = 1.57

$$-\frac{\frac{2\,a\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{b^3}-\frac{\tan\left(dx+c\right)}{b^2}-\frac{2\,ab\tan\left(dx+c\right)+a^2-b^2}{\left(b\tan\left(dx+c\right)+a\right)b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*tan(d*x + c) + a))/b^3 - tan(d*x + c)/b^2 - (2*a*b*tan(d*x
+ c) + a^2 - b^2)/((b*tan(d*x + c) + a)*b^3))/d
```

3.557 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2} dx$

Optimal. Leaf size=20

$$-\frac{1}{bd(a+b\tan(c+dx))}$$

```
[Out] -(1/(b*d*(a + b*Tan[c + d*x])))
```

Rubi [A] time = 0.0391909, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 32}

$$-\frac{1}{bd(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]

[Out] -(1/(b*d*(a + b*Tan[c + d*x])))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2} dx, x, b\tan(c+dx)\right)}{bd} \\ &= -\frac{1}{bd(a+b\tan(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0443234, size = 32, normalized size = 1.6

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]

[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Maple [A] time = 0.043, size = 21, normalized size = 1.1

$$-\frac{1}{bd\left(a+b\tan\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/b/d/(a+b*tan(d*x+c))
```

Maxima [A] time = 1.12928, size = 27, normalized size = 1.35

$$-\frac{1}{\left(b\tan\left(dx+c\right)+a\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

Fricas [B] time = 1.76802, size = 132, normalized size = 6.6

$$-\frac{b\cos\left(dx+c\right)-a\sin\left(dx+c\right)}{\left(a^{3}+ab^{2}\right)d\cos\left(dx+c\right)+\left(a^{2}b+b^{3}\right)d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b +
 b^3)*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{2}\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x))**2, x)
```

Giac [A] time = 1.33803, size = 27, normalized size = 1.35

$$-\frac{1}{\left(b\tan\left(dx+c\right)+a\right)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

3.558 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$\frac{b\left(a^2-3b^2\right)}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^3}+\frac{x\left(6a^2b^2\right.}{2\left(a\right.}$$

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/(2*(a^2 + b^2)^3) + (4*a*b^3*Log[a*Cos[c + d*
x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (b*(a^2 - 3*b^2))/(2*(a^2 + b^2)^
2*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.162816, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3506, 741, 801, 635, 203, 260}

$$\frac{b\left(a^2-3b^2\right)}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^3}+\frac{x\left(6a^2b^2\right.}{2\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/(2*(a^2 + b^2)^3) + (4*a*b^3*Log[a*Cos[c + d*
x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (b*(a^2 - 3*b^2))/(2*(a^2 + b^2)^
2*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{b\,\text{Subst}\left(\int \frac{-3-\frac{a^2}{b^2}-\frac{2ax}{b^2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)}\,dx, x, b\tan(c+dx)\right)}{2\left(a^2+b^2\right)d}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{b\,\text{Subst}\left(\int \left(\frac{a^2-3b^2}{\left(a^2+b^2\right)(a+x)^2} - \frac{8ab^2}{\left(a^2+b^2\right)^2(a+x)} + \frac{-a^4-6a^2b^2+3b^4}{\left(a^2+b^2\right)^2\left(b^2\right.}\right.\right.}{2\left(a^2+b^2\right)d}$$

$$= \frac{4ab^3\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3 d} + \frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \frac{\cos^2(c+dx)(b+a\tan}{2\left(a^2+b^2\right)d(a+b\tan}$$

$$= \frac{4ab^3\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3 d} + \frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \frac{\cos^2(c+dx)(b+a\tan}{2\left(a^2+b^2\right)d(a+b\tan}$$

$$= \frac{\left(a^4+6a^2b^2-3b^4\right)x}{2\left(a^2+b^2\right)^3} + \frac{4ab^3\log(\cos(c+dx))}{\left(a^2+b^2\right)^3 d} + \frac{4ab^3\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3 d} + \frac{}{2\left(a^2+b\right.}$$

Mathematica [A] time = 3.75131, size = 304, normalized size = 2.

$$\frac{-\frac{ab\left(\left(\sqrt{-b^2}-a\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-2\sqrt{-b^2}\log(a+b\tan(c+dx))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)\right)}{\sqrt{-b^2}\left(a^2+b^2\right)} + \frac{b\left(a^2-3b^2\right)\left(\frac{2\left(a^2+b^2\right)}{a+b\tan(c+dx)}+\left(\frac{b^2-a^2}{\sqrt{-b^2}}+2a\right)\log\left(\sqrt{\right.\right.}{}}{2d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-((a*b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*Sqrt[-b^2]*
Log[a + b*Tan[c + d*x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]
))/(Sqrt[-b^2]*(a^2 + b^2))) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(a + b
*Tan[c + d*x]) + (b*(a^2 - 3*b^2)*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt
[-b^2] - b*Tan[c + d*x]] - 4*a*Log[a + b*Tan[c + d*x]] + (2*a + (a^2 - b^2)
/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 + b^2))/(a + b*Tan[
```

```
c + d*x])))/(2*(a^2 + b^2)^2))/(2*(a^2 + b^2)*d)
```

Maple [A] time = 0.102, size = 292, normalized size = 1.9

$$\frac{\tan(dx+c)\,a^4}{2d\,(a^2+b^2)^3\left(1+(\tan(dx+c))^2\right)}-\frac{\tan(dx+c)\,b^4}{2d\,(a^2+b^2)^3\left(1+(\tan(dx+c))^2\right)}+\frac{ba^3}{d\,(a^2+b^2)^3\left(1+(\tan(dx+c))^2\right)}+\frac{}{d\,(a^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/2/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)*tan(d*x+c)*a^4-1/2/d/(a^2+b^2)^3/(1+tan(
d*x+c)^2)*tan(d*x+c)*b^4+1/d/(a^2+b^2)^3/(1+tan(d*x+c)^2)*b*a^3+1/d/(a^2+b^
2)^3/(1+tan(d*x+c)^2)*b^3*a-2/d/(a^2+b^2)^3*b^3*a*ln(1+tan(d*x+c)^2)+3/d/(a
^2+b^2)^3*arctan(tan(d*x+c))*a^2*b^2-3/2/d/(a^2+b^2)^3*arctan(tan(d*x+c))*b
^4+1/2/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^4-1/d*b^3/(a^2+b^2)^2/(a+b*tan(d*
x+c))+4/d*b^3/(a^2+b^2)^3*a*ln(a+b*tan(d*x+c))
```

Maxima [A] time = 1.66835, size = 381, normalized size = 2.51

$$\frac{\frac{8\,ab^3\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^4+6\,a^2b^2-3\,b^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,a^2b-2\,b^3+\left(a^2b-3\,b^3\right)\tan(dx+c)^2+\left(a^3+ab^2\right)\tan}{a^5+2\,a^3b^2+ab^4+\left(a^4b+2\,a^2b^3+b^5\right)\tan(dx+c)^3+\left(a^5+2\,a^3b^2+ab^4\right)\tan(dx+}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a*b^3*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^4
+ 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*a^2
*b - 2*b^3 + (a^2*b - 3*b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d*x + c))/(
a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^3 + (a^5 +
 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)
))/d
```

Fricas [A] time = 2.12177, size = 614, normalized size = 4.04

$$\frac{\left(a^4b+2\,a^2b^3+b^5\right)\cos(dx+c)^3-\left(a^2b^3+3\,b^5-\left(a^5+6\,a^3b^2-3\,ab^4\right)dx\right)\cos(dx+c)+4\left(a^2b^3\cos(dx+c)+ab^4\sin(d}{2\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 - (a^2*b^3 + 3*b^5 - (a^5 + 6
*a^3*b^2 - 3*a*b^4)*d*x)*cos(d*x + c) + 4*(a^2*b^3*cos(d*x + c) + a*b^4*sin
(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) - (a^3*b^2 - a*b^4 - (a^4*b + 6*a^2*b^3 - 3*b^5)*d*x - (a^5 + 2*a^3
*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
 a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x +
```

```
c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.43003, size = 338, normalized size = 2.22

$$\frac{\frac{8\,ab^4\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{\left(a^4+6\,a^2b^2-3\,b^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{a^2b\tan(dx+c)^2-3\,b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan(dx+c)+2\,a}{\left(a^4+2\,a^2b^2+b^4\right)\left(b\tan(dx+c)^3+a\tan(dx+c)^2+b\tan(dx+c)+a\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(8*a*b^4*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7) - 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ (a^4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 (a^2*b*tan(d*x + c)^2 - 3*b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*ta
n(d*x + c) + 2*a^2*b - 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x + c)^3 +
a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

3.559 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=235

$$\frac{3b\left(a^2-b^2\right)\left(a^2+5b^2\right)}{8d\left(a^2+b^2\right)^3\left(a+b\tan(c+dx)\right)}-\frac{\cos^2(c+dx)\left(b\left(a^2-5b^2\right)-3a\left(a^2+3b^2\right)\tan(c+dx)\right)}{8d\left(a^2+b^2\right)^2\left(a+b\tan(c+dx)\right)}+\frac{\cos^4(c+dx)(a\tan(c+dx)}{4d\left(a^2+b^2\right)(a+b\tan(c+}$$

```
[Out] (3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*x)/(8*(a^2 + b^2)^4) + (6*a*b^5*L
og[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) + (3*b*(a^2 - b^2)*(
a^2 + 5*b^2))/(8*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(b
 + a*Tan[c + d*x]))/(4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^
2*(b*(a^2 - 5*b^2) - 3*a*(a^2 + 3*b^2)*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d*(a
 + b*Tan[c + d*x]))
```

Rubi [A] time = 0.265665, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3506, 741, 823, 801, 635, 203, 260}

$$\frac{3b\left(a^2-b^2\right)\left(a^2+5b^2\right)}{8d\left(a^2+b^2\right)^3\left(a+b\tan(c+dx)\right)}-\frac{\cos^2(c+dx)\left(b\left(a^2-5b^2\right)-3a\left(a^2+3b^2\right)\tan(c+dx)\right)}{8d\left(a^2+b^2\right)^2\left(a+b\tan(c+dx)\right)}+\frac{\cos^4(c+dx)(a\tan(c+dx)}{4d\left(a^2+b^2\right)(a+b\tan(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*x)/(8*(a^2 + b^2)^4) + (6*a*b^5*L
og[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) + (3*b*(a^2 - b^2)*(
a^2 + 5*b^2))/(8*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(b
 + a*Tan[c + d*x]))/(4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^
2*(b*(a^2 - 5*b^2) - 3*a*(a^2 + 3*b^2)*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d*(a
 + b*Tan[c + d*x]))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
```

```
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{b\,\text{Subst}\left(\int \frac{-5-\frac{3a^2}{b^2}-\frac{4ax}{b^2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{4\left(a^2+b^2\right)d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b\left(a^2-5b^2\right)-3a\left(a^2+3b^2\right)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b\left(a^2-5b^2\right)-3a\left(a^2+3b^2\right)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))}$$

$$= \frac{6ab^5\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{3b\left(a^2-b^2\right)\left(a^2+5b^2\right)}{8\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)(b+a\tan}{4\left(a^2+b^2\right)d(a+b\tan}$$

$$= \frac{6ab^5\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{3b\left(a^2-b^2\right)\left(a^2+5b^2\right)}{8\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)(b+a\tan}{4\left(a^2+b^2\right)d(a+b\tan}$$

$$= \frac{3\left(a^6+5a^4b^2+15a^2b^4-5b^6\right)x}{8\left(a^2+b^2\right)^4} + \frac{6ab^5\log(\cos(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{6ab^5\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d}$$

Mathematica [A] time = 3.16456, size = 416, normalized size = 1.77

$$-\frac{\sqrt{-b^2}\left(6a\left(a^2+b^2\right)\left(a^2+3b^2\right)(a+b\tan(c+dx))\left(\left(a-\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2\sqrt{-b^2}\log(a+b\tan(c+dx))-\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)\right)+3\left(4a^2b^2+a^4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (4*b*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) + (2*b*Cos[c + d*x]^2*(-(a^2*b) +
5*b^3 + 3*a*(a^2 + 3*b^2)*Tan[c + d*x]))/(a^2 + b^2) - (Sqrt[-b^2]*(6*a*(a^
2 + b^2)*(a^2 + 3*b^2)*((a - Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] +
 2*Sqrt[-b^2]*Log[a + b*Tan[c + d*x]] - (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b
*Tan[c + d*x]])*(a + b*Tan[c + d*x]) + 3*(a^4 + 4*a^2*b^2 - 5*b^4)*(2*Sqrt[
-b^2]*(a^2 + b^2) + (-a^2 + b^2 + 2*a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c
+ d*x]]*(a + b*Tan[c + d*x]) - 4*a*Sqrt[-b^2]*Log[a + b*Tan[c + d*x]]*(a +
b*Tan[c + d*x]) + (a^2 - b^2 + 2*a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d
*x]]*(a + b*Tan[c + d*x]))))/(a^2 + b^2)^3)/(16*b*(a^2 + b^2)*d*(a + b*Tan[
c + d*x]))
```

Maple [B] time = 0.103, size = 661, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 3/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^6+15/8/d/(a^2+b^2)^4/(1
+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^4*b^2+5/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*
tan(d*x+c)^3*a^2*b^4-7/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*b^6+
2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^3*b^3+2/d/(a^2+b^2)^4/(1+
tan(d*x+c)^2)^2*tan(d*x+c)^2*a*b^5+17/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*ta
n(d*x+c)*a^4*b^2+3/8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^2*b^4-9/
8/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*tan(d*x+c)*b^6+5/8/d/(a^2+b^2)^4/(1+tan(
d*x+c)^2)^2*tan(d*x+c)*a^6+1/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*a^5*b+3/d/(
a^2+b^2)^4/(1+tan(d*x+c)^2)^2*a^3*b^3+5/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)^2*
a*b^5-3/d/(a^2+b^2)^4*a*b^5*ln(1+tan(d*x+c)^2)+45/8/d/(a^2+b^2)^4*arctan(ta
n(d*x+c))*a^2*b^4-15/8/d/(a^2+b^2)^4*arctan(tan(d*x+c))*b^6+3/8/d/(a^2+b^2)
^4*arctan(tan(d*x+c))*a^6+15/8/d/(a^2+b^2)^4*arctan(tan(d*x+c))*a^4*b^2-1/d
*b^5/(a^2+b^2)^3/(a+b*tan(d*x+c))+6/d*b^5/(a^2+b^2)^4*a*ln(a+b*tan(d*x+c))
```

Maxima [B] time = 1.73441, size = 678, normalized size = 2.89

$$\frac{\frac{48\,ab^5\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{24\,ab^5\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\left(a^6+5\,a^4b^2+15\,a^2b^4-5\,b^6\right)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{4\,a^4b+20\,a^2b^3-8\,b^5+3\left(a^4\right.}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6+\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\tan(}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(48*a*b^5*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8) - 24*a*b^5*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) + 3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*(d*x + c)/(a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (4*a^4*b + 20*a^2*b^3 - 8*b^
5 + 3*(a^4*b + 4*a^2*b^3 - 5*b^5)*tan(d*x + c)^4 + 3*(a^5 + 4*a^3*b^2 + 3*a
*b^4)*tan(d*x + c)^3 + (5*a^4*b + 28*a^2*b^3 - 25*b^5)*tan(d*x + c)^2 + (5*
a^5 + 16*a^3*b^2 + 11*a*b^4)*tan(d*x + c))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a
*b^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)^5 + (a^7 + 3*a^5*
b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5
+ b^7)*tan(d*x + c)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c
)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)))/d
```

Fricas [A] time = 2.54553, size = 944, normalized size = 4.02

$$\frac{4\left(a^6b+3a^4b^3+3a^2b^5+b^7\right)\cos(dx+c)^5-2\left(a^6b-3a^4b^3-9a^2b^5-5b^7\right)\cos(dx+c)^3+\left(3a^6b+8a^4b^3-9a^2b^5-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^5 - 2*(a^6*b - 3
*a^4*b^3 - 9*a^2*b^5 - 5*b^7)*cos(d*x + c)^3 + (3*a^6*b + 8*a^4*b^3 - 9*a^2
*b^5 - 30*b^7 + 6*(a^7 + 5*a^5*b^2 + 15*a^3*b^4 - 5*a*b^6)*d*x)*cos(d*x + c
) + 48*(a^2*b^5*cos(d*x + c) + a*b^6*sin(d*x + c))*log(2*a*b*cos(d*x + c)*s
in(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (3*a^5*b^2 + 22*a^3*b^4 +
 3*a*b^6 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^4 - 6*(a^6*
b + 5*a^4*b^3 + 15*a^2*b^5 - 5*b^7)*d*x - 6*(a^7 + 5*a^5*b^2 + 7*a^3*b^4 +
3*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^
3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7
+ b^9)*d*sin(d*x + c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.38562, size = 626, normalized size = 2.66

$$\frac{\frac{48\,ab^6\log(|b\tan(dx+c)+a|)}{a^8b+4a^6b^3+6a^4b^5+4a^2b^7+b^9}-\frac{24\,ab^5\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{3\left(a^6+5a^4b^2+15a^2b^4-5b^6\right)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{8\left(6ab^6\tan(dx+c)+7a^2b^5+b^7\right)}{\left(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8\right)(b\tan(dx+c)+a)}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(48*a*b^6*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 +
 4*a^2*b^7 + b^9) - 24*a*b^5*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a
^4*b^4 + 4*a^2*b^6 + b^8) + 3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*(d*x +
 c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(6*a*b^6*tan(d*x +
c) + 7*a^2*b^5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*t
an(d*x + c) + a)) + (36*a*b^5*tan(d*x + c)^4 + 3*a^6*tan(d*x + c)^3 + 15*a^
4*b^2*tan(d*x + c)^3 + 5*a^2*b^4*tan(d*x + c)^3 - 7*b^6*tan(d*x + c)^3 + 16
*a^3*b^3*tan(d*x + c)^2 + 88*a*b^5*tan(d*x + c)^2 + 5*a^6*tan(d*x + c) + 17
*a^4*b^2*tan(d*x + c) + 3*a^2*b^4*tan(d*x + c) - 9*b^6*tan(d*x + c) + 4*a^5
*b + 24*a^3*b^3 + 56*a*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*(tan(d*x + c)^2 + 1)^2))/d
```

3.560 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=235

$$-\frac{5\sec(c+dx)\left(8a\left(a^2+b^2\right)-b\left(4a^2+3b^2\right)\tan(c+dx)\right)}{8b^5d}+\frac{5a\left(a^2+b^2\right)^{3/2}\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{b^6d\sqrt{\sec^2(c+dx)}}+5$$

```
[Out] (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(8*b^6*
d*Sqrt[Sec[c + d*x]^2]) + (5*a*(a^2 + b^2)^(3/2)*ArcTanh[(b - a*Tan[c + d*x
])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(b^6*d*Sqrt[Sec[c
+ d*x]^2]) - (5*Sec[c + d*x]^3*(4*a - 3*b*Tan[c + d*x]))/(12*b^3*d) - Sec[c
 + d*x]^5/(b*d*(a + b*Tan[c + d*x])) - (5*Sec[c + d*x]*(8*a*(a^2 + b^2) - b
*(4*a^2 + 3*b^2)*Tan[c + d*x]))/(8*b^5*d)
```

Rubi [A] time = 0.268137, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3512, 733, 815, 844, 215, 725, 206}

$$-\frac{5\sec(c+dx)\left(8a\left(a^2+b^2\right)-b\left(4a^2+3b^2\right)\tan(c+dx)\right)}{8b^5d}+\frac{5a\left(a^2+b^2\right)^{3/2}\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{b^6d\sqrt{\sec^2(c+dx)}}+5$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^2,x]
```


```
[Out] (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(8*b^6*
d*Sqrt[Sec[c + d*x]^2]) + (5*a*(a^2 + b^2)^(3/2)*ArcTanh[(b - a*Tan[c + d*x
])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(b^6*d*Sqrt[Sec[c
+ d*x]^2]) - (5*Sec[c + d*x]^3*(4*a - 3*b*Tan[c + d*x]))/(12*b^3*d) - Sec[c
 + d*x]^5/(b*d*(a + b*Tan[c + d*x])) - (5*Sec[c + d*x]*(8*a*(a^2 + b^2) - b
*(4*a^2 + 3*b^2)*Tan[c + d*x]))/(8*b^5*d)
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
```

```
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{5/2}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(5\sec(c+dx))\,\text{Subst}\left(\int \frac{x\left(1+\frac{x^2}{b^2}\right)^{3/2}}{a+x}\,dx, x, b\tan(c+dx)\right)}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(5\sec(c+dx))\,\text{Subst}\left(\right.}{}$$

$$= -\frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} - \frac{5\sec(c+dx)\left(8a\left(a^2+b\right.\right.}{}$$

$$= -\frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} - \frac{5\sec(c+dx)\left(8a\left(a^2+b\right.\right.}{}$$

$$= \frac{5\left(8a^4+12a^2b^2+3b^4\right)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{8b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sec^3(c+dx)(4a-3b\tan(c+\right.}{12b^3d}$$

$$= \frac{5\left(8a^4+12a^2b^2+3b^4\right)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{8b^6d\sqrt{\sec^2(c+dx)}} + \frac{5a\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b(1-}{\sqrt{a^2+b^2}}\right.}{b^6d\sqrt{\sec^2(c}}$$

Mathematica [C] time = 6.17926, size = 1152, normalized size = 4.9

$$\frac{10ia(a+ib)(ia+b)\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{\sqrt{a^2+b^2}\left(a\sin\left(\frac{1}{2}(c+dx)\right)-b\cos\left(\frac{1}{2}(c+dx)\right)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)a^2+b^2\cos\left(\frac{1}{2}(c+dx)\right)}\right)(a\cos(c+dx)+b\sin(c+dx))^2\sec^2(c+dx)}{b^6d(a+b\tan(c+dx))^2} -$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a - I*b)^2*(a + I*b)^2*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])
)/(b^5*d*(a + b*Tan[c + d*x])^2)) - (a*(12*a^2 + 13*b^2)*Sec[c + d*x]^2*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^5*d*(a + b*Tan[c + d*x])^2) + ((10*I
)*a*(a + I*b)*(I*a + b)*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a^2 + b^2]*(-(b*Cos[(
c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Cos[(c + d*x)/2] + b^2*Cos[(c + d*
x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(b^6*d*(a + b*T
an[c + d*x])^2) - (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(8*b^6*
d*(a + b*Tan[c + d*x])^2) + (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^
2)/(8*b^6*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*S
in[c + d*x])^2)/(16*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a + b*Ta
```

```
n[c + d*x])^2) + ((36*a^2 - 8*a*b + 21*b^2)*Sec[c + d*x]^2*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^2)/(48*b^4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a +
 b*Tan[c + d*x])^2) - (a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] +
b*Sin[c + d*x])^2)/(3*b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a + b*
Tan[c + d*x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(16
*b^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a + b*Tan[c + d*x])^2) + (a
*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^
3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a + b*Tan[c + d*x])^2) + ((-36
*a^2 - 8*a*b - 21*b^2)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(48*b^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2) +
 (Sec[c + d*x]^2*(-12*a^3*Sin[(c + d*x)/2] - 13*a*b^2*Sin[(c + d*x)/2])*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^5*d*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(12*a^3*Sin[(c + d*x)/2] +
13*a*b^2*Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^5*d*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)
```

Maple [B] time = 0.129, size = 989, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 11/8/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2-15/8/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)+9/8
/d/b^2/(tan(1/2*d*x+1/2*c)-1)-1/4/d/b^2/(tan(1/2*d*x+1/2*c)+1)^4+1/2/d/b^2/
(tan(1/2*d*x+1/2*c)+1)^3-11/8/d/b^2/(tan(1/2*d*x+1/2*c)+1)^2+15/8/d/b^2*ln(
tan(1/2*d*x+1/2*c)+1)+9/8/d/b^2/(tan(1/2*d*x+1/2*c)+1)+2/d/b/(tan(1/2*d*x+1
/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)+1/4/d/b^2/(tan(1/2*d*x+1/2*c)-1)^4+1/2/
d/b^2/(tan(1/2*d*x+1/2*c)-1)^3-2/3/d/b^3/(tan(1/2*d*x+1/2*c)+1)^3*a+2/3/d/b
^3/(tan(1/2*d*x+1/2*c)-1)^3*a+3/2/d/b^4/(tan(1/2*d*x+1/2*c)-1)^2*a^2+1/d/b^
3/(tan(1/2*d*x+1/2*c)-1)^2*a-5/d/b^6*ln(tan(1/2*d*x+1/2*c)-1)*a^4-15/2/d/b^
4*ln(tan(1/2*d*x+1/2*c)-1)*a^2+4/d/b^5/(tan(1/2*d*x+1/2*c)-1)*a^3+3/2/d/b^4
/(tan(1/2*d*x+1/2*c)-1)*a^2-20/d/b^4*a^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*t
an(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-10/d/b^2*a/(a^2+b^2)^(1/2)*arctanh(
1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+2/d/b^4/(tan(1/2*d*x+1/2*
c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)*a^3*tan(1/2*d*x+1/2*c)+4/d/b^2/(tan(1/2*d*
x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)*a*tan(1/2*d*x+1/2*c)-10/d/b^6*a^5/(a
^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+5/d
/b^6*ln(tan(1/2*d*x+1/2*c)+1)*a^4+15/2/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)*a^2-4
/d/b^5/(tan(1/2*d*x+1/2*c)+1)*a^3+3/2/d/b^4/(tan(1/2*d*x+1/2*c)+1)*a^2-5/d/
b^3/(tan(1/2*d*x+1/2*c)+1)*a+5/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a+2/d/b^5/(tan(
1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)*a^4+4/d/b^3/(tan(1/2*d*x+1/2*c
)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)*a^2-3/2/d/b^4/(tan(1/2*d*x+1/2*c)+1)^2*a^2+
1/d/b^3/(tan(1/2*d*x+1/2*c)+1)^2*a+2/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*
x+1/2*c)*b-a)/a*tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.88144, size = 1118, normalized size = 4.76

$$\frac{12\,b^5 - 30\left(8\,a^4b + 12\,a^2b^3 + 3\,b^5\right)\cos\left(dx+c\right)^4 + 10\left(4\,a^2b^3 + 3\,b^5\right)\cos\left(dx+c\right)^2 + 120\left(\left(a^4 + a^2b^2\right)\cos\left(dx+c\right)^5 + \left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(12*b^5 - 30*(8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4 + 10*(4*a^2
*b^3 + 3*b^5)*cos(d*x + c)^2 + 120*((a^4 + a^2*b^2)*cos(d*x + c)^5 + (a^3*b
 + a*b^3)*cos(d*x + c)^4*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 15*((8*a^5 + 12*a^3*b^2 + 3*a*b^4)*cos
(d*x + c)^5 + (8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*sin(d*x + c))*l
og(sin(d*x + c) + 1) - 15*((8*a^5 + 12*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^5 +
(8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*sin(d*x + c))*log(-sin(d*x +
c) + 1) - 10*(2*a*b^4*cos(d*x + c) + 3*(4*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3
)*sin(d*x + c))/(a*b^6*d*cos(d*x + c)^5 + b^7*d*cos(d*x + c)^4*sin(d*x + c)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^7\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**7/(a + b*tan(c + d*x))**2, x)
```

Giac [B] time = 1.6479, size = 716, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^
6 - 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6
+ 120*(a^5 + 2*a^3*b^2 + a*b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*
sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*b^6) + 48*(a^4*b*tan(1/2*d*x + 1/2*c) + 2*a^2*b^3*tan(1/2*d
*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + a^5 + 2*a^3*b^2 + a*b^4)/((a*tan(1
/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)*a*b^5) + 2*(36*a^2*b*tan(
```

```
1/2*d*x + 1/2*c)^7 + 27*b^3*tan(1/2*d*x + 1/2*c)^7 + 96*a^3*tan(1/2*d*x + 1
/2*c)^6 + 144*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^
5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 - 288*a^3*tan(1/2*d*x + 1/2*c)^4 - 336*a*b
^2*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*b^3*tan(1/2
*d*x + 1/2*c)^3 + 288*a^3*tan(1/2*d*x + 1/2*c)^2 + 304*a*b^2*tan(1/2*d*x +
1/2*c)^2 + 36*a^2*b*tan(1/2*d*x + 1/2*c) + 27*b^3*tan(1/2*d*x + 1/2*c) - 96
*a^3 - 112*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*b^5))/d
```

3.561 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=176

$$\frac{3a\sqrt{a^2+b^2}\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{b^4d\sqrt{\sec^2(c+dx)}}+\frac{3\left(2a^2+b^2\right)\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{2b^4d\sqrt{\sec^2(c+dx)}}-\frac{3\sec(c+dx)(2a\cdots}{2b\cdots}$$

```
[Out] (3*(2*a^2 + b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(2*b^4*d*Sqrt[Sec[c +
d*x]^2]) + (3*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^
2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(b^4*d*Sqrt[Sec[c + d*x]^2]) - (3*S
ec[c + d*x]*(2*a - b*Tan[c + d*x]))/(2*b^3*d) - Sec[c + d*x]^3/(b*d*(a + b*
Tan[c + d*x]))
```

Rubi [A] time = 0.167884, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3512, 733, 815, 844, 215, 725, 206}

$$\frac{3a\sqrt{a^2+b^2}\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{b^4d\sqrt{\sec^2(c+dx)}}+\frac{3\left(2a^2+b^2\right)\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{2b^4d\sqrt{\sec^2(c+dx)}}-\frac{3\sec(c+dx)(2a\cdots}{2b\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (3*(2*a^2 + b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(2*b^4*d*Sqrt[Sec[c +
d*x]^2]) + (3*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^
2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(b^4*d*Sqrt[Sec[c + d*x]^2]) - (3*S
ec[c + d*x]*(2*a - b*Tan[c + d*x]))/(2*b^3*d) - Sec[c + d*x]^3/(b*d*(a + b*
Tan[c + d*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/2}}{(a+x)^2}dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(3\sec(c+dx))\,\text{Subst}\left(\int \frac{x\sqrt{1+\frac{x^2}{b^2}}}{a+x}dx, x, b\tan(c+dx)\right)}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d} - \frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(3\sec(c+dx))\,\text{Subst}\left(\int \cdots\right)}{2bd\sqrt{\cdots}}$$

$$= -\frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d} - \frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))} - \frac{\left(3a\left(a^2+b^2\right)\sec(c+dx)\right)\cdots}{b\cdots}$$

$$= \frac{3\left(2a^2+b^2\right)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2b^4d\sqrt{\sec^2(c+dx)}} - \frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d} - \frac{\cdots}{bd(a\cdots}$$

$$= \frac{3\left(2a^2+b^2\right)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2b^4d\sqrt{\sec^2(c+dx)}} + \frac{3a\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\text{se}\cdots}{b^4d\sqrt{\sec^2(c+dx)}}$$

Mathematica [C] time = 6.12165, size = 709, normalized size = 4.03

$$-\frac{3\left(2a^2+b^2\right)\sec^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{2b^4d(a+b\tan(c+dx))^2} + \frac{3\left(2a^2+b^2\right)\sec^2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a - I*b)*(a + I*b)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(b
^3*d*(a + b*Tan[c + d*x])^2)) - (2*a*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(b^3*d*(a + b*Tan[c + d*x])^2) - (6*a*Sqrt[a^2 + b^2]*ArcTanh
[(Sqrt[a^2 + b^2]*(-(b*Cos[(c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Cos[(c
 + d*x)/2] + b^2*Cos[(c + d*x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(b^4*d*(a + b*Tan[c + d*x])^2) - (3*(2*a^2 + b^2)*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x
])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (3*(2*a^2 + b^2)*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a + b*Tan[c
 + d*x])^2) - (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a + b*Tan[c + d*
x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(4*b^2*d*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2) + (2*a*Sec[c +
d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(b^3*d*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)
```

Maple [B] time = 0.111, size = 440, normalized size = 2.5

$$-\frac{1}{2\,b^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-2\,\frac{a}{db^3\left(\tan\left(1/2\,dx+c/2\right)+1\right)}+\frac{1}{2\,b^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+3\,\frac{\ln\left(\tan\left(1/2\,dx+c/2\right)+\right.}{db^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)^2-2/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a+1/2/d/
b^2/(tan(1/2*d*x+1/2*c)+1)+3/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)*a^2+3/2/d/b^2*l
n(tan(1/2*d*x+1/2*c)+1)+2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c
)*b-a)*a*tan(1/2*d*x+1/2*c)+2/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c
)*b-a)/a*tan(1/2*d*x+1/2*c)+2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1
/2*c)*b-a)*a^2+2/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)-6/d/
b^4*(a^2+b^2)^(1/2)*a*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1
/2))+1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2+2/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a+1/
2/d/b^2/(tan(1/2*d*x+1/2*c)-1)-3/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)*a^2-3/2/d/b
^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.62475, size = 860, normalized size = 4.89

$$-\frac{6\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,b^3+6\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)^2-6\left(a^2\cos\left(dx+c\right)^3+ab\cos\left(dx+c\right)^2\sin\left(dx+c\right)\right)\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*cos(d*x + c)*sin(d*x + c) - 2*b^3 + 6*(2*a^2*b + b^3)*cos(d*x
 + c)^2 - 6*(a^2*cos(d*x + c)^3 + a*b*cos(d*x + c)^2*sin(d*x + c))*sqrt(a^2
 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 -
 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*
cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 3*((2*a^3
+ a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos(d*x + c)^2*sin(d*x + c))*log(
sin(d*x + c) + 1) + 3*((2*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos
(d*x + c)^2*sin(d*x + c))*log(-sin(d*x + c) + 1))/(a*b^4*d*cos(d*x + c)^3 +
 b^5*d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.6443, size = 378, normalized size = 2.15

$$\frac{\frac{3\left(2a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}-\frac{3\left(2a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}+\frac{6\left(a^3+ab^2\right)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\cdots\right.}{\left(\cdots\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^2 + b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^3 + a*b^2)*log(abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) -
 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(b*tan(1/2*d*x + 1/2*c
)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 4*a)/((tan(1/2*
d*x + 1/2*c)^2 - 1)^2*b^3) + 4*(a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*
x + 1/2*c) + a^3 + a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/
2*c) - a)*a*b^3))/d
```

3.562 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{\sec(c+dx)}{bd(a+b\tan(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x]
)/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^2]*d) - Sec[c + d*x]/(b*d*(a + b*Tan[
c + d*x]))
```

Rubi [A] time = 0.107103, antiderivative size = 132, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3512, 733, 844, 215, 725, 206}

$$\frac{a\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{b^2d\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}-\frac{\sec(c+dx)}{bd(a+b\tan(c+dx))}+\frac{\sec(c+dx)\sinh^{-1}(\tan(c+dx))}{b^2d\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(b^2*d*Sqrt[Sec[c + d*x]^2]) + (a*ArcT
anh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*
x])/(b^2*Sqrt[a^2 + b^2]*d*Sqrt[Sec[c + d*x]^2]) - Sec[c + d*x]/(b*d*(a + b
*Tan[c + d*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x^2}{b^2}}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec(c+dx)}{bd(a+b\tan(c+dx))} + \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{x}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec(c+dx)}{bd(a+b\tan(c+dx))} + \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{b^3d\sqrt{\sec^2(c+dx)}} - \frac{(a\sec(c+}{}$$

$$= \frac{\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{b^2d\sqrt{\sec^2(c+dx)}} - \frac{\sec(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(a\sec(c+dx))\,\text{Subst}\left(\int \frac{}{1+\frac{a}{b}}\right.}{b^3d\sqrt{\sec^2(c}$$

$$= \frac{\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{b^2d\sqrt{\sec^2(c+dx)}} + \frac{a\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{b^2\sqrt{a^2+b^2}\,d\sqrt{\sec^2(c+dx)}} - \frac{\sec(c}{bd(a+b\tan}$$

Mathematica [A] time = 0.748003, size = 120, normalized size = 1.32

$$-\frac{\frac{2a\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{b\sec(c+dx)}{a+b\tan(c+dx)} + \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+\right.\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
 + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + (b*Sec[c + d*x])/(a + b*Tan[c + d*x]))/(b^2*d))
```

Maple [A] time = 0.102, size = 174, normalized size = 1.9

$$\frac{1}{b^2 d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\frac{\tan(1/2\,dx+c/2)}{d\left((\tan(1/2\,dx+c/2))^2a-2\tan(1/2\,dx+c/2)\,b-a\right)a}+2\frac{1}{bd\left((\tan(1/2\,dx+c/2))^2a-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+
1/2*c)*b-a)/a*tan(1/2*d*x+1/2*c)+2/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*
x+1/2*c)*b-a)-2/d/b^2*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)
-2*b)/(a^2+b^2)^(1/2))-1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.31909, size = 699, normalized size = 7.68

$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos(dx+c)+ab\sin(dx+c)\right)\sqrt{a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(dx}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right.}{2\left(\left(a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^
2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d
*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^3 + a*b^2)*
cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((a^3 +
a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x))**2, x)
```

Giac [A] time = 1.65698, size = 224, normalized size = 2.46

$$\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + log(ab
s(tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 +
 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*
x + 1/2*c) - a)*a*b))/d
```

3.563 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2} dx$

Optimal. Leaf size=82

$$-\frac{b\sec(c+dx)}{d(a^2+b^2)(a+b\tan(c+dx))}-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}$$

```
[Out] -((a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^
2)^(3/2)*d)) - (b*Sec[c + d*x])/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.0733768, antiderivative size = 105, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3512, 731, 725, 206}

$$-\frac{b\sec(c+dx)}{d(a^2+b^2)(a+b\tan(c+dx))}-\frac{a\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{d(a^2+b^2)^{3/2}\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*S
ec[c + d*x])/((a^2 + b^2)^(3/2)*d*Sqrt[Sec[c + d*x]^2])) - (b*Sec[c + d*x])
/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 731

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{1}{(a+x)^2\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{(a\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+d\right.}{b(a^2+b^2)\,d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{(a\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{b(a^2+b^2)\,d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{a\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{(a^2+b^2)^{3/2}\,d\sqrt{\sec^2(c+dx)}} - \frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

Mathematica [A] time = 0.356037, size = 78, normalized size = 0.95

$$\frac{\frac{2a\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} - \frac{b\sec(c+dx)}{(a^2+b^2)(a+b\tan(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)
 - (b*Sec[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])))/d
```

Maple [A] time = 0.048, size = 118, normalized size = 1.4

$$\frac{1}{d}\left(-2\,\frac{1}{(\tan(1/2\,dx+c/2))^2\,a-2\,\tan(1/2\,dx+c/2)\,b-a}\left(-\frac{b^2\tan(1/2\,dx+c/2)}{(a^2+b^2)\,a}-\frac{b}{a^2+b^2}\right)+2\,\frac{a}{(a^2+b^2)^{3/2}}\text{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-2*(-b^2/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)-b/(a^2+b^2))/(tan(1/2*d*x+1/2*
c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.81042, size = 502, normalized size = 6.12

$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right.}{2\left(\left(a^5+2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b+2\,a^2b^3+b^5\right)d\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a
^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^5 + 2*a^3*b
^2 + a*b^4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x))**2, x)
```

Giac [A] time = 1.43339, size = 186, normalized size = 2.27

$$-\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab\right)}{\left(a^3+ab^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b^2*ta
n(1/2*d*x + 1/2*c) + a*b)/((a^3 + a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*ta
n(1/2*d*x + 1/2*c) - a)))/d
```

3.564 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$\frac{\cos(c+dx)(a\tan(c+dx)+b)}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{b\left(a^2-2b^2\right)\sec(c+dx)}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{3ab^2\cos(c+dx)\sqrt{\sec^2(c+dx)}\tanh^{-1}\left(\frac{b-a\tan(}{\sqrt{a^2+b^2}\sqrt{\sec}}\right.}{d\left(a^2+b^2\right)^{5/2}}$$

```
[Out] (-3*a*b^2*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2
])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/((a^2 + b^2)^(5/2)*d) + (b*(a^2 - 2*
b^2)*Sec[c + d*x])/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]*(
b + a*Tan[c + d*x]))/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.126985, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3512, 741, 807, 725, 206}

$$\frac{\cos(c+dx)(a\tan(c+dx)+b)}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{b\left(a^2-2b^2\right)\sec(c+dx)}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{3ab^2\cos(c+dx)\sqrt{\sec^2(c+dx)}\tanh^{-1}\left(\frac{b-a\tan(}{\sqrt{a^2+b^2}\sqrt{\sec}}\right.}{d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a*b^2*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2
])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/((a^2 + b^2)^(5/2)*d) + (b*(a^2 - 2*
b^2)*Sec[c + d*x])/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]*(
b + a*Tan[c + d*x]))/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos(c+dx)(b+a\tan(c+dx))}{\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{-2-\frac{ax}{b^2}}{(a+x)^2\sqrt{1+\frac{x^2}{b^2}}}\,dx, x,\right.}{\left(a^2+b^2\right)d}$$

$$= \frac{b\left(a^2-2b^2\right)\sec(c+dx)}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{\left(a^2+b^2\right)d(a+b\tan(c+dx))} + \frac{\left(3ab\cos(c+dx)\sqrt{\sec^2(\right.}{}$$

$$= \frac{b\left(a^2-2b^2\right)\sec(c+dx)}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\left(3ab\cos(c+dx)\sqrt{\sec^2(\right.}{}$$

$$= -\frac{3ab^2\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{\left(a^2+b^2\right)^{5/2}d} + \frac{b\left(a^2-2b^2\right)\sec(c+dx)}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx}$$

Mathematica [A] time = 0.529381, size = 153, normalized size = 0.97

$$\frac{\sec(c+dx)\left(\left(a^2+b^2\right)\left(a\left(a^2+b^2\right)\sin(2(c+dx))+b\left(a^2+b^2\right)\cos(2(c+dx))+3b\left(a^2-b^2\right)\right)+12ab^2\sqrt{a^2+b^2}(a\cos(c+\right.}{2d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(12*a*b^2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/S
qrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + (a^2 + b^2)*(3*b*(a^2 -
 b^2) + b*(a^2 + b^2)*Cos[2*(c + d*x)] + a*(a^2 + b^2)*Sin[2*(c + d*x)])))/
(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Maple [A] time = 0.098, size = 172, normalized size = 1.1

$$\frac{1}{d}\left(-2\,\frac{\left(-a^2+b^2\right)\tan\left(1/2\,dx+c/2\right)-2\,ab}{\left(a^4+2\,a^2b^2+b^4\right)\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)}-2\,\frac{b^2}{\left(a^2+b^2\right)^2}\left(\frac{1}{\left(\tan\left(1/2\,dx+c/2\right)\right)^2a-2\,\tan\left(1/2\,dx+c/2\right)b-a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2*d*x+1/2*c)-2*a*b)/(1+tan(1/
2*d*x+1/2*c)^2)-2*b^2/(a^2+b^2)^2*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(tan(1/2*d
*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*
a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.94141, size = 697, normalized size = 4.44

$$\frac{2\,a^4b-2\,a^2b^3-4\,b^5+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^2+2\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+3\left(a^2b^2\cos\right.}{2\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)
^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(a^2*b^2*cos
(d*x + c) + a*b^3*sin(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*si
n(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(
b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
 b^2)*cos(d*x + c)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(
d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x))**2, x)
```

Giac [A] time = 1.49833, size = 386, normalized size = 2.46

$$-\frac{\frac{3\,ab^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2\right.}{\left(a^5+2\,a^3b^2+ab^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b\tan\left(\frac{1}{2}\,dx\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(3*a*b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2
*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4
)*sqrt(a^2 + b^2)) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - a^2*b^2*tan(1/2*d*x +
1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 - a^
4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x +
 1/2*c) - 2*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*
c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/d
```

3.565 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=241

$$\frac{\cos^3(c+dx)(a\tan(c+dx)+b)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\cos(c+dx)\left(b\left(a^2-4b^2\right)-a\left(2a^2+7b^2\right)\tan(c+dx)\right)}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{b\left(9a^2b^2+2a^4-8b^4\right)\sec(}{3d\left(a^2+b^2\right)^3(a+b\tan(c}$$

```
[Out] (-5*a*b^4*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2
])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/((a^2 + b^2)^(7/2)*d) + (b*(2*a^4 +
9*a^2*b^2 - 8*b^4)*Sec[c + d*x])/(3*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) +
 (Cos[c + d*x]^3*(b + a*Tan[c + d*x]))/(3*(a^2 + b^2)*d*(a + b*Tan[c + d*x]
)) - (Cos[c + d*x]*(b*(a^2 - 4*b^2) - a*(2*a^2 + 7*b^2)*Tan[c + d*x]))/(3*(
a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.257687, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3512, 741, 823, 807, 725, 206}

$$\frac{\cos^3(c+dx)(a\tan(c+dx)+b)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))}-\frac{\cos(c+dx)\left(b\left(a^2-4b^2\right)-a\left(2a^2+7b^2\right)\tan(c+dx)\right)}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}+\frac{b\left(9a^2b^2+2a^4-8b^4\right)\sec(}{3d\left(a^2+b^2\right)^3(a+b\tan(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-5*a*b^4*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2
])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/((a^2 + b^2)^(7/2)*d) + (b*(2*a^4 +
9*a^2*b^2 - 8*b^4)*Sec[c + d*x])/(3*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) +
 (Cos[c + d*x]^3*(b + a*Tan[c + d*x]))/(3*(a^2 + b^2)*d*(a + b*Tan[c + d*x]
)) - (Cos[c + d*x]*(b*(a^2 - 4*b^2) - a*(2*a^2 + 7*b^2)*Tan[c + d*x]))/(3*(
a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{-2\left(2+\frac{a^2}{b^2}\right)-\frac{3ax}{b^2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx,\right.}{3\left(a^2+b^2\right)d}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b\left(a^2-4b^2\right)-a\left(2a^2+7b^2\right)\tan(c+dx)\right)}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} +$$

$$= \frac{b\left(2a^4+9a^2b^2-8b^4\right)\sec(c+dx)}{3\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b\left(a^2-\right.\right.}{3\left(a^2+b^2\right.}$$

$$= \frac{b\left(2a^4+9a^2b^2-8b^4\right)\sec(c+dx)}{3\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b\left(a^2-\right.\right.}{3\left(a^2+b^2\right.}$$

$$= -\frac{5ab^4\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{\left(a^2+b^2\right)^{7/2}d} + \frac{b\left(2a^4+9a^2b^2-8b^4\right)\sec(c+\right.}{3\left(a^2+b^2\right)^3 d(a+b\tan(c+\right.}$$

Mathematica [A] time = 1.16903, size = 249, normalized size = 1.03

$$\frac{\sec(c+dx)\left(\left(a^2+b^2\right)\left(40a^3b^2\sin(2(c+dx))+2a^3b^2\sin(4(c+dx))+20b^3\left(a^2+b^2\right)\cos(2(c+dx))+b\left(a^2+b^2\right)^2\cos\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(240*a*b^4*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + (a^2 + b^2)*(15*a^4*b
+ 90*a^2*b^3 - 45*b^5 + 20*b^3*(a^2 + b^2)*Cos[2*(c + d*x)] + b*(a^2 + b^2)
^2*Cos[4*(c + d*x)] + 10*a^5*Sin[2*(c + d*x)] + 40*a^3*b^2*Sin[2*(c + d*x)]
 + 30*a*b^4*Sin[2*(c + d*x)] + a^5*Sin[4*(c + d*x)] + 2*a^3*b^2*Sin[4*(c +
d*x)] + a*b^4*Sin[4*(c + d*x)])))/(24*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x]))
```

Maple [A] time = 0.116, size = 320, normalized size = 1.3

$$\frac{1}{d}\left(-2\frac{\left(-a^4-3a^2b^2+2b^4\right)(\tan(1/2\,dx+c/2))^5+\left(-2ba^3-6b^3a\right)(\tan(1/2\,dx+c/2))^4+\left(-2/3\,a^4-6\,a^2b^2+8/3\,b^4\right)}{\left(a^2+b^2\right)\left(a^4+2\,a^2b^2+b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^4-3*a^2*b^2+2*b^4)*tan(1/2*d*x+1
/2*c)^5+(-2*a^3*b-6*a*b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^4-6*a^2*b^2+8/3*b^4
)*tan(1/2*d*x+1/2*c)^3-8*b^3*a*tan(1/2*d*x+1/2*c)^2+(-a^4-3*a^2*b^2+2*b^4)*
tan(1/2*d*x+1/2*c)-2/3*b*a^3-14/3*b^3*a)/(1+tan(1/2*d*x+1/2*c)^2)^3-2*b^4/(
a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(tan(1/2*d*x+1/
2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)-5*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan
(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2147, size = 946, normalized size = 3.93

$$\frac{4\,a^6b+22\,a^4b^3+2\,a^2b^5-16\,b^7+2\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos(dx+c)^4-2\left(a^6b-2\,a^4b^3-7\,a^2b^5-4\,b^7\right)\cos(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(4*a^6*b + 22*a^4*b^3 + 2*a^2*b^5 - 16*b^7 + 2*(a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(a^6*b - 2*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*co
s(d*x + c)^2 + 15*(a^2*b^4*cos(d*x + c) + a*b^5*sin(d*x + c))*sqrt(a^2 + b^
2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a
^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*((a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (2*a^7 + 11*a^5*b^2 + 16*a^3*b^
4 + 7*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^
7 + b^9)*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.6133, size = 591, normalized size = 2.45

$$-\frac{\frac{15\,ab^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{6\left(b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab^5\right)}{\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}-\frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a^2b^2\tan\left(\frac{1}{2}\,d\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(15*a*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/
abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 6*(b^6*tan(1/2*d*x + 1/2*c) + a*b^5)/
((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(
1/2*d*x + 1/2*c) - a)) - 2*(3*a^4*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b^2*tan(1/
2*d*x + 1/2*c)^5 - 6*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b*tan(1/2*d*x + 1/2
*c)^4 + 18*a*b^3*tan(1/2*d*x + 1/2*c)^4 + 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 18
*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*b^4*tan(1/2*d*x + 1/2*c)^3 + 24*a*b^3*t
an(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x + 1/2*c) + 9*a^2*b^2*tan(1/2*d*x
+ 1/2*c) - 6*b^4*tan(1/2*d*x + 1/2*c) + 2*a^3*b + 14*a*b^3)/((a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

3.566 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=185

$$\frac{3(2a^2+b^2)\tan^2(c+dx)}{2b^5d}-\frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d}+\frac{6a(a^2+b^2)^2}{b^7d(a+b\tan(c+dx))}-\frac{(a^2+b^2)^3}{2b^7d(a+b\tan(c+dx))^2}+\frac{3(a^2\cdots}{}$$

```
[Out] (3*(a^2 + b^2)*(5*a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^7*d) - (a*(10*a^2
+ 9*b^2)*Tan[c + d*x])/(b^6*d) + (3*(2*a^2 + b^2)*Tan[c + d*x]^2)/(2*b^5*d)
 - (a*Tan[c + d*x]^3)/(b^4*d) + Tan[c + d*x]^4/(4*b^3*d) - (a^2 + b^2)^3/(2
*b^7*d*(a + b*Tan[c + d*x])^2) + (6*a*(a^2 + b^2)^2)/(b^7*d*(a + b*Tan[c +
d*x]))
```

Rubi [A] time = 0.155936, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$\frac{3(2a^2+b^2)\tan^2(c+dx)}{2b^5d}-\frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d}+\frac{6a(a^2+b^2)^2}{b^7d(a+b\tan(c+dx))}-\frac{(a^2+b^2)^3}{2b^7d(a+b\tan(c+dx))^2}+\frac{3(a^2\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^3,x]
```


```
[Out] (3*(a^2 + b^2)*(5*a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^7*d) - (a*(10*a^2
+ 9*b^2)*Tan[c + d*x])/(b^6*d) + (3*(2*a^2 + b^2)*Tan[c + d*x]^2)/(2*b^5*d)
 - (a*Tan[c + d*x]^3)/(b^4*d) + Tan[c + d*x]^4/(4*b^3*d) - (a^2 + b^2)^3/(2
*b^7*d*(a + b*Tan[c + d*x])^2) + (6*a*(a^2 + b^2)^2)/(b^7*d*(a + b*Tan[c +
d*x]))
```


Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^3}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-10a^3-9ab^2}{b^6} + \frac{3(2a^2+b^2)x}{b^6} - \frac{3ax^2}{b^6} + \frac{x^3}{b^6} + \frac{(a^2+b^2)^3}{b^6(a+x)^3} - \frac{6a(a^2+b^2)^2}{b^6(a+x)^2} + \frac{3(5a^4+6a^2b^2+b^4)}{b^6(a+x)}\right) dx, x, b\right.}{bd}$$

$$= \frac{3(a^2+b^2)(5a^2+b^2)\log(a+b\tan(c+dx))}{b^7d} - \frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d} + \frac{3(2a^2+b^2)\,\mathrm{ta}}{2b^5}$$

Mathematica [A] time = 1.33094, size = 272, normalized size = 1.47

$$\frac{4a^2b^4\tan^4(c+dx) - 20ab^3(a^2+b^2)\tan^3(c+dx) + 4b^2\tan^2(c+dx)\left(3\left(6a^2b^2+5a^4+b^4\right)\log(a+b\tan(c+dx)) - 10a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (2*(a^2 + b^2)*(19*a^4 + 16*a^2*b^2 - 3*b^4 + 6*a^2*(5*a^2 + b^2)*Log[a + b
*Tan[c + d*x]]) + b^6*Sec[c + d*x]^6 + 4*a*b*(4*a^4 + 17*a^2*b^2 + 11*b^4 +
 6*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x] + 4*b^2*
(-13*a^4 - 10*a^2*b^2 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]]
)*Tan[c + d*x]^2 - 20*a*b^3*(a^2 + b^2)*Tan[c + d*x]^3 + 4*a^2*b^4*Tan[c +
d*x]^4 + b^4*Sec[c + d*x]^4*(a^2 + 3*b^2 - 2*a*b*Tan[c + d*x]))/(4*b^7*d*(a
 + b*Tan[c + d*x])^2)
```

Maple [A] time = 0.128, size = 321, normalized size = 1.7

$$\frac{(\tan(dx+c))^4}{4b^3d} - \frac{a(\tan(dx+c))^3}{b^4d} + 3\frac{(\tan(dx+c))^2a^2}{db^5} + \frac{3(\tan(dx+c))^2}{2b^3d} - 10\frac{a^3\tan(dx+c)}{db^6} - 9\frac{a\tan(dx+c)}{b^4d} - \frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/4*tan(d*x+c)^4/b^3/d-a*tan(d*x+c)^3/b^4/d+3/d/b^5*tan(d*x+c)^2*a^2+3/2*ta
n(d*x+c)^2/b^3/d-10/d/b^6*a^3*tan(d*x+c)-9*a*tan(d*x+c)/b^4/d-1/2/d/b^7/(a+
b*tan(d*x+c))^2*a^6-3/2/d/b^5/(a+b*tan(d*x+c))^2*a^4-3/2/d/b^3/(a+b*tan(d*x
+c))^2*a^2-1/2/b/d/(a+b*tan(d*x+c))^2+6/d*a^5/b^7/(a+b*tan(d*x+c))+12/d*a^3
/b^5/(a+b*tan(d*x+c))+6*a/b^3/d/(a+b*tan(d*x+c))+15/d/b^7*ln(a+b*tan(d*x+c)
)*a^4+18/d/b^5*ln(a+b*tan(d*x+c))*a^2+3*ln(a+b*tan(d*x+c))/b^3/d
```

Maxima [A] time = 1.03506, size = 270, normalized size = 1.46

$$\frac{\frac{2\left(11a^6+21a^4b^2+9a^2b^4-b^6+12\left(a^5b+2a^3b^3+ab^5\right)\tan(dx+c)\right)}{b^9\tan(dx+c)^2+2ab^8\tan(dx+c)+a^2b^7} + \frac{b^3\tan(dx+c)^4-4ab^2\tan(dx+c)^3+6\left(2a^2b+b^3\right)\tan(dx+c)^2-4\left(10a^3+9ab^2\right)\tan(dx+c)}{b^6} + \frac{12}{}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(11*a^6 + 21*a^4*b^2 + 9*a^2*b^4 - b^6 + 12*(a^5*b + 2*a^3*b^3 + a*b
^5)*tan(d*x + c))/(b^9*tan(d*x + c)^2 + 2*a*b^8*tan(d*x + c) + a^2*b^7) + (
b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*(2*a^2*b + b^3)*tan(d*x + c
)^2 - 4*(10*a^3 + 9*a*b^2)*tan(d*x + c))/b^6 + 12*(5*a^4 + 6*a^2*b^2 + b^4)
*log(b*tan(d*x + c) + a)/b^7)/d
```

Fricas [B] time = 2.5378, size = 1076, normalized size = 5.82

$$\frac{8\left(15\,a^4b^2+13\,a^2b^4\right)\cos\left(dx+c\right)^6+b^6-2\left(45\,a^4b^2+44\,a^2b^4+3\,b^6\right)\cos\left(dx+c\right)^4+\left(5\,a^2b^4+3\,b^6\right)\cos\left(dx+c\right)^2+6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*(15*a^4*b^2 + 13*a^2*b^4)*cos(d*x + c)^6 + b^6 - 2*(45*a^4*b^2 + 44*
a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + (5*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 + 6*((
5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b + 6*a^3*b^3
+ a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4 + b^6)*cos(d*
x + c)^4)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - 6*((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b
 + 6*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4
+ b^6)*cos(d*x + c)^4)*log(cos(d*x + c)^2) - 2*(a*b^5*cos(d*x + c) + 2*(15*
a^5*b - 2*a^3*b^3 - 13*a*b^5)*cos(d*x + c)^5 + 10*(a^3*b^3 + a*b^5)*cos(d*x
 + c)^3)*sin(d*x + c))/(2*a*b^8*d*cos(d*x + c)^5*sin(d*x + c) + b^9*d*cos(d
*x + c)^4 + (a^2*b^7 - b^9)*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43227, size = 328, normalized size = 1.77

$$\frac{\frac{12\left(5\,a^4+6\,a^2b^2+b^4\right)\log\left(|b\tan(dx+c)+a|\right)}{b^7}-\frac{2\left(45\,a^4b^2\tan(dx+c)^2+54\,a^2b^4\tan(dx+c)^2+9\,b^6\tan(dx+c)^2+78\,a^5b\tan(dx+c)+84\,a^3b^3\tan(dx+c)+6\,ab^5\tan(\right.}{(b\tan(dx+c)+a)^2b^7}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(12*(5*a^4 + 6*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 2*(45*
a^4*b^2*tan(d*x + c)^2 + 54*a^2*b^4*tan(d*x + c)^2 + 9*b^6*tan(d*x + c)^2 +
```

```
 78*a^5*b*tan(d*x + c) + 84*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c) + 3
4*a^6 + 33*a^4*b^2 + b^6)/((b*tan(d*x + c) + a)^2*b^7) + (b^9*tan(d*x + c)^
4 - 4*a*b^8*tan(d*x + c)^3 + 12*a^2*b^7*tan(d*x + c)^2 + 6*b^9*tan(d*x + c)
^2 - 40*a^3*b^6*tan(d*x + c) - 36*a*b^8*tan(d*x + c))/b^12)/d
```

3.567 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=121

$$-\frac{\left(a^2+b^2\right)^2}{2b^5d(a+b\tan(c+dx))^2}+\frac{4a\left(a^2+b^2\right)}{b^5d(a+b\tan(c+dx))}+\frac{2\left(3a^2+b^2\right)\log(a+b\tan(c+dx))}{b^5d}-\frac{3a\tan(c+dx)}{b^4d}+\frac{\tan^2(c}{2b}$$

```
[Out] (2*(3*a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^5*d) - (3*a*Tan[c + d*x])/(b^4
*d) + Tan[c + d*x]^2/(2*b^3*d) - (a^2 + b^2)^2/(2*b^5*d*(a + b*Tan[c + d*x]
)^2) + (4*a*(a^2 + b^2))/(b^5*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.101935, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$-\frac{\left(a^2+b^2\right)^2}{2b^5d(a+b\tan(c+dx))^2}+\frac{4a\left(a^2+b^2\right)}{b^5d(a+b\tan(c+dx))}+\frac{2\left(3a^2+b^2\right)\log(a+b\tan(c+dx))}{b^5d}-\frac{3a\tan(c+dx)}{b^4d}+\frac{\tan^2(c}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (2*(3*a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^5*d) - (3*a*Tan[c + d*x])/(b^4
*d) + Tan[c + d*x]^2/(2*b^3*d) - (a^2 + b^2)^2/(2*b^5*d*(a + b*Tan[c + d*x]
)^2) + (4*a*(a^2 + b^2))/(b^5*d*(a + b*Tan[c + d*x]))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^2}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{3a}{b^4}+\frac{x}{b^4}+\frac{\left(a^2+b^2\right)^2}{b^4(a+x)^3}-\frac{4a\left(a^2+b^2\right)}{b^4(a+x)^2}+\frac{2\left(3a^2+b^2\right)}{b^4(a+x)}\right)dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{2\left(3a^2+b^2\right)\log(a+b\tan(c+dx))}{b^5d}-\frac{3a\tan(c+dx)}{b^4d}+\frac{\tan^2(c+dx)}{2b^3d}-\frac{\left(a^2+b^2\right)}{2b^5d(a+b\tan(c}\end{aligned}$$

Mathematica [A] time = 3.10909, size = 140, normalized size = 1.16

$$\frac{-2a\left(-\frac{a^2+b^2}{a+b\tan(c+dx)}-2a\log(a+b\tan(c+dx))+b\tan(c+dx)\right)+2\left(a^2+b^2\right)\left(\frac{3a^2+4ab\tan(c+dx)-b^2}{2(a+b\tan(c+dx))^2}+\log(a+b\tan(c+dx)\right.}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((b^4*Sec[c + d*x]^4)/(2*(a + b*Tan[c + d*x])^2) - 2*a*(-2*a*Log[a + b*Tan[
c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c + d*x])) + 2*(a^2 + b
^2)*(Log[a + b*Tan[c + d*x]] + (3*a^2 - b^2 + 4*a*b*Tan[c + d*x])/(2*(a + b
*Tan[c + d*x])^2)))/(b^5*d)
```

Maple [A] time = 0.124, size = 184, normalized size = 1.5

$$\frac{(\tan(dx+c))^2}{2b^3d}-3\frac{a\tan(dx+c)}{b^4d}-\frac{a^4}{2db^5(a+b\tan(dx+c))^2}-\frac{a^2}{b^3d(a+b\tan(dx+c))^2}-\frac{1}{2bd(a+b\tan(dx+c))^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2*tan(d*x+c)^2/b^3/d-3*a*tan(d*x+c)/b^4/d-1/2/d/b^5/(a+b*tan(d*x+c))^2*a^
4-1/d/b^3/(a+b*tan(d*x+c))^2*a^2-1/2/b/d/(a+b*tan(d*x+c))^2+6/d/b^5*ln(a+b*
tan(d*x+c))*a^2+2*ln(a+b*tan(d*x+c))/b^3/d+4/d*a^3/b^5/(a+b*tan(d*x+c))+4*a
/b^3/d/(a+b*tan(d*x+c))
```

Maxima [A] time = 1.1771, size = 173, normalized size = 1.43

$$\frac{\frac{7a^4+6a^2b^2-b^4+8\left(a^3b+ab^3\right)\tan(dx+c)}{b^7\tan(dx+c)^2+2ab^6\tan(dx+c)+a^2b^5}+\frac{b\tan(dx+c)^2-6a\tan(dx+c)}{b^4}+\frac{4\left(3a^2+b^2\right)\log(b\tan(dx+c)+a)}{b^5}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((7*a^4 + 6*a^2*b^2 - b^4 + 8*(a^3*b + a*b^3)*tan(d*x + c))/(b^7*tan(d*
x + c)^2 + 2*a*b^6*tan(d*x + c) + a^2*b^5) + (b*tan(d*x + c)^2 - 6*a*tan(d*
x + c))/b^4 + 4*(3*a^2 + b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

Fricas [B] time = 2.13414, size = 814, normalized size = 6.73

$$\frac{24a^2b^2\cos(dx+c)^4+b^4-2\left(9a^2b^2+b^4\right)\cos(dx+c)^2+2\left(\left(3a^4-2a^2b^2-b^4\right)\cos(dx+c)^4+2\left(3a^3b+ab^3\right)\cos(dx-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(24*a^2*b^2*cos(d*x + c)^4 + b^4 - 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 +
 2*((3*a^4 - 2*a^2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((3*a^4 - 2*a^2*b^
2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x + c)^3*sin(d*x + c) +
 (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(cos(d*x + c)^2) - 4*(a*b^3*cos(d*x +
 c) + 3*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^6*d*cos(d*x +
c)^3*sin(d*x + c) + b^7*d*cos(d*x + c)^2 + (a^2*b^5 - b^7)*d*cos(d*x + c)^4
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45648, size = 189, normalized size = 1.56

$$\frac{\frac{4\left(3a^2+b^2\right)\log\left(|b\tan(dx+c)+a|\right)}{b^5}+\frac{b^3\tan(dx+c)^2-6ab^2\tan(dx+c)}{b^6}-\frac{18a^2b^2\tan(dx+c)^2+6b^4\tan(dx+c)^2+28a^3b\tan(dx+c)+4ab^3\tan(dx+c)+11a^4+b^4}{(b\tan(dx+c)+a)^2b^5}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*(3*a^2 + b^2)*log(abs(b*tan(d*x + c) + a))/b^5 + (b^3*tan(d*x + c)^2
 - 6*a*b^2*tan(d*x + c))/b^6 - (18*a^2*b^2*tan(d*x + c)^2 + 6*b^4*tan(d*x +
 c)^2 + 28*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 11*a^4 + b^4)/((b*ta
n(d*x + c) + a)^2*b^5))/d
```

3.568 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3} dx$

Optimal. Leaf size=69

$$-\frac{a^2+b^2}{2b^3d(a+b\tan(c+dx))^2}+\frac{2a}{b^3d(a+b\tan(c+dx))}+\frac{\log(a+b\tan(c+dx))}{b^3d}$$

```
[Out] Log[a + b*Tan[c + d*x]]/(b^3*d) - (a^2 + b^2)/(2*b^3*d*(a + b*Tan[c + d*x])
^2) + (2*a)/(b^3*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.0727754, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$-\frac{a^2+b^2}{2b^3d(a+b\tan(c+dx))^2}+\frac{2a}{b^3d(a+b\tan(c+dx))}+\frac{\log(a+b\tan(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] Log[a + b*Tan[c + d*x]]/(b^3*d) - (a^2 + b^2)/(2*b^3*d*(a + b*Tan[c + d*x])
^2) + (2*a)/(b^3*d*(a + b*Tan[c + d*x]))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1+\frac{x^2}{b^2}}{(a+x)^3} dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2+b^2}{b^2(a+x)^3}-\frac{2a}{b^2(a+x)^2}+\frac{1}{b^2(a+x)}\right) dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\tan(c+dx))}{b^3d}-\frac{a^2+b^2}{2b^3d(a+b\tan(c+dx))^2}+\frac{2a}{b^3d(a+b\tan(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.176622, size = 57, normalized size = 0.83

$$\frac{-\frac{a^2+b^2}{2(a+b\tan(c+dx))^2}+\frac{2a}{a+b\tan(c+dx)}+\log(a+b\tan(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Log[a + b*Tan[c + d*x]] - (a^2 + b^2)/(2*(a + b*Tan[c + d*x])^2) + (2*a)/(
a + b*Tan[c + d*x]))/(b^3*d)
```

Maple [A] time = 0.116, size = 84, normalized size = 1.2

$$-\frac{a^2}{2\,db^3\,(a+b\tan(dx+c))^2}-\frac{1}{2\,bd\,(a+b\tan(dx+c))^2}+2\,\frac{a}{db^3\,(a+b\tan(dx+c))}+\frac{\ln(a+b\tan(dx+c))}{db^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/d/b^3/(a+b*tan(d*x+c))^2*a^2-1/2/b/d/(a+b*tan(d*x+c))^2+2*a/b^3/d/(a+b
*tan(d*x+c))+ln(a+b*tan(d*x+c))/b^3/d
```

Maxima [A] time = 1.14823, size = 105, normalized size = 1.52

$$\frac{\frac{4\,ab\tan(dx+c)+3\,a^2-b^2}{b^5\tan(dx+c)^2+2\,ab^4\tan(dx+c)+a^2b^3}+\frac{2\log(b\tan(dx+c)+a)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((4*a*b*tan(d*x + c) + 3*a^2 - b^2)/(b^5*tan(d*x + c)^2 + 2*a*b^4*tan(d
*x + c) + a^2*b^3) + 2*log(b*tan(d*x + c) + a)/b^3)/d
```

Fricas [B] time = 2.26396, size = 647, normalized size = 9.38

$$\frac{4\,a^2b^2\cos(dx+c)^2-3\,a^2b^2-b^4-2\left(a^3b-ab^3\right)\cos(dx+c)\sin(dx+c)+\left(a^2b^2+b^4+\left(a^4-b^4\right)\cos(dx+c)^2+2\left(a\right.\right.}{2\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*b^2*cos(d*x + c)^2 - 3*a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*x
 + c)*sin(d*x + c) + (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x + c)^2 + 2*(a^3*b
 + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x +
c)^2 + 2*(a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2))/((
a^4*b^3 - b^7)*d*cos(d*x + c)^2 + 2*(a^3*b^4 + a*b^6)*d*cos(d*x + c)*sin(d*
x + c) + (a^2*b^5 + b^7)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x))**3, x)
```

Giac [A] time = 1.40997, size = 84, normalized size = 1.22

$$\frac{\frac{2\log(|b\tan(dx+c)+a|)}{b^3} - \frac{3b\tan(dx+c)^2+2a\tan(dx+c)+b}{(b\tan(dx+c)+a)^2b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(b*tan(d*x + c) + a))/b^3 - (3*b*tan(d*x + c)^2 + 2*a*tan(d*x
 + c) + b)/((b*tan(d*x + c) + a)^2*b^2))/d
```

3.569 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2bd(a+b\tan(c+dx))^2}$$

```
[Out] -1/(2*b*d*(a + b*Tan[c + d*x])^2)
```

Rubi [A] time = 0.0429776, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 32}

$$-\frac{1}{2bd(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/(2*b*d*(a + b*Tan[c + d*x])^2)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= -\frac{1}{2bd(a+b\tan(c+dx))^2}\end{aligned}$$

Mathematica [B] time = 0.174164, size = 58, normalized size = 2.64

$$\frac{2\tan(c+dx)(a+b\tan(c+dx))-b\sec^2(c+dx)}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-(b*Sec[c + d*x]^2) + 2*Tan[c + d*x]*(a + b*Tan[c + d*x]))/(2*(a^2 + b^2)*
d*(a + b*Tan[c + d*x])^2)
```

Maple [A] time = 0.051, size = 21, normalized size = 1.

$$-\frac{1}{2bd\,(a+b\tan(dx+c))^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/b/d/(a+b*tan(d*x+c))^2
```

Maxima [A] time = 1.19315, size = 27, normalized size = 1.23

$$-\frac{1}{2\,(b\tan(dx+c)+a)^{2}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*tan(d*x + c) + a)^2*b*d)
```

Fricas [B] time = 2.16357, size = 313, normalized size = 14.23

$$-\frac{4\,a^{2}b\cos(dx+c)^{2}-a^{2}b+b^{3}-2\left(a^{3}-ab^{2}\right)\cos(dx+c)\sin(dx+c)}{2\left(\left(a^{6}+a^{4}b^{2}-a^{2}b^{4}-b^{6}\right)d\cos(dx+c)^{2}+2\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}\right)d\cos(dx+c)\sin(dx+c)+\left(a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*b*cos(d*x + c)^2 - a^2*b + b^3 - 2*(a^3 - a*b^2)*cos(d*x + c)*s
in(d*x + c))/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^
6)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^{2}(c+dx)}{(a+b\tan(c+dx))^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x))**3, x)
```

Giac [A] time = 1.48829, size = 27, normalized size = 1.23

$$-\frac{1}{2\,(b\tan\left(dx+c\right)+a)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*tan(d*x + c) + a)^2*b*d)
```

3.570 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$\frac{ab\left(a^2-11b^2\right)}{2d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{b\left(a^2-2b^2\right)}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}+\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{2b^3\left(5a^2-b^2\right)}{}$$

```
[Out] (a*(a^4 + 10*a^2*b^2 - 15*b^4)*x)/(2*(a^2 + b^2)^4) + (2*b^3*(5*a^2 - b^2)*
Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) + (b*(a^2 - 2*b^2))
/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) + (Cos[c + d*x]^2*(b + a*Tan[c
+ d*x]))/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (a*b*(a^2 - 11*b^2))/(2
*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.233042, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3506, 741, 801, 635, 203, 260}

$$\frac{ab\left(a^2-11b^2\right)}{2d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{b\left(a^2-2b^2\right)}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}+\frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{2b^3\left(5a^2-b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^4 + 10*a^2*b^2 - 15*b^4)*x)/(2*(a^2 + b^2)^4) + (2*b^3*(5*a^2 - b^2)*
Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) + (b*(a^2 - 2*b^2))
/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) + (Cos[c + d*x]^2*(b + a*Tan[c
+ d*x]))/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (a*b*(a^2 - 11*b^2))/(2
*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{b\,\text{Subst}\left(\int \frac{-4-\frac{a^2}{b^2}-\frac{3ax}{b^2}}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)}\,dx, x, b\tan(c+dx)\right)}{2\left(a^2+b^2\right)d}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{b\,\text{Subst}\left(\int \left(\frac{2\left(a^2-2b^2\right)}{\left(a^2+b^2\right)(a+x)^3} + \frac{a^3-11ab^2}{\left(a^2+b^2\right)^2(a+x)^2} + \frac{4b^2\left(-5a^2+b^2\right)}{\left(a^2+b^2\right)^3(a+x)}\right.\right.}{2\left(a^2\right.}$$

$$= \frac{2b^3\left(5a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{b\left(a^2-2b^2\right)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos^2(c+d}{2\left(a^2+b^2\right)}$$

$$= \frac{2b^3\left(5a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{b\left(a^2-2b^2\right)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos^2(c+d}{2\left(a^2+b^2\right)}$$

$$= \frac{a\left(a^4+10a^2b^2-15b^4\right)x}{2\left(a^2+b^2\right)^4} + \frac{2b^3\left(5a^2-b^2\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{2b^3\left(5a^2-b^2\right)\log(a+b\,\text{ta}}{\left(a^2+b^2\right)^4 d}$$

Mathematica [B] time = 6.27209, size = 458, normalized size = 2.27

$$b^3\left(\frac{\cos^2(c+dx)\left(ab\tan(c+dx)+b^2\right)}{2b^4\left(a^2+b^2\right)(a+b\tan(c+dx))^2} - \frac{\left(2a^2-4b^2\right)\left(-\frac{2a}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))} - \frac{1}{2\left(a^2+b^2\right)(a+b\tan(c+dx))^2} - \frac{\left(-\frac{a^3-3ab^2}{\sqrt{-b^2}}+3a^2-b^2\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3} + \frac{\left(3a^2-b^2\right)\log(}{\left(a^2+\right.}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^3*((Cos[c + d*x]^2*(b^2 + a*b*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)*(a + b*T
an[c + d*x])^2) - ((2*a^2 - 4*b^2)*(-((3*a^2 - b^2 - (a^3 - 3*a*b^2)/Sqrt[-
```

```
b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^3) + ((3*a^2 - b^2)*
Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 - ((3*a^2 - b^2 + (a^3 - 3*a*b^2)/Sq
rt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^3) - 1/(2*(a^2 +
 b^2)*(a + b*Tan[c + d*x])^2) - (2*a)/((a^2 + b^2)^2*(a + b*Tan[c + d*x]))
 - 3*a*(-((2*a - (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/
(2*(a^2 + b^2)^2) + (2*a*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - ((2*a + (
a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^2)
- 1/((a^2 + b^2)*(a + b*Tan[c + d*x]))))/(2*b^2*(a^2 + b^2))))/d
```

Maple [B] time = 0.122, size = 453, normalized size = 2.2

$$\frac{a^5\tan(dx+c)}{2d\left(a^2+b^2\right)^4\left(1+(\tan(dx+c))^2\right)}-\frac{a^3\tan(dx+c)\,b^2}{d\left(a^2+b^2\right)^4\left(1+(\tan(dx+c))^2\right)}-\frac{3\,a\tan(dx+c)\,b^4}{2d\left(a^2+b^2\right)^4\left(1+(\tan(dx+c))^2\right)}+\frac{}{2d\left(a^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)*tan(d*x+c)*a^5-1/d/(a^2+b^2)^4/(1+tan(d*
x+c)^2)*tan(d*x+c)*a^3*b^2-3/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)*tan(d*x+c)*a*
b^4+3/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)*a^4*b+1/d/(a^2+b^2)^4/(1+tan(d*x+c)^
2)*a^2*b^3-1/2/d/(a^2+b^2)^4/(1+tan(d*x+c)^2)*b^5-5/d/(a^2+b^2)^4*ln(1+tan(
d*x+c)^2)*a^2*b^3+1/d/(a^2+b^2)^4*ln(1+tan(d*x+c)^2)*b^5+5/d/(a^2+b^2)^4*ar
ctan(tan(d*x+c))*a^3*b^2-15/2/d/(a^2+b^2)^4*arctan(tan(d*x+c))*a*b^4+1/2/d/
(a^2+b^2)^4*arctan(tan(d*x+c))*a^5-1/2/d*b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))^2
-4/d*b^3/(a^2+b^2)^3*a/(a+b*tan(d*x+c))+10/d*b^3/(a^2+b^2)^4*ln(a+b*tan(d*x
+c))*a^2-2/d*b^5/(a^2+b^2)^4*ln(a+b*tan(d*x+c))
```

Maxima [B] time = 1.82646, size = 618, normalized size = 3.06

$$\frac{\frac{\left(a^5+10\,a^3b^2-15\,ab^4\right)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{4\left(5\,a^2b^3-b^5\right)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{2\left(5\,a^2b^3-b^5\right)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\,a^4b-10\,a^2}{a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6+\left(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^5 + 10*a^3*b^2 - 15*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8) + 4*(5*a^2*b^3 - b^5)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 2*(5*a^2*b^3 - b^5)*log(tan(d*x + c)
^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (3*a^4*b - 10*a^2
*b^3 - b^5 + (a^3*b^2 - 11*a*b^4)*tan(d*x + c)^3 + 2*(a^4*b - 6*a^2*b^3 - b
^5)*tan(d*x + c)^2 + (a^5 + 3*a^3*b^2 - 10*a*b^4)*tan(d*x + c))/(a^8 + 3*a^
6*b^2 + 3*a^4*b^4 + a^2*b^6 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*tan(d
*x + c)^4 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)^3 + (a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^7*b + 3*a
^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)))/d
```

Fricas [B] time = 2.8846, size = 1102, normalized size = 5.46

$$-\frac{3\,a^4b^3-16\,a^2b^5+b^7-2\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos(dx+c)^4-2\left(a^5b^2+10\,a^3b^4-15\,ab^6\right)dx-\left(a^6b-a^4b^3-45\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*a^4*b^3 - 16*a^2*b^5 + b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7
)*cos(d*x + c)^4 - 2*(a^5*b^2 + 10*a^3*b^4 - 15*a*b^6)*d*x - (a^6*b - a^4*b
^3 - 45*a^2*b^5 - 3*b^7 + 2*(a^7 + 9*a^5*b^2 - 25*a^3*b^4 + 15*a*b^6)*d*x)*
cos(d*x + c)^2 - 4*(5*a^2*b^5 - b^7 + (5*a^4*b^3 - 6*a^2*b^5 + b^7)*cos(d*x
 + c)^2 + 2*(5*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((a^7 + 3*a^5*b
^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 2*(a^5*b^2 - 3*a^3*b^4 + 6*a*b^6 -
 (a^6*b + 10*a^4*b^3 - 15*a^2*b^5)*d*x)*cos(d*x + c))*sin(d*x + c))/((a^10
+ 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 +
2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*
x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.26748, size = 593, normalized size = 2.94

$$\frac{(a^5+10a^3b^2-15ab^4)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} - \frac{2(5a^2b^3-b^5)\log(\tan(dx+c)^2+1)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8} + \frac{4(5a^2b^4-b^6)\log(|b\tan(dx+c)+a|)}{a^8b+4a^6b^3+6a^4b^5+4a^2b^7+b^9} + \frac{10a^2b^3\tan(dx+c)^2-2b^5\tan(dx+c)^2+a^5\tan(}{(a^8+4a^6b^2+6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^5 + 10*a^3*b^2 - 15*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8) - 2*(5*a^2*b^3 - b^5)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 4*(5*a^2*b^4 - b^6)*log(abs(b*tan(d*
x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (10*a^2*b^
3*tan(d*x + c)^2 - 2*b^5*tan(d*x + c)^2 + a^5*tan(d*x + c) - 2*a^3*b^2*tan(
d*x + c) - 3*a*b^4*tan(d*x + c) + 3*a^4*b + 12*a^2*b^3 - 3*b^5)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(d*x + c)^2 + 1)) - (30*a^2*b^5*t
an(d*x + c)^2 - 6*b^7*tan(d*x + c)^2 + 68*a^3*b^4*tan(d*x + c) - 4*a*b^6*ta
n(d*x + c) + 39*a^4*b^3 + 4*a^2*b^5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^2))/d
```

3.571 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=295

$$\frac{3ab\left(6a^2b^2+a^4-27b^4\right)}{8d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}+\frac{3b\left(5a^2b^2+a^4-4b^4\right)}{8d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{\cos^2(c+dx)\left(2b\left(a^2-3b^2\right)-a\left(3a^2+11b^2\right)\tan(\ldots\right)}{8d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}$$

```
[Out] (3*a*(a^6 + 7*a^4*b^2 + 35*a^2*b^4 - 35*b^6)*x)/(8*(a^2 + b^2)^5) + (3*b^5*
(7*a^2 - b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^5*d) + (3*
b*(a^4 + 5*a^2*b^2 - 4*b^4))/(8*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) + (
Cos[c + d*x]^4*(b + a*Tan[c + d*x]))/(4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^
2) + (3*a*b*(a^4 + 6*a^2*b^2 - 27*b^4))/(8*(a^2 + b^2)^4*d*(a + b*Tan[c + d
*x])) - (Cos[c + d*x]^2*(2*b*(a^2 - 3*b^2) - a*(3*a^2 + 11*b^2)*Tan[c + d*x
]))/(8*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2)
```

Rubi [A] time = 0.349339, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3506, 741, 823, 801, 635, 203, 260}

$$\frac{3ab\left(6a^2b^2+a^4-27b^4\right)}{8d\left(a^2+b^2\right)^4(a+b\tan(c+dx))}+\frac{3b\left(5a^2b^2+a^4-4b^4\right)}{8d\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{\cos^2(c+dx)\left(2b\left(a^2-3b^2\right)-a\left(3a^2+11b^2\right)\tan(\ldots\right)}{8d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```


```
[Out] (3*a*(a^6 + 7*a^4*b^2 + 35*a^2*b^4 - 35*b^6)*x)/(8*(a^2 + b^2)^5) + (3*b^5*
(7*a^2 - b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^5*d) + (3*
b*(a^4 + 5*a^2*b^2 - 4*b^4))/(8*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) + (
Cos[c + d*x]^4*(b + a*Tan[c + d*x]))/(4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^
2) + (3*a*b*(a^4 + 6*a^2*b^2 - 27*b^4))/(8*(a^2 + b^2)^4*d*(a + b*Tan[c + d
*x])) - (Cos[c + d*x]^2*(2*b*(a^2 - 3*b^2) - a*(3*a^2 + 11*b^2)*Tan[c + d*x
]))/(8*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2)
```


Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
```

```
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^3}dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{b\,\text{Subst}\left(\int \frac{-3\left(2+\frac{a^2}{b^2}\right)-\frac{5ax}{b^2}}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^2}dx, x, b\tan(c+dx)\right)}{4(a^2+b^2)\,d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{\cos^2(c+dx)\left(2b\left(a^2-3b^2\right)-a\left(3a^2+11b^2\right)\tan(c+dx)\right.}{8\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{\cos^2(c+dx)\left(2b\left(a^2-3b^2\right)-a\left(3a^2+11b^2\right)\tan(c+dx)\right.}{8\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2}$$

$$= \frac{3b^5\left(7a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{3b\left(a^4+5a^2b^2-4b^4\right)}{8\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} + \frac{\cos^4(c+dx)(b}{4\left(a^2+b^2\right)d(a}$$

$$= \frac{3b^5\left(7a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{3b\left(a^4+5a^2b^2-4b^4\right)}{8\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} + \frac{\cos^4(c+dx)(b}{4\left(a^2+b^2\right)d(a}$$

$$= \frac{3a\left(a^6+7a^4b^2+35a^2b^4-35b^6\right)x}{8\left(a^2+b^2\right)^5} + \frac{3b^5\left(7a^2-b^2\right)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5 d} + \frac{3b^5\left(7a^2-b^2\right)\log(a}{\left(a^2+b\right.}$$

Mathematica [B] time = 6.24788, size = 596, normalized size = 2.02

$$b^5\left(\frac{\cos^4(c+dx)\left(ab\tan(c+dx)+b^2\right)}{4b^6\left(a^2+b^2\right)(a+b\tan(c+dx))^2} - \frac{\frac{\cos^2(c+dx)\left(b\left(-3a\left(a^2+2b^2\right)-5ab^2\right)\tan(c+dx)+5a^2b^2-3b^2\left(a^2+2b^2\right)\right)}{2b^4\left(a^2+b^2\right)(a+b\tan(c+dx))^2} - \frac{\left(3\left(a^2b^2+a^4+8b^4\right)-3a^2\left(3a^2+11b^2\right)\right)\left(-\frac{2a}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{}{2(a^2}\right.}{}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^5*((Cos[c + d*x]^4*(b^2 + a*b*Tan[c + d*x]))/(4*b^6*(a^2 + b^2)*(a + b*T
an[c + d*x])^2) - ((Cos[c + d*x]^2*(5*a^2*b^2 - 3*b^2*(a^2 + 2*b^2) + b*(-5
*a*b^2 - 3*a*(a^2 + 2*b^2))*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^2) - ((-3*a^2*(3*a^2 + 11*b^2) + 3*(a^4 + a^2*b^2 + 8*b^4))*(-((3*a
^2 - b^2 - (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2
*(a^2 + b^2)^3) + ((3*a^2 - b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 - (
(3*a^2 - b^2 + (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]
)/(2*(a^2 + b^2)^3) - 1/(2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) - (2*a)/((a^
2 + b^2)^2*(a + b*Tan[c + d*x]))) + 3*a*(3*a^2 + 11*b^2)*(-((2*a - (a^2 - b
^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(2*(a^2 + b^2)^2) + (2*a*
Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - ((2*a + (a^2 - b^2)/Sqrt[-b^2])*Lo
g[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^2) - 1/((a^2 + b^2)*(a + b*T
```

```
an[c + d*x]))))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))/d
```

Maple [B] time = 0.13, size = 824, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -3/d*b^7/(a^2+b^2)^5*ln(a+b*tan(d*x+c))+3/2/d/(a^2+b^2)^5*ln(1+tan(d*x+c)^2
)*b^7+3/8/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^7+4/d/(a^2+b^2)^5/(1+tan(d*x+c
)^2)^2*tan(d*x+c)^2*a^2*b^5+25/4/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*a^4*b^3+1
7/4/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*a^2*b^5-21/2/d/(a^2+b^2)^5*ln(1+tan(d*
x+c)^2)*a^2*b^5+3/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)^3*a^7-1/d/(
a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*b^7+5/8/d/(a^2+b^2)^5/(1+tan(d*x
+c)^2)^2*tan(d*x+c)*a^7-105/8/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a*b^6+21/8/d
/(a^2+b^2)^5*arctan(tan(d*x+c))*a^5*b^2-6/d*b^5/(a^2+b^2)^4*a/(a+b*tan(d*x+
c))+21/d*b^5/(a^2+b^2)^5*ln(a+b*tan(d*x+c))*a^2+3/4/d/(a^2+b^2)^5/(1+tan(d*
x+c)^2)^2*a^6*b+105/8/d/(a^2+b^2)^5*arctan(tan(d*x+c))*a^3*b^4-1/2/d*b^5/(a
^2+b^2)^3/(a+b*tan(d*x+c))^2+21/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+
c)^3*a^5*b^2+5/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)^2*a^4*b^3+19/8/d
/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a^5*b^2-15/8/d/(a^2+b^2)^5/(1+ta
n(d*x+c)^2)^2*tan(d*x+c)^3*a^3*b^4-33/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*ta
n(d*x+c)^3*a*b^6-5/4/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2*b^7-39/8/d/(a^2+b^2)^
5/(1+tan(d*x+c)^2)^2*tan(d*x+c)*a*b^6-25/8/d/(a^2+b^2)^5/(1+tan(d*x+c)^2)^2
*tan(d*x+c)*a^3*b^4
```

Maxima [B] time = 1.81573, size = 996, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^7 + 7*a^5*b^2 + 35*a^3*b^4 - 35*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(7*a^2*b^5 - b^7)*log(
b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
 + b^10) - 12*(7*a^2*b^5 - b^7)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 +
 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + (6*a^6*b + 44*a^4*b^3 - 62*a
^2*b^5 - 4*b^7 + 3*(a^5*b^2 + 6*a^3*b^4 - 27*a*b^6)*tan(d*x + c)^5 + 6*(a^6
*b + 6*a^4*b^3 - 13*a^2*b^5 - 2*b^7)*tan(d*x + c)^4 + (3*a^7 + 23*a^5*b^2 +
 61*a^3*b^4 - 151*a*b^6)*tan(d*x + c)^3 + 2*(5*a^6*b + 37*a^4*b^3 - 73*a^2*
b^5 - 9*b^7)*tan(d*x + c)^2 + (5*a^7 + 26*a^5*b^2 + 49*a^3*b^4 - 68*a*b^6)*
tan(d*x + c))/(a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8 + (a^8*b^
2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*tan(d*x + c)^6 + 2*(a^9*b + 4
*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^5 + (a^10 + 6*a^8*b^
2 + 14*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + 2*b^10)*tan(d*x + c)^4 + 4*(a^9*b
 + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^3 + (2*a^10 + 9*
a^8*b^2 + 16*a^6*b^4 + 14*a^4*b^6 + 6*a^2*b^8 + b^10)*tan(d*x + c)^2 + 2*(a
^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)))/d
```

Fricas [B] time = 2.87083, size = 1497, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/32*(9*a^6*b^3 + 95*a^4*b^5 - 141*a^2*b^7 - 3*b^9 - 8*(a^8*b + 4*a^6*b^3
+ 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^6 + 8*(a^8*b - 6*a^4*b^5 - 8*a^
2*b^7 - 3*b^9)*cos(d*x + c)^4 - 12*(a^7*b^2 + 7*a^5*b^4 + 35*a^3*b^6 - 35*a
*b^8)*d*x - (15*a^8*b + 82*a^6*b^3 + 68*a^4*b^5 - 498*a^2*b^7 - 51*b^9 + 12
*(a^9 + 6*a^7*b^2 + 28*a^5*b^4 - 70*a^3*b^6 + 35*a*b^8)*d*x)*cos(d*x + c)^2
 - 48*(7*a^2*b^7 - b^9 + (7*a^4*b^5 - 8*a^2*b^7 + b^9)*cos(d*x + c)^2 + 2*(
7*a^3*b^6 - a*b^8)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*
x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*(4*(a^9 + 4*a^7*b^2 + 6*a^5*
b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x + c)^5 + 2*(3*a^9 + 20*a^7*b^2 + 42*a^5*b^
4 + 36*a^3*b^6 + 11*a*b^8)*cos(d*x + c)^3 - (3*a^7*b^2 + 53*a^5*b^4 - 15*a^
3*b^6 + 159*a*b^8 - 12*(a^8*b + 7*a^6*b^3 + 35*a^4*b^5 - 35*a^2*b^7)*d*x)*c
os(d*x + c))*sin(d*x + c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*
a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10
*a^5*b^7 + 5*a^3*b^9 + a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 5*
a^8*b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.38893, size = 792, normalized size = 2.68

$$\frac{\frac{3\left(a^7+7a^5b^2+35a^3b^4-35ab^6\right)(dx+c)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}-\frac{12\left(7a^2b^5-b^7\right)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5a^8b^2+10a^6b^4+10a^4b^6+5a^2b^8+b^{10}}+\frac{24\left(7a^2b^6-b^8\right)\log(|b\tan(dx+c)+a|)}{a^{10}b+5a^8b^3+10a^6b^5+10a^4b^7+5a^2b^9+b^{11}}+\frac{3a^5b^2\tan(dx+c)^5+18}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^7 + 7*a^5*b^2 + 35*a^3*b^4 - 35*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(7*a^2*b^5 - b^7)*log(
tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
 + b^10) + 24*(7*a^2*b^6 - b^8)*log(abs(b*tan(d*x + c) + a))/(a^10*b + 5*a^
8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11) + (3*a^5*b^2*tan(d*x +
c)^5 + 18*a^3*b^4*tan(d*x + c)^5 - 81*a*b^6*tan(d*x + c)^5 + 6*a^6*b*tan(d*
x + c)^4 + 36*a^4*b^3*tan(d*x + c)^4 - 78*a^2*b^5*tan(d*x + c)^4 - 12*b^7*t
an(d*x + c)^4 + 3*a^7*tan(d*x + c)^3 + 23*a^5*b^2*tan(d*x + c)^3 + 61*a^3*b
^4*tan(d*x + c)^3 - 151*a*b^6*tan(d*x + c)^3 + 10*a^6*b*tan(d*x + c)^2 + 74
*a^4*b^3*tan(d*x + c)^2 - 146*a^2*b^5*tan(d*x + c)^2 - 18*b^7*tan(d*x + c)^
```

```
2 + 5*a^7*tan(d*x + c) + 26*a^5*b^2*tan(d*x + c) + 49*a^3*b^4*tan(d*x + c)
- 68*a*b^6*tan(d*x + c) + 6*a^6*b + 44*a^4*b^3 - 62*a^2*b^5 - 4*b^7)/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c)^3 + a*tan(d*x +
c)^2 + b*tan(d*x + c) + a)^2))/d
```

3.572 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=239

$$\frac{5\sec(c+dx)\left(4a^2-2ab\tan(c+dx)+b^2\right)}{2b^5d}-\frac{5\sqrt{a^2+b^2}\left(4a^2+b^2\right)\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2b^6d\sqrt{\sec^2(c+dx)}}-\frac{5a\left(4a^2+3b^2\right)}{}$$

```
[Out] (-5*a*(4*a^2 + 3*b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(2*b^6*d*Sqrt[Sec
[c + d*x]^2]) - (5*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(b - a*Tan[c + d*x
])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(2*b^6*d*Sqrt[Sec[
c + d*x]^2]) - Sec[c + d*x]^5/(2*b*d*(a + b*Tan[c + d*x])^2) + (5*Sec[c + d
*x]^3*(4*a + b*Tan[c + d*x]))/(6*b^3*d*(a + b*Tan[c + d*x])) + (5*Sec[c + d
*x]*(4*a^2 + b^2 - 2*a*b*Tan[c + d*x]))/(2*b^5*d)
```

Rubi [A] time = 0.243291, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3512, 733, 813, 815, 844, 215, 725, 206}

$$\frac{5\sec(c+dx)\left(4a^2-2ab\tan(c+dx)+b^2\right)}{2b^5d}-\frac{5\sqrt{a^2+b^2}\left(4a^2+b^2\right)\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2b^6d\sqrt{\sec^2(c+dx)}}-\frac{5a\left(4a^2+3b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-5*a*(4*a^2 + 3*b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(2*b^6*d*Sqrt[Sec
[c + d*x]^2]) - (5*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(b - a*Tan[c + d*x
])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(2*b^6*d*Sqrt[Sec[
c + d*x]^2]) - Sec[c + d*x]^5/(2*b*d*(a + b*Tan[c + d*x])^2) + (5*Sec[c + d
*x]^3*(4*a + b*Tan[c + d*x]))/(6*b^3*d*(a + b*Tan[c + d*x])) + (5*Sec[c + d
*x]*(4*a^2 + b^2 - 2*a*b*Tan[c + d*x]))/(2*b^5*d)
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
```

```
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{5/2}}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{(5\sec(c+dx))\,\text{Subst}\left(\int \frac{x\left(1+\frac{x^2}{b^2}\right)^{3/2}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{2b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))} - \frac{(5\sec(c+dx))\,\text{Subst}\left(\int \right.}{4b^3d}$$

$$= -\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))} + \frac{5\sec(c+dx)\left(4a^2+b^2-\right.}{2b^5d}$$

$$= -\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))} + \frac{5\sec(c+dx)\left(4a^2+b^2-\right.}{2b^5d}$$

$$= -\frac{5a\left(4a^2+3b^2\right)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} - \frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{5\sec^3(c+dx}{6b^3d(a}$$

$$= -\frac{5a\left(4a^2+3b^2\right)\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sqrt{a^2+b^2}\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan}{b}\right.}{\sqrt{a^2+b^2}\sqrt{\sec}}\right.}{2b^6d\sqrt{\sec^2(c+dx)}}$$

Mathematica [C] time = 2.21019, size = 688, normalized size = 2.88

$$\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{6b^2\left(a^2+b^2\right)^2\sin(c+dx)}{a} + 2b\left(36a^2+13b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2 + \frac{2b(36a}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((6*b^2*(a^2 + b^2)^2*Sin
[c + d*x])/a + (6*(a - I*b)*(a + I*b)*b*(8*a^2 - b^2)*(a*Cos[c + d*x] + b*S
in[c + d*x]))/a + 2*b*(36*a^2 + 13*b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
 + 60*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[
a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 30*a*(4*a^2 + 3*b^2)*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 -
30*a*(4*a^2 + 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2 + (b^2*(-9*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^
2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^3*Sin[(c + d*x)/2]*(a*Cos
[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (2
*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^3*Sin[(c + d*x)/2]*(a*Cos[c +
d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (b^2*(9
```

```
*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2 - (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*b^6*d*(a + b*Tan[
c + d*x])^3)
```

Maple [B] time = 0.148, size = 1125, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x)
```

```
[Out] -2/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2/a^2*tan(1/2*d*x+
1/2*c)^2+25/d/b^4/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*a^3*t
an(1/2*d*x+1/2*c)-7/d/b^4/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)
^2*a^3*tan(1/2*d*x+1/2*c)^3-5/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1
/2*c)*b-a)^2*a*tan(1/2*d*x+1/2*c)^3-1/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2
*d*x+1/2*c)*b-a)^2-1/3/d/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/b^3/(tan(1/2*d*
x+1/2*c)-1)^2-5/2/d/b^3/(tan(1/2*d*x+1/2*c)-1)+1/3/d/b^3/(tan(1/2*d*x+1/2*c
)+1)^3-1/2/d/b^3/(tan(1/2*d*x+1/2*c)+1)^2+5/2/d/b^3/(tan(1/2*d*x+1/2*c)+1)+
25/d/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)
^(1/2))*a^2+23/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*a*
tan(1/2*d*x+1/2*c)-8/d/b^5/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a
)^2*a^4*tan(1/2*d*x+1/2*c)^2+9/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+
1/2*c)*b-a)^2*a^2*tan(1/2*d*x+1/2*c)^2+8/d/b^5/(tan(1/2*d*x+1/2*c)^2*a-2*ta
n(1/2*d*x+1/2*c)*b-a)^2*a^4+7/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1
/2*c)*b-a)^2*a^2+5/d/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c
)-2*b)/(a^2+b^2)^(1/2))-3/2/d/b^4/(tan(1/2*d*x+1/2*c)-1)^2*a-6/d/b^5/(tan(1
/2*d*x+1/2*c)-1)*a^2-3/2/d/b^4/(tan(1/2*d*x+1/2*c)-1)*a+10/d*a^3/b^6*ln(tan
(1/2*d*x+1/2*c)-1)+15/2/d*a/b^4*ln(tan(1/2*d*x+1/2*c)-1)+2/d/(tan(1/2*d*x+1
/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2/a*tan(1/2*d*x+1/2*c)^3-2/d/(tan(1/2*d
*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2/a*tan(1/2*d*x+1/2*c)-10/d*a^3/b^6
*ln(tan(1/2*d*x+1/2*c)+1)-15/2/d*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)+3/2/d/b^4/(
tan(1/2*d*x+1/2*c)+1)^2*a+6/d/b^5/(tan(1/2*d*x+1/2*c)+1)*a^2-3/2/d/b^4/(tan
(1/2*d*x+1/2*c)+1)*a+15/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-
a)^2*tan(1/2*d*x+1/2*c)^2+20/d/b^6/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2
*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.12065, size = 1299, normalized size = 5.44

$$4\,b^{5} + 30\left(4\,a^{4}b + a^{2}b^{3} - b^{5}\right)\cos\left(dx + c\right)^{4} + 20\left(2\,a^{2}b^{3} + b^{5}\right)\cos\left(dx + c\right)^{2} + 15\left(\left(4\,a^{4} - 3\,a^{2}b^{2} - b^{4}\right)\cos\left(dx + c\right)^{5} + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^5 + 30*(4*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^4 + 20*(2*a^2*b^3 +
 b^5)*cos(d*x + c)^2 + 15*((4*a^4 - 3*a^2*b^2 - b^4)*cos(d*x + c)^5 + 2*(4*
a^3*b + a*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^2*b^2 + b^4)*cos(d*x + c)
^3)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x +
 c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))
 - 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4*b + 3*a^2*b^3)
*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^3)*log(si
n(d*x + c) + 1) + 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4
*b + 3*a^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x
 + c)^3)*log(-sin(d*x + c) + 1) - 10*(a*b^4*cos(d*x + c) - 6*(3*a^3*b^2 + 2
*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^7*d*cos(d*x + c)^4*sin(d*x + c
) + b^8*d*cos(d*x + c)^3 + (a^2*b^6 - b^8)*d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.63557, size = 689, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(15*(4*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*a
^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^4 + 5*a^2*b^
2 + b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*
a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) +
2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 + 18*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x +
 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 + 14*b^2)/((tan(1/2*d*x + 1
/2*c)^2 - 1)^3*b^5) + 6*(7*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*b^3*tan(1/2
*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d*x + 1/2*
c)^2 - 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2
 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 - 25*a^5*b*tan(1/2*d*x + 1/2*c) - 23*a^3*b^
3*tan(1/2*d*x + 1/2*c) + 2*a*b^5*tan(1/2*d*x + 1/2*c) - 8*a^6 - 7*a^4*b^2 +
 a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a^2*
b^5))/d
```

3.573 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=148

$$-\frac{3\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^4d\sqrt{a^2+b^2}}-\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))}-\frac{\sec}{2bd(a+}$$

```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(b^4*d) - (3*(2*a^2 + b^2)*ArcTanh[(b*Cos[c +
d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(2*b^4*Sqrt[a^2 + b^2]*d) - Sec[c
+ d*x]^3/(2*b*d*(a + b*Tan[c + d*x])^2) + (3*Sec[c + d*x]*(2*a + b*Tan[c +
d*x]))/(2*b^3*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.160246, antiderivative size = 189, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3512, 733, 813, 844, 215, 725, 206}

$$-\frac{3\left(2a^2+b^2\right)\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2b^4d\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}+\frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))}-\frac{3a\sec(c+dx)\sinh^{-1}(\tan}{b^4d\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(b^4*d*Sqrt[Sec[c + d*x]^2]) - (3
*(2*a^2 + b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d
*x]^2])]*Sec[c + d*x])/(2*b^4*Sqrt[a^2 + b^2]*d*Sqrt[Sec[c + d*x]^2]) - Sec
[c + d*x]^3/(2*b*d*(a + b*Tan[c + d*x])^2) + (3*Sec[c + d*x]*(2*a + b*Tan[c
 + d*x]))/(2*b^3*d*(a + b*Tan[c + d*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
```

```
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/2}}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{(3\sec(c+dx))\,\text{Subst}\left(\int \frac{x\sqrt{1+\frac{x^2}{b^2}}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{2b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))} - \frac{(3\sec(c+dx))\,\text{Subst}\left(\int - \ldots\right.}{4b^3d\sqrt{\text{s}\ldots}}$$

$$= -\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))} - \frac{(3a\sec(c+dx))\,\text{Subst}\left(\int \ldots\right.}{b^5d\sqrt{\sec\ldots}}$$

$$= -\frac{3a\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{b^4d\sqrt{\sec^2(c+dx)}} - \frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+dx)(2a+b\tan(\ldots}{2b^3d(a+b\tan(c+\ldots}$$

$$= -\frac{3a\sinh^{-1}(\tan(c+dx))\sec(c+dx)}{b^4d\sqrt{\sec^2(c+dx)}} - \frac{3\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2b^4\sqrt{a^2+b^2}\,d\sqrt{\sec^2(c+dx)}} - \ldots$$

Mathematica [B] time = 2.34927, size = 396, normalized size = 2.68

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{b^2(a^2+b^2)\sin(c+dx)}{a}+\frac{6(2a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{2b\,s}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((b^2*(a^2 + b^2)*Sin[c +
 d*x])/a + ((2*a - b)*b*(2*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x]))/a + 2*
b*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + (6*(2*a^2 + b^2)*ArcTanh[(-b + a*Ta
n[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/Sqrt[
a^2 + b^2] + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2 - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^2 + (2*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b*Sin[(c + d*x)/2]*
(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
)/(2*b^4*d*(a + b*Tan[c + d*x])^3)
```

Maple [B] time = 0.127, size = 611, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d/b^3/(tan(1/2*d*x+1/2*c)+1)-3/d*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)-3/d/b^2/(
tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*a*tan(1/2*d*x+1/2*c)^3+2
/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2/a*tan(1/2*d*x+1/2*c)
^3-4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*a^2*tan(1/2*
d*x+1/2*c)^2+9/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*tan(
1/2*d*x+1/2*c)^2-2/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2/
a^2*tan(1/2*d*x+1/2*c)^2+13/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2
*c)*b-a)^2*a*tan(1/2*d*x+1/2*c)-2/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1
/2*c)*b-a)^2/a*tan(1/2*d*x+1/2*c)+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2
*d*x+1/2*c)*b-a)^2*a^2-1/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b
-a)^2+6/d/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2
+b^2)^(1/2))*a^2+3/d/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c
)-2*b)/(a^2+b^2)^(1/2))-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)+3/d*a/b^4*ln(tan(1/2
*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.53068, size = 1175, normalized size = 7.94

$$\frac{4\,a^2b^3+4\,b^5+6\left(2\,a^4b+a^2b^3-b^5\right)\cos\left(dx+c\right)^2+18\left(a^3b^2+ab^4\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+3\left(\left(2\,a^4-a^2b^2-b^4\right)\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*b^3 + 4*b^5 + 6*(2*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^2 + 18*(a
^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*((2*a^4 - a^2*b^2 - b^4)*cos(
d*x + c)^3 + 2*(2*a^3*b + a*b^3)*cos(d*x + c)^2*sin(d*x + c) + (2*a^2*b^2 +
 b^4)*cos(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x +
c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 + b^2)) - 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*co
s(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(sin(d*x + c
) + 1) + 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*cos(d*x + c)
^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^4*b^4 - b^8)*d*cos(d*x + c)^3 + 2*(a^3*b^5 + a*b^7)*d*cos(d*x + c)^2*sin
(d*x + c) + (a^2*b^6 + b^8)*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x))**3, x)
```

Giac [B] time = 1.71835, size = 424, normalized size = 2.86

$$-\frac{\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{3\left(2\,a^2+b^2\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{4}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b^3}+\frac{2\left(3\,a^3b\tan\left(\right.\right.}{}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*a*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/b^4 + 3*(2*a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b
 - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2
)))/(sqrt(a^2 + b^2)*b^4) + 4/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3) + 2*(3*a^3
```

```
*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan(1/2*
d*x + 1/2*c)^2 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2
*c)^2 - 13*a^3*b*tan(1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c) - 4*a^
4 + a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a
^2*b^3))/d
```

3.574 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=95

$$-\frac{\sec(c+dx)(b-a\tan(c+dx))}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(2*(a^2 + b^2)^
(3/2)*d) - (Sec[c + d*x]*(b - a*Tan[c + d*x]))/(2*(a^2 + b^2)*d*(a + b*Tan[
c + d*x])^2)
```

Rubi [A] time = 0.0916806, antiderivative size = 118, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3512, 721, 725, 206}

$$-\frac{\sec(c+dx)(b-a\tan(c+dx))}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2d\left(a^2+b^2\right)^{3/2}\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -(ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[
c + d*x])/(2*(a^2 + b^2)^(3/2)*d*Sqrt[Sec[c + d*x]^2]) - (Sec[c + d*x]*(b -
 a*Tan[c + d*x]))/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 721

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(-2*a*e + (2*c*d)*x)*(a + c*x^2)^p)/(2*(m + 1)*(c*d^2 +
 a*e^2)), x] - Dist[(4*a*c*p)/(2*(m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m
 + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{x^2}{b^2}}}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx\right.}{2b\left(a^2+b^2\right)d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{2b\left(a^2+b^2\right)d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2\left(a^2+b^2\right)^{3/2}d\sqrt{\sec^2(c+dx)}} - \frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}$$

Mathematica [C] time = 0.293105, size = 132, normalized size = 1.39

$$\frac{\left(a^2+b^2\right)(a\sin(c+dx)-b\cos(c+dx))+2\sqrt{a^2+b^2}(a\cos(c+dx)+b\sin(c+dx))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{2d(a-ib)^2(a+ib)^2(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcTa
nh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d
*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Maple [B] time = 0.118, size = 191, normalized size = 2.

$$\frac{1}{d}\left(-2\frac{1}{\left((\tan(1/2\,dx+c/2))^2a-2\tan(1/2\,dx+c/2)\,b-a\right)^2}\left(-1/2\frac{\left(a^2+2\,b^2\right)(\tan(1/2\,dx+c/2))^3}{\left(a^2+b^2\right)a}-1/2\frac{b\left(a^2-2\,b^2\right)(}{\left(a^2\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^2
)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*d*
x+1/2*c)+1/2*b/(a^2+b^2))/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)
^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01829, size = 679, normalized size = 7.15

$$\frac{\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\cdots}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos\cdots}\cdots\right.}{4\left(\left(a^6+a^4b^2-a^2b^4-b^6\right)d\cos\left(dx+c\right)^2+2\left(a^5b+2\,a^3b^3+ab^5\right)d\cos\left(dx+c\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*s
qrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))
/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*
(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4*b^2
- a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*cos(d*x
 + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x))**3, x)
```

Giac [B] time = 1.91071, size = 298, normalized size = 3.14

$$-\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^2\tan\cdots\right.}{\left(a^4+a^2b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2}\cdots}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^3*
tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*d*x
 + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) - 2*a
*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)
^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

3.575 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$-\frac{3ab\sec(c+dx)}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{b\sec(c+dx)}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\left(2a^2-b^2\right)\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2d\left(a^2+b^2\right)^{5/2}\sqrt{\sec^2(c+dx)}}$$

```
[Out] -((2*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c +
d*x]^2])]*Sec[c + d*x])/(2*(a^2 + b^2)^(5/2)*d*Sqrt[Sec[c + d*x]^2]) - (b*S
ec[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (3*a*b*Sec[c + d*x]
)/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.114242, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3512, 745, 807, 725, 206}

$$-\frac{3ab\sec(c+dx)}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{b\sec(c+dx)}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\left(2a^2-b^2\right)\sec(c+dx)\tanh^{-1}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2d\left(a^2+b^2\right)^{5/2}\sqrt{\sec^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```



```
[Out] -((2*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c +
d*x]^2])]*Sec[c + d*x])/(2*(a^2 + b^2)^(5/2)*d*Sqrt[Sec[c + d*x]^2]) - (b*S
ec[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (3*a*b*Sec[c + d*x]
)/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
```

```
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{1}{(a+x)^3\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{-2a+x}{(a+x)^2\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+d\right.}{2b\left(a^2+b^2\right)d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{3ab\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \frac{\left(\left(2a^2-b^2\right)\sec\right.}{}$$

$$= -\frac{b\sec(c+dx)}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{3ab\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} - \frac{\left(\left(2a^2-b^2\right)\sec\right.}{}$$

$$= -\frac{\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2\left(a^2+b^2\right)^{5/2}d\sqrt{\sec^2(c+dx)}} - \frac{b\sec(c+dx)}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} -$$

Mathematica [A] time = 0.894272, size = 110, normalized size = 0.71

$$\frac{\frac{2\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{b\sec(c+dx)\left(4a^2+3ab\tan(c+dx)+b^2\right)}{\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((2*(2*a^2 - b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2
+ b^2)^(5/2) - (b*Sec[c + d*x]*(4*a^2 + b^2 + 3*a*b*Tan[c + d*x]))/((a^2 +
b^2)^2*(a + b*Tan[c + d*x])^2))/(2*d)
```

Maple [A] time = 0.069, size = 280, normalized size = 1.8

$$\frac{1}{d}\left(-2\frac{1}{\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-2\,\tan\left(1/2\,dx+c/2\right)b-a\right)^{2}}\left(-1/2\,\frac{b^{2}\left(5\,a^{2}+2\,b^{2}\right)\left(\tan\left(1/2\,dx+c/2\right)\right)^{3}}{a\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}-1/2\,\frac{b\left(4\,a^{4}-7\,a^{2}\right.}{\left(\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3-
1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tan(1/2*d*x+1/2*c)^2+
1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)+1/2*b*(4*a^
2+b^2)/(a^4+2*a^2*b^2+b^4))/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-
a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.03376, size = 795, normalized size = 5.13

$$-\frac{\left(2\,a^{2}b^{2}-b^{4}+\left(2\,a^{4}-3\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{2}+2\left(2\,a^{3}b-ab^{3}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{2\,ab\cos(dx+c)\sin(\ldots}{\ldots}\right.}{4\left(\left(a^{8}+2\,a^{6}b^{2}-2\,a^{2}b^{6}-b^{8}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{7}b+3\,a^{5}b^{3}+3\,a\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*a^2*b^2 - b^4 + (2*a^4 - 3*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a^
3*b - a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x
+ c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
 b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cos(d*x
 + c) + 6*(a^3*b^2 + a*b^4)*sin(d*x + c))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b
^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos(d*x
+ c)*sin(d*x + c) + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x))**3, x)
```

Giac [B] time = 2.2486, size = 396, normalized size = 2.55

$$-\frac{\frac{\left(2\,a^2-b^2\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(5\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-7\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^5\tan\cdots}{\left(a^6+2\,a^4b^2+a^2b^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b
^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2
*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^
4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b^3*tan(1
/2*d*x + 1/2*c)^2 - 2*b^5*tan(1/2*d*x + 1/2*c)^2 - 11*a^3*b^2*tan(1/2*d*x +
 1/2*c) - 2*a*b^4*tan(1/2*d*x + 1/2*c) - 4*a^4*b - a^2*b^3)/((a^6 + 2*a^4*b
^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))
/d
```

3.576 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$\frac{\cos(c+dx)(a\tan(c+dx)+b)}{d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{ab\left(2a^2-13b^2\right)\sec(c+dx)}{2d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{b\left(2a^2-3b^2\right)\sec(c+dx)}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{3b^2\left(4a^2-b^2\right)}{}$$

```
[Out] (-3*b^2*(4*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Se
c[c + d*x]^2])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/(2*(a^2 + b^2)^(7/2)*d)
+ (b*(2*a^2 - 3*b^2)*Sec[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^
2) + (Cos[c + d*x]*(b + a*Tan[c + d*x]))/((a^2 + b^2)*d*(a + b*Tan[c + d*x]
)^2) + (a*b*(2*a^2 - 13*b^2)*Sec[c + d*x])/(2*(a^2 + b^2)^3*d*(a + b*Tan[c
+ d*x]))
```

Rubi [A] time = 0.213629, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3512, 741, 835, 807, 725, 206}

$$\frac{\cos(c+dx)(a\tan(c+dx)+b)}{d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{ab\left(2a^2-13b^2\right)\sec(c+dx)}{2d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}+\frac{b\left(2a^2-3b^2\right)\sec(c+dx)}{2d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{3b^2\left(4a^2-b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*b^2*(4*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Se
c[c + d*x]^2])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/(2*(a^2 + b^2)^(7/2)*d)
+ (b*(2*a^2 - 3*b^2)*Sec[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^
2) + (Cos[c + d*x]*(b + a*Tan[c + d*x]))/((a^2 + b^2)*d*(a + b*Tan[c + d*x]
)^2) + (a*b*(2*a^2 - 13*b^2)*Sec[c + d*x])/(2*(a^2 + b^2)^3*d*(a + b*Tan[c
+ d*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
```

```
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos(c+dx)(b+a\tan(c+dx))}{\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\operatorname{Subst}\left(\int \frac{-3-\frac{2ax}{b^2}}{(a+x)^3\sqrt{1+\frac{x^2}{b^2}}}\,dx\right.}{\left(a^2+b^2\right)d}$$

$$= \frac{b\left(2a^2-3b^2\right)\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\left(b^3\cos(c+dx)\sqrt{\right.}{}$$

$$= \frac{b\left(2a^2-3b^2\right)\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{ab\left(2a^2-13b^2\right.}{2\left(a^2+b^2\right)^3 d(a+}$$

$$= \frac{b\left(2a^2-3b^2\right)\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{ab\left(2a^2-13b^2\right.}{2\left(a^2+b^2\right)^3 d(a+}$$

$$= -\frac{3b^2\left(4a^2-b^2\right)\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{2\left(a^2+b^2\right)^{7/2} d} + \frac{b\left(2a^2-3b^2\right.}{2\left(a^2+b^2\right)^2 d(a-}$$

Mathematica [A] time = 1.84247, size = 183, normalized size = 0.83

$$\frac{\frac{\sec^2(c+dx)\left(b\left(a^2+b^2\right)^2\cos(3(c+dx))+b\left(-22a^2b^2+11a^4-3b^4\right)\cos(c+dx)+2a\sin(c+dx)\left(\left(a^2+b^2\right)^2\cos(2(c+dx))+4a^2b^2+a^4-12b^4\right)\right)}{\left(a^2+b^2\right)^3(a+b\tan(c+dx))^2}-\frac{12b^2\left(b^2-4a^2\right)\tanh^{-1}\left(\frac{a\tan}{\cdots}\right.}{\left(a^2+b^2\right)^{7/2}}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((-12*b^2*(-4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]
)/(a^2 + b^2)^(7/2) + (Sec[c + d*x]^2*(b*(11*a^4 - 22*a^2*b^2 - 3*b^4)*Cos[
c + d*x] + b*(a^2 + b^2)^2*Cos[3*(c + d*x)] + 2*a*(a^4 + 4*a^2*b^2 - 12*b^4
 + (a^2 + b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*x]))/((a^2 + b^2)^3*(a + b*Tan
[c + d*x])^2))/(4*d)
```

Maple [A] time = 0.129, size = 283, normalized size = 1.3

$$\frac{1}{d}\left(-2\frac{\left(-a^3+3ab^2\right)\tan(1/2\,dx+c/2)-3\,ba^2+b^3}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\frac{b^2}{\left(a^2+b^2\right)^3}\left(\frac{1}{\left((\tan(1/2\,dx+c/2))^2a-2\tan(1/2\,dx+c/2\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^3+3*a*b^2)*tan(1/2*d*x+1/2*c)-3*
b*a^2+b^3)/(1+tan(1/2*d*x+1/2*c)^2)-2*b^2/(a^2+b^2)^3*((-1/2*b^2*(9*a^2+2*b
^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(8*a^4-15*a^2*b^2-2*b^4)/a^2*tan(1/2*d*x+1
/2*c)^2+1/2*b^2*(23*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)+4*b*a^2+1/2*b^3)/(tan(1
/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2-3/2*(4*a^2-b^2)/(a^2+b^2)^(1/
2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30406, size = 1073, normalized size = 4.86

$$\frac{4\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos(dx+c)^3-3\left(4\,a^2b^4-b^6+\left(4\,a^4b^2-5\,a^2b^4+b^6\right)\cos(dx+c)^2+2\left(4\,a^3b^3-ab^5\right)\cos(\cdots\right.}{4\left(\left(a^{10}+3\,a^8b^2+2\,a^6b^4-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*(4*a^2*b^4
- b^6 + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(4*a^3*b^3 - a*b^5
)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*
x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*co
s(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2)) + 2*(4*a^6*b - 10*a^4*b^3 - 17*a^2*b^5 - 3*b^7)*co
s(d*x + c) + 2*(2*a^5*b^2 - 11*a^3*b^4 - 13*a*b^6 + 2*(a^7 + 3*a^5*b^2 + 3*
a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^8*b^2 + 2*a^6*b
^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3
+ 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4
*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x))**3, x)
```

Giac [A] time = 2.27872, size = 539, normalized size = 2.44

$$-\frac{\frac{3\left(4a^2b^2-b^4\right)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{4\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^2b-b^3\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}-\frac{2\left(9a^3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^6\tan\left(\frac{1}{2}dx\right.\right.}{}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(4*a^2*b^2 - b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a
^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 4*(a^3*tan(1/2*d*x + 1/2*c
) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b - b^3)/((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*(9*a^3*b^4*tan(1/2*d*x + 1/2*
c)^3 + 2*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 -
15*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 - 23*a^3*b
^4*tan(1/2*d*x + 1/2*c) - 2*a*b^6*tan(1/2*d*x + 1/2*c) - 8*a^4*b^3 - a^2*b^
5)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b
*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

3.577 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=310

$$\frac{\cos^3(c+dx)(a\tan(c+dx)+b)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\cos(c+dx)\left(b\left(2a^2-5b^2\right)-a\left(2a^2+9b^2\right)\tan(c+dx)\right)}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}+\frac{ab\left(28a^2b^2+4a^4-81b^4\right)\mathrm{se}}{6d\left(a^2+b^2\right)^4(a+b\tan(}$$

```
[Out] (-5*b^4*(6*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Se
c[c + d*x]^2])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/(2*(a^2 + b^2)^(9/2)*d)
+ (b*(4*a^4 + 24*a^2*b^2 - 15*b^4)*Sec[c + d*x])/(6*(a^2 + b^2)^3*d*(a + b*
Tan[c + d*x])^2) + (Cos[c + d*x]^3*(b + a*Tan[c + d*x]))/(3*(a^2 + b^2)*d*(
a + b*Tan[c + d*x])^2) + (a*b*(4*a^4 + 28*a^2*b^2 - 81*b^4)*Sec[c + d*x])/(
6*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]*(b*(2*a^2 - 5*b^2)
- a*(2*a^2 + 9*b^2)*Tan[c + d*x]))/(3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^
2)
```

Rubi [A] time = 0.384532, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3512, 741, 823, 835, 807, 725, 206}

$$\frac{\cos^3(c+dx)(a\tan(c+dx)+b)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}-\frac{\cos(c+dx)\left(b\left(2a^2-5b^2\right)-a\left(2a^2+9b^2\right)\tan(c+dx)\right)}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}+\frac{ab\left(28a^2b^2+4a^4-81b^4\right)\mathrm{se}}{6d\left(a^2+b^2\right)^4(a+b\tan(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-5*b^4*(6*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Se
c[c + d*x]^2])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/(2*(a^2 + b^2)^(9/2)*d)
+ (b*(4*a^4 + 24*a^2*b^2 - 15*b^4)*Sec[c + d*x])/(6*(a^2 + b^2)^3*d*(a + b*
Tan[c + d*x])^2) + (Cos[c + d*x]^3*(b + a*Tan[c + d*x]))/(3*(a^2 + b^2)*d*(
a + b*Tan[c + d*x])^2) + (a*b*(4*a^4 + 28*a^2*b^2 - 81*b^4)*Sec[c + d*x])/(
6*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]*(b*(2*a^2 - 5*b^2)
- a*(2*a^2 + 9*b^2)*Tan[c + d*x]))/(3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^
2)
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{-5-\frac{2a^2}{b^2}-\frac{4ax}{b^2}}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx\right.}{3\left(a^2+b^2\right)d}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\cos(c+dx)\left(b\left(2a^2-5b^2\right)-a\left(2a^2+9b^2\right)\tan(c+dx)\right)}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2}$$

$$= \frac{b\left(4a^4+24a^2b^2-15b^4\right)\sec(c+dx)}{6\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\cos(c+dx)\left(b\left(2a\right.\right.}{3\left(a^2\right.}$$

$$= \frac{b\left(4a^4+24a^2b^2-15b^4\right)\sec(c+dx)}{6\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{ab\left(4a^4+28a^2b^2\right.}{6\left(a^2+b^2\right)^4 d(}$$

$$= \frac{b\left(4a^4+24a^2b^2-15b^4\right)\sec(c+dx)}{6\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{ab\left(4a^4+28a^2b^2\right.}{6\left(a^2+b^2\right)^4 d(}$$

$$= -\frac{5b^4\left(6a^2-b^2\right)\tanh^{-1}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{2\left(a^2+b^2\right)^{9/2}d} + \frac{b\left(4a^4+24a^2b^2-1\right.}{6\left(a^2+b^2\right)^3 d(a+b}$$

Mathematica [A] time = 1.83266, size = 371, normalized size = 1.2

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{6b^6\tan(c+dx)}{a\left(a^2+b^2\right)^3} - \frac{6b^5\left(12a^2+b^2\right)(a+b\tan(c+dx))}{a\left(a^2+b^2\right)^4} + \frac{9b\left(14a^2b^2+a^4-3b^4\right)(a\cos(c+dx)+b\sin(c+dx))^2}{\left(a^2+b^2\right)^4}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*((9*b*(a^4 + 14*a^2*b^2 -
 3*b^4)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^2 + b^2)^4 + (6*b^6*Tan[c +
 d*x])/(a*(a^2 + b^2)^3) + (9*a*(a^4 + 6*a^2*b^2 - 11*b^4)*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^2*Tan[c + d*x])/(a^2 + b^2)^4 - (6*b^5*(12*a^2 + b^2)*(a
+ b*Tan[c + d*x]))/(a*(a^2 + b^2)^4) - (60*b^4*(-6*a^2 + b^2)*ArcTanh[(-b +
 a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*Cos[c + d*x]*(a + b*Tan[c + d*x])^2)/
(a^2 + b^2)^(9/2) - (b*(-3*a^2 + b^2)*Cos[c + d*x]*Cos[3*(c + d*x)]*(a + b*
Tan[c + d*x])^2)/(a^2 + b^2)^3 + (a*(a^2 - 3*b^2)*Cos[c + d*x]*Sin[3*(c + d
*x)]*(a + b*Tan[c + d*x])^2)/(a^2 + b^2)^3))/(12*d*(a + b*Tan[c + d*x])^3)
```

Maple [A] time = 0.155, size = 457, normalized size = 1.5

$$\frac{1}{d}\left(-2\,\frac{1}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(a^2+b^2\right)\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)^3}\left(\left(-a^5-4\,a^3b^2+9\,ab^4\right)\left(\tan\left(1/2\,dx+c/2\right)\right)^5+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)*((-a^5-4*a^3*b^2+9*a*b^4)*t
an(1/2*d*x+1/2*c)^5+(-3*a^4*b-12*a^2*b^3+3*b^5)*tan(1/2*d*x+1/2*c)^4+(-2/3*
a^5-32/3*a^3*b^2+14*a*b^4)*tan(1/2*d*x+1/2*c)^3+(-20*a^2*b^3+4*b^5)*tan(1/2
*d*x+1/2*c)^2+(-a^5-4*a^3*b^2+9*a*b^4)*tan(1/2*d*x+1/2*c)-a^4*b-32/3*a^2*b^
3+7/3*b^5)/(1+tan(1/2*d*x+1/2*c)^2)^3-2*b^4/(a^2+b^2)/(a^6+3*a^4*b^2+3*a^2*
b^4+b^6)*((-1/2*b^2*(13*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(12*a^4-23*
a^2*b^2-2*b^4)/a^2*tan(1/2*d*x+1/2*c)^2+1/2*b^2*(35*a^2+2*b^2)/a*tan(1/2*d*
x+1/2*c)+6*b*a^2+1/2*b^3)/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)
^2-5/2*(6*a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)
/(a^2+b^2)^(1/2))))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.79722, size = 1391, normalized size = 4.49

$$\frac{4\left(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9\right)\cos\left(dx+c\right)^5-4\left(2\,a^8b+a^6b^3-9\,a^4b^5-13\,a^2b^7-5\,b^9\right)\cos\left(dx+c\right)^3-15\left(6\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^5 -
4*(2*a^8*b + a^6*b^3 - 9*a^4*b^5 - 13*a^2*b^7 - 5*b^9)*cos(d*x + c)^3 - 15*
(6*a^2*b^6 - b^8 + (6*a^4*b^4 - 7*a^2*b^6 + b^8)*cos(d*x + c)^2 + 2*(6*a^3*
b^5 - a*b^7)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x
+ c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
 b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(8*a^8*b + 64*a^6*b^3 - 16*a^4*b^5
- 87*a^2*b^7 - 15*b^9)*cos(d*x + c) + 2*(4*a^7*b^2 + 32*a^5*b^4 - 53*a^3*b^
6 - 81*a*b^8 + 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x
+ c)^4 + 2*(2*a^9 + 15*a^7*b^2 + 33*a^5*b^4 + 29*a^3*b^6 + 9*a*b^8)*cos(d*x
 + c)^2)*sin(d*x + c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*a^2*
```

```
b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5
*b^7 + 5*a^3*b^9 + a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 5*a^8*
b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.19951, size = 864, normalized size = 2.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(15*(6*a^2*b^4 - b^6)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(
a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*sqrt(a^2 + b^2)) - 6*(13*a^3*b^6
*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*b^5*tan(1
/2*d*x + 1/2*c)^2 - 23*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 - 2*b^9*tan(1/2*d*x +
 1/2*c)^2 - 35*a^3*b^6*tan(1/2*d*x + 1/2*c) - 2*a*b^8*tan(1/2*d*x + 1/2*c)
- 12*a^4*b^5 - a^2*b^7)/((a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^
8)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2) - 4*(3*a^5*
tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*a*b^4*tan(1
/2*d*x + 1/2*c)^5 + 9*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^3*tan(1/2*d*x
 + 1/2*c)^4 - 9*b^5*tan(1/2*d*x + 1/2*c)^4 + 2*a^5*tan(1/2*d*x + 1/2*c)^3 +
 32*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 42*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 60*a
^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 12*b^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1
/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*a*b^4*tan(1/2*d*x +
1/2*c) + 3*a^4*b + 32*a^2*b^3 - 7*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

3.578 $\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=121

$$\frac{6ad^3\sin(e+fx)\sqrt{d\sec(e+fx)}}{5f}-\frac{6ad^4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2ad\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}+\frac{2b(d\sec(e+}{7f}$$

```
[Out] (-6*a*d^4*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e +
 f*x]]) + (2*b*(d*Sec[e + f*x])^(7/2))/(7*f) + (6*a*d^3*Sqrt[d*Sec[e + f*x]
]*Sin[e + f*x])/(5*f) + (2*a*d*(d*Sec[e + f*x])^(5/2)*Sin[e + f*x])/(5*f)
```

Rubi [A] time = 0.0928726, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3486, 3768, 3771, 2639}

$$\frac{6ad^3\sin(e+fx)\sqrt{d\sec(e+fx)}}{5f}-\frac{6ad^4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2ad\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}+\frac{2b(d\sec(e+}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(7/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (-6*a*d^4*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e +
 f*x]]) + (2*b*(d*Sec[e + f*x])^(7/2))/(7*f) + (6*a*d^3*Sqrt[d*Sec[e + f*x]
]*Sin[e + f*x])/(5*f) + (2*a*d*(d*Sec[e + f*x])^(5/2)*Sin[e + f*x])/(5*f)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \frac{2b(d\sec(e+fx))^{7/2}}{7f} + a\int (d\sec(e+fx))^{7/2}\,dx$$
$$= \frac{2b(d\sec(e+fx))^{7/2}}{7f} + \frac{2ad(d\sec(e+fx))^{5/2}\sin(e+fx)}{5f} + \frac{1}{5}\left(3ad^2\right)\int (d\sec$$
$$= \frac{2b(d\sec(e+fx))^{7/2}}{7f} + \frac{6ad^3\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f} + \frac{2ad(d\sec(e+fx))}{5f}$$
$$= \frac{2b(d\sec(e+fx))^{7/2}}{7f} + \frac{6ad^3\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f} + \frac{2ad(d\sec(e+fx))}{5f}$$
$$= -\frac{6ad^4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2b(d\sec(e+fx))^{7/2}}{7f} + \frac{6ad^3\sqrt{d\sec(e+f}}{5f}$$

Mathematica [A] time = 0.618749, size = 69, normalized size = 0.57

$$\frac{(d\sec(e+fx))^{7/2}\left(70a\sin(2(e+fx)) + 21a\sin(4(e+fx)) - 168a\cos^{\frac{7}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right) + 40b\right)}{140f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^(7/2)*(40*b - 168*a*Cos[e + f*x]^(7/2)*EllipticE[(e + f*x
)/2, 2] + 70*a*Sin[2*(e + f*x)] + 21*a*Sin[4*(e + f*x)]))/(140*f)
```

Maple [C] time = 0.327, size = 371, normalized size = 3.1

$$\frac{2\left(\cos\left(fx+e\right)+1\right)^2\left(\cos\left(fx+e\right)-1\right)^2}{35f\left(\sin\left(fx+e\right)\right)^5}\left(21i\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^4\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\text{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] 2/35/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(21*I*sin(f*x+e)*cos(f*x+e)^4*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x
+e)-1)/sin(f*x+e),I)*a-21*I*sin(f*x+e)*cos(f*x+e)^4*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I
)*a+21*I*sin(f*x+e)*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a-21*I*sin(f*x+e)
*cos(f*x+e)^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a-21*cos(f*x+e)^4*a+14*cos(f*x+e)^3*
a+5*b*sin(f*x+e)+7*a*cos(f*x+e))*(d/cos(f*x+e))^(7/2)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}\left(b\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)*(b*tan(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bd^3\sec\left(fx+e\right)^3\tan\left(fx+e\right)+ad^3\sec\left(fx+e\right)^3\right)\sqrt{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*d^3*sec(f*x + e)^3*tan(f*x + e) + a*d^3*sec(f*x + e)^3)*sqrt(d*
sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(7/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}\left(b\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)*(b*tan(f*x + e) + a), x)
```

3.579 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=92

$$\frac{2ad^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{2ad\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

```
[Out] (2*a*d^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])
/(3*f) + (2*b*(d*Sec[e + f*x])^(5/2))/(5*f) + (2*a*d*(d*Sec[e + f*x])^(3/2)
*Sin[e + f*x])/(3*f)
```

Rubi [A] time = 0.0691213, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3486, 3768, 3771, 2641}

$$\frac{2ad^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{2ad\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f}+\frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*a*d^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])
/(3*f) + (2*b*(d*Sec[e + f*x])^(5/2))/(5*f) + (2*a*d*(d*Sec[e + f*x])^(3/2)
*Sin[e + f*x])/(3*f)
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \frac{2b(d\sec(e+fx))^{5/2}}{5f} + a\int (d\sec(e+fx))^{5/2}\,dx$$
$$= \frac{2b(d\sec(e+fx))^{5/2}}{5f} + \frac{2ad(d\sec(e+fx))^{3/2}\sin(e+fx)}{3f} + \frac{1}{3}\left(ad^2\right)\int \sqrt{d$$
$$= \frac{2b(d\sec(e+fx))^{5/2}}{5f} + \frac{2ad(d\sec(e+fx))^{3/2}\sin(e+fx)}{3f} + \frac{1}{3}\left(ad^2\sqrt{\cos(e$$
$$= \frac{2ad^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3f} + \frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

Mathematica [A] time = 0.408076, size = 58, normalized size = 0.63

$$\frac{(d\sec(e+fx))^{5/2}\left(5a\sin(2(e+fx))+10a\cos^{\frac{5}{2}}(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)+6b\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^(5/2)*(6*b + 10*a*Cos[e + f*x]^(5/2)*EllipticF[(e + f*x)/
2, 2] + 5*a*Sin[2*(e + f*x)]))/(15*f)
```

Maple [C] time = 0.214, size = 195, normalized size = 2.1

$$\frac{2\left(\cos\left(fx+e\right)+1\right)^2\left(\cos\left(fx+e\right)-1\right)^2}{15f\left(\sin\left(fx+e\right)\right)^4}\left(5i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\left(\cos\left(fx+e\right)\right)^3\mathrm{EllipticF}\left(\frac{i(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] 2/15/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*EllipticF(I*(cos(f*x+e)-1)/sin(f
*x+e),I)*a+5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c
os(f*x+e)^2*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a+5*sin(f*x+e)*cos(f*x
+e)*a+3*b)*(d/cos(f*x+e))^(5/2)/sin(f*x+e)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bd^2\sec\left(fx+e\right)^2\tan\left(fx+e\right)+ad^2\sec\left(fx+e\right)^2\right)\sqrt{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*d^2*sec(f*x + e)^2*tan(f*x + e) + a*d^2*sec(f*x + e)^2)*sqrt(d*
sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a), x)
```

3.580 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=88

$$-\frac{2ad^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2ad\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

```
[Out] (-2*a*d^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f
*x]]) + (2*b*(d*Sec[e + f*x])^(3/2))/(3*f) + (2*a*d*Sqrt[d*Sec[e + f*x]]*Si
n[e + f*x])/f
```

Rubi [A] time = 0.0694522, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3486, 3768, 3771, 2639}

$$-\frac{2ad^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2ad\sin(e+fx)\sqrt{d\sec(e+fx)}}{f}+\frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (-2*a*d^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f
*x]]) + (2*b*(d*Sec[e + f*x])^(3/2))/(3*f) + (2*a*d*Sqrt[d*Sec[e + f*x]]*Si
n[e + f*x])/f
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \frac{2b(d\sec(e+fx))^{3/2}}{3f} + a\int (d\sec(e+fx))^{3/2}\,dx$$
$$= \frac{2b(d\sec(e+fx))^{3/2}}{3f} + \frac{2ad\sqrt{d\sec(e+fx)}\sin(e+fx)}{f} - \left(ad^2\right)\int \frac{1}{\sqrt{d\sec(e+}$$
$$= \frac{2b(d\sec(e+fx))^{3/2}}{3f} + \frac{2ad\sqrt{d\sec(e+fx)}\sin(e+fx)}{f} - \frac{\left(ad^2\right)\int \sqrt{\cos(e+}}{\sqrt{\cos(e+fx)}\sqrt{d\,\mathrm{se}}}$$
$$= -\frac{2ad^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2b(d\sec(e+fx))^{3/2}}{3f} + \frac{2ad\sqrt{d\sec(e+fx}}{f}$$

Mathematica [A] time = 0.287762, size = 58, normalized size = 0.66

$$\frac{(d\sec(e+fx))^{3/2}\left(3a\sin(2(e+fx)) - 6a\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right) + 2b\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^(3/2)*(2*b - 6*a*Cos[e + f*x]^(3/2)*EllipticE[(e + f*x)/2
, 2] + 3*a*Sin[2*(e + f*x)]))/(3*f)
```

Maple [C] time = 0.211, size = 355, normalized size = 4.

$$\frac{2\left(\cos\left(fx+e\right)+1\right)^2\left(\cos\left(fx+e\right)-1\right)^2}{3f\left(\sin\left(fx+e\right)\right)^5}\left(3i\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^2\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] 2/3/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(3*I*sin(f*x+e)*cos(f*x+e)^2*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e
)-1)/sin(f*x+e),I)*a-3*I*sin(f*x+e)*cos(f*x+e)^2*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a
+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I
*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*a*sin(f*x+e)-3*I*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*
x+e),I)*cos(f*x+e)*a*sin(f*x+e)-3*cos(f*x+e)^2*a+b*sin(f*x+e)+3*a*cos(f*x+e
))*(d/cos(f*x+e))^(3/2)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bd\sec\left(fx+e\right)\tan\left(fx+e\right)+ad\sec\left(fx+e\right)\right)\sqrt{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*d*sec(f*x + e)*tan(f*x + e) + a*d*sec(f*x + e))*sqrt(d*sec(f*x
+ e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}\left(a+b\tan\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a), x)
```

3.581 $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=58

$$\frac{2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{f}+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$

```
[Out] (2*b*Sqrt[d*Sec[e + f*x]])/f + (2*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/
2, 2]*Sqrt[d*Sec[e + f*x]])/f
```

Rubi [A] time = 0.0474153, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3486, 3771, 2641}

$$\frac{2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{f}+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*b*Sqrt[d*Sec[e + f*x]])/f + (2*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/
2, 2]*Sqrt[d*Sec[e + f*x]])/f
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx &= \frac{2b\sqrt{d\sec(e+fx)}}{f}+a\int \sqrt{d\sec(e+fx)}\,dx\\ &= \frac{2b\sqrt{d\sec(e+fx)}}{f}+\left(a\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx\\ &= \frac{2b\sqrt{d\sec(e+fx)}}{f}+\frac{2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.192757, size = 42, normalized size = 0.72

$$\frac{2\sqrt{d\sec(e+fx)}\left(a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)+b\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*(b + a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[d*Sec[e + f*x]
])/f
```

Maple [C] time = 0.25, size = 168, normalized size = 2.9

$$2\frac{\left(\cos\left(fx+e\right)+1\right)^{2}\left(\cos\left(fx+e\right)-1\right)^{2}}{f\left(\sin\left(fx+e\right)\right)^{4}}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\left(i\cos\left(fx+e\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] 2/f*(d/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(I*cos(f*x+e)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f
*x+e)-1)/sin(f*x+e),I)*a+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a+b)/sin(f*x+e)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d \sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a), x)
```

3.582 $\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

```
[Out] (-2*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*a*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[C
os[e + f*x]]*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.0492027, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3486, 3771, 2639}

$$\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{2b}{f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-2*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*a*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[C
os[e + f*x]]*Sqrt[d*Sec[e + f*x]])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx &= -\frac{2b}{f\sqrt{d\sec(e+fx)}}+a\int \frac{1}{\sqrt{d\sec(e+fx)}}\,dx\\ &= -\frac{2b}{f\sqrt{d\sec(e+fx)}}+\frac{a\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\\ &= -\frac{2b}{f\sqrt{d\sec(e+fx)}}+\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.27413, size = 54, normalized size = 0.93

$$\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)-2b\sqrt{\cos(e+fx)}}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-2*b*Sqrt[Cos[e + f*x]] + 2*a*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f
*x]]*Sqrt[d*Sec[e + f*x]])
```

Maple [C] time = 0.258, size = 916, normalized size = 15.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(cos(f*x+e)-1)*(4*I*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*(cos(f*x+e)-
1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a-4*I*sin(f*x+e)*cos(f*x+e)^2*Ellipti
cE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a+8*I*sin(f*x+e)*cos
(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a-8*I
*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)*a+4*I*a*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e
),I)*sin(f*x+e)-4*I*a*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f
*x+e),I)*sin(f*x+e)-4*sin(f*x+e)*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*b-4*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a+b*ln(-2*(2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*x
+e)-b*cos(f*x+e)*ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-c
os(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+
e)^2)*sin(f*x+e)-4*sin(f*x+e)*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*b+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a)/sin(f*x+e)^3/cos(
f*x+e)/(d/cos(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\tan\left(fx+e\right)+a}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/sqrt(d*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)}{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)/(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\tan\left(e+fx\right)}{\sqrt{d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/sqrt(d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\tan\left(fx+e\right)+a}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/sqrt(d*sec(f*x + e)), x)
```

3.583 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

```
[Out] (-2*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + (2*a*Sqrt[Cos[e + f*x]]*EllipticF[(e
+ f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*a*Sin[e + f*x])/(3*d*f*Sq
rt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.0693648, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3486, 3769, 3771, 2641}

$$\frac{2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + (2*a*Sqrt[Cos[e + f*x]]*EllipticF[(e
+ f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*a*Sin[e + f*x])/(3*d*f*Sq
rt[d*Sec[e + f*x]])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{2b}{3f(d\sec(e+fx))^{3/2}} + a\int \frac{1}{(d\sec(e+fx))^{3/2}}\,dx$$
$$= -\frac{2b}{3f(d\sec(e+fx))^{3/2}} + \frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} + \frac{a\int\sqrt{d\sec(e+fx)}\,dx}{3d^2}$$
$$= -\frac{2b}{3f(d\sec(e+fx))^{3/2}} + \frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} + \frac{\left(a\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}}{3d^2}$$
$$= -\frac{2b}{3f(d\sec(e+fx))^{3/2}} + \frac{2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f} + \frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+}}$$

Mathematica [A] time = 0.195126, size = 69, normalized size = 0.73

$$-\frac{\sqrt{d\sec(e+fx)}\left(-a\sin(2(e+fx))-2a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)+b\cos(2(e+fx))+b\right)}{3d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(Sqrt[d*Sec[e + f*x]]*(b + b*Cos[2*(e + f*x)] - 2*a*Sqrt[Cos[e + f*x]]*Ell
ipticF[(e + f*x)/2, 2] - a*Sin[2*(e + f*x)]))/(3*d^2*f)
```

Maple [C] time = 0.197, size = 172, normalized size = 1.8

$$\frac{2}{3f\left(\cos\left(fx+e\right)\right)^2}\left(i\cos\left(fx+e\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)a+i\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/3/f*(I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a+I*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a
+sin(f*x+e)*cos(f*x+e)*a-b*cos(f*x+e)^2)/cos(f*x+e)^2/(d/cos(f*x+e))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)}{d^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)/(d^2*sec(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\tan\left(e+fx\right)}{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(3/2), x)
```

3.584 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=94

$$\frac{6aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

```
[Out] (-2*b)/(5*f*(d*Sec[e + f*x])^(5/2)) + (6*a*EllipticE[(e + f*x)/2, 2])/(5*d^
2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*a*Sin[e + f*x])/(5*d*f*(d
*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.0704951, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3486, 3769, 3771, 2639}

$$\frac{6aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}-\frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/2),x]
```


```
[Out] (-2*b)/(5*f*(d*Sec[e + f*x])^(5/2)) + (6*a*EllipticE[(e + f*x)/2, 2])/(5*d^
2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*a*Sin[e + f*x])/(5*d*f*(d
*Sec[e + f*x])^(3/2))
```


Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = -\frac{2b}{5f(d\sec(e+fx))^{5/2}} + a\int \frac{1}{(d\sec(e+fx))^{5/2}}\,dx$$
$$= -\frac{2b}{5f(d\sec(e+fx))^{5/2}} + \frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}} + \frac{(3a)\int \frac{1}{\sqrt{d\sec(e+fx)}}\,dx}{5d^2}$$
$$= -\frac{2b}{5f(d\sec(e+fx))^{5/2}} + \frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}} + \frac{(3a)\int \sqrt{\cos(e+fx)}\,dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$
$$= -\frac{2b}{5f(d\sec(e+fx))^{5/2}} + \frac{6aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.605078, size = 74, normalized size = 0.79

$$\frac{2\sqrt{d\sec(e+fx)}\left(\cos^2(e+fx)(a\sin(e+fx)-b\cos(e+fx))+3a\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{5d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*Sqrt[d*Sec[e + f*x]]*(3*a*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] +
 Cos[e + f*x]^2*(-(b*Cos[e + f*x]) + a*Sin[e + f*x])))/(5*d^3*f)
```

Maple [C] time = 0.204, size = 345, normalized size = 3.7

$$-\frac{2}{5f\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^3}\left(3i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{EllipticE}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\cos\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x)
```

```
[Out] -2/5/f*(3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*a*sin(f*x+e)-3*I*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)
/sin(f*x+e),I)*cos(f*x+e)*a*sin(f*x+e)+3*I*sin(f*x+e)*EllipticE(I*(cos(f*x+
e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*a-3*I*sin(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a+sin(f*x+e)*cos(f*x+e)^3*b+c
os(f*x+e)^4*a+2*cos(f*x+e)^2*a-3*a*cos(f*x+e))/sin(f*x+e)/cos(f*x+e)^3/(d/c
os(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)}{d^3\sec\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)/(d^3*sec(f*x + e)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\tan\left(e+fx\right)}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/2), x)
```

3.585 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=123

$$\frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}}+\frac{10a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f}+\frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

```
[Out] (-2*b)/(7*f*(d*Sec[e + f*x])^(7/2)) + (10*a*Sqrt[Cos[e + f*x]]*EllipticF[(e
 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(21*d^4*f) + (2*a*Sin[e + f*x])/(7*d*f*
(d*Sec[e + f*x])^(5/2)) + (10*a*Sin[e + f*x])/(21*d^3*f*Sqrt[d*Sec[e + f*x]
])
```

Rubi [A] time = 0.0905019, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3486, 3769, 3771, 2641}

$$\frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}}+\frac{10a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f}+\frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}}-\frac{2b}{7f(d\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-2*b)/(7*f*(d*Sec[e + f*x])^(7/2)) + (10*a*Sqrt[Cos[e + f*x]]*EllipticF[(e
 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(21*d^4*f) + (2*a*Sin[e + f*x])/(7*d*f*
(d*Sec[e + f*x])^(5/2)) + (10*a*Sin[e + f*x])/(21*d^3*f*Sqrt[d*Sec[e + f*x]
])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + a\int \frac{1}{(d\sec(e+fx))^{7/2}}\,dx$$

$$= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{(5a)\int \frac{1}{(d\sec(e+fx))^{3/2}}\,dx}{7d^2}$$

$$= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} + \frac{(5a)\int \sqrt{d\sec(e+}}{21d^4}$$

$$= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} + \frac{\left(5a\sqrt{\cos(e+fx)}\right.}{}$$

$$= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{10a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f} + \frac{2a\sin(e+}{7df(d\sec(e+}$$

Mathematica [A] time = 0.348078, size = 94, normalized size = 0.76

$$\frac{\sqrt{d\sec(e+fx)}\left(26a\sin(2(e+fx)) + 3a\sin(4(e+fx)) + 40a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right) - 12b\cos(2(e+fx)) - 3b\right.}{84d^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (Sqrt[d*Sec[e + f*x]]*(-9*b - 12*b*Cos[2*(e + f*x)] - 3*b*Cos[4*(e + f*x)]
+ 40*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + 26*a*Sin[2*(e + f*x)]
 + 3*a*Sin[4*(e + f*x)]))/(84*d^4*f)
```

Maple [C] time = 0.199, size = 190, normalized size = 1.5

$$\frac{2}{21f\left(\cos\left(fx+e\right)\right)^4}\left(5i\cos\left(fx+e\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\mathrm{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)a+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x)
```

```
[Out] 2/21/f*(5*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a+3*sin(f*x+e)*cos(f*x+e)^3
*a-3*b*cos(f*x+e)^4+5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a+5*sin(f*x+e)*cos(f*x+e)*
a)/cos(f*x+e)^4/(d/cos(f*x+e))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)}{d^4\sec\left(fx+e\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)/(d^4*sec(f*x + e)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(7/2), x)
```

3.586 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=143

$$\frac{2d^2\left(7a^2-2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21f}+\frac{2d\left(7a^2-2b^2\right)\sin(e+fx)(d\sec(e+fx))^{3/2}}{21f}+\frac{18ab(d$$

```
[Out] (2*(7*a^2 - 2*b^2)*d^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*
Sec[e + f*x]])/(21*f) + (18*a*b*(d*Sec[e + f*x])^(5/2))/(35*f) + (2*(7*a^2
- 2*b^2)*d*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(21*f) + (2*b*(d*Sec[e + f*
x])^(5/2)*(a + b*Tan[e + f*x]))/(7*f)
```

Rubi [A] time = 0.160003, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3508, 3486, 3768, 3771, 2641}

$$\frac{2d^2\left(7a^2-2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21f}+\frac{2d\left(7a^2-2b^2\right)\sin(e+fx)(d\sec(e+fx))^{3/2}}{21f}+\frac{18ab(d$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2,x]
```


```
[Out] (2*(7*a^2 - 2*b^2)*d^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*
Sec[e + f*x]])/(21*f) + (18*a*b*(d*Sec[e + f*x])^(5/2))/(35*f) + (2*(7*a^2
- 2*b^2)*d*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(21*f) + (2*b*(d*Sec[e + f*
x])^(5/2)*(a + b*Tan[e + f*x]))/(7*f)
```


Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}+\frac{2}{7}\int (d\sec(e+fx))^{5/2}\left(\frac{7a^2}{2}-b^2\right.$$

$$=\frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}+\frac{1}{7}\left(7a^2\right.$$

$$=\frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2\left(7a^2-2b^2\right)d(d\sec(e+fx))^{3/2}\sin(e+fx)}{21f}+\frac{2b}{}$$

$$=\frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2\left(7a^2-2b^2\right)d(d\sec(e+fx))^{3/2}\sin(e+fx)}{21f}+\frac{2b}{}$$

$$=\frac{2\left(7a^2-2b^2\right)d^2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21f}+\frac{18ab(d\sec}{35}$$

Mathematica [A] time = 0.771574, size = 127, normalized size = 0.89

$$\frac{2d^2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\left(\frac{5}{2}\left(7a^2-2b^2\right)\sin(2(e+fx))+5\left(7a^2-2b^2\right)\cos^{\frac{5}{2}}(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)+3b(1\right.}{105f(a\cos(e+fx)+b\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*d^2*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2*(5*(7*a^2 - 2*b^2)*Cos[e
 + f*x]^(5/2)*EllipticF[(e + f*x)/2, 2] + (5*(7*a^2 - 2*b^2)*Sin[2*(e + f*x
)])/2 + 3*b*(14*a + 5*b*Tan[e + f*x])))/(105*f*(a*Cos[e + f*x] + b*Sin[e +
f*x])^2)
```

Maple [C] time = 0.313, size = 382, normalized size = 2.7

$$\frac{2\left(\cos\left(fx+e\right)+1\right)^2\left(\cos\left(fx+e\right)-1\right)^2}{105\,f\left(\sin\left(fx+e\right)\right)^4\cos\left(fx+e\right)}\left(35\,i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\left(\cos\left(fx+e\right)\right)^4\mathrm{EllipticF}\left(\frac{i\left(\cos\right.}{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x)
```

```
[Out] 2/105/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(35*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4*EllipticF(I*(cos(f*x+e)-1)/sin
(f*x+e),I)*a^2-10*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*cos(f*x+e)^4*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2+35*I*(1/(cos(
f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*EllipticF(I
*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2-10*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I
)*b^2+35*sin(f*x+e)*cos(f*x+e)^2*a^2-10*sin(f*x+e)*cos(f*x+e)^2*b^2+42*cos(
```

```
f*x+e)*a*b+15*sin(f*x+e)*b^2)*(d/cos(f*x+e))^(5/2)/sin(f*x+e)^4/cos(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2d^2\sec\left(fx+e\right)^2\tan\left(fx+e\right)^2+2\,abd^2\sec\left(fx+e\right)^2\tan\left(fx+e\right)+a^2d^2\sec\left(fx+e\right)^2\right)\sqrt{d\sec\left(fx+e\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d^2*sec(f*x + e)^2*tan(f*x + e)^2 + 2*a*b*d^2*sec(f*x + e)^2*
tan(f*x + e) + a^2*d^2*sec(f*x + e)^2)*sqrt(d*sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2, x)
```

3.587 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=143

$$-\frac{2d^2\left(5a^2-2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2d\left(5a^2-2b^2\right)\sin(e+fx)\sqrt{d\sec(e+fx)}}{5f}+\frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2b(d\sec(e+f$$

```
[Out] (-2*(5*a^2 - 2*b^2)*d^2*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*
Sqrt[d*Sec[e + f*x]]) + (14*a*b*(d*Sec[e + f*x])^(3/2))/(15*f) + (2*(5*a^2
- 2*b^2)*d*Sqrt[d*Sec[e + f*x]]*Sin[e + f*x])/(5*f) + (2*b*(d*Sec[e + f*x])
^(3/2)*(a + b*Tan[e + f*x]))/(5*f)
```

Rubi [A] time = 0.162875, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3508, 3486, 3768, 3771, 2639}

$$-\frac{2d^2\left(5a^2-2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2d\left(5a^2-2b^2\right)\sin(e+fx)\sqrt{d\sec(e+fx)}}{5f}+\frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2b(d\sec(e+f$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2,x]
```


```
[Out] (-2*(5*a^2 - 2*b^2)*d^2*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*
Sqrt[d*Sec[e + f*x]]) + (14*a*b*(d*Sec[e + f*x])^(3/2))/(15*f) + (2*(5*a^2
- 2*b^2)*d*Sqrt[d*Sec[e + f*x]]*Sin[e + f*x])/(5*f) + (2*b*(d*Sec[e + f*x])
^(3/2)*(a + b*Tan[e + f*x]))/(5*f)
```


Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}+\frac{2}{5}\int (d\sec(e+fx))^{3/2}\left(\frac{5a^2}{2}-\right.$$

$$=\frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}+\frac{1}{5}(5$$

$$=\frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2\left(5a^2-2b^2\right)d\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f}+\frac{2b}{}$$

$$=\frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2\left(5a^2-2b^2\right)d\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f}+\frac{2b}{}$$

$$=-\frac{2\left(5a^2-2b^2\right)d^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2\left(5a^2-2\right.}{}$$

Mathematica [A] time = 0.636578, size = 126, normalized size = 0.88

$$-\frac{2d^2(a+b\tan(e+fx))^2\left(\left(3b^2-\frac{15a^2}{2}\right)\sin(2(e+fx))+3\left(5a^2-2b^2\right)\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)-b(10a+3b\tan(\right.}{15f\sqrt{d\sec(e+fx)}(a\cos(e+fx)+b\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-2*d^2*(a + b*Tan[e + f*x])^2*(3*(5*a^2 - 2*b^2)*Cos[e + f*x]^(3/2)*Ellipt
icE[(e + f*x)/2, 2] + ((-15*a^2)/2 + 3*b^2)*Sin[2*(e + f*x)] - b*(10*a + 3*
b*Tan[e + f*x])))/(15*f*Sqrt[d*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*
x])^2)
```

Maple [C] time = 0.296, size = 712, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x)
```

```
[Out] -2/15/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(15*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*co
s(f*x+e)^3*sin(f*x+e)*a^2-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^3*sin(f*x
+e)*b^2-15*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos
(f*x+e)^3*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a^2+6*I*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*Elliptic
E(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*b^2+15*I*(1/(cos(f*x+e)+1))^(1/
```

```
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),
I)*cos(f*x+e)^2*sin(f*x+e)*a^2-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(co
s(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^2*si
n(f*x+e)*b^2-15*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*cos(f*x+e)^2*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a^2+6*I*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*Ell
ipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*b^2+15*cos(f*x+e)^3*a^2-6*
cos(f*x+e)^3*b^2-15*a^2*cos(f*x+e)^2+9*b^2*cos(f*x+e)^2-10*a*cos(f*x+e)*b*s
in(f*x+e)-3*b^2)*(d/cos(f*x+e))^(3/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 d \sec\left(fx+e\right)\tan\left(fx+e\right)^2 + 2\,abd\sec\left(fx+e\right)\tan\left(fx+e\right) + a^2 d\sec\left(fx+e\right)\right)\sqrt{d\sec\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d*sec(f*x + e)*tan(f*x + e)^2 + 2*a*b*d*sec(f*x + e)*tan(f*x
+ e) + a^2*d*sec(f*x + e))*sqrt(d*sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2, x)
```

3.588 $\int \sqrt{d \sec(e+fx)}(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=103

$$\frac{2\left(3a^2-2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

```
[Out] (10*a*b*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*(3*a^2 - 2*b^2)*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*b*Sqrt[d*Sec[e
+ f*x]]*(a + b*Tan[e + f*x]))/(3*f)
```

Rubi [A] time = 0.123328, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3508, 3486, 3771, 2641}

$$\frac{2\left(3a^2-2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3f}+\frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2,x]
```


```
[Out] (10*a*b*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*(3*a^2 - 2*b^2)*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*b*Sqrt[d*Sec[e
+ f*x]]*(a + b*Tan[e + f*x]))/(3*f)
```


Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}+\frac{2}{3}\int \sqrt{d\sec(e+fx)}\left(\frac{3a^2}{2}-b^2+\frac{5}{2}ab\right.$$
$$=\frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}+\frac{1}{3}\left(3a^2-2b^2\right)$$
$$=\frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}+\frac{1}{3}\left(\left(3a^2-2b^2\right)\right.$$
$$=\frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2\left(3a^2-2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+}}{3f}$$

Mathematica [A] time = 0.617032, size = 87, normalized size = 0.84

$$\frac{2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\left(\left(3a^2-2b^2\right)\cos^{\frac{5}{2}}(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)+b\cos(e+fx)(6a\cos(e+fx)+b\sin(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*Sec[e + f*x]^2*Sqrt[d*Sec[e + f*x]]*((3*a^2 - 2*b^2)*Cos[e + f*x]^(5/2)*
EllipticF[(e + f*x)/2, 2] + b*Cos[e + f*x]*(6*a*Cos[e + f*x] + b*Sin[e + f*
x])))/(3*f)
```

Maple [C] time = 0.282, size = 339, normalized size = 3.3

$$\frac{2\left(\cos\left(fx+e\right)-1\right)^2\left(\cos\left(fx+e\right)+1\right)^2}{3f\cos\left(fx+e\right)\left(\sin\left(fx+e\right)\right)^4}\sqrt{\frac{d}{\cos\left(fx+e\right)}}\left(3i\left(\cos\left(fx+e\right)\right)^2\mathrm{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\frac{\cos(}{\cos(fx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x)
```

```
[Out] 2/3/f*(d/cos(f*x+e))^(1/2)*(cos(f*x+e)-1)^2*(3*I*cos(f*x+e)^2*EllipticF(I*(
cos(f*x+e)-1)/sin(f*x+e),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e
)+1))^(1/2)*a^2-2*I*cos(f*x+e)^2*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*b^2+3*I*cos(f*x+e
)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*(1/(cos(f*x+e)+1))^(1/2)*a^2-2*I*cos(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/si
n(f*x+e),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*b^2+
6*cos(f*x+e)*a*b+sin(f*x+e)*b^2)*(cos(f*x+e)+1)^2/cos(f*x+e)/sin(f*x+e)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2\right)\sqrt{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*sqrt(d*sec(f*x + e
)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2, x)
```

3.589 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2\left(a^2-2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

```
[Out] (-6*a*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*(a^2 - 2*b^2)*EllipticE[(e + f*x)/2,
 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*b*(a + b*Tan[e + f*x]
))/(f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.125565, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3508, 3486, 3771, 2639}

$$\frac{2\left(a^2-2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-6*a*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*(a^2 - 2*b^2)*EllipticE[(e + f*x)/2,
 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*b*(a + b*Tan[e + f*x]
))/(f*Sqrt[d*Sec[e + f*x]])
```

Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}} + 2\int \frac{\frac{a^2}{2}-b^2+\frac{3}{2}ab\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$$
$$= -\frac{6ab}{f\sqrt{d\sec(e+fx)}} + \frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}} + \left(a^2-2b^2\right)\int \frac{1}{\sqrt{d\sec(e+fx)}}\,dx$$
$$= -\frac{6ab}{f\sqrt{d\sec(e+fx)}} + \frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}} + \frac{\left(a^2-2b^2\right)\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$
$$= -\frac{6ab}{f\sqrt{d\sec(e+fx)}} + \frac{2\left(a^2-2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

Mathematica [A] time = 0.885828, size = 64, normalized size = 0.67

$$\frac{\frac{2\left(a^2-2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}} + 2b(b\tan(e+fx)-2a)}{f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] ((2*(a^2 - 2*b^2)*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 2*b*(-2*a
 + b*Tan[e + f*x]))/(f*Sqrt[d*Sec[e + f*x]])
```

Maple [C] time = 0.309, size = 2564, normalized size = 27.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(cos(f*x+e)-1)*(16*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/si
n(f*x+e),I)*cos(f*x+e)^3*sin(f*x+e)*b^2+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*cos(f*x+e)^2*a^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)^5*a^
2+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)^5*b^2-4*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*cos(f*x+e)^4*a^2+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*cos(f*x+e)^4*b^2-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)^2*b^2
-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)^3*b^2+2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*cos(f*x+e)*b^2+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*
cos(f*x+e)*a^2+12*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x
+e),I)*cos(f*x+e)^2*sin(f*x+e)*a^2-24*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2
)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(c
os(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*b^2-12*I*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*a^2+
24*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+
e)^2*sin(f*x+e)*b^2+8*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin
```

```
(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*a^2-16*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3
/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*
(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*b^2+4*I*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2*sin(f*x+e)+2*I*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2*sin(f*x+e)-4*I*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*b^2*sin(f*x+e)-2*
I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^2*sin(f*x
+e)-8*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f
*x+e)*sin(f*x+e)*a^2+16*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/s
in(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*b^2+2*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I
*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^4*sin(f*x+e)*a^2-4*I*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^4*sin(f*x+e)*b^
2-2*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x
+e)^4*sin(f*x+e)*a^2+4*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e
)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/si
n(f*x+e),I)*cos(f*x+e)^4*sin(f*x+e)*b^2+8*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(
I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^3*sin(f*x+e)*a^2-16*I*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^3*sin(f*x+e)*
b^2-8*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f
*x+e)^3*sin(f*x+e)*a^2+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*b^2-a*b*ln(-2
*(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*s
in(f*x+e)+a*b*cos(f*x+e)^2*ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(
f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1
)/sin(f*x+e)^2)*sin(f*x+e)-12*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+
e)^2*sin(f*x+e)*a*b-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)*sin(f
*x+e)*a*b-4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)^4*sin(f*x+e)*a*
b-12*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*cos(f*x+e)^3*sin(f*x+e)*a*b)*(cos
(f*x+e)+1)^4*(d/cos(f*x+e))^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/cos(
f*x+e)^3/d/sin(f*x+e)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/sqrt(d*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2\right)\sqrt{d\sec\left(fx+e\right)}}{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*sqrt(d*sec(f*x + e
))/(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\tan\left(e+fx\right)\right)^2}{\sqrt{d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/sqrt(d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/sqrt(d*sec(f*x + e)), x)
```

3.590 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{2\left(a^2+2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2\left(a^2+2b^2\right)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

```
[Out] (2*a*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + (2*(a^2 + 2*b^2)*Sqrt[Cos[e + f*x]]*
EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*(a^2 + 2*b^2
)*Sin[e + f*x])/(3*d*f*Sqrt[d*Sec[e + f*x]]) - (2*b*(a + b*Tan[e + f*x]))/(
f*(d*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.162471, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3508, 3486, 3769, 3771, 2641}

$$\frac{2\left(a^2+2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2\left(a^2+2b^2\right)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}+\frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(3/2),x]
```


```
[Out] (2*a*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + (2*(a^2 + 2*b^2)*Sqrt[Cos[e + f*x]]*
EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*(a^2 + 2*b^2
)*Sin[e + f*x])/(3*d*f*Sqrt[d*Sec[e + f*x]]) - (2*b*(a + b*Tan[e + f*x]))/(
f*(d*Sec[e + f*x])^(3/2))
```


Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}} - 2\int \frac{-\frac{a^2}{2}-b^2+\frac{1}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx$$
$$= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}} - \left(-a^2-2b^2\right)\int \frac{1}{(d\sec(e+fx))^{3/2}}\,dx$$
$$= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} + \frac{2\left(a^2+2b^2\right)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}} + \frac{\left(a^2+2b^2\right)\int}{}$$
$$= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} + \frac{2\left(a^2+2b^2\right)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}} + \frac{\left(\left(a^2+2b^2\right)\right.}{}$$
$$= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} + \frac{2\left(a^2+2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{3d^2f} + \frac{2(a}{3}$$

Mathematica [A] time = 0.519407, size = 101, normalized size = 0.73

$$\frac{\sec^2(e+fx)\left(2\left(a^2+2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)+a^2\sin(2(e+fx))-2ab\cos(2(e+fx))-2ab-b^2\sin(2(e+\right.}{3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(-2*a*b - 2*a*b*Cos[2*(e + f*x)] + 2*(a^2 + 2*b^2)*Sqrt[Cos
[e + f*x]]*EllipticF[(e + f*x)/2, 2] + a^2*Sin[2*(e + f*x)] - b^2*Sin[2*(e
+ f*x)]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Maple [C] time = 0.277, size = 320, normalized size = 2.3

$$\frac{2}{3f\left(\cos\left(fx+e\right)\right)^2}\left(i\mathrm{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\cos\left(fx+e\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}a^2+2i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/3/f*(I*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2+2*I*EllipticF(I*(cos(f*x+e
)-1)/sin(f*x+e),I)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*b^2+I*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2+2*I*EllipticF(I*(cos(f*x+e
)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
```

```
2)*b^2-2*cos(f*x+e)^2*a*b+cos(f*x+e)*sin(f*x+e)*a^2-cos(f*x+e)*sin(f*x+e)*b
^2)/(d/cos(f*x+e))^(3/2)/cos(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2\right)\sqrt{d\sec\left(fx+e\right)}}{d^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*sqrt(d*sec(f*x + e
))/(d^2*sec(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan\left(e+fx\right)\right)^2}{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(3/2), x)
```

3.591 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$\frac{2\left(3a^2+2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\left(3a^2+2b^2\right)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}}-\frac{2ab}{15f(d\sec(e+fx))^{5/2}}-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

```
[Out] (-2*a*b)/(15*f*(d*Sec[e + f*x])^(5/2)) + (2*(3*a^2 + 2*b^2)*EllipticE[(e +
f*x)/2, 2])/(5*d^2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*(3*a^2 +
 2*b^2)*Sin[e + f*x])/(15*d*f*(d*Sec[e + f*x])^(3/2)) - (2*b*(a + b*Tan[e +
 f*x]))/(3*f*(d*Sec[e + f*x])^(5/2))
```

Rubi [A] time = 0.165958, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3508, 3486, 3769, 3771, 2639}

$$\frac{2\left(3a^2+2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\left(3a^2+2b^2\right)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}}-\frac{2ab}{15f(d\sec(e+fx))^{5/2}}-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*b)/(15*f*(d*Sec[e + f*x])^(5/2)) + (2*(3*a^2 + 2*b^2)*EllipticE[(e +
f*x)/2, 2])/(5*d^2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*(3*a^2 +
 2*b^2)*Sin[e + f*x])/(15*d*f*(d*Sec[e + f*x])^(3/2)) - (2*b*(a + b*Tan[e +
 f*x]))/(3*f*(d*Sec[e + f*x])^(5/2))
```

Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = -\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}} - \frac{2}{3}\int \frac{-\frac{3a^2}{2}-b^2-\frac{1}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx$$
$$= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}} - \frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}} - \frac{1}{3}\left(-3a^2-2b^2\right)\int \frac{1}{(d\sec(e+fx))^{5/2}}\,dx$$
$$= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}} + \frac{2\left(3a^2+2b^2\right)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}} + \frac{\left(3a^2+2b^2\right)}{}$$
$$= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}} + \frac{2\left(3a^2+2b^2\right)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}} + \frac{\left(3a^2+2b^2\right)}{5d^2\sqrt{\cos(e+}}$$
$$= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}} + \frac{2\left(3a^2+2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2\left(3a^2+2b^2\right)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}} -$$

Mathematica [A] time = 0.922147, size = 92, normalized size = 0.63

$$\frac{\left(6a^2+4b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)+2\cos^{\frac{3}{2}}(e+fx)\left(\left(a^2-b^2\right)\sin(e+fx)-2ab\cos(e+fx)\right)}{5f\cos^{\frac{5}{2}}(e+fx)(d\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((6*a^2 + 4*b^2)*EllipticE[(e + f*x)/2, 2] + 2*Cos[e + f*x]^(3/2)*(-2*a*b*C
os[e + f*x] + (a^2 - b^2)*Sin[e + f*x]))/(5*f*Cos[e + f*x]^(5/2)*(d*Sec[e +
 f*x])^(5/2))
```

Maple [C] time = 0.275, size = 670, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/5/f*(3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*a^2+2*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1
)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*b^2-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(
f*x+e)*sin(f*x+e)*a^2-2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*b^
2+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(
```

```
I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a^2+2*I*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*
sin(f*x+e)*b^2-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a^2-2*I*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/s
in(f*x+e),I)*sin(f*x+e)*b^2-cos(f*x+e)^4*a^2+cos(f*x+e)^4*b^2-2*cos(f*x+e)^
3*sin(f*x+e)*a*b-2*a^2*cos(f*x+e)^2-3*b^2*cos(f*x+e)^2+3*cos(f*x+e)*a^2+2*c
os(f*x+e)*b^2)/cos(f*x+e)^3/sin(f*x+e)/(d/cos(f*x+e))^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2\right)\sqrt{d\sec\left(fx+e\right)}}{d^3\sec\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*sqrt(d*sec(f*x + e
))/(d^3*sec(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/2), x)
```

3.592 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=184

$$\frac{2\left(5a^2+2b^2\right)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}}+\frac{2\left(5a^2+2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f}+\frac{2\left(5a^2+2b^2\right)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}}-$$

```
[Out] (-6*a*b)/(35*f*(d*Sec[e + f*x])^(7/2)) + (2*(5*a^2 + 2*b^2)*Sqrt[Cos[e + f*
x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(21*d^4*f) + (2*(5*a^2
+ 2*b^2)*Sin[e + f*x])/(35*d*f*(d*Sec[e + f*x])^(5/2)) + (2*(5*a^2 + 2*b^2)
*Sin[e + f*x])/(21*d^3*f*Sqrt[d*Sec[e + f*x]]) - (2*b*(a + b*Tan[e + f*x]))
/(5*f*(d*Sec[e + f*x])^(7/2))
```

Rubi [A] time = 0.191904, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3508, 3486, 3769, 3771, 2641}

$$\frac{2\left(5a^2+2b^2\right)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}}+\frac{2\left(5a^2+2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f}+\frac{2\left(5a^2+2b^2\right)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(7/2),x]
```


```
[Out] (-6*a*b)/(35*f*(d*Sec[e + f*x])^(7/2)) + (2*(5*a^2 + 2*b^2)*Sqrt[Cos[e + f*
x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(21*d^4*f) + (2*(5*a^2
+ 2*b^2)*Sin[e + f*x])/(35*d*f*(d*Sec[e + f*x])^(5/2)) + (2*(5*a^2 + 2*b^2)
*Sin[e + f*x])/(21*d^3*f*Sqrt[d*Sec[e + f*x]]) - (2*b*(a + b*Tan[e + f*x]))
/(5*f*(d*Sec[e + f*x])^(7/2))
```


Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}} - \frac{2}{5}\int \frac{-\frac{5a^2}{2}-b^2-\frac{3}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}} - \frac{1}{5}\left(-5a^2-2b^2\right)\int \frac{1}{(d\sec(e+fx))^{7/2}}\,dx$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2\left(5a^2+2b^2\right)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}} + \frac{\left(5a^2+2b^2\right)}{}$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2\left(5a^2+2b^2\right)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} + \frac{2\left(5a^2+2b^2\right)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} - \frac{2b(a+b}{5f(d\sec}$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2\left(5a^2+2b^2\right)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} + \frac{2\left(5a^2+2b^2\right)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} - \frac{2b(a+b}{5f(d\sec}$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2\left(5a^2+2b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{d\sec(e+fx)}}{21d^4f} + \frac{2(}{35}$$

Mathematica [A] time = 2.22214, size = 127, normalized size = 0.69

$$\frac{\frac{4(5a^2+2b^2)F\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}} + 23a^2\sin(e+fx) + 3a^2\sin(3(e+fx)) - 18ab\cos(e+fx) - 6ab\cos(3(e+fx)) + 5b^2\sin(e+fx) -}{42d^3f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-18*a*b*Cos[e + f*x] - 6*a*b*Cos[3*(e + f*x)] + (4*(5*a^2 + 2*b^2)*Ellipti
cF[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 23*a^2*Sin[e + f*x] + 5*b^2*Sin[e
+ f*x] + 3*a^2*Sin[3*(e + f*x)] - 3*b^2*Sin[3*(e + f*x)])/(42*d^3*f*Sqrt[d*
Sec[e + f*x]])
```

Maple [C] time = 0.281, size = 359, normalized size = 2.

$$\frac{2}{21f\left(\cos\left(fx+e\right)\right)^4}\left(5\,i\mathrm{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\cos\left(fx+e\right)\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}a^2+2\,i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x)
```

```
[Out] 2/21/f*(5*I*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2+2*I*EllipticF(I*(cos(f*
x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*b^2+5*I*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2+2*I*EllipticF(I*(cos(
f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*b^2-6*a*b*cos(f*x+e)^4+3*cos(f*x+e)^3*sin(f*x+e)*a^2-3*cos(f*x+e)^3
*sin(f*x+e)*b^2+5*cos(f*x+e)*sin(f*x+e)*a^2+2*cos(f*x+e)*sin(f*x+e)*b^2)/co
s(f*x+e)^4/(d/cos(f*x+e))^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2\right)\sqrt{d\sec\left(fx+e\right)}}{d^4\sec\left(fx+e\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*sqrt(d*sec(f*x + e
))/(d^4*sec(f*x + e)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(7/2), x)
```

3.593 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=184

$$\frac{2\left(7a^2+2b^2\right)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}}+\frac{2\left(7a^2+2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15d^4f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\left(7a^2+2b^2\right)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}}-\frac{10ab}{63f(d\sec(e+fx))^{9/2}}-$$

```
[Out] (-10*a*b)/(63*f*(d*Sec[e + f*x])^(9/2)) + (2*(7*a^2 + 2*b^2)*EllipticE[(e +
 f*x)/2, 2])/(15*d^4*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*(7*a^2
 + 2*b^2)*Sin[e + f*x])/(63*d*f*(d*Sec[e + f*x])^(7/2)) + (2*(7*a^2 + 2*b^2
)*Sin[e + f*x])/(45*d^3*f*(d*Sec[e + f*x])^(3/2)) - (2*b*(a + b*Tan[e + f*x
]))/(7*f*(d*Sec[e + f*x])^(9/2))
```

Rubi [A] time = 0.191764, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3508, 3486, 3769, 3771, 2639}

$$\frac{2\left(7a^2+2b^2\right)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}}+\frac{2\left(7a^2+2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15d^4f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2\left(7a^2+2b^2\right)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}}-\frac{10ab}{63f(d\sec(e+fx))^{9/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(9/2),x]
```


```
[Out] (-10*a*b)/(63*f*(d*Sec[e + f*x])^(9/2)) + (2*(7*a^2 + 2*b^2)*EllipticE[(e +
 f*x)/2, 2])/(15*d^4*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*(7*a^2
 + 2*b^2)*Sin[e + f*x])/(63*d*f*(d*Sec[e + f*x])^(7/2)) + (2*(7*a^2 + 2*b^2
)*Sin[e + f*x])/(45*d^3*f*(d*Sec[e + f*x])^(3/2)) - (2*b*(a + b*Tan[e + f*x
]))/(7*f*(d*Sec[e + f*x])^(9/2))
```


Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}} - \frac{2}{7}\int \frac{-\frac{7a^2}{2} - b^2 - \frac{5}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{9/2}}\,dx$$

$$= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}} - \frac{1}{7}\left(-7a^2-2b^2\right)\int \frac{1}{(d\sec(e+fx))^{9/2}}\,dx$$

$$= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2\left(7a^2+2b^2\right)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}} + \frac{\left(7a^2+2b^2\right)\ldots}{\ldots}$$

$$= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2\left(7a^2+2b^2\right)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} + \frac{2\left(7a^2+2b^2\right)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\ldots}{7f(d\sec\ldots}$$

$$= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2\left(7a^2+2b^2\right)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} + \frac{2\left(7a^2+2b^2\right)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\ldots}{7f(d\sec\ldots}$$

$$= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2\left(7a^2+2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15d^4f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2\left(7a^2+2b^2\right)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} + \ldots$$

Mathematica [A] time = 2.84275, size = 126, normalized size = 0.68

$$\frac{4\cos(e+fx)\left(2\sin(e+fx)\left(5\left(a^2-b^2\right)\cos(2(e+fx))+19a^2-b^2\right)-30ab\cos(e+fx)-10ab\cos(3(e+fx))\right)+\frac{48\left(7a^2+2\ldots\right.}{\sqrt{\ldots}}}{360d^4f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] ((48*(7*a^2 + 2*b^2)*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 4*Cos[
e + f*x]*(-30*a*b*Cos[e + f*x] - 10*a*b*Cos[3*(e + f*x)] + 2*(19*a^2 - b^2
+ 5*(a^2 - b^2)*Cos[2*(e + f*x)])*Sin[e + f*x]))/(360*d^4*f*Sqrt[d*Sec[e +
f*x]])
```

Maple [C] time = 0.352, size = 697, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x)
```

```
[Out] -2/45/f*(5*cos(f*x+e)^6*a^2-5*cos(f*x+e)^6*b^2+10*cos(f*x+e)^5*sin(f*x+e)*a
*b-21*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Elliptic
```

```
F(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a^2-6*I*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I
)*sin(f*x+e)*b^2+21*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^
(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*a^2+6*
I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(c
os(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*b^2-6*I*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(
f*x+e)*sin(f*x+e)*b^2+6*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+
1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*b^
2+21*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticE
(I*(cos(f*x+e)-1)/sin(f*x+e),I)*sin(f*x+e)*a^2-21*I*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I
)*cos(f*x+e)*sin(f*x+e)*a^2+2*cos(f*x+e)^4*a^2+7*cos(f*x+e)^4*b^2+14*a^2*co
s(f*x+e)^2+4*b^2*cos(f*x+e)^2-21*cos(f*x+e)*a^2-6*cos(f*x+e)*b^2)/cos(f*x+e
)^5/sin(f*x+e)/(d/cos(f*x+e))^(9/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2\right)\sqrt{d\sec\left(fx+e\right)}}{d^5\sec\left(fx+e\right)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*sqrt(d*sec(f*x + e
))/(d^5*sec(f*x + e)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(9/2), x)
```

3.594 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=198

$$\frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\left(14\left(11a^2-2b^2\right)+65ab\tan(e+fx)\right)}{315f}+\frac{2ad^2\left(7a^2-6b^2\right)\tan(e+fx)\sqrt{d\sec(e+fx)}}{21f}$$

```
[Out] (2*a*(7*a^2 - 6*b^2)*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e
+ f*x]])/(21*f*(Sec[e + f*x]^2)^(1/4)) + (2*a*(7*a^2 - 6*b^2)*d^2*Sqrt[d*Se
c[e + f*x]]*Tan[e + f*x])/(21*f) + (2*b*d^2*Sec[e + f*x]^2*Sqrt[d*Sec[e + f
*x]]*(a + b*Tan[e + f*x])^2)/(9*f) + (2*b*d^2*Sec[e + f*x]^2*Sqrt[d*Sec[e +
 f*x]]*(14*(11*a^2 - 2*b^2) + 65*a*b*Tan[e + f*x]))/(315*f)
```

Rubi [A] time = 0.154041, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3512, 743, 780, 195, 231}

$$\frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\left(14\left(11a^2-2b^2\right)+65ab\tan(e+fx)\right)}{315f}+\frac{2ad^2\left(7a^2-6b^2\right)\tan(e+fx)\sqrt{d\sec(e+fx)}}{21f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (2*a*(7*a^2 - 6*b^2)*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e
+ f*x]])/(21*f*(Sec[e + f*x]^2)^(1/4)) + (2*a*(7*a^2 - 6*b^2)*d^2*Sqrt[d*Se
c[e + f*x]]*Tan[e + f*x])/(21*f) + (2*b*d^2*Sec[e + f*x]^2*Sqrt[d*Sec[e + f
*x]]*(a + b*Tan[e + f*x])^2)/(9*f) + (2*b*d^2*Sec[e + f*x]^2*Sqrt[d*Sec[e +
 f*x]]*(14*(11*a^2 - 2*b^2) + 65*a*b*Tan[e + f*x]))/(315*f)
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int (a+x)^3\sqrt[4]{1+\frac{x^2}{b^2}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{9f} + \frac{\left(2bd^2\sqrt{d\sec(e+f\ldots\right.}{\ldots}$$

$$= \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{9f} + \frac{2bd^2\sec^2(e+fx)\sqrt{\ldots}}{\ldots}$$

$$= \frac{2a\left(7a^2-6b^2\right)d^2\sqrt{d\sec(e+fx)}\tan(e+fx)}{21f} + \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e\ldots}}{9f\ldots}$$

$$= \frac{2a\left(7a^2-6b^2\right)d^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{21f\sqrt[4]{\sec^2(e+fx)}} + \frac{2a\left(7a^2-6\ldots\right.}{\ldots}$$

Mathematica [A] time = 1.69054, size = 157, normalized size = 0.79

$$-\frac{2d(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\left(63b\left(b^2-3a^2\right)\cos^2(e+fx)-15a\left(7a^2-6b^2\right)\sin(e+fx)\cos^3(e+fx)-15a\left(\ldots\right.}{315f(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-2*d*(d*Sec[e + f*x])^(3/2)*(63*b*(-3*a^2 + b^2)*Cos[e + f*x]^2 - 15*a*(7*
a^2 - 6*b^2)*Cos[e + f*x]^(9/2)*EllipticF[(e + f*x)/2, 2] - 15*a*(7*a^2 - 6
*b^2)*Cos[e + f*x]^3*Sin[e + f*x] - (5*b^2*(14*b + 27*a*Sin[2*(e + f*x)]))/
2)*(a + b*Tan[e + f*x])^3)/(315*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [C] time = 0.382, size = 414, normalized size = 2.1

$$\frac{2\left(\cos\left(fx+e\right)+1\right)^2\left(\cos\left(fx+e\right)-1\right)^2}{315f\left(\cos\left(fx+e\right)\right)^2\left(\sin\left(fx+e\right)\right)^4}\left(105\,i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\text{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)}\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x)
```

```
[Out] 2/315/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(105*I*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*c
os(f*x+e)^5*a^3-90*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^5*a*b^2+105*I*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x
+e)-1)/sin(f*x+e),I)*cos(f*x+e)^4*a^3-90*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x
+e)^4*a*b^2+105*sin(f*x+e)*cos(f*x+e)^3*a^3-90*sin(f*x+e)*cos(f*x+e)^3*a*b^
2+189*a^2*cos(f*x+e)^2*b-63*b^3*cos(f*x+e)^2+135*sin(f*x+e)*cos(f*x+e)*a*b^
2+35*b^3)*(d/cos(f*x+e))^(5/2)/cos(f*x+e)^2/sin(f*x+e)^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3d^2\sec\left(fx+e\right)^2\tan\left(fx+e\right)^3+3\,ab^2d^2\sec\left(fx+e\right)^2\tan\left(fx+e\right)^2+3\,a^2bd^2\sec\left(fx+e\right)^2\tan\left(fx+e\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*d^2*sec(f*x + e)^2*tan(f*x + e)^3 + 3*a*b^2*d^2*sec(f*x + e)^
2*tan(f*x + e)^2 + 3*a^2*b*d^2*sec(f*x + e)^2*tan(f*x + e) + a^3*d^2*sec(f*
x + e)^2)*sqrt(d*sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)+a\right)^{3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3, x)
```

3.595 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=176

$$\frac{2b(d\sec(e+fx))^{3/2}\left(10\left(9a^2-2b^2\right)+33ab\tan(e+fx)\right)}{105f}+\frac{2a\left(5a^2-6b^2\right)\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{5f}-$$

```
[Out] (-2*a*(5*a^2 - 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])
^(3/2))/(5*f*(Sec[e + f*x]^2)^(3/4)) + (2*a*(5*a^2 - 6*b^2)*Cos[e + f*x]*(d
*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(5*f) + (2*b*(d*Sec[e + f*x])^(3/2)*(a +
 b*Tan[e + f*x])^2)/(7*f) + (2*b*(d*Sec[e + f*x])^(3/2)*(10*(9*a^2 - 2*b^2)
 + 33*a*b*Tan[e + f*x]))/(105*f)
```

Rubi [A] time = 0.14558, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3512, 743, 780, 227, 196}

$$\frac{2b(d\sec(e+fx))^{3/2}\left(10\left(9a^2-2b^2\right)+33ab\tan(e+fx)\right)}{105f}+\frac{2a\left(5a^2-6b^2\right)\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{5f}-$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3,x]
```


```
[Out] (-2*a*(5*a^2 - 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])
^(3/2))/(5*f*(Sec[e + f*x]^2)^(3/4)) + (2*a*(5*a^2 - 6*b^2)*Cos[e + f*x]*(d
*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(5*f) + (2*b*(d*Sec[e + f*x])^(3/2)*(a +
 b*Tan[e + f*x])^2)/(7*f) + (2*b*(d*Sec[e + f*x])^(3/2)*(10*(9*a^2 - 2*b^2)
 + 33*a*b*Tan[e + f*x]))/(105*f)
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \frac{(d\sec(e+fx))^{3/2}\,\text{Subst}\left(\int \frac{(a+x)^3}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f} + \frac{\left(2b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \right.}{7f\,\text{se}}$$

$$= \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f} + \frac{2b(d\sec(e+fx))^{3/2}\left(10\left(9a^2-2\right.\right.}{105f}$$

$$= \frac{2a\left(5a^2-6b^2\right)\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{5f} + \frac{2b(d\sec(e+fx)}{}$$

$$= -\frac{2a\left(5a^2-6b^2\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{5f\sec^2(e+fx)^{3/4}} + \frac{2a\left(5a^2-\right.}{}$$

Mathematica [A] time = 1.78392, size = 155, normalized size = 0.88

$$-\frac{d\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\left(70b\left(b^2-3a^2\right)\cos^2(e+fx)-42a\left(5a^2-6b^2\right)\sin(e+fx)\cos^3(e+fx)+42a\left(5a^2\right.\right.}{105f(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] -(d*Sqrt[d*Sec[e + f*x]]*(70*b*(-3*a^2 + b^2)*Cos[e + f*x]^2 + 42*a*(5*a^2
- 6*b^2)*Cos[e + f*x]^(7/2)*EllipticE[(e + f*x)/2, 2] - 42*a*(5*a^2 - 6*b^2
)*Cos[e + f*x]^3*Sin[e + f*x] - 3*b^2*(10*b + 21*a*Sin[2*(e + f*x)]))*(a +
b*Tan[e + f*x])^3)/(105*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [C] time = 0.34, size = 759, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x)
```

```
[Out] 2/105/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(-105*I*cos(f*x+e)^4*sin(f*x+e)*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(
f*x+e)-1)/sin(f*x+e),I)*a^3-105*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*
x+e),I)*a^3+126*I*cos(f*x+e)^4*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2-12
6*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2-126*I*cos(f*x+e)
^4*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2+105*I*cos(f*x+e)^4*sin(f*x+e)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos
(f*x+e)-1)/sin(f*x+e),I)*a^3+126*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f
*x+e),I)*a*b^2+105*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3-1
05*cos(f*x+e)^4*a^3+126*cos(f*x+e)^4*a*b^2+105*a^3*cos(f*x+e)^3-189*a*b^2*c
os(f*x+e)^3+105*a^2*cos(f*x+e)^2*b*sin(f*x+e)-35*cos(f*x+e)^2*sin(f*x+e)*b^
3+63*a*cos(f*x+e)*b^2+15*sin(f*x+e)*b^3)*(d/cos(f*x+e))^(3/2)/cos(f*x+e)^2/
sin(f*x+e)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 d \sec\left(fx+e\right) \tan\left(fx+e\right)^3 + 3\,ab^2 d \sec\left(fx+e\right) \tan\left(fx+e\right)^2 + 3\,a^2 bd \sec\left(fx+e\right) \tan\left(fx+e\right) + a^3 d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*d*sec(f*x + e)*tan(f*x + e)^3 + 3*a*b^2*d*sec(f*x + e)*tan(f*
x + e)^2 + 3*a^2*b*d*sec(f*x + e)*tan(f*x + e) + a^3*d*sec(f*x + e))*sqrt(d
*sec(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3, x)
```

3.596 $\int \sqrt{d \sec(e+fx)}(a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=129

$$\frac{2b\sqrt{d\sec(e+fx)}\left(2\left(7a^2-2b^2\right)+3ab\tan(e+fx)\right)}{5f}+\frac{2a\left(a^2-2b^2\right)\sqrt{d\sec(e+fx)}F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)}{f\sqrt[4]{\sec^2(e+fx)}}+\frac{2b\cdot}{}$$

```
[Out] (2*a*(a^2 - 2*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]
])/(f*(Sec[e + f*x]^2)^(1/4)) + (2*b*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*
x])^2)/(5*f) + (2*b*Sqrt[d*Sec[e + f*x]]*(2*(7*a^2 - 2*b^2) + 3*a*b*Tan[e +
 f*x]))/(5*f)
```

Rubi [A] time = 0.112089, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3512, 743, 780, 231}

$$\frac{2b\sqrt{d\sec(e+fx)}\left(2\left(7a^2-2b^2\right)+3ab\tan(e+fx)\right)}{5f}+\frac{2a\left(a^2-2b^2\right)\sqrt{d\sec(e+fx)}F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)}{f\sqrt[4]{\sec^2(e+fx)}}+\frac{2b\cdot}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3,x]
```


```
[Out] (2*a*(a^2 - 2*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]
])/(f*(Sec[e + f*x]^2)^(1/4)) + (2*b*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*
x])^2)/(5*f) + (2*b*Sqrt[d*Sec[e + f*x]]*(2*(7*a^2 - 2*b^2) + 3*a*b*Tan[e +
 f*x]))/(5*f)
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx = \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{5f} + \frac{\left(2b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)\left(\ldots\right.}{\ldots}\right.}{5f\sqrt[4]{\sec^2(\ldots}}$$

$$= \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{5f} + \frac{2b\sqrt{d\sec(e+fx)}\left(2\left(7a^2-2b^2\right)+3\ldots\right.}{5f}$$

$$= \frac{2a\left(a^2-2b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{f\sqrt[4]{\sec^2(e+fx)}} + \frac{2b\sqrt{d\sec(e+fx}\ldots}{\ldots}$$

Mathematica [A] time = 2.03126, size = 132, normalized size = 1.02

$$-\frac{2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\left(5b\left(b^2-3a^2\right)\cos^3(e+fx)-5a\left(a^2-2b^2\right)\cos^{\frac{7}{2}}(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)-\frac{1}{2}b^2\cos\ldots\right.}{5f(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]]*(5*b*(-3*a^2 + b^2)*Cos[e + f*x]^3 - 5*a*(a^2 - 2*
b^2)*Cos[e + f*x]^(7/2)*EllipticF[(e + f*x)/2, 2] - (b^2*Cos[e + f*x]*(2*b
+ 5*a*Sin[2*(e + f*x)]))/2)*(a + b*Tan[e + f*x])^3)/(5*f*(a*Cos[e + f*x] +
b*Sin[e + f*x])^3)
```

Maple [C] time = 0.327, size = 373, normalized size = 2.9

$$\frac{2\left(\cos\left(fx+e\right)+1\right)^2\left(\cos\left(fx+e\right)-1\right)^2}{5f\left(\cos\left(fx+e\right)\right)^2\left(\sin\left(fx+e\right)\right)^4}\left(5i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\text{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\ldots\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x)
```

```
[Out] 2/5/f*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)^2*(5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f
*x+e)^3*a^3-10*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^3*a*b^2+5*I*(1/(cos(f*
```

```
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)
/sin(f*x+e),I)*cos(f*x+e)^2*a^3-10*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)^2*
a*b^2+15*a^2*cos(f*x+e)^2*b-5*b^3*cos(f*x+e)^2+5*sin(f*x+e)*cos(f*x+e)*a*b^
2+b^3)*(d/cos(f*x+e))^(1/2)/cos(f*x+e)^2/sin(f*x+e)^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^3\tan\left(fx+e\right)^3+3\,ab^2\tan\left(fx+e\right)^2+3\,a^2b\tan\left(fx+e\right)+a^3\right)\sqrt{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*sqrt(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3, x)
```

3.597 $\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=178

$$-\frac{2b\sec^2(e+fx)\left(2\left(3a^2-2b^2\right)+3ab\tan(e+fx)\right)}{3f\sqrt{d\sec(e+fx)}}-\frac{2a\left(a^2-6b^2\right)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}}+\frac{2a\left(a^2-6b^2\right)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\tan^{-}\right.}{f\sqrt{d\sec(e+fx)}}$$

```
[Out] (2*a*(a^2 - 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1
/4))/(f*Sqrt[d*Sec[e + f*x]]) - (2*a*(a^2 - 6*b^2)*Tan[e + f*x])/(f*Sqrt[d*
Sec[e + f*x]]) - (2*(b - a*Tan[e + f*x])*(a + b*Tan[e + f*x])^2)/(f*Sqrt[d*
Sec[e + f*x]]) - (2*b*Sec[e + f*x]^2*(2*(3*a^2 - 2*b^2) + 3*a*b*Tan[e + f*x
]))/(3*f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.140579, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3512, 739, 780, 227, 196}

$$-\frac{2b\sec^2(e+fx)\left(2\left(3a^2-2b^2\right)+3ab\tan(e+fx)\right)}{3f\sqrt{d\sec(e+fx)}}-\frac{2a\left(a^2-6b^2\right)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}}+\frac{2a\left(a^2-6b^2\right)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\tan^{-}\right.}{f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (2*a*(a^2 - 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1
/4))/(f*Sqrt[d*Sec[e + f*x]]) - (2*a*(a^2 - 6*b^2)*Tan[e + f*x])/(f*Sqrt[d*
Sec[e + f*x]]) - (2*(b - a*Tan[e + f*x])*(a + b*Tan[e + f*x])^2)/(f*Sqrt[d*
Sec[e + f*x]]) - (2*b*Sec[e + f*x]^2*(2*(3*a^2 - 2*b^2) + 3*a*b*Tan[e + f*x
]))/(3*f*Sqrt[d*Sec[e + f*x]])
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
```

```
Q[p, -1]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} + \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)\left(\frac{1}{2}\left(4-\frac{a^2}{b^2}\right)-\right.}{\sqrt[4]{1+\frac{x^2}{b^2}}}\right.}{f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} - \frac{2b\sec^2(e+fx)\left(2\left(3a^2-2b^2\right)+3ab\tan(e\right.}{3f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2a\left(a^2-6b^2\right)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}} - \frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} - \frac{2b\sec^2(e+fx}{}$$

$$= \frac{2a\left(a^2-6b^2\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{f\sqrt{d\sec(e+fx)}} - \frac{2a\left(a^2-6b^2\right)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}} -$$

Mathematica [A] time = 1.89124, size = 130, normalized size = 0.73

$$\frac{d(a+b\tan(e+fx))^3\left(6a\left(a^2-6b^2\right)\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+b\left(\left(3b^2-9a^2\right)\cos(2(e+fx))-9a^2+9ab\sin(2(e\right.\right.}{3f(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (d*(6*a*(a^2 - 6*b^2)*Cos[e + f*x]^(3/2)*EllipticE[(e + f*x)/2, 2] + b*(-9*
a^2 + 5*b^2 + (-9*a^2 + 3*b^2)*Cos[2*(e + f*x)] + 9*a*b*Sin[2*(e + f*x)]))*
(a + b*Tan[e + f*x])^3)/(3*f*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin
[e + f*x])^3)
```

Maple [C] time = 0.346, size = 5006, normalized size = 28.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^3}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/sqrt(d*sec(f*x + e)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^3\tan\left(fx+e\right)^3+3\,ab^2\tan\left(fx+e\right)^2+3\,a^2b\tan\left(fx+e\right)+a^3\right)\sqrt{d\sec\left(fx+e\right)}}{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*sqrt(d*sec(f*x + e))/(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan\left(e+fx\right)\right)^3}{\sqrt{d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3/sqrt(d*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^{3}}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/sqrt(d*sec(f*x + e)), x)
```

3.598 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2b\sec^2(e+fx)\left(2\left(a^2-2b^2\right)+ab\tan(e+fx)\right)}{3f(d\sec(e+fx))^{3/2}}+\frac{2a\left(a^2+6b^2\right)\sec^2(e+fx)^{3/4}F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)}{3f(d\sec(e+fx))^{3/2}}-\frac{2(b-a\ta$$

```
[Out] (2*a*(a^2 + 6*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3
/4))/(3*f*(d*Sec[e + f*x])^(3/2)) - (2*(b - a*Tan[e + f*x])*(a + b*Tan[e +
f*x])^2)/(3*f*(d*Sec[e + f*x])^(3/2)) - (2*b*Sec[e + f*x]^2*(2*(a^2 - 2*b^2
) + a*b*Tan[e + f*x]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.124466, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3512, 739, 780, 231}

$$-\frac{2b\sec^2(e+fx)\left(2\left(a^2-2b^2\right)+ab\tan(e+fx)\right)}{3f(d\sec(e+fx))^{3/2}}+\frac{2a\left(a^2+6b^2\right)\sec^2(e+fx)^{3/4}F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)}{3f(d\sec(e+fx))^{3/2}}-\frac{2(b-a\ta$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(3/2),x]
```



```
[Out] (2*a*(a^2 + 6*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3
/4))/(3*f*(d*Sec[e + f*x])^(3/2)) - (2*(b - a*Tan[e + f*x])*(a + b*Tan[e +
f*x])^2)/(3*f*(d*Sec[e + f*x])^(3/2)) - (2*b*Sec[e + f*x]^2*(2*(a^2 - 2*b^2
) + a*b*Tan[e + f*x]))/(3*f*(d*Sec[e + f*x])^(3/2))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)^{3/4}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}} + \frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{(a+x)\left(\frac{1}{2}\left(4+\frac{a^2}{b^2}\right)\right.}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\right.}{3f(d\sec(e+fx))^{3/}}$$

$$= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}} - \frac{2b\sec^2(e+fx)\left(2\left(a^2-2b^2\right)+ab\tan(e+\right.}{3f(d\sec(e+fx))^{3/2}}$$

$$= \frac{2a\left(a^2+6b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3f(d\sec(e+fx))^{3/2}} - \frac{2(b-a\tan(e+fx))(a+b\,\text{t}}{3f(d\sec(e+fx))^{3}}$$

Mathematica [A] time = 1.30836, size = 117, normalized size = 0.8

$$\frac{\sec^2(e+fx)\left(\left(b^3-3a^2b\right)\cos(2(e+fx))+2a\left(a^2+6b^2\right)\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)-3a^2b+a^3\sin(2(e+fx))-3a\right.}{3f(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(-3*a^2*b + 7*b^3 + (-3*a^2*b + b^3)*Cos[2*(e + f*x)] + 2*a
*(a^2 + 6*b^2)*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + a^3*Sin[2*(e
+ f*x)] - 3*a*b^2*Sin[2*(e + f*x)]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Maple [C] time = 0.303, size = 342, normalized size = 2.3

$$\frac{2}{3f\left(\cos\left(fx+e\right)\right)^2}\left(i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\text{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\cos\left(fx+e\right)a^3+6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/3/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipti
cF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*a^3+6*I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),
I)*cos(f*x+e)*a*b^2+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3+6*I*(1/(cos(f*x+e)+1))^(
1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e
```

```
),I)*a*b^2-3*a^2*cos(f*x+e)^2*b+b^3*cos(f*x+e)^2+cos(f*x+e)*a^3*sin(f*x+e)-
3*sin(f*x+e)*cos(f*x+e)*a*b^2+3*b^3)/(d/cos(f*x+e))^(3/2)/cos(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^3}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^3\tan\left(fx+e\right)^3+3\,ab^2\tan\left(fx+e\right)^2+3\,a^2b\tan\left(fx+e\right)+a^3\right)\sqrt{d\sec\left(fx+e\right)}}{d^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*sqrt(d*sec(f*x + e))/(d^2*sec(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tan\left(e+fx\right)\right)^3}{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^3}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(3/2), x)
```

3.599 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{6a\left(a^2+2b^2\right)\tan(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}}-\frac{2\left(2b\left(a^2+2b^2\right)-a\left(3a^2+5b^2\right)\tan(e+fx)\right)}{5d^2f\sqrt{d\sec(e+fx)}}+\frac{6a\left(a^2+2b^2\right)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\tan^{-1}\right.}{5d^2f\sqrt{d\sec(e+fx)}}$$

```
[Out] (6*a*(a^2 + 2*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1
/4))/(5*d^2*f*Sqrt[d*Sec[e + f*x]]) - (6*a*(a^2 + 2*b^2)*Tan[e + f*x])/(5*d
^2*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*x]^2*(b - a*Tan[e + f*x])*(a + b*
Tan[e + f*x])^2)/(5*d^2*f*Sqrt[d*Sec[e + f*x]]) - (2*(2*b*(a^2 + 2*b^2) - a
*(3*a^2 + 5*b^2)*Tan[e + f*x]))/(5*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.157326, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3512, 739, 778, 227, 196}

$$-\frac{6a\left(a^2+2b^2\right)\tan(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}}-\frac{2\left(2b\left(a^2+2b^2\right)-a\left(3a^2+5b^2\right)\tan(e+fx)\right)}{5d^2f\sqrt{d\sec(e+fx)}}+\frac{6a\left(a^2+2b^2\right)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\tan^{-1}\right.}{5d^2f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (6*a*(a^2 + 2*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1
/4))/(5*d^2*f*Sqrt[d*Sec[e + f*x]]) - (6*a*(a^2 + 2*b^2)*Tan[e + f*x])/(5*d
^2*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*x]^2*(b - a*Tan[e + f*x])*(a + b*
Tan[e + f*x])^2)/(5*d^2*f*Sqrt[d*Sec[e + f*x]]) - (2*(2*b*(a^2 + 2*b^2) - a
*(3*a^2 + 5*b^2)*Tan[e + f*x]))/(5*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2f\sqrt{d\sec(e+fx)}} + \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x}{}\right.}{5d^2f\sqrt{d\,s}}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{2\left(2b\left(a^2+2b^2\right)-a\left(3a^2+5b^2\right)\text{t}\right.}{5d^2f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{6a\left(a^2+2b^2\right)\tan(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{2\left(2l\right.}{}$$

$$= \frac{6a\left(a^2+2b^2\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{6a\left(a^2+2b^2\right)\tan(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{2\,\text{c}}{}$$

Mathematica [A] time = 1.4614, size = 150, normalized size = 0.74

$$\frac{\sqrt{d\sec(e+fx)}\left(-b\left(9a^2+17b^2\right)\cos(e+fx)+12a\left(a^2+2b^2\right)\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)-3a^2b\cos(3(e+fx))+a^3\,\text{si}\right.}{10d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (Sqrt[d*Sec[e + f*x]]*(-(b*(9*a^2 + 17*b^2)*Cos[e + f*x]) - 3*a^2*b*Cos[3*(
e + f*x)] + b^3*Cos[3*(e + f*x)] + 12*a*(a^2 + 2*b^2)*Sqrt[Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2] + a^3*Sin[e + f*x] - 3*a*b^2*Sin[e + f*x] + a^3*Sin
[3*(e + f*x)] - 3*a*b^2*Sin[3*(e + f*x)]))/(10*d^3*f)
```

Maple [C] time = 0.314, size = 1920, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/10/f*(cos(f*x+e)+1)*(cos(f*x+e)-1)^2*(12*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*a*b^2+4*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*b^3+12*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a*b^2+
4*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*b^3-36*cos(f
*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a*b^2-20*cos(f*x+e)^2*sin(f*x+
e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*b^3-12*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*a*b^2-20*cos(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)*b^3+24*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a*b^2
+5*cos(f*x+e)*ln(-2*(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-co
s(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e
)^2)*b^3*sin(f*x+e)-5*cos(f*x+e)*ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*
x+e)-1)/sin(f*x+e)^2)*b^3*sin(f*x+e)-12*I*cos(f*x+e)^2*sin(f*x+e)*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3+24*I*cos(f*x+e)*si
n(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3-
24*I*cos(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/
sin(f*x+e),I)*a^3+24*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(
f*x+e),I)*a*b^2*sin(f*x+e)-24*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e
)-1)/sin(f*x+e),I)*a*b^2*sin(f*x+e)+12*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*
sin(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3+48*I*cos(f*x+e)*sin
(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2
-48*I*cos(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)
/sin(f*x+e),I)*a*b^2+24*I*cos(f*x+e)^2*sin(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2-24*I*cos(f*x+e)^2*sin(f*x+e)*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2+12*cos(f*x
+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^3-4*cos(f*x+e)^5*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*a^3-4*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)*a^3-8*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^3+4*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^3-12*cos(f*x+e)^4*sin(f*x+e)*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^2*b-12*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*a^2*b+12*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(co
s(f*x+e)-1)/sin(f*x+e),I)*a^3*sin(f*x+e)-12*I*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
E(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3*sin(f*x+e))/cos(f*x+e)^3/(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)/sin(f*x+e)^5/(d/cos(f*x+e))^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^3\tan\left(fx+e\right)^3+3\,ab^2\tan\left(fx+e\right)^2+3\,a^2b\tan\left(fx+e\right)+a^3\right)\sqrt{d\sec\left(fx+e\right)}}{d^3\sec\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*sqrt(d*sec(f*x + e))/(d^3*sec(f*x + e)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^3}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(5/2), x)
```

3.600 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{2\left(2b\left(3a^2+2b^2\right)-a\left(5a^2+3b^2\right)\tan(e+fx)\right)}{21d^2f(d\sec(e+fx))^{3/2}}+\frac{2a\left(5a^2+6b^2\right)\sec^2(e+fx)^{3/4}F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)}{21d^2f(d\sec(e+fx))^{3/2}}-\frac{2\cos^2}{}$$

```
[Out] (2*a*(5*a^2 + 6*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^
(3/4))/(21*d^2*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e + f*x]^2*(b - a*Tan[e +
 f*x])*(a + b*Tan[e + f*x])^2)/(7*d^2*f*(d*Sec[e + f*x])^(3/2)) - (2*(2*b*(
3*a^2 + 2*b^2) - a*(5*a^2 + 3*b^2)*Tan[e + f*x]))/(21*d^2*f*(d*Sec[e + f*x]
)^(3/2))
```

Rubi [A] time = 0.142954, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3512, 739, 778, 231}

$$-\frac{2\left(2b\left(3a^2+2b^2\right)-a\left(5a^2+3b^2\right)\tan(e+fx)\right)}{21d^2f(d\sec(e+fx))^{3/2}}+\frac{2a\left(5a^2+6b^2\right)\sec^2(e+fx)^{3/4}F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)}{21d^2f(d\sec(e+fx))^{3/2}}-\frac{2\cos^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(7/2),x]
```


```
[Out] (2*a*(5*a^2 + 6*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^
(3/4))/(21*d^2*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e + f*x]^2*(b - a*Tan[e +
 f*x])*(a + b*Tan[e + f*x])^2)/(7*d^2*f*(d*Sec[e + f*x])^(3/2)) - (2*(2*b*(
3*a^2 + 2*b^2) - a*(5*a^2 + 3*b^2)*Tan[e + f*x]))/(21*d^2*f*(d*Sec[e + f*x]
)^(3/2))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sec^2(e+fx)^{3/4}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{11/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{7d^2f(d\sec(e+fx))^{3/2}} + \frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{(a+}{}\right.}{7d^2f(d\,\text{se}}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{7d^2f(d\sec(e+fx))^{3/2}} - \frac{2\left(2b\left(3a^2+2b^2\right)-a\left(5a^2+3b^2\right)\right.}{21d^2f(d\sec(e+fx))^{3}}$$

$$= \frac{2a\left(5a^2+6b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{21d^2f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)(b-a\tan(e+f}{7d^2f(d\sec(e+}$$

Mathematica [A] time = 2.44114, size = 150, normalized size = 0.88

$$\frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\left(4\left(5a^3+6ab^2\right)F\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}\left(-b\left(27a^2+19b^2\right)\cos(e+fx)+\left(3b^3-9a^2\right.\right.\right.}{42d^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]*(4*(5*a^3 + 6*a*b^2)*EllipticF[(e
+ f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(-(b*(27*a^2 + 19*b^2)*Cos[e + f*x]) + (-
9*a^2*b + 3*b^3)*Cos[3*(e + f*x)] + 2*a*(13*a^2 + 3*b^2 + 3*(a^2 - 3*b^2)*C
os[2*(e + f*x)])*Sin[e + f*x])))/(42*d^4*f)
```

Maple [C] time = 0.304, size = 391, normalized size = 2.3

$$-\frac{2}{21f\left(\cos\left(fx+e\right)\right)^4}\left(-5i\sqrt{\left(\cos\left(fx+e\right)+1\right)^{-1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\text{EllipticF}\left(\frac{i\left(\cos\left(fx+e\right)-1\right)}{\sin\left(fx+e\right)},i\right)\cos\left(fx+e\right)a^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x)
```

```
[Out] -2/21/f*(-5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*a^3-6*I*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticF(I*(cos(f*x+e
```

```
)-1)/sin(f*x+e),I)*a*b^2+9*cos(f*x+e)^4*a^2*b-3*cos(f*x+e)^4*b^3-3*sin(f*x+
e)*cos(f*x+e)^3*a^3+9*sin(f*x+e)*cos(f*x+e)^3*a*b^2-5*I*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+
e),I)*a^3-6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2+7*b^3*cos(f*x+e)^2-5*cos(f*x+e
)*a^3*sin(f*x+e)-6*sin(f*x+e)*cos(f*x+e)*a*b^2)/(d/cos(f*x+e))^(7/2)/cos(f*
x+e)^4
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^3\tan\left(fx+e\right)^3+3\,ab^2\tan\left(fx+e\right)^2+3\,a^2b\tan\left(fx+e\right)+a^3\right)\sqrt{d\sec\left(fx+e\right)}}{d^4\sec\left(fx+e\right)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*sqrt(d*sec(f*x + e))/(d^4*sec(f*x + e)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^3}{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="giac")

[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(7/2), x)
```

3.601 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{2\cos^2(e+fx)\left(2b\left(5a^2+2b^2\right)-a\left(7a^2+b^2\right)\tan(e+fx)\right)}{45d^4f\sqrt{d\sec(e+fx)}}+\frac{2a\left(7a^2+6b^2\right)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\right|}{15d^4f\sqrt{d\sec(e+fx)}}$$

```
[Out] (2*a*(7*a^2 + 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^
(1/4))/(15*d^4*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*x]^4*(b - a*Tan[e + f
*x])*(a + b*Tan[e + f*x])^2)/(9*d^4*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*
x]^2*(2*b*(5*a^2 + 2*b^2) - a*(7*a^2 + b^2)*Tan[e + f*x]))/(45*d^4*f*Sqrt[d
*Sec[e + f*x]])
```

Rubi [A] time = 0.139476, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3512, 739, 778, 196}

$$-\frac{2\cos^2(e+fx)\left(2b\left(5a^2+2b^2\right)-a\left(7a^2+b^2\right)\tan(e+fx)\right)}{45d^4f\sqrt{d\sec(e+fx)}}+\frac{2a\left(7a^2+6b^2\right)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\right|}{15d^4f\sqrt{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (2*a*(7*a^2 + 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^
(1/4))/(15*d^4*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*x]^4*(b - a*Tan[e + f
*x])*(a + b*Tan[e + f*x])^2)/(9*d^4*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*
x]^2*(2*b*(5*a^2 + 2*b^2) - a*(7*a^2 + b^2)*Tan[e + f*x]))/(45*d^4*f*Sqrt[d
*Sec[e + f*x]])
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{13/4}}\,dx, x, b\tan(e+fx)\right)}{bd^4f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{9d^4f\sqrt{d\sec(e+fx)}} + \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x}{\ldots}\right.}{9d^4f\sqrt{d\,s}}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{9d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)\left(2b\left(5a^2+2b^2\right)-a\right.}{45d^4f\sqrt{d\sec(e}}$$

$$= \frac{2a\left(7a^2+6b^2\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^4(e+fx)(b-a\tan(e+f}{9d^4f\sqrt{d\sec(e}}$$

Mathematica [B] time = 6.37808, size = 372, normalized size = 2.11

$$\frac{\sec^2(e+fx)(a+b\tan(e+fx))^3\left(\frac{1}{180}a\left(19a^2-3b^2\right)\sin(e+fx)+\frac{1}{360}a\left(43a^2-21b^2\right)\sin(3(e+fx))+\frac{1}{72}a\left(a^2-3b^2\right)\sin(\right.}{f(d\sec(e+fx))^{9/2}(a\cos(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (Sec[e + f*x]^(3/2)*((2*(56*a^3 + 48*a*b^2)*EllipticE[(e + f*x)/2, 2])/(Sqr
t[Cos[e + f*x]]*Sqrt[Sec[e + f*x]]) - (2*(15*a^2*b + 7*b^3)*Sin[e + f*x]^2)
/(Sqrt[1 - Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]]*Sqrt[Cos[e + f*x]^2*(-1 + Sec
[e + f*x]^2)]))*(a + b*Tan[e + f*x])^3)/(120*f*(d*Sec[e + f*x])^(9/2)*(a*Co
s[e + f*x] + b*Sin[e + f*x])^3) + (Sec[e + f*x]^2*(-(b*(15*a^2 + 4*b^2)*Cos
[e + f*x])/90 - (b*(75*a^2 + 11*b^2)*Cos[3*(e + f*x)])/360 - (b*(3*a^2 - b^
2)*Cos[5*(e + f*x)])/72 + (a*(19*a^2 - 3*b^2)*Sin[e + f*x])/180 + (a*(43*a^
2 - 21*b^2)*Sin[3*(e + f*x)])/360 + (a*(a^2 - 3*b^2)*Sin[5*(e + f*x)])/72)*
(a + b*Tan[e + f*x])^3)/(f*(d*Sec[e + f*x])^(9/2)*(a*Cos[e + f*x] + b*Sin[e
 + f*x])^3)
```

Maple [C] time = 0.384, size = 745, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x)
```

```
[Out] -2/45/f*(-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c
os(f*x+e)*sin(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3+18*I*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e
)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2+18*I*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticE(I*(cos(f*x+e)-1)
/sin(f*x+e),I)*a*b^2+21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*sin(f*x+e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3+5*cos(f*x
+e)^6*a^3-15*cos(f*x+e)^6*a*b^2+15*cos(f*x+e)^5*sin(f*x+e)*a^2*b-5*cos(f*x+
e)^5*sin(f*x+e)*b^3-18*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*sin(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a*b^2+21*I*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+
e)*EllipticE(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3-18*I*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticF(I*(cos
(f*x+e)-1)/sin(f*x+e),I)*a*b^2-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*sin(f*x+e)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*a^3
+2*cos(f*x+e)^4*a^3+21*cos(f*x+e)^4*a*b^2+9*cos(f*x+e)^3*sin(f*x+e)*b^3+14*
cos(f*x+e)^2*a^3+12*cos(f*x+e)^2*a*b^2-21*a^3*cos(f*x+e)-18*a*cos(f*x+e)*b^
2)/cos(f*x+e)^5/sin(f*x+e)/(d/cos(f*x+e))^(9/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^3\tan\left(fx+e\right)^3+3\,ab^2\tan\left(fx+e\right)^2+3\,a^2b\tan\left(fx+e\right)+a^3\right)\sqrt{d\sec\left(fx+e\right)}}{d^5\sec\left(fx+e\right)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*sqrt(d*sec(f*x + e))/(d^5*sec(f*x + e)^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^{3}}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(9/2), x)
```

3.602 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=218

$$\frac{10a\left(3a^2+2b^2\right)\tan(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}}-\frac{2\cos^2(e+fx)\left(2b\left(7a^2+2b^2\right)-a\left(9a^2-b^2\right)\tan(e+fx)\right)}{77d^4f(d\sec(e+fx))^{3/2}}+\frac{10a\left(3a^2+2b^2\right)\sec^2(e+\ldots}{77d^4f(\ldots}$$

```
[Out] (10*a*(3*a^2 + 2*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)
^(3/4))/(77*d^4*f*(d*Sec[e + f*x])^(3/2)) + (10*a*(3*a^2 + 2*b^2)*Tan[e + f
*x])/(77*d^4*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e + f*x]^4*(b - a*Tan[e + f
*x])*(a + b*Tan[e + f*x])^2)/(11*d^4*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e +
 f*x]^2*(2*b*(7*a^2 + 2*b^2) - a*(9*a^2 - b^2)*Tan[e + f*x]))/(77*d^4*f*(d*
Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.166802, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3512, 739, 778, 199, 231}

$$\frac{10a\left(3a^2+2b^2\right)\tan(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}}-\frac{2\cos^2(e+fx)\left(2b\left(7a^2+2b^2\right)-a\left(9a^2-b^2\right)\tan(e+fx)\right)}{77d^4f(d\sec(e+fx))^{3/2}}+\frac{10a\left(3a^2+2b^2\right)\sec^2(e+\ldots}{77d^4f(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(11/2),x]
```


```
[Out] (10*a*(3*a^2 + 2*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)
^(3/4))/(77*d^4*f*(d*Sec[e + f*x])^(3/2)) + (10*a*(3*a^2 + 2*b^2)*Tan[e + f
*x])/(77*d^4*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e + f*x]^4*(b - a*Tan[e + f
*x])*(a + b*Tan[e + f*x])^2)/(11*d^4*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e +
 f*x]^2*(2*b*(7*a^2 + 2*b^2) - a*(9*a^2 - b^2)*Tan[e + f*x]))/(77*d^4*f*(d*
Sec[e + f*x])^(3/2))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 739

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
```

```
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \frac{\sec^2(e+fx)^{3/4}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{15/4}}\,dx, x, b\tan(e+fx)\right)}{bd^4 f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4 f(d\sec(e+fx))^{3/2}} + \frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{(a+}{}\right.}{11d^4 f(d\,\text{se}}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4 f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)\left(2b\left(7a^2+2b^2\right)-a\right.}{77d^4 f(d\sec(e+}$$

$$= \frac{10a\left(3a^2+2b^2\right)\tan(e+fx)}{77d^4 f(d\sec(e+fx))^{3/2}} - \frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4 f(d\sec(e+fx))^{3/2}} - \frac{2\,\text{co}}{}$$

$$= \frac{10a\left(3a^2+2b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{77d^4 f(d\sec(e+fx))^{3/2}} + \frac{10a\left(3a^2+2b^2\right)\tan(e+fx)}{77d^4 f(d\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 6.43815, size = 296, normalized size = 1.36

$$\frac{\sec^3(e+fx)(a+b\tan(e+fx))^3\left(\frac{a\left(347a^2+103b^2\right)\sin(2(e+fx))}{1232} + \frac{1}{308}a\left(16a^2-15b^2\right)\sin(4(e+fx)) + \frac{1}{176}a\left(a^2-3b^2\right)\sin(6(e+\right.}{f(d\sec(e+fx))^{11/2}(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(11/2),x]
```

```
[Out] (10*a*(3*a^2 + 2*b^2)*EllipticF[(e + f*x)/2, 2]*(a + b*Tan[e + f*x])^3)/(77
*f*Cos[e + f*x]^(5/2)*(d*Sec[e + f*x])^(11/2)*(a*Cos[e + f*x] + b*Sin[e + f
*x])^3) + (Sec[e + f*x]^3*(-(b*(105*a^2 + 31*b^2))/616 - (b*(315*a^2 + 71*b
^2)*Cos[2*(e + f*x)])/1232 - (b*(63*a^2 + b^2)*Cos[4*(e + f*x)])/616 - (b*(
3*a^2 - b^2)*Cos[6*(e + f*x)])/176 + (a*(347*a^2 + 103*b^2)*Sin[2*(e + f*x)
```

```
])/1232 + (a*(16*a^2 - 15*b^2)*Sin[4*(e + f*x)])/308 + (a*(a^2 - 3*b^2)*Sin
[6*(e + f*x)])/176)*(a + b*Tan[e + f*x])^3)/(f*(d*Sec[e + f*x])^(11/2)*(a*C
os[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [C] time = 0.387, size = 430, normalized size = 2.

$$\frac{2}{77 f \left(\cos\left(fx+e\right)\right)^{6}}\left(-21 \left(\cos\left(fx+e\right)\right)^{6} a^{2} b+7 \left(\cos\left(fx+e\right)\right)^{6} b^{3}+7 \left(\cos\left(fx+e\right)\right)^{5} \sin\left(fx+e\right) a^{3}-21 \left(\cos\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x)
```

```
[Out] 2/77/f*(-21*cos(f*x+e)^6*a^2*b+7*cos(f*x+e)^6*b^3+7*cos(f*x+e)^5*sin(f*x+e)
*a^3-21*cos(f*x+e)^5*sin(f*x+e)*a*b^2+15*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x
+e)*a^3+10*I*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*cos(f*x+e)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2+15*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(
f*x+e),I)*a^3+10*I*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2-11*cos(f*x+e)^4*b^3+9*sin
(f*x+e)*cos(f*x+e)^3*a^3+6*sin(f*x+e)*cos(f*x+e)^3*a*b^2+15*cos(f*x+e)*a^3*
sin(f*x+e)+10*sin(f*x+e)*cos(f*x+e)*a*b^2)/cos(f*x+e)^6/(d/cos(f*x+e))^(11/
2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^{3} \tan\left(fx+e\right)^{3}+3 a b^{2} \tan\left(fx+e\right)^{2}+3 a^{2} b \tan\left(fx+e\right)+a^{3}\right) \sqrt{d \sec\left(fx+e\right)}}{d^{6} \sec\left(fx+e\right)^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*sqrt(d*sec(f*x + e))/(d^6*sec(f*x + e)^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^{3}}{\left(d\sec\left(fx+e\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(11/2), x)
```

3.603 $\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=456

$$\frac{d^2\left(a^2+b^2\right)^{3/4}(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{5/2}f\sec^2(e+fx)^{3/4}}-\frac{d^2\left(a^2+b^2\right)^{3/4}(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{5/2}f\sec^2(e+fx)^{3/4}}-ad$$

```
[Out] (2*d^2*(d*Sec[e + f*x])^(3/2))/(3*b*f) + ((a^2 + b^2)^(3/4)*d^2*ArcTan[(Sqr
t[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(b^
(5/2)*f*(Sec[e + f*x]^2)^(3/4)) - ((a^2 + b^2)^(3/4)*d^2*ArcTanh[(Sqrt[b]*(
Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(b^(5/2)*
f*(Sec[e + f*x]^2)^(3/4)) + (2*a*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(
d*Sec[e + f*x])^(3/2))/(b^2*f*(Sec[e + f*x]^2)^(3/4)) - (2*a*d^2*Cos[e + f*
x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(b^2*f) - (a*Sqrt[a^2 + b^2]*d^2*Co
t[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)],
 -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b^3*f*(Sec[e + f*x]^2)^
(3/4)) + (a*Sqrt[a^2 + b^2]*d^2*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2],
ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*
x]^2])/(b^3*f*(Sec[e + f*x]^2)^(3/4))
```

Rubi [A] time = 0.408458, antiderivative size = 456, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3512, 735, 844, 227, 196, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{d^2\left(a^2+b^2\right)^{3/4}(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{5/2}f\sec^2(e+fx)^{3/4}}-\frac{d^2\left(a^2+b^2\right)^{3/4}(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{5/2}f\sec^2(e+fx)^{3/4}}-ad$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x]),x]
```


```
[Out] (2*d^2*(d*Sec[e + f*x])^(3/2))/(3*b*f) + ((a^2 + b^2)^(3/4)*d^2*ArcTan[(Sqr
t[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(b^
(5/2)*f*(Sec[e + f*x]^2)^(3/4)) - ((a^2 + b^2)^(3/4)*d^2*ArcTanh[(Sqrt[b]*(
Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(b^(5/2)*
f*(Sec[e + f*x]^2)^(3/4)) + (2*a*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(
d*Sec[e + f*x])^(3/2))/(b^2*f*(Sec[e + f*x]^2)^(3/4)) - (2*a*d^2*Cos[e + f*
x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(b^2*f) - (a*Sqrt[a^2 + b^2]*d^2*Co
t[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)],
 -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b^3*f*(Sec[e + f*x]^2)^
(3/4)) + (a*Sqrt[a^2 + b^2]*d^2*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2],
ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*
x]^2])/(b^3*f*(Sec[e + f*x]^2)^(3/4))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/4}}{a+x}dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} + \frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1-\frac{ax}{b^2}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} - \frac{\left(ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{b^3f\sec^2(e+fx)^{3/4}} + \frac{\left(\left(1+\right.\right.}{}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} - \frac{2ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f} + \frac{\left(ad^2(d\sec(e+fx))^{3/2}\right.}{}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} + \frac{2ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} - \frac{2ad^2\cos(e+fx)}{}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} + \frac{2ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} - \frac{2ad^2\cos(e+fx)}{}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} + \frac{2ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} - \frac{2ad^2\cos(e+fx)}{}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} + \frac{\left(a^2+b^2\right)^{3/4}d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{b^{5/2}f\sec^2(e+fx)^{3/4}} - \frac{\left(a^2+b^2\right)^{3/4}d}{}$$

Mathematica [B] time = 29.3312, size = 11149, normalized size = 24.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.654, size = 26371, normalized size = 57.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a), x)
```

3.604 $\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=396

$$-\frac{d^2\sqrt[4]{a^2+b^2}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}-\frac{d^2\sqrt[4]{a^2+b^2}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}+ad^2\sqrt{-\tan^2(e}$$

```
[Out] (2*d^2*Sqrt[d*Sec[e + f*x]])/(b*f) - ((a^2 + b^2)^(1/4)*d^2*ArcTan[(Sqrt[b]
*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(b^(3/2)*
f*(Sec[e + f*x]^2)^(1/4)) - ((a^2 + b^2)^(1/4)*d^2*ArcTanh[(Sqrt[b]*(Sec[e
+ f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(b^(3/2)*f*(Sec[e
 + f*x]^2)^(1/4)) - (2*a*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Se
c[e + f*x]])/(b^2*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^2*Cot[e + f*x]*EllipticP
i[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e +
f*x]]*Sqrt[-Tan[e + f*x]^2])/(b^2*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^2*Cot[e
+ f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sq
rt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(b^2*f*(Sec[e + f*x]^2)^(1/4))
```

Rubi [A] time = 0.384003, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3512, 735, 844, 231, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{d^2\sqrt[4]{a^2+b^2}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}-\frac{d^2\sqrt[4]{a^2+b^2}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}+ad^2\sqrt{-\tan^2(e}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*d^2*Sqrt[d*Sec[e + f*x]])/(b*f) - ((a^2 + b^2)^(1/4)*d^2*ArcTan[(Sqrt[b]
*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(b^(3/2)*
f*(Sec[e + f*x]^2)^(1/4)) - ((a^2 + b^2)^(1/4)*d^2*ArcTanh[(Sqrt[b]*(Sec[e
+ f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(b^(3/2)*f*(Sec[e
 + f*x]^2)^(1/4)) - (2*a*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Se
c[e + f*x]])/(b^2*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^2*Cot[e + f*x]*EllipticP
i[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e +
f*x]]*Sqrt[-Tan[e + f*x]^2])/(b^2*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^2*Cot[e
+ f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sq
rt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(b^2*f*(Sec[e + f*x]^2)^(1/4))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m
```

```
+ 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x^2}{b^2}}}{a+x}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1-\frac{ax}{b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{\left(ad^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{b^3f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(\left(1+\right.\right.}{}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{2ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(\left(1+\frac{a^2}{b^2}\right)d^2\sqrt{d\,\text{se}\right.}{}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{2ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(\left(1+\frac{a^2}{b^2}\right)d^2\sqrt{d\,\text{se}\right.}{}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{2ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(2\left(1+\frac{a^2}{b^2}\right)bd^2\sqrt{d\right.}{}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{2ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(\left(1+\frac{a^2}{b^2}\right)bd^2\sqrt{d\,\text{s}\right.}{}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{\sqrt[4]{a^2+b^2}d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}} - \frac{\sqrt[4]{a^2+b^2}d^2\tanh}{b^3}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{\sqrt[4]{a^2+b^2}d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}} - \frac{\sqrt[4]{a^2+b^2}d^2\tanh}{b^3}$$

Mathematica [B] time = 24.6276, size = 6631, normalized size = 16.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.497, size = 10704, normalized size = 27.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a), x)
```

3.605 $\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=334

$$\frac{(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{\sqrt{b}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}-\frac{(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{\sqrt{b}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}-\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d}{b}$$

```
[Out] (ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x]
)^(3/2))/(Sqrt[b]*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) - (ArcTanh[(S
qrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(
Sqrt[b]*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) - (a*Cot[e + f*x]*Ellip
ticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e +
f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b*Sqrt[a^2 + b^2]*f*(Sec[e + f*x]^2)^(3
/4)) + (a*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2
)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b*Sqrt[a^2 + b
^2]*f*(Sec[e + f*x]^2)^(3/4))
```

Rubi [A] time = 0.294234, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {3512, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{\sqrt{b}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}-\frac{(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{\sqrt{b}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}-\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x]
)^(3/2))/(Sqrt[b]*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) - (ArcTanh[(S
qrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(
Sqrt[b]*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) - (a*Cot[e + f*x]*Ellip
ticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e +
f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b*Sqrt[a^2 + b^2]*f*(Sec[e + f*x]^2)^(3
/4)) + (a*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2
)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b*Sqrt[a^2 + b
^2]*f*(Sec[e + f*x]^2)^(3/4))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \frac{(d\sec(e+fx))^{3/2}\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{(d\sec(e+fx))^{3/2}\,\text{Subst}\left(\int \frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}} + \frac{\left(a(d\sec(e+fx))^{3/2}\right)\text{Sub}}{b}$$

$$= -\frac{(d\sec(e+fx))^{3/2}\,\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx, x, b^2\tan^2(e+fx)\right)}{2bf\sec^2(e+fx)^{3/4}} + \frac{\left(2a\cot(e+fx)(d\sec(e\right.}{}$$

$$= -\frac{\left(2b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sec^2(e+fx)^{3/4}} + \frac{\left(a\cot(e+fx)(d\sec(e\right.}{}$$

$$= -\frac{(d\sec(e+fx))^{3/2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sec^2(e+fx)^{3/4}} + \frac{(d\sec(e+fx))^{3/2}\,\text{Subst}\left(\int\right.}{f\sec}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} - \frac{a\cot}{}$$

Mathematica [C] time = 21.676, size = 6301, normalized size = 18.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.326, size = 3737, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] 1/2/f/b/(a^2+b^2)^(1/2)/(-b+(a^2+b^2)^(1/2))/a^2/(b*((a^2+b^2)^(1/2)*a^2+2*
(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)/(b+(a^2+b^2)^(1/2))/(-b*((a^2
+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(cos(f*x+e)
+1)^2*(-4*I*b*a^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*EllipticPi(I*(cos(f*x+e)-1)/sin(f*x+e),-1/(b+(a^2+b^2)^(1/2))^2*a^2,I)*(b
*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*(-b*(
(a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(a^2+b^
```

2)^(1/2)+4*I*(a^2+b^2)^(3/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2
-2*b*a^2-2*b^3)/a^4)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(b*((a^2+b^2)^(1/2)*a^2
+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*a*b-4*I*b*a^3*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticPi(I*(cos(f*x+e)-1)/
sin(f*x+e),-1/(-b+(a^2+b^2)^(1/2))^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+
b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^
2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(a^2+b^2)^(1/2)-4*I*(a^2+b^2)^(1/2)*
(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f
*x+e)-1)/sin(f*x+e),I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^
2+2*b^3)/a^4)^(1/2)*a*b^3+(a^2+b^2)^(3/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b
^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*ln(2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(
1/2)*a^2-(a^2+b^2)^(3/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b
*a^2-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)*(b*((a^2+b^2)^(1/2)*a^2+
2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*a^2+(a^2+b^2)^(3/2)*(-b*((a
^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos
(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-
1)/sin(f*x+e)^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^
3)/a^4)^(1/2)*a^2-(a^2+b^2)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2
)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2)*
(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*a^2
b^2+(a^2+b^2)^(1/2)(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2
-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a
^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*a^2*b^2-(a^2+b^2)^(1/2)*(-b*((a
^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos
(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-
1)/sin(f*x+e)^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^
3)/a^4)^(1/2)*a^2*b^2-(a^2+b^2)^(3/2)*arctanh(1/2/(b*((a^2+b^2)^(1/2)*a^2+2
*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)/a^2*(cos(f*x+e)-1)*((a^2+b^2
)^(1/2)*cos(f*x+e)*b+cos(f*x+e)*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b^2)/s
in(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-b*((a^2+b^2)^(1/2)*a^2+
2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)*b^2+(a^2+b^2)^(3/2)*arctanh(1/2/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b
^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)/a^2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)
*cos(f*x+e)*b-cos(f*x+e)*a^2-cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f*x+
e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2
)*b^2+(a^2+b^2)^(1/2)*arctanh(1/2/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)
*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)/a^2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+
e)*b+cos(f*x+e)*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b^2)/sin(f*x+e)^2/(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2
)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*b^4-(a
^2+b^2)^(1/2)*arctanh(1/2/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*
b*a^2-2*b^3)/a^4)^(1/2)/a^2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b-co
s(f*x+e)*a^2-cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f*x+e)^2/(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(b*((a^2+b^
2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*b^4-arctanh(1/
2/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)/a
^2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b+cos(f*x+e)*a^2+cos(f*x+e)*b
^2-b*(a^2+b^2)^(1/2)-b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*

```
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^4*b-arctanh(1/2/(b*((a^2+b^2)^(1/2)*
a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)/a^2*(cos(f*x+e)-1)*((a^
2+b^2)^(1/2)*cos(f*x+e)*b+cos(f*x+e)*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b
^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-b*((a^2+b^2)^(1/2)
*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)*a^2*b^3-arctanh(1/2/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/
2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)/a^2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*
x+e)*b-cos(f*x+e)*a^2-cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f*x+e)^2/(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(b
*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*a^4*b
-arctanh(1/2/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/
a^4)^(1/2)/a^2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b-cos(f*x+e)*a^2-
cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+
2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*a^2*b^3)*(cos(f*x+e)-1)*(d/
cos(f*x+e))^(3/2)*cos(f*x+e)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}}{a+b\tan\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a), x)
```

3.606 $\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=324

$$-\frac{\sqrt{b}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{\sqrt{b}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}+\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+f}{}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[
d*Sec[e + f*x]])/((a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4))) - (Sqrt[b]*A
rcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f
*x]])/((a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (a*Cot[e + f*x]*Ellipt
icPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e
 + f*x]]*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) + (a
*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)],
 -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)*f*(Sec[e + f*
x]^2)^(1/4))
```

Rubi [A] time = 0.301219, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3512, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{\sqrt{b}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{\sqrt{b}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}+\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[
d*Sec[e + f*x]])/((a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4))) - (Sqrt[b]*A
rcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f
*x]])/((a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (a*Cot[e + f*x]*Ellipt
icPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e
 + f*x]]*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) + (a
*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)],
 -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)*f*(Sec[e + f*
x]^2)^(1/4))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{x}{\left(a^2-x^2\right)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(a\sqrt{d\sec(e+fx)}\right)\text{Subst}}{bj}$$

$$= -\frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{\left(a^2-x\right)\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx, x, b^2\tan^2(e+fx)\right)}{2bf\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(a\cot(e+fx)\sqrt{d\sec(e\right.}{}$$

$$= -\frac{\left(2b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(2a\cot(e+fx)\sqrt{d\sec(e\right.}{}$$

$$= -\frac{\left(b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{\sqrt{a^2+b^2}\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(b\sqrt{d\sec(e+fx)}\right)\text{Subst}}{\sqrt{a^2+l}}$$

$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} + \cdots$$

$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} + \cdots$$

Mathematica [C] time = 24.0671, size = 4648, normalized size = 14.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[d*Sec[e + f*x]]*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^(3/2)*(-1 + Tan[(e + f*x)/2]^2)*(EllipticF[ArcSin[Tan
[(e + f*x)/2]], -1] + (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 -
I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])
*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^
2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a +
I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(
```

```
a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sq
rt[2]], 2])*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e +
f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2])^2)/(Sqrt[2]*(a - I*b)*Sqrt[a
^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])))/(a*f*Sqrt[Sec[(e + f*x)/
2]^2]*(a + b*Tan[e + f*x])*((-2*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[e + f*x]*
Sec[(e + f*x)/2]^4]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2)*Tan[(e + f*x)/2
]*(EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] + (((-2*I)*b*Sqrt[a^2 + b^2]*Ell
ipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a -
 I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))
/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/
Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b
 + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e +
f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt
[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2])^2)/(S
qrt[2]*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])))/a
 + (Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])
^(3/2)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)*(EllipticF[ArcSin[Tan[(e
+ f*x)/2]], -1] + (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Co
s[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*Ell
ipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]),
 ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b
+ Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a +
b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2
]], 2])*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x]
 + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2])^2)/(Sqrt[2]*(a - I*b)*Sqrt[a^2 +
 b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])))/(a*Sqrt[Sec[(e + f*x)/2]^2])
 - ((Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2)*(-(Sec[(e + f*x)/2]^4*Sin[e + f
*x]) + 2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2])*(-1 + Tan[(e + f
*x)/2]^2)*(EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] + (((-2*I)*b*Sqrt[a^2 +
b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]
+ a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 +
 b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e
+ f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(
a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*
Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sqrt[I*Cos[e + f*x] - Sin[e + f*
x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2
])^2)/(Sqrt[2]*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]
^4])))/(a*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]) -
 (2*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])
^(3/2)*(-1 + Tan[(e + f*x)/2]^2)*((((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcS
in[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt
[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - S
qrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2
] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2
 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[
e + f*x]]/Sqrt[2]], 2])*Sec[(e + f*x)/2]^2*Sqrt[I*Cos[e + f*x] - Sin[e + f*
x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2
]))/(Sqrt[2]*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4
]) + (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] +
Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 +
 I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[
1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 +
b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2
+ b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*(-Cos
[e + f*x] - I*Sin[e + f*x])*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x
])]*(I + Tan[(e + f*x)/2])^2)/(2*Sqrt[2]*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos
[e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]) + (((-2*
I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x
]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*
```

```
(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e
 + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*Ellip
ticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), Ar
cSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sqrt[I*Cos[e + f
*x] - Sin[e + f*x]]*(Cos[e + f*x]*(I*Cos[e + f*x] - Sin[e + f*x]) - (Cos[e
+ f*x] + I*Sin[e + f*x])*Sin[e + f*x])*(I + Tan[(e + f*x)/2])^2)/(2*Sqrt[2]
*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[Cos[e
 + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]) + (Sqrt[I*Cos[e + f*x] - Sin[e +
f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(((-I)*b*Sqrt[a^2
+ b^2]*(Cos[e + f*x] + I*Sin[e + f*x]))/(Sqrt[2]*Sqrt[1 + (-1 + I*Cos[e + f
*x] - Sin[e + f*x])/2]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[1 - I*Cos[e
 + f*x] + Sin[e + f*x]]) + (a*(a - I*b + Sqrt[a^2 + b^2])*(Cos[e + f*x] + I
*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[1 + (-1 + I*Cos[e + f*x] - Sin[e + f*x])/2]
*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]
]*(1 - ((1/2 + I/2)*(a + I*(-b + Sqrt[a^2 + b^2]))*(1 - I*Cos[e + f*x] + Si
n[e + f*x]))/(a + b - Sqrt[a^2 + b^2]))) + (a*(-a + I*b + Sqrt[a^2 + b^2])*
(Cos[e + f*x] + I*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[1 + (-1 + I*Cos[e + f*x] -
 Sin[e + f*x])/2]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[1 - I*Cos[e + f*
x] + Sin[e + f*x]]*(1 - ((1/2 + I/2)*(a - I*(b + Sqrt[a^2 + b^2]))*(1 - I*C
os[e + f*x] + Sin[e + f*x]))/(a + b + Sqrt[a^2 + b^2]))))*(I + Tan[(e + f*x
)/2])^2)/(Sqrt[2]*(a - I*b)*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)
/2]^4]) - (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*
x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[
((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[
Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a
^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt
[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*
Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin
[e + f*x])]*(I + Tan[(e + f*x)/2])^2*(-(Sec[(e + f*x)/2]^4*Sin[e + f*x]) +
2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2]))/(2*Sqrt[2]*(a - I*b)*S
qrt[a^2 + b^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^4)^(3/2)) + Sec[(e + f*x)/2]^
2/(2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e + f*x)/2]^2])))/(a*Sqrt[S
ec[(e + f*x)/2]^2]) - (3*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[Cos[(e
+ f*x)/2]^2*Sec[e + f*x]]*(-1 + Tan[(e + f*x)/2]^2)*(EllipticF[ArcSin[Tan[(
e + f*x)/2]], -1] + (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*
Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*E
llipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]
), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*
b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a
+ b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt
[2]], 2])*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*
x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2])^2)/(Sqrt[2]*(a - I*b)*Sqrt[a^2
 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]))*(-(Cos[(e + f*x)/2]*Sec[e +
 f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(a
*Sqrt[Sec[(e + f*x)/2]^2])))
```

Maple [B] time = 0.357, size = 3130, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] -1/2/f/(a^2+b^2)/a^2/(-b+(a^2+b^2)^(1/2))/(b+(a^2+b^2)^(1/2))/(b*((a^2+b^2)
^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)/(-b*((a^2+b^2)^(
1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(d/cos(f*x+e))^(1/
```

2)*(cos(f*x+e)+1)^2*(cos(f*x+e)-1)*(4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)-1)/sin(f*x+e),I)*(b*((a^2+b
^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2
)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*a^5+4*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cos(f*x+e)
-1)/sin(f*x+e),I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b
^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3
)/a^4)^(1/2)*a^3*b^2+4*I*b*a^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticPi(I*(cos(f*x+e)-1)/sin(f*x+e),-1/(-b+(a^2+b^2)^(1/2
))^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^
4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)
^(1/2)*(a^2+b^2)^(1/2)-4*I*b*a^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticPi(I*(cos(f*x+e)-1)/sin(f*x+e),-1/(b+(a^2+b^2)^(1/
2))^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a
^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4
)^(1/2)*(a^2+b^2)^(1/2)+(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2)*(b*((a^2
+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*(-b*((a^2+b
^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*a^4*b-(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*cos(f*x
+e)-1)/sin(f*x+e)^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+
2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*
b^3)/a^4)^(1/2)*a^4*b+(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2-cos(f*x+e)^2-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)+2*cos(f*x+e)-1)/sin(f*x+e)^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(
a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^
2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*a^4*b-(a^2+b^2)^(3/2)*arctanh(1/
2/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)/
a^2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b-cos(f*x+e)*a^2-cos(f*x+e)*
b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)
^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*b^2+(a^2+b^2)^(1/2)*arctanh(1/2/(-b*((
a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)/a^2*(cos
(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b-cos(f*x+e)*a^2-cos(f*x+e)*b^2-b*(a
^2+b^2)^(1/2)+b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b
^2+2*b*a^2+2*b^3)/a^4)^(1/2)*b^4+arctanh(1/2/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^
2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)/a^2*(cos(f*x+e)-1)*((a^2+b^2)^(1
/2)*cos(f*x+e)*b-cos(f*x+e)*a^2-cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f
*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(
1/2)*a^4*b+arctanh(1/2/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a
^2-2*b^3)/a^4)^(1/2)/a^2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b-cos(f
*x+e)*a^2-cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)+b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(b*((a^2+b^2)^
(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)*a^2*b^3-(a^2+b^2)
^(3/2)*arctanh(1/2/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*
b^3)/a^4)^(1/2)/a^2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b+cos(f*x+e)
*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f
x+e)+1)^2)^(1/2))(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2
b^3)/a^4)^(1/2)(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*b^2+(a^2+b^2)^(1/2)*a
rctanh(1/2/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4
)^(1/2)/a^2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b+cos(f*x+e)*a^2+cos
(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2))*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^
4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*b^4-arctanh(1/2/(b*((a^2+b^2)
^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)/a^2*(cos(f*x+e)-
1)*((a^2+b^2)^(1/2)*cos(f*x+e)*b+cos(f*x+e)*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^
(1/2)-b^2)/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-b*((a^2+b^2

```
)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*a^4*b-arctanh(1/2/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)
^(1/2)*b^2+2*b*a^2+2*b^3)/a^4)^(1/2)/a^2*(cos(f*x+e)-1)*((a^2+b^2)^(1/2)*co
s(f*x+e)*b+cos(f*x+e)*a^2+cos(f*x+e)*b^2-b*(a^2+b^2)^(1/2)-b^2)/sin(f*x+e)^
2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2
)^(1/2)*b^2-2*b*a^2-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*
a^2*b^3)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sec \left(fx+e\right)}}{b \tan \left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sec \left(e+fx\right)}}{a+b \tan \left(e+fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sec \left(fx+e\right)}}{b \tan \left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a), x)
```

3.607 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

Optimal. Leaf size=451

$$\frac{b^{3/2}\sqrt[4]{\sec^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{5/4}\sqrt{d\sec(e+fx)}}-\frac{b^{3/2}\sqrt[4]{\sec^2(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{5/4}\sqrt{d\sec(e+fx)}}-\frac{2a\tan(e+fx)}{f\left(a^2+b^2\right)\sqrt{d\sec(e+fx)}}+\cdot$$

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e
+ f*x]^2)^(1/4))/((a^2 + b^2)^(5/4)*f*Sqrt[d*Sec[e + f*x]]) - (b^(3/2)*ArcT
anh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1
/4))/((a^2 + b^2)^(5/4)*f*Sqrt[d*Sec[e + f*x]]) + (2*a*EllipticE[ArcTan[Tan
[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/((a^2 + b^2)*f*Sqrt[d*Sec[e + f*x]
]) - (2*a*Tan[e + f*x])/((a^2 + b^2)*f*Sqrt[d*Sec[e + f*x]]) - (a*b*Cot[e +
 f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*
(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(3/2)*f*Sqrt[d*S
ec[e + f*x]]) + (a*b*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec
[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^
2 + b^2)^(3/2)*f*Sqrt[d*Sec[e + f*x]]) + (2*(b + a*Tan[e + f*x]))/((a^2 + b
^2)*f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.422755, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3512, 741, 844, 227, 196, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{b^{3/2}\sqrt[4]{\sec^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{5/4}\sqrt{d\sec(e+fx)}}-\frac{b^{3/2}\sqrt[4]{\sec^2(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{5/4}\sqrt{d\sec(e+fx)}}-\frac{2a\tan(e+fx)}{f\left(a^2+b^2\right)\sqrt{d\sec(e+fx)}}+\cdot$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])),x]
```


```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e
+ f*x]^2)^(1/4))/((a^2 + b^2)^(5/4)*f*Sqrt[d*Sec[e + f*x]]) - (b^(3/2)*ArcT
anh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1
/4))/((a^2 + b^2)^(5/4)*f*Sqrt[d*Sec[e + f*x]]) + (2*a*EllipticE[ArcTan[Tan
[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/((a^2 + b^2)*f*Sqrt[d*Sec[e + f*x]
]) - (2*a*Tan[e + f*x])/((a^2 + b^2)*f*Sqrt[d*Sec[e + f*x]]) - (a*b*Cot[e +
 f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*
(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(3/2)*f*Sqrt[d*S
ec[e + f*x]]) + (a*b*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec
[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^
2 + b^2)^(3/2)*f*Sqrt[d*Sec[e + f*x]]) + (2*(b + a*Tan[e + f*x]))/((a^2 + b
^2)*f*Sqrt[d*Sec[e + f*x]])
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$= \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-1+\frac{a^2}{b^2}\right)+\frac{ax}{2b^2}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x,\right.}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{\left(a\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e\right.}{b\,(a^2+b^2)\,f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2a\tan(e+fx)}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} + \frac{\left(a\sqrt[4]{\sec^2(e+fx)}\right)}{b(}$$

$$= \frac{2aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{2a\tan(e+fx)}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} +$$

$$= \frac{2aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{2a\tan(e+fx)}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} +$$

$$= \frac{2aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{2a\tan(e+fx)}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} +$$

$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{5/4}\,f\sqrt{d\sec(e+fx)}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2}}{(a^2+b^2)^{5/4}\,f\sqrt{d\sec(e+fx}}$$

Mathematica [C] time = 30.3493, size = 4693, normalized size = 10.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])),x]
```

```
[Out] (-2*Sec[e + f*x]^(3/2)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2)*(a*Cos[e + f
*x] + b*Sin[e + f*x])*(Sqrt[Sec[e + f*x]]/(2*(a*Cos[e + f*x] + b*Sin[e + f*
x])) + (Cos[2*(e + f*x)]*Sqrt[Sec[e + f*x]])/(2*(a*Cos[e + f*x] + b*Sin[e +
 f*x])))*(-1 + Tan[(e + f*x)/2]^2)^2*(-((a*Sqrt[(1 + Cos[e + f*x])^(-1)]*El
lipticE[ArcSin[Tan[(e + f*x)/2]], -1])/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])
]) + ((a^2 + b^2)*Sqrt[(1 + Cos[e + f*x])^(-1)]*EllipticF[ArcSin[Tan[(e + f
*x)/2]], -1])/(a*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]) + (-2*Sqrt[2]*b^2*S
qrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqr
t[2]], 2]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]] + a*(b*(a + I*b + Sqrt[a^2 +
b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2
 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*Sqrt[
```

```
(2*I)*Cos[e + f*x] - 2*Sin[e + f*x]] + b*(-a - I*b + Sqrt[a^2 + b^2])*Ellip
ticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), Ar
cSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*Sqrt[(2*I)*Cos[e
+ f*x] - 2*Sin[e + f*x]] - 2*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*(Cos[e + f*x
] + I*Sin[e + f*x])]*(b + a*Tan[(e + f*x)/2])))/(2*a*Sqrt[a^2 + b^2]*Sqrt[C
os[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]))))/((a^2 + b^2)*f*Sqrt[Sec[(e
+ f*x)/2]^2]*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])*((-4*Sqrt[Sec[(e + f
*x)/2]^2]*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2)*Tan[(e + f*x)/2]*(-1 + Ta
n[(e + f*x)/2]^2)*(-((a*Sqrt[(1 + Cos[e + f*x])^(-1)]*EllipticE[ArcSin[Tan[
(e + f*x)/2]], -1])/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]) + ((a^2 + b^2)*S
qrt[(1 + Cos[e + f*x])^(-1)]*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1])/(a*Sq
rt[Cos[e + f*x]/(1 + Cos[e + f*x])]) + (-2*Sqrt[2]*b^2*Sqrt[a^2 + b^2]*Elli
pticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*Sqrt[I*Co
s[e + f*x] - Sin[e + f*x]] + a*(b*(a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((
1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sq
rt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*Sqrt[(2*I)*Cos[e + f*x]
- 2*Sin[e + f*x]] + b*(-a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a -
 I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos
[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*Sqrt[(2*I)*Cos[e + f*x] - 2*Sin[e +
f*x]] - 2*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])
]*(b + a*Tan[(e + f*x)/2])))/(2*a*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*(Cos[e
+ f*x] + I*Sin[e + f*x])]))))/(a^2 + b^2) + ((Cos[(e + f*x)/2]^2*Sec[e + f*x
])^(3/2)*Tan[(e + f*x)/2]*(-1 + Tan[(e + f*x)/2]^2)^2*(-((a*Sqrt[(1 + Cos[e
 + f*x])^(-1)]*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1])/Sqrt[Cos[e + f*x]/(
1 + Cos[e + f*x])]) + ((a^2 + b^2)*Sqrt[(1 + Cos[e + f*x])^(-1)]*EllipticF[
ArcSin[Tan[(e + f*x)/2]], -1])/(a*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]) +
(-2*Sqrt[2]*b^2*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] +
Sin[e + f*x]]/Sqrt[2]], 2]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]] + a*(b*(a +
I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/
(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/S
qrt[2]], 2]*Sqrt[(2*I)*Cos[e + f*x] - 2*Sin[e + f*x]] + b*(-a - I*b + Sqrt[
a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqr
t[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*
Sqrt[(2*I)*Cos[e + f*x] - 2*Sin[e + f*x]] - 2*Sqrt[a^2 + b^2]*Sqrt[Cos[e +
f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(b + a*Tan[(e + f*x)/2])))/(2*a*Sqrt[
a^2 + b^2]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]))))/((a^2 + b^
2)*Sqrt[Sec[(e + f*x)/2]^2]) - (2*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(3/2)*(
-1 + Tan[(e + f*x)/2]^2)^2*(-(a*((1 + Cos[e + f*x])^(-1))^(3/2)*EllipticE[A
rcSin[Tan[(e + f*x)/2]], -1]*Sin[e + f*x])/(2*Sqrt[Cos[e + f*x]/(1 + Cos[e
+ f*x])]) + ((a^2 + b^2)*((1 + Cos[e + f*x])^(-1))^(3/2)*EllipticF[ArcSin[T
an[(e + f*x)/2]], -1]*Sin[e + f*x])/(2*a*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x
])]) + (a*Sqrt[(1 + Cos[e + f*x])^(-1)]*EllipticE[ArcSin[Tan[(e + f*x)/2]],
 -1]*((Cos[e + f*x]*Sin[e + f*x])/(1 + Cos[e + f*x])^2 - Sin[e + f*x]/(1 +
Cos[e + f*x])))/(2*(Cos[e + f*x]/(1 + Cos[e + f*x]))^(3/2)) - ((a^2 + b^2)*
Sqrt[(1 + Cos[e + f*x])^(-1)]*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1]*((Cos
[e + f*x]*Sin[e + f*x])/(1 + Cos[e + f*x])^2 - Sin[e + f*x]/(1 + Cos[e + f*
x])))/(2*a*(Cos[e + f*x]/(1 + Cos[e + f*x]))^(3/2)) + ((a^2 + b^2)*Sqrt[(1
+ Cos[e + f*x])^(-1)]*Sec[(e + f*x)/2]^2)/(2*a*Sqrt[Cos[e + f*x]/(1 + Cos[e
 + f*x])]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e + f*x)/2]^2]) - (a*S
qrt[(1 + Cos[e + f*x])^(-1)]*Sec[(e + f*x)/2]^2*Sqrt[1 + Tan[(e + f*x)/2]^2
])/(2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[1 - Tan[(e + f*x)/2]^2]) -
 ((Cos[e + f*x]*(I*Cos[e + f*x] - Sin[e + f*x]) - (Cos[e + f*x] + I*Sin[e +
 f*x])*Sin[e + f*x])*(-2*Sqrt[2]*b^2*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[
1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*Sqrt[I*Cos[e + f*x] - Sin[e
 + f*x]] + a*(b*(a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b
+ Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f
*x] + Sin[e + f*x]]/Sqrt[2]], 2]*Sqrt[(2*I)*Cos[e + f*x] - 2*Sin[e + f*x]]
+ b*(-a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 +
 b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e
```

```
+ f*x]]/Sqrt[2]], 2]*Sqrt[(2*I)*Cos[e + f*x] - 2*Sin[e + f*x]] - 2*Sqrt[a^2
 + b^2]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(b + a*Tan[(e +
f*x)/2]))))/(4*a*Sqrt[a^2 + b^2]*(Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*
x]))^(3/2)) + (-((Sqrt[2]*b^2*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*C
os[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*(-Cos[e + f*x] - I*Sin[e + f*x]))/
Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]) - (b^2*Sqrt[a^2 + b^2]*(Cos[e + f*x] +
 I*Sin[e + f*x]))/(Sqrt[1 + (-1 + I*Cos[e + f*x] - Sin[e + f*x])/2]*Sqrt[1
- I*Cos[e + f*x] + Sin[e + f*x]]) + a*(-(a*Sqrt[a^2 + b^2]*Sec[(e + f*x)/2]
^2*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]) + (b*(a + I*b + Sqrt
[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - S
qrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2
]*(-2*Cos[e + f*x] - (2*I)*Sin[e + f*x]))/(2*Sqrt[(2*I)*Cos[e + f*x] - 2*Si
n[e + f*x]]) + (b*(-a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(
b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e +
 f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*(-2*Cos[e + f*x] - (2*I)*Sin[e + f*x]))/
(2*Sqrt[(2*I)*Cos[e + f*x] - 2*Sin[e + f*x]]) + (b*(a + I*b + Sqrt[a^2 + b^
2])*Sqrt[(2*I)*Cos[e + f*x] - 2*Sin[e + f*x]]*(Cos[e + f*x] + I*Sin[e + f*x
]))/(2*Sqrt[2]*Sqrt[1 + (-1 + I*Cos[e + f*x] - Sin[e + f*x])/2]*Sqrt[I*Cos[
e + f*x] - Sin[e + f*x]]*Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]*(1 - ((1/2
 + I/2)*(a + I*(-b + Sqrt[a^2 + b^2]))*(1 - I*Cos[e + f*x] + Sin[e + f*x]))
/(a + b - Sqrt[a^2 + b^2]))) + (b*(-a - I*b + Sqrt[a^2 + b^2])*Sqrt[(2*I)*C
os[e + f*x] - 2*Sin[e + f*x]]*(Cos[e + f*x] + I*Sin[e + f*x]))/(2*Sqrt[2]*S
qrt[1 + (-1 + I*Cos[e + f*x] - Sin[e + f*x])/2]*Sqrt[I*Cos[e + f*x] - Sin[e
 + f*x]]*Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]*(1 - ((1/2 + I/2)*(a - I*(
b + Sqrt[a^2 + b^2]))*(1 - I*Cos[e + f*x] + Sin[e + f*x]))/(a + b + Sqrt[a^
2 + b^2]))) - (Sqrt[a^2 + b^2]*(Cos[e + f*x]*(I*Cos[e + f*x] - Sin[e + f*x]
) - (Cos[e + f*x] + I*Sin[e + f*x])*Sin[e + f*x])*(b + a*Tan[(e + f*x)/2]))
/Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]))/(2*a*Sqrt[a^2 + b^2]*
Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])])))/((a^2 + b^2)*Sqrt[Sec
[(e + f*x)/2]^2]) - (3*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*(-1 + Tan[(e +
 f*x)/2]^2)^2*(-((a*Sqrt[(1 + Cos[e + f*x])^(-1)]*EllipticE[ArcSin[Tan[(e +
 f*x)/2]], -1])/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]) + ((a^2 + b^2)*Sqrt[
(1 + Cos[e + f*x])^(-1)]*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1])/(a*Sqrt[C
os[e + f*x]/(1 + Cos[e + f*x])]) + (-2*Sqrt[2]*b^2*Sqrt[a^2 + b^2]*Elliptic
F[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*Sqrt[I*Cos[e
+ f*x] - Sin[e + f*x]] + a*(b*(a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 +
I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1
 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*Sqrt[(2*I)*Cos[e + f*x] - 2*
Sin[e + f*x]] + b*(-a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(
b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e +
 f*x] + Sin[e + f*x]]/Sqrt[2]], 2]*Sqrt[(2*I)*Cos[e + f*x] - 2*Sin[e + f*x]
] - 2*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(b
 + a*Tan[(e + f*x)/2])))/(2*a*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*(Cos[e + f*
x] + I*Sin[e + f*x])]))*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2])
+ Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/((a^2 + b^2)*Sqrt[Sec[(e +
 f*x)/2]^2])))
```

Maple [B] time = 0.433, size = 8821, normalized size = 19.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)), x)
```

3.608 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

Optimal. Leaf size=422

$$-\frac{b^{5/2}\sec^2(e+fx)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{7/4}(d\sec(e+fx))^{3/2}}-\frac{b^{5/2}\sec^2(e+fx)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{7/4}(d\sec(e+fx))^{3/2}}+\frac{2(a\tan(e+fx)+b)}{3f\left(a^2+b^2\right)(d\sec(e+fx))^{3/2}}$$

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[
e + f*x]^2)^(3/4))/((a^2 + b^2)^(7/4)*f*(d*Sec[e + f*x])^(3/2))) - (b^(5/2)
*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^
2)^(3/4))/((a^2 + b^2)^(7/4)*f*(d*Sec[e + f*x])^(3/2)) + (2*a*EllipticF[Arc
Tan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3/4))/(3*(a^2 + b^2)*f*(d*Sec[e +
 f*x])^(3/2)) + (a*b^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin
[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])
/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (a*b^2*Cot[e + f*x]*EllipticPi[
b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/
4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (2*(b
+ a*Tan[e + f*x]))/(3*(a^2 + b^2)*f*(d*Sec[e + f*x])^(3/2))
```

Rubi [A] time = 0.431916, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3512, 741, 844, 231, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{b^{5/2}\sec^2(e+fx)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{7/4}(d\sec(e+fx))^{3/2}}-\frac{b^{5/2}\sec^2(e+fx)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{7/4}(d\sec(e+fx))^{3/2}}+\frac{2(a\tan(e+fx)+b)}{3f\left(a^2+b^2\right)(d\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])),x]
```

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[
e + f*x]^2)^(3/4))/((a^2 + b^2)^(7/4)*f*(d*Sec[e + f*x])^(3/2))) - (b^(5/2)
*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^
2)^(3/4))/((a^2 + b^2)^(7/4)*f*(d*Sec[e + f*x])^(3/2)) + (2*a*EllipticF[Arc
Tan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3/4))/(3*(a^2 + b^2)*f*(d*Sec[e +
 f*x])^(3/2)) + (a*b^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin
[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])
/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (a*b^2*Cot[e + f*x]*EllipticPi[
b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/
4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (2*(b
+ a*Tan[e + f*x]))/(3*(a^2 + b^2)*f*(d*Sec[e + f*x])^(3/2))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \frac{\sec^2(e+fx)^{3/4}\,\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$= \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} - \frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3-\frac{a^2}{b^2}\right)}{(a+x)\left(1+\frac{x^2}{b^2}\right)}\right.}{3\left(a^2+b^2\right)f(d\sec(e+}$$

$$= \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} + \frac{\left(a\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\right.}{3b\left(a^2+b^2\right)f(d\sec(e+}$$

$$= \frac{2aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} + \frac{2(b+a\tan(e+fx}{3\left(a^2+b^2\right)f(d\sec(e+}$$

$$= \frac{2aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} + \frac{2(b+a\tan(e+fx}{3\left(a^2+b^2\right)f(d\sec(e+}$$

$$= \frac{2aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} + \frac{2(b+a\tan(e+fx}{3\left(a^2+b^2\right)f(d\sec(e+}$$

$$= \frac{2aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} + \frac{2(b+a\tan(e+fx}{3\left(a^2+b^2\right)f(d\sec(e+}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{\left(a^2+b^2\right)^{7/4}f(d\sec(e+fx))^{3/2}} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/4}f(d\sec(}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{\left(a^2+b^2\right)^{7/4}f(d\sec(e+fx))^{3/2}} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/4}f(d\sec(}$$

Mathematica [C] time = 26.5739, size = 9313, normalized size = 22.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.313, size = 6252, normalized size = 14.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{3}{2}}\left(a+b\tan\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)), x)
```

3.609 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

Optimal. Leaf size=568

$$\frac{b^{7/2}\sqrt[4]{\sec^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{d^2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}-\frac{b^{7/2}\sqrt[4]{\sec^2(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{d^2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}-\frac{2a\left(3a^2+8b^2\right)\tan(e+fx)}{5d^2f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}}+$$

```
[Out] (b^(7/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e
+ f*x]^2)^(1/4))/((a^2 + b^2)^(9/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) - (b^(7/2)*
ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2
)^(1/4))/((a^2 + b^2)^(9/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*a*(3*a^2 + 8*b
^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/(5*(a^2 +
b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]) - (2*a*(3*a^2 + 8*b^2)*Tan[e + f*x])/(5*
(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]) - (a*b^3*Cot[e + f*x]*EllipticPi[
-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^
(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(5/2)*d^2*f*Sqrt[d*Sec[e + f*x]])
 + (a*b^3*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2
)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(5
/2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*Cos[e + f*x]^2*(b + a*Tan[e + f*x]))/(
5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*(5*b^3 + a*(3*a^2 + 8*b^2)*T
an[e + f*x]))/(5*(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rubi [A] time = 0.6085, antiderivative size = 568, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3512, 741, 823, 844, 227, 196, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{b^{7/2}\sqrt[4]{\sec^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{d^2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}-\frac{b^{7/2}\sqrt[4]{\sec^2(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{d^2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}-\frac{2a\left(3a^2+8b^2\right)\tan(e+fx)}{5d^2f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])),x]
```


```
[Out] (b^(7/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e
+ f*x]^2)^(1/4))/((a^2 + b^2)^(9/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) - (b^(7/2)*
ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2
)^(1/4))/((a^2 + b^2)^(9/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*a*(3*a^2 + 8*b
^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/(5*(a^2 +
b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]) - (2*a*(3*a^2 + 8*b^2)*Tan[e + f*x])/(5*
(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]) - (a*b^3*Cot[e + f*x]*EllipticPi[
-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^
(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(5/2)*d^2*f*Sqrt[d*Sec[e + f*x]])
 + (a*b^3*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2
)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(5
/2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*Cos[e + f*x]^2*(b + a*Tan[e + f*x]))/(
5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*(5*b^3 + a*(3*a^2 + 8*b^2)*T
an[e + f*x]))/(5*(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]])
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
```

```
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5-\right.}{(a+x)\left(\right.}\right.}{5\left(a^2+b^2\right)d^2f\sqrt{d}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}} + \frac{2\left(5b^3+a\left(3a^2+8b^2\right)\tan(e+fx)\right)}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}} + \frac{2\left(5b^3+a\left(3a^2+8b^2\right)\tan(e+fx)\right)}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2a\left(3a^2+8b^2\right)\tan(e+fx)}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}} + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}} +$$

$$= \frac{2a\left(3a^2+8b^2\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}} - \frac{2a\left(3a^2+\right.}{5\left(a^2+b^2\right)}$$

$$= \frac{2a\left(3a^2+8b^2\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}} - \frac{2a\left(3a^2+\right.}{5\left(a^2+b^2\right)}$$

$$= \frac{2a\left(3a^2+8b^2\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}} - \frac{2a\left(3a^2+\right.}{5\left(a^2+b^2\right)}$$

$$= \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^{9/4}d^2f\sqrt{d\sec(e+fx)}} - \frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{9/4}d^2f\sqrt{d\sec}}$$

Mathematica [C] time = 32.6737, size = 17838, normalized size = 31.4

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.536, size = 14547, normalized size = 25.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d \sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b \tan\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d \sec\left(e+fx\right)\right)^{\frac{5}{2}}\left(a+b \tan\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d \sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b \tan\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="giac")

[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)), x)
```

3.610 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=480

$$-\frac{3ad^2(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{5/2}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}+\frac{3ad^2(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{5/2}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}+\frac{3a^2d^2\sqrt{-\tan^2(e+fx)}\ \cdots}{\cdots}$$

```
[Out] (-3*a*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec
[e + f*x])^(3/2))/(2*b^(5/2)*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) +
(3*a*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec
[e + f*x])^(3/2))/(2*b^(5/2)*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) -
(3*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(3/2))/(b^2*f*
(Sec[e + f*x]^2)^(3/4)) + (3*d^2*Cos[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e
+ f*x])/(b^2*f) + (3*a^2*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]),
ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*
x]^2])/(2*b^3*Sqrt[a^2 + b^2]*f*(Sec[e + f*x]^2)^(3/4)) - (3*a^2*d^2*Cot[e
+ f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d
*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(2*b^3*Sqrt[a^2 + b^2]*f*(Sec[e
 + f*x]^2)^(3/4)) - (d^2*(d*Sec[e + f*x])^(3/2))/(b*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.382219, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3512, 733, 844, 227, 196, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$-\frac{3ad^2(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{5/2}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}+\frac{3ad^2(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{5/2}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}+\frac{3a^2d^2\sqrt{-\tan^2(e+fx)}\ \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-3*a*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec
[e + f*x])^(3/2))/(2*b^(5/2)*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) +
(3*a*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec
[e + f*x])^(3/2))/(2*b^(5/2)*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) -
(3*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(3/2))/(b^2*f*
(Sec[e + f*x]^2)^(3/4)) + (3*d^2*Cos[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e
+ f*x])/(b^2*f) + (3*a^2*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]),
ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*
x]^2])/(2*b^3*Sqrt[a^2 + b^2]*f*(Sec[e + f*x]^2)^(3/4)) - (3*a^2*d^2*Cot[e
+ f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d
*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(2*b^3*Sqrt[a^2 + b^2]*f*(Sec[e
 + f*x]^2)^(3/4)) - (d^2*(d*Sec[e + f*x])^(3/2))/(b*f*(a + b*Tan[e + f*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
```

```
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/4}}{(a+x)^2}\,dx,x,b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))} + \frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+f\right.}{2b^3f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))} + \frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{2b^3f\sec^2(e+fx)^{3/4}}$$

$$= \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f} - \frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))} - \frac{\left(3d^2(d\sec(e+\right.}{}$$

$$= -\frac{3d^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} + \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\text{s}}{b^2f}$$

$$= -\frac{3d^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} + \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\text{s}}{b^2f}$$

$$= -\frac{3d^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}} + \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\text{s}}{b^2f}$$

$$= -\frac{3ad^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2b^{5/2}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} + \frac{3ad^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+}{2b^{5/2}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}}$$

Mathematica [C] time = 22.1452, size = 1129, normalized size = 2.35

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*(d*Sec[e + f*x])^(7/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((
3*Cos[e + f*x])/(a*b) + (3*Sin[e + f*x])/b^2 - 1/(b*(a*Cos[e + f*x] + b*Sin
[e + f*x]))))/(f*(a + b*Tan[e + f*x])^2) + (3*(d*Sec[e + f*x])^(7/2)*(a*Cos
[e + f*x] + b*Sin[e + f*x])^2*(-((a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]
*Sqrt[1 + Tan[(e + f*x)/2]^2])/Sqrt[1 - Tan[(e + f*x)/2]^2]) + (2*a*Ellipti
cF[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 + Tan[(e + f*x)/2]^2])/Sqrt[1 - Tan
[(e + f*x)/2]^2] + (-2*Sqrt[2]*a*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[((
1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-(
(1 + I*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] + Sqrt[2]*a^2*Sqrt[a^2 +
b^2]*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 +
 b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])
]/Sqrt[2]], 2]*Sqrt[-((1 + I*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] + a
^2*(a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 +
b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/
```

```
2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((2 + (2*I)*Tan[(e + f*x)/2
])/(I + Tan[(e + f*x)/2]))] - a^3*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2
+ b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x
)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((2 + (2*I)*Tan[(e + f*x)
/2])/(I + Tan[(e + f*x)/2]))] - I*a^2*b*EllipticPi[((1 + I)*(a - I*(b + Sqr
t[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e
 + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((2 + (2*I)*Tan[(e
+ f*x)/2])/(I + Tan[(e + f*x)/2]))] - 2*b^2*Sqrt[a^2 + b^2]*Sqrt[(-1 + Tan[
(e + f*x)/2]^2)/(I + Tan[(e + f*x)/2])^2] - 2*a*b*Sqrt[a^2 + b^2]*Tan[(e +
f*x)/2]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)/(I + Tan[(e + f*x)/2])^2])/(2*b*Sqrt
[a^2 + b^2]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)/(I + Tan[(e + f*x)/2])^2])))/(a*
b^2*f*Sec[e + f*x]^(3/2)*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2
]^2)]*(a + b*Tan[e + f*x])^2)
```

Maple [B] time = 4.405, size = 44463, normalized size = 92.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{7}{2}}}{\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a)^2, x)
```

3.611 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=440

$$\frac{ad^2\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{3/2}f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}+\frac{ad^2\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{3/2}f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{a^2d^2\sqrt{-\tan^2(e+fx)}\cot(e+f\ldots}{2b\ldots}$$

```
[Out] (a*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Se
c[e + f*x]])/(2*b^(3/2)*(a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^
2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e
+ f*x]])/(2*b^(3/2)*(a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (d^2*Elli
pticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]])/(b^2*f*(Sec[e + f*x]
^2)^(1/4)) - (a^2*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[
(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(2
*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (a^2*d^2*Cot[e + f*x]*Elliptic
Pi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*
x]]*Sqrt[-Tan[e + f*x]^2])/(2*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (
d^2*Sqrt[d*Sec[e + f*x]])/(b*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.394753, antiderivative size = 440, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3512, 733, 844, 231, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$\frac{ad^2\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{3/2}f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}+\frac{ad^2\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{3/2}f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{a^2d^2\sqrt{-\tan^2(e+fx)}\cot(e+f\ldots}{2b\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```


```
[Out] (a*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Se
c[e + f*x]])/(2*b^(3/2)*(a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^
2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e
+ f*x]])/(2*b^(3/2)*(a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (d^2*Elli
pticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]])/(b^2*f*(Sec[e + f*x]
^2)^(1/4)) - (a^2*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[
(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(2
*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (a^2*d^2*Cot[e + f*x]*Elliptic
Pi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*
x]]*Sqrt[-Tan[e + f*x]^2])/(2*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (
d^2*Sqrt[d*Sec[e + f*x]])/(b*f*(a + b*Tan[e + f*x]))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
```

```
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x^2}{b^2}}}{(a+x)^2}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx\cdots\right.}{2b^3f\sqrt[4]{\sec^2(e+fx)}\cdots}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2b^3f\sqrt[4]{\sec^2(e+fx)}} - \cdots$$

$$= \frac{d^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \frac{\left(ad^2\sqrt{d\sec(e\cdots\right.}{\cdots}$$

$$= \frac{d^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \frac{\left(ad^2\sqrt{d\sec(e\cdots\right.}{\cdots}$$

$$= \frac{d^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \frac{\left(ad^2\sqrt{d\sec(e\cdots\right.}{\cdots}$$

$$= \frac{d^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \frac{\left(ad^2\sqrt{d\sec(e\cdots\right.}{\cdots}$$

$$= \frac{ad^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} + \cdots$$

$$= \frac{ad^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4}f\sqrt[4]{\sec^2(e+fx)}} + \cdots$$

Mathematica [C] time = 21.8024, size = 3091, normalized size = 7.02

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^(5/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(-(1/(a*b)) + S
in[e + f*x]/(a*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x])
^2) - (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] +
 Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1
+ I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt
[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 +
```

b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*(d*Sec[e + f*x])^(5/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*Sin[e + f*x]*(a*Cos[e + f*x] + b*Sin[e + f*x])*(I + Tan[(e + f*x)/2])^2)/(4*(a - I*b)*b^3*Sqrt[a^2 + b^2]*f*Sqrt[(1 + Cos[e + f*x])^(-1)]*(a + b*Tan[e + f*x])^2*(-(((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sec[(e + f*x)/2]^2*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2]))/(2*(a - I*b)*b^2*Sqrt[a^2 + b^2]*Sqrt[(1 + Cos[e + f*x])^(-1)]) - (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*(-Cos[e + f*x] - I*Sin[e + f*x])*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(I + Tan[(e + f*x)/2])^2)/(4*(a - I*b)*b^2*Sqrt[a^2 + b^2]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]) + (Sqrt[(1 + Cos[e + f*x])^(-1)]*((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*Sin[e + f*x]*(I + Tan[(e + f*x)/2])^2)/(4*(a - I*b)*b^2*Sqrt[a^2 + b^2]) - (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*(Cos[e + f*x]*(I*Cos[e + f*x] - Sin[e + f*x]) - (Cos[e + f*x] + I*Sin[e + f*x])*Sin[e + f*x])*(I + Tan[(e + f*x)/2])^2)/(4*(a - I*b)*b^2*Sqrt[a^2 + b^2]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]) - (Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Sin[e + f*x])]*(((-I)*b*Sqrt[a^2 + b^2]*(Cos[e + f*x] + I*Sin[e + f*x]))/(Sqrt[2]*Sqrt[1 + (-1 + I*Cos[e + f*x] - Sin[e + f*x])/2]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]) + (a*(a - I*b + Sqrt[a^2 + b^2])*(Cos[e + f*x] + I*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[1 + (-1 + I*Cos[e + f*x] - Sin[e + f*x])/2]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]*(1 - ((1/2 + I/2)*(a + I*(-b + Sqrt[a^2 + b^2]))*(1 - I*Cos[e + f*x] + Sin[e + f*x]))/(a + b - Sqrt[a^2 + b^2]))) + (a*(-a + I*b + Sqrt[a^2 + b^2])*(Cos[e + f*x] + I*Sin[e + f*x]))/(2*Sqrt[2]*Sqrt[1 + (-1 + I*Cos[e + f*x] - Sin[e + f*x])/2]*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]*(1 - ((1/2 + I/2)*(a - I*(b + Sqrt[a^2 + b^2]))*(1 - I*Cos[e + f*x] + Sin[e + f*x]))/(a + b + Sqrt[a^2 + b^2]))))*(I + Tan[(e + f*x)/2])^2)/(2*(a - I*b)*b^2*Sqrt[a^2 + b^2]*Sqrt[(1 + Cos[e + f*x])^(-1)]) - (((-2*I)*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(a - I*b + Sqrt[a^2 + b^2])*EllipticPi

```
[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin
[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2] + a*(-a + I*b + Sqrt[
a^2 + b^2])*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqr
t[a^2 + b^2]), ArcSin[Sqrt[1 - I*Cos[e + f*x] + Sin[e + f*x]]/Sqrt[2]], 2])
*Sqrt[I*Cos[e + f*x] - Sin[e + f*x]]*Sqrt[Cos[e + f*x]*(Cos[e + f*x] + I*Si
n[e + f*x])]*(I + Tan[(e + f*x)/2])^2*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[
(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(4*(a - I*b)
*b^2*Sqrt[a^2 + b^2]*Sqrt[(1 + Cos[e + f*x])^(-1)]*Sqrt[Cos[(e + f*x)/2]^2*
Sec[e + f*x]]))
```

Maple [B] time = 0.607, size = 5329, normalized size = 12.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^2, x)
```

3.612 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=477

$$\frac{a(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2\sqrt{b}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}-\frac{a(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2\sqrt{b}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}-\frac{b(d\sec(e+fx))^{3/2}}{f\left(a^2+b^2\right)(a+b\tan(e+fx}$$

```
[Out] (a*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*
x])^(3/2))/(2*Sqrt[b]*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2)^(3/4)) - (a*ArcT
anh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3
/2))/(2*Sqrt[b]*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2)^(3/4)) - (EllipticE[Ar
cTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(3/2))/((a^2 + b^2)*f*(Sec[e + f*
x]^2)^(3/4)) + (Cos[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/((a^2 + b
^2)*f) - (a^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e +
 f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(2*b*(a^
2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) + (a^2*Cot[e + f*x]*EllipticPi[b/S
qrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*
Sqrt[-Tan[e + f*x]^2])/(2*b*(a^2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) - (
b*(d*Sec[e + f*x])^(3/2))/((a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.385205, antiderivative size = 477, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3512, 745, 844, 227, 196, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{a(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2\sqrt{b}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}-\frac{a(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2\sqrt{b}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}-\frac{b(d\sec(e+fx))^{3/2}}{f\left(a^2+b^2\right)(a+b\tan(e+fx}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (a*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*
x])^(3/2))/(2*Sqrt[b]*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2)^(3/4)) - (a*ArcT
anh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3
/2))/(2*Sqrt[b]*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2)^(3/4)) - (EllipticE[Ar
cTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(3/2))/((a^2 + b^2)*f*(Sec[e + f*
x]^2)^(3/4)) + (Cos[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/((a^2 + b
^2)*f) - (a^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e +
 f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(2*b*(a^
2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) + (a^2*Cot[e + f*x]*EllipticPi[b/S
qrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*
Sqrt[-Tan[e + f*x]^2])/(2*b*(a^2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) - (
b*(d*Sec[e + f*x])^(3/2))/((a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{(d\sec(e+fx))^{3/2}\,\text{Subst}\left(\int \frac{1}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{(d\sec(e+fx))^{3/2}\,\text{Subst}\left(\int \frac{-a-\frac{x}{2}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+f\right.}{b\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \frac{(d\sec(e+fx))^{3/2}\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2b\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}}$$

$$= \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{\left(a^2+b^2\right)f} - \frac{b(d\sec(e+fx))^{3/2}}{\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{(d\sec(e+fx)}{2}$$

$$= -\frac{E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}} + \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{\left(a^2+b^2\right)f}$$

$$= -\frac{E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}} + \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{\left(a^2+b^2\right)f}$$

$$= -\frac{E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}} + \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{\left(a^2+b^2\right)f}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}} - \frac{E}{}$$

Mathematica [C] time = 29.786, size = 6560, normalized size = 13.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 1.04, size = 25422, normalized size = 53.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec \left(e + f x\right)\right)^{\frac{3}{2}}}{\left(a + b \tan \left(e + f x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec \left(f x + e\right)\right)^{\frac{3}{2}}}{\left(b \tan \left(f x + e\right) + a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a)^2, x)
```

3.613 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=430

$$-\frac{3a\sqrt{b}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{7/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{3a\sqrt{b}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{7/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{b\sqrt{d\sec(e+fx)}}{f\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

```
[Out] (-3*a*Sqrt[b]*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sq
rt[d*Sec[e + f*x]])/(2*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1/4)) - (3*a*S
qrt[b]*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*S
ec[e + f*x]])/(2*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1/4)) - (EllipticF[A
rcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]])/((a^2 + b^2)*f*(Sec[e + f*x
]^2)^(1/4)) + (3*a^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(
Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(2*
(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) + (3*a^2*Cot[e + f*x]*EllipticPi[b/
Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*S
qrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) - (b*Sqrt[
d*Sec[e + f*x]])/((a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.37999, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3512, 745, 844, 231, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{3a\sqrt{b}\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{7/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{3a\sqrt{b}\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{7/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{b\sqrt{d\sec(e+fx)}}{f\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-3*a*Sqrt[b]*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sq
rt[d*Sec[e + f*x]])/(2*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1/4)) - (3*a*S
qrt[b]*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*S
ec[e + f*x]])/(2*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1/4)) - (EllipticF[A
rcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]])/((a^2 + b^2)*f*(Sec[e + f*x
]^2)^(1/4)) + (3*a^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(
Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(2*
(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) + (3*a^2*Cot[e + f*x]*EllipticPi[b/
Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*S
qrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) - (b*Sqrt[
d*Sec[e + f*x]])/((a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
```

```
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{-a+\frac{x}{2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+\right.}{b\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right.}{2b\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\left(3a\sqrt{d}\right.}{}$$

$$= -\frac{F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\left(3a\sqrt{d}\right.}{}$$

$$= -\frac{F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\left(3ab\sqrt{\right.}}{}$$

$$= -\frac{F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\left(3ab\sqrt{\right.}}{}$$

$$= -\frac{3a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} - \frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+}}{2\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{3a\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} - \frac{3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+}}{2\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}$$

Mathematica [C] time = 26.1265, size = 8876, normalized size = 20.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.877, size = 14318, normalized size = 33.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(e+fx\right)}}{\left(a+b\tan\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d\sec\left(fx+e\right)}}{\left(b\tan\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")

[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^2, x)
```

3.614 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=555

$$\frac{5ab^{3/2}\sqrt[4]{\sec^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}+\frac{b\left(2a^2-3b^2\right)\sec^2(e+fx)}{f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}-\frac{5ab^{3/2}\sqrt[4]{\sec^2(e+fx)}\tanh}{2f\left(a^2+b^2\right)^{9/4}\sqrt{d}}$$

```
[Out] (5*a*b^(3/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Se
c[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(9/4)*f*Sqrt[d*Sec[e + f*x]]) - (5*a*b^
(3/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e +
f*x]^2)^(1/4))/(2*(a^2 + b^2)^(9/4)*f*Sqrt[d*Sec[e + f*x]]) + ((2*a^2 - 3*b
^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/((a^2 + b^
2)^2*f*Sqrt[d*Sec[e + f*x]]) - ((2*a^2 - 3*b^2)*Tan[e + f*x])/((a^2 + b^2)^
2*f*Sqrt[d*Sec[e + f*x]]) - (5*a^2*b*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 +
 b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Ta
n[e + f*x]^2])/(2*(a^2 + b^2)^(5/2)*f*Sqrt[d*Sec[e + f*x]]) + (5*a^2*b*Cot[
e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*
(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^(5/2)*f*Sqrt[d
*Sec[e + f*x]]) + (b*(2*a^2 - 3*b^2)*Sec[e + f*x]^2)/((a^2 + b^2)^2*f*Sqrt[
d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) + (2*(b + a*Tan[e + f*x]))/((a^2 + b^
2)*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.537244, antiderivative size = 555, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3512, 741, 835, 844, 227, 196, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{5ab^{3/2}\sqrt[4]{\sec^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}+\frac{b\left(2a^2-3b^2\right)\sec^2(e+fx)}{f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}-\frac{5ab^{3/2}\sqrt[4]{\sec^2(e+fx)}\tanh}{2f\left(a^2+b^2\right)^{9/4}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (5*a*b^(3/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Se
c[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(9/4)*f*Sqrt[d*Sec[e + f*x]]) - (5*a*b^
(3/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e +
f*x]^2)^(1/4))/(2*(a^2 + b^2)^(9/4)*f*Sqrt[d*Sec[e + f*x]]) + ((2*a^2 - 3*b
^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/((a^2 + b^
2)^2*f*Sqrt[d*Sec[e + f*x]]) - ((2*a^2 - 3*b^2)*Tan[e + f*x])/((a^2 + b^2)^
2*f*Sqrt[d*Sec[e + f*x]]) - (5*a^2*b*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 +
 b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Ta
n[e + f*x]^2])/(2*(a^2 + b^2)^(5/2)*f*Sqrt[d*Sec[e + f*x]]) + (5*a^2*b*Cot[
e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*
(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^(5/2)*f*Sqrt[d
*Sec[e + f*x]]) + (b*(2*a^2 - 3*b^2)*Sec[e + f*x]^2)/((a^2 + b^2)^2*f*Sqrt[
d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) + (2*(b + a*Tan[e + f*x]))/((a^2 + b^
2)*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
```

```
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
```

```
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\mathrm{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$= \frac{2(b+a\tan(e+fx))}{\left(a^2+b^2\right)f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\mathrm{Subs}}{\left(a^2+b\right.}$$

$$= \frac{b\left(2a^2-3b^2\right)\sec^2(e+fx)}{\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2(b+a\tan}{\left(a^2+b^2\right)f\sqrt{d\sec(e+f}}$$

$$= \frac{b\left(2a^2-3b^2\right)\sec^2(e+fx)}{\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2(b+a\tan}{\left(a^2+b^2\right)f\sqrt{d\sec(e+f}}$$

$$= -\frac{\left(2a^2-3b^2\right)\tan(e+fx)}{\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}} + \frac{b\left(2a^2-3b^2\right)\sec^2(e+fx)}{\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+}$$

$$= \frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}} - \frac{\left(2a^2-3b^2\right)\mathrm{ta}}{\left(a^2+b^2\right)^2 f\sqrt{d}}$$

$$= \frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}} - \frac{\left(2a^2-3b^2\right)\mathrm{ta}}{\left(a^2+b^2\right)^2 f\sqrt{d}}$$

$$= \frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}} - \frac{\left(2a^2-3b^2\right)\mathrm{ta}}{\left(a^2+b^2\right)^2 f\sqrt{d}}$$

$$= \frac{5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{9/4} f\sqrt{d\sec(e+fx)}} - \frac{5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx}}{\sqrt[4]{a^2+b^2}}\right.}{2\left(a^2+b^2\right)^{9/4} f\sqrt{d\,\mathrm{se}}}$$

Mathematica [C] time = 32.1259, size = 17812, normalized size = 32.09

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 2.362, size = 38627, normalized size = 69.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2), x)
```

3.615 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=520

$$-\frac{7ab^{5/2}\sec^2(e+fx)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{11/4}(d\sec(e+fx))^{3/2}}+\frac{b\left(2a^2-5b^2\right)\sec^2(e+fx)}{3f\left(a^2+b^2\right)^2(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}-\frac{7ab^{5/2}\sec^2(e+f}{2f\left(a^2+b^2\right.}$$

```
[Out] (-7*a*b^(5/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(S
ec[e + f*x]^2)^(3/4))/(2*(a^2 + b^2)^(11/4)*f*(d*Sec[e + f*x])^(3/2)) - (7*
a*b^(5/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[
e + f*x]^2)^(3/4))/(2*(a^2 + b^2)^(11/4)*f*(d*Sec[e + f*x])^(3/2)) + ((2*a^
2 - 5*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3/4))/(3*
(a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (7*a^2*b^2*Cot[e + f*x]*EllipticP
i[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2
)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)) +
 (7*a^2*b^2*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]
^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2
)^3*f*(d*Sec[e + f*x])^(3/2)) + (b*(2*a^2 - 5*b^2)*Sec[e + f*x]^2)/(3*(a^2
+ b^2)^2*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])) + (2*(b + a*Tan[e +
 f*x]))/(3*(a^2 + b^2)*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.566093, antiderivative size = 520, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3512, 741, 835, 844, 231, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{7ab^{5/2}\sec^2(e+fx)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{11/4}(d\sec(e+fx))^{3/2}}+\frac{b\left(2a^2-5b^2\right)\sec^2(e+fx)}{3f\left(a^2+b^2\right)^2(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}-\frac{7ab^{5/2}\sec^2(e+f}{2f\left(a^2+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (-7*a*b^(5/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(S
ec[e + f*x]^2)^(3/4))/(2*(a^2 + b^2)^(11/4)*f*(d*Sec[e + f*x])^(3/2)) - (7*
a*b^(5/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[
e + f*x]^2)^(3/4))/(2*(a^2 + b^2)^(11/4)*f*(d*Sec[e + f*x])^(3/2)) + ((2*a^
2 - 5*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3/4))/(3*
(a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (7*a^2*b^2*Cot[e + f*x]*EllipticP
i[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2
)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)) +
 (7*a^2*b^2*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]
^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2
)^3*f*(d*Sec[e + f*x])^(3/2)) + (b*(2*a^2 - 5*b^2)*Sec[e + f*x]^2)/(3*(a^2
+ b^2)^2*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])) + (2*(b + a*Tan[e +
 f*x]))/(3*(a^2 + b^2)*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
```

```
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \frac{\sec^2(e+fx)^{3/4}\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$= \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} - \frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{S}}{3\left(a^2\right.}$$

$$= \frac{b\left(2a^2-5b^2\right)\sec^2(e+fx)}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{2(b+}{3\left(a^2+b^2\right)f(d\sec(e}$$

$$= \frac{b\left(2a^2-5b^2\right)\sec^2(e+fx)}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{2(b+}{3\left(a^2+b^2\right)f(d\sec(e}$$

$$= \frac{\left(2a^2-5b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{b\left(2\right.}{3\left(a^2+b^2\right)^2 f(d}$$

$$= \frac{\left(2a^2-5b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{b\left(2\right.}{3\left(a^2+b^2\right)^2 f(d}$$

$$= \frac{\left(2a^2-5b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{b\left(2\right.}{3\left(a^2+b^2\right)^2 f(d}$$

$$= \frac{\left(2a^2-5b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{b\left(2\right.}{3\left(a^2+b^2\right)^2 f(d}$$

$$= -\frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4} f(d\sec(e+fx))^{3/2}} - \frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+f}}{\sqrt[4]{a^2+b^2}}\right.}{2\left(a^2+b^2\right)^{11/4} f(d\sec}$$

$$= -\frac{7ab^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4} f(d\sec(e+fx))^{3/2}} - \frac{7ab^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+f}}{\sqrt[4]{a^2+b^2}}\right.}{2\left(a^2+b^2\right)^{11/4} f(d\sec}$$

Mathematica [C] time = 27.6345, size = 11962, normalized size = 23.

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2),x]
```

[Out] Result too large to show

Maple [B] time = 0.838, size = 15455, normalized size = 29.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d \sec\left(fx+e\right)\right)^{\frac{3}{2}} \left(b \tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d \sec\left(e+fx\right)\right)^{\frac{3}{2}} \left(a+b \tan\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2), x)
```

3.616 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=700

$$\frac{9ab^{7/2}\sqrt[4]{\sec^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2d^2f\left(a^2+b^2\right)^{13/4}\sqrt{d\sec(e+fx)}}+\frac{3b\left(10a^2b^2+2a^4-7b^4\right)\sec^2(e+fx)}{5d^2f\left(a^2+b^2\right)^3\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}-\frac{9ab^{7/2}\sqrt[4]{\sec^2(e+f}}{2d^2f\left(a^2+b^2\right.}$$

```
[Out] (9*a*b^(7/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Se
c[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(13/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) - (9
*a*b^(7/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec
[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(13/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (3*
(2*a^4 + 10*a^2*b^2 - 7*b^4)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e +
f*x]^2)^(1/4))/(5*(a^2 + b^2)^3*d^2*f*Sqrt[d*Sec[e + f*x]]) - (3*(2*a^4 + 1
0*a^2*b^2 - 7*b^4)*Tan[e + f*x])/(5*(a^2 + b^2)^3*d^2*f*Sqrt[d*Sec[e + f*x]
]) - (9*a^2*b^3*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e
 + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^
2 + b^2)^(7/2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (9*a^2*b^3*Cot[e + f*x]*Ellipt
icPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2
)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^(7/2)*d^2*f*Sqrt[d*Sec[e + f*
x]]) + (3*b*(2*a^4 + 10*a^2*b^2 - 7*b^4)*Sec[e + f*x]^2)/(5*(a^2 + b^2)^3*d
^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) + (2*Cos[e + f*x]^2*(b + a*
Tan[e + f*x]))/(5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x
])) - (2*(b*(2*a^2 - 7*b^2) - 3*a*(a^2 + 4*b^2)*Tan[e + f*x]))/(5*(a^2 + b^
2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.714032, antiderivative size = 700, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3512, 741, 823, 835, 844, 227, 196, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{9ab^{7/2}\sqrt[4]{\sec^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2d^2f\left(a^2+b^2\right)^{13/4}\sqrt{d\sec(e+fx)}}+\frac{3b\left(10a^2b^2+2a^4-7b^4\right)\sec^2(e+fx)}{5d^2f\left(a^2+b^2\right)^3\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}-\frac{9ab^{7/2}\sqrt[4]{\sec^2(e+f}}{2d^2f\left(a^2+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2),x]
```


```
[Out] (9*a*b^(7/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Se
c[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(13/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) - (9
*a*b^(7/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec
[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(13/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (3*
(2*a^4 + 10*a^2*b^2 - 7*b^4)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e +
f*x]^2)^(1/4))/(5*(a^2 + b^2)^3*d^2*f*Sqrt[d*Sec[e + f*x]]) - (3*(2*a^4 + 1
0*a^2*b^2 - 7*b^4)*Tan[e + f*x])/(5*(a^2 + b^2)^3*d^2*f*Sqrt[d*Sec[e + f*x]
]) - (9*a^2*b^3*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e
 + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^
2 + b^2)^(7/2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (9*a^2*b^3*Cot[e + f*x]*Ellipt
icPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2
)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^(7/2)*d^2*f*Sqrt[d*Sec[e + f*
x]]) + (3*b*(2*a^4 + 10*a^2*b^2 - 7*b^4)*Sec[e + f*x]^2)/(5*(a^2 + b^2)^3*d
^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) + (2*Cos[e + f*x]^2*(b + a*
Tan[e + f*x]))/(5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x
])) - (2*(b*(2*a^2 - 7*b^2) - 3*a*(a^2 + 4*b^2)*Tan[e + f*x]))/(5*(a^2 + b^
```

```
2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Su}}{5\left(a^2\right.}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{2\left(b\left(2a^2-7b^2\right)-3\right.}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\,\text{se}}}$$

$$= \frac{3b\left(2a^4+10a^2b^2-7b^4\right)\sec^2(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2\cos^2(e+f}{5\left(a^2+b^2\right)d^2f\sqrt{d\,\text{se}}}$$

$$= \frac{3b\left(2a^4+10a^2b^2-7b^4\right)\sec^2(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2\cos^2(e+f}{5\left(a^2+b^2\right)d^2f\sqrt{d\,\text{se}}}$$

$$= -\frac{3\left(2a^4+10a^2b^2-7b^4\right)\tan(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} + \frac{3b\left(2a^4+10a^2b^2-7b^4\right)\text{sec}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a\,-}$$

$$= \frac{3\left(2a^4+10a^2b^2-7b^4\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} - \frac{3\left(2a\right.}{5\left(\right.}$$

$$= \frac{3\left(2a^4+10a^2b^2-7b^4\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} - \frac{3\left(2a\right.}{5\left(\right.}$$

$$= \frac{3\left(2a^4+10a^2b^2-7b^4\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} - \frac{3\left(2a\right.}{5\left(\right.}$$

$$= \frac{9ab^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\sec(e+fx)}} - \frac{9ab^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right.}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\,\text{s}}}$$

Mathematica [C] time = 31.9224, size = 18542, normalized size = 26.49

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 1.676, size = 44337, normalized size = 63.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2), x)
```

3.617 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=583

$$\frac{3d^2\left(a^2+2b^2\right)(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{5/2}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}-\frac{3d^2\left(a^2+2b^2\right)(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{5/2}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}+\frac{\cdots}{4b\cdots}$$

```
[Out] (3*(a^2 + 2*b^2)*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1
/4)]*(d*Sec[e + f*x])^(3/2))/(8*b^(5/2)*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2
)^(3/4)) - (3*(a^2 + 2*b^2)*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a
^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(8*b^(5/2)*(a^2 + b^2)^(5/4)*f*(Se
c[e + f*x]^2)^(3/4)) + (3*a*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec
[e + f*x])^(3/2))/(4*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(3/4)) - (3*a*d^2*C
os[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(4*b^2*(a^2 + b^2)*f) - (3
*a*(a^2 + 2*b^2)*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(
Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(
8*b^3*(a^2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) + (3*a*(a^2 + 2*b^2)*d^2*
Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)],
-1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(8*b^3*(a^2 + b^2)^(3/2)*
f*(Sec[e + f*x]^2)^(3/4)) - (d^2*(d*Sec[e + f*x])^(3/2))/(2*b*f*(a + b*Tan[
e + f*x])^2) + (3*a*d^2*(d*Sec[e + f*x])^(3/2))/(4*b*(a^2 + b^2)*f*(a + b*T
an[e + f*x]))
```

Rubi [A] time = 0.517093, antiderivative size = 583, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3512, 733, 835, 844, 227, 196, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{3d^2\left(a^2+2b^2\right)(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{5/2}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}-\frac{3d^2\left(a^2+2b^2\right)(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{5/2}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}+\frac{\cdots}{4b\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (3*(a^2 + 2*b^2)*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1
/4)]*(d*Sec[e + f*x])^(3/2))/(8*b^(5/2)*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2
)^(3/4)) - (3*(a^2 + 2*b^2)*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a
^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(8*b^(5/2)*(a^2 + b^2)^(5/4)*f*(Se
c[e + f*x]^2)^(3/4)) + (3*a*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec
[e + f*x])^(3/2))/(4*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(3/4)) - (3*a*d^2*C
os[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(4*b^2*(a^2 + b^2)*f) - (3
*a*(a^2 + 2*b^2)*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(
Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(
8*b^3*(a^2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) + (3*a*(a^2 + 2*b^2)*d^2*
Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)],
-1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(8*b^3*(a^2 + b^2)^(3/2)*
f*(Sec[e + f*x]^2)^(3/4)) - (d^2*(d*Sec[e + f*x])^(3/2))/(2*b*f*(a + b*Tan[
e + f*x])^2) + (3*a*d^2*(d*Sec[e + f*x])^(3/2))/(4*b*(a^2 + b^2)*f*(a + b*T
an[e + f*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/4}}{(a+x)^3}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+f\right.}{4b^3f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\sec(e+fx))^{3/2}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\text{Sul}}{4b\left(a^2+\right.}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\sec(e+fx))^{3/2}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\left(3ad^2(d\sec(e+fx))^{3/2}\right)\text{Su}}{8b^3\left(a^2+\right.}$$

$$= -\frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4b^2\left(a^2+b^2\right)f} - \frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\,\mathrm{s}}{4b\left(a^2+b^2\right)f}$$

$$= \frac{3ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}} - \frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin}{4b^2\left(a^2+b^2\right)f}$$

$$= \frac{3ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}} - \frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin}{4b^2\left(a^2+b^2\right)f}$$

$$= \frac{3ad^2E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}} - \frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin}{4b^2\left(a^2+b^2\right)f}$$

$$= \frac{3\left(a^2+2b^2\right)d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8b^{5/2}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}} - \frac{3\left(a^2+2b^2\right)d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+}}{\sqrt[4]{a^2+b^2}}\right.}{8b^{5/2}\left(a^2+b^2\right)^{5/4}f\sec^2}$$

Mathematica [C] time = 29.0176, size = 14257, normalized size = 24.45

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 5.917, size = 101372, normalized size = 173.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec \left(fx+e\right)\right)^{\frac{7}{2}}}{\left(b \tan \left(fx+e\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a)^3, x)
```

3.618 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=532

$$\frac{d^2(a^2-2b^2)\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{3/2}f(a^2+b^2)^{7/4}\sqrt[4]{\sec^2(e+fx)}}+\frac{d^2(a^2-2b^2)\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{3/2}f(a^2+b^2)^{7/4}\sqrt[4]{\sec^2(e+fx)}}+\frac{ad^2\sqrt{}}{4bf(a^2+b^2}$$

```
[Out] ((a^2 - 2*b^2)*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4
)]*Sqrt[d*Sec[e + f*x]])/(8*b^(3/2)*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1
/4)) + ((a^2 - 2*b^2)*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b
^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*b^(3/2)*(a^2 + b^2)^(7/4)*f*(Sec[e + f*
x]^2)^(1/4)) + (a*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f
*x]])/(4*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (a*(a^2 - 2*b^2)*d^2*C
ot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)]
, -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(8*b^2*(a^2 + b^2)^2*f*(S
ec[e + f*x]^2)^(1/4)) - (a*(a^2 - 2*b^2)*d^2*Cot[e + f*x]*EllipticPi[b/Sqrt
[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])/(8*b^2*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) - (d^2*Sqr
t[d*Sec[e + f*x]])/(2*b*f*(a + b*Tan[e + f*x])^2) + (a*d^2*Sqrt[d*Sec[e + f
*x]])/(4*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.497527, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3512, 733, 835, 844, 231, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$\frac{d^2(a^2-2b^2)\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{3/2}f(a^2+b^2)^{7/4}\sqrt[4]{\sec^2(e+fx)}}+\frac{d^2(a^2-2b^2)\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{3/2}f(a^2+b^2)^{7/4}\sqrt[4]{\sec^2(e+fx)}}+\frac{ad^2\sqrt{}}{4bf(a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```


```
[Out] ((a^2 - 2*b^2)*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4
)]*Sqrt[d*Sec[e + f*x]])/(8*b^(3/2)*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1
/4)) + ((a^2 - 2*b^2)*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b
^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*b^(3/2)*(a^2 + b^2)^(7/4)*f*(Sec[e + f*
x]^2)^(1/4)) + (a*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f
*x]])/(4*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (a*(a^2 - 2*b^2)*d^2*C
ot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)]
, -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(8*b^2*(a^2 + b^2)^2*f*(S
ec[e + f*x]^2)^(1/4)) - (a*(a^2 - 2*b^2)*d^2*Cot[e + f*x]*EllipticPi[b/Sqrt
[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])/(8*b^2*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) - (d^2*Sqr
t[d*Sec[e + f*x]])/(2*b*f*(a + b*Tan[e + f*x])^2) + (a*d^2*Sqrt[d*Sec[e + f
*x]])/(4*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
```

```
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x^2}{b^2}}}{(a+x)^3}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+\right.}{4b^3f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Sul}}{4b\left(a^2\right.}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))} + \frac{\left(ad^2\sqrt{d\sec(e+fx)}\right)\text{Su}}{8b^3\left(a^2\right.}$$

$$= \frac{ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2}{4b\left(a^2+b^2\right.}$$

$$= \frac{ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2}{4b\left(a^2+b^2\right.}$$

$$= \frac{ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2}{4b\left(a^2+b^2\right.}$$

$$= \frac{ad^2F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2}{4b\left(a^2+b^2\right.}$$

$$= \frac{\left(a^2-2b^2\right)d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(a^2-2b^2\right)d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx}}{\sqrt[4]{a^2+b^2}}\right.}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2}}$$

$$= \frac{\left(a^2-2b^2\right)d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(a^2-2b^2\right)d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx}}{\sqrt[4]{a^2+b^2}}\right.}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2}}$$

Mathematica [C] time = 25.7548, size = 4504, normalized size = 8.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```

[Out] (Sec[e + f*x]*(d*Sec[e + f*x])^(5/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(-1/(4*(a - I*b)*(a + I*b)*b) - b/(2*(a - I*b)*(a + I*b)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2) + (3*Sin[e + f*x])/(4*(a - I*b)*(a + I*b)*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x])^3) + (b^(9/2)*Sqrt[a^2 + b^2]*(8*a*b^(3/2)*(a^2 + b^2)^(3/2)*Sqrt[b^2*(a^2 + b^2)]*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] + (a^2 - 2*b^2)*(Sqrt[b^2*(a^2 + b^2)]*(Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*(a^2 + b*(b + Sqrt[a^2 + b^2]))*ArcTan[(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])] + (-a^2 + b*(-b + Sqrt[a^2 + b^2]))*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])]) + 4*a*b^(3/2)*(a^2 + b^2)^(3/2)*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), -ArcSin[Tan[(e + f*x)/2]], -1] - 4*a*b^(3/2)*(a^2 + b^2)^(3/2)*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), -ArcSin[Tan[(e + f*x)/2]], -1]))*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[Sec[e + f*x]]*(d*Sec[e + f*x])^(5/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*(1/(4*(a - I*b)*(a + I*b)*Sqrt[Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])) + (a*Sqrt[Sec[e + f*x]]*Sin[e + f*x])/(8*(a - I*b)*(a + I*b)*b*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(16*a^2*(b^2*(a^2 + b^2))^(7/2)*f*Sqrt[Sec[(e + f*x)/2]^2]*(a + b*Tan[e + f*x])^3*(-(b^(9/2)*Sqrt[a^2 + b^2]*(8*a*b^(3/2)*(a^2 + b^2)^(3/2)*Sqrt[b^2*(a^2 + b^2)]*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] + (a^2 - 2*b^2)*(Sqrt[b^2*(a^2 + b^2)]*(Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*(a^2 + b*(b + Sqrt[a^2 + b^2]))*ArcTan[(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])] + (-a^2 + b*(-b + Sqrt[a^2 + b^2]))*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])]) + 4*a*b^(3/2)*(a^2 + b^2)^(3/2)*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), -ArcSin[Tan[(e + f*x)/2]], -1] - 4*a*b^(3/2)*(a^2 + b^2)^(3/2)*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), -ArcSin[Tan[(e + f*x)/2]], -1]))*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Tan[(e + f*x)/2])/(32*a^2*(b^2*(a^2 + b^2))^(7/2)*Sqrt[Sec[(e + f*x)/2]^2]) + (b^(9/2)*Sqrt[a^2 + b^2]*(8*a*b^(3/2)*(a^2 + b^2)^(3/2)*Sqrt[b^2*(a^2 + b^2)]*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] + (a^2 - 2*b^2)*(Sqrt[b^2*(a^2 + b^2)]*(Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*(a^2 + b*(b + Sqrt[a^2 + b^2]))*ArcTan[(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])] + (-a^2 + b*(-b + Sqrt[a^2 + b^2]))*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])]) + 4*a*b^(3/2)*(a^2 + b^2)^(3/2)*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), -ArcSin[Tan[(e + f*x)/2]], -1] - 4*a*b^(3/2)*(a^2 + b^2)^(3/2)*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), -ArcSin[Tan[(e + f*x)/2]], -1]))*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*(-(Sec[(e + f*x)/2]^4*Sin[e + f*x]) + 2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2]))/(32*a^2*(b^2*(a^2 + b^2))^(7/2)*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]) + (b^(9/2)*Sqrt[a^2 + b^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*((4*a*b^(3/2)*(a^2 + b^2)^(3/2)*Sqrt[b^2*(a^2 + b^2)]*Sec[(e + f*x)/2]^2)/(Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e + f*x)/2]^2]) + (a^2 - 2*b^2)*((-2*a*b^(3/2)*(a^2 + b^2)^(3/2)*Sec[(e + f*x)/2]^2)/(Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e + f*x)/2]^2]*(1 - (a^2*Tan[(e + f*x)/2]^2)/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]))) + (2*a*b^(3/2)*(a^2 + b^2)^

```
(3/2)*Sec[(e + f*x)/2]^2)/(Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e + f
*x)/2]^2]*(1 - (a^2*Tan[(e + f*x)/2]^2)/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2
)])))) + Sqrt[b^2*(a^2 + b^2)]*((Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2
*b + Sqrt[a^2 + b^2])]*(a^2 + b*(b + Sqrt[a^2 + b^2]))*(-((a^2 + 2*b*(b - S
qrt[a^2 + b^2]))*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2*Sqrt[b]*Sqrt[2*b^2
*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Se
c[(e + f*x)/2]^4]) - ((-(Sec[(e + f*x)/2]^4*Sin[e + f*x]) + 2*Cos[e + f*x]*
Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2])*(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]
))*Tan[(e + f*x)/2]^2))/(4*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(
-2*b + Sqrt[a^2 + b^2])]*(Cos[e + f*x]*Sec[(e + f*x)/2]^4)^(3/2))))/(1 + (C
os[(e + f*x)/2]^4*Sec[e + f*x]*(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan
[(e + f*x)/2]^2)^2)/(4*b*(2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^
2 + b^2])))) + ((-a^2 + b*(-b + Sqrt[a^2 + b^2]))*Sqrt[2*b^2*(b + Sqrt[a^2
+ b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*(-((a^2 + 2*b*(b + Sqrt[a^2 + b^2]))
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2*Sqrt[b]*Sqrt[2*b^2*(b + Sqrt[a^2 +
 b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]
) - ((-(Sec[(e + f*x)/2]^4*Sin[e + f*x]) + 2*Cos[e + f*x]*Sec[(e + f*x)/2]^
4*Tan[(e + f*x)/2])*(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/
2]^2))/(4*Sqrt[b]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 +
b^2])]*(Cos[e + f*x]*Sec[(e + f*x)/2]^4)^(3/2))))/(1 + (Cos[(e + f*x)/2]^4*
Sec[e + f*x]*(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)^2
)/(4*b*(2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2]))))))))/(1
6*a^2*(b^2*(a^2 + b^2))^(7/2)*Sqrt[Sec[(e + f*x)/2]^2]) + (b^(9/2)*Sqrt[a^2
 + b^2]*(8*a*b^(3/2)*(a^2 + b^2)^(3/2)*Sqrt[b^2*(a^2 + b^2)]*EllipticF[ArcS
in[Tan[(e + f*x)/2]], -1] + (a^2 - 2*b^2)*(Sqrt[b^2*(a^2 + b^2)]*(Sqrt[2*b^
2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*(a^2 + b*(b + Sqrt[
a^2 + b^2]))*ArcTan[(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x)/
2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 +
b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])] + (-a^2 + b*(-b + Sqrt[a^2 +
 b^2]))*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*Arc
Tan[(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]
*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e
 + f*x]*Sec[(e + f*x)/2]^4])]) + 4*a*b^(3/2)*(a^2 + b^2)^(3/2)*EllipticPi[a
^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), -ArcSin[Tan[(e + f*x)/2]], -1]
- 4*a*b^(3/2)*(a^2 + b^2)^(3/2)*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^
2 + b^2)])), -ArcSin[Tan[(e + f*x)/2]], -1]))*Sqrt[Cos[e + f*x]*Sec[(e + f*
x)/2]^4]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)
/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(32*a^2*(b^2*(a^2 + b^2))^(7/2)*Sqrt[Sec[
(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]))
```

Maple [B] time = 3.185, size = 45973, normalized size = 86.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec \left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b \tan \left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^3, x)
```

3.619 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=566

$$\frac{(3a^2-2b^2)(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8\sqrt{b}f(a^2+b^2)^{9/4}\sec^2(e+fx)^{3/4}}-\frac{(3a^2-2b^2)(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8\sqrt{b}f(a^2+b^2)^{9/4}\sec^2(e+fx)^{3/4}}-\frac{5a}{4f(a^2+}$$

```
[Out] ((3*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]
*(d*Sec[e + f*x])^(3/2))/(8*Sqrt[b]*(a^2 + b^2)^(9/4)*f*(Sec[e + f*x]^2)^(3
/4)) - ((3*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2
)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(8*Sqrt[b]*(a^2 + b^2)^(9/4)*f*(Sec[e + f*
x]^2)^(3/4)) - (5*a*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(
3/2))/(4*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(3/4)) + (5*a*Cos[e + f*x]*(d*Sec
[e + f*x])^(3/2)*Sin[e + f*x])/(4*(a^2 + b^2)^2*f) - (a*(3*a^2 - 2*b^2)*Cot
[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)],
-1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(8*b*(a^2 + b^2)^(5/2)*f*
(Sec[e + f*x]^2)^(3/4)) + (a*(3*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[b/Sqrt
[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqr
t[-Tan[e + f*x]^2])/(8*b*(a^2 + b^2)^(5/2)*f*(Sec[e + f*x]^2)^(3/4)) - (b*(
d*Sec[e + f*x])^(3/2))/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (5*a*b*(d
*Sec[e + f*x])^(3/2))/(4*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.539162, antiderivative size = 566, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3512, 745, 835, 844, 227, 196, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{(3a^2-2b^2)(d\sec(e+fx))^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8\sqrt{b}f(a^2+b^2)^{9/4}\sec^2(e+fx)^{3/4}}-\frac{(3a^2-2b^2)(d\sec(e+fx))^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8\sqrt{b}f(a^2+b^2)^{9/4}\sec^2(e+fx)^{3/4}}-\frac{5a}{4f(a^2+}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
```


```
[Out] ((3*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]
*(d*Sec[e + f*x])^(3/2))/(8*Sqrt[b]*(a^2 + b^2)^(9/4)*f*(Sec[e + f*x]^2)^(3
/4)) - ((3*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2
)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(8*Sqrt[b]*(a^2 + b^2)^(9/4)*f*(Sec[e + f*
x]^2)^(3/4)) - (5*a*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(
3/2))/(4*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(3/4)) + (5*a*Cos[e + f*x]*(d*Sec
[e + f*x])^(3/2)*Sin[e + f*x])/(4*(a^2 + b^2)^2*f) - (a*(3*a^2 - 2*b^2)*Cot
[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)],
-1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(8*b*(a^2 + b^2)^(5/2)*f*
(Sec[e + f*x]^2)^(3/4)) + (a*(3*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[b/Sqrt
[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqr
t[-Tan[e + f*x]^2])/(8*b*(a^2 + b^2)^(5/2)*f*(Sec[e + f*x]^2)^(3/4)) - (b*(
d*Sec[e + f*x])^(3/2))/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (5*a*b*(d
*Sec[e + f*x])^(3/2))/(4*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
```

```
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{(d\sec(e+fx))^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{(a+x)^3\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{(d\sec(e+fx))^{3/2}\,\mathrm{Subst}\left(\int \frac{-2a+\frac{x}{2}}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e\right.}{2b\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4\left(a^2+b^2\right)^2 f(a+b\tan(e+fx))} + \frac{\left(b(d\sec(e+fx))^3\right.}{}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4\left(a^2+b^2\right)^2 f(a+b\tan(e+fx))} + \frac{\left(5a(d\sec(e+fx)\right.}{8b}$$

$$= \frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4\left(a^2+b^2\right)^2 f} - \frac{b(d\sec(e+fx))^{3/2}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{5ab}{4\left(a^2+b\right.}$$

$$= -\frac{5aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4\left(a^2+b^2\right)^2 f\sec^2(e+fx)^{3/4}} + \frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e}{4\left(a^2+b^2\right)^2 f}$$

$$= -\frac{5aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4\left(a^2+b^2\right)^2 f\sec^2(e+fx)^{3/4}} + \frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e}{4\left(a^2+b^2\right)^2 f}$$

$$= -\frac{5aE\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4\left(a^2+b^2\right)^2 f\sec^2(e+fx)^{3/4}} + \frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e}{4\left(a^2+b^2\right)^2 f}$$

$$= -\frac{\left(2-\frac{3a^2}{b^2}\right)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8\left(a^2+b^2\right)^{9/4} f\sec^2(e+fx)^{3/4}} + \frac{\left(2-\frac{3a^2}{b^2}\right)b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+f}}{\sqrt[4]{a^2+b^2}}\right.}{8\left(a^2+b^2\right)^{9/4} f\sec^2(e}$$

Mathematica [C] time = 29.1131, size = 14396, normalized size = 25.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] Result too large to show
```

Maple [B] time = 4.376, size = 80250, normalized size = 141.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec\left(e + fx\right)\right)^{\frac{3}{2}}}{\left(a + b \tan\left(e + fx\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec\left(fx + e\right)\right)^{\frac{3}{2}}}{\left(b \tan\left(fx + e\right) + a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a)^3, x)
```

3.620 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=515

$$-\frac{3\sqrt{b}\left(5a^2-2b^2\right)\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f\left(a^2+b^2\right)^{11/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{3\sqrt{b}\left(5a^2-2b^2\right)\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f\left(a^2+b^2\right)^{11/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{\quad}{4f(}$$

```
[Out] (-3*Sqrt[b]*(5*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*(a^2 + b^2)^(11/4)*f*(Sec[e + f*x]^2)^
(1/4)) - (3*Sqrt[b]*(5*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4)
)/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*(a^2 + b^2)^(11/4)*f*(Sec[e +
 f*x]^2)^(1/4)) - (7*a*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e +
f*x]])/(4*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) + (3*a*(5*a^2 - 2*b^2)*Co
t[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)],
 -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^3*f*(Sec[e
+ f*x]^2)^(1/4)) + (3*a*(5*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2
+ b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[
e + f*x]^2])/(8*(a^2 + b^2)^3*f*(Sec[e + f*x]^2)^(1/4)) - (b*Sqrt[d*Sec[e +
 f*x]])/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (7*a*b*Sqrt[d*Sec[e + f*
x]])/(4*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.521928, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3512, 745, 835, 844, 231, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{3\sqrt{b}\left(5a^2-2b^2\right)\sqrt{d\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f\left(a^2+b^2\right)^{11/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{3\sqrt{b}\left(5a^2-2b^2\right)\sqrt{d\sec(e+fx)}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f\left(a^2+b^2\right)^{11/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{\quad}{4f(}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```


```
[Out] (-3*Sqrt[b]*(5*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*(a^2 + b^2)^(11/4)*f*(Sec[e + f*x]^2)^
(1/4)) - (3*Sqrt[b]*(5*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4)
)/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*(a^2 + b^2)^(11/4)*f*(Sec[e +
 f*x]^2)^(1/4)) - (7*a*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e +
f*x]])/(4*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) + (3*a*(5*a^2 - 2*b^2)*Co
t[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)],
 -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^3*f*(Sec[e
+ f*x]^2)^(1/4)) + (3*a*(5*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2
+ b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[
e + f*x]^2])/(8*(a^2 + b^2)^3*f*(Sec[e + f*x]^2)^(1/4)) - (b*Sqrt[d*Sec[e +
 f*x]])/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (7*a*b*Sqrt[d*Sec[e + f*
x]])/(4*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
```

```
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
```

```
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{\sqrt{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{-2a+\frac{3x}{2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan\right.}{2b\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2 f(a+b\tan(e+fx))} + \frac{\left(b\sqrt{d\sec(e+f\right.}}{}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2 f(a+b\tan(e+fx))} - \frac{\left(7a\sqrt{d\sec(e+\right.}}{}$$

$$= -\frac{7aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{}{4}$$

$$= -\frac{7aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{}{4}$$

$$= -\frac{7aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{}{4}$$

$$= -\frac{7aF\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{}{4}$$

$$= \frac{3\left(2-\frac{5a^2}{b^2}\right)b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\left(a^2+b^2\right)^{11/4} f\sqrt[4]{\sec^2(e+fx)}} + \frac{3\left(2-\frac{5a^2}{b^2}\right)b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e}}{\sqrt[4]{a^2+b}}\right.}{8\left(a^2+b^2\right)^{11/4} f\sqrt[4]{\sec}}$$

$$= \frac{3\left(2-\frac{5a^2}{b^2}\right)b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\left(a^2+b^2\right)^{11/4} f\sqrt[4]{\sec^2(e+fx)}} + \frac{3\left(2-\frac{5a^2}{b^2}\right)b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e}}{\sqrt[4]{a^2+b}}\right.}{8\left(a^2+b^2\right)^{11/4} f\sqrt[4]{\sec}}$$

Mathematica [C] time = 25.2449, size = 4471, normalized size = 8.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (Sec[e + f*x]^3*Sqrt[d*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*((
-9*b)/(4*(a - I*b)^2*(a + I*b)^2) - b^3/(2*(a - I*b)^2*(a + I*b)^2*(a*Cos[e
 + f*x] + b*Sin[e + f*x])^2) + (11*b^2*Sin[e + f*x])/(4*(a - I*b)^2*(a + I*
b)^2*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x])^3) - ((8*
a*Sqrt[b^2*(a^2 + b^2)]*(-4*a^4 - a^2*b^2 + 3*b^4)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], -1] - 3*Sqrt[b]*(-5*a^2 + 2*b^2)*(Sqrt[b^2*(a^2 + b^2)]*((b + S
qrt[a^2 + b^2])*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b
^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(
2*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*
Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])] + (b - Sqrt[a^2 + b^2])*Sqrt[2*b^2*
(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2
*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b + Sq
rt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*
x)/2]^4])]) + 4*a*b^(3/2)*(a^2 + b^2)*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[
b^2*(a^2 + b^2)]), -ArcSin[Tan[(e + f*x)/2]], -1] - 4*a*b^(3/2)*(a^2 + b^2)
*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), -ArcSin[Tan[(e + f
*x)/2]], -1]))*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sec[e + f*x]^(5/2)*Sqr
t[d*Sec[e + f*x]]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*(a*Cos[e + f*x] + b
*Sin[e + f*x])^3*(a^2/((a - I*b)^2*(a + I*b)^2*Sqrt[Sec[e + f*x]]*(a*Cos[e
+ f*x] + b*Sin[e + f*x])) - (3*b^2)/(4*(a - I*b)^2*(a + I*b)^2*Sqrt[Sec[e +
 f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])) - (7*a*b*Sqrt[Sec[e + f*x]]*Sin[e
 + f*x])/(8*(a - I*b)^2*(a + I*b)^2*(a*Cos[e + f*x] + b*Sin[e + f*x])))))/(1
6*a^2*(a^2 + b^2)^3*Sqrt[b^2*(a^2 + b^2)]*f*Sqrt[Sec[(e + f*x)/2]^2]*(a + b
*Tan[e + f*x])^3*(((8*a*Sqrt[b^2*(a^2 + b^2)]*(-4*a^4 - a^2*b^2 + 3*b^4)*El
lipticF[ArcSin[Tan[(e + f*x)/2]], -1] - 3*Sqrt[b]*(-5*a^2 + 2*b^2)*(Sqrt[b^
2*(a^2 + b^2)]*((b + Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^
2*(-2*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))
*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*
b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])] + (b - Sqrt[a
^2 + b^2])*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*
ArcTan[(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt
[b]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*Sqrt[Co
s[e + f*x]*Sec[(e + f*x)/2]^4])]) + 4*a*b^(3/2)*(a^2 + b^2)*EllipticPi[a^2/
(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), -ArcSin[Tan[(e + f*x)/2]], -1] - 4
*a*b^(3/2)*(a^2 + b^2)*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)]
)), -ArcSin[Tan[(e + f*x)/2]], -1]))*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*
Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Tan[(e + f*x)/2])/(32*a^2*(a^2 + b^2)
^3*Sqrt[b^2*(a^2 + b^2)]*Sqrt[Sec[(e + f*x)/2]^2]) - ((8*a*Sqrt[b^2*(a^2 +
b^2)]*(-4*a^4 - a^2*b^2 + 3*b^4)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] -
3*Sqrt[b]*(-5*a^2 + 2*b^2)*(Sqrt[b^2*(a^2 + b^2)]*((b + Sqrt[a^2 + b^2])*Sq
rt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2
- (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b
^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*
Sec[(e + f*x)/2]^4])] + (b - Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b + Sqrt[a^2 + b^
2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 +
 b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a
^2*(2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])]) + 4*a*
b^(3/2)*(a^2 + b^2)*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]),
 -ArcSin[Tan[(e + f*x)/2]], -1] - 4*a*b^(3/2)*(a^2 + b^2)*EllipticPi[a^2/(a
^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), -ArcSin[Tan[(e + f*x)/2]], -1]))*Sqr
t[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*(-(Sec[(e + f*x)/2]^4*Sin[e + f*x]) + 2*
Cos[e + f*x]*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2]))/(32*a^2*(a^2 + b^2)^3*Sq
rt[b^2*(a^2 + b^2)]*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[e + f*x]*Sec[(e + f*x
)/2]^4]) - (Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sqrt[Cos[(e + f*x)/2]^2*S
ec[e + f*x]]*((4*a*Sqrt[b^2*(a^2 + b^2)]*(-4*a^4 - a^2*b^2 + 3*b^4)*Sec[(e
+ f*x)/2]^2)/(Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e + f*x)/2]^2]) -
3*Sqrt[b]*(-5*a^2 + 2*b^2)*((-2*a*b^(3/2)*(a^2 + b^2)*Sec[(e + f*x)/2]^2)/(
Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e + f*x)/2]^2]*(1 - (a^2*Tan[(e
+ f*x)/2]^2)/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]))) + (2*a*b^(3/2)*(a^2
```

```
+ b^2)*Sec[(e + f*x)/2]^2)/(Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e +
f*x)/2]^2]*(1 - (a^2*Tan[(e + f*x)/2]^2)/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^
2)])))) + Sqrt[b^2*(a^2 + b^2)]*(((b + Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b - Sqr
t[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*(-((a^2 + 2*b*(b - Sqrt[a^2 +
 b^2]))*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2*Sqrt[b]*Sqrt[2*b^2*(b - Sqr
t[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*
x)/2]^4]) - ((-(Sec[(e + f*x)/2]^4*Sin[e + f*x]) + 2*Cos[e + f*x]*Sec[(e +
f*x)/2]^4*Tan[(e + f*x)/2])*(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e
 + f*x)/2]^2))/(4*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sq
rt[a^2 + b^2])]*(Cos[e + f*x]*Sec[(e + f*x)/2]^4)^(3/2))))/(1 + (Cos[(e + f
*x)/2]^4*Sec[e + f*x]*(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x
)/2]^2)^2)/(4*b*(2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])
))) + ((b - Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b +
Sqrt[a^2 + b^2])]*(-((a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2])/(2*Sqrt[b]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + S
qrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]) - ((-(Sec[(e + f*x)
/2]^4*Sin[e + f*x]) + 2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2])*(
a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2))/(4*Sqrt[b]*Sqr
t[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*(Cos[e + f*x]*
Sec[(e + f*x)/2]^4)^(3/2))))/(1 + (Cos[(e + f*x)/2]^4*Sec[e + f*x]*(a^2 - (
a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)^2)/(4*b*(2*b^2*(b + Sq
rt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2]))))))))/(16*a^2*(a^2 + b^2)^3*S
qrt[b^2*(a^2 + b^2)]*Sqrt[Sec[(e + f*x)/2]^2]) - ((8*a*Sqrt[b^2*(a^2 + b^2)
]*(-4*a^4 - a^2*b^2 + 3*b^4)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] - 3*Sq
rt[b]*(-5*a^2 + 2*b^2)*(Sqrt[b^2*(a^2 + b^2)]*((b + Sqrt[a^2 + b^2])*Sqrt[2
*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a
^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(
b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[
(e + f*x)/2]^4])] + (b - Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2])
+ a^2*(2*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2
]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(
2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])]) + 4*a*b^(3
/2)*(a^2 + b^2)*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), -Ar
cSin[Tan[(e + f*x)/2]], -1] - 4*a*b^(3/2)*(a^2 + b^2)*EllipticPi[a^2/(a^2 +
 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), -ArcSin[Tan[(e + f*x)/2]], -1]))*Sqrt[Co
s[e + f*x]*Sec[(e + f*x)/2]^4]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*
x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(32*a^2*(a^2 + b^2)
^3*Sqrt[b^2*(a^2 + b^2)]*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*S
ec[e + f*x]])))
```

Maple [B] time = 4.334, size = 82060, normalized size = 159.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x)

[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sec \left(e+f x\right)}}{\left(a+b \tan \left(e+f x\right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d \sec \left(f x+e\right)}}{\left(b \tan \left(f x+e\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^3, x)
```

3.621 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=664

$$\frac{5b^{3/2}\left(7a^2-2b^2\right)\sqrt[4]{\sec^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f\left(a^2+b^2\right)^{13/4}\sqrt{d\sec(e+fx)}}+\frac{ab\left(8a^2-37b^2\right)\sec^2(e+fx)}{4f\left(a^2+b^2\right)^3\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}+\frac{\cdots}{2f\left(a^2+b\cdots\right.}$$

```
[Out] (5*b^(3/2)*(7*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b
^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(13/4)*f*Sqrt[d*Sec[e + f
*x]]) - (5*b^(3/2)*(7*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))
/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(13/4)*f*Sqrt[d*
Sec[e + f*x]]) + (a*(8*a^2 - 37*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(
Sec[e + f*x]^2)^(1/4))/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]) - (a*(8*a^2
 - 37*b^2)*Tan[e + f*x])/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]) - (5*a*b*
(7*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e
 + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^
2 + b^2)^(7/2)*f*Sqrt[d*Sec[e + f*x]]) + (5*a*b*(7*a^2 - 2*b^2)*Cot[e + f*x
]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e
+ f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(7/2)*f*Sqrt[d*Sec[e
+ f*x]]) + (b*(4*a^2 - 5*b^2)*Sec[e + f*x]^2)/(2*(a^2 + b^2)^2*f*Sqrt[d*Sec
[e + f*x]]*(a + b*Tan[e + f*x])^2) + (2*(b + a*Tan[e + f*x]))/((a^2 + b^2)*
f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2) + (a*b*(8*a^2 - 37*b^2)*Sec[
e + f*x]^2)/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rubi [A] time = 0.761036, antiderivative size = 664, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3512, 741, 835, 844, 227, 196, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{5b^{3/2}\left(7a^2-2b^2\right)\sqrt[4]{\sec^2(e+fx)}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f\left(a^2+b^2\right)^{13/4}\sqrt{d\sec(e+fx)}}+\frac{ab\left(8a^2-37b^2\right)\sec^2(e+fx)}{4f\left(a^2+b^2\right)^3\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}+\frac{\cdots}{2f\left(a^2+b\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3),x]
```


```
[Out] (5*b^(3/2)*(7*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b
^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(13/4)*f*Sqrt[d*Sec[e + f
*x]]) - (5*b^(3/2)*(7*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))
/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(13/4)*f*Sqrt[d*
Sec[e + f*x]]) + (a*(8*a^2 - 37*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(
Sec[e + f*x]^2)^(1/4))/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]) - (a*(8*a^2
 - 37*b^2)*Tan[e + f*x])/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]) - (5*a*b*
(7*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e
 + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^
2 + b^2)^(7/2)*f*Sqrt[d*Sec[e + f*x]]) + (5*a*b*(7*a^2 - 2*b^2)*Cot[e + f*x
]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e
+ f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(7/2)*f*Sqrt[d*Sec[e
+ f*x]]) + (b*(4*a^2 - 5*b^2)*Sec[e + f*x]^2)/(2*(a^2 + b^2)^2*f*Sqrt[d*Sec
[e + f*x]]*(a + b*Tan[e + f*x])^2) + (2*(b + a*Tan[e + f*x]))/((a^2 + b^2)*
f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2) + (a*b*(8*a^2 - 37*b^2)*Sec[
e + f*x]^2)/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$= \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\right.}{\left(a^2+b^2\right.}$$

$$= \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} + \frac{2(b+a\tan}{(a^2+b^2)\,f\sqrt{d\sec(e+f}}$$

$$= \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} + \frac{2(b+a\tan}{(a^2+b^2)\,f\sqrt{d\sec(e+f}}$$

$$= \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} + \frac{2(b+a\tan}{(a^2+b^2)\,f\sqrt{d\sec(e+f}}$$

$$= -\frac{a(8a^2-37b^2)\tan(e+fx)}{4(a^2+b^2)^3 f\sqrt{d\sec(e+fx)}} + \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+}$$

$$= \frac{a(8a^2-37b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{4(a^2+b^2)^3 f\sqrt{d\sec(e+fx)}} - \frac{a(8a^2-37b^2)}{4(a^2+b^2)^3 f\sqrt{}}$$

$$= \frac{a(8a^2-37b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{4(a^2+b^2)^3 f\sqrt{d\sec(e+fx)}} - \frac{a(8a^2-37b^2)}{4(a^2+b^2)^3 f\sqrt{}}$$

$$= \frac{a(8a^2-37b^2)\,E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{4(a^2+b^2)^3 f\sqrt{d\sec(e+fx)}} - \frac{a(8a^2-37b^2)}{4(a^2+b^2)^3 f\sqrt{}}$$

$$= \frac{5b^{3/2}(7a^2-2b^2)\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8(a^2+b^2)^{13/4} f\sqrt{d\sec(e+fx)}} - \frac{5b^{3/2}(7a^2-2b^2)\,\text{ta}}{8(a^2+}$$

Mathematica [C] time = 29.2325, size = 14684, normalized size = 22.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 11.363, size = 100394, normalized size = 151.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3), x)
```

3.622 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=620

$$-\frac{7b^{5/2}\left(9a^2-2b^2\right)\sec^2(e+fx)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f\left(a^2+b^2\right)^{15/4}(d\sec(e+fx))^{3/2}}+\frac{ab\left(8a^2-69b^2\right)\sec^2(e+fx)}{12f\left(a^2+b^2\right)^3(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}+\frac{\cdots}{6f(\cdots}$$

```
[Out] (-7*b^(5/2)*(9*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4))/(8*(a^2 + b^2)^(15/4)*f*(d*Sec[e + f*x]
)^(3/2)) - (7*b^(5/2)*(9*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/
4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4))/(8*(a^2 + b^2)^(15/4)*f*(d*S
ec[e + f*x])^(3/2)) + (a*(8*a^2 - 69*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2,
 2]*(Sec[e + f*x]^2)^(3/4))/(12*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)) + (
7*a*b^2*(9*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSi
n[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2]
)/(8*(a^2 + b^2)^4*f*(d*Sec[e + f*x])^(3/2)) + (7*a*b^2*(9*a^2 - 2*b^2)*Cot
[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]
*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^4*f*(d*Sec[e
+ f*x])^(3/2)) + (b*(4*a^2 - 7*b^2)*Sec[e + f*x]^2)/(6*(a^2 + b^2)^2*f*(d*S
ec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2) + (2*(b + a*Tan[e + f*x]))/(3*(a
^2 + b^2)*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2) + (a*b*(8*a^2 -
69*b^2)*Sec[e + f*x]^2)/(12*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)*(a + b*T
an[e + f*x]))
```

Rubi [A] time = 0.758669, antiderivative size = 620, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3512, 741, 835, 844, 231, 747, 401, 108, 409, 1213, 537, 444, 63, 212, 208, 205}

$$-\frac{7b^{5/2}\left(9a^2-2b^2\right)\sec^2(e+fx)^{3/4}\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f\left(a^2+b^2\right)^{15/4}(d\sec(e+fx))^{3/2}}+\frac{ab\left(8a^2-69b^2\right)\sec^2(e+fx)}{12f\left(a^2+b^2\right)^3(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}+\frac{\cdots}{6f(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3),x]
```


```
[Out] (-7*b^(5/2)*(9*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4))/(8*(a^2 + b^2)^(15/4)*f*(d*Sec[e + f*x]
)^(3/2)) - (7*b^(5/2)*(9*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/
4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4))/(8*(a^2 + b^2)^(15/4)*f*(d*S
ec[e + f*x])^(3/2)) + (a*(8*a^2 - 69*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2,
 2]*(Sec[e + f*x]^2)^(3/4))/(12*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)) + (
7*a*b^2*(9*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSi
n[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2]
)/(8*(a^2 + b^2)^4*f*(d*Sec[e + f*x])^(3/2)) + (7*a*b^2*(9*a^2 - 2*b^2)*Cot
[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]
*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^4*f*(d*Sec[e
+ f*x])^(3/2)) + (b*(4*a^2 - 7*b^2)*Sec[e + f*x]^2)/(6*(a^2 + b^2)^2*f*(d*S
ec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2) + (2*(b + a*Tan[e + f*x]))/(3*(a
^2 + b^2)*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2) + (a*b*(8*a^2 -
69*b^2)*Sec[e + f*x]^2)/(12*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)*(a + b*T
an[e + f*x]))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 747

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[-((b*x^2)/a)]/(2*x), Subst[Int[1/(Sqrt[-((b*x)/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 108

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - (d*e
)/f + (d*x^4)/f]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-(f/(d*e - c*f)), 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \frac{\sec^2(e+fx)^{3/4}\,\mathrm{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$= \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} - \frac{\left(2b\sec^2(e+fx)^{3/4}\right) \mathrm{S}}{3\left(a\right.}$$

$$= \frac{b\left(4a^2-7b^2\right)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{2(b+}{3\left(a^2+b^2\right)f(d\sec(}$$

$$= \frac{b\left(4a^2-7b^2\right)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{2(b+}{3\left(a^2+b^2\right)f(d\sec(}$$

$$= \frac{b\left(4a^2-7b^2\right)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{2(b+}{3\left(a^2+b^2\right)f(d\sec(}$$

$$= \frac{a\left(8a^2-69b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}} + \frac{}{6\left(a^2+b^2\right)^2}$$

$$= \frac{a\left(8a^2-69b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}} + \frac{}{6\left(a^2+b^2\right)^2}$$

$$= \frac{a\left(8a^2-69b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}} + \frac{}{6\left(a^2+b^2\right)^2}$$

$$= \frac{a\left(8a^2-69b^2\right)F\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sec^2(e+fx)^{3/4}}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}} + \frac{}{6\left(a^2+b^2\right)^2}$$

$$= -\frac{7b^{5/2}\left(9a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4} f(d\sec(e+fx))^{3/2}} - \frac{7b^{5/2}\left(9a^2-2b^2\right.}{8\left(a^2\right.}$$

$$= -\frac{7b^{5/2}\left(9a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4} f(d\sec(e+fx))^{3/2}} - \frac{7b^{5/2}\left(9a^2-2b^2\right.}{8\left(a^2\right.}$$

Mathematica [C] time = 24.8826, size = 4725, normalized size = 7.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] (Sec[e + f*x]^5*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*((b*(12*a^2 - 55*b^2))/
(12*(a - I*b)^3*(a + I*b)^3) + (b*(3*a^2 - b^2)*Cos[2*(e + f*x)])/(3*(a - I
*b)^3*(a + I*b)^3) - b^5/(2*(a - I*b)^3*(a + I*b)^3*(a*Cos[e + f*x] + b*Sin
[e + f*x])^2) + (19*b^4*Sin[e + f*x])/(4*(a - I*b)^3*(a + I*b)^3*(a*Cos[e +
 f*x] + b*Sin[e + f*x])) + (a*(a^2 - 3*b^2)*Sin[2*(e + f*x)])/(3*(a - I*b)^
3*(a + I*b)^3)))/(f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3) - ((8*a*
Sqrt[b^2*(a^2 + b^2)]*(-4*a^6 - 64*a^4*b^2 - 39*a^2*b^4 + 21*b^6)*EllipticF
[ArcSin[Tan[(e + f*x)/2]], -1] - 21*b^(5/2)*(-9*a^2 + 2*b^2)*(Sqrt[b^2*(a^2
 + b^2)]*((b + Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*
b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(
e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sq
rt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])] + (b - Sqrt[a^2 + b
^2])*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*ArcTan
[(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sq
rt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e +
f*x]*Sec[(e + f*x)/2]^4])]) + 4*a*b^(3/2)*(a^2 + b^2)*EllipticPi[a^2/(a^2 +
 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), -ArcSin[Tan[(e + f*x)/2]], -1] - 4*a*b^(
3/2)*(a^2 + b^2)*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), -A
rcSin[Tan[(e + f*x)/2]], -1]))*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]*Sec[e
+ f*x]^(9/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[
e + f*x])^3*(a^4/(3*(a - I*b)^3*(a + I*b)^3*Sqrt[Sec[e + f*x]]*(a*Cos[e + f
*x] + b*Sin[e + f*x])) + (5*a^2*b^2)/((a - I*b)^3*(a + I*b)^3*Sqrt[Sec[e +
f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])) - (7*b^4)/(4*(a - I*b)^3*(a + I*b)
^3*Sqrt[Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])) + (a^3*b*Sqrt[Sec[
e + f*x]]*Sin[e + f*x])/(3*(a - I*b)^3*(a + I*b)^3*(a*Cos[e + f*x] + b*Sin[
e + f*x])) - (23*a*b^3*Sqrt[Sec[e + f*x]]*Sin[e + f*x])/(8*(a - I*b)^3*(a +
 I*b)^3*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(48*a^2*(a^2 + b^2)^4*Sqrt[b^2
*(a^2 + b^2)]*f*Sqrt[Sec[(e + f*x)/2]^2]*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[
e + f*x])^3*(((8*a*Sqrt[b^2*(a^2 + b^2)]*(-4*a^6 - 64*a^4*b^2 - 39*a^2*b^4
+ 21*b^6)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] - 21*b^(5/2)*(-9*a^2 + 2*
b^2)*(Sqrt[b^2*(a^2 + b^2)]*((b + Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b - Sqrt[a^2
 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b - Sqrt
[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2
]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])]
+ (b - Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[
a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/
2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b
^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])]) + 4*a*b^(3/2)*(a^2 + b^2)*El
lipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), -ArcSin[Tan[(e + f*x)
/2]], -1] - 4*a*b^(3/2)*(a^2 + b^2)*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2
*(a^2 + b^2)])), -ArcSin[Tan[(e + f*x)/2]], -1]))*Sqrt[Cos[e + f*x]*Sec[(e
+ f*x)/2]^4]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*Tan[(e + f*x)/2])/(96*a^
2*(a^2 + b^2)^4*Sqrt[b^2*(a^2 + b^2)]*Sqrt[Sec[(e + f*x)/2]^2]) - ((8*a*Sqr
t[b^2*(a^2 + b^2)]*(-4*a^6 - 64*a^4*b^2 - 39*a^2*b^4 + 21*b^6)*EllipticF[Ar
cSin[Tan[(e + f*x)/2]], -1] - 21*b^(5/2)*(-9*a^2 + 2*b^2)*(Sqrt[b^2*(a^2 +
b^2)]*((b + Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b +
 Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e +
 f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[
a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])] + (b - Sqrt[a^2 + b^2]
)*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*ArcTan[(a
^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[
2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x
]*Sec[(e + f*x)/2]^4])]) + 4*a*b^(3/2)*(a^2 + b^2)*EllipticPi[a^2/(a^2 + 2*
b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), -ArcSin[Tan[(e + f*x)/2]], -1] - 4*a*b^(3/2
)*(a^2 + b^2)*EllipticPi[a^2/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])), -ArcS
```

```
in[Tan[(e + f*x)/2]], -1]))*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*(-(Sec[(e
 + f*x)/2]^4*Sin[e + f*x]) + 2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Tan[(e + f*x
)/2]))/(96*a^2*(a^2 + b^2)^4*Sqrt[b^2*(a^2 + b^2)]*Sqrt[Sec[(e + f*x)/2]^2]
*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]) - (Sqrt[Cos[e + f*x]*Sec[(e + f*x)/
2]^4]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*((4*a*Sqrt[b^2*(a^2 + b^2)]*(-4
*a^6 - 64*a^4*b^2 - 39*a^2*b^4 + 21*b^6)*Sec[(e + f*x)/2]^2)/(Sqrt[1 - Tan[
(e + f*x)/2]^2]*Sqrt[1 + Tan[(e + f*x)/2]^2]) - 21*b^(5/2)*(-9*a^2 + 2*b^2)
*((-2*a*b^(3/2)*(a^2 + b^2)*Sec[(e + f*x)/2]^2)/(Sqrt[1 - Tan[(e + f*x)/2]^
2]*Sqrt[1 + Tan[(e + f*x)/2]^2]*(1 - (a^2*Tan[(e + f*x)/2]^2)/(a^2 + 2*b^2
- 2*Sqrt[b^2*(a^2 + b^2)]))) + (2*a*b^(3/2)*(a^2 + b^2)*Sec[(e + f*x)/2]^2)
/(Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[1 + Tan[(e + f*x)/2]^2]*(1 - (a^2*Tan[(
e + f*x)/2]^2)/(a^2 + 2*(b^2 + Sqrt[b^2*(a^2 + b^2)])))) + Sqrt[b^2*(a^2 +
b^2)]*(((b + Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b
+ Sqrt[a^2 + b^2])]*(-((a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2])/(2*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b
+ Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]) - ((-(Sec[(e + f
*x)/2]^4*Sin[e + f*x]) + 2*Cos[e + f*x]*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2]
)*(a^2 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2))/(4*Sqrt[b]*
Sqrt[2*b^2*(b - Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*(Cos[e + f
*x]*Sec[(e + f*x)/2]^4)^(3/2))))/(1 + (Cos[(e + f*x)/2]^4*Sec[e + f*x]*(a^2
 - (a^2 + 2*b*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)^2)/(4*b*(2*b^2*(b
- Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])))) + ((b - Sqrt[a^2 + b^2
])*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*(-((a^2
+ 2*b*(b + Sqrt[a^2 + b^2]))*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2*Sqrt[b
]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[
e + f*x]*Sec[(e + f*x)/2]^4]) - ((-(Sec[(e + f*x)/2]^4*Sin[e + f*x]) + 2*Co
s[e + f*x]*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2])*(a^2 - (a^2 + 2*b*(b + Sqrt
[a^2 + b^2]))*Tan[(e + f*x)/2]^2))/(4*Sqrt[b]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^
2]) + a^2*(2*b + Sqrt[a^2 + b^2])]*(Cos[e + f*x]*Sec[(e + f*x)/2]^4)^(3/2))
))/(1 + (Cos[(e + f*x)/2]^4*Sec[e + f*x]*(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 +
b^2]))*Tan[(e + f*x)/2]^2)^2)/(4*b*(2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b
+ Sqrt[a^2 + b^2]))))))))/(48*a^2*(a^2 + b^2)^4*Sqrt[b^2*(a^2 + b^2)]*Sqrt[
Sec[(e + f*x)/2]^2]) - ((8*a*Sqrt[b^2*(a^2 + b^2)]*(-4*a^6 - 64*a^4*b^2 - 3
9*a^2*b^4 + 21*b^6)*EllipticF[ArcSin[Tan[(e + f*x)/2]], -1] - 21*b^(5/2)*(-
9*a^2 + 2*b^2)*(Sqrt[b^2*(a^2 + b^2)]*((b + Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b
- Sqrt[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b
*(b - Sqrt[a^2 + b^2]))*Tan[(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b - Sqrt
[a^2 + b^2]) - a^2*(-2*b + Sqrt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x
)/2]^4])] + (b - Sqrt[a^2 + b^2])*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2
*b + Sqrt[a^2 + b^2])]*ArcTan[(a^2 - (a^2 + 2*b*(b + Sqrt[a^2 + b^2]))*Tan[
(e + f*x)/2]^2)/(2*Sqrt[b]*Sqrt[2*b^2*(b + Sqrt[a^2 + b^2]) + a^2*(2*b + Sq
rt[a^2 + b^2])]*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4])]) + 4*a*b^(3/2)*(a^2
 + b^2)*EllipticPi[a^2/(a^2 + 2*b^2 - 2*Sqrt[b^2*(a^2 + b^2)]), -ArcSin[Tan
[(e + f*x)/2]], -1] - 4*a*b^(3/2)*(a^2 + b^2)*EllipticPi[a^2/(a^2 + 2*(b^2
+ Sqrt[b^2*(a^2 + b^2)])), -ArcSin[Tan[(e + f*x)/2]], -1]))*Sqrt[Cos[e + f*
x]*Sec[(e + f*x)/2]^4]*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) +
 Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/(96*a^2*(a^2 + b^2)^4*Sqrt[
b^2*(a^2 + b^2)]*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f
*x]])))
```

Maple [B] time = 2.92, size = 82289, normalized size = 132.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d \sec \left(fx+e\right)\right)^{\frac{3}{2}} \left(b \tan \left(fx+e\right)+a\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3), x)
```

3.623 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

Optimal. Leaf size=814

$$\frac{9\left(11a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}b^{7/2}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}}-\frac{9\left(11a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}b^{7/2}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}}-\frac{9a(}{}$$

```
[Out] (9*b^(7/2)*(11*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(17/4)*d^2*f*Sqrt[d*Sec[
e + f*x]]) - (9*b^(7/2)*(11*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^
(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(17/4)*d^2
*f*Sqrt[d*Sec[e + f*x]]) + (3*a*(8*a^4 + 64*a^2*b^2 - 139*b^4)*EllipticE[Ar
cTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/(20*(a^2 + b^2)^4*d^2*f*Sq
rt[d*Sec[e + f*x]]) - (3*a*(8*a^4 + 64*a^2*b^2 - 139*b^4)*Tan[e + f*x])/(20
*(a^2 + b^2)^4*d^2*f*Sqrt[d*Sec[e + f*x]]) - (9*a*b^3*(11*a^2 - 2*b^2)*Cot[
e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -
1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(9/2)*d^2*f
*Sqrt[d*Sec[e + f*x]]) + (9*a*b^3*(11*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[
b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/
4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(9/2)*d^2*f*Sqrt[d*Sec[e + f*x]])
+ (3*b*(4*a^4 + 28*a^2*b^2 - 15*b^4)*Sec[e + f*x]^2)/(10*(a^2 + b^2)^3*d^2*
f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2) + (2*Cos[e + f*x]^2*(b + a*T
an[e + f*x]))/(5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]
)^2) + (3*a*b*(8*a^4 + 64*a^2*b^2 - 139*b^4)*Sec[e + f*x]^2)/(20*(a^2 + b^2
)^4*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) - (2*(b*(4*a^2 - 9*b^2
) - a*(3*a^2 + 16*b^2)*Tan[e + f*x]))/(5*(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e +
 f*x]]*(a + b*Tan[e + f*x])^2)
```

Rubi [A] time = 0.915303, antiderivative size = 814, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 17, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.68, Rules used = {3512, 741, 823, 835, 844, 227, 196, 746, 399, 490, 1213, 537, 444, 63, 298, 205, 208}

$$\frac{9\left(11a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}b^{7/2}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}}-\frac{9\left(11a^2-2b^2\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}b^{7/2}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}}-\frac{9a(}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] (9*b^(7/2)*(11*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(17/4)*d^2*f*Sqrt[d*Sec[
e + f*x]]) - (9*b^(7/2)*(11*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^
(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(17/4)*d^2
*f*Sqrt[d*Sec[e + f*x]]) + (3*a*(8*a^4 + 64*a^2*b^2 - 139*b^4)*EllipticE[Ar
cTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/(20*(a^2 + b^2)^4*d^2*f*Sq
rt[d*Sec[e + f*x]]) - (3*a*(8*a^4 + 64*a^2*b^2 - 139*b^4)*Tan[e + f*x])/(20
*(a^2 + b^2)^4*d^2*f*Sqrt[d*Sec[e + f*x]]) - (9*a*b^3*(11*a^2 - 2*b^2)*Cot[
e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -
1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(9/2)*d^2*f
*Sqrt[d*Sec[e + f*x]]) + (9*a*b^3*(11*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[
b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/
4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(9/2)*d^2*f*Sqrt[d*Sec[e + f*x]])
```

```
+ (3*b*(4*a^4 + 28*a^2*b^2 - 15*b^4)*Sec[e + f*x]^2)/(10*(a^2 + b^2)^3*d^2*
f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2) + (2*Cos[e + f*x]^2*(b + a*T
an[e + f*x]))/(5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]
)^2) + (3*a*b*(8*a^4 + 64*a^2*b^2 - 139*b^4)*Sec[e + f*x]^2)/(20*(a^2 + b^2
)^4*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) - (2*(b*(4*a^2 - 9*b^2
) - a*(3*a^2 + 16*b^2)*Tan[e + f*x]))/(5*(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e +
 f*x]]*(a + b*Tan[e + f*x])^2)
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 227

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2*x)/(a + b*x^2)^(1/4)
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2*EllipticE[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(5/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 746

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 399

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[(2*Sqrt[-((b*x^2)/a)])/x, Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1213

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{S}}{5\left(a\right.}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} - \frac{2\left(b\left(4a^2-9b^2\right)-\right.}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\,\text{se}}}$$

$$= \frac{3b\left(4a^4+28a^2b^2-15b^4\right)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} + \frac{2\cos^2(e+}{5\left(a^2+b^2\right)d^2f\sqrt{d}}$$

$$= \frac{3b\left(4a^4+28a^2b^2-15b^4\right)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} + \frac{2\cos^2(e+}{5\left(a^2+b^2\right)d^2f\sqrt{d}}$$

$$= \frac{3b\left(4a^4+28a^2b^2-15b^4\right)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} + \frac{2\cos^2(e+}{5\left(a^2+b^2\right)d^2f\sqrt{d}}$$

$$= -\frac{3a\left(8a^4+64a^2b^2-139b^4\right)\tan(e+fx)}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}} + \frac{3b\left(4a^4+28a^2b^2-15b^4\right.}{10\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+f}}$$

$$= \frac{3a\left(8a^4+64a^2b^2-139b^4\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}} - \frac{3}{}$$

$$= \frac{3a\left(8a^4+64a^2b^2-139b^4\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}} - \frac{3}{}$$

$$= \frac{3a\left(8a^4+64a^2b^2-139b^4\right)E\left(\frac{1}{2}\tan^{-1}(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}} - \frac{3}{}$$

$$= \frac{9b^{7/2}\left(11a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}} - \frac{9b^{7/2}\left(11a^2-2b^2\right.}{8\left(a^2\right.}$$

Mathematica [C] time = 28.7014, size = 15513, normalized size = 19.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 4.79, size = 114407, normalized size = 140.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{2}}\left(b\tan\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3), x)
```

3.624 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=78

$$\frac{3ad\sin(e+fx)(d\sec(e+fx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

```
[Out] (3*b*(d*Sec[e + f*x])^(5/3))/(5*f) + (3*a*d*Hypergeometric2F1[-1/3, 1/2, 2/
3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])/(2*f*Sqrt[Sin[e + f
*x]^2])
```

Rubi [A] time = 0.0642202, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3486, 3772, 2643}

$$\frac{3ad\sin(e+fx)(d\sec(e+fx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}}+\frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (3*b*(d*Sec[e + f*x])^(5/3))/(5*f) + (3*a*d*Hypergeometric2F1[-1/3, 1/2, 2/
3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])/(2*f*Sqrt[Sin[e + f
*x]^2])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \frac{3b(d\sec(e+fx))^{5/3}}{5f} + a\int (d\sec(e+fx))^{5/3}\,dx$$
$$= \frac{3b(d\sec(e+fx))^{5/3}}{5f} + \left(a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(e+fx)}{d}\right)^{5/}}$$
$$= \frac{3b(d\sec(e+fx))^{5/3}}{5f} + \frac{3ad\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin}{2f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.506981, size = 126, normalized size = 1.62

$$\frac{d(d\sec(e+fx))^{2/3}(a+b\tan(e+fx))\left(3\cos^2(e+fx)^{2/3}(5a\sin(2(e+fx))+4b)-10a\sin(e+fx)\cos^3(e+fx)\text{Hyperge}\right.}{20f\cos^2(e+fx)^{2/3}(a\cos(e+fx)+b\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (d*(d*Sec[e + f*x])^(2/3)*(-10*a*Cos[e + f*x]^3*Hypergeometric2F1[1/3, 1/2,
 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + 3*(Cos[e + f*x]^2)^(2/3)*(4*b + 5*a*Si
n[2*(e + f*x)]))*(a + b*Tan[e + f*x]))/(20*f*(Cos[e + f*x]^2)^(2/3)*(a*Cos[
e + f*x] + b*Sin[e + f*x]))
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(a+b\tan\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(b\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bd\sec\left(fx+e\right)\tan\left(fx+e\right)+ad\sec\left(fx+e\right)\right)\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*d*sec(f*x + e)*tan(f*x + e) + a*d*sec(f*x + e))*(d*sec(f*x + e)
)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+b*tan(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d \sec\left(fx+e\right)\right)^{\frac{5}{3}} \left(b \tan\left(fx+e\right)+a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a), x)
```

3.625 $\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=76

$$\frac{3b\sqrt[3]{d\sec(e+fx)}}{f}-\frac{3ad\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{2/3}}$$

```
[Out] (3*b*(d*Sec[e + f*x])^(1/3))/f - (3*a*d*Hypergeometric2F1[1/3, 1/2, 4/3, Co
s[e + f*x]^2]*Sin[e + f*x])/(2*f*(d*Sec[e + f*x])^(2/3)*Sqrt[Sin[e + f*x]^2
])
```

Rubi [A] time = 0.0598606, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3486, 3772, 2643}

$$\frac{3b\sqrt[3]{d\sec(e+fx)}}{f}-\frac{3ad\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (3*b*(d*Sec[e + f*x])^(1/3))/f - (3*a*d*Hypergeometric2F1[1/3, 1/2, 4/3, Co
s[e + f*x]^2]*Sin[e + f*x])/(2*f*(d*Sec[e + f*x])^(2/3)*Sqrt[Sin[e + f*x]^2
])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} + a\int \sqrt[3]{d\sec(e+fx)}\,dx$$
$$= \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} + \left(a\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\right)\int \frac{1}{\sqrt[3]{\frac{\cos(e+fx)}{d}}}\,dx$$
$$= \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} - \frac{3a\cos(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(e+fx)\right)\sqrt[3]{d\sec(e+f}}{2f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.174404, size = 58, normalized size = 0.76

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(a\cos^2(e+fx)^{2/3}\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\sin^2(e+fx)\right)+3b\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^(1/3)*(3*b + a*(Cos[e + f*x]^2)^(2/3)*Hypergeometric2F1[1
/2, 2/3, 3/2, Sin[e + f*x]^2]*Tan[e + f*x]))/f
```

Maple [F] time = 0.124, size = 0, normalized size = 0.

$$\int \sqrt[3]{d\sec(fx+e)}\left(a+b\tan(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec(fx+e)\right)^{\frac{1}{3}}\left(b\tan(fx+e)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sec(fx+e)\right)^{\frac{1}{3}}\left(b\tan(fx+e)+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(b\tan\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

3.626 $\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=76

$$-\frac{3ad\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{4/3}}-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] (-3*b)/(f*(d*Sec[e + f*x])^(1/3)) - (3*a*d*Hypergeometric2F1[1/2, 2/3, 5/3,
 Cos[e + f*x]^2]*Sin[e + f*x])/(4*f*(d*Sec[e + f*x])^(4/3)*Sqrt[Sin[e + f*x
]^2])
```

Rubi [A] time = 0.0598214, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3486, 3772, 2643}

$$-\frac{3ad\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{4/3}}-\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (-3*b)/(f*(d*Sec[e + f*x])^(1/3)) - (3*a*d*Hypergeometric2F1[1/2, 2/3, 5/3,
 Cos[e + f*x]^2]*Sin[e + f*x])/(4*f*(d*Sec[e + f*x])^(4/3)*Sqrt[Sin[e + f*x
]^2])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}} + a\int \frac{1}{\sqrt[3]{d\sec(e+fx)}}\,dx$$
$$= -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}} + \left(a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(e+fx)}{d}}\,dx$$
$$= -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}} - \frac{3a\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{4df\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.929204, size = 119, normalized size = 1.57

$$-\frac{3(a\cot(e+fx)+b)\left(a\sqrt{\sin^2(e+fx)}\sqrt{-\tan^2(e+fx)}\cot(e+fx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(e+fx)\right)+b\sin(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}\left(a\sqrt{\sin^2(e+fx)}\cot(e+fx)+b\sin(e+fx)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (-3*(b + a*Cot[e + f*x])*(b*Sin[e + f*x] + a*Cot[e + f*x]*Hypergeometric2F1
[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*Sqrt[Sin[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]
))/(f*(d*Sec[e + f*x])^(1/3)*(b*Sin[e + f*x] + a*Cot[e + f*x]*Sqrt[Sin[e +
f*x]^2]))
```

Maple [F] time = 0.138, size = 0, normalized size = 0.

$$\int (a+b\tan(fx+e))\frac{1}{\sqrt[3]{d\sec(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}\left(b\tan\left(fx+e\right)+a\right)}{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(2/3)*(b*tan(f*x + e) + a)/(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\tan\left(e+fx\right)}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(1/3),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

3.627 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=78

$$-\frac{3ad\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{8/3}}-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$

```
[Out] (-3*b)/(5*f*(d*Sec[e + f*x])^(5/3)) - (3*a*d*Hypergeometric2F1[1/2, 4/3, 7/
3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(8/3)*Sqrt[Sin[e + f
*x]^2])
```

Rubi [A] time = 0.0639063, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3486, 3772, 2643}

$$-\frac{3ad\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{8/3}}-\frac{3b}{5f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (-3*b)/(5*f*(d*Sec[e + f*x])^(5/3)) - (3*a*d*Hypergeometric2F1[1/2, 4/3, 7/
3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(8/3)*Sqrt[Sin[e + f
*x]^2])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3b}{5f(d\sec(e+fx))^{5/3}} + a\int \frac{1}{(d\sec(e+fx))^{5/3}}\,dx$$
$$= -\frac{3b}{5f(d\sec(e+fx))^{5/3}} + \left(a\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\right)\int \left(\frac{\cos(e+fx)}{d}\right)^{5/3} dx$$
$$= -\frac{3b}{5f(d\sec(e+fx))^{5/3}} - \frac{3a\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sin(e+}{8d^2f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.433942, size = 94, normalized size = 1.21

$$\frac{2a\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\sin^2(e+fx)\right)+3\sqrt[3]{\cos^2(e+fx)}(a\sin(e+fx)-b\cos(e+fx))}{5df\sqrt[3]{\cos^2(e+fx)}(d\sec(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (2*a*Hypergeometric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^2]*Sin[e + f*x] + 3*(Cos
[e + f*x]^2)^(1/3)*(-(b*Cos[e + f*x]) + a*Sin[e + f*x]))/(5*d*f*(Cos[e + f*
x]^2)^(1/3)*(d*Sec[e + f*x])^(2/3))
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int (a+b\tan(fx+e))(d\sec(fx+e))^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

```
[Out] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)}{d^2\sec(fx+e)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)/(d^2*sec(f*x + e)^2),
x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b\tan\left(e + fx\right)}{\left(d\sec\left(e + fx\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(5/3),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\tan\left(fx + e\right) + a}{\left(d\sec\left(fx + e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

3.628 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=119

$$\frac{3d(8a^2-3b^2)\sin(e+fx)(d\sec(e+fx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)}{16f\sqrt{\sin^2(e+fx)}}+\frac{33ab(d\sec(e+fx))^{5/3}}{40f}+$$

```
[Out] (33*a*b*(d*Sec[e + f*x])^(5/3))/(40*f) + (3*(8*a^2 - 3*b^2)*d*Hypergeometri
c2F1[-1/3, 1/2, 2/3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])/(
16*f*Sqrt[Sin[e + f*x]^2]) + (3*b*(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x
]))/(8*f)
```

Rubi [A] time = 0.152344, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3508, 3486, 3772, 2643}

$$\frac{3d(8a^2-3b^2)\sin(e+fx)(d\sec(e+fx))^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)}{16f\sqrt{\sin^2(e+fx)}}+\frac{33ab(d\sec(e+fx))^{5/3}}{40f}+$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (33*a*b*(d*Sec[e + f*x])^(5/3))/(40*f) + (3*(8*a^2 - 3*b^2)*d*Hypergeometri
c2F1[-1/3, 1/2, 2/3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])/(
16*f*Sqrt[Sin[e + f*x]^2]) + (3*b*(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x
]))/(8*f)
```

Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}+\frac{3}{8}\int (d\sec(e+fx))^{5/3}\left(\frac{8a^2}{3}-b^2\right.$$

$$= \frac{33ab(d\sec(e+fx))^{5/3}}{40f}+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}+\frac{1}{8}\left(8a^2\right.$$

$$= \frac{33ab(d\sec(e+fx))^{5/3}}{40f}+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}+\frac{1}{8}\left(\left(8a^2\right.\right.$$

$$= \frac{33ab(d\sec(e+fx))^{5/3}}{40f}+\frac{3\left(8a^2-3b^2\right)d\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(e+fx)\right)(d\sec(}{16f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 1.73674, size = 108, normalized size = 0.91

$$\frac{(d\sec(e+fx))^{5/3}\left(-5\left(8a^2-3b^2\right)\sin(2(e+fx))\sqrt[3]{\cos^2(e+fx)}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\sin^2(e+fx)\right)+15\left(8a^2-3\right.\right.}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^(5/3)*(15*(8*a^2 - 3*b^2)*Sin[2*(e + f*x)] - 5*(8*a^2 - 3
*b^2)*(Cos[e + f*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, Sin[e + f*x]^
2]*Sin[2*(e + f*x)] + 12*b*(16*a + 5*b*Tan[e + f*x])))/(160*f)
```

Maple [F] time = 0.147, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(a+b\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 d\sec\left(fx+e\right)\tan\left(fx+e\right)^2+2\,abd\sec\left(fx+e\right)\tan\left(fx+e\right)+a^2 d\sec\left(fx+e\right)\right)\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d*sec(f*x + e)*tan(f*x + e)^2 + 2*a*b*d*sec(f*x + e)*tan(f*x
+ e) + a^2*d*sec(f*x + e))*(d*sec(f*x + e))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(b\tan\left(fx+e\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2, x)
```

3.629 $\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=119

$$-\frac{3d\left(4a^2-3b^2\right)\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{2/3}}+\frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f}+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+}{4f}$$

```
[Out] (21*a*b*(d*Sec[e + f*x])^(1/3))/(4*f) - (3*(4*a^2 - 3*b^2)*d*Hypergeometric
2F1[1/3, 1/2, 4/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(2/3
)*Sqrt[Sin[e + f*x]^2]) + (3*b*(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]))
/(4*f)
```

Rubi [A] time = 0.136399, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3508, 3486, 3772, 2643}

$$-\frac{3d\left(4a^2-3b^2\right)\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{2/3}}+\frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f}+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2,x]
```


```
[Out] (21*a*b*(d*Sec[e + f*x])^(1/3))/(4*f) - (3*(4*a^2 - 3*b^2)*d*Hypergeometric
2F1[1/3, 1/2, 4/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(2/3
)*Sqrt[Sin[e + f*x]^2]) + (3*b*(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]))
/(4*f)
```


Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f} + \frac{3}{4}\int \sqrt[3]{d\sec(e+fx)}\left(\frac{4a^2}{3} - b^2 + \right.$$
$$= \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f} + \frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f} + \frac{1}{4}\left(4a^2 - 3\right.$$
$$= \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f} + \frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f} + \frac{1}{4}\left(\left(4a^2 - \right.\right.$$
$$= \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f} - \frac{3\left(4a^2-3b^2\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.460638, size = 83, normalized size = 0.7

$$\frac{\sqrt[3]{d\sec(e+fx)}\left(\left(4a^2-3b^2\right)\cos^2(e+fx)^{2/3}\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\sin^2(e+fx)\right)+3b(8a+b\tan\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^(1/3)*((4*a^2 - 3*b^2)*(Cos[e + f*x]^2)^(2/3)*Hypergeomet
ric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^2]*Tan[e + f*x] + 3*b*(8*a + b*Tan[e + f
*x])))/(4*f)
```

Maple [F] time = 0.141, size = 0, normalized size = 0.

$$\int \sqrt[3]{d\sec(fx+e)}\left(a+b\tan(fx+e)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec(fx+e)\right)^{\frac{1}{3}}\left(b\tan(fx+e)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2\right)\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^(
1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(b\tan\left(fx+e\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2, x)
```

3.630 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{3d(2a^2-3b^2)\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{4/3}}-\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}}+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] (-15*a*b)/(2*f*(d*Sec[e + f*x])^(1/3)) - (3*(2*a^2 - 3*b^2)*d*Hypergeometri
c2F1[1/2, 2/3, 5/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(4/
3)*Sqrt[Sin[e + f*x]^2]) + (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e + f*x])
^(1/3))
```

Rubi [A] time = 0.142824, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3508, 3486, 3772, 2643}

$$-\frac{3d(2a^2-3b^2)\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{4/3}}-\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}}+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (-15*a*b)/(2*f*(d*Sec[e + f*x])^(1/3)) - (3*(2*a^2 - 3*b^2)*d*Hypergeometri
c2F1[1/2, 2/3, 5/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(4/
3)*Sqrt[Sin[e + f*x]^2]) + (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e + f*x])
^(1/3))
```

Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}} + \frac{3}{2}\int \frac{\frac{2a^2}{3} - b^2 + \frac{5}{3}ab\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

$$= -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}} + \frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}} + \frac{1}{2}\left(2a^2-3b^2\right)\int \frac{1}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

$$= -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}} + \frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}} + \frac{1}{2}\left(\left(2a^2-3b^2\right)\left(\frac{\cos(e+fx)}{d}\right)^{2/3} (d\sec(e+f$$

$$= -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}} - \frac{3\left(2a^2-3b^2\right)\cos^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(e+fx)\right)(d\sec(e+fx))^{2}}{8df\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 3.91785, size = 209, normalized size = 1.76

$$\frac{3d\sin(e+fx)(a+b\tan(e+fx))^2\left(\frac{\left(\left(2a^2-3b^2\right)\cot(e+fx)+4ab\right)\left(\left(2a^2-3b^2\right)\sqrt{\sin^2(e+fx)}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(e+fx)\right)-4ab\cos(e+fx)\right.}{\sqrt{-\tan^2(e+fx)}\left(\left(2a^2-3b^2\right)\sqrt{\sin^2(e+fx)}\cot(e+fx)+4ab\sin(e+fx)\right)}\right.}{2f(d\sec(e+fx))^{4/3}(a\cos(e+fx)+b\sin(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (3*d*Sin[e + f*x]*(a + b*Tan[e + f*x])^2*(b^2 + ((4*a*b + (2*a^2 - 3*b^2)*C
ot[e + f*x])*((2*a^2 - 3*b^2)*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x
]^2]*Sqrt[Sin[e + f*x]^2] - 4*a*b*Cos[e + f*x]*Sqrt[-Tan[e + f*x]^2]))/((4*
a*b*Sin[e + f*x] + (2*a^2 - 3*b^2)*Cot[e + f*x]*Sqrt[Sin[e + f*x]^2])*Sqrt[
-Tan[e + f*x]^2])))/(2*f*(d*Sec[e + f*x])^(4/3)*(a*Cos[e + f*x] + b*Sin[e +
 f*x])^2)
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(fx+e\right)\right)^2 \frac{1}{\sqrt[3]{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2\right)\left(d\sec\left(fx+e\right)\right)^{\frac{2}{3}}}{d\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^(
2/3)/(d*sec(f*x + e)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\tan\left(e+fx\right)\right)^2}{\sqrt[3]{d\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(1/3),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

3.631 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=119

$$-\frac{3d\left(2a^2+3b^2\right)\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)}{16f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{8/3}}+\frac{3ab}{10f(d\sec(e+fx))^{5/3}}-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

```
[Out] (3*a*b)/(10*f*(d*Sec[e + f*x])^(5/3)) - (3*(2*a^2 + 3*b^2)*d*Hypergeometric
2F1[1/2, 4/3, 7/3, Cos[e + f*x]^2]*Sin[e + f*x])/(16*f*(d*Sec[e + f*x])^(8/
3)*Sqrt[Sin[e + f*x]^2]) - (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e + f*x])
^(5/3))
```

Rubi [A] time = 0.153647, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3508, 3486, 3772, 2643}

$$-\frac{3d\left(2a^2+3b^2\right)\sin(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)}{16f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{8/3}}+\frac{3ab}{10f(d\sec(e+fx))^{5/3}}-\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```


```
[Out] (3*a*b)/(10*f*(d*Sec[e + f*x])^(5/3)) - (3*(2*a^2 + 3*b^2)*d*Hypergeometric
2F1[1/2, 4/3, 7/3, Cos[e + f*x]^2]*Sin[e + f*x])/(16*f*(d*Sec[e + f*x])^(8/
3)*Sqrt[Sin[e + f*x]^2]) - (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e + f*x])
^(5/3))
```


Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}} - \frac{3}{2}\int \frac{-\frac{2a^2}{3}-b^2+\frac{1}{3}ab\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$$
$$= \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}} - \frac{1}{2}\left(-2a^2-3b^2\right)\int \frac{1}{(d\sec(e+fx))^{5/3}}\,dx$$
$$= \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}} - \frac{1}{2}\left(\left(-2a^2-3b^2\right)\sqrt[3]{\frac{\cos(e+fx)}{d}}\,\sqrt[3]{d\sec(\ldots}\right.$$
$$= \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3\left(2a^2+3b^2\right)\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(e+fx)\right)\sqrt[3]{d\sec(e+\ldots}}{16d^2f\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.487629, size = 119, normalized size = 1.

$$\frac{\sec^2(e+fx)\left(2\left(2a^2+3b^2\right)\cos^2(e+fx)^{2/3}\tan(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},\sin^2(e+fx)\right)+3a^2\sin(2(e+f\ldots\right.}{10f(d\sec(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (Sec[e + f*x]^2*(-6*a*b - 6*a*b*Cos[2*(e + f*x)] + 3*a^2*Sin[2*(e + f*x)] -
 3*b^2*Sin[2*(e + f*x)] + 2*(2*a^2 + 3*b^2)*(Cos[e + f*x]^2)^(2/3)*Hypergeo
metric2F1[1/2, 2/3, 3/2, Sin[e + f*x]^2]*Tan[e + f*x]))/(10*f*(d*Sec[e + f*
x])^(5/3))
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int \left(a+b\tan\left(fx+e\right)\right)^2\left(d\sec\left(fx+e\right)\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

```
[Out] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2\right)\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}{d^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^(
1/3)/(d^2*sec(f*x + e)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\tan\left(e+fx\right)\right)^2}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(5/3),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\tan\left(fx+e\right)+a\right)^2}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

3.632 $\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=552

$$\frac{\tan(e+fx)(d\sec(e+fx))^{5/3}F_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sec^2(e+fx)^{5/6}}+\frac{(d\sec(e+fx))^{5/3}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(}\right.}{4b^{2/3}f\sqrt[6]{a^2+b^2}\,\mathrm{s}}$$

```
[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a
^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*b^(2/3)*(a^2 + b^2)^(1/6)*f*(S
ec[e + f*x]^2)^(5/6)) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x
]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*b^(2/3)
*(a^2 + b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) - (ArcTanh[(b^(1/3)*(Sec[e + f
*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(5/3))/(b^(2/3)*(a^2 + b^
2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) + (Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2
+ b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Se
c[e + f*x])^(5/3))/(4*b^(2/3)*(a^2 + b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) -
 (Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6)
+ b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(5/3))/(4*b^(2/3)*(a^2 +
 b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) + (AppellF1[1/2, 1, 1/6, 3/2, (b^2*Ta
n[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*Tan[e + f*x])/(a
*f*(Sec[e + f*x]^2)^(5/6))
```

Rubi [A] time = 0.84942, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {3512, 757, 429, 444, 63, 296, 634, 618, 204, 628, 208}

$$\frac{\tan(e+fx)(d\sec(e+fx))^{5/3}F_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sec^2(e+fx)^{5/6}}+\frac{(d\sec(e+fx))^{5/3}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(}\right.}{4b^{2/3}f\sqrt[6]{a^2+b^2}\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x]),x]
```


```
[Out] -(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a
^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*b^(2/3)*(a^2 + b^2)^(1/6)*f*(S
ec[e + f*x]^2)^(5/6)) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x
]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*b^(2/3)
*(a^2 + b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) - (ArcTanh[(b^(1/3)*(Sec[e + f
*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(5/3))/(b^(2/3)*(a^2 + b^
2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) + (Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2
+ b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Se
c[e + f*x])^(5/3))/(4*b^(2/3)*(a^2 + b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) -
 (Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6)
+ b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(5/3))/(4*b^(2/3)*(a^2 +
 b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) + (AppellF1[1/2, 1, 1/6, 3/2, (b^2*Ta
n[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*Tan[e + f*x])/(a
*f*(Sec[e + f*x]^2)^(5/6))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
```

```
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \frac{(d\sec(e+fx))^{5/3}\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{(d\sec(e+fx))^{5/3}\,\text{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\sqrt[6]{1+\frac{x^2}{b^2}}} + \frac{x}{(-a^2+x^2)\sqrt[6]{1+\frac{x^2}{b^2}}}\right) dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{(d\sec(e+fx))^{5/3}\,\text{Subst}\left(\int \frac{x}{(-a^2+x^2)\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}} + \frac{\left(a(d\sec(e+fx))^{5/3}\right)\text{Sub}}{b}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}} + \frac{(d\sec(e+fx))^{5}}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}} + \frac{\left(3b(d\sec(e+f\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}} - \frac{(d\sec(e+fx))^{5}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} + \frac{F_1\left(\frac{1}{2};1,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d}{af\sec^2(e+fx)^{5/6}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} + \frac{\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\right.}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)(d\sec(e+fx))^{5/3}}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)(d\sec(e+fx)}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$

Mathematica [C] time = 5.83324, size = 276, normalized size = 0.5

$$-\frac{24d^2(a+b\tan(e+fx))F_1\left(\frac{1}{3};\frac{1}{6},\frac{1}{6};\frac{4}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)}{bf\sqrt[3]{d\sec(e+fx)}\left((a+ib)F_1\left(\frac{4}{3};\frac{1}{6},\frac{7}{6};\frac{7}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+(a-ib)F_1\left(\frac{4}{3};\frac{7}{6},\frac{1}{6};\frac{7}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+8\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (-24*d^2*AppellF1[1/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(b*f*(d*Sec[e + f*x])^(1/3
)*((a + I*b)*AppellF1[4/3, 1/6, 7/6, 7/3, (a - I*b)/(a + b*Tan[e + f*x]), (
a + I*b)/(a + b*Tan[e + f*x])] + (a - I*b)*AppellF1[4/3, 7/6, 1/6, 7/3, (a
- I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 8*AppellF1[1
/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
 f*x])]*(a + b*Tan[e + f*x])))
```

Maple [F] time = 0.174, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\tan\left(fx+e\right)}\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a), x)
```

3.633 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=552

$$\frac{\tan(e+fx)\sqrt[3]{d\sec(e+fx)}F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sqrt[6]{\sec^2(e+fx)}}+\frac{b^{2/3}\sqrt[3]{d\sec(e+fx)}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+}\right.}{4f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec}}$$

```
[Out] (Sqrt[3]*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqr
t[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*(a^2 + b^2)^(5/6)*f*(Se
c[e + f*x]^2)^(1/6)) - (Sqrt[3]*b^(2/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[
e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*
(a^2 + b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) - (b^(2/3)*ArcTanh[(b^(1/3)*(Se
c[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(1/3))/((a^2 + b^2
)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) + (b^(2/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3
)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)
]*(d*Sec[e + f*x])^(1/3))/(4*(a^2 + b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) -
(b^(2/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)
^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(1/3))/(4*(a^2 +
b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) + (AppellF1[1/2, 1, 5/6, 3/2, (b^2*Tan
[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(1/3)*Tan[e + f*x])/(a*
f*(Sec[e + f*x]^2)^(1/6))
```

Rubi [A] time = 0.768554, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {3512, 757, 429, 444, 63, 210, 634, 618, 204, 628, 208}

$$\frac{\tan(e+fx)\sqrt[3]{d\sec(e+fx)}F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sqrt[6]{\sec^2(e+fx)}}+\frac{b^{2/3}\sqrt[3]{d\sec(e+fx)}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+}\right.}{4f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (Sqrt[3]*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqr
t[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*(a^2 + b^2)^(5/6)*f*(Se
c[e + f*x]^2)^(1/6)) - (Sqrt[3]*b^(2/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[
e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*
(a^2 + b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) - (b^(2/3)*ArcTanh[(b^(1/3)*(Se
c[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(1/3))/((a^2 + b^2
)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) + (b^(2/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3
)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)
]*(d*Sec[e + f*x])^(1/3))/(4*(a^2 + b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) -
(b^(2/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)
^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(1/3))/(4*(a^2 +
b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) + (AppellF1[1/2, 1, 5/6, 3/2, (b^2*Tan
[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(1/3)*Tan[e + f*x])/(a*
f*(Sec[e + f*x]^2)^(1/6))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
```

```
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \frac{\sqrt[3]{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt[3]{d\sec(e+fx)}\,\text{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{5/6}} + \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{5/6}}\right) dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt[3]{d\sec(e+fx)}\,\text{Subst}\left(\int \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}} + \frac{\left(a\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \right.}{bf\sqrt[6]{s}}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}} + \frac{\sqrt[3]{d\sec(e+fx)}\,\text{Subs}}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}} + \frac{\left(3b\sqrt[3]{d\sec(e+fx)}\right)\text{S}}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}} - \frac{\left(b\sqrt[3]{d\sec(e+fx)}\right)\text{Su}}{}$$

$$= -\frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} + \frac{F_1\left(\frac{1}{2};1,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\,\text{s}}}{af\sqrt[6]{\sec^2(e+fx)}}$$

$$= -\frac{b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx}\right.}{4\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\text{se}}}$$

$$= \frac{\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[3]{d\sec(e+fx)}}{2\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} - \frac{\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[3]{d\sec(e+f}}{2\left(a^2+b^2\right)^{5/6}f\sqrt[6]{\sec^2(e+fx)}}$$

Mathematica [C] time = 4.34673, size = 280, normalized size = 0.51

$$-\frac{48d^2(a+b\tan(e+fx))F_1\left(\frac{5}{3};\frac{5}{6},\frac{5}{6};\frac{8}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{\quad}{a+\cdots}\right.}{5bf(d\sec(e+fx))^{5/3}\left(5(a+ib)F_1\left(\frac{8}{3};\frac{5}{6},\frac{11}{6};\frac{11}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+5(a-ib)F_1\left(\frac{8}{3};\frac{11}{6},\frac{5}{6};\frac{11}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{\quad}{a+b\cdots}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (-48*d^2*AppellF1[5/3, 5/6, 5/6, 8/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(5*b*f*(d*Sec[e + f*x])^(5
/3)*(5*(a + I*b)*AppellF1[8/3, 5/6, 11/6, 11/3, (a - I*b)/(a + b*Tan[e + f*
x]), (a + I*b)/(a + b*Tan[e + f*x])] + 5*(a - I*b)*AppellF1[8/3, 11/6, 5/6,
 11/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 16
*AppellF1[5/3, 5/6, 5/6, 8/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a
+ b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\tan\left(fx+e\right)}\sqrt[3]{d\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{d\sec\left(e+fx\right)}}{a+b\tan\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**(1/3)/(a + b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a), x)
```

3.634 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

Optimal. Leaf size=579

$$\frac{\tan(e+fx)\sqrt[6]{\sec^2(e+fx)}F_1\left(\frac{1}{2};1,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sqrt[3]{d\sec(e+fx)}}+\frac{3b}{f\left(a^2+b^2\right)\sqrt[3]{d\sec(e+fx)}}+\frac{b^{4/3}\sqrt[6]{\sec^2(e+f}}{}$$

```
[Out] (3*b)/((a^2 + b^2)*f*(d*Sec[e + f*x])^(1/3)) - (Sqrt[3]*b^(4/3)*ArcTan[1/Sq
rt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Se
c[e + f*x]^2)^(1/6))/(2*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(1/3)) + (Sqrt
[3]*b^(4/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*
(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(1/6))/(2*(a^2 + b^2)^(7/6)*f*(d*Sec[e
 + f*x])^(1/3)) - (b^(4/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^2 +
b^2)^(1/6)]*(Sec[e + f*x]^2)^(1/6))/((a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(
1/3)) + (b^(4/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e +
 f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(4
*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(1/3)) - (b^(4/3)*Log[(a^2 + b^2)^(1/
3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*
x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(4*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x]
)^(1/3)) + (AppellF1[1/2, 1, 7/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f
*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x])/(a*f*(d*Sec[e + f*x])^(1/3))
```

Rubi [A] time = 0.850136, antiderivative size = 579, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3512, 757, 429, 444, 51, 63, 296, 634, 618, 204, 628, 208}

$$\frac{\tan(e+fx)\sqrt[6]{\sec^2(e+fx)}F_1\left(\frac{1}{2};1,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sqrt[3]{d\sec(e+fx)}}+\frac{3b}{f\left(a^2+b^2\right)\sqrt[3]{d\sec(e+fx)}}+\frac{b^{4/3}\sqrt[6]{\sec^2(e+f}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (3*b)/((a^2 + b^2)*f*(d*Sec[e + f*x])^(1/3)) - (Sqrt[3]*b^(4/3)*ArcTan[1/Sq
rt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Se
c[e + f*x]^2)^(1/6))/(2*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(1/3)) + (Sqrt
[3]*b^(4/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*
(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(1/6))/(2*(a^2 + b^2)^(7/6)*f*(d*Sec[e
 + f*x])^(1/3)) - (b^(4/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^2 +
b^2)^(1/6)]*(Sec[e + f*x]^2)^(1/6))/((a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(
1/3)) + (b^(4/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e +
 f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(4
*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(1/3)) - (b^(4/3)*Log[(a^2 + b^2)^(1/
3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*
x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(4*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x]
)^(1/3)) + (AppellF1[1/2, 1, 7/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f
*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x])/(a*f*(d*Sec[e + f*x])^(1/3))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
```

```
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{7/6}} + \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{7/6}}\right) dx, x, b\tan(e+f\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}} + \frac{\left(a\sqrt[6]{\sec^2(e+\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}} + \frac{\sqrt[6]{\sec}}{}$$

$$= \frac{3b}{(a^2+b^2)f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2};1,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2}}{af\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{3b}{(a^2+b^2)f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2};1,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2}}{af\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{3b}{(a^2+b^2)f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2};1,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2}}{af\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{3b}{(a^2+b^2)f\sqrt[3]{d\sec(e+fx)}} - \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{(a^2+b^2)^{7/6}f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1}{}$$

$$= \frac{3b}{(a^2+b^2)f\sqrt[3]{d\sec(e+fx)}} - \frac{b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{(a^2+b^2)^{7/6}f\sqrt[3]{d\sec(e+fx)}} + \frac{b^{4/}}{}$$

$$= \frac{3b}{(a^2+b^2)f\sqrt[3]{d\sec(e+fx)}} - \frac{\sqrt{3}b^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[6]{\sec^2(e+fx)}}{2(a^2+b^2)^{7/6}f\sqrt[3]{d\sec(e+fx)}} +$$

Mathematica [C] time = 21.3437, size = 285, normalized size = 0.49

$$-\frac{60dF_1\left(\frac{7}{3};\frac{7}{6},\frac{7}{6};\frac{10}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)(a\cos(e+fx)+}{7bf(d\sec(e+fx))^{4/3}\left(7(a+ib)F_1\left(\frac{10}{3};\frac{7}{6},\frac{13}{6};\frac{13}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+7(a-ib)F_1\left(\frac{10}{3};\frac{13}{6},\frac{7}{6};\frac{13}{3};\frac{a-ib}{a+b\tan(e+fx)},\frac{a}{a+b\,\text{ta}}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (-60*d*AppellF1[7/3, 7/6, 7/6, 10/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I
*b)/(a + b*Tan[e + f*x])]*(a*Cos[e + f*x] + b*Sin[e + f*x]))/(7*b*f*(d*Sec[
```

```
e + f*x])^(4/3)*(7*(a + I*b)*AppellF1[10/3, 7/6, 13/6, 13/3, (a - I*b)/(a +
 b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 7*(a - I*b)*AppellF1[10
/3, 13/6, 7/6, 13/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e
 + f*x])] + 20*AppellF1[7/3, 7/6, 7/6, 10/3, (a - I*b)/(a + b*Tan[e + f*x])
, (a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Maple [F] time = 0.167, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\tan\left(fx+e\right)}\frac{1}{\sqrt[3]{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(b\tan\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(b\tan\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)), x)
```

3.635 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$

Optimal. Leaf size=581

$$\frac{\tan(e+fx)\sec^2(e+fx)^{5/6}F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af(d\sec(e+fx))^{5/3}}+\frac{3b}{5f\left(a^2+b^2\right)(d\sec(e+fx))^{5/3}}+\frac{b^{8/3}\sec^2(e\cdot}{}$$

```
[Out] (3*b)/(5*(a^2 + b^2)*f*(d*Sec[e + f*x])^(5/3)) + (Sqrt[3]*b^(8/3)*ArcTan[1/
Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(
Sec[e + f*x]^2)^(5/6))/(2*(a^2 + b^2)^(11/6)*f*(d*Sec[e + f*x])^(5/3)) - (S
qrt[3]*b^(8/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[
3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(5/6))/(2*(a^2 + b^2)^(11/6)*f*(d*S
ec[e + f*x])^(5/3)) - (b^(8/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^
2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(5/6))/((a^2 + b^2)^(11/6)*f*(d*Sec[e + f*
x])^(5/3)) + (b^(8/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Se
c[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6
))/(4*(a^2 + b^2)^(11/6)*f*(d*Sec[e + f*x])^(5/3)) - (b^(8/3)*Log[(a^2 + b^
2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[
e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6))/(4*(a^2 + b^2)^(11/6)*f*(d*Sec[e
 + f*x])^(5/3)) + (AppellF1[1/2, 1, 11/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -T
an[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan[e + f*x])/(a*f*(d*Sec[e + f*x])^(
5/3))
```

Rubi [A] time = 0.819225, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3512, 757, 429, 444, 51, 63, 210, 634, 618, 204, 628, 208}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{5/6}F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af(d\sec(e+fx))^{5/3}}+\frac{3b}{5f\left(a^2+b^2\right)(d\sec(e+fx))^{5/3}}+\frac{b^{8/3}\sec^2(e\cdot}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])),x]
```


```
[Out] (3*b)/(5*(a^2 + b^2)*f*(d*Sec[e + f*x])^(5/3)) + (Sqrt[3]*b^(8/3)*ArcTan[1/
Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(
Sec[e + f*x]^2)^(5/6))/(2*(a^2 + b^2)^(11/6)*f*(d*Sec[e + f*x])^(5/3)) - (S
qrt[3]*b^(8/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[
3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(5/6))/(2*(a^2 + b^2)^(11/6)*f*(d*S
ec[e + f*x])^(5/3)) - (b^(8/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^
2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(5/6))/((a^2 + b^2)^(11/6)*f*(d*Sec[e + f*
x])^(5/3)) + (b^(8/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Se
c[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6
))/(4*(a^2 + b^2)^(11/6)*f*(d*Sec[e + f*x])^(5/3)) - (b^(8/3)*Log[(a^2 + b^
2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[
e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6))/(4*(a^2 + b^2)^(11/6)*f*(d*Sec[e
 + f*x])^(5/3)) + (AppellF1[1/2, 1, 11/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -T
an[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan[e + f*x])/(a*f*(d*Sec[e + f*x])^(
5/3))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
```

```
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \frac{\sec^2(e+fx)^{5/6}\operatorname{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\sec^2(e+fx)^{5/6}\operatorname{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{11/6}} + \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{11/6}}\right)dx, x, b\tan(\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\sec^2(e+fx)^{5/6}\operatorname{Subst}\left(\int \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}} + \frac{(a\sec^2(e}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{af(d\sec(e+fx))^{5/3}} + $$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right.}{af(d\sec(e+fx)}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right.}{af(d\sec(e+fx)}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2};1,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right.}{af(d\sec(e+fx)}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} - \frac{b^{8/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} - \frac{b^{8/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} + \frac{\sqrt{3}b^{8/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sec^2(e+f}{2(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$

Mathematica [B] time = 31.7372, size = 6862, normalized size = 11.81

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.169, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\tan\left(fx+e\right)}\left(d\sec\left(fx+e\right)\right)^{-\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(b\tan\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}\left(a+b\tan\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral(1/((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(b\tan\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)), x)
```

3.636 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=687

$$\frac{b^2\tan^3(e+fx)(d\sec(e+fx))^{5/3}F_1\left(\frac{3}{2};2,\frac{1}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f\sec^2(e+fx)^{5/6}}+\frac{\tan(e+fx)(d\sec(e+fx))^{5/3}F_1\left(\frac{1}{2};2,\frac{1}{6};\right.}{a^2f\sec^2(e+}$$

```
[Out] -(a*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b
^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*Sqrt[3]*b^(2/3)*(a^2 + b^2)^(7/6)*f*
(Sec[e + f*x]^2)^(5/6)) + (a*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)
^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*Sqrt[3]*b^(
2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) - (a*ArcTanh[(b^(1/3)*(Sec
[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(5/3))/(3*b^(2/3)*(
a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) + (a*Log[(a^2 + b^2)^(1/3) - b^(
1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1
/3)]*(d*Sec[e + f*x])^(5/3))/(12*b^(2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^
2)^(5/6)) - (a*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f
*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(5/3))/(12*
b^(2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) + (AppellF1[1/2, 2, 1/6
, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*Ta
n[e + f*x])/(a^2*f*(Sec[e + f*x]^2)^(5/6)) + (b^2*AppellF1[3/2, 2, 1/6, 5/2
, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*Tan[e +
 f*x]^3)/(3*a^4*f*(Sec[e + f*x]^2)^(5/6)) - (a*b*(d*Sec[e + f*x])^(5/3))/((
a^2 + b^2)*f*(a^2 - b^2*Tan[e + f*x]^2))
```

Rubi [A] time = 0.944485, antiderivative size = 687, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3512, 757, 429, 444, 51, 63, 296, 634, 618, 204, 628, 208, 510}

$$\frac{b^2\tan^3(e+fx)(d\sec(e+fx))^{5/3}F_1\left(\frac{3}{2};2,\frac{1}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f\sec^2(e+fx)^{5/6}}+\frac{\tan(e+fx)(d\sec(e+fx))^{5/3}F_1\left(\frac{1}{2};2,\frac{1}{6};\right.}{a^2f\sec^2(e+}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -(a*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b
^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*Sqrt[3]*b^(2/3)*(a^2 + b^2)^(7/6)*f*
(Sec[e + f*x]^2)^(5/6)) + (a*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)
^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*Sqrt[3]*b^(
2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) - (a*ArcTanh[(b^(1/3)*(Sec
[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(5/3))/(3*b^(2/3)*(
a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) + (a*Log[(a^2 + b^2)^(1/3) - b^(
1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1
/3)]*(d*Sec[e + f*x])^(5/3))/(12*b^(2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^
2)^(5/6)) - (a*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f
*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(5/3))/(12*
b^(2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) + (AppellF1[1/2, 2, 1/6
, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*Ta
n[e + f*x])/(a^2*f*(Sec[e + f*x]^2)^(5/6)) + (b^2*AppellF1[3/2, 2, 1/6, 5/2
, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*Tan[e +
 f*x]^3)/(3*a^4*f*(Sec[e + f*x]^2)^(5/6)) - (a*b*(d*Sec[e + f*x])^(5/3))/((
```

```
a^2 + b^2)*f*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \frac{(d\sec(e+fx))^{5/3}\operatorname{Subst}\left(\int \frac{1}{(a+x)^2\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{(d\sec(e+fx))^{5/3}\operatorname{Subst}\left(\int \left(\frac{a^2}{\left(a^2-x^2\right)^2\sqrt[6]{1+\frac{x^2}{b^2}}} - \frac{2ax}{\left(a^2-x^2\right)^2\sqrt[6]{1+\frac{x^2}{b^2}}} + \frac{x^2}{\left(-a^2+x^2\right)^2\sqrt[6]{1+\frac{x^2}{b^2}}}\right) dx, x, b\tan(e}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{(d\sec(e+fx))^{5/3}\operatorname{Subst}\left(\int \frac{x^2}{\left(-a^2+x^2\right)^2\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}} - \frac{\left(2a(d\sec(e+fx))^{5/3}\right)\operatorname{Su}}{b}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}} + \frac{b^2F_1\left(\frac{3}{2};2,\frac{1}{6};\frac{5}{2};\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}} + \frac{b^2F_1\left(\frac{3}{2};2,\frac{1}{6};\frac{5}{2};\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}} + \frac{b^2F_1\left(\frac{3}{2};2,\frac{1}{6};\frac{5}{2};\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}} + \frac{b^2F_1\left(\frac{3}{2};2,\frac{1}{6};\frac{5}{2};\right.}{}$$

$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{3b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}} + \frac{F_1\left(\frac{1}{2};2,\frac{1}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d}{a^2f\sec^2(e+fx)^{5}}$$

$$= -\frac{a\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{3b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}} + \frac{a\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx}\right.}{12b^{2/3}\left(a^2+b^2\right)^{7/6}f}$$

$$= -\frac{a\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}} + \frac{a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}b^{2/3}\left(a^2+b^2\right)^{7/6}f\sec^2(e+fx)^{5/6}}$$

Mathematica [C] time = 39.1889, size = 3398, normalized size = 4.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]*(d*Sec[e + f*x])^(5/3)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((
b*Cos[e + f*x])/(a*(a - I*b)*(a + I*b)) + Sin[e + f*x]/((a - I*b)*(a + I*b)
) - b/((a - I*b)*(a + I*b)*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*T
an[e + f*x])^2) - (4*AppellF1[1/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e +
f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(d*Sec[e + f*x])^(5/3)*(a*Cos[e + f*
```

```
x] + b*Sin[e + f*x])^2)/(a*b*f*(a + b*Tan[e + f*x])*((a + I*b)*AppellF1[4/3
, 1/6, 7/6, 7/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f
*x])] + (a - I*b)*AppellF1[4/3, 7/6, 1/6, 7/3, (a - I*b)/(a + b*Tan[e + f*x
]), (a + I*b)/(a + b*Tan[e + f*x])] + 8*AppellF1[1/3, 1/6, 1/6, 4/3, (a - I
*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*
x]))) - (Sec[e + f*x]*(d*Sec[e + f*x])^(5/3)*((6*(b + a*Sqrt[1 - Cos[e + f*
x]^2]*Sec[e + f*x]))/((a^2 + b^2)*Sec[e + f*x]^(1/3)) + (132*a*b^(2/3)*(5*a
^2 + 3*b^2)*AppellF1[5/6, 1/2, 1, 11/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2
)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(8/3) - 240*a*b^(8/3)*
AppellF1[11/6, 1/2, 1, 17/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^
2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(14/3) - 55*(-1)^(1/6)*(a^2 - b^2
)*(a^2 + b^2)^(5/6)*(-2*ArcTan[Sqrt[3] - (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]
^(1/3))/(a^2 + b^2)^(1/6)] + 2*ArcTan[Sqrt[3] + (2*(-1)^(1/6)*b^(1/3)*Sec[e
 + f*x]^(1/3))/(a^2 + b^2)^(1/6)] + 4*ArcTan[((-1)^(1/6)*b^(1/3)*Sec[e + f*
x]^(1/3))/(a^2 + b^2)^(1/6)] + Sqrt[3]*Log[(a^2 + b^2)^(1/3) - (-1)^(1/6)*S
qrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Se
c[e + f*x]^(2/3)] - Sqrt[3]*Log[(a^2 + b^2)^(1/3) + (-1)^(1/6)*Sqrt[3]*b^(1
/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^
(2/3)])*Sqrt[1 - Sec[e + f*x]^2])/(220*b^(2/3)*(a^2 + b^2)^2*Sqrt[1 - Sec[e
 + f*x]^2]))*(Cos[e + f*x] - Sin[e + f*x])*(Cos[e + f*x] + Sin[e + f*x])*(a
*Cos[e + f*x] + b*Sin[e + f*x]))/(6*a*f*(a + b*Tan[e + f*x])^2*((-2*Sec[e +
 f*x]^(2/3)*(b + a*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])*Sin[e + f*x])/(a^
2 + b^2) + (Sec[e + f*x]^2*(132*a*b^(2/3)*(5*a^2 + 3*b^2)*AppellF1[5/6, 1/2
, 1, 11/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e
 + f*x]^2]*Sec[e + f*x]^(8/3) - 240*a*b^(8/3)*AppellF1[11/6, 1/2, 1, 17/6,
Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*
Sec[e + f*x]^(14/3) - 55*(-1)^(1/6)*(a^2 - b^2)*(a^2 + b^2)^(5/6)*(-2*ArcTa
n[Sqrt[3] - (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)] +
2*ArcTan[Sqrt[3] + (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1
/6)] + 4*ArcTan[((-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)]
+ Sqrt[3]*Log[(a^2 + b^2)^(1/3) - (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1
/6)*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)] - Sqrt[3]*L
og[(a^2 + b^2)^(1/3) + (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e +
 f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)])*Sqrt[1 - Sec[e + f*x]
^2])*Tan[e + f*x])/(220*b^(2/3)*(a^2 + b^2)^2*(1 - Sec[e + f*x]^2)^(3/2)) +
 (6*((a*Sin[e + f*x])/Sqrt[1 - Cos[e + f*x]^2] + a*Sqrt[1 - Cos[e + f*x]^2]
*Sec[e + f*x]*Tan[e + f*x]))/((a^2 + b^2)*Sec[e + f*x]^(1/3)) + ((132*a*b^(
2/3)*(5*a^2 + 3*b^2)*AppellF1[5/6, 1/2, 1, 11/6, Sec[e + f*x]^2, (b^2*Sec[e
 + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^(5/3)*Sin[e + f*x])/Sqrt[1 - Cos[e + f
*x]^2] - (240*a*b^(8/3)*AppellF1[11/6, 1/2, 1, 17/6, Sec[e + f*x]^2, (b^2*S
ec[e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^(11/3)*Sin[e + f*x])/Sqrt[1 - Cos[
e + f*x]^2] + 352*a*b^(2/3)*(5*a^2 + 3*b^2)*AppellF1[5/6, 1/2, 1, 11/6, Sec
[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec
[e + f*x]^(11/3)*Sin[e + f*x] - 1120*a*b^(8/3)*AppellF1[11/6, 1/2, 1, 17/6,
 Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]
*Sec[e + f*x]^(17/3)*Sin[e + f*x] - 55*(-1)^(1/6)*(a^2 - b^2)*(a^2 + b^2)^(
5/6)*Sqrt[1 - Sec[e + f*x]^2]*((4*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(4/3)*Sin
[e + f*x])/(3*(a^2 + b^2)^(1/6)*(1 + (Sqrt[3] - (2*(-1)^(1/6)*b^(1/3)*Sec[e
 + f*x]^(1/3))/(a^2 + b^2)^(1/6))^2)) + (4*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^
(4/3)*Sin[e + f*x])/(3*(a^2 + b^2)^(1/6)*(1 + (Sqrt[3] + (2*(-1)^(1/6)*b^(1
/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6))^2)) + (4*(-1)^(1/6)*b^(1/3)*Sec[
e + f*x]^(4/3)*Sin[e + f*x])/(3*(a^2 + b^2)^(1/6)*(1 + ((-1)^(1/3)*b^(2/3)*
Sec[e + f*x]^(2/3))/(a^2 + b^2)^(1/3))) + (Sqrt[3]*(-(((-1)^(1/6)*b^(1/3)*(
a^2 + b^2)^(1/6)*Sec[e + f*x]^(4/3)*Sin[e + f*x])/Sqrt[3]) + (2*(-1)^(1/3)*
b^(2/3)*Sec[e + f*x]^(5/3)*Sin[e + f*x])/3))/((a^2 + b^2)^(1/3) - (-1)^(1/6
)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)
*Sec[e + f*x]^(2/3)) - (Sqrt[3]*(((-1)^(1/6)*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[
e + f*x]^(4/3)*Sin[e + f*x])/Sqrt[3] + (2*(-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(
5/3)*Sin[e + f*x])/3))/((a^2 + b^2)^(1/3) + (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2
```

```
 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)))
+ (55*(-1)^(1/6)*(a^2 - b^2)*(a^2 + b^2)^(5/6)*(-2*ArcTan[Sqrt[3] - (2*(-1)
^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)] + 2*ArcTan[Sqrt[3] +
(2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)] + 4*ArcTan[((-
1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)] + Sqrt[3]*Log[(a^2
+ b^2)^(1/3) - (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1
/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)] - Sqrt[3]*Log[(a^2 + b^2)^(1/3
) + (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1)^
(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)])*Sec[e + f*x]^2*Tan[e + f*x])/Sqrt[1 - Se
c[e + f*x]^2] + 132*a*b^(2/3)*(5*a^2 + 3*b^2)*Sqrt[1 - Cos[e + f*x]^2]*Sec[
e + f*x]^(8/3)*((10*b^2*AppellF1[11/6, 1/2, 2, 17/6, Sec[e + f*x]^2, (b^2*S
ec[e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/(11*(a^2 + b^2)) +
 (5*AppellF1[11/6, 3/2, 1, 17/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2
+ b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/11) - 240*a*b^(8/3)*Sqrt[1 - Cos[e + f
*x]^2]*Sec[e + f*x]^(14/3)*((22*b^2*AppellF1[17/6, 1/2, 2, 23/6, Sec[e + f*
x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/(17*(a
^2 + b^2)) + (11*AppellF1[17/6, 3/2, 1, 23/6, Sec[e + f*x]^2, (b^2*Sec[e +
f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/17))/(220*b^(2/3)*(a^2 +
b^2)^2*Sqrt[1 - Sec[e + f*x]^2])))
```

Maple [F] time = 0.278, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\tan\left(fx+e\right)\right)^{2}}\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}{\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a)^2, x)
```

3.637 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=687

$$\frac{b^2\tan^3(e+fx)\sqrt[3]{d\sec(e+fx)}F_1\left(\frac{3}{2};2,\frac{5}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f\sqrt[6]{\sec^2(e+fx)}}+\frac{\tan(e+fx)\sqrt[3]{d\sec(e+fx)}F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};\frac{b^2\tan\ldots\right.}{a^2f\sqrt[6]{\sec^2(e+fx)}}$$

```
[Out] (5*a*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]
*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*Sqrt[3]*(a^2 + b^2)^(11/6)*
f*(Sec[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec
[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2
*Sqrt[3]*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*ArcTan
h[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(1/3
))/(3*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) + (5*a*b^(2/3)*Log[(a^2
+ b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(
Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(1/3))/(12*(a^2 + b^2)^(11/6)*f*(Se
c[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 +
b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[
e + f*x])^(1/3))/(12*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) + (Appell
F1[1/2, 2, 5/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e +
f*x])^(1/3)*Tan[e + f*x])/(a^2*f*(Sec[e + f*x]^2)^(1/6)) + (b^2*AppellF1[3/
2, 2, 5/6, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])
^(1/3)*Tan[e + f*x]^3)/(3*a^4*f*(Sec[e + f*x]^2)^(1/6)) - (a*b*(d*Sec[e + f
*x])^(1/3))/((a^2 + b^2)*f*(a^2 - b^2*Tan[e + f*x]^2))
```

Rubi [A] time = 0.883918, antiderivative size = 687, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3512, 757, 429, 444, 51, 63, 210, 634, 618, 204, 628, 208, 510}

$$\frac{b^2\tan^3(e+fx)\sqrt[3]{d\sec(e+fx)}F_1\left(\frac{3}{2};2,\frac{5}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f\sqrt[6]{\sec^2(e+fx)}}+\frac{\tan(e+fx)\sqrt[3]{d\sec(e+fx)}F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};\frac{b^2\tan\ldots\right.}{a^2f\sqrt[6]{\sec^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x])^2,x]
```


```
[Out] (5*a*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]
*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*Sqrt[3]*(a^2 + b^2)^(11/6)*
f*(Sec[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec
[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2
*Sqrt[3]*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*ArcTan
h[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(1/3
))/(3*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) + (5*a*b^(2/3)*Log[(a^2
+ b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(
Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(1/3))/(12*(a^2 + b^2)^(11/6)*f*(Se
c[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 +
b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[
e + f*x])^(1/3))/(12*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) + (Appell
F1[1/2, 2, 5/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e +
f*x])^(1/3)*Tan[e + f*x])/(a^2*f*(Sec[e + f*x]^2)^(1/6)) + (b^2*AppellF1[3/
2, 2, 5/6, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])
^(1/3)*Tan[e + f*x]^3)/(3*a^4*f*(Sec[e + f*x]^2)^(1/6)) - (a*b*(d*Sec[e + f
```

```
*x])^(1/3))/((a^2 + b^2)*f*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt[3]{d\sec(e+fx)}\,\mathrm{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt[3]{d\sec(e+fx)}\,\mathrm{Subst}\left(\int \left(\frac{a^2}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}} - \frac{2ax}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}} + \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}}\right)dx, x, b\right.}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt[3]{d\sec(e+fx)}\,\mathrm{Subst}\left(\int \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}} - \frac{\left(2a\sqrt[3]{d\sec(e+fx)}\right)\mathrm{Su}}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2F_1\left(\frac{3}{2};2,\frac{5}{6};\frac{5}{2}\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2F_1\left(\frac{3}{2};2,\frac{5}{6};\frac{5}{2}\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2F_1\left(\frac{3}{2};2,\frac{5}{6};\frac{5}{2}\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2F_1\left(\frac{3}{2};2,\frac{5}{6};\frac{5}{2}\right.}{}$$

$$= -\frac{5ab^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{3\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}} + \frac{F_1\left(\frac{1}{2};2,\frac{5}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+\right.}{a^2f\sqrt[6]{\sec^2(e}}$$

$$= -\frac{5ab^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{3\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}} + \frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{s}\right.}{12\left(a^2+b\right.}$$

$$= \frac{5ab^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[3]{d\sec(e+fx)}}{2\sqrt{3}\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}} - \frac{5ab^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[3]{d\sec(e}}{2\sqrt{3}\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}}$$

Mathematica [C] time = 25.4959, size = 4485, normalized size = 6.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^(1/3)*((5*(-1)^(5/6)*a*b^(2/3)*(-2*ArcTan[Sqrt[3] - (2*(-
1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)] + 2*ArcTan[Sqrt[3]
+ (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)] + 4*ArcTan[(
```

```
(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6)] - Sqrt[3]*Log[(a^
2 + b^2)^(1/3) - (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^
(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)] + Sqrt[3]*Log[(a^2 + b^2)^(1
/3) + (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) + (-1
)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)]))/(12*(a - I*b)*(a + I*b)*(a^2 + b^2)^(
5/6)) + 3*((-2*b^2*AppellF1[7/6, 1/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[e +
 f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(10/3))/(21*(a^
2 + b^2)^2*Sqrt[1 - Sec[e + f*x]^2]) + (Sec[e + f*x]^(1/3)*((-(a*b) + b^2*S
qrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/(a^2 + b^2) + (7*(3*a^2 - 2*b^2)*Appe
llF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sq
rt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/((-1 + Sec[e + f*x]^2)*(7*(a^2 + b^2)*
AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)
] + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^
2)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6, 3/2, 1, 13/6, Sec[e + f*x]^2, (
b^2*Sec[e + f*x]^2)/(a^2 + b^2)])*Sec[e + f*x]^2))))/(3*(a^2 - b^2*(-1 + Se
c[e + f*x]^2))))))/(f*(a + b*Tan[e + f*x])^2*((5*(-1)^(5/6)*a*b^(2/3)*((4*(
-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(4/3)*Sin[e + f*x])/(3*(a^2 + b^2)^(1/6)*(1
+ (Sqrt[3] - (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)^(1/6))^2
)) + (4*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(4/3)*Sin[e + f*x])/(3*(a^2 + b^2)^
(1/6)*(1 + (Sqrt[3] + (2*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(1/3))/(a^2 + b^2)
^(1/6))^2)) + (4*(-1)^(1/6)*b^(1/3)*Sec[e + f*x]^(4/3)*Sin[e + f*x])/(3*(a^
2 + b^2)^(1/6)*(1 + ((-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3))/(a^2 + b^2)^(1/
3))) - (Sqrt[3]*(-(((-1)^(1/6)*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(4/3)
*Sin[e + f*x])/Sqrt[3]) + (2*(-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(5/3)*Sin[e +
f*x])/3))/((a^2 + b^2)^(1/3) - (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)
*Sec[e + f*x]^(1/3) + (-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3)) + (Sqrt[3]*(((
-1)^(1/6)*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(4/3)*Sin[e + f*x])/Sqrt[3
] + (2*(-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(5/3)*Sin[e + f*x])/3))/((a^2 + b^2)
^(1/3) + (-1)^(1/6)*Sqrt[3]*b^(1/3)*(a^2 + b^2)^(1/6)*Sec[e + f*x]^(1/3) +
(-1)^(1/3)*b^(2/3)*Sec[e + f*x]^(2/3))))/(12*(a - I*b)*(a + I*b)*(a^2 + b^2
)^(5/6)) + 3*((-2*b^2*AppellF1[7/6, 1/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[
e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(19/3)*Sin[e
 + f*x])/(21*(a^2 + b^2)^2*(1 - Sec[e + f*x]^2)^(3/2)) - (2*b^2*AppellF1[7/
6, 1/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sec[e +
f*x]^(7/3)*Sin[e + f*x])/(21*(a^2 + b^2)^2*Sqrt[1 - Cos[e + f*x]^2]*Sqrt[1
- Sec[e + f*x]^2]) - (20*b^2*AppellF1[7/6, 1/2, 1, 13/6, Sec[e + f*x]^2, (b
^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(13/3
)*Sin[e + f*x])/(63*(a^2 + b^2)^2*Sqrt[1 - Sec[e + f*x]^2]) + (2*b^2*Sec[e
+ f*x]^(10/3)*((-(a*b) + b^2*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/(a^2 +
b^2) + (7*(3*a^2 - 2*b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*S
ec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/((-1 + S
ec[e + f*x]^2)*(7*(a^2 + b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b
^2*Sec[e + f*x]^2)/(a^2 + b^2)] + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[
e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6, 3
/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)])*Sec[e + f*x
]^2)))*Sin[e + f*x])/(3*(a^2 - b^2*(-1 + Sec[e + f*x]^2))^2) + (Sec[e + f*x
]^(4/3)*((-(a*b) + b^2*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/(a^2 + b^2) +
 (7*(3*a^2 - 2*b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e +
 f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x])/((-1 + Sec[e +
 f*x]^2)*(7*(a^2 + b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec
[e + f*x]^2)/(a^2 + b^2)] + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[e + f*
x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6, 3/2, 1,
 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)])*Sec[e + f*x]^2)))
*Sin[e + f*x])/(9*(a^2 - b^2*(-1 + Sec[e + f*x]^2))) - (2*b^2*Sqrt[1 - Cos[
e + f*x]^2]*Sec[e + f*x]^(10/3)*((14*b^2*AppellF1[13/6, 1/2, 2, 19/6, Sec[e
 + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/(
13*(a^2 + b^2)) + (7*AppellF1[13/6, 3/2, 1, 19/6, Sec[e + f*x]^2, (b^2*Sec[
e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/13))/(21*(a^2 + b^2)^
2*Sqrt[1 - Sec[e + f*x]^2]) + (Sec[e + f*x]^(1/3)*((7*(3*a^2 - 2*b^2)*Appel
```

```
lF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sin
[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + Sec[e + f*x]^2)*(7*(a^2 + b^2)*A
ppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]
 + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2
)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6, 3/2, 1, 13/6, Sec[e + f*x]^2, (b
^2*Sec[e + f*x]^2)/(a^2 + b^2)])*Sec[e + f*x]^2)) - (14*(3*a^2 - 2*b^2)*App
ellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*S
qrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^3*Tan[e + f*x])/((-1 + Sec[e + f*x]^2)
^2*(7*(a^2 + b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f
*x]^2)/(a^2 + b^2)] + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[e + f*x]^2,
(b^2*Sec[e + f*x]^2)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6, 3/2, 1, 13/6,
 Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)])*Sec[e + f*x]^2)) + (7*(
3*a^2 - 2*b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]
^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]*Tan[e + f*x])/((-1 +
 Sec[e + f*x]^2)*(7*(a^2 + b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2,
(b^2*Sec[e + f*x]^2)/(a^2 + b^2)] + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Se
c[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6,
 3/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)])*Sec[e + f
*x]^2)) + ((b^2*Sin[e + f*x])/Sqrt[1 - Cos[e + f*x]^2] + b^2*Sqrt[1 - Cos[e
 + f*x]^2]*Sec[e + f*x]*Tan[e + f*x])/(a^2 + b^2) + (7*(3*a^2 - 2*b^2)*Sqrt
[1 - Cos[e + f*x]^2]*Sec[e + f*x]*((2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[e
 + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/(
7*(a^2 + b^2)) + (AppellF1[7/6, 3/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[e +
f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/7))/((-1 + Sec[e + f*x]^2
)*(7*(a^2 + b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*
x]^2)/(a^2 + b^2)] + 3*(2*b^2*AppellF1[7/6, 1/2, 2, 13/6, Sec[e + f*x]^2, (
b^2*Sec[e + f*x]^2)/(a^2 + b^2)] + (a^2 + b^2)*AppellF1[7/6, 3/2, 1, 13/6,
Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)])*Sec[e + f*x]^2)) - (7*(3
*a^2 - 2*b^2)*AppellF1[1/6, 1/2, 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^
2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]*(6*(2*b^2*AppellF1[7/
6, 1/2, 2, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)] + (a^2 +
 b^2)*AppellF1[7/6, 3/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2
 + b^2)])*Sec[e + f*x]^2*Tan[e + f*x] + 7*(a^2 + b^2)*((2*b^2*AppellF1[7/6,
 1/2, 2, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*
x]^2*Tan[e + f*x])/(7*(a^2 + b^2)) + (AppellF1[7/6, 3/2, 1, 13/6, Sec[e + f
*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/7) +
3*Sec[e + f*x]^2*(2*b^2*((28*b^2*AppellF1[13/6, 1/2, 3, 19/6, Sec[e + f*x]^
2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/(13*(a^2
+ b^2)) + (7*AppellF1[13/6, 3/2, 2, 19/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]
^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Tan[e + f*x])/13) + (a^2 + b^2)*((14*b^2*Ap
pellF1[13/6, 3/2, 2, 19/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)
]*Sec[e + f*x]^2*Tan[e + f*x])/(13*(a^2 + b^2)) + (21*AppellF1[13/6, 5/2, 1
, 19/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sec[e + f*x]^2*Ta
n[e + f*x])/13))))/((-1 + Sec[e + f*x]^2)*(7*(a^2 + b^2)*AppellF1[1/6, 1/2,
 1, 7/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)] + 3*(2*b^2*Appel
lF1[7/6, 1/2, 2, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)] +
(a^2 + b^2)*AppellF1[7/6, 3/2, 1, 13/6, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2
)/(a^2 + b^2)])*Sec[e + f*x]^2)^2)))/(3*(a^2 - b^2*(-1 + Sec[e + f*x]^2))))
))
```

Maple [F] time = 0.274, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\tan\left(fx+e\right)\right)^{2}}\sqrt[3]{d\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{d \sec \left(e + fx\right)}}{\left(a + b \tan \left(e + fx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**(1/3)/(a + b*tan(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec \left(fx + e\right)\right)^{\frac{1}{3}}}{\left(b \tan \left(fx + e\right) + a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a)^2, x)
```

3.638 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=715

$$\frac{b^2\tan^3(e+fx)\sqrt[6]{\sec^2(e+fx)}F_1\left(\frac{3}{2};2,\frac{7}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f\sqrt[3]{d\sec(e+fx)}}+\frac{\tan(e+fx)\sqrt[6]{\sec^2(e+fx)}F_1\left(\frac{1}{2};2,\frac{7}{6};\frac{3}{2};\frac{b^2}{}\right.}{a^2f\sqrt[3]{d\sec(e+fx}}$$

```
[Out] (7*a*b)/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(1/3)) - (7*a*b^(4/3)*ArcTan[1/Sq
rt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Se
c[e + f*x]^2)^(1/6))/(2*Sqrt[3]*(a^2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)
) + (7*a*b^(4/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqr
t[3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(1/6))/(2*Sqrt[3]*(a^2 + b^2)^(13
/6)*f*(d*Sec[e + f*x])^(1/3)) - (7*a*b^(4/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]
^2)^(1/6))/(a^2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(1/6))/(3*(a^2 + b^2)^(13/6)
*f*(d*Sec[e + f*x])^(1/3)) + (7*a*b^(4/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(
a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(
Sec[e + f*x]^2)^(1/6))/(12*(a^2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)) - (
7*a*b^(4/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]
^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(12*(a^
2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)) + (AppellF1[1/2, 2, 7/6, 3/2, (b^
2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x]
)/(a^2*f*(d*Sec[e + f*x])^(1/3)) + (b^2*AppellF1[3/2, 2, 7/6, 5/2, (b^2*Tan
[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x]^3)/(
3*a^4*f*(d*Sec[e + f*x])^(1/3)) - (a*b)/((a^2 + b^2)*f*(d*Sec[e + f*x])^(1/
3)*(a^2 - b^2*Tan[e + f*x]^2))
```

Rubi [A] time = 0.99437, antiderivative size = 715, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3512, 757, 429, 444, 51, 63, 296, 634, 618, 204, 628, 208, 510}

$$\frac{b^2\tan^3(e+fx)\sqrt[6]{\sec^2(e+fx)}F_1\left(\frac{3}{2};2,\frac{7}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f\sqrt[3]{d\sec(e+fx)}}+\frac{\tan(e+fx)\sqrt[6]{\sec^2(e+fx)}F_1\left(\frac{1}{2};2,\frac{7}{6};\frac{3}{2};\frac{b^2}{}\right.}{a^2f\sqrt[3]{d\sec(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (7*a*b)/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(1/3)) - (7*a*b^(4/3)*ArcTan[1/Sq
rt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Se
c[e + f*x]^2)^(1/6))/(2*Sqrt[3]*(a^2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)
) + (7*a*b^(4/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqr
t[3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(1/6))/(2*Sqrt[3]*(a^2 + b^2)^(13
/6)*f*(d*Sec[e + f*x])^(1/3)) - (7*a*b^(4/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]
^2)^(1/6))/(a^2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(1/6))/(3*(a^2 + b^2)^(13/6)
*f*(d*Sec[e + f*x])^(1/3)) + (7*a*b^(4/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(
a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(
Sec[e + f*x]^2)^(1/6))/(12*(a^2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)) - (
7*a*b^(4/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]
^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(12*(a^
2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)) + (AppellF1[1/2, 2, 7/6, 3/2, (b^
2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x]
)/(a^2*f*(d*Sec[e + f*x])^(1/3)) + (b^2*AppellF1[3/2, 2, 7/6, 5/2, (b^2*Tan
[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x]^3)/(
```

```
3*a^4*f*(d*Sec[e + f*x])^(1/3)) - (a*b)/((a^2 + b^2)*f*(d*Sec[e + f*x])^(1/
3)*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
```

```
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \left(\frac{a^2}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}} - \frac{2ax}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}} + \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)}\right.\right.}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}} - \frac{\left(2a\sqrt[6]{\sec^2(\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2f\sqrt[3]{d\sec(e+fx)}} + \frac{b^2F}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2f\sqrt[3]{d\sec(e+fx)}} + \frac{b^2F}{}$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2};2,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\text{se}}}{a^2f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2};2,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\text{se}}}{a^2f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} + \frac{F_1\left(\frac{1}{2};2,\frac{7}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\text{se}}}{a^2f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{3(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} -$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{3(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} -$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\sqrt{3}(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} -$$

Mathematica [C] time = 51.3887, size = 9626, normalized size = 13.46

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.278, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\tan\left(fx+e\right)\right)^{2}}\frac{1}{\sqrt[3]{d\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)

[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")

[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e))**2,x)

[Out] Integral(1/((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2), x)
```

3.639 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=717

$$\frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{5/6}F_1\left(\frac{3}{2};2,\frac{11}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f(d\sec(e+fx))^{5/3}}+\frac{\tan(e+fx)\sec^2(e+fx)^{5/6}F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2}\right.}{a^2f(d\sec(e+f}$$

```
[Out] (11*a*b)/(5*(a^2 + b^2)^2*f*(d*Sec[e + f*x])^(5/3)) + (11*a*b^(8/3)*ArcTan[
1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]
*(Sec[e + f*x]^2)^(5/6))/(2*Sqrt[3]*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(
5/3)) - (11*a*b^(8/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))
/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(5/6))/(2*Sqrt[3]*(a^2 + b^2
)^(17/6)*f*(d*Sec[e + f*x])^(5/3)) - (11*a*b^(8/3)*ArcTanh[(b^(1/3)*(Sec[e
+ f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(5/6))/(3*(a^2 + b^2)^
(17/6)*f*(d*Sec[e + f*x])^(5/3)) + (11*a*b^(8/3)*Log[(a^2 + b^2)^(1/3) - b^
(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(
1/3)]*(Sec[e + f*x]^2)^(5/6))/(12*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(5/
3)) - (11*a*b^(8/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[
e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6))
/(12*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(5/3)) + (AppellF1[1/2, 2, 11/6,
 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan
[e + f*x])/(a^2*f*(d*Sec[e + f*x])^(5/3)) + (b^2*AppellF1[3/2, 2, 11/6, 5/2
, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan[e +
 f*x]^3)/(3*a^4*f*(d*Sec[e + f*x])^(5/3)) - (a*b)/((a^2 + b^2)*f*(d*Sec[e +
 f*x])^(5/3)*(a^2 - b^2*Tan[e + f*x]^2))
```

Rubi [A] time = 0.968849, antiderivative size = 717, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3512, 757, 429, 444, 51, 63, 210, 634, 618, 204, 628, 208, 510}

$$\frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{5/6}F_1\left(\frac{3}{2};2,\frac{11}{6};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f(d\sec(e+fx))^{5/3}}+\frac{\tan(e+fx)\sec^2(e+fx)^{5/6}F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2}\right.}{a^2f(d\sec(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2),x]
```


```
[Out] (11*a*b)/(5*(a^2 + b^2)^2*f*(d*Sec[e + f*x])^(5/3)) + (11*a*b^(8/3)*ArcTan[
1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]
*(Sec[e + f*x]^2)^(5/6))/(2*Sqrt[3]*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(
5/3)) - (11*a*b^(8/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))
/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(5/6))/(2*Sqrt[3]*(a^2 + b^2
)^(17/6)*f*(d*Sec[e + f*x])^(5/3)) - (11*a*b^(8/3)*ArcTanh[(b^(1/3)*(Sec[e
+ f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(5/6))/(3*(a^2 + b^2)^
(17/6)*f*(d*Sec[e + f*x])^(5/3)) + (11*a*b^(8/3)*Log[(a^2 + b^2)^(1/3) - b^
(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(
1/3)]*(Sec[e + f*x]^2)^(5/6))/(12*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(5/
3)) - (11*a*b^(8/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[
e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6))
/(12*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(5/3)) + (AppellF1[1/2, 2, 11/6,
 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan
[e + f*x])/(a^2*f*(d*Sec[e + f*x])^(5/3)) + (b^2*AppellF1[3/2, 2, 11/6, 5/2
, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan[e +
```

```
f*x]^3)/(3*a^4*f*(d*Sec[e + f*x])^(5/3)) - (a*b)/((a^2 + b^2)*f*(d*Sec[e +
f*x])^(5/3)*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \frac{\sec^2(e+fx)^{5/6}\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\sec^2(e+fx)^{5/6}\,\text{Subst}\left(\int \left(\frac{a^2}{\left(a^2-x^2\right)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}} - \frac{2ax}{\left(a^2-x^2\right)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}} + \frac{x^2}{\left(-a^2+x^2\right)^2\left(1+\right.}\right.\right.}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\sec^2(e+fx)^{5/6}\,\text{Subst}\left(\int \frac{x^2}{\left(-a^2+x^2\right)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}} - \frac{\left(2a\sec^2\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2f(d\sec(e+fx))^{5/3}} + \cdots$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2f(d\sec(e+fx))^{5/3}} + \cdots$$

$$= \frac{11ab}{5\left(a^2+b^2\right)^2f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+f\right.}{a^2f(d\sec(e+f}$$

$$= \frac{11ab}{5\left(a^2+b^2\right)^2f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+f\right.}{a^2f(d\sec(e+f}$$

$$= \frac{11ab}{5\left(a^2+b^2\right)^2f(d\sec(e+fx))^{5/3}} + \frac{F_1\left(\frac{1}{2};2,\frac{11}{6};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+f\right.}{a^2f(d\sec(e+f}$$

$$= \frac{11ab}{5\left(a^2+b^2\right)^2f(d\sec(e+fx))^{5/3}} - \frac{11ab^{8/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+}{3\left(a^2+b^2\right)^{17/6}f(d\sec(e+fx))^{5/3}}$$

$$= \frac{11ab}{5\left(a^2+b^2\right)^2f(d\sec(e+fx))^{5/3}} - \frac{11ab^{8/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+}{3\left(a^2+b^2\right)^{17/6}f(d\sec(e+fx))^{5/3}}$$

$$= \frac{11ab}{5\left(a^2+b^2\right)^2f(d\sec(e+fx))^{5/3}} + \frac{11ab^{8/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sec^2(e+}{2\sqrt{3}\left(a^2+b^2\right)^{17/6}f(d\sec(e+fx))^{5}}$$

Mathematica [C] time = 27.4658, size = 5235, normalized size = 7.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.272, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\tan\left(fx+e\right)\right)^{2}}\left(d\sec\left(fx+e\right)\right)^{-\frac{5}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2), x)
```

3.640 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=173

$$-\frac{a\left(3b^2-a^2(m+1)\right)\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\,{}_2F_1\left(\frac{1}{2},1-\frac{m}{2};\frac{3}{2};-\tan^2(e+fx)\right)}{f(m+1)}-\frac{b(d\sec(e+fx)}{}$$

```
[Out] -((a*(3*b^2 - a^2*(1 + m))*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*
x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + m)*(Sec[e + f*x]^2)^(m/2)))
+ (b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^2)/(f*(2 + m)) - (b*(d*Sec[e +
 f*x])^m*(2*(1 + m)*(b^2 - a^2*(3 + m)) - a*b*m*(4 + m)*Tan[e + f*x]))/(f*m
*(2 + 3*m + m^2))
```

Rubi [A] time = 0.19662, antiderivative size = 167, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3512, 743, 780, 245}

$$\frac{a\left(a^2-\frac{3b^2}{m+1}\right)\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\,{}_2F_1\left(\frac{1}{2},1-\frac{m}{2};\frac{3}{2};-\tan^2(e+fx)\right)}{f}-\frac{b(d\sec(e+fx))^m\left(2(m\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (a*(a^2 - (3*b^2)/(1 + m))*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*
x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x])/(f*(Sec[e + f*x]^2)^(m/2)) + (b*(d*S
ec[e + f*x])^m*(a + b*Tan[e + f*x])^2)/(f*(2 + m)) - (b*(d*Sec[e + f*x])^m*
(2*(1 + m)*(b^2 - a^2*(3 + m)) - a*b*m*(4 + m)*Tan[e + f*x]))/(f*m*(2 + 3*m
 + m^2))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx = \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}dx,x,b\,\text{t}\right.}{bf}$$

$$= \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))^2}{f(2+m)} + \frac{\left(b(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right.}{}$$

$$= \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))^2}{f(2+m)} - \frac{b(d\sec(e+fx))^m\left(2(1+m)\left(b^2-a\right.\right.}{fm\left(2+\right.}$$

$$= -\frac{a\left(3b^2-a^2(1+m)\right)\,{}_2F_1\left(\frac{1}{2},1-\frac{m}{2};\frac{3}{2};-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2}{f(1+m)}$$

Mathematica [A] time = 57.2557, size = 175, normalized size = 1.01

$$\frac{\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\left(b\left(\left(3a^2(m+2)-2b^2\right)\left(\sec^2(e+fx)^{m/2}-1\right)+abm(m+2)\tan^3(e+fx)\,{}_2F_1\left(\frac{3}{2},1-\frac{m}{2};\right.\right.\right.}{fm(m+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((d*Sec[e + f*x])^m*(a^3*m*(2 + m)*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Ta
n[e + f*x]^2]*Tan[e + f*x] + b*((-2*b^2 + 3*a^2*(2 + m))*(-1 + (Sec[e + f*x
]^2)^(m/2)) + b^2*m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^2 + a*b*m*(2 + m)*H
ypergeometric2F1[3/2, 1 - m/2, 5/2, -Tan[e + f*x]^2]*Tan[e + f*x]^3)))/(f*m
*(2 + m)*(Sec[e + f*x]^2)^(m/2))
```

Maple [F] time = 0.543, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^m\left(a+b\tan\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)+a\right)^3\left(d\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3*(d*sec(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\tan\left(fx+e\right)^3+3\,ab^2\tan\left(fx+e\right)^2+3\,a^2b\tan\left(fx+e\right)+a^3\right)\left(d\sec\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*(d*sec(f*x + e))^m, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)+a\right)^3\left(d\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3*(d*sec(f*x + e))^m, x)
```

3.641 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=147

$$\frac{d\left(b^2-a^2(m+1)\right)\sin(e+fx)(d\sec(e+fx))^{m-1}{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)}{f(1-m)(m+1)\sqrt{\sin^2(e+fx)}}+\frac{ab(m+2)(d\sec(e+fx))^m}{fm(m+1)}+\frac{b(a+b\,\mathrm{t}}{}$$

```
[Out] (a*b*(2 + m)*(d*Sec[e + f*x])^m)/(f*m*(1 + m)) + (d*(b^2 - a^2*(1 + m))*Hyp
ergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
(-1 + m)*Sin[e + f*x])/(f*(1 - m)*(1 + m)*Sqrt[Sin[e + f*x]^2]) + (b*(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(1 + m))
```

Rubi [A] time = 0.166241, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3508, 3486, 3772, 2643}

$$\frac{d\left(b^2-a^2(m+1)\right)\sin(e+fx)(d\sec(e+fx))^{m-1}{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)}{f(1-m)(m+1)\sqrt{\sin^2(e+fx)}}+\frac{ab(m+2)(d\sec(e+fx))^m}{fm(m+1)}+\frac{b(a+b\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (a*b*(2 + m)*(d*Sec[e + f*x])^m)/(f*m*(1 + m)) + (d*(b^2 - a^2*(1 + m))*Hyp
ergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
(-1 + m)*Sin[e + f*x])/(f*(1 - m)*(1 + m)*Sqrt[Sin[e + f*x]^2]) + (b*(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(1 + m))
```

Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))}{f(1+m)} + \frac{\int (d\sec(e+fx))^m\left(-b^2+a^2(1+\right.}{1+}$$

$$= \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)} + \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))}{f(1+m)} + \left(a^2\right.$$

$$= \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)} + \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))}{f(1+m)} + \left(\left(a\right.\right.$$

$$= \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)} - \frac{\left(a^2-\frac{b^2}{1+m}\right)\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos\right.}{f(1-m)\sqrt{\sin}}$$

Mathematica [C] time = 26.5429, size = 11095, normalized size = 75.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.325, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^m\left(a+b\tan\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)+a\right)^2\left(d\sec\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2*(d*sec(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2\right)\left(d\sec\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^m
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(e+fx\right)\right)^{m}\left(a+b\tan\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)+a\right)^{2}\left(d\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2*(d*sec(f*x + e))^m, x)
```

3.642 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=93

$$\frac{b(d\sec(e+fx))^m}{fm}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{m-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)}{f(1-m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] (b*(d*Sec[e + f*x])^m)/(f*m) - (a*d*Hypergeometric2F1[1/2, (1 - m)/2, (3 -
m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + m)*Sin[e + f*x])/(f*(1 - m)*Sq
rt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.0615625, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3486, 3772, 2643}

$$\frac{b(d\sec(e+fx))^m}{fm}-\frac{ad\sin(e+fx)(d\sec(e+fx))^{m-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)}{f(1-m)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

```
[Out] (b*(d*Sec[e + f*x])^m)/(f*m) - (a*d*Hypergeometric2F1[1/2, (1 - m)/2, (3 -
m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + m)*Sin[e + f*x])/(f*(1 - m)*Sq
rt[Sin[e + f*x]^2])
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \frac{b(d\sec(e+fx))^m}{fm} + a\int (d\sec(e+fx))^m\,dx$$
$$= \frac{b(d\sec(e+fx))^m}{fm} + \left(a\left(\frac{\cos(e+fx)}{d}\right)^m (d\sec(e+fx))^m\right)\int \left(\frac{\cos(e+fx)}{d}\right)^{-m}$$
$$= \frac{b(d\sec(e+fx))^m}{fm} - \frac{a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(e+fx)\right)(d\sec(e+f}{f(1-m)\sqrt{\sin^2(e+fx)}}$$

<u>Mathematica</u> [C] time = 16.8281, size = 3302, normalized size = 35.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

```
[Out] -((Sec[e + f*x]^(-1 - m)*(d*Sec[e + f*x])^m*(Cos[(e + f*x)/2]^2*Sec[e + f*x
])^m*(a*Sec[e + f*x]^m + b*Sec[e + f*x]^(1 + m)*Sin[e + f*x])*Tan[(e + f*x)
/2]*(-(b*AppellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2]) - b*AppellF1[1, 1 + m
, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*
x)/2]^2)^m*Tan[(e + f*x)/2] - (6*a*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + m))/(3*AppellF1[1/
2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*Ap
pellF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*App
ellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan
[(e + f*x)/2]^2))*(a + b*Tan[e + f*x]))/(f*(a*Cos[e + f*x] + b*Sin[e + f*x]
)*(-(Sec[(e + f*x)/2]^2*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*(-(b*AppellF1[1
, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(
e + f*x)/2]^2)^m*Tan[(e + f*x)/2]) - b*AppellF1[1, 1 + m, -m, 2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m*Tan[(e +
 f*x)/2] - (6*a*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + m))/(3*AppellF1[1/2, m, 1 - m, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m, 2 -
m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 1
 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))/
2 - (Cos[(e + f*x)/2]^2*Sec[e + f*x])^m*Tan[(e + f*x)/2]*(-(b*AppellF1[1, m
, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(Co
s[e + f*x]*Sec[(e + f*x)/2]^2)^m)/2 - (b*AppellF1[1, 1 + m, -m, 2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e +
f*x)/2]^2)^m)/2 - b*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2]*(-
((1 - m)*AppellF1[2, m, 2 - m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*
Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2 + (m*AppellF1[2, 1 + m, 1 - m, 3, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])
/2) - b*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2]*((m*AppellF1[2
, 1 + m, 1 - m, 3, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2])/2 + ((1 + m)*AppellF1[2, 2 + m, -m, 3, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/2) - b*m*Ap
pellF1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*
x]*Sec[(e + f*x)/2]^2)^(-1 + m)*Tan[(e + f*x)/2]*(-(Sec[(e + f*x)/2]^2*Sin[
e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - b*m*AppellF
1[1, 1 + m, -m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*S
ec[(e + f*x)/2]^2)^(-1 + m)*Tan[(e + f*x)/2]*(-(Sec[(e + f*x)/2]^2*Sin[e +
f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - (6*a*(-1 + m)*A
ppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(
```

```
e + f*x)/2]^2)^(-1 + m)*Tan[(e + f*x)/2])/(3*AppellF1[1/2, m, 1 - m, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m, 2 -
m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 1
 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) -
(6*a*(Sec[(e + f*x)/2]^2)^(-1 + m)*(-((1 - m)*AppellF1[3/2, m, 2 - m, 5/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2
])/3 + (m*AppellF1[3/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, m, 1 - m
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2,
 m, 2 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2,
1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/
2]^2) + (6*a*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + m)*(2*((-1 + m)*AppellF1[3/2, m, 2 - m,
5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 1 -
m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e
 + f*x)/2] + 3*(-((1 - m)*AppellF1[3/2, m, 2 - m, 5/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (m*AppellF1[3
/2, 1 + m, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*
x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + m)*((-3*(2 - m)*
AppellF1[5/2, m, 3 - m, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*m*AppellF1[5/2, 1 + m, 2 - m, 7/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
)/5) + m*((-3*(1 - m)*AppellF1[5/2, 1 + m, 2 - m, 7/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + m)*Ap
pellF1[5/2, 2 + m, 1 - m, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, m, 1 - m, 3/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m, 2 - m,
 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m, 1 -
 m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2) -
 m*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + m)*Tan[(e + f*x)/2]*(-(b*AppellF
1[1, m, 1 - m, 2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Se
c[(e + f*x)/2]^2)^m*Tan[(e + f*x)/2]) - b*AppellF1[1, 1 + m, -m, 2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^m*Tan[(
e + f*x)/2] - (6*a*AppellF1[1/2, m, 1 - m, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(-1 + m))/(3*AppellF1[1/2, m, 1 - m, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + m)*AppellF1[3/2, m, 2
 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + m*AppellF1[3/2, 1 + m
, 1 - m, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)
)*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*S
ec[e + f*x]*Tan[e + f*x]))))
```

Maple [F] time = 0.501, size = 0, normalized size = 0.

$$\int \left(d \sec\left(fx+e\right)\right)^{m} \left(a+b \tan\left(fx+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \tan\left(fx+e\right)+a\right) \left(d \sec\left(fx+e\right)\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)+a\right)\left(d\sec\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\sec\left(e+fx\right)\right)^{m}\left(a+b\tan\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)+a\right)\left(d\sec\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

3.643 $\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=141

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m F_1\left(\frac{1}{2};1,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af}-\frac{b(d\sec(e+fx))^m\,{}_2F_1\left(1\right.}{fm\left(a^2\right.}$$

```
[Out] -((b*Hypergeometric2F1[1, m/2, (2 + m)/2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]
*(d*Sec[e + f*x])^m)/((a^2 + b^2)*f*m)) + (AppellF1[1/2, 1, 1 - m/2, 3/2, (
b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x])/
(a*f*(Sec[e + f*x]^2)^(m/2))
```

Rubi [A] time = 0.157447, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3512, 757, 429, 444, 68}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m F_1\left(\frac{1}{2};1,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af}-\frac{b(d\sec(e+fx))^m\,{}_2F_1\left(1\right.}{fm\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] -((b*Hypergeometric2F1[1, m/2, (2 + m)/2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]
*(d*Sec[e + f*x])^m)/((a^2 + b^2)*f*m)) + (AppellF1[1/2, 1, 1 - m/2, 3/2, (
b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x])/
(a*f*(Sec[e + f*x]^2)^(m/2))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{a+x}\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int \left(\frac{a\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{a^2-x^2}+\frac{x\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{-a^2+x^2}\right)dx, x, b\tan(e+fx)\right.}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int \frac{x\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{-a^2+x^2}\,dx, x, b\tan(e+fx)\right)}{bf} + \frac{\left(a(d\sec(e+\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e+fx)}{af} +$$

$$= -\frac{b\,{}_2F_1\left(1,\frac{m}{2};\frac{2+m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{\left(a^2+b^2\right)fm} + \frac{F_1\left(\frac{1}{2};1,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+f\right.}{}$$

Mathematica [C] time = 15.053, size = 1158, normalized size = 8.21

$$\frac{}{f(a+b\tan(e+fx))\left(-\frac{1}{2}bmF_1\left(-m;-\frac{m}{2},-\frac{m}{2};1-m;\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)\sec^2(e+fx)^{m/2}\left(\frac{b(\tan(e+fx)+i)}{a+b\tan(e+fx)}\right)^{-m/2}\left(\frac{b\sec^2(e}{a+b\tan(}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^m*(b - b*(Sec[e + f*x]^2)^(m/2) + a*m*Hypergeometric2F1[1
/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (b*AppellF1[-m, -m/2, -m/
2, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(
Sec[e + f*x]^2)^(m/2))/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2
)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))))/(f*(a + b*Tan[e +
f*x])*(a*m*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Sec[e + f*
x]^2 - b*m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x] + (b*m*AppellF1[-m, -m/2, -m
/2, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*
(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x])/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e
```

```
 + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)) + (b*(
Sec[e + f*x]^2)^(m/2)*(-((a - I*b)*b*m^2*AppellF1[1 - m, 1 - m/2, -m/2, 2 -
 m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e +
 f*x]^2)/(2*(1 - m)*(a + b*Tan[e + f*x])^2) - ((a + I*b)*b*m^2*AppellF1[1 -
 m, -m/2, 1 - m/2, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*
Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(1 - m)*(a + b*Tan[e + f*x])^2)))/(((b*(-
I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a +
 b*Tan[e + f*x]))^(m/2)) - (b*m*AppellF1[-m, -m/2, -m/2, 1 - m, (a - I*b)/(
a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)
*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 - m/2)*(-((b^2*Sec[e +
f*x]^2*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a
 + b*Tan[e + f*x])))/(2*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)
) - (b*m*AppellF1[-m, -m/2, -m/2, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a
 + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*((b*(I + Tan[e + f*x])
)/(a + b*Tan[e + f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*x]^2*(I + Tan[e + f*x]
))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2*(
(b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)) + a*m*Sec[e + f*x]^2*(
-Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^
2)^(-1 + m/2))))
```

Maple [F] time = 0.228, size = 0, normalized size = 0.

$$\int \frac{\left(d \sec \left(f x + e\right)\right)^{m}}{a + b \tan \left(f x + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d \sec \left(f x + e\right)\right)^{m}}{b \tan \left(f x + e\right) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d \sec \left(f x + e\right)\right)^{m}}{b \tan \left(f x + e\right) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^m}{a+b\tan\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*sec(e + f*x))**m/(a + b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^m}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

3.644 $\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=227

$$\frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m F_1\left(\frac{3}{2};2,1-\frac{m}{2};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f}+\frac{\tan(e+fx)\sec^2(e\cdots}{}$$

```
[Out] (-2*a*b*Hypergeometric2F1[2, m/2, (2 + m)/2, (b^2*Sec[e + f*x]^2)/(a^2 + b^
2)]*(d*Sec[e + f*x])^m)/((a^2 + b^2)^2*f*m) + (AppellF1[1/2, 2, 1 - m/2, 3/
2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*
x])/(a^2*f*(Sec[e + f*x]^2)^(m/2)) + (b^2*AppellF1[3/2, 2, 1 - m/2, 5/2, (b
^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]^3)
/(3*a^4*f*(Sec[e + f*x]^2)^(m/2))
```

Rubi [A] time = 0.197627, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3512, 757, 429, 444, 68, 510}

$$\frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m F_1\left(\frac{3}{2};2,1-\frac{m}{2};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f}+\frac{\tan(e+fx)\sec^2(e\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```


```
[Out] (-2*a*b*Hypergeometric2F1[2, m/2, (2 + m)/2, (b^2*Sec[e + f*x]^2)/(a^2 + b^
2)]*(d*Sec[e + f*x])^m)/((a^2 + b^2)^2*f*m) + (AppellF1[1/2, 2, 1 - m/2, 3/
2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*
x])/(a^2*f*(Sec[e + f*x]^2)^(m/2)) + (b^2*AppellF1[3/2, 2, 1 - m/2, 5/2, (b
^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]^3)
/(3*a^4*f*(Sec[e + f*x]^2)^(m/2))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(a+x)^2}\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int \left(\frac{a^2\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{\left(a^2-x^2\right)^2} - \frac{2ax\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{\left(a^2-x^2\right)^2} + \frac{x^2\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{\left(-a^2+x^2\right)^2}\right.\right.}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{\left(-a^2+x^2\right)^2}\,dx, x, b\tan(e+fx)\right)}{bf} - \frac{(2a(d\sec}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e+fx}{a^2f}$$

$$= -\frac{2ab\,{}_2F_1\left(2,\frac{m}{2};\frac{2+m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{\left(a^2+b^2\right)^2fm} + \frac{F_1\left(\frac{1}{2};2,1-\frac{m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e\right.}{}$$

Mathematica [C] time = 3.24919, size = 356, normalized size = 1.57

$$\frac{2(m-4)(d\sec(e+fx))^m(a\cos(e+fx)+b\sin}{bf(m-3)(a+b\tan(e+fx))^2\left((m-2)\left((a+ib)F_1\left(4-m;1-\frac{m}{2},2-\frac{m}{2};5-m;\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+(a-ib)F_1\left(4\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*(-4 + m)*AppellF1[3 - m, 1 - m/2, 1 - m/2, 4 - m, (a - I*b)/(a + b*Tan[e
 + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(d*Sec[e + f*x])^m*(a*Cos[e + f*x
] + b*Sin[e + f*x])^2)/(b*f*(-3 + m)*(a + b*Tan[e + f*x])^2*((-2 + m)*((a +
 I*b)*AppellF1[4 - m, 1 - m/2, 2 - m/2, 5 - m, (a - I*b)/(a + b*Tan[e + f*x
]), (a + I*b)/(a + b*Tan[e + f*x])] + (a - I*b)*AppellF1[4 - m, 2 - m/2, 1
- m/2, 5 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x]
)]) + 2*(-4 + m)*AppellF1[3 - m, 1 - m/2, 1 - m/2, 4 - m, (a - I*b)/(a + b*
Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Maple [F] time = 0.267, size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{m}}{\left(a+b\tan\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{m}}{\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d\sec\left(fx+e\right)\right)^{m}}{b^{2}\tan\left(fx+e\right)^{2}+2\,ab\tan\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^m/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(e+fx\right)\right)^{m}}{\left(a+b\tan\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*sec(e + f*x))**m/(a + b*tan(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\sec\left(fx+e\right)\right)^{m}}{\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

3.645 $\int (d\sec(e+fx))^m (a+b\tan(e+fx))^n\,dx$

Optimal. Leaf size=181

$$\frac{b(d\sec(e+fx))^m\left(\frac{a+b\tan(e+fx)}{\sqrt{-b^2}-a}+1\right)^{-m/2}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{-m/2}(a+b\tan(e+fx))^{n+1}F_1\left(n+1;1-\frac{m}{2},1-\frac{m}{2};n+2;\frac{a+b}{\ }\right.}{f(n+1)\left(a^2+b^2\right)}$$

```
[Out] (b*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a + Sqrt[
-b^2]), (a + b*Tan[e + f*x])/(a - Sqrt[-b^2])]*(d*Sec[e + f*x])^m*(a + b*Ta
n[e + f*x])^(1 + n))/((a^2 + b^2)*f*(1 + n)*(1 + (a + b*Tan[e + f*x])/(-a +
 Sqrt[-b^2]))^(m/2)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^(m/2))
```

Rubi [A] time = 0.167858, antiderivative size = 187, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3512, 760, 133}

$$\frac{\cos^2(e+fx)(d\sec(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{1-\frac{m}{2}}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{1-\frac{m}{2}}(a+b\tan(e+fx))^{n+1}F_1\left(n+1;1-\frac{m}{2},1-\frac{m}{2}\right.}{bf(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - Sqrt[-b
^2]), (a + b*Tan[e + f*x])/(a + Sqrt[-b^2])]*Cos[e + f*x]^2*(d*Sec[e + f*x]
)^m*(a + b*Tan[e + f*x])^(1 + n)*(1 - (a + b*Tan[e + f*x])/(a - Sqrt[-b^2])
)^(1 - m/2)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^(1 - m/2))/(b*f*(1
+ n))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx = \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int (a+x)^n\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}dx, x, b\ldots\right.}{bf}$$

$$= \frac{\left(\cos^2(e+fx)(d\sec(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{1-\frac{m}{2}}\left(1-\frac{a+b\tan(e+fx)}{a+\frac{b^2}{\sqrt{-b^2}}}\right)^{1-\frac{m}{2}}\ldots\right.}{bf}$$

$$= \frac{F_1\left(1+n;1-\frac{m}{2},1-\frac{m}{2};2+n;\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)\cos^2(e+fx)(d\sec(\ldots}{bf(1\ldots}$$

Mathematica [C] time = 6.2025, size = 699, normalized size = 3.86

$$\frac{}{f\left(2n(b-a\tan(e+fx))F_1\left(n+1;1-\frac{m}{2},1-\frac{m}{2};n+2;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)+2(m+n)\tan(e+fx)(a+b\tan(e+f\ldots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (2*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b),
 (a + b*Tan[e + f*x])/(a + I*b)]*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(1
 + n))/(f*(2*b*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x]
)/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2 + 2*n*AppellF1[
1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[
e + f*x])/(a + I*b)]*(b - a*Tan[e + f*x]) - (b*(-2 + m)*(1 + n)*((a - I*b)*
AppellF1[2 + n, 1 - m/2, 2 - m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a
 + b*Tan[e + f*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 2 - m/2, 1 - m/2,
 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)])*Se
c[e + f*x]^2*(a + b*Tan[e + f*x]))/((a - I*b)*(a + I*b)*(2 + n)) + 2*(m + n
)*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b),
(a + b*Tan[e + f*x])/(a + I*b)]*Tan[e + f*x]*(a + b*Tan[e + f*x]) - (m*Appe
llF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b
*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(a + b*Tan[e + f*x]))/(-I + Tan[e
+ f*x]) - (m*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/
(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(a + b*Tan[e + f*
x]))/(I + Tan[e + f*x])))
```

Maple [F] time = 0.284, size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^m\left(a+b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)+a\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d\sec\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)+a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\sec\left(fx+e\right)\right)^m \left(b\tan\left(fx+e\right)+a\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

3.646 $\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=161

$$\frac{(a^2+b^2)^2(a+b\tan(c+dx))^{n+1}}{b^5d(n+1)}-\frac{4a(a^2+b^2)(a+b\tan(c+dx))^{n+2}}{b^5d(n+2)}+\frac{2(3a^2+b^2)(a+b\tan(c+dx))^{n+3}}{b^5d(n+3)}-\frac{4a(a+b\cdots}{b^5\cdots}$$

```
[Out] ((a^2 + b^2)^2*(a + b*Tan[c + d*x])^(1 + n))/(b^5*d*(1 + n)) - (4*a*(a^2 +
b^2)*(a + b*Tan[c + d*x])^(2 + n))/(b^5*d*(2 + n)) + (2*(3*a^2 + b^2)*(a +
b*Tan[c + d*x])^(3 + n))/(b^5*d*(3 + n)) - (4*a*(a + b*Tan[c + d*x])^(4 + n
))/(b^5*d*(4 + n)) + (a + b*Tan[c + d*x])^(5 + n)/(b^5*d*(5 + n))
```

Rubi [A] time = 0.123486, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$\frac{(a^2+b^2)^2(a+b\tan(c+dx))^{n+1}}{b^5d(n+1)}-\frac{4a(a^2+b^2)(a+b\tan(c+dx))^{n+2}}{b^5d(n+2)}+\frac{2(3a^2+b^2)(a+b\tan(c+dx))^{n+3}}{b^5d(n+3)}-\frac{4a(a+b\cdots}{b^5\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a^2 + b^2)^2*(a + b*Tan[c + d*x])^(1 + n))/(b^5*d*(1 + n)) - (4*a*(a^2 +
b^2)*(a + b*Tan[c + d*x])^(2 + n))/(b^5*d*(2 + n)) + (2*(3*a^2 + b^2)*(a +
b*Tan[c + d*x])^(3 + n))/(b^5*d*(3 + n)) - (4*a*(a + b*Tan[c + d*x])^(4 + n
))/(b^5*d*(4 + n)) + (a + b*Tan[c + d*x])^(5 + n)/(b^5*d*(5 + n))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx=\frac{\text{Subst}\left(\int(a+x)^n\left(1+\frac{x^2}{b^2}\right)^2dx,x,b\tan(c+dx)\right)}{bd}$$

$$=\frac{\text{Subst}\left(\int\left(\frac{(a^2+b^2)^2(a+x)^n}{b^4}-\frac{4a(a^2+b^2)(a+x)^{1+n}}{b^4}+\frac{2(3a^2+b^2)(a+x)^{2+n}}{b^4}-\frac{4a(a+x)^{3+n}}{b^4}+\frac{(a+x)^{4}\cdots}{b^4}\cdots\right.\right.}{bd}$$

$$=\frac{(a^2+b^2)^2(a+b\tan(c+dx))^{1+n}}{b^5d(1+n)}-\frac{4a(a^2+b^2)(a+b\tan(c+dx))^{2+n}}{b^5d(2+n)}+\frac{2(3a^2\cdots}{\cdots}$$

Mathematica [A] time = 2.96177, size = 161, normalized size = 1.

$$\frac{(a+b\tan(c+dx))^{n+1}\left(4\left(a^2+b^2\right)\left(\frac{a^2+b^2}{n+1}+\frac{(a+b\tan(c+dx))^2}{n+3}-\frac{2a(a+b\tan(c+dx))}{n+2}\right)-4a(a+b\tan(c+dx))\left(\frac{a^2+b^2}{n+2}+\frac{(a+b\tan(c+dx))^2}{n+4}\right.}{b^5d(n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a + b*Tan[c + d*x])^(1 + n)*(b^4*Sec[c + d*x]^4 + 4*(a^2 + b^2)*((a^2 + b
^2)/(1 + n) - (2*a*(a + b*Tan[c + d*x]))/(2 + n) + (a + b*Tan[c + d*x])^2/(
3 + n)) - 4*a*(a + b*Tan[c + d*x])*((a^2 + b^2)/(2 + n) - (2*a*(a + b*Tan[c
 + d*x]))/(3 + n) + (a + b*Tan[c + d*x])^2/(4 + n))))/(b^5*d*(5 + n))
```

Maple [F] time = 0.245, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^6 (a+b\tan(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.44236, size = 899, normalized size = 5.58

$$\frac{\left(8\left(3a^5+10a^3b^2+15ab^4-\left(a^3b^2-3ab^4\right)n^2+3\left(a^3b^2+5ab^4\right)n\right)\cos(dx+c)^5+4\left(2ab^4n^3+3\left(a^3b^2+3ab^4\right)n^2+\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4 - (a^3*b^2 - 3*a*b^4)*n^2 + 3*(a^3*b^2 +
5*a*b^4)*n)*cos(d*x + c)^5 + 4*(2*a*b^4*n^3 + 3*(a^3*b^2 + 3*a*b^4)*n^2 + (
3*a^3*b^2 + 7*a*b^4)*n)*cos(d*x + c)^3 + (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^
4*n^2 + 6*a*b^4*n)*cos(d*x + c) + (b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b
^5*n + 24*b^5 + 8*(8*b^5 - (3*a^2*b^3 - b^5)*n^2 - 3*(a^4*b + 3*a^2*b^3 - 2
*b^5)*n)*cos(d*x + c)^4 + 4*(8*b^5 - (a^2*b^3 - b^5)*n^3 - (3*a^2*b^3 - 7*b
```

```
^5)*n^2 - 2*(a^2*b^3 - 7*b^5)*n)*cos(d*x + c)^2)*sin(d*x + c))*((a*cos(d*x
+ c) + b*sin(d*x + c))/cos(d*x + c))^n/((b^5*d*n^5 + 15*b^5*d*n^4 + 85*b^5*
d*n^3 + 225*b^5*d*n^2 + 274*b^5*d*n + 120*b^5*d)*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.647 $\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=88

$$\frac{(a^2+b^2)(a+b\tan(c+dx))^{n+1}}{b^3d(n+1)}-\frac{2a(a+b\tan(c+dx))^{n+2}}{b^3d(n+2)}+\frac{(a+b\tan(c+dx))^{n+3}}{b^3d(n+3)}$$

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^(1 + n))/(b^3*d*(1 + n)) - (2*a*(a + b*Ta
n[c + d*x])^(2 + n))/(b^3*d*(2 + n)) + (a + b*Tan[c + d*x])^(3 + n)/(b^3*d*
(3 + n))
```

Rubi [A] time = 0.0764361, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 697}

$$\frac{(a^2+b^2)(a+b\tan(c+dx))^{n+1}}{b^3d(n+1)}-\frac{2a(a+b\tan(c+dx))^{n+2}}{b^3d(n+2)}+\frac{(a+b\tan(c+dx))^{n+3}}{b^3d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^(1 + n))/(b^3*d*(1 + n)) - (2*a*(a + b*Ta
n[c + d*x])^(2 + n))/(b^3*d*(2 + n)) + (a + b*Tan[c + d*x])^(3 + n)/(b^3*d*
(3 + n))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx&=\frac{\text{Subst}\left(\int (a+x)^n\left(1+\frac{x^2}{b^2}\right)dx,x,b\tan(c+dx)\right)}{bd}\\&=\frac{\text{Subst}\left(\int\left(\frac{(a^2+b^2)(a+x)^n}{b^2}-\frac{2a(a+x)^{1+n}}{b^2}+\frac{(a+x)^{2+n}}{b^2}\right)dx,x,b\tan(c+dx)\right)}{bd}\\&=\frac{(a^2+b^2)(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)}-\frac{2a(a+b\tan(c+dx))^{2+n}}{b^3d(2+n)}+\frac{(a+b\tan(c+dx))^{3+n}}{b^3d(3+n)}\end{aligned}$$

Mathematica [A] time = 0.240196, size = 71, normalized size = 0.81

$$\frac{(a+b\tan(c+dx))^{n+1}\left(\frac{a^2+b^2}{n+1}+\frac{(a+b\tan(c+dx))^2}{n+3}-\frac{2a(a+b\tan(c+dx))}{n+2}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a + b*Tan[c + d*x])^(1 + n)*((a^2 + b^2)/(1 + n) - (2*a*(a + b*Tan[c + d*
x]))/(2 + n) + (a + b*Tan[c + d*x])^2/(3 + n)))/(b^3*d)
```

Maple [F] time = 0.197, size = 0, normalized size = 0.

$$\int (\sec (dx+c))^4 (a+b\tan (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.77677, size = 392, normalized size = 4.45

$$\frac{\left(2\left(2\,ab^2n+a^3+3\,ab^2\right)\cos(dx+c)^3+\left(ab^2n^2+ab^2n\right)\cos(dx+c)+\left(b^3n^2+3\,b^3n+2\,b^3+2\left(2\,b^3-\left(a^2b-b^3\right)n\right)\cos(dx+c)^2\right)\sin(dx+c)\right)\left(\frac{a\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)^n}{\left(b^3dn^3+6\,b^3dn^2+11\,b^3dn+6\,b^3d\right)\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (2*(2*a*b^2*n + a^3 + 3*a*b^2)*cos(d*x + c)^3 + (a*b^2*n^2 + a*b^2*n)*cos(d
*x + c) + (b^3*n^2 + 3*b^3*n + 2*b^3 + 2*(2*b^3 - (a^2*b - b^3)*n)*cos(d*x
+ c)^2)*sin(d*x + c))*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^n/((
b^3*d*n^3 + 6*b^3*d*n^2 + 11*b^3*d*n + 6*b^3*d)*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.648 $\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=26

$$\frac{(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

```
[Out] (a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n))
```

Rubi [A] time = 0.0435077, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3506, 32}

$$\frac{(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n))
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx &= \frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A] time = 0.178291, size = 26, normalized size = 1.

$$\frac{(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n))
```

Maple [A] time = 0.017, size = 27, normalized size = 1.

$$\frac{(a+b\tan(dx+c))^{1+n}}{bd(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

```
[Out] (a+b*tan(d*x+c))^(1+n)/b/d/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.55939, size = 155, normalized size = 5.96

$$\frac{(a\cos(dx+c)+b\sin(dx+c))\left(\frac{a\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)^n}{(bdn+bd)\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (a*cos(d*x + c) + b*sin(d*x + c))*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*
x + c))^n/((b*d*n + b*d)*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\tan(c+dx))^n\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**2, x)
```

Giac [A] time = 2.96348, size = 35, normalized size = 1.35

$$\frac{(b\tan(dx+c)+a)^{n+1}}{bd(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] (b*tan(d*x + c) + a)^(n + 1)/(b*d*(n + 1))
```

3.649 $\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=272

$$-\frac{\left(\sqrt{-b^2}\left(\frac{a^2}{b^2}-n+1\right)-an\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4bd(n+1)\left(\frac{a^2}{b^2}+1\right)\left(a-\sqrt{-b^2}\right)}+\frac{b\left(\sqrt{-b^2}\left(\frac{a^2}{b^2}-n+1\right)+an\right)(a+}{4d(n}$$

```
[Out] -((Sqrt[-b^2]*(1 + a^2/b^2 - n) - a*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (
a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(4*(1 +
 a^2/b^2)*b*(a - Sqrt[-b^2])*d*(1 + n)) + (b*(Sqrt[-b^2]*(1 + a^2/b^2 - n)
+ a*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b
^2])]*(a + b*Tan[c + d*x])^(1 + n))/(4*(a^2 + b^2)*(a + Sqrt[-b^2])*d*(1 +
n)) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/(2
*(a^2 + b^2)*d)
```

Rubi [A] time = 0.457396, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3506, 741, 831, 68}

$$-\frac{\left(\sqrt{-b^2}\left(\frac{a^2}{b^2}-n+1\right)-an\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4bd(n+1)\left(\frac{a^2}{b^2}+1\right)\left(a-\sqrt{-b^2}\right)}+\frac{b\left(\sqrt{-b^2}\left(\frac{a^2}{b^2}-n+1\right)+an\right)(a+}{4d(n}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -((Sqrt[-b^2]*(1 + a^2/b^2 - n) - a*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (
a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(4*(1 +
 a^2/b^2)*b*(a - Sqrt[-b^2])*d*(1 + n)) + (b*(Sqrt[-b^2]*(1 + a^2/b^2 - n)
+ a*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b
^2])]*(a + b*Tan[c + d*x])^(1 + n))/(4*(a^2 + b^2)*(a + Sqrt[-b^2])*d*(1 +
n)) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/(2
*(a^2 + b^2)*d)
```

Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
```

```
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d} - \frac{b\,\text{Subst}\left(\int \frac{(a+x)^n\left(-1-\frac{a^2}{b^2}\right.}{1+\frac{x^2}{b^2}}\right.}{2\left(a\right.}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d} - \frac{b\,\text{Subst}\left(\int\left(\frac{\left(-an+\sqrt{-b^2}\right.}{2(}\right.\right.}{}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d} + \frac{\left(b\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)\right.\right.}{}$$

$$= -\frac{b\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)-an\right)\,{}_2F_1\left(1,1+n;2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx}{4\left(a^2+b^2\right)\left(a-\sqrt{-b^2}\right)d(1+n)}$$

Mathematica [A] time = 1.24, size = 225, normalized size = 0.83

$$\frac{(a+b\tan(c+dx))^{n+1}\left(-\frac{\left(\sqrt{-b^2}\left(a^2-b^2(n-1)\right)-ab^2n\right){}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{(n+1)\left(a-\sqrt{-b^2}\right)} + \frac{\left(a^2\sqrt{-b^2}+ab^2n+\left(-b^2\right)^{3/2}(n-1)\right){}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{(n+1)\left(a+\sqrt{-b^2}\right)} -\right.}{4bd\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a + b*Tan[c + d*x])^(1 + n)*(-(((Sqrt[-b^2]*(a^2 - b^2*(-1 + n)) - a*b^2*
n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])
])/((a - Sqrt[-b^2])*(1 + n))) + ((a^2*Sqrt[-b^2] + (-b^2)^(3/2)*(-1 + n) +
 a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt
[-b^2])])/((a + Sqrt[-b^2])*(1 + n)) + 2*b*Cos[c + d*x]^2*(b + a*Tan[c + d*
x])))/(4*b*(a^2 + b^2)*d)
```

Maple [F] time = 0.377, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n \cos(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\tan(dx+c)+a)^n \cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n \cos(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

3.650 $\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=434

$$\frac{b\left(\frac{an\left(\frac{3a^2}{b^2}-2n+5\right)}{b^2}-\frac{\sqrt{-b^2}\left(a^2b^2\left(-n^2-2n+6\right)+3a^4+b^4\left(n^2-4n+3\right)\right)}{b^6}\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{16d(n+1)\left(\frac{a^2}{b^2}+1\right)^2\left(a-\sqrt{-b^2}\right)}+\frac{b\left(\frac{\sqrt{-b^2}\left(a^2\right.}{}\right.}{}$$

```
[Out] (b*((a*(5 + (3*a^2)/b^2 - 2*n)*n)/b^2 - (Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2
*n - n^2) + b^4*(3 - 4*n + n^2)))/b^6)*Hypergeometric2F1[1, 1 + n, 2 + n, (
a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*(1
+ a^2/b^2)^2*(a - Sqrt[-b^2])*d*(1 + n)) + (b*((a*(5 + (3*a^2)/b^2 - 2*n)*n
)/b^2 + (Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2))
)/b^6)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b
^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*(1 + a^2/b^2)^2*(a + Sqrt[-b^2])*d*
(1 + n)) + (Cos[c + d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n
))/(4*(a^2 + b^2)*d) + (b*Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2*
(3 - n) + a^2*(1 + n) + a*b*(5 + (3*a^2)/b^2 - 2*n)*Tan[c + d*x]))/(8*(a^2
+ b^2)^2*d)
```

Rubi [A] time = 0.686767, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3506, 741, 823, 831, 68}

$$\frac{b\left(\frac{an\left(\frac{3a^2}{b^2}-2n+5\right)}{b^2}-\frac{\sqrt{-b^2}\left(a^2b^2\left(-n^2-2n+6\right)+3a^4+b^4\left(n^2-4n+3\right)\right)}{b^6}\right)(a+b\tan(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{16d(n+1)\left(\frac{a^2}{b^2}+1\right)^2\left(a-\sqrt{-b^2}\right)}+\frac{b\left(\frac{\sqrt{-b^2}\left(a^2\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```


```
[Out] (b*((a*(5 + (3*a^2)/b^2 - 2*n)*n)/b^2 - (Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2
*n - n^2) + b^4*(3 - 4*n + n^2)))/b^6)*Hypergeometric2F1[1, 1 + n, 2 + n, (
a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*(1
+ a^2/b^2)^2*(a - Sqrt[-b^2])*d*(1 + n)) + (b*((a*(5 + (3*a^2)/b^2 - 2*n)*n
)/b^2 + (Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2))
)/b^6)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b
^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*(1 + a^2/b^2)^2*(a + Sqrt[-b^2])*d*
(1 + n)) + (Cos[c + d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n
))/(4*(a^2 + b^2)*d) + (b*Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2*
(3 - n) + a^2*(1 + n) + a*b*(5 + (3*a^2)/b^2 - 2*n)*Tan[c + d*x]))/(8*(a^2
+ b^2)^2*d)
```


Rule 3506

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 831

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^3}dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} - \frac{b\,\text{Subst}\left(\int \frac{(a+x)^n\left(-3-\frac{3a}{b^{}}\right.}{\left(1+\right.}\right.}{4}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} + \frac{b\cos^2(c+dx)(a+b\,\text{ta}}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} + \frac{b\cos^2(c+dx)(a+b\,\text{ta}}{}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d} + \frac{b\cos^2(c+dx)(a+b\,\text{ta}}{}$$

$$= \frac{b^5\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2} - \frac{\sqrt{-b^2}\left(3a^4+a^2b^2\left(6-2n-n^2\right)+b^4\left(3-4n+n^2\right)\right)}{b^6}\right){}_2F_1\left(1,1+n;2+n;\frac{a+b\tan}{a-\sqrt{}}\right.}{16\left(a^2+b^2\right)^2\left(a-\sqrt{-b^2}\right)d(1+n)}$$

Mathematica [A] time = 4.34034, size = 360, normalized size = 0.83

$$\frac{(a+b\tan(c+dx))^{n+1}\left(\frac{\frac{\left(\sqrt{-b^2}\left(a^2b^2\left(n^2+2n-6\right)-3a^4-b^4\left(n^2-4n+3\right)\right)+ab^2n\left(3a^2+b^2(5-2n)\right)\right){}_2F_1\left(1,n+1;n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{a-\sqrt{-b^2}}+\frac{\left(\sqrt{-b^2}\left(-a^2b^2\left(n^2+2n-6\right)+3a^4+b^4\left(n^2-4n+3\right)\right)+a\right.}{a}}{(n+1)\left(a^2+b^2\right)}\right.}{16bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a + b*Tan[c + d*x])^(1 + n)*((((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b
^2]*(-3*a^4 - b^4*(3 - 4*n + n^2) + a^2*b^2*(-6 + 2*n + n^2)))*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])])/(a - Sqrt[-b
^2]) + ((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 + b^4*(3 - 4*n
 + n^2) - a^2*b^2*(-6 + 2*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a
+ b*Tan[c + d*x])/(a + Sqrt[-b^2])])/(a + Sqrt[-b^2]))/((a^2 + b^2)*(1 + n)
) + 4*b*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) - (2*b*Cos[c + d*x]^2*(b^3*(-3
+ n) - a^2*b*(1 + n) - a*(3*a^2 + b^2*(5 - 2*n))*Tan[c + d*x]))/(a^2 + b^2)
))/(16*b*(a^2 + b^2)*d)
```

Maple [F] time = 0.328, size = 0, normalized size = 0.

$$\int (\cos (dx + c))^4 (a + b \tan (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x)

[Out] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan (dx + c) + a)^n \cos (dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \tan (dx + c) + a)^n \cos (dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")

[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**n,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \tan (dx + c) + a)^n \cos (dx + c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")

[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

3.651 $\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=159

$$\frac{\sec(c+dx)(a+b\tan(c+dx))^{n+1}F_1\left(n+1;-\frac{1}{2},-\frac{1}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

```
[Out] (AppellF1[1 + n, -1/2, -1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]),
(a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^(1
 + n))/(b*d*(1 + n)*Sqrt[1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1
- (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])])
```

Rubi [A] time = 0.163687, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3512, 760, 133}

$$\frac{\sec(c+dx)(a+b\tan(c+dx))^{n+1}F_1\left(n+1;-\frac{1}{2},-\frac{1}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, -1/2, -1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]),
(a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^(1
 + n))/(b*d*(1 + n)*Sqrt[1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1
- (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])])
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int (a+x)^n\sqrt{1+\frac{x^2}{b^2}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$
$$= \frac{\sec(c+dx)\,\text{Subst}\left(\int x^n\sqrt{1-\frac{x}{a-\sqrt{-b^2}}}\sqrt{1-\frac{x}{a+\sqrt{-b^2}}}\,dx, x, a+b\tan(c+dx)\right)}{bd\sqrt{1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\frac{b^2}{\sqrt{-b^2}}}}}$$
$$= \frac{F_1\left(1+n;-\frac{1}{2},-\frac{1}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\sec(c+dx)(a+b\tan(c+}{bd(1+n)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

Mathematica [C] time = 3.91497, size = 306, normalized size = 1.92

$$\frac{2(n+2)(a-ib)(a+ib)\sec(c+dx)(a+b\tan(c+dx))^{n+1}F_1(}{bd(n+1)\left(2(n+2)\left(a^2+b^2\right)F_1\left(n+1;-\frac{1}{2},-\frac{1}{2};n+2;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)-(a+b\tan(c+dx))\left((a-ib)F_1\left(n+\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a - I*b)*(a + I*b)*(2 + n)*AppellF1[1 + n, -1/2, -1/2, 2 + n, (a + b*Ta
n[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Sec[c + d*x]*(a + b*
Tan[c + d*x])^(1 + n))/(b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n,
-1/2, -1/2, 2 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a
+ I*b)] - ((a - I*b)*AppellF1[2 + n, -1/2, 1/2, 3 + n, (a + b*Tan[c + d*x])
/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 1/2
, -1/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*
b)])*(a + b*Tan[c + d*x])))
```

Maple [F] time = 0.183, size = 0, normalized size = 0.

$$\int (\sec(dx+c))^3(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\tan(dx+c)+a)^n\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^n\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

3.652 $\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=159

$$\frac{\cos(c+dx)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}(a+b\tan(c+dx))^{n+1}F_1\left(n+1;\frac{1}{2},\frac{1}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)}$$

```
[Out] (AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]*(a + b*Tan[c + d*x])^(1 +
 n)*Sqrt[1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1 - (a + b*Tan[c +
 d*x])/(a + Sqrt[-b^2])])/(b*d*(1 + n))
```

Rubi [A] time = 0.113323, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3512, 760, 133}

$$\frac{\cos(c+dx)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}(a+b\tan(c+dx))^{n+1}F_1\left(n+1;\frac{1}{2},\frac{1}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]*(a + b*Tan[c + d*x])^(1 +
 n)*Sqrt[1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1 - (a + b*Tan[c +
 d*x])/(a + Sqrt[-b^2])])/(b*d*(1 + n))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\sec(c+dx)\,\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{\left(\cos(c+dx)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\frac{b^2}{\sqrt{-b^2}}}}\right)\text{Subst}\left(\int \frac{x^n}{\sqrt{1-\frac{x}{a-\sqrt{-b^2}}}\sqrt{1-\frac{x}{a+\sqrt{-b^2}}}}\,d\right.}{bd}$$

$$= \frac{F_1\left(1+n;\frac{1}{2},\frac{1}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(1+n)}$$

Mathematica [C] time = 3.75919, size = 340, normalized size = 2.14

$$\frac{2(n+2)\left(a^2+b^2\right)^2\cos^3(c+dx)(\tan(c+dx)-i)(\tan(c+dx)+i)(a+b\,\text{ta}}{bd(n+1)(a-ib)(a+ib)\left(2(n+2)\left(a^2+b^2\right)F_1\left(n+1;\frac{1}{2},\frac{1}{2};n+2;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+(a+b\tan(c+dx))\left((a-ib\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Tan[c + d*
x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^3*(-I + Tan[c +
 d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a + I*b
)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 1/2, 1/2, 2 + n, (a +
b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + ((a - I*b)*App
ellF1[2 + n, 1/2, 3/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c
+ d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 3/2, 1/2, 3 + n, (a + b*Tan[
c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c + d*x]))
)
```

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int \sec(dx+c)\,(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\tan(dx+c)+a)^n\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\tan(c+dx))^n\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(b\tan(dx+c)+a)^n\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

3.653 $\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=161

$$\frac{\cos^3(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{3/2}(a+b\tan(c+dx))^{n+1}F_1\left(n+1;\frac{3}{2},\frac{3}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)}$$

```
[Out] (AppellF1[1 + n, 3/2, 3/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^(1
 + n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(3/2)*(1 - (a + b*Tan[c +
 d*x])/(a + Sqrt[-b^2]))^(3/2))/(b*d*(1 + n))
```

Rubi [A] time = 0.124967, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3512, 760, 133}

$$\frac{\cos^3(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{3/2}(a+b\tan(c+dx))^{n+1}F_1\left(n+1;\frac{3}{2},\frac{3}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 3/2, 3/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^(1
 + n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(3/2)*(1 - (a + b*Tan[c +
 d*x])/(a + Sqrt[-b^2]))^(3/2))/(b*d*(1 + n))
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^{3/2}}dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\left(\cos^3(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\frac{b^2}{\sqrt{-b^2}}}\right)^{3/2}\right)\text{Subst}\left(\int \frac{x}{\left(1-\frac{x}{a-\sqrt{-b^2}}\right)^{3/2}}\right.}{bd}$$

$$= \frac{F_1\left(1+n;\frac{3}{2},\frac{3}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^3(c+dx)(a+b\tan(c+dx}{bd(1+n)}$$

Mathematica [C] time = 4.78599, size = 341, normalized size = 2.12

$$\frac{2(n+2)\left(a^2+b^2\right)^2\cos^5(c+dx)(\tan(c+dx)-i)(\tan(c+dx)+i)(a+}{bd(n+1)(a-ib)(a+ib)\left(2(n+2)\left(a^2+b^2\right)F_1\left(n+1;\frac{3}{2},\frac{3}{2};n+2;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+3(a+b\tan(c+dx))\left((a\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 3/2, 3/2, 2 + n, (a + b*Tan[c + d*
x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^5*(-I + Tan[c +
 d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a + I*b
)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 3/2, 3/2, 2 + n, (a +
b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + 3*((a - I*b)*A
ppellF1[2 + n, 3/2, 5/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[
c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 5/2, 3/2, 3 + n, (a + b*Ta
n[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c + d*x]
)))
```

Maple [F] time = 0.156, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\tan(dx+c)+a)^n\cos(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\tan(c+dx))^n\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

3.654 $\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal. Leaf size=161

$$\frac{\cos^5(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{5/2}(a+b\tan(c+dx))^{n+1}F_1\left(n+1;\frac{5}{2},\frac{5}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(}{a+\sqrt{-}}\right.}{bd(n+1)}$$

```
[Out] (AppellF1[1 + n, 5/2, 5/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^(1
 + n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(5/2)*(1 - (a + b*Tan[c +
 d*x])/(a + Sqrt[-b^2]))^(5/2))/(b*d*(1 + n))
```

Rubi [A] time = 0.134122, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3512, 760, 133}

$$\frac{\cos^5(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{5/2}(a+b\tan(c+dx))^{n+1}F_1\left(n+1;\frac{5}{2},\frac{5}{2};n+2;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(}{a+\sqrt{-}}\right.}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```


```
[Out] (AppellF1[1 + n, 5/2, 5/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^(1
 + n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(5/2)*(1 - (a + b*Tan[c +
 d*x])/(a + Sqrt[-b^2]))^(5/2))/(b*d*(1 + n))
```


Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\left(\cos^5(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\frac{b^2}{\sqrt{-b^2}}}\right)^{5/2}\right)\text{Subst}\left(\int \frac{x^n}{\left(1-\frac{x}{a-\sqrt{-b^2}}\right)^{5/2}\left(1\cdots\right.}\right.}{bd}$$

$$= \frac{F_1\left(1+n;\frac{5}{2},\frac{5}{2};2+n;\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^5(c+dx)(a+b\tan(c+dx))^{1\cdots}}{bd(1+n)}$$

Mathematica [C] time = 6.1758, size = 341, normalized size = 2.12

$$\frac{2(n+2)\left(a^2+b^2\right)^2\cos^7(c+dx)(\tan(c+dx)-i)(\tan(c+dx)+i)(a+b\,\text{ta}\cdots}{bd(n+1)(a-ib)(a+ib)\left(2(n+2)\left(a^2+b^2\right)F_1\left(n+1;\frac{5}{2},\frac{5}{2};n+2;\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+5(a+b\tan(c+dx))\left((a-i\cdots\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 5/2, 5/2, 2 + n, (a + b*Tan[c + d*
x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^7*(-I + Tan[c +
 d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a + I*b
)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 5/2, 5/2, 2 + n, (a +
b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + 5*((a - I*b)*A
ppellF1[2 + n, 5/2, 7/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[
c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 7/2, 5/2, 3 + n, (a + b*Ta
n[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c + d*x]
)))
```

Maple [F] time = 0.613, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^3(a+b\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\tan(dx+c)+a)^n\cos(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

3.655 $\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=124

$$-\frac{2ia(e\cos(c+dx))^{7/2}}{7d}+\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{7/2}}{21d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a\tan(c+dx)(e\cos(c+dx))^{7/2}}{7d}+\frac{10a\tan(c+dx)\sec^2(}{}$$

```
[Out] (((-2*I)/7)*a*(e*Cos[c + d*x])^(7/2))/d + (10*a*(e*Cos[c + d*x])^(7/2)*Elli
pticF[(c + d*x)/2, 2])/(21*d*Cos[c + d*x]^(7/2)) + (2*a*(e*Cos[c + d*x])^(7
/2)*Tan[c + d*x])/(7*d) + (10*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2*Tan[c
 + d*x])/(21*d)
```

Rubi [A] time = 0.13542, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3515, 3486, 3769, 3771, 2641}

$$-\frac{2ia(e\cos(c+dx))^{7/2}}{7d}+\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{7/2}}{21d\cos^{\frac{7}{2}}(c+dx)}+\frac{2a\tan(c+dx)(e\cos(c+dx))^{7/2}}{7d}+\frac{10a\tan(c+dx)\sec^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```


```
[Out] (((-2*I)/7)*a*(e*Cos[c + d*x])^(7/2))/d + (10*a*(e*Cos[c + d*x])^(7/2)*Elli
pticF[(c + d*x)/2, 2])/(21*d*Cos[c + d*x]^(7/2)) + (2*a*(e*Cos[c + d*x])^(7
/2)*Tan[c + d*x])/(7*d) + (10*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2*Tan[c
 + d*x])/(21*d)
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \left((e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \left(a(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int \frac{}{(e\sec(c}$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d} + \frac{(5a(e\cos(c+}{}$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d} + \frac{10a(e\cos(c+}{}$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d} + \frac{10a(e\cos(c+}{}$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{10a(e\cos(c+dx))^{7/2}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(e\cos}{}$$

Mathematica [A] time = 0.689215, size = 133, normalized size = 1.07

$$\frac{ae^3(\cos(dx)-i\sin(dx))\sqrt{e\cos(c+dx)}(\cos(c+2dx)+i\sin(c+2dx))\left(\sqrt{\cos(c+dx)}(5\sin(2(c+dx))+2i\cos(2(c+d}{21d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*e^3*Sqrt[e*Cos[c + d*x]]*(Cos[d*x] - I*Sin[d*x])*(10*EllipticF[(c + d*x)
/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + Sqrt[Cos[c + d*x]]*(-8*I + (2*I)*C
os[2*(c + d*x)] + 5*Sin[2*(c + d*x)]))*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x]))
/(21*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.749, size = 241, normalized size = 1.9

$$-\frac{2ae^4}{21d}\left(48i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+48\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^8-96i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-72(\sin(1/2dx+c/$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] -2/21/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^4*(48*I*si
n(1/2*d*x+1/2*c)^9+48*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-96*I*sin(1/2*
d*x+1/2*c)^7-72*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+72*I*sin(1/2*d*x+1/
2*c)^5+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-24*I*sin(1/2*d*x+1/2*c)^3
+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+3*I*
```

```
sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{2}}(ia\tan(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")

[Out] integrate((e*cos(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(\sqrt{\frac{1}{2}}\left(-3i\,ae^{3}e^{(4i\,dx+4i\,c)}-16i\,ae^{3}e^{(2i\,dx+2i\,c)}+7i\,ae^{3}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+42\,de^{(i\,dx+i\,c)}\mathrm{integral}\left(-\frac{10i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+}}{21\left(de^{(2i\,dx+}\right.}\right.\right.}{42\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/42*(sqrt(1/2)*(-3*I*a*e^3*e^(4*I*d*x + 4*I*c) - 16*I*a*e^3*e^(2*I*d*x + 2
*I*c) + 7*I*a*e^3)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c)
 + 42*d*e^(I*d*x + I*c)*integral(-10/21*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I
*c) + e)*a*e^3*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^(2*I*d*x + 2*I*c) + d), x))*e^
(-I*d*x - I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+I*a*tan(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{7}{2}}(ia\tan(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")

[Out] integrate((e*cos(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

3.656 $\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=90

$$-\frac{2ia(e\cos(c+dx))^{5/2}}{5d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{5/2}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\tan(c+dx)(e\cos(c+dx))^{5/2}}{5d}$$

```
[Out] (((-2*I)/5)*a*(e*Cos[c + d*x])^(5/2))/d + (6*a*(e*Cos[c + d*x])^(5/2)*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(e*Cos[c + d*x])^(5/2
)*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.111353, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3515, 3486, 3769, 3771, 2639}

$$-\frac{2ia(e\cos(c+dx))^{5/2}}{5d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{5/2}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\tan(c+dx)(e\cos(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((-2*I)/5)*a*(e*Cos[c + d*x])^(5/2))/d + (6*a*(e*Cos[c + d*x])^(5/2)*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(e*Cos[c + d*x])^(5/2
)*Tan[c + d*x])/(5*d)
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx$$
$$= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d} + \left(a(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{1}{(e\sec(c+\ldots}$$
$$= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d} + \frac{2a(e\cos(c+dx))^{5/2}\tan(c+dx)}{5d} + \frac{(3a(e\cos(c+dx\ldots}{\ldots}$$
$$= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d} + \frac{2a(e\cos(c+dx))^{5/2}\tan(c+dx)}{5d} + \frac{(3a(e\cos(c+dx\ldots}{5c\ldots}$$
$$= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d} + \frac{6a(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(e\cos(c+\ldots}{\ldots}$$

Mathematica [C] time = 12.4758, size = 387, normalized size = 4.3

$$\frac{(\cos(dx)-i\sin(dx))(a+ia\tan(c+dx))(e\cos(c+dx))^{5/2}\left(\frac{2\sqrt{2}(\cot(c)-i)e^{-idx}\left(e^{2idx}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+3e^{2i(c+dx)}-3\sqrt{1-ie^{i(c+\ldots}}\ldots}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(Cos[d*x] - I*Sin[d*x])*((2*Sqrt[2]*(-I + Cot[c])*(
3 + 3*E^((2*I)*(c + d*x)) + 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*
x))*(-I + E^(I*(c + d*x)))]*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c
 + d*x]]], -1] - 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E
^(I*(c + d*x)))]*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]],
-1] + E^((2*I)*d*x)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))]))/(5*E^(I*d*x)*Sqrt[(1 + E^((2*I)*(c + d*x)))
/E^(I*(c + d*x))]) + (2*Sqrt[Cos[c + d*x]]*Sin[c]*(-1 + Cos[2*d*x]*(1 - I*C
ot[c]) - 6*Cot[c]^2 + I*Sin[2*d*x] + Cot[c]*(5*I + Sin[2*d*x])))/5)*(a + I*
a*Tan[c + d*x]))/(2*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 2.346, size = 205, normalized size = 2.3

$$\frac{2\,ae^3}{5\,d}\left(8\,i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+8\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)-12\,i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-8\,(\sin(1/2\,dx+c/2))^4\cos(\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/5/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^3*(8*I*sin(1
/2*d*x+1/2*c)^7+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*I*sin(1/2*d*x+
1/2*c)^5-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+6*I*sin(1/2*d*x+1/2*c)^3
```

```
+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-I*sin
(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}} (i a\tan(dx+c)+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left(-i ae^2 e^{(3 i dx+3 i c)} + i ae^2 e^{(2 i dx+2 i c)} - 7 i ae^2 e^{(i dx+i c)} - 5 i ae^2\right)\sqrt{ee^{(2 i dx+2 i c)} + e}e^{\left(-\frac{1}{2} i dx-\frac{1}{2} i c\right)} + 5\left(de^{(i dx+i c)} - d\right)\text{integral}}{5\left(de^{(i dx+i c)} - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(1/2)*(-I*a*e^2*e^(3*I*d*x + 3*I*c) + I*a*e^2*e^(2*I*d*x + 2*I*c)
- 7*I*a*e^2*e^(I*d*x + I*c) - 5*I*a*e^2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^
(-1/2*I*d*x - 1/2*I*c) + 5*(d*e^(I*d*x + I*c) - d)*integral(1/5*sqrt(1/2)*(
-6*I*a*e^2*e^(2*I*d*x + 2*I*c) - 12*I*a*e^2*e^(I*d*x + I*c) - 6*I*a*e^2)*sq
rt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^(4*I*d*x + 4*I*
c) - 2*d*e^(3*I*d*x + 3*I*c) + 2*d*e^(2*I*d*x + 2*I*c) - 2*d*e^(I*d*x + I*c
) + d), x))/(d*e^(I*d*x + I*c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{5}{2}} (i a\tan(dx+c)+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

3.657 $\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=90

$$-\frac{2ia(e\cos(c+dx))^{3/2}}{3d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\tan(c+dx)(e\cos(c+dx))^{3/2}}{3d}$$

```
[Out] (((-2*I)/3)*a*(e*Cos[c + d*x])^(3/2))/d + (2*a*(e*Cos[c + d*x])^(3/2)*Ellip
ticF[(c + d*x)/2, 2])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(e*Cos[c + d*x])^(3/2
)*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.109314, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3515, 3486, 3769, 3771, 2641}

$$-\frac{2ia(e\cos(c+dx))^{3/2}}{3d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\tan(c+dx)(e\cos(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((-2*I)/3)*a*(e*Cos[c + d*x])^(3/2))/d + (2*a*(e*Cos[c + d*x])^(3/2)*Ellip
ticF[(c + d*x)/2, 2])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(e*Cos[c + d*x])^(3/2
)*Tan[c + d*x])/(3*d)
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx$$
$$= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \left(a(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{1}{(e\sec(c+}$$
$$= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \frac{2a(e\cos(c+dx))^{3/2}\tan(c+dx)}{3d} + \frac{\left(a(e\cos(c+dx)\right.}{}$$
$$= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \frac{2a(e\cos(c+dx))^{3/2}\tan(c+dx)}{3d} + \frac{\left(a(e\cos(c+dx)\right.}{3\cos}$$
$$= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \frac{2a(e\cos(c+dx))^{3/2}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(e\cos(c+}{}$$

Mathematica [A] time = 0.342929, size = 100, normalized size = 1.11

$$\frac{2ae\sqrt{\cos(c+dx)}(\tan(c+dx)-i)(\cos(dx)-i\sin(dx))\sqrt{e\cos(c+dx)}\left(\sqrt{\cos(c+dx)}(\cos(dx)+i\sin(dx))+(\sin(c)+i\cos\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*e*Sqrt[Cos[c + d*x]]*Sqrt[e*Cos[c + d*x]]*(EllipticF[(c + d*x)/2, 2]*(
I*Cos[c] + Sin[c]) + Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x]))*(Cos[d*x]
- I*Sin[d*x])*(-I + Tan[c + d*x]))/(3*d)
```

Maple [A] time = 2.457, size = 168, normalized size = 1.9

$$-\frac{2\,ae^2}{3\,d}\left(4\,i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+4\,(\sin(1/2\,dx+c/2))^4\cos(1/2\,dx+c/2)-4\,i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\,(\mathrm{si}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] -2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^2*(4*I*sin(
1/2*d*x+1/2*c)^5+4*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*I*sin(1/2*d*x+
1/2*c)^3+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{-2i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}aee^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\,d\mathrm{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}aee^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)},x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(-2*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*a*e*e^(1/2*I*d*x + 1/2*
I*c) + 3*d*integral(-2/3*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*a*e*e^
(-1/2*I*d*x - 1/2*I*c)/(d*e^(2*I*d*x + 2*I*c) + d), x))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

3.658 $\int \sqrt{e \cos(c+dx)}(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2ia\sqrt{e\cos(c+dx)}}{d}$$

```
[Out] ((-2*I)*a*Sqrt[e*Cos[c + d*x]])/d + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0782001, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3515, 3486, 3771, 2639}

$$\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{2ia\sqrt{e\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-2*I)*a*Sqrt[e*Cos[c + d*x]])/d + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = \left(\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx$$
$$= -\frac{2ia\sqrt{e\cos(c+dx)}}{d} + \left(a\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{e\sec(c+dx)}}$$
$$= -\frac{2ia\sqrt{e\cos(c+dx)}}{d} + \frac{\left(a\sqrt{e\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{2ia\sqrt{e\cos(c+dx)}}{d} + \frac{2a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 0.690825, size = 244, normalized size = 4.07

$$\frac{ae(\cot(c)+i)e^{-i(c+dx)}\left(e^{2idx}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-3\sqrt{1-ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}\left(e^{i(c+dx)}-i\right)}F\left(\sin^{-1}\left(\sqrt{\sin(c\right.\right.}{3d\sqrt{e\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*e*(I + Cot[c])*(3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I +
 E^(I*(c + d*x)))]*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]]
, -1] - 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c +
d*x)))]*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1] + E^(
(2*I)*d*x)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))]))/(3*d*E^(I*(c + d*x))*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 1.352, size = 108, normalized size = 1.8

$$2\,\frac{ae\left(2\,i\left(\sin\left(1/2\,dx+c/2\right)\right)^{3}+\mathrm{EllipticE}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}-\right.}{\sin\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}e+ed}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e*(2*I*sin(1/2*d
*x+1/2*c)^3+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos(dx+c)}(i\,a\tan(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{-4i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}ae^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\left(de^{(i\,dx+i\,c)}-d\right)\mathrm{integral}\left(\frac{\sqrt{\frac{1}{2}}\left(-2i\,ae^{(2i\,dx+2i\,c)}-4i\,ae^{(i\,dx+i\,c)}-2i\,a\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{de^{(4i\,dx+4i\,c)}-2\,de^{(3i\,dx+3i\,c)}+2\,de^{(2i\,dx+2i\,c)}-2\,de^{(i\,dx+i\,c)}+d},x\right)}{de^{(i\,dx+i\,c)}-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-4*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*a*e^(1/2*I*d*x + 1/2*I*c) +
 (d*e^(I*d*x + I*c) - d)*integral(sqrt(1/2)*(-2*I*a*e^(2*I*d*x + 2*I*c) - 4
*I*a*e^(I*d*x + I*c) - 2*I*a)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x
 - 1/2*I*c)/(d*e^(4*I*d*x + 4*I*c) - 2*d*e^(3*I*d*x + 3*I*c) + 2*d*e^(2*I*d
*x + 2*I*c) - 2*d*e^(I*d*x + I*c) + d), x))/(d*e^(I*d*x + I*c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e\cos\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

3.659 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}+\frac{2ia}{d\sqrt{e\cos(c+dx)}}$$

```
[Out] ((2*I)*a)/(d*Sqrt[e*Cos[c + d*x]]) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```

Rubi [A] time = 0.0793143, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3515, 3486, 3771, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}+\frac{2ia}{d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] ((2*I)*a)/(d*Sqrt[e*Cos[c + d*x]]) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx = \frac{\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= \frac{2ia}{d\sqrt{e\cos(c+dx)}} + \frac{a\int \sqrt{e\sec(c+dx)}\,dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$
$$= \frac{2ia}{d\sqrt{e\cos(c+dx)}} + \frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}}$$
$$= \frac{2ia}{d\sqrt{e\cos(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{e\cos(c+dx)}}$$

Mathematica [C] time = 1.11397, size = 143, normalized size = 2.38

$$-\frac{\sqrt{2}a\sin(c)(\cot(c)-i)(\tan(c+dx)-i)(\cos(dx)-i\sin(dx))\sqrt{e\cos(c+dx)}\left(\sqrt{2}\sqrt{\csc^2(c)}+i\csc(c)\cos(c+dx)\sqrt{\cos\left(2dx\right.}\right.}{de\sqrt{\csc^2(c)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*a*Sqrt[e*Cos[c + d*x]]*(-I + Cot[c])*(Sqrt[2]*Sqrt[Csc[c]^2] + I
*Cos[c + d*x]*Sqrt[1 + Cos[2*d*x - 2*ArcTan[Cot[c]]]]*Csc[c]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]
]])*Sin[c]*(Cos[d*x] - I*Sin[d*x])*(-I + Tan[c + d*x]))/(d*e*Sqrt[Csc[c]^2]
))
```

Maple [A] time = 2.592, size = 94, normalized size = 1.6

$$2\,\frac{\left(-\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)+i\sin\left(1/2\,dx+c/2\right)\right)a}{\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2e+e}\sin\left(1/2\,dx+c/2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/sin(1/2*d*x+1/2*c)*(-(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+I*sin(1/2*d*x+1/2*c))*a/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,a\tan\left(dx+c\right)+a}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{4i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}ae^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\left(dee^{(2i\,dx+2i\,c)}+de\right)\mathrm{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}ae^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{dee^{(2i\,dx+2i\,c)}+de},x\right)}{dee^{(2i\,dx+2i\,c)}+de}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (4*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*a*e^(1/2*I*d*x + 1/2*I*c) +
(d*e*e^(2*I*d*x + 2*I*c) + d*e)*integral(-2*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x +
 2*I*c) + e)*a*e^(-1/2*I*d*x - 1/2*I*c)/(d*e*e^(2*I*d*x + 2*I*c) + d*e), x)
)/(d*e*e^(2*I*d*x + 2*I*c) + d*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,a\tan\left(dx+c\right)+a}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

3.660 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{2ia}{3d(e\cos(c+dx))^{3/2}}-\frac{2a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] (((2*I)/3)*a)/(d*(e*Cos[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^(3/2)*Elliptic
E[(c + d*x)/2, 2])/(d*(e*Cos[c + d*x])^(3/2)) + (2*a*Sin[c + d*x])/(d*e*Sqr
t[e*Cos[c + d*x]])
```

Rubi [A] time = 0.104711, antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3515, 3486, 3768, 3771, 2639}

$$\frac{2ia}{3d(e\cos(c+dx))^{3/2}}-\frac{2a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d(e\cos(c+dx))^{3/2}}+\frac{2a\sin(c+dx)\cos(c+dx)}{d(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/3)*a)/(d*(e*Cos[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^(3/2)*Elliptic
E[(c + d*x)/2, 2])/(d*(e*Cos[c + d*x])^(3/2)) + (2*a*Cos[c + d*x]*Sin[c + d
*x])/(d*(e*Cos[c + d*x])^(3/2))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx &= \frac{\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} + \frac{a\int (e\sec(c+dx))^{3/2}\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{d(e\cos(c+dx))^{3/2}} - \frac{\left(ae^2\right)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}\\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{d(e\cos(c+dx))^{3/2}} - \frac{\left(a\cos^{\frac{3}{2}}(c+dx)\right)\int \sqrt{\cos(c+dx)}\,dx}{(e\cos(c+dx))^{3/2}}\\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} - \frac{2a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d(e\cos(c+dx))^{3/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{d(e\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 3.9527, size = 369, normalized size = 4.15

$$\cos^{\frac{5}{2}}(c+dx)(\cos(dx)-i\sin(dx))(a+ia\tan(c+dx))\left(\frac{2\sin(c)(\cot(c)-i)(3\csc(c)\cos(c+2dx)+3\cot(c)+2i)}{3\cos^{\frac{3}{2}}(c+dx)} - \frac{2\sqrt{2}(\cot(c)-i)e^{-idx}\left(e^{2idx}\sqrt{1\cdots}\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*((-2*Sqrt[2]*(-I + Cot[c])*(3 + 3*E^((2*I)*(c + d*x)) +
 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c + d*x)))]
*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1] - 3*Sqrt[1 -
 I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c + d*x)))]*EllipticF[
ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1] + E^((2*I)*d*x)*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)]))/(3*E^(I*d*x)*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) + (2*(-I
 + Cot[c])*(2*I + 3*Cot[c] + 3*Cos[c + 2*d*x]*Csc[c])*Sin[c])/(3*Cos[c + d*
x]^(3/2)))*(Cos[d*x] - I*Sin[d*x])*(a + I*a*Tan[c + d*x]))/(2*d*(e*Cos[c +
d*x])^(3/2))
```

Maple [B] time = 3.768, size = 214, normalized size = 2.4

$$-\frac{2a}{3de}\left(6\sqrt{2(\sin(1/2dx+c/2))^2-1}\sqrt{(\sin(1/2dx+c/2))^2}\mathrm{EllipticE}\left(\cos(1/2dx+c/2),\sqrt{2}\right)(\sin(1/2dx+c/2))^2-12\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*e+e)^(1/2)/e*(6*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-12*sin(1/2*d
```

```
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+6*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+I*sin(1/2*d*x+1/2*c))*a/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left(-12i\,ae^{(4i\,dx+4i\,c)}-4i\,ae^{(2i\,dx+2i\,c)}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+3\left(de^{2}e^{(4i\,dx+4i\,c)}+2\,de^{2}e^{(2i\,dx+2i\,c)}+de^{2}\right)\mathrm{integral}\left(\frac{2i\sqrt{\cdots}}{\cdots}\right)}{3\left(de^{2}e^{(4i\,dx+4i\,c)}+2\,de^{2}e^{(2i\,dx+2i\,c)}+de^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(1/2)*(-12*I*a*e^(4*I*d*x + 4*I*c) - 4*I*a*e^(2*I*d*x + 2*I*c))*sq
rt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c) + 3*(d*e^2*e^(4*I*d*
x + 4*I*c) + 2*d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)*integral(2*I*sqrt(1/2)*sq
rt(e*e^(2*I*d*x + 2*I*c) + e)*a*e^(1/2*I*d*x + 1/2*I*c)/(d*e^2*e^(2*I*d*x +
 2*I*c) + d*e^2), x))/(d*e^2*e^(4*I*d*x + 4*I*c) + 2*d*e^2*e^(2*I*d*x + 2*I
*c) + d*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

3.661 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2ia}{5d(e\cos(c+dx))^{5/2}}+\frac{2a\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)\cos(c+dx)}{3d(e\cos(c+dx))^{5/2}}$$

```
[Out] (((2*I)/5)*a)/(d*(e*Cos[c + d*x])^(5/2)) + (2*a*Cos[c + d*x]^(5/2)*Elliptic
F[(c + d*x)/2, 2])/(3*d*(e*Cos[c + d*x])^(5/2)) + (2*a*Cos[c + d*x]*Sin[c +
 d*x])/(3*d*(e*Cos[c + d*x])^(5/2))
```

Rubi [A] time = 0.103749, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3515, 3486, 3768, 3771, 2641}

$$\frac{2ia}{5d(e\cos(c+dx))^{5/2}}+\frac{2a\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d(e\cos(c+dx))^{5/2}}+\frac{2a\sin(c+dx)\cos(c+dx)}{3d(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/5)*a)/(d*(e*Cos[c + d*x])^(5/2)) + (2*a*Cos[c + d*x]^(5/2)*Elliptic
F[(c + d*x)/2, 2])/(3*d*(e*Cos[c + d*x])^(5/2)) + (2*a*Cos[c + d*x]*Sin[c +
 d*x])/(3*d*(e*Cos[c + d*x])^(5/2))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx &= \frac{\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&= \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{a\int (e\sec(c+dx))^{5/2}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&= \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}} + \frac{\left(ae^2\right)\int \sqrt{e\sec(c+dx)}\,dx}{3(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&= \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}} + \frac{\left(a\cos^{\frac{5}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3(e\cos(c+dx))^{5/2}} \\
&= \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d(e\cos(c+dx))^{5/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.495286, size = 57, normalized size = 0.59

$$\frac{a\left(5\sin(2(c+dx))+10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+6i\right)}{15d(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (a*(6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*(c + d*
x)]))/(15*d*(e*Cos[c + d*x])^(5/2))
```

Maple [B] time = 4.727, size = 283, normalized size = 3.

$$\frac{2a}{15e^2d}\left(-20\sqrt{2(\sin(1/2dx+c/2))^2-1}\sqrt{(\sin(1/2dx+c/2))^2}\text{EllipticF}\left(\cos(1/2dx+c/2),\sqrt{2}\right)(\sin(1/2dx+c/2))^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/15/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)/(
-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^2*(-20*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/
2*d*x+1/2*c)^4+20*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-20*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+10*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)+3*I*sin(1/2*d*x+1/2*c))*a/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i a \tan(dx + c) + a}{(e \cos(dx + c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left(-20 i a e^{(5 i dx+5 i c)} + 48 i a e^{(3 i dx+3 i c)} + 20 i a e^{(i dx+i c)}\right)\sqrt{e e^{(2 i dx+2 i c)} + e}\, e^{\left(-\frac{1}{2} i dx-\frac{1}{2} i c\right)} + 15\left(d e^3 e^{(6 i dx+6 i c)} + 3\, d e^3 e^{(4 i dx+4 i c)} + \right.}{15\left(d e^3 e^{(6 i dx+6 i c)} + 3\, d e^3 e^{(4 i dx+4 i c)} + 3\, d e^3 e^{(2 i dx+2 i c)} + d e^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(sqrt(1/2)*(-20*I*a*e^(5*I*d*x + 5*I*c) + 48*I*a*e^(3*I*d*x + 3*I*c) +
 20*I*a*e^(I*d*x + I*c))*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/
2*I*c) + 15*(d*e^3*e^(6*I*d*x + 6*I*c) + 3*d*e^3*e^(4*I*d*x + 4*I*c) + 3*d*
e^3*e^(2*I*d*x + 2*I*c) + d*e^3)*integral(-2/3*I*sqrt(1/2)*sqrt(e*e^(2*I*d*
x + 2*I*c) + e)*a*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^3*e^(2*I*d*x + 2*I*c) + d*e
^3), x))/(d*e^3*e^(6*I*d*x + 6*I*c) + 3*d*e^3*e^(4*I*d*x + 4*I*c) + 3*d*e^3
*e^(2*I*d*x + 2*I*c) + d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i a \tan(dx + c) + a}{(e \cos(dx + c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

3.662 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=130

$$\frac{2ia}{7d(e\cos(c+dx))^{7/2}}+\frac{6a\sin(c+dx)\cos^3(c+dx)}{5d(e\cos(c+dx))^{7/2}}-\frac{6a\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d(e\cos(c+dx))^{7/2}}+\frac{2a\sin(c+dx)\cos(c+dx)}{5d(e\cos(c+dx))^{7/2}}$$

```
[Out] (((2*I)/7)*a)/(d*(e*Cos[c + d*x])^(7/2)) - (6*a*Cos[c + d*x]^(7/2)*Elliptic
E[(c + d*x)/2, 2])/(5*d*(e*Cos[c + d*x])^(7/2)) + (2*a*Cos[c + d*x]*Sin[c +
 d*x])/(5*d*(e*Cos[c + d*x])^(7/2)) + (6*a*Cos[c + d*x]^3*Sin[c + d*x])/(5*
d*(e*Cos[c + d*x])^(7/2))
```

Rubi [A] time = 0.127311, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3515, 3486, 3768, 3771, 2639}

$$\frac{2ia}{7d(e\cos(c+dx))^{7/2}}+\frac{6a\sin(c+dx)\cos^3(c+dx)}{5d(e\cos(c+dx))^{7/2}}-\frac{6a\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d(e\cos(c+dx))^{7/2}}+\frac{2a\sin(c+dx)\cos(c+dx)}{5d(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (((2*I)/7)*a)/(d*(e*Cos[c + d*x])^(7/2)) - (6*a*Cos[c + d*x]^(7/2)*Elliptic
E[(c + d*x)/2, 2])/(5*d*(e*Cos[c + d*x])^(7/2)) + (2*a*Cos[c + d*x]*Sin[c +
 d*x])/(5*d*(e*Cos[c + d*x])^(7/2)) + (6*a*Cos[c + d*x]^3*Sin[c + d*x])/(5*
d*(e*Cos[c + d*x])^(7/2))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \frac{\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{a\int (e\sec(c+dx))^{7/2}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{\left(3ae^2\right)\int (e\sec(c+dx))^{3/2}\,dx}{5(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{6a\cos^3(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} - \frac{\left(3ae^4\right.}{5(e\cos(c+}$$
$$= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{6a\cos^3(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} - \frac{\left(3a\cos^{\frac{7}{2}}(c\right.}{5(}$$
$$= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} - \frac{6a\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{6a\cos}{5d}$$

Mathematica [C] time = 6.47058, size = 596, normalized size = 4.58

$$\frac{\cos^5(c+dx)(a+ia\tan(c+dx))\left(\left(\frac{2\sin(c)}{7}+\frac{2}{7}i\cos(c)\right)\sec^4(c+dx)+\sec(c)\left(\frac{2\cos(c)}{5}-\frac{2}{5}i\sin(c)\right)\sin(dx)\sec^3(c+dx)+\mathrm{se}\right.}{d(\cos(dx)+i\sin(d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((-I/2)*Cos[c + d*x]^(9/2)*((-3*I)/5 + (3*Cot[c])/5)*((((-2*I)/3)*Sqrt[2]*E
^(I*d*x)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))])/Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))] - (2*(C
os[c/2] + I*Sin[c/2])^2*((2*I)*Cos[c + d*x] + EllipticF[ArcSin[Sqrt[(-I)*Co
s[c + d*x] + Sin[c + d*x]]], -1]*((-I)*Cos[c + d*x] - Sin[c + d*x])*Sqrt[1
- I*Cos[c + d*x] + Sin[c + d*x]]*Sqrt[(-I)*Cos[c + d*x] + Cos[2*(c + d*x)]
+ Sin[c + d*x] + I*Sin[2*(c + d*x)]] + EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d
*x] + Sin[c + d*x]]], -1]*Sqrt[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(I*Cos[c
+ d*x] + Sin[c + d*x])*Sqrt[(-I)*Cos[c + d*x] + Cos[2*(c + d*x)] + Sin[c +
d*x] + I*Sin[2*(c + d*x)]]]))/Sqrt[Cos[c + d*x]])*(a + I*a*Tan[c + d*x]))/(d
*(e*Cos[c + d*x])^(7/2)*(Cos[d*x] + I*Sin[d*x])) + (Cos[c + d*x]^5*(Csc[c]*
Sec[c]*((6*Cos[c])/5 - ((6*I)/5)*Sin[c]) + Sec[c + d*x]^4*(((2*I)/7)*Cos[c]
 + (2*Sin[c])/7) + Sec[c]*Sec[c + d*x]^3*((2*Cos[c])/5 - ((2*I)/5)*Sin[c])*
Sin[d*x] + Sec[c]*Sec[c + d*x]*((6*Cos[c])/5 - ((6*I)/5)*Sin[c])*Sin[d*x] +
 Sec[c + d*x]^2*((2*Cos[c])/5 - ((2*I)/5)*Sin[c])*Tan[c])*(a + I*a*Tan[c +
d*x]))/(d*(e*Cos[c + d*x])^(7/2)*(Cos[d*x] + I*Sin[d*x]))
```

Maple [B] time = 8.012, size = 396, normalized size = 3.1

$$-\frac{2a}{35e^3d}\left(168\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\left(\sin(1/2\,dx+c/2)\right)^6-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/35/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^
2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^3*(168*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-336*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+
1/2*c)^8-252*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+504*sin(1/2*d*x+
1/2*c)^6*cos(1/2*d*x+1/2*c)+126*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)
^2-280*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-21*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+
56*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*I*sin(1/2*d*x+1/2*c))*a/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left(-84i\,ae^{(8i\,dx+8i\,c)}-308i\,ae^{(6i\,dx+6i\,c)}-92i\,ae^{(4i\,dx+4i\,c)}-28i\,ae^{(2i\,dx+2i\,c)}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+35\left(de^4e^{(8i\,dx+\cdots}}{35\left(de^4e^{(8i\,dx+8i\,c)}+4\,de^4e^{(6i\,dx+6i\,c)}+6\,de^4e^{(4i\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/35*(sqrt(1/2)*(-84*I*a*e^(8*I*d*x + 8*I*c) - 308*I*a*e^(6*I*d*x + 6*I*c)
- 92*I*a*e^(4*I*d*x + 4*I*c) - 28*I*a*e^(2*I*d*x + 2*I*c))*sqrt(e*e^(2*I*d*
x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c) + 35*(d*e^4*e^(8*I*d*x + 8*I*c) +
4*d*e^4*e^(6*I*d*x + 6*I*c) + 6*d*e^4*e^(4*I*d*x + 4*I*c) + 4*d*e^4*e^(2*I*
d*x + 2*I*c) + d*e^4)*integral(6/5*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) +
 e)*a*e^(1/2*I*d*x + 1/2*I*c)/(d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4), x))/(d*e
^4*e^(8*I*d*x + 8*I*c) + 4*d*e^4*e^(6*I*d*x + 6*I*c) + 6*d*e^4*e^(4*I*d*x +
 4*I*c) + 4*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ia\tan(dx+c)+a}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

3.663 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=190

$$\frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{7/2}}{7a^2d\cos^{\frac{7}{2}}(c+dx)}+\frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{7/2}}{15a^2d}+$$

```
[Out] (2*(e*Cos[c + d*x])^(7/2)*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Cos[c + d*x]^
(7/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(15*a^2*d) +
(6*(e*Cos[c + d*x])^(7/2)*Tan[c + d*x])/(35*a^2*d) + (2*(e*Cos[c + d*x])^(7
/2)*Sec[c + d*x]^2*Tan[c + d*x])/(7*a^2*d) + (((4*I)/15)*Cos[c + d*x]^2*(e*
Cos[c + d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.220811, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3515, 3500, 3769, 3771, 2641}

$$\frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{7/2}}{7a^2d\cos^{\frac{7}{2}}(c+dx)}+\frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{7/2}}{15a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```


```
[Out] (2*(e*Cos[c + d*x])^(7/2)*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Cos[c + d*x]^
(7/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(15*a^2*d) +
(6*(e*Cos[c + d*x])^(7/2)*Tan[c + d*x])/(35*a^2*d) + (2*(e*Cos[c + d*x])^(7
/2)*Sec[c + d*x]^2*Tan[c + d*x])/(7*a^2*d) + (((4*I)/15)*Cos[c + d*x]^2*(e*
Cos[c + d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \left((e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$$

$$= \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(11e^2(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int \frac{1}{(e\sec(c+dx))^{11/2}}}{15a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{(3(e\cos(c}{}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} + \frac{4i\cos^2(c+}{15d\left(a^2+\right.}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} + \frac{2(e\cos(c+}{}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} + \frac{2(e\cos(c+}{}$$

$$= \frac{2(e\cos(c+dx))^{7/2}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(}{}$$

Mathematica [A] time = 1.23597, size = 156, normalized size = 0.82

$$\frac{e^3\sqrt{e\cos(c+dx)}\left(\sqrt{\cos(c+dx)}(134\sin(c+dx)-117\sin(3(c+dx))-11\sin(5(c+dx))-296i\cos(c+dx)+68i\cos(3(c\right.}{840a^2d\cos^{\frac{5}{2}}(c+dx)(\tan(c+dx)-i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (e^3*Sqrt[e*Cos[c + d*x]]*(-240*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)]
 + I*Sin[2*(c + d*x)]) + Sqrt[Cos[c + d*x]]*((-296*I)*Cos[c + d*x] + (68*I)
*Cos[3*(c + d*x)] + (4*I)*Cos[5*(c + d*x)] + 134*Sin[c + d*x] - 117*Sin[3*(
c + d*x)] - 11*Sin[5*(c + d*x)])))/(840*a^2*d*Cos[c + d*x]^(5/2)*(-I + Tan[
c + d*x])^2)
```

Maple [B] time = 3.086, size = 387, normalized size = 2.

$$\frac{2e^4}{105a^2d}\left(14i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)-3584\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^{16}+1568i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+12544(\sin(1/2dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/105/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(14*I*
sin(1/2*d*x+1/2*c)-3584*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^16+1568*I*sin
(1/2*d*x+1/2*c)^5+12544*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)-25088*I*si
n(1/2*d*x+1/2*c)^11-19264*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)-224*I*si
n(1/2*d*x+1/2*c)^3+16800*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+25088*I*s
in(1/2*d*x+1/2*c)^13-9104*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+3584*I*si
n(1/2*d*x+1/2*c)^17+3128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-6272*I*sin
(1/2*d*x+1/2*c)^7-700*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-14336*I*sin(1
/2*d*x+1/2*c)^15-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+90*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+15680*I*sin(1/2*d*x+1/2*c)^9)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(1680\,a^2 d e^{(7i\,dx+7i\,c)}\text{integral}\left(-\frac{2i\,\sqrt{\frac{1}{2}}\,\sqrt{ee^{(2i\,dx+2i\,c)}+e}\,e^3 e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{7\left(a^2 d e^{(2i\,dx+2i\,c)}+a^2 d\right)},x\right)+\sqrt{\frac{1}{2}}\left(-15i\,e^3 e^{(10i\,dx+10i\,c)}-185i\,e^3 e^{(8i\,dx+8i\,c)}+430i\,e^3 e\right.\right.}{1680\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1680*(1680*a^2*d*e^(7*I*d*x + 7*I*c)*integral(-2/7*I*sqrt(1/2)*sqrt(e*e^(
2*I*d*x + 2*I*c) + e)*e^3*e^(-1/2*I*d*x - 1/2*I*c)/(a^2*d*e^(2*I*d*x + 2*I*
c) + a^2*d), x) + sqrt(1/2)*(-15*I*e^3*e^(10*I*d*x + 10*I*c) - 185*I*e^3*e^
(8*I*d*x + 8*I*c) + 430*I*e^3*e^(6*I*d*x + 6*I*c) + 162*I*e^3*e^(4*I*d*x +
4*I*c) + 49*I*e^3*e^(2*I*d*x + 2*I*c) + 7*I*e^3)*sqrt(e*e^(2*I*d*x + 2*I*c)
 + e)*e^(-1/2*I*d*x - 1/2*I*c))*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(ia\tan(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.664 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=154

$$\frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{5/2}}{65a^2d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{5/2}}{13a^2d}$$

```
[Out] (42*(e*Cos[c + d*x])^(5/2)*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*Cos[c + d*x
]^(5/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(13*a^2*d)
+ (14*(e*Cos[c + d*x])^(5/2)*Tan[c + d*x])/(65*a^2*d) + (((4*I)/13)*Cos[c +
 d*x]^2*(e*Cos[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.19137, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3515, 3500, 3769, 3771, 2639}

$$\frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{5/2}}{65a^2d\cos^{\frac{5}{2}}(c+dx)}+\frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{5/2}}{13a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (42*(e*Cos[c + d*x])^(5/2)*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*Cos[c + d*x
]^(5/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(13*a^2*d)
+ (14*(e*Cos[c + d*x])^(5/2)*Tan[c + d*x])/(65*a^2*d) + (((4*I)/13)*Cos[c +
 d*x]^2*(e*Cos[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$$

$$= \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(9e^2(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{1}{(e\sec(c+dx))^{9/2}}\,d}{13a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(7(e\cos(c\right.}{}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{14(e\cos(c+dx))^{5/2}\tan(c+dx)}{65a^2d} + \frac{4i\cos^2(c+}{13d\left(a^2\right.}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{14(e\cos(c+dx))^{5/2}\tan(c+dx)}{65a^2d} + \frac{4i\cos^2(c+}{13d\left(a^2\right.}$$

$$= \frac{42(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{14(e\,co}{}$$

Mathematica [C] time = 3.12367, size = 471, normalized size = 3.06

$$\frac{(\cos(dx)+i\sin(dx))^2(e\cos(c+dx))^{5/2}\left(\frac{14\sqrt{2}\csc(c)e^{-idx}(\cos(2c)+i\sin(2c))\left(e^{2idx}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+3e^{2i(c+dx)}-3\sqrt{1-ie^{i(c+dx)}}\sqrt{e^{i(c}}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^2*((14*Sqrt[2]*Csc[c]*(3 +
3*E^((2*I)*(c + d*x)) + 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*
(-I + E^(I*(c + d*x)))]*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d
*x]]], -1] - 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*
(c + d*x)))]*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1]
+ E^((2*I)*d*x)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7
/4, -E^((2*I)*(c + d*x))])*(Cos[2*c] + I*Sin[2*c]))/(65*E^(I*d*x)*Sqrt[(1 +
 E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]) - (Sqrt[Cos[c + d*x]]*Csc[c]*(Cos[2
*d*x] - I*Sin[2*d*x])*(178*Cos[c + 2*d*x] + 158*Cos[3*c + 2*d*x] - 9*Cos[3*
c + 4*d*x] + 9*Cos[5*c + 4*d*x] - (88*I)*Sin[c] + (208*I)*Sin[c + 2*d*x] +
(128*I)*Sin[3*c + 2*d*x] - (4*I)*Sin[3*c + 4*d*x] + (4*I)*Sin[5*c + 4*d*x])
)/260))/(2*d*Cos[c + d*x]^(9/2)*(a + I*a*Tan[c + d*x])^2)
```

Maple [B] time = 3.112, size = 351, normalized size = 2.3

$$-\frac{2\,e^3}{65\,a^2d}\left(1280\,i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{15}-1280\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{14}\cos\left(1/2\,dx+c/2\right)-5600\,i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}+3840\,(\mathrm{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -2/65/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(1280*
I*sin(1/2*d*x+1/2*c)^15-1280*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)-5600*
I*sin(1/2*d*x+1/2*c)^9+3840*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)-10*I*s
in(1/2*d*x+1/2*c)-4960*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-4480*I*sin(
1/2*d*x+1/2*c)^13+3520*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+140*I*sin(1/
2*d*x+1/2*c)^3-1496*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+2800*I*sin(1/2*
d*x+1/2*c)^7+376*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-840*I*sin(1/2*d*x+
1/2*c)^5-21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-44*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/
2*c)+6720*I*sin(1/2*d*x+1/2*c)^11)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\sqrt{\tfrac{1}{2}}\Big(-13i\,e^2e^{(9i\,dx+9i\,c)}+13i\,e^2e^{(8i\,dx+8i\,c)}-286i\,e^2e^{(7i\,dx+7i\,c)}-386i\,e^2e^{(6i\,dx+6i\,c)}+88i\,e^2e^{(5i\,dx+5i\,c)}-88i\,e^2e^{(4i\,dx+4i\,c)}+30$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/520*(sqrt(1/2)*(-13*I*e^2*e^(9*I*d*x + 9*I*c) + 13*I*e^2*e^(8*I*d*x + 8*I
*c) - 286*I*e^2*e^(7*I*d*x + 7*I*c) - 386*I*e^2*e^(6*I*d*x + 6*I*c) + 88*I*
e^2*e^(5*I*d*x + 5*I*c) - 88*I*e^2*e^(4*I*d*x + 4*I*c) + 30*I*e^2*e^(3*I*d*
x + 3*I*c) - 30*I*e^2*e^(2*I*d*x + 2*I*c) + 5*I*e^2*e^(I*d*x + I*c) - 5*I*e
^2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c) + 520*(a^2*d*e
^(7*I*d*x + 7*I*c) - a^2*d*e^(6*I*d*x + 6*I*c))*integral(1/65*sqrt(1/2)*(-4
2*I*e^2*e^(2*I*d*x + 2*I*c) - 84*I*e^2*e^(I*d*x + I*c) - 42*I*e^2)*sqrt(e*e
^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c)/(a^2*d*e^(4*I*d*x + 4*I*c)
 - 2*a^2*d*e^(3*I*d*x + 3*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) - 2*a^2*d*e^(I
*d*x + I*c) + a^2*d), x))/(a^2*d*e^(7*I*d*x + 7*I*c) - a^2*d*e^(6*I*d*x + 6
*I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos{(dx+c)})^{\frac{5}{2}}}{(i\,a\tan{(dx+c)}+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.665 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=154

$$\frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{3/2}}{33a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{3/2}}{11a^2d}+$$

```
[Out] (10*(e*Cos[c + d*x])^(3/2)*EllipticF[(c + d*x)/2, 2])/(33*a^2*d*Cos[c + d*x
]^(3/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*a^2*d)
+ (10*(e*Cos[c + d*x])^(3/2)*Tan[c + d*x])/(33*a^2*d) + (((4*I)/11)*Cos[c +
 d*x]^2*(e*Cos[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.198362, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3515, 3500, 3769, 3771, 2641}

$$\frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{3/2}}{33a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{3/2}}{11a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```


```
[Out] (10*(e*Cos[c + d*x])^(3/2)*EllipticF[(c + d*x)/2, 2])/(33*a^2*d*Cos[c + d*x
]^(3/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*a^2*d)
+ (10*(e*Cos[c + d*x])^(3/2)*Tan[c + d*x])/(33*a^2*d) + (((4*I)/11)*Cos[c +
 d*x]^2*(e*Cos[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$$

$$= \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(7e^2(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{1}{(e\sec(c+dx))^{7/2}}\,d}{11a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(5(e\cos(c\right.}{}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{10(e\cos(c+dx))^{3/2}\tan(c+dx)}{33a^2d} + \frac{4i\cos^2(c+}{11d\left(a^2\right.}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{10(e\cos(c+dx))^{3/2}\tan(c+dx)}{33a^2d} + \frac{4i\cos^2(c+}{11d\left(a^2\right.}$$

$$= \frac{10(e\cos(c+dx))^{3/2}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{33a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{10(e\,\mathrm{co}}{}$$

Mathematica [A] time = 0.661062, size = 131, normalized size = 0.85

$$\frac{(e\cos(c+dx))^{3/2}\left(\sqrt{\cos(c+dx)}(13\sin(c+dx)-7\sin(3(c+dx))-28i\cos(c+dx)+4i\cos(3(c+dx)))-20F\left(\frac{1}{2}(c+dx)\right.\right.}{66a^2d\cos^{\frac{7}{2}}(c+dx)(\tan(c+dx)-i)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(-20*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] +
I*Sin[2*(c + d*x)]) + Sqrt[Cos[c + d*x]]*((-28*I)*Cos[c + d*x] + (4*I)*Cos[
3*(c + d*x)] + 13*Sin[c + d*x] - 7*Sin[3*(c + d*x)])))/(66*a^2*d*Cos[c + d*
x]^(7/2)*(-I + Tan[c + d*x])^2)
```

Maple [A] time = 2.975, size = 315, normalized size = 2.1

$$\frac{2e^2}{33a^2d}\left(384i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{13} - 384\,(\sin(1/2\,dx+c/2))^{12}\cos(1/2\,dx+c/2) - 1152i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11} + 960\,(\sin(1/2\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/33/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(384*I*
sin(1/2*d*x+1/2*c)^13-384*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)-1152*I*s
in(1/2*d*x+1/2*c)^11+960*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+1440*I*si
n(1/2*d*x+1/2*c)^9-1008*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-960*I*sin(1
/2*d*x+1/2*c)^7+552*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+360*I*sin(1/2*d
*x+1/2*c)^5-176*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-72*I*sin(1/2*d*x+1/
2*c)^3-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+28*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)+6*I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(132\,a^2de^{(5i\,dx+5i\,c)}\text{integral}\left(-\frac{10i\,\sqrt{\frac{1}{2}}\,\sqrt{ee^{(2i\,dx+2i\,c)}+e}ee^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{33\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)},x\right)+\sqrt{\frac{1}{2}}\left(-11i\,ee^{(6i\,dx+6i\,c)}+41i\,ee^{(4i\,dx+4i\,c)}+15i\,ee^{(2i\,dx+2i\,c)}\right.\right.}{132\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/132*(132*a^2*d*e^(5*I*d*x + 5*I*c)*integral(-10/33*I*sqrt(1/2)*sqrt(e*e^(
2*I*d*x + 2*I*c) + e)*e*e^(-1/2*I*d*x - 1/2*I*c)/(a^2*d*e^(2*I*d*x + 2*I*c)
 + a^2*d), x) + sqrt(1/2)*(-11*I*e*e^(6*I*d*x + 6*I*c) + 41*I*e*e^(4*I*d*x
+ 4*I*c) + 15*I*e*e^(2*I*d*x + 2*I*c) + 3*I*e)*sqrt(e*e^(2*I*d*x + 2*I*c) +
 e)*e^(-1/2*I*d*x - 1/2*I*c))*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^2, x)
```

3.666 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{2i\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3a^2d\sqrt{\cos(c+dx)}}+\frac{2i\sqrt{e\cos(c+dx)}}{9d(a+ia\tan(c+dx))^2}$$

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[Cos[c + d*
x]]) + (((2*I)/9)*Sqrt[e*Cos[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^2) + (((2
*I)/9)*Sqrt[e*Cos[c + d*x]])/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.167294, antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3515, 3500, 3769, 3771, 2639}

$$\frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{2\sin(c+dx)\cos(c+dx)\sqrt{e\cos(c+dx)}}{9a^2d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3a^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[Cos[c + d*
x]]) + (2*Cos[c + d*x]*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(9*a^2*d) + (((4*
I)/9)*Cos[c + d*x]^2*Sqrt[e*Cos[c + d*x]])/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \left(\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(5e^2\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{9a^2}$$
$$= \frac{2\cos(c+dx)\sqrt{e\cos(c+dx)}\sin(c+dx)}{9a^2d} + \frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(\sqrt{e\cos(c+dx\right.}}{}$$
$$= \frac{2\cos(c+dx)\sqrt{e\cos(c+dx)}\sin(c+dx)}{9a^2d} + \frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\sqrt{e\cos(c+dx)}}{3a^2\sqrt{c}}$$
$$= \frac{2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}} + \frac{2\cos(c+dx)\sqrt{e\cos(c+dx)}\sin(c+dx)}{9a^2d} + \frac{4i\cos^2(c+d}{9d\left(a^2+ia\right.}$$

Mathematica [C] time = 1.75048, size = 420, normalized size = 3.5

$$\frac{(\cos(dx)+i\sin(dx))^2\sqrt{e\cos(c+dx)}\left(\frac{2\sqrt{2}\csc(c)e^{-idx}(\cos(2c)+i\sin(2c))\left(e^{2idx}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+3e^{2i(c+dx)}-3\sqrt{1-ie^{i(c+dx)}}\sqrt{e^{i(c+dx)}}\right.}{9\sqrt{}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2*((2*Sqrt[2]*Csc[c]*(3 + 3*E
^((2*I)*(c + d*x)) + 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I
 + E^(I*(c + d*x)))]*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]
]], -1] - 3*Sqrt[1 - I*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c
+ d*x)))]*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1] + E
^((2*I)*d*x)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])*(Cos[2*c] + I*Sin[2*c]))/(9*E^(I*d*x)*Sqrt[(1 + E^(
(2*I)*(c + d*x)))/E^(I*(c + d*x))]) - (Sqrt[Cos[c + d*x]]*Csc[c]*(Cos[2*d*x
] - I*Sin[2*d*x])*(7*Cos[c + 2*d*x] + 5*Cos[3*c + 2*d*x] - (4*I)*(Sin[c] -
2*Sin[c + 2*d*x] - Sin[3*c + 2*d*x]))))/9))/(2*d*Cos[c + d*x]^(5/2)*(a + I*a
*Tan[c + d*x])^2)
```

Maple [B] time = 2.736, size = 277, normalized size = 2.3

$$-\frac{2e}{9a^2d}\left(64i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11} - 64\,(\sin(1/2\,dx+c/2))^{10}\cos(1/2\,dx+c/2) - 160i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9} + 128\,\cos(1/2\,dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -2/9/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(64*I*sin
(1/2*d*x+1/2*c)^11-64*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-160*I*sin(1/
2*d*x+1/2*c)^9+128*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+160*I*sin(1/2*d*
x+1/2*c)^7-104*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-80*I*sin(1/2*d*x+1/2
*c)^5+40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+20*I*sin(1/2*d*x+1/2*c)^3-
3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)-6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*I*si
n(1/2*d*x+1/2*c))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-9i\,e^{(5i\,dx+5i\,c)}-15i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(3i\,dx+3i\,c)}-4i\,e^{(2i\,dx+2i\,c)}+i\,e^{(i\,dx+i\,c)}-i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+18\left(a^2de^{\cdots}\right.}{18\left(a^2de^{(5i\,dx+5i\,c)}-a^2de^{(4i\,dx+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-9*I*e^(5*I*d*x + 5*I*c) -
 15*I*e^(4*I*d*x + 4*I*c) + 4*I*e^(3*I*d*x + 3*I*c) - 4*I*e^(2*I*d*x + 2*I*
c) + I*e^(I*d*x + I*c) - I)*e^(-1/2*I*d*x - 1/2*I*c) + 18*(a^2*d*e^(5*I*d*x
 + 5*I*c) - a^2*d*e^(4*I*d*x + 4*I*c))*integral(1/3*sqrt(1/2)*sqrt(e*e^(2*I
*d*x + 2*I*c) + e)*(-2*I*e^(2*I*d*x + 2*I*c) - 4*I*e^(I*d*x + I*c) - 2*I)*e
^(-1/2*I*d*x - 1/2*I*c)/(a^2*d*e^(4*I*d*x + 4*I*c) - 2*a^2*d*e^(3*I*d*x + 3
*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) - 2*a^2*d*e^(I*d*x + I*c) + a^2*d), x))
/(a^2*d*e^(5*I*d*x + 5*I*c) - a^2*d*e^(4*I*d*x + 4*I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos(dx+c)}}{(ia\tan(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/(I*a*tan(d*x + c) + a)^2, x)
```

3.667 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{2i}{7d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\cos(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}+\frac{2i}{7d(a+ia\tan(c+dx))^2\sqrt{e\cos(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c + d*
x]]) + ((2*I)/7)/(d*Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x])^2) + ((2*I)
/7)/(d*Sqrt[e*Cos[c + d*x]]*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.158404, antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3515, 3500, 3769, 3771, 2641}

$$\frac{4i\cos^2(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)\sqrt{e\cos(c+dx)}}+\frac{2\sin(c+dx)\cos(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c + d*
x]]) + (2*Cos[c + d*x]*Sin[c + d*x])/(7*a^2*d*Sqrt[e*Cos[c + d*x]]) + (((4*
I)/7)*Cos[c + d*x]^2)/(d*Sqrt[e*Cos[c + d*x]]*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{4i\cos^2(c+dx)}{7d\sqrt{e\cos(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(3e^2\right)\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7a^2\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{2\cos(c+dx)\sin(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{4i\cos^2(c+dx)}{7d\sqrt{e\cos(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{}{7a^2\sqrt{}}$$

$$= \frac{2\cos(c+dx)\sin(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{4i\cos^2(c+dx)}{7d\sqrt{e\cos(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\sqrt{\cos}}{}$$

$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{2\cos(c+dx)\sin(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{4i}{7d\sqrt{e\cos(c+}}$$

Mathematica [A] time = 0.640359, size = 158, normalized size = 1.32

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)-i\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sqrt{\cos(c+dx)}\left(4i\sin^3\left(\frac{1}{2}(c+dx)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)\right)+2F\left(\frac{1}{2}(c\right.\right.}{7a^2d\cos^{\frac{3}{2}}(c+dx)(\tan(c+dx)-i)^2\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (((-I)*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sqrt[Cos[c + d*x]]*(3*Cos[(c +
 d*x)/2] + Cos[(3*(c + d*x))/2] + (4*I)*Sin[(c + d*x)/2]^3) + 2*EllipticF[(
c + d*x)/2, 2]*((-I)*Cos[(3*(c + d*x))/2] + Sin[(3*(c + d*x))/2])))/(7*a^2*
d*Cos[c + d*x]^(3/2)*Sqrt[e*Cos[c + d*x]]*(-I + Tan[c + d*x])^2)
```

Maple [A] time = 2.564, size = 240, normalized size = 2.

$$\frac{2}{7a^2d}\left(32i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9 - 32\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^8 - 64i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 + 48(\sin(1/2\,dx+c/2))^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/7/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(32*I*sin(1/
2*d*x+1/2*c)^9-32*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-64*I*sin(1/2*d*x+
1/2*c)^7+48*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+48*I*sin(1/2*d*x+1/2*c)
```

```
^5-28*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-16*I*sin(1/2*d*x+1/2*c)^3-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+6*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+2*I*sin(1/2
*d*x+1/2*c))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(7 a^2 d e e^{(3 i d x+3 i c)} \text{integral}\left(-\frac{2 i \sqrt{\frac{1}{2}} \sqrt{e e^{(2 i d x+2 i c)}+e}\, e^{\left(-\frac{1}{2} i d x-\frac{1}{2} i c\right)}}{7 \left(a^2 d e e^{(2 i d x+2 i c)}+a^2 d e\right)}, x\right)+\sqrt{\frac{1}{2}} \sqrt{e e^{(2 i d x+2 i c)}+e}\left(3 i e^{(2 i d x+2 i c)}+i\right) e^{\left(-\frac{1}{2} i d x-\frac{1}{2} i c\right)}\right) e^{(-3 i d x-3 i c)}}{7 a^2 d e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/7*(7*a^2*d*e*e^(3*I*d*x + 3*I*c)*integral(-2/7*I*sqrt(1/2)*sqrt(e*e^(2*I*
d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c)/(a^2*d*e*e^(2*I*d*x + 2*I*c) + a
^2*d*e), x) + sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(3*I*e^(2*I*d*x + 2
*I*c) + I)*e^(-1/2*I*d*x - 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^2*d*e)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e \cos(dx+c)}(i a \tan(dx+c)+a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(I*a*tan(d*x + c) + a)^2), x)
```

3.668 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$\frac{2\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{3/2}}+\frac{4i\cos^2(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{3/2}}$$

```
[Out] (2*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*(e*Cos[c + d*x])^
(3/2)) + (((4*I)/5)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(3/2)*(a^2 + I*a^2*
Tan[c + d*x]))
```

Rubi [A] time = 0.15422, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3515, 3500, 3771, 2639}

$$\frac{2\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{3/2}}+\frac{4i\cos^2(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```


```
[Out] (2*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*(e*Cos[c + d*x])^
(3/2)) + (((4*I)/5)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(3/2)*(a^2 + I*a^2*
Tan[c + d*x]))
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$= \frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a^2(e\cos(c+dx))^{3/2}(e\sec(c$$

$$= \frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\cos^{\frac{3}{2}}(c+dx)\int\sqrt{\cos(c+dx)}}{5a^2(e\cos(c+dx))^{3/2}}$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{3/2}} + \frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx\right.}$$

Mathematica [B] time = 0.675349, size = 244, normalized size = 2.65

$$\frac{(\sin(c+dx)+i\cos(c+dx))\left(i\sin(2(c+dx))+3\cos(2(c+dx))-2\sqrt{\sin(c+dx)-i\cos(c+dx)+1}\sqrt{\sin(c+dx)+i\sin(}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((I*Cos[c + d*x] + Sin[c + d*x])*(3 + 3*Cos[2*(c + d*x)] + 2*EllipticE[ArcS
in[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1]*Sqrt[1 - I*Cos[c + d*x] + S
in[c + d*x]]*Sqrt[(-I)*Cos[c + d*x] + Cos[2*(c + d*x)] + Sin[c + d*x] + I*S
in[2*(c + d*x)]] - 2*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]
]], -1]*Sqrt[1 - I*Cos[c + d*x] + Sin[c + d*x]]*Sqrt[(-I)*Cos[c + d*x] + Co
s[2*(c + d*x)] + Sin[c + d*x] + I*Sin[2*(c + d*x)]] + I*Sin[2*(c + d*x)]))/
(5*a^2*d*e*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 2.392, size = 207, normalized size = 2.3

$$-\frac{2}{5\,a^2ed}\left(16\,i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-16\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)-24\,i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+16\,(\sin(1/2\,dx+c/2))\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -2/5/e/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(16*I*sin
(1/2*d*x+1/2*c)^7-16*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-24*I*sin(1/2*d
*x+1/2*c)^5+16*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*I*sin(1/2*d*x+1/2
*c)^3-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*
I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(5\,a^2de^2e^{(2i\,dx+2i\,c)}\text{integral}\left(-\frac{2i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{5\left(a^2de^2e^{(2i\,dx+2i\,c)}+a^2de^2\right)},x\right)+\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(4i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right)e^{(-2i\,d\cdots)}}{5\,a^2de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/5*(5*a^2*d*e^2*e^(2*I*d*x + 2*I*c)*integral(-2/5*I*sqrt(1/2)*sqrt(e*e^(2*
I*d*x + 2*I*c) + e)*e^(1/2*I*d*x + 1/2*I*c)/(a^2*d*e^2*e^(2*I*d*x + 2*I*c)
+ a^2*d*e^2), x) + sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(4*I*e^(2*I*d*
x + 2*I*c) + 2*I)*e^(-1/2*I*d*x - 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^2*d*e^2
)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.669 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$-\frac{2\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(e\cos(c+dx))^{5/2}}+\frac{4i\cos^2(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{5/2}}$$

```
[Out] (-2*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*(e*Cos[c + d*x])
^(5/2)) + (((4*I)/3)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(5/2)*(a^2 + I*a^2
*Tan[c + d*x]))
```

Rubi [A] time = 0.14994, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3515, 3500, 3771, 2641}

$$-\frac{2\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(e\cos(c+dx))^{5/2}}+\frac{4i\cos^2(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*(e*Cos[c + d*x])
^(5/2)) + (((4*I)/3)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(5/2)*(a^2 + I*a^2
*Tan[c + d*x]))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$
$$= \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)} - \frac{e^2\int \sqrt{e\sec(c+dx)}}{3a^2(e\cos(c+dx))^{5/2}(e\sec}$$
$$= \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)} - \frac{\cos^{\frac{5}{2}}(c+dx)\int \frac{1}{\sqrt{\cos(c+dx)}}}{3a^2(e\cos(c+dx))^{5/2}}$$
$$= -\frac{2\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(e\cos(c+dx))^{5/2}} + \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\left(a^2+ia^2\tan(c\right.}$$

Mathematica [A] time = 0.368917, size = 116, normalized size = 1.26

$$\frac{2\sqrt{\cos(c+dx)}(\cos(dx)+i\sin(dx))^2\left(2\sqrt{\cos(c+dx)}(\sin(c-dx)-i\cos(c-dx))+(\cos(2c)+i\sin(2c))F\left(\frac{1}{2}(c+dx)\middle|2\right.\right.}{3a^2d(\tan(c+dx)-i)^2(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2*(EllipticF[(c + d*x)/2, 2]*
(Cos[2*c] + I*Sin[2*c]) + 2*Sqrt[Cos[c + d*x]]*((-I)*Cos[c - d*x] + Sin[c -
 d*x])))/(3*a^2*d*(e*Cos[c + d*x])^(5/2)*(-I + Tan[c + d*x])^2)
```

Maple [A] time = 2.291, size = 170, normalized size = 1.9

$$\frac{2}{3e^2a^2d}\left(8i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - 8\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2) - 8i\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/3/e^2/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(8*I*sin
(1/2*d*x+1/2*c)^5-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-8*I*sin(1/2*d*x
+1/2*c)^3+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)+2*I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,a^2de^3e^{(i\,dx+i\,c)}\text{integral}\left(\frac{2i\,\sqrt{\frac{1}{2}}\,\sqrt{ee^{(2i\,dx+2i\,c)}+ee}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\left(a^2de^3e^{(2i\,dx+2i\,c)}+a^2de^3\right)},x\right)+4i\,\sqrt{\frac{1}{2}}\,\sqrt{ee^{(2i\,dx+2i\,c)}+ee}\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right)e^{(-i\,dx-i\,c)}}{3\,a^2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/3*(3*a^2*d*e^3*e^(I*d*x + I*c)*integral(2/3*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x
 + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c)/(a^2*d*e^3*e^(2*I*d*x + 2*I*c) + a^
2*d*e^3), x) + 4*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x
- 1/2*I*c))*e^(-I*d*x - I*c)/(a^2*d*e^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}\,(i\,a\tan(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.670 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=122

$$-\frac{6\sin(c+dx)\cos^3(c+dx)}{a^2d(e\cos(c+dx))^{7/2}}+\frac{4i\cos^2(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{7/2}}+\frac{6\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(e\cos(c+dx))^{7/2}}$$

```
[Out] (6*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2])/(a^2*d*(e*Cos[c + d*x])^(7
/2)) - (6*Cos[c + d*x]^3*Sin[c + d*x])/(a^2*d*(e*Cos[c + d*x])^(7/2)) + ((4
*I)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rubi [A] time = 0.171009, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3515, 3500, 3768, 3771, 2639}

$$-\frac{6\sin(c+dx)\cos^3(c+dx)}{a^2d(e\cos(c+dx))^{7/2}}+\frac{4i\cos^2(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{7/2}}+\frac{6\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(e\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```


```
[Out] (6*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2])/(a^2*d*(e*Cos[c + d*x])^(7
/2)) - (6*Cos[c + d*x]^3*Sin[c + d*x])/(a^2*d*(e*Cos[c + d*x])^(7/2)) + ((4
*I)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$= \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)} - \frac{\left(3e^2\right)\int(e\sec(c+dx))^{3/2}\,d}{a^2(e\cos(c+dx))^{7/2}(e\sec(c+d}$$
$$= -\frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)} +$$
$$= -\frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)} +$$
$$= \frac{6\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(e\cos(c+dx))^{7/2}} - \frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{}{d(e\cos(c+}$$

Mathematica [B] time = 0.984883, size = 255, normalized size = 2.09

$$\frac{-2\sin(c+dx)+10i\cos(c+dx)-6i(\cos(c+dx)-i\sin(c+dx))\sqrt{\sin(c+dx)-i\cos(c+dx)+1}\sqrt{\sin(c+dx)+i\sin(2(c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] ((10*I)*Cos[c + d*x] - 2*Sin[c + d*x] - (6*I)*EllipticF[ArcSin[Sqrt[(-I)*Co
s[c + d*x] + Sin[c + d*x]]], -1]*(Cos[c + d*x] - I*Sin[c + d*x])*Sqrt[1 - I
*Cos[c + d*x] + Sin[c + d*x]]*Sqrt[(-I)*Cos[c + d*x] + Cos[2*(c + d*x)] + S
in[c + d*x] + I*Sin[2*(c + d*x)]] + 6*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*
x] + Sin[c + d*x]]], -1]*Sqrt[1 - I*Cos[c + d*x] + Sin[c + d*x]]*(I*Cos[c +
 d*x] + Sin[c + d*x])*Sqrt[(-I)*Cos[c + d*x] + Cos[2*(c + d*x)] + Sin[c + d
*x] + I*Sin[2*(c + d*x)]])/(a^2*d*e^3*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 1.97, size = 135, normalized size = 1.1

$$-2\,\frac{4\,i\,(\sin(1/2\,dx+c/2))^3-3\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2+2\,(s}}{e^3a^2\sin(1/2\,dx+c/2)\sqrt{-2\,(\sin(1/2\,dx+c/2))^2e+ed}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -2/e^3/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(4*I*sin(
1/2*d*x+1/2*c)^3-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+2*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)-2*I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(12i\,e^{(2i\,dx+2i\,c)}+8i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+\left(a^2de^4e^{(2i\,dx+2i\,c)}+a^2de^4\right)\mathrm{integral}\left(-\frac{6i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{a^2de^4e^{(2i\,dx+2i\,c)}+a^2de^4},\right.}{a^2de^4e^{(2i\,dx+2i\,c)}+a^2de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] (sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(12*I*e^(2*I*d*x + 2*I*c) + 8*I)
*e^(-1/2*I*d*x - 1/2*I*c) + (a^2*d*e^4*e^(2*I*d*x + 2*I*c) + a^2*d*e^4)*int
egral(-6*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(1/2*I*d*x + 1/2*I*c
)/(a^2*d*e^4*e^(2*I*d*x + 2*I*c) + a^2*d*e^4), x))/(a^2*d*e^4*e^(2*I*d*x +
2*I*c) + a^2*d*e^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.671 $\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=126

$$\frac{10\sin(c+dx)\cos^3(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}}-\frac{4i\cos^2(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{9/2}}+\frac{10\cos^{\frac{9}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(e\cos(c+dx))^{9/2}}$$

```
[Out] (10*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*(e*Cos[c + d*x])
^(9/2)) + (10*Cos[c + d*x]^3*Sin[c + d*x])/(3*a^2*d*(e*Cos[c + d*x])^(9/2))
 - ((4*I)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(9/2)*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rubi [A] time = 0.172323, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3515, 3500, 3768, 3771, 2641}

$$\frac{10\sin(c+dx)\cos^3(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}}-\frac{4i\cos^2(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{9/2}}+\frac{10\cos^{\frac{9}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(e\cos(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(9/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (10*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*(e*Cos[c + d*x])
^(9/2)) + (10*Cos[c + d*x]^3*Sin[c + d*x])/(3*a^2*d*(e*Cos[c + d*x])^(9/2))
 - ((4*I)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(9/2)*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}$$

$$= -\frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(5e^2\right)\int (e\sec(c+dx))^{5/2}}{a^2(e\cos(c+dx))^{9/2}(e\sec(c+}$$

$$= \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}} - \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)} +$$

$$= \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}} - \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)} +$$

$$= \frac{10\cos^{\frac{9}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(e\cos(c+dx))^{9/2}} + \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}} - \frac{}{d(e\cos(c}$$

Mathematica [A] time = 0.358312, size = 67, normalized size = 0.53

$$\frac{2\left(-\sin(c+dx)-6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3a^2de^3(e\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(9/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*((-6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] -
Sin[c + d*x]))/(3*a^2*d*e^3*(e*Cos[c + d*x])^(3/2))
```

Maple [A] time = 3.309, size = 208, normalized size = 1.7

$$-\frac{2}{3e^4a^2d}\left(10\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\text{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\sqrt{2}\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] -2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*
c)^2*e+e)^(1/2)/e^4*(10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+12*I*
sin(1/2*d*x+1/2*c)^3-5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
```

```
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c)-6*I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)

[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-20i\,e^{(3i\,dx+3i\,c)}-28i\,e^{(i\,dx+i\,c)}\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+3\left(a^2de^5e^{(4i\,dx+4i\,c)}+2\,a^2de^5e^{(2i\,dx+2i\,c)}+a^2de^5\right)\mathrm{integr}}{3\left(a^2de^5e^{(4i\,dx+4i\,c)}+2\,a^2de^5e^{(2i\,dx+2i\,c)}+a^2de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)

[Out] 1/3*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-20*I*e^(3*I*d*x + 3*I*c) -
 28*I*e^(I*d*x + I*c))*e^(-1/2*I*d*x - 1/2*I*c) + 3*(a^2*d*e^5*e^(4*I*d*x +
 4*I*c) + 2*a^2*d*e^5*e^(2*I*d*x + 2*I*c) + a^2*d*e^5)*integral(-10/3*I*sqr
t(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c)/(a^2*d*e^5*
e^(2*I*d*x + 2*I*c) + a^2*d*e^5), x))/(a^2*d*e^5*e^(4*I*d*x + 4*I*c) + 2*a^
2*d*e^5*e^(2*I*d*x + 2*I*c) + a^2*d*e^5)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**2,x)

[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{9}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")

[Out] integrate(1/((e*cos(d*x + c))^(9/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.672 $\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$\frac{14\sin(c+dx)\cos^5(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}}+\frac{14\sin(c+dx)\cos^3(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}}-\frac{4i\cos^2(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{11/2}}-\frac{14\cos^{\frac{11}{2}}(c\cdots}{5a^2d(e\cdots}$$

```
[Out] (-14*Cos[c + d*x]^(11/2)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*(e*Cos[c + d*x
])^(11/2)) + (14*Cos[c + d*x]^3*Sin[c + d*x])/(15*a^2*d*(e*Cos[c + d*x])^(1
1/2)) + (14*Cos[c + d*x]^5*Sin[c + d*x])/(5*a^2*d*(e*Cos[c + d*x])^(11/2))
- (((4*I)/3)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(11/2)*(a^2 + I*a^2*Tan[c
+ d*x]))
```

Rubi [A] time = 0.190808, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3515, 3500, 3768, 3771, 2639}

$$\frac{14\sin(c+dx)\cos^5(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}}+\frac{14\sin(c+dx)\cos^3(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}}-\frac{4i\cos^2(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{11/2}}-\frac{14\cos^{\frac{11}{2}}(c\cdots}{5a^2d(e\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d*x])^2),x]
```



```
[Out] (-14*Cos[c + d*x]^(11/2)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*(e*Cos[c + d*x
])^(11/2)) + (14*Cos[c + d*x]^3*Sin[c + d*x])/(15*a^2*d*(e*Cos[c + d*x])^(1
1/2)) + (14*Cos[c + d*x]^5*Sin[c + d*x])/(5*a^2*d*(e*Cos[c + d*x])^(11/2))
- (((4*I)/3)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(11/2)*(a^2 + I*a^2*Tan[c
+ d*x]))
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3500

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(2*d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
 f*x])^(n + 1))/(b*f*(m + 2*n)), x] - Dist[(d^2*(m - 2))/(b^2*(m + 2*n)), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$= -\frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(7e^2\right)\int(e\sec(c+d\ldots}{3a^2(e\cos(c+dx))^{11/2}(e\,\mathrm{se}\ldots}$$

$$= \frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} - \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

$$= \frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos^5(c+dx)\sin(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}} - \frac{\ldots}{3d(e\cos(c+\ldots}$$

$$= \frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos^5(c+dx)\sin(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}} - \frac{\ldots}{3d(e\cos(c+\ldots}$$

$$= -\frac{14\cos^{\frac{11}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos\ldots}{5a^2\ldots}$$

Mathematica [C] time = 5.81227, size = 406, normalized size = 2.48

$$2\sqrt{2}\csc(c)e^{3ic+2idx}(\cos(2c)+i\sin(2c))\cos^{\frac{7}{2}}(c+dx)(\cos(dx)+i\sin(dx))^2\left(-\frac{1}{2}e^{-2ic}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(7\left(1+e^{2i(c+dx}\ldots\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[2]*E^((3*I)*c + (2*I)*d*x)*Cos[c + d*x]^(7/2)*Csc[c]*(-42*Sqrt[2 -
(2*I)*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c + d*x)))]*Cos[c +
 d*x]^(5/2)*EllipticE[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c + d*x]]], -1] +
 42*Sqrt[2 - (2*I)*E^(I*(c + d*x))]*Sqrt[E^(I*(c + d*x))*(-I + E^(I*(c + d*
x)))]*Cos[c + d*x]^(5/2)*EllipticF[ArcSin[Sqrt[(-I)*Cos[c + d*x] + Sin[c +
d*x]]], -1] - (Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*((-1 + E^((2
*I)*c))*(47 + 56*E^((2*I)*(c + d*x)) + 21*E^((4*I)*(c + d*x))) + 7*(1 + E^(
(2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))]))/(2*E^((2*I)*c)))*(Cos[2*c] + I*Sin[2*c])*(Cos[d*x] + I*Sin[d*x])^2)/(
15*d*(1 + E^((2*I)*(c + d*x)))^3*(e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d
*x])^2)
```

Maple [A] time = 5.024, size = 321, normalized size = 2.

$$\frac{2}{15\,e^{5}a^{2}d}\left(-84\sqrt{2\,(\sin(1/2\,dx+c/2))^{2}-1}\sqrt{(\sin(1/2\,dx+c/2))^{2}}\,\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)(\sin(1/2\,dx+c/2))\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] 2/15/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^2/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^5*(-84*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*si
n(1/2*d*x+1/2*c)^4+168*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+84*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-168*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1
/2*c)+20*I*sin(1/2*d*x+1/2*c)^3-21*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+36*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c)-10*I*sin(1/2*d*x+1/2*c))/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-84i\,e^{(6i\,dx+6i\,c)}-224i\,e^{(4i\,dx+4i\,c)}-188i\,e^{(2i\,dx+2i\,c)}\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+15\left(a^{2}de^{6}e^{(6i\,dx+6i\,c)}+3\,a^{2}de^{6}e^{(4i\,d}\right.}{15\left(a^{2}de^{6}e^{(6i\,dx+6i\,c)}+3\,a^{2}de^{6}e^{(4i\,dx+4i\,c)}+3\,a^{2}de^{6}e^{(2i\,dx+2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/15*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-84*I*e^(6*I*d*x + 6*I*c)
- 224*I*e^(4*I*d*x + 4*I*c) - 188*I*e^(2*I*d*x + 2*I*c))*e^(-1/2*I*d*x - 1/
2*I*c) + 15*(a^2*d*e^6*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^6*e^(4*I*d*x + 4*I*c
) + 3*a^2*d*e^6*e^(2*I*d*x + 2*I*c) + a^2*d*e^6)*integral(14/5*I*sqrt(1/2)*
sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(1/2*I*d*x + 1/2*I*c)/(a^2*d*e^6*e^(2*I*d
*x + 2*I*c) + a^2*d*e^6), x))/(a^2*d*e^6*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^6*
e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^6*e^(2*I*d*x + 2*I*c) + a^2*d*e^6)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{11}{2}}\left(ia\tan\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(11/2)*(I*a*tan(d*x + c) + a)^2), x)
```

3.673 $\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=179

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}{7d}+\frac{32ia\sec^4(c+dx)(e\cos(c+dx))^{7/2}}{35d\sqrt{a+ia\tan(c+dx)}}-\frac{16i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}(e}{35d}$$

```
[Out] (((12*I)/35)*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c
 + d*x]]) + (((32*I)/35)*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^4)/(d*Sqrt[a
 + I*a*Tan[c + d*x]]) - (((2*I)/7)*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[
c + d*x]])/d - (((16*I)/35)*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2*Sqrt[a +
I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.382654, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3515, 3497, 3502, 3488}

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}{7d}+\frac{32ia\sec^4(c+dx)(e\cos(c+dx))^{7/2}}{35d\sqrt{a+ia\tan(c+dx)}}-\frac{16i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}(e}{35d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((12*I)/35)*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c
 + d*x]]) + (((32*I)/35)*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^4)/(d*Sqrt[a
 + I*a*Tan[c + d*x]]) - (((2*I)/7)*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[
c + d*x]])/d - (((16*I)/35)*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2*Sqrt[a +
I*a*Tan[c + d*x]])/d
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \left((e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx$$

$$= -\frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}{7d} + \frac{\left(6a(e\cos(c+dx))^{7/2}(e\sec(c+d\right.}{}$$

$$= \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx}}{7d}$$

$$= \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx}}{7d}$$

$$= \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{32ia(e\cos(c+dx))^{7/2}\sec^4(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - 2$$

Mathematica [A] time = 0.536439, size = 80, normalized size = 0.45

$$\frac{e^3\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}(70\sin(c+dx)+6\sin(3(c+dx))+35i\cos(c+dx)+i\cos(3(c+dx)))}{70d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (e^3*Sqrt[e*Cos[c + d*x]]*((35*I)*Cos[c + d*x] + I*Cos[3*(c + d*x)] + 70*Si
n[c + d*x] + 6*Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(70*d)
```

Maple [A] time = 0.388, size = 97, normalized size = 0.5

$$\frac{2i(\cos(dx+c))^3+12(\cos(dx+c))^2\sin(dx+c)+16i\cos(dx+c)+32\sin(dx+c)}{35d(\cos(dx+c))^3}(e\cos(dx+c))^{\frac{7}{2}}\sqrt{\frac{a(i\sin(dx+c}{\cos(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/35/d*(I*cos(d*x+c)^3+6*cos(d*x+c)^2*sin(d*x+c)+8*I*cos(d*x+c)+16*sin(d*x+
c))*(e*cos(d*x+c))^(7/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos
(d*x+c)^3
```

Maxima [A] time = 2.92401, size = 273, normalized size = 1.53

$$\left(7ie^3\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)-5ie^3\cos\left(\frac{7}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)-35ie^3\cos\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/140*(7*I*e^3*cos(5/2*d*x + 5/2*c) - 5*I*e^3*cos(7/5*arctan2(sin(5/2*d*x +
 5/2*c), cos(5/2*d*x + 5/2*c))) - 35*I*e^3*cos(3/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))) + 105*I*e^3*cos(1/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 7*e^3*sin(5/2*d*x + 5/2*c) + 5*e^3*sin(7/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 35*e^3*sin(3/5*arctan2(
sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 105*e^3*sin(1/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(a)*sqrt(e)/d
```

Fricas [A] time = 2.07285, size = 300, normalized size = 1.68

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-5i\,e^3e^{(6i\,dx+6i\,c)} - 35i\,e^3e^{(4i\,dx+4i\,c)} + 105i\,e^3e^{(2i\,dx+2i\,c)} + 7i\,e^3\right)\sqrt{ee^{(2i\,dx+2i\,c)} + e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/140*sqrt(2)*sqrt(1/2)*(-5*I*e^3*e^(6*I*d*x + 6*I*c) - 35*I*e^3*e^(4*I*d*x
 + 4*I*c) + 105*I*e^3*e^(2*I*d*x + 2*I*c) + 7*I*e^3)*sqrt(e*e^(2*I*d*x + 2*
I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2*I*d*x - 5/2*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.674 $\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=132

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}{5d}-\frac{16i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}{15d}+\frac{8ia\sec^2(c+dx)(e\cos}{15d\sqrt{a+ia\tan(}}$$

```
[Out] (((8*I)/15)*a*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((2*I)/5)*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
- (((16*I)/15)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d
*x]])/d
```

Rubi [A] time = 0.288011, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3515, 3497, 3502, 3488}

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}{5d}-\frac{16i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}{15d}+\frac{8ia\sec^2(c+dx)(e\cos}{15d\sqrt{a+ia\tan(}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((8*I)/15)*a*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((2*I)/5)*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
- (((16*I)/15)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d
*x]])/d
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
```

```
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx$$
$$= -\frac{2i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{5d} + \frac{\left(4a(e\cos(c+dx))^{5/2}(e\sec(c\right.}{}$$
$$= \frac{8ia(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+d}}{5d}$$
$$= \frac{8ia(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+d}}{5d}$$

Mathematica [A] time = 0.336596, size = 63, normalized size = 0.48

$$\frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}(-4i\sin(2(c+dx))+\cos(2(c+dx))-15)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((I/15)*e^2*Sqrt[e*Cos[c + d*x]]*(-15 + Cos[2*(c + d*x)] - (4*I)*Sin[2*(c +
 d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A] time = 0.373, size = 80, normalized size = 0.6

$$\frac{2\,i\,(\cos(dx+c))^2+8\,\cos(dx+c)\sin(dx+c)-16\,i}{15\,d\,(\cos(dx+c))^2}(e\cos(dx+c))^{\frac{5}{2}}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d*(I*cos(d*x+c)^2+4*cos(d*x+c)*sin(d*x+c)-8*I)*(e*cos(d*x+c))^(5/2)*(a
*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2
```

Maxima [A] time = 3.11623, size = 200, normalized size = 1.52

$$\frac{\left(5ie^2\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3ie^2\cos\left(\frac{5}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)-30ie^2\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/30*(5*I*e^2*cos(3/2*d*x + 3/2*c) - 3*I*e^2*cos(5/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 30*I*e^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 5*e^2*sin(3/2*d*x + 3/2*c) + 3*e^2*sin(5/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 30*e^2*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)*sqrt(e)/d
```

Fricas [A] time = 2.06499, size = 255, normalized size = 1.93

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-3i\,e^2e^{(4i\,dx+4i\,c)}-30i\,e^2e^{(2i\,dx+2i\,c)}+5i\,e^2\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/30*sqrt(2)*sqrt(1/2)*(-3*I*e^2*e^(4*I*d*x + 4*I*c) - 30*I*e^2*e^(2*I*d*x
+ 2*I*c) + 5*I*e^2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*e^(-3/2*I*d*x - 3/2*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.675 $\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{4iae\sec(c+dx)\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}{3d}$$

```
[Out] (((4*I)/3)*a*e*Sqrt[e*Cos[c + d*x]]*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d
*x]]) - (((2*I)/3)*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.212166, antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3515, 3497, 3488}

$$\frac{4ia\sec^2(c+dx)(e\cos(c+dx))^{3/2}}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((4*I)/3)*a*(e*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c +
 d*x]]) - (((2*I)/3)*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx$$

$$=-\frac{2i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{\left(2a(e\cos(c+dx))^{3/2}(e\sec(c\right.}{3\text{}}$$

$$=\frac{4ia(e\cos(c+dx))^{3/2}\sec^2(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{2i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+d}}{3d}$$

Mathematica [A] time = 0.203282, size = 56, normalized size = 0.66

$$\frac{2e\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}(2\sin(c+dx)+i\cos(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*e*Sqrt[e*Cos[c + d*x]]*(I*Cos[c + d*x] + 2*Sin[c + d*x])*Sqrt[a + I*a*Ta
n[c + d*x]])/(3*d)
```

Maple [A] time = 0.338, size = 70, normalized size = 0.8

$$\frac{2\,i\cos(dx+c)+4\,\sin(dx+c)}{3\,d\cos(dx+c)}(e\cos(dx+c))^{\frac{3}{2}}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d*(I*cos(d*x+c)+2*sin(d*x+c))*(e*cos(d*x+c))^(3/2)*(a*(I*sin(d*x+c)+cos
(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)
```

Maxima [A] time = 3.02176, size = 80, normalized size = 0.94

$$\frac{\left(-i\,e\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3i\,e\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+e\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,e\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}\sqrt{e}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*(-I*e*cos(3/2*d*x + 3/2*c) + 3*I*e*cos(1/2*d*x + 1/2*c) + e*sin(3/2*d*x
 + 3/2*c) + 3*e*sin(1/2*d*x + 1/2*c))*sqrt(a)*sqrt(e)/d
```

Fricas [A] time = 2.06136, size = 204, normalized size = 2.4

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-i\,ee^{(2i\,dx+2i\,c)}+3i\,e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/3*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-I*e*e^(2*I*d*x + 2*
I*c) + 3*I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.676 $\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=36

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{d}$$

```
[Out] ((-2*I)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rubi [A] time = 0.129311, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3515, 3488}

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx &= \left(\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx\\ &= -\frac{2i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.171395, size = 36, normalized size = 1.

$$-\frac{2i\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A] time = 0.322, size = 45, normalized size = 1.3

$$\frac{-2\,i}{d}\sqrt{e\cos\left(dx+c\right)}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -2*I/d*(e*cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.36209, size = 103, normalized size = 2.86

$$-\frac{2i\,\sqrt{a}\sqrt{e}\sqrt{-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{d\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2*I*sqrt(a)*sqrt(e)*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 - 1)/(d*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 - 1))
```

Fricas [A] time = 2.10329, size = 157, normalized size = 4.36

$$-\frac{2i\,\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

3.677 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal. Leaf size=335

$$\frac{i\sqrt{2}\sqrt{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}}-\frac{i\sqrt{2}\sqrt{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}}-\frac{i\sqrt{a}\log\left(-\sqrt{2}\sqrt{a}\sqrt{a}\right.}{}$$

```
[Out] (I*Sqrt[2]*Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Ta
n[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(d*Sqrt[e]) - (I*Sqrt[2]*Sqrt[a]*ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e
])])/(d*Sqrt[e]) - (I*Sqrt[a]*Log[a*Sqrt[e] - Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a + I*a*Tan[c +
d*x])])/(Sqrt[2]*d*Sqrt[e]) + (I*Sqrt[a]*Log[a*Sqrt[e] + Sqrt[2]*Sqrt[a]*Sq
rt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a + I
*a*Tan[c + d*x])])/(Sqrt[2]*d*Sqrt[e])
```

Rubi [A] time = 0.214058, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {3513, 297, 1162, 617, 204, 1165, 628}

$$\frac{i\sqrt{2}\sqrt{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}}-\frac{i\sqrt{2}\sqrt{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}}-\frac{i\sqrt{a}\log\left(-\sqrt{2}\sqrt{a}\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Ta
n[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(d*Sqrt[e]) - (I*Sqrt[2]*Sqrt[a]*ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e
])])/(d*Sqrt[e]) - (I*Sqrt[a]*Log[a*Sqrt[e] - Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a + I*a*Tan[c +
d*x])])/(Sqrt[2]*d*Sqrt[e]) + (I*Sqrt[a]*Log[a*Sqrt[e] + Sqrt[2]*Sqrt[a]*Sq
rt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a + I
*a*Tan[c + d*x])])/(Sqrt[2]*d*Sqrt[e])
```

Rule 3513

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[(-4*b)/f, Subst[Int[x^2/(a^2*d^2 + x^4), x], x,
Sqrt[d*Cos[e + f*x]]*Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}
, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{(4ia)\,\text{Subst}\left(\int \frac{x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)}{d}$$

$$= \frac{(2ia)\,\text{Subst}\left(\int \frac{ae-x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)}{d} - \frac{(2ia)\,\text{Subst}\left(\int \frac{ae+x^2}{a^2e^2+x^4}\,dx \ldots\right.}{}$$

$$= -\frac{(ia)\,\text{Subst}\left(\int \frac{1}{ae-\sqrt{2}\sqrt{a}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)}{d} - \frac{(ia)\,\text{Subst}\left(\int \frac{}{ae+\ldots}\right.}{}$$

$$= -\frac{i\sqrt{a}\log\left(a\sqrt{e}-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+\ldots\right.}{\sqrt{2}d\sqrt{e}}$$

$$= \frac{i\sqrt{2}\sqrt{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}} - \frac{i\sqrt{2}\sqrt{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+d\ldots}}{\sqrt{a}\sqrt{e}}\right.}{d\sqrt{e}}$$

Mathematica [A] time = 0.849434, size = 125, normalized size = 0.37

$$\frac{i\left(-e^{-2ic}\right)^{3/4} e^{-\frac{3}{2}idx}\left(1+e^{2i(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}\left(\tan^{-1}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)-\tanh^{-1}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)\right)}{d\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (I*(-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))*(ArcTan[E^((I/2)*d*x)/(-
E^((-2*I)*c))^(1/4)] - ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)])*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*E^(((3*I)/2)*d*x)*Sqrt[e*Cos[c + d*x]])
```

Maple [A] time = 0.371, size = 226, normalized size = 0.7

$$-\frac{\cos(dx+c)(\cos(dx+c)-1)}{d\sin(dx+c)(i\sin(dx+c)+\cos(dx+c)-1)}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(i\mathrm{Artanh}\left(\frac{-\cos(dx+c)-1+\sin}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(cos(d*x+c)-
1)*(I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))+I*ar
ctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+arctanh(1/2*(
1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))-arctanh(1/2*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))))/sin(d*x+c)/(e*cos(d*x+c))^(1/2)/(
I*sin(d*x+c)+cos(d*x+c)-1)/(1/(cos(d*x+c)+1))^(1/2)
```

Maxima [B] time = 3.13678, size = 1890, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/4*(-2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt
(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 1) - 2*sqrt(2)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)
 + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + I*sqr
t(2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt
(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(2
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
```

```
/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(
2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + sqrt(2)*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + sqrt(2)*log(2*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*sqrt(a)/(d*sqrt(e))
```

Fricas [A] time = 2.26279, size = 917, normalized size = 2.74

$$\frac{1}{2}\sqrt{\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\frac{1}{2}\,de\sqrt{\frac{4i\,a}{d^2e}}\right)-\frac{1}{2}\sqrt{\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(4*I*a/(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) +
e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 1/2*d*e*sqrt
(4*I*a/(d^2*e))) - 1/2*sqrt(4*I*a/(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqrt(e*e^(
2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*
I*c) - 1/2*d*e*sqrt(4*I*a/(d^2*e))) - 1/2*sqrt(-4*I*a/(d^2*e))*log(sqrt(2)*
sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*e^(1/2*I*d*x + 1/2*I*c) + 1/2*d*e*sqrt(-4*I*a/(d^2*e))) + 1/2*sqrt(-4*I*a/
(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 1/2*d*e*sqrt(-4*I*a/(d^2*e)
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\,(i\tan{(c+dx)}+1)}}{\sqrt{e\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(I*tan(c + d*x) + 1))/sqrt(e*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i a \tan(dx+c) + a}}{\sqrt{e \cos(dx+c)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

3.678 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=524

$$-\frac{ia^{3/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia^{3/2}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia^{3/}}{}$$

```
[Out] (I*a)/(d*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) - (I*a^(3/2)*Ar
cTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]
*Sqrt[e])]*Sec[c + d*x])/(Sqrt[2]*d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + (I*a^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e*Cos[c + d*x
]]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]*Sec[c + d*x])/(Sqrt[2]*d*
e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + ((I/2)*a^(
3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a*Tan[c + d*x
]])/Sqrt[e] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d
*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^
(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a*Tan[c + d*
x]])/Sqrt[e] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*
d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.590468, antiderivative size = 620, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3515, 3498, 3499, 3495, 297, 1162, 617, 204, 1165, 628}

$$-\frac{ia^{3/2}e^{3/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}+\frac{ia^{3/2}e^{3/2}\sec(c+dx)\tan}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (I*a)/(d*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) - (I*a^(3/2)*e^
(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt
[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(3/2)*(e*Sec[c
 + d*x])^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (I*
a^(3/2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(3/2
)*(e*Sec[c + d*x])^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*
x]]) + ((I/2)*a^(3/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a
*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]
*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*Sqr
t[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^(3/2)*e^(3/2
)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c
 + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e
*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]])
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3499

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[(d*Sec[e + f*x])/(Sqrt[a - b*Tan[e + f*x]]*Sqr
t[a + b*Tan[e + f*x]]), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{(ae\sec(c+dx))\int \sqrt{e\sec(c+dx)}\sqrt{a-ia}}{2(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+}}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(2ia^2e^3\sec(c+dx)\right)\mathrm{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx,x\right.}{d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+}}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{\left(ia^2e^2\sec(c+dx)\right)\mathrm{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx,x\right.}{d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+}}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(ia^2e\sec(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx\right.}{2d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c}}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{3/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx\right.}{2\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia}}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}e^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right.}{\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan}}$$

Mathematica [A] time = 3.89304, size = 274, normalized size = 0.52

$$\frac{ie^{-\frac{1}{2}i(c+dx)}\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}(\cos(c+dx)+i\sin(c+dx))\left(-2i\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)+2\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (I*Cos[c + d*x]^2*(2*Sqrt[2]*Cos[(c + d*x)/2] + 2*ArcTan[1 - Sqrt[2]*E^((I/
2)*(c + d*x))]*Cos[c + d*x] - 2*ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d*x))]*Cos
[c + d*x] + Cos[c + d*x]*Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*
x))] - Cos[c + d*x]*Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))]
- (2*I)*Sqrt[2]*Sin[(c + d*x)/2])*(Cos[c + d*x] + I*Sin[c + d*x])*Sqrt[a +
I*a*Tan[c + d*x]])/(Sqrt[2]*d*E^((I/2)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))
*(e*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.412, size = 308, normalized size = 0.6

$$\frac{\cos(dx+c)(\cos(dx+c)-1)^2}{2d(\sin(dx+c))^3(i\sin(dx+c)+\cos(dx+c)-1)}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(2i\sin(dx+c)\sqrt{(\cos(dx+c)+1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(cos(d*x+c)
-1)^2*(2*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+I*cos(d*x+c)*arctanh(1/2*(1/
(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))+I*arctanh(1/2*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)-2*cos(d*x+c)*(1/(cos(d*
x+c)+1))^(1/2)-cos(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)
-1+sin(d*x+c)))+arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+
c)))*cos(d*x+c)-2*(1/(cos(d*x+c)+1))^(1/2))/sin(d*x+c)^3/(I*sin(d*x+c)+cos(
d*x+c)-1)/(1/(cos(d*x+c)+1))^(3/2)/(e*cos(d*x+c))^(3/2)
```

Maxima [B] time = 3.33966, size = 2476, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -(16*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arct
an2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(
2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 16*(sqrt(2)*
cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 16*(sqrt(2)*cos(2*d*x + 2
*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1) + 16*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2
)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + (16*I*sqrt(2)*cos(2*d*x + 2*c) - 16*sqrt(2)*sin(2*d*
x + 2*c) + 16*I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))),
sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (-16*I*sqrt(2)*cos(2*d*x +
2*c) + 16*sqrt(2)*sin(2*d*x + 2*c) - 16*I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
 8*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*log(2*
sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 1) - 8*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) +
 sqrt(2))*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
```

```
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) + (8*I*sqrt(2)*cos(2*d*x + 2*c) - 8*sqrt(2)
*sin(2*d*x + 2*c) + 8*I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqr
t(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + (-8*I*sqrt
(2)*cos(2*d*x + 2*c) + 8*sqrt(2)*sin(2*d*x + 2*c) - 8*I*sqrt(2))*log(2*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2) + (8*I*sqrt(2)*cos(2*d*x + 2*c) - 8*sqrt(2)*sin(2*d*x +
 2*c) + 8*I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + (-8*I*sqrt(2)*cos(2*d*
x + 2*c) + 8*sqrt(2)*sin(2*d*x + 2*c) - 8*I*sqrt(2))*log(2*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2) - 128*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 128*I*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*sqrt(e)/((-64
*I*e^2*cos(2*d*x + 2*c) + 64*e^2*sin(2*d*x + 2*c) - 64*I*e^2)*d)
```

Fricas [A] time = 2.24811, size = 1303, normalized size = 2.49

$$4i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2idx+2ic)}+e}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}+\left(de^2e^{(2idx+2ic)}+de^2\right)\sqrt{\frac{ia}{d^2e^3}}\log\left(ide^2\sqrt{\frac{ia}{d^2e^3}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2idx+2ic)}+e}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(4*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + (d*e^2*e^(2*I*d*x + 2*I*c) + d*
e^2)*sqrt(I*a/(d^2*e^3))*log(I*d*e^2*sqrt(I*a/(d^2*e^3)) + sqrt(2)*sqrt(1/2
)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*
I*d*x + 1/2*I*c)) - (d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)*sqrt(I*a/(d^2*e^3))
*log(-I*d*e^2*sqrt(I*a/(d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2
*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)) + (d*
e^2*e^(2*I*d*x + 2*I*c) + d*e^2)*sqrt(-I*a/(d^2*e^3))*log(I*d*e^2*sqrt(-I*a
/(d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)) - (d*e^2*e^(2*I*d*x + 2*I*c
) + d*e^2)*sqrt(-I*a/(d^2*e^3))*log(-I*d*e^2*sqrt(-I*a/(d^2*e^3)) + sqrt(2)
*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(1/2*I*d*x + 1/2*I*c)))/(d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i a \tan(dx + c) + a}}{(e \cos(dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

3.679 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=512

$$\frac{3i\sqrt{a}e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{3i\sqrt{a}e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{3i\sqrt{a}e^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right.}{8\sqrt{2}d(e\cos(c+dx}$$

```
[Out] (((3*I)/4)*Sqrt[a]*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d*(e*Cos[c + d*x])^(5/2)*
(e*Sec[c + d*x])^(5/2)) - (((3*I)/4)*Sqrt[a]*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]
*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - (((3*I)/8)*Sqrt[a]*e^(5
/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec
[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d*(e*Cos[c + d*
x])^(5/2)*(e*Sec[c + d*x])^(5/2)) + (((3*I)/8)*Sqrt[a]*e^(5/2)*Log[a + (Sqr
t[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos
[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec
[c + d*x])^(5/2)) + ((I/2)*a)/(d*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((3*I)/4)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Cos[
c + d*x])^(5/2))
```

Rubi [A] time = 0.564371, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3515, 3498, 3501, 3495, 297, 1162, 617, 204, 1165, 628}

$$\frac{3i\sqrt{a}e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{3i\sqrt{a}e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{3i\sqrt{a}e^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right.}{8\sqrt{2}d(e\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (((3*I)/4)*Sqrt[a]*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d*(e*Cos[c + d*x])^(5/2)*
(e*Sec[c + d*x])^(5/2)) - (((3*I)/4)*Sqrt[a]*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]
*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - (((3*I)/8)*Sqrt[a]*e^(5
/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec
[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d*(e*Cos[c + d*
x])^(5/2)*(e*Sec[c + d*x])^(5/2)) + (((3*I)/8)*Sqrt[a]*e^(5/2)*Log[a + (Sqr
t[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos
[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec
[c + d*x])^(5/2)) + ((I/2)*a)/(d*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((3*I)/4)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Cos[
c + d*x])^(5/2))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\int (e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{(3a)\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{4(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}} + \frac{(3e^2)\int \sqrt{e}\cdots}{8(e\cos\cdots}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}} - \frac{(3iae^4)\,\mathrm{Sub}\cdots}{2d(e\cos\cdots}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}} + \frac{(3iae^3)\,\mathrm{Sub}\cdots}{4d(e\cos\cdots}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}} - \frac{(3iae^2)\,\mathrm{Sub}\cdots}{8d(e\cdots}$$

$$= -\frac{3i\sqrt{a}e^{5/2}\log\left(a - \frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}} + \cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} + \frac{3i\sqrt{a}e^{5/2}\log\left(\cdots\right.}{\cdots}$$

$$= \frac{3i\sqrt{a}e^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{3i\sqrt{a}e^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{3i\sqrt{a}e^{\cdots}}{\cdots}$$

Mathematica [A] time = 2.3867, size = 227, normalized size = 0.44

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\left(-3i\cos^{\frac{3}{2}}(c+dx) + 2\sqrt{\cos(c+dx)}(\sin(c+dx)+i\cos(c+dx)) + \frac{3i\left(-e^{-2ic}\right)^{3/4}e^{-\frac{1}{2}i(2c+5dx)}\sqrt{e\cdots}}{\cdots}\right.}{4d(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*((((3*I)/4)*(-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*
x)))^2*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(ArcTan[E^((I/2)*d*x
)/(-E^((-2*I)*c))^(1/4)] - ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)]))/(
Sqrt[2]*E^((I/2)*(2*c + 5*d*x))) - (3*I)*Cos[c + d*x]^(3/2) + 2*Sqrt[Cos[c
+ d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))*Sqrt[a + I*a*Tan[c + d*x]])/(4*d*(
e*Cos[c + d*x])^(5/2))
```

Maple [A] time = 0.432, size = 366, normalized size = 0.7

$$-\frac{\cos(dx+c)(\cos(dx+c)-1)^3}{8d(\sin(dx+c))^5(i\sin(dx+c)+\cos(dx+c)-1)}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(6i\sin(dx+c)\sqrt{(\cos(dx+c)+1\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/8/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(cos(d*x+c
)-1)^3*(6*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*I*cos(d*x+c)^2
*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))+3*I*arctan
h(1/2*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*cos(d*x+c)^2+4*I
*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+6*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2
)-3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x
+c)^2+3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))*co
s(d*x+c)^2+2*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-4*(1/(cos(d*x+c)+1))^(1/2)
)/sin(d*x+c)^5/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(cos(d*x+c)+1))^(5/2)/(e*cos(
d*x+c))^(5/2)
```

Maxima [B] time = 3.60334, size = 3058, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -((192*sqrt(2)*cos(4*d*x + 4*c) + 384*sqrt(2)*cos(2*d*x + 2*c) + 192*I*sqrt
(2)*sin(4*d*x + 4*c) + 384*I*sqrt(2)*sin(2*d*x + 2*c) + 192*sqrt(2))*arctan
2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (192*sqrt(2)*c
os(4*d*x + 4*c) + 384*sqrt(2)*cos(2*d*x + 2*c) + 192*I*sqrt(2)*sin(4*d*x +
4*c) + 384*I*sqrt(2)*sin(2*d*x + 2*c) + 192*sqrt(2))*arctan2(sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (192*sqrt(2)*cos(4*d*x + 4*c)
 + 384*sqrt(2)*cos(2*d*x + 2*c) + 192*I*sqrt(2)*sin(4*d*x + 4*c) + 384*I*sq
rt(2)*sin(2*d*x + 2*c) + 192*sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1) + (192*sqrt(2)*cos(4*d*x + 4*c) + 384*sqrt(2)*c
os(2*d*x + 2*c) + 192*I*sqrt(2)*sin(4*d*x + 4*c) + 384*I*sqrt(2)*sin(2*d*x
+ 2*c) + 192*sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 1) - (192*I*sqrt(2)*cos(4*d*x + 4*c) + 384*I*sqrt(2)*cos(2*d*x +
2*c) - 192*sqrt(2)*sin(4*d*x + 4*c) - 384*sqrt(2)*sin(2*d*x + 2*c) + 192*I*
sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1) - (-192*I*sqrt(2)*cos(4*d*x + 4*c) - 384*I*sq
rt(2)*cos(2*d*x + 2*c) + 192*sqrt(2)*sin(4*d*x + 4*c) + 384*sqrt(2)*sin(2*d
*x + 2*c) - 192*I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (96*sqrt(2)*cos(4*d*x +
 4*c) + 192*sqrt(2)*cos(2*d*x + 2*c) + 96*I*sqrt(2)*sin(4*d*x + 4*c) + 192*
I*sqrt(2)*sin(2*d*x + 2*c) + 96*sqrt(2))*log(2*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
```

```
*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (96*sqrt(2)
*cos(4*d*x + 4*c) + 192*sqrt(2)*cos(2*d*x + 2*c) + 96*I*sqrt(2)*sin(4*d*x +
 4*c) + 192*I*sqrt(2)*sin(2*d*x + 2*c) + 96*sqrt(2))*log(-2*sqrt(2)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
- (-96*I*sqrt(2)*cos(4*d*x + 4*c) - 192*I*sqrt(2)*cos(2*d*x + 2*c) + 96*sqr
t(2)*sin(4*d*x + 4*c) + 192*sqrt(2)*sin(2*d*x + 2*c) - 96*I*sqrt(2))*log(2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2) - (96*I*sqrt(2)*cos(4*d*x + 4*c) + 192*I*sqrt(2)*co
s(2*d*x + 2*c) - 96*sqrt(2)*sin(4*d*x + 4*c) - 192*sqrt(2)*sin(2*d*x + 2*c)
 + 96*I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (-96*I*sqrt(2)*cos(4*d*x +
 4*c) - 192*I*sqrt(2)*cos(2*d*x + 2*c) + 96*sqrt(2)*sin(4*d*x + 4*c) + 192*
sqrt(2)*sin(2*d*x + 2*c) - 96*I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (9
6*I*sqrt(2)*cos(4*d*x + 4*c) + 192*I*sqrt(2)*cos(2*d*x + 2*c) - 96*sqrt(2)*
sin(4*d*x + 4*c) - 192*sqrt(2)*sin(2*d*x + 2*c) + 96*I*sqrt(2))*log(2*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2) + 1536*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 512*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1536*I*sin
(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 512*I*sin(3/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*sqrt(e)/((-1024*I*e^3*cos(4*d*
x + 4*c) - 2048*I*e^3*cos(2*d*x + 2*c) + 1024*e^3*sin(4*d*x + 4*c) + 2048*e
^3*sin(2*d*x + 2*c) - 1024*I*e^3)*d)
```

Fricas [A] time = 2.57679, size = 1628, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(-3*I*e^(3*I*d*x + 3*I*c) + I*e^(I*d*x + I*c))*e^(1/2*I*d*x +
 1/2*I*c) + (d*e^3*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^
3)*sqrt(9/16*I*a/(d^2*e^5))*log(4/3*d*e^3*sqrt(9/16*I*a/(d^2*e^5)) + sqrt(2
)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*e^(1/2*I*d*x + 1/2*I*c)) - (d*e^3*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d
*x + 2*I*c) + d*e^3)*sqrt(9/16*I*a/(d^2*e^5))*log(-4/3*d*e^3*sqrt(9/16*I*a/
(d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)) - (d*e^3*e^(4*I*d*x + 4*I*c)
```

```
 + 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)*sqrt(-9/16*I*a/(d^2*e^5))*log(4/3*d
*e^3*sqrt(-9/16*I*a/(d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*
c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)) + (d*e^3
*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)*sqrt(-9/16*I*a/
(d^2*e^5))*log(-4/3*d*e^3*sqrt(-9/16*I*a/(d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqr
t(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
 + 1/2*I*c)))/(d*e^3*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*
e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

3.680 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=719

$$-\frac{5ia^{3/2}e^{7/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}+\frac{5ia^{3/2}e^{7/2}\sec(c+dx)\,\mathrm{ta}}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{tan}}}$$

```
[Out] ((I/3)*a)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/8
)*a*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((5*I)/8)*a^(3/2)*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c
+ d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c
+ d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*
a*Tan[c + d*x]]) + (((5*I)/8)*a^(3/2)*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*S
qrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(S
qrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/16)*a^(3/2)*e^(7/2)*Log[a - (S
qrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + C
os[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x
])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan
[c + d*x]]) - (((5*I)/16)*a^(3/2)*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Ta
n[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x
])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((5*I)/1
2)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Cos[c + d*x])^(7/2))
```

Rubi [A] time = 0.859819, antiderivative size = 719, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {3515, 3498, 3501, 3499, 3495, 297, 1162, 617, 204, 1165, 628}

$$-\frac{5ia^{3/2}e^{7/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}+\frac{5ia^{3/2}e^{7/2}\sec(c+dx)\,\mathrm{ta}}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{tan}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((I/3)*a)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/8
)*a*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((5*I)/8)*a^(3/2)*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c
+ d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c
+ d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*
a*Tan[c + d*x]]) + (((5*I)/8)*a^(3/2)*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*S
qrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(S
qrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/16)*a^(3/2)*e^(7/2)*Log[a - (S
qrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + C
os[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x
])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan
[c + d*x]]) - (((5*I)/16)*a^(3/2)*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Ta
n[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x
])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((5*I)/1
2)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Cos[c + d*x])^(7/2))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3499

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[(d*Sec[e + f*x])/(Sqrt[a - b*Tan[e + f*x]]*Sqr
t[a + b*Tan[e + f*x]]), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{\int (e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{(5a)\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{6(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}} + \frac{(5e^2)\int (e\,\mathrm{s}\cdots}{8(e\,\mathrm{c}\cdots}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos^{\cdots}}{\cdots}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos^{\cdots}}{\cdots}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos^{\cdots}}{\cdots}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos^{\cdots}}{\cdots}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos^{\cdots}}{\cdots}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia^{3/2}\cdots}{16\sqrt{\cdots}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{\cdots}{8\sqrt{2}d(\cdots}$$

Mathematica [A] time = 2.98095, size = 305, normalized size = 0.42

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\left(-\frac{40}{3}i\cos^{\frac{3}{2}}(c+dx)+\frac{5}{8}ie^{-\frac{7}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(1+e^{2i(c+dx)}\right)^3\left(\log\left(-\sqrt{2}e^{\frac{1}{2}\dots}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(((-40*I)/3)*Cos[c + d*x]^(3/2) + (((5*I)/8)*(1 + E^((2
*I)*(c + d*x)))^3*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(2*ArcTan
[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d*x)
)] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))] - Log[1 + Sqrt[
2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))]))/E^(((7*I)/2)*(c + d*x)) + (32*S
qrt[Cos[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))/3 + 20*Cos[c + d*x]^(5/2
)*(I*Cos[c + d*x] + Sin[c + d*x]))*Sqrt[a + I*a*Tan[c + d*x]])/(32*d*(e*Cos
[c + d*x])^(7/2))
```

Maple [A] time = 0.379, size = 417, normalized size = 0.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x)
```

```
[Out] 1/48/d*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(cos(d*x+c
)-1)^4*(30*I*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)+15*I*cos(d*x+
c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))+15*I*
cos(d*x+c)^3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))
)+20*I*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-15*cos(d*x+c)^3*arcta
nh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))+15*cos(d*x+c)^3
*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-30*cos(d*x
+c)^3*(1/(cos(d*x+c)+1))^(1/2)+16*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-10*
cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)+4*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)
-16*(1/(cos(d*x+c)+1))^(1/2))/sin(d*x+c)^7/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(
cos(d*x+c)+1))^(7/2)/(e*cos(d*x+c))^(7/2)
```

Maxima [B] time = 4.3269, size = 3606, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -((5760*sqrt(2)*cos(6*d*x + 6*c) + 17280*sqrt(2)*cos(4*d*x + 4*c) + 17280*s
qrt(2)*cos(2*d*x + 2*c) + 5760*I*sqrt(2)*sin(6*d*x + 6*c) + 17280*I*sqrt(2)
*sin(4*d*x + 4*c) + 17280*I*sqrt(2)*sin(2*d*x + 2*c) + 5760*sqrt(2))*arctan
2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (5760*sqrt(2)*
cos(6*d*x + 6*c) + 17280*sqrt(2)*cos(4*d*x + 4*c) + 17280*sqrt(2)*cos(2*d*x
```

```
 + 2*c) + 5760*I*sqrt(2)*sin(6*d*x + 6*c) + 17280*I*sqrt(2)*sin(4*d*x + 4*c
) + 17280*I*sqrt(2)*sin(2*d*x + 2*c) + 5760*sqrt(2))*arctan2(sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (5760*sqrt(2)*cos(6*d*x + 6*c
) + 17280*sqrt(2)*cos(4*d*x + 4*c) + 17280*sqrt(2)*cos(2*d*x + 2*c) + 5760*
I*sqrt(2)*sin(6*d*x + 6*c) + 17280*I*sqrt(2)*sin(4*d*x + 4*c) + 17280*I*sqr
t(2)*sin(2*d*x + 2*c) + 5760*sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1) + (5760*sqrt(2)*cos(6*d*x + 6*c) + 17280*sqrt(2
)*cos(4*d*x + 4*c) + 17280*sqrt(2)*cos(2*d*x + 2*c) + 5760*I*sqrt(2)*sin(6*
d*x + 6*c) + 17280*I*sqrt(2)*sin(4*d*x + 4*c) + 17280*I*sqrt(2)*sin(2*d*x +
 2*c) + 5760*sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 1) + (5760*I*sqrt(2)*cos(6*d*x + 6*c) + 17280*I*sqrt(2)*cos(4*d*x
 + 4*c) + 17280*I*sqrt(2)*cos(2*d*x + 2*c) - 5760*sqrt(2)*sin(6*d*x + 6*c)
- 17280*sqrt(2)*sin(4*d*x + 4*c) - 17280*sqrt(2)*sin(2*d*x + 2*c) + 5760*I*
sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1) + (-5760*I*sqrt(2)*cos(6*d*x + 6*c) - 17280*I
*sqrt(2)*cos(4*d*x + 4*c) - 17280*I*sqrt(2)*cos(2*d*x + 2*c) + 5760*sqrt(2)
*sin(6*d*x + 6*c) + 17280*sqrt(2)*sin(4*d*x + 4*c) + 17280*sqrt(2)*sin(2*d*
x + 2*c) - 5760*I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2880*sqrt(2)*cos(6*d*x
 + 6*c) + 8640*sqrt(2)*cos(4*d*x + 4*c) + 8640*sqrt(2)*cos(2*d*x + 2*c) + 2
880*I*sqrt(2)*sin(6*d*x + 6*c) + 8640*I*sqrt(2)*sin(4*d*x + 4*c) + 8640*I*s
qrt(2)*sin(2*d*x + 2*c) + 2880*sqrt(2))*log(2*sqrt(2)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (2880*sqrt(2
)*cos(6*d*x + 6*c) + 8640*sqrt(2)*cos(4*d*x + 4*c) + 8640*sqrt(2)*cos(2*d*x
 + 2*c) + 2880*I*sqrt(2)*sin(6*d*x + 6*c) + 8640*I*sqrt(2)*sin(4*d*x + 4*c)
 + 8640*I*sqrt(2)*sin(2*d*x + 2*c) + 2880*sqrt(2))*log(-2*sqrt(2)*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
(2880*I*sqrt(2)*cos(6*d*x + 6*c) + 8640*I*sqrt(2)*cos(4*d*x + 4*c) + 8640*I
*sqrt(2)*cos(2*d*x + 2*c) - 2880*sqrt(2)*sin(6*d*x + 6*c) - 8640*sqrt(2)*si
n(4*d*x + 4*c) - 8640*sqrt(2)*sin(2*d*x + 2*c) + 2880*I*sqrt(2))*log(2*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2) + (-2880*I*sqrt(2)*cos(6*d*x + 6*c) - 8640*I*sqrt(2)*co
s(4*d*x + 4*c) - 8640*I*sqrt(2)*cos(2*d*x + 2*c) + 2880*sqrt(2)*sin(6*d*x +
 6*c) + 8640*sqrt(2)*sin(4*d*x + 4*c) + 8640*sqrt(2)*sin(2*d*x + 2*c) - 288
0*I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + (2880*I*sqrt(2)*cos(6*d*x + 6*
```

```
c) + 8640*I*sqrt(2)*cos(4*d*x + 4*c) + 8640*I*sqrt(2)*cos(2*d*x + 2*c) - 28
80*sqrt(2)*sin(6*d*x + 6*c) - 8640*sqrt(2)*sin(4*d*x + 4*c) - 8640*sqrt(2)*
sin(2*d*x + 2*c) + 2880*I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + (-2880*I
*sqrt(2)*cos(6*d*x + 6*c) - 8640*I*sqrt(2)*cos(4*d*x + 4*c) - 8640*I*sqrt(2
)*cos(2*d*x + 2*c) + 2880*sqrt(2)*sin(6*d*x + 6*c) + 8640*sqrt(2)*sin(4*d*x
 + 4*c) + 8640*sqrt(2)*sin(2*d*x + 2*c) - 2880*I*sqrt(2))*log(2*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 2) + 15360*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 129024*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 46080*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 15360*I*sin(9/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 129024*I*sin(5/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 46080*I*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*sqrt(a)*sqrt(e)/((-36864*I*e^4*cos(6*d*x + 6*c) - 110592*I*e^4
*cos(4*d*x + 4*c) - 110592*I*e^4*cos(2*d*x + 2*c) + 36864*e^4*sin(6*d*x + 6
*c) + 110592*e^4*sin(4*d*x + 4*c) + 110592*e^4*sin(2*d*x + 2*c) - 36864*I*e
^4)*d)
```

Fricas [A] time = 2.72937, size = 1883, normalized size = 2.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/12*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*(-5*I*e^(4*I*d*x + 4*I*c) + 42*I*e^(2*I*d*x + 2*I*c) + 15*I)
*e^(1/2*I*d*x + 1/2*I*c) + 6*(d*e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*d*
x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)*sqrt(25/64*I*a/(d^2*e^7))
*log(8/5*I*d*e^4*sqrt(25/64*I*a/(d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*
I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*
c)) - 6*(d*e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*
e^(2*I*d*x + 2*I*c) + d*e^4)*sqrt(25/64*I*a/(d^2*e^7))*log(-8/5*I*d*e^4*sqr
t(25/64*I*a/(d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*
sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)) + 6*(d*e^4*e^(6*
I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c)
+ d*e^4)*sqrt(-25/64*I*a/(d^2*e^7))*log(8/5*I*d*e^4*sqrt(-25/64*I*a/(d^2*e^
7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)) - 6*(d*e^4*e^(6*I*d*x + 6*I*c) + 3*
d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)*sqrt(-25/6
4*I*a/(d^2*e^7))*log(-8/5*I*d*e^4*sqrt(-25/64*I*a/(d^2*e^7)) + sqrt(2)*sqrt
(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(
1/2*I*d*x + 1/2*I*c)))/(d*e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*d*x + 4*
I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ia\tan(dx+c)+a}}{(e\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

3.681 $\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=175

$$-\frac{12i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}{35ad}+\frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{32i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}(e\cos(c+d}{35ad}$$

```
[Out] (((2*I)/7)*(e*Cos[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I
)/35)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
 - (((12*I)/35)*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) -
(((32*I)/35)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x
]])/(a*d)
```

Rubi [A] time = 0.378285, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3515, 3502, 3497, 3488}

$$-\frac{12i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}{35ad}+\frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}}-\frac{32i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}(e\cos(c+d}{35ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```



```
[Out] (((2*I)/7)*(e*Cos[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I
)/35)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
 - (((12*I)/35)*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) -
(((32*I)/35)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x
]])/(a*d)
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}dx = \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}dx$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(6(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}dx}{7a}$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} - \frac{12i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{35ad} + \frac{\left(24(e\cos(c+dx))^{5/2}(e\right.}{}$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{12i(e\cos(c+dx))^{5/2}\sqrt{a+ia\,\mathrm{ta}}}{35ad}$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{12i(e\cos(c+dx))^{5/2}\sqrt{a+ia\,\mathrm{ta}}}{35ad}$$

Mathematica [A] time = 0.530145, size = 80, normalized size = 0.46

$$-\frac{ie^3(70i\sin(c+dx)+6i\sin(3(c+dx))+35\cos(c+dx)+\cos(3(c+dx)))}{70d\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I/70)*e^3*(35*Cos[c + d*x] + Cos[3*(c + d*x)] + (70*I)*Sin[c + d*x] + (6
*I)*Sin[3*(c + d*x)]))/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.336, size = 110, normalized size = 0.6

$$\frac{10\,i(\cos(dx+c))^4+10\,(\cos(dx+c))^3\sin(dx+c)+4\,i(\cos(dx+c))^2+16\,\cos(dx+c)\sin(dx+c)-32\,i}{35\,ad\,(\cos(dx+c))^2}\sqrt{\frac{a\,(i\sin(dx}{\mathrm{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/35/d/a*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e*cos(d*x+c))^(5/2
)*(5*I*cos(d*x+c)^4+5*cos(d*x+c)^3*sin(d*x+c)+2*I*cos(d*x+c)^2+8*cos(d*x+c)
*sin(d*x+c)-16*I)/cos(d*x+c)^2
```

Maxima [A] time = 3.24811, size = 273, normalized size = 1.56

$$\left(5ie^2\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)-7ie^2\cos\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right),\cos\left(\frac{7}{2}dx+\frac{7}{2}c\right)\right)\right)+35ie^2\cos\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/140*(5*I*e^2*cos(7/2*d*x + 7/2*c) - 7*I*e^2*cos(5/7*arctan2(sin(7/2*d*x +
 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*I*e^2*cos(3/7*arctan2(sin(7/2*d*x + 7/
2*c), cos(7/2*d*x + 7/2*c))) - 105*I*e^2*cos(1/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c))) + 5*e^2*sin(7/2*d*x + 7/2*c) + 7*e^2*sin(5/7*arc
tan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*e^2*sin(3/7*arctan2(
sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 105*e^2*sin(1/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(e)/(sqrt(a)*d)
```

Fricas [A] time = 2.13451, size = 305, normalized size = 1.74

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-7i\,e^{2}e^{(6i\,dx+6i\,c)}-105i\,e^{2}e^{(4i\,dx+4i\,c)}+35i\,e^{2}e^{(2i\,dx+2i\,c)}+5i\,e^{2}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{7}{2}i\,dx-\frac{7}{2}i\,c\right)}}{140\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/140*sqrt(2)*sqrt(1/2)*(-7*I*e^2*e^(6*I*d*x + 6*I*c) - 105*I*e^2*e^(4*I*d*
x + 4*I*c) + 35*I*e^2*e^(2*I*d*x + 2*I*c) + 5*I*e^2)*sqrt(e*e^(2*I*d*x + 2*
I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2*I*d*x - 7/2*I*c)/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.682 $\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{8i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}{15ad}+\frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}}+\frac{16i\sec^2(c+dx)(e\cos(c+dx))^{3/2}}{15d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] (((2*I)/5)*(e*Cos[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I
)/15)*(e*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
 - (((8*I)/15)*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rubi [A] time = 0.312992, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3515, 3502, 3497, 3488}

$$-\frac{8i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}{15ad}+\frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}}+\frac{16i\sec^2(c+dx)(e\cos(c+dx))^{3/2}}{15d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```


```
[Out] (((2*I)/5)*(e*Cos[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I
)/15)*(e*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
 - (((8*I)/15)*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3497

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)/
(a*f*m), x] + Dist[(a*(m + n))/(m*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
```

```
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(4(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{5a}$$
$$= \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} - \frac{8i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{15ad} + \frac{\left(8(e\cos(c+dx))^{3/2}(e\right.}{}$$
$$= \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{3/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{8i(e\cos(c+dx))^{3/2}\sqrt{a+ia}}{15ad}$$

Mathematica [A] time = 0.335607, size = 63, normalized size = 0.5

$$-\frac{ie^2(4i\sin(2(c+dx))+\cos(2(c+dx))-15)}{15d\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I/15)*e^2*(-15 + Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)]))/(d*Sqrt[e*C
os[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.35, size = 100, normalized size = 0.8

$$\frac{6\,i(\cos(dx+c))^3+6\,(\cos(dx+c))^2\sin(dx+c)+8\,i\cos(dx+c)+16\,\sin(dx+c)}{15\,ad\cos(dx+c)}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/15/d/a*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)*(e*cos(d*x+c))^(3/2
)*(3*I*cos(d*x+c)^3+3*cos(d*x+c)^2*sin(d*x+c)+4*I*cos(d*x+c)+8*sin(d*x+c))/
cos(d*x+c)
```

Maxima [A] time = 3.56851, size = 184, normalized size = 1.46

$$\frac{\left(3i\,e\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,e\cos\left(\frac{3}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)+30i\,e\cos\left(\frac{1}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/30*(3*I*e*cos(5/2*d*x + 5/2*c) - 5*I*e*cos(3/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 30*I*e*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
os(5/2*d*x + 5/2*c))) + 3*e*sin(5/2*d*x + 5/2*c) + 5*e*sin(3/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 30*e*sin(1/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(e)/(sqrt(a)*d)
```

Fricas [A] time = 2.0634, size = 252, normalized size = 2.

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-5i\,ee^{(4i\,dx+4i\,c)}+30i\,ee^{(2i\,dx+2i\,c)}+3i\,e\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/30*sqrt(2)*sqrt(1/2)*(-5*I*e*e^(4*I*d*x + 4*I*c) + 30*I*e*e^(2*I*d*x + 2*
I*c) + 3*I*e)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*e^(-5/2*I*d*x - 5/2*I*c)/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

3.683 $\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{3ad}$$

```
[Out] (((2*I)/3)*Sqrt[e*Cos[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3
)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rubi [A] time = 0.208784, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3515, 3502, 3488}

$$\frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{4i\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Cos[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((2*I)/3)*Sqrt[e*Cos[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3
)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3502

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(a*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(b*f*(m + 2*n)), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \left(\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(2\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{3a}$$
$$= \frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3ad}$$

Mathematica [A] time = 0.18951, size = 48, normalized size = 0.6

$$\frac{2(2\tan(c+dx)-i)\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*(-I + 2*Tan[c + d*x]))/(3*d*Sqrt[a + I*a*Tan[c + d*
x]])
```

Maple [A] time = 0.364, size = 74, normalized size = 0.9

$$\frac{2\,i\left(\cos\left(dx+c\right)\right)^{2}+2\,\cos\left(dx+c\right)\sin\left(dx+c\right)-4\,i}{3\,ad}\sqrt{e\cos\left(dx+c\right)}\sqrt{\frac{a\left(i\sin\left(dx+c\right)+\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 2/3/d/a*(e*cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2)
*(I*cos(d*x+c)^2+cos(d*x+c)*sin(d*x+c)-2*I)
```

Maxima [A] time = 3.144, size = 108, normalized size = 1.35

$$\frac{\sqrt{e}\left(i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\cos\left(\frac{1}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\right.\right.}{3\,\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*sqrt(e)*(I*cos(3/2*d*x + 3/2*c) - 3*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

Fricas [A] time = 2.09765, size = 204, normalized size = 2.55

$$\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/3*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*(-3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3/2*I*d*x - 3/2*I*c)/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos\left(c+dx\right)}}{\sqrt{a\left(i\tan\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\cos\left(dx+c\right)}}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/sqrt(I*a*tan(d*x + c) + a), x)
```

3.684 $\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=36

$$\frac{2i}{d\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}$$

```
[Out] (2*I)/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.139454, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3515, 3488}

$$\frac{2i}{d\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (2*I)/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3488

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n)
/(a*f*m), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\ &= \frac{2i}{d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.172804, size = 36, normalized size = 1.

$$\frac{2i}{d\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (2*I)/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] time = 0.45, size = 69, normalized size = 1.9

$$\frac{-2\,i\,(i\sin(dx+c)-\cos(dx+c))}{ade}\sqrt{e\cos(dx+c)}\sqrt{\frac{a\,(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -2*I/d/a*(e*cos(d*x+c))^(1/2)*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c))^(1/2
)*(I*sin(d*x+c)-cos(d*x+c))/e
```

Maxima [B] time = 2.30743, size = 103, normalized size = 2.86

$$\frac{2i\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{\sqrt{a}d\sqrt{e}\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 2*I*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)/(sqrt(a)*d*sqrt(e)*sqrt(
-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 1))
```

Fricas [B] time = 2.04861, size = 165, normalized size = 4.58

$$\frac{2i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{ade}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a*d*e)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\cos(dx+c)}\sqrt{ia\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.685 $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=495

$$-\frac{i\sqrt{2}\sqrt{a}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{2}\sqrt{a}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right.}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a
*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]*Sec[c + d*x])/(d*e^(3/2)*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (I*Sqrt[2]*Sqrt[a]*ArcTan[1 + (Sq
rt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]*S
ec[c + d*x])/(d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]]) + (I*Sqrt[a]*Log[a*Sqrt[e] - Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c + d*x]]*Sqrt[
a - I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c
+ d*x])/(Sqrt[2]*d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c +
d*x]]) - (I*Sqrt[a]*Log[a*Sqrt[e] + Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c + d*x]]*Sq
rt[a - I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec
[c + d*x])/(Sqrt[2]*d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c
 + d*x]])
```

Rubi [A] time = 0.332187, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3514, 3513, 297, 1162, 617, 204, 1165, 628}

$$-\frac{i\sqrt{2}\sqrt{a}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{i\sqrt{2}\sqrt{a}\sec(c+dx)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right.}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a
*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]*Sec[c + d*x])/(d*e^(3/2)*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (I*Sqrt[2]*Sqrt[a]*ArcTan[1 + (Sq
rt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]*S
ec[c + d*x])/(d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]]) + (I*Sqrt[a]*Log[a*Sqrt[e] - Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c + d*x]]*Sqrt[
a - I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c
+ d*x])/(Sqrt[2]*d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c +
d*x]]) - (I*Sqrt[a]*Log[a*Sqrt[e] + Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c + d*x]]*Sq
rt[a - I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec
[c + d*x])/(Sqrt[2]*d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c
 + d*x]])
```

Rule 3514

```
Int[1/((cos[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)*Sqrt[(a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]]), x_Symbol] :> Dist[1/(d*Cos[e + f*x]*Sqrt[a - b*Tan[e + f*x]]
*Sqrt[a + b*Tan[e + f*x]]), Int[Sqrt[a - b*Tan[e + f*x]]/Sqrt[d*Cos[e + f*x
]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3513

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[(-4*b)/f, Subst[Int[x^2/(a^2*d^2 + x^4), x], x,
```

```
Sqrt[d*Cos[e + f*x]]*Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}
, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sec(c+dx)\int \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$= \frac{(4ia\sec(c+dx))\,\text{Subst}\left(\int \frac{x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$= -\frac{(2ia\sec(c+dx))\,\text{Subst}\left(\int \frac{ae-x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$= \frac{\left(i\sqrt{a}\sec(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}+2x}{-ae-\sqrt{2}\sqrt{a}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$= \frac{i\sqrt{a}\log\left(a\sqrt{e}-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)\ldots\right.}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c\ldots}}$$
$$= -\frac{i\sqrt{2}\sqrt{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{i\sqrt{2}\sqrt{a}\tan\ldots}{de^{3}\ldots}$$

Mathematica [A] time = 9.51568, size = 209, normalized size = 0.42

$$\frac{ie^{\frac{1}{2}i(c+dx)}\left(\log\left(-\sqrt{2}e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}+1\right)-\log\left(\sqrt{2}e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}+1\right)+2\tan^{-1}\left(1-\sqrt{2}e^{\frac{1}{2}i(c+dx)}\right)-2\tan^{-1}\left(1+\ldots\right.\right.}{\sqrt{2}de\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (I*E^((I/2)*(c + d*x))*(2*ArcTan[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*ArcTa
n[1 + Sqrt[2]*E^((I/2)*(c + d*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) +
E^(I*(c + d*x))] - Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))]))
/(Sqrt[2]*d*e*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[
(e*(1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x))])
```

Maple [A] time = 0.362, size = 232, normalized size = 0.5

$$\frac{(\cos(dx+c))^2(\cos(dx+c)-1)^2}{ad(\sin(dx+c))^3(i\sin(dx+c)+\cos(dx+c)-1)}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(i\text{Artanh}\left(\frac{-\cos(dx+c)-1+\text{s}\ldots}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*cos(d*x+c)^2*(cos(d*x+c)-1)^2*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+c)
)^(1/2)*(I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))
+I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))-arctanh(
1/2*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))+arctanh(1/2*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c))))/sin(d*x+c)^3/(e*cos(d*x+c))^
(3/2)/(1/(cos(d*x+c)+1))^(3/2)/(I*sin(d*x+c)+cos(d*x+c)-1)
```

Maxima [A] time = 3.32039, size = 964, normalized size = 1.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] -1/4*(2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2
*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1,
-sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*
x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqr
t(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 2*sqrt(
2)*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x
 + 1/2*c) + cos(d*x + c) + 1) + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/2*d*x + 1/
2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + I*
sqrt(2)*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(sqrt(2)*cos(1/
2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^
2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) - 2*(
sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2
*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - s
qrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))/(sqrt(a)*d*e^(3/2))
```

Fricas [A] time = 2.29791, size = 973, normalized size = 1.97

$$\frac{1}{2}\sqrt{\frac{4i}{ad^2e^3}}\log\left(\frac{1}{2}i\,ade^2\sqrt{\frac{4i}{ad^2e^3}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right)-\frac{1}{2}\sqrt{\frac{4i}{ad^2e^3}}\log\left(-\frac{1}{2}i\,ade^2\sqrt{\frac{4i}{ad^2e^3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*sqrt(4*I/(a*d^2*e^3))*log(1/2*I*a*d*e^2*sqrt(4*I/(a*d^2*e^3)) + sqrt(2)
*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*e^(1/2*I*d*x + 1/2*I*c)) - 1/2*sqrt(4*I/(a*d^2*e^3))*log(-1/2*I*a*d*e^2*s
qrt(4*I/(a*d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)) + 1/2*sqrt(-4*I/(a
*d^2*e^3))*log(1/2*I*a*d*e^2*sqrt(-4*I/(a*d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqr
t(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
 + 1/2*I*c)) - 1/2*sqrt(-4*I/(a*d^2*e^3))*log(-1/2*I*a*d*e^2*sqrt(-4*I/(a*d
^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}\sqrt{ia\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.686 $\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=470

$$\frac{ie^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{ie^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{ie^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right.}{2\sqrt{2}\sqrt{a}d(e\cos(c+}$$

```
[Out] (I*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]
*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c
 + d*x])^(5/2)) - (I*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c
 + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*
x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - ((I/2)*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]
*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a
 + I*a*Tan[c + d*x])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c +
 d*x])^(5/2)) + ((I/2)*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*
a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])
])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - (I*C
os[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Cos[c + d*x])^(5/2))
```

Rubi [A] time = 0.440715, antiderivative size = 470, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3515, 3501, 3495, 297, 1162, 617, 204, 1165, 628}

$$\frac{ie^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{ie^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}-\frac{ie^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right.}{2\sqrt{2}\sqrt{a}d(e\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (I*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]
*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c
 + d*x])^(5/2)) - (I*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c
 + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*
x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - ((I/2)*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]
*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a
 + I*a*Tan[c + d*x])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c +
 d*x])^(5/2)) + ((I/2)*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*
a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])
])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - (I*C
os[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Cos[c + d*x])^(5/2))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
```

```
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} + \frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+}}{2a(e\cos(c+dx))^{5/2}(e\sec(c+dx)}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} - \frac{(2ie^4)\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\,\text{ta}}}{\sqrt{e\sec}}\right.}{d(e\cos(c+dx))^{5/2}(e\sec(c+dx)}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} + \frac{(ie^3)\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan}}{\sqrt{e\sec(c}}\right.}{d(e\cos(c+dx))^{5/2}(e\sec(c+dx))}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} - \frac{(ie^2)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+i}}{\sqrt{e}}\right.}{2d(e\cos(c+dx))^{5/2}(e\sec(c+d}$$

$$= -\frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} + -\frac{ie}{}$$

$$= \frac{ie^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{ie^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\,\text{ta}}}{\sqrt{a}\sqrt{e\sec(}}\right.}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec}$$

Mathematica [A] time = 1.6913, size = 250, normalized size = 0.53

$$\frac{ie^{ic-\frac{idx}{2}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)\tan^{-1}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)-\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)-2e^{\frac{3idx}{2}}\right)}{d\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (I*E^(I*c - (I/2)*d*x)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-2*
E^(((3*I)/2)*d*x) + (-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))*ArcTan[
E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)] - (-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*
(c + d*x)))*ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)]))/(d*Sqrt[(1 + E^(
(2*I)*(c + d*x)))/E^(I*(c + d*x))]*Sqrt[Cos[c + d*x]]*(e*Cos[c + d*x])^(5/2
)*Sec[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.356, size = 313, normalized size = 0.7

$$-\frac{(\cos(dx+c))^2(\cos(dx+c)-1)^3}{2\,ad(\sin(dx+c))^5(i\sin(dx+c)+\cos(dx+c)-1)}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(i\cos(dx+c)\,\text{Artanh}\left(\frac{-\cos(d}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d/a*cos(d*x+c)^2*(cos(d*x+c)-1)^3*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x
+c))^(1/2)*(I*cos(d*x+c)*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-
1+sin(d*x+c)))+I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x
```

```
+c)))*cos(d*x+c)+2*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)*arctanh
(1/2*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+c)))-arctanh(1/2*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)+2*cos(d*x+c)*(1/(
cos(d*x+c)+1))^(1/2)+2*(1/(cos(d*x+c)+1))^(1/2))/sin(d*x+c)^5/(I*sin(d*x+c)
+cos(d*x+c)-1)/(1/(cos(d*x+c)+1))^(5/2)/(e*cos(d*x+c))^(5/2)
```

Maxima [B] time = 3.32576, size = 2909, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] -(16*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 1) + 16*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 16*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*sin(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 16*(sqrt(2)*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, -sqrt(2)*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (16*I*sqrt(
2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 16*sqrt(2
)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*I*sqrt(
2))*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (-16*I*sqrt(2)*c
os(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*sqrt(2)*si
n(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 16*I*sqrt(2))*
arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), -sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 8*(sqrt(2)*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*log(2*sqrt(2
)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(2/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + 1) + 8*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + sqrt(2))*log(-2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
```

```
d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
- 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 1) - (-8*I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*I*sqrt(2))*log(2*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 2) - (8*I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*I*sqrt(2))*log(2*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - (-8*I*sqrt(2)*cos(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*sqrt(2)*sin(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*I*sqrt(2))*log(2*cos(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - (8*I*sqrt(2)*cos(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*sqrt(2)*sin(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 8*I*sqrt(2))*log(2*cos(1/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 128*cos(3/2*d*x + 3/2*c) + 128*I*s
in(3/2*d*x + 3/2*c))*sqrt(a)*sqrt(e)/((-64*I*a*e^3*cos(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 64*a*e^3*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) - 64*I*a*e^3)*d)
```

Fricas [A] time = 2.25852, size = 1331, normalized size = 2.83

$$-4i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2idx+2ic)}+e}\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}e^{\left(\frac{3}{2}idx+\frac{3}{2}ic\right)}+\left(ade^3e^{(2idx+2ic)}+ade^3\right)\sqrt{\frac{i}{ad^2e^5}}\log\left(ade^3\sqrt{\frac{i}{ad^2e^5}}+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2idx+2ic)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(-4*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) + (a*d*e^3*e^(2*I*d*x + 2*I*c) +
 a*d*e^3)*sqrt(I/(a*d^2*e^5))*log(a*d*e^3*sqrt(I/(a*d^2*e^5)) + sqrt(2)*sqr
t(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^
(1/2*I*d*x + 1/2*I*c)) - (a*d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)*sqrt(I/(a*
d^2*e^5))*log(-a*d*e^3*sqrt(I/(a*d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*
I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*
c)) - (a*d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)*sqrt(-I/(a*d^2*e^5))*log(a*d*
e^3*sqrt(-I/(a*d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)) + (a*d*e^3*e^(
```

```
2*I*d*x + 2*I*c) + a*d*e^3)*sqrt(-I/(a*d^2*e^5))*log(-a*d*e^3*sqrt(-I/(a*d^
2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)))/(a*d*e^3*e^(2*I*d*x + 2*I*c) +
 a*d*e^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}\sqrt{ia\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.687 $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=682

$$-\frac{3i\sqrt{a}e^{7/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}+\frac{3i\sqrt{a}e^{7/2}\sec(c+dx)\,\mathrm{ta}}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{tan}}}$$

```
[Out] (((3*I)/4)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d
*x]]) - (((3*I)/4)*Sqrt[a]*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a
*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e
*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + (((3*I)/4)*Sqrt[a]*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d
*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*
Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((3*I)/8)*Sqrt[a]*e^(7/2)*Log[
a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x
]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c
 + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I
*a*Tan[c + d*x]]) - (((3*I)/8)*Sqrt[a]*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqr
t[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I
*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c
+ d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/
2)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Cos[c + d*x])^(7/2))
```

Rubi [A] time = 0.778134, antiderivative size = 682, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {3515, 3501, 3498, 3499, 3495, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3i\sqrt{a}e^{7/2}\sec(c+dx)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}+\frac{3i\sqrt{a}e^{7/2}\sec(c+dx)\,\mathrm{ta}}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\,\mathrm{tan}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```


```
[Out] (((3*I)/4)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d
*x]]) - (((3*I)/4)*Sqrt[a]*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a
*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e
*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + (((3*I)/4)*Sqrt[a]*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d
*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*
Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((3*I)/8)*Sqrt[a]*e^(7/2)*Log[
a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x
]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c
 + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I
*a*Tan[c + d*x]]) - (((3*I)/8)*Sqrt[a]*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqr
t[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I
*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c
+ d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/
2)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Cos[c + d*x])^(7/2))
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3501

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(d^2*(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e +
f*x])^(n + 1))/(b*f*(m + n - 1)), x] + Dist[(d^2*(m - 2))/(a*(m + n - 1)),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3498

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(n
 - 1))/(f*(m + n - 1)), x] + Dist[(a*(m + 2*n - 2))/(m + n - 1), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3499

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[(d*Sec[e + f*x])/(Sqrt[a - b*Tan[e + f*x]]*Sqr
t[a + b*Tan[e + f*x]]), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3495

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[(-4*b*d^2)/f, Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}} + \frac{\left(3e^2\right)\int(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan}}{4a(e\cos(c+dx))^{7/2}(e\sec(c+d}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+d}}{2ad(e\cos(c+dx))^{7/2}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+d}}{2ad(e\cos(c+dx))^{7/2}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+d}}{2ad(e\cos(c+dx))^{7/2}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+d}}{2ad(e\cos(c+dx))^{7/2}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+d}}{2ad(e\cos(c+dx))^{7/2}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{3i\sqrt{a}e^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan}}{\sqrt{e\sec(c+dx}}\right.}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\sqrt{a}e^{7/2}\tan^{-1}\left(1\right.}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+}$$

Mathematica [A] time = 1.57739, size = 245, normalized size = 0.36

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}ie^{\frac{1}{2}i(c+dx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{5/2}\left(\log\left(-\sqrt{2}e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}+1\right)-\log\left(\sqrt{2}e^{\frac{1}{2}i(c+dx)}+e^{i(c+dx)}+1\right)+2\right.}{16d\sqrt{a+ia\tan(c+dx)}(e\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(((3*I)/4)*E^((I/2)*(c + d*x))*((1 + E^((2*I)*(c + d*x)
))/E^(I*(c + d*x)))^(5/2)*(2*ArcTan[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*Ar
cTan[1 + Sqrt[2]*E^((I/2)*(c + d*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x))
 + E^(I*(c + d*x))] - Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))
]) + 4*Sqrt[Cos[c + d*x]]*(I*Cos[c + d*x] + 2*Sin[c + d*x])))/(16*d*(e*Cos[
c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] time = 0.392, size = 371, normalized size = 0.5

$$\frac{(\cos(dx+c))^2(\cos(dx+c)-1)^4}{8\,ad(\sin(dx+c))^7(i\sin(dx+c)+\cos(dx+c)-1)}\sqrt{\frac{a(i\sin(dx+c)+\cos(dx+c))}{\cos(dx+c)}}\left(3\,i\mathrm{Artanh}\left(\frac{-\cos(dx+c)-1}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] 1/8/d/a*cos(d*x+c)^2*(cos(d*x+c)-1)^4*(a*(I*sin(d*x+c)+cos(d*x+c))/cos(d*x+
c))^(1/2)*(3*I*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)-1+sin(d*x+
c)))*cos(d*x+c)^2+3*I*cos(d*x+c)^2*arctanh(1/2*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)+1+sin(d*x+c)))+6*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+
4*I*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-3*arctanh(1/2*(1/(cos(d*x+c)+1))^(1
/2)*(-cos(d*x+c)-1+sin(d*x+c)))*cos(d*x+c)^2+3*arctanh(1/2*(1/(cos(d*x+c)+1
))^(1/2)*(cos(d*x+c)+1+sin(d*x+c)))*cos(d*x+c)^2-6*cos(d*x+c)^2*(1/(cos(d*x
+c)+1))^(1/2)-2*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+4*(1/(cos(d*x+c)+1))^(1
/2))/sin(d*x+c)^7/(I*sin(d*x+c)+cos(d*x+c)-1)/(1/(cos(d*x+c)+1))^(7/2)/(e*c
os(d*x+c))^(7/2)
```

Maxima [B] time = 3.93079, size = 3056, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] -((192*sqrt(2)*cos(4*d*x + 4*c) + 384*sqrt(2)*cos(2*d*x + 2*c) + 192*I*sqrt
(2)*sin(4*d*x + 4*c) + 384*I*sqrt(2)*sin(2*d*x + 2*c) + 192*sqrt(2))*arctan
2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (192*sqrt(2)*c
os(4*d*x + 4*c) + 384*sqrt(2)*cos(2*d*x + 2*c) + 192*I*sqrt(2)*sin(4*d*x +
4*c) + 384*I*sqrt(2)*sin(2*d*x + 2*c) + 192*sqrt(2))*arctan2(sqrt(2)*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (192*sqrt(2)*cos(4*d*x + 4*c)
 + 384*sqrt(2)*cos(2*d*x + 2*c) + 192*I*sqrt(2)*sin(4*d*x + 4*c) + 384*I*sq
rt(2)*sin(2*d*x + 2*c) + 192*sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 1) + (192*sqrt(2)*cos(4*d*x + 4*c) + 384*sqrt(2)*c
os(2*d*x + 2*c) + 192*I*sqrt(2)*sin(4*d*x + 4*c) + 384*I*sqrt(2)*sin(2*d*x
+ 2*c) + 192*sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
```

```
 2*c))) + 1) + (192*I*sqrt(2)*cos(4*d*x + 4*c) + 384*I*sqrt(2)*cos(2*d*x +
2*c) - 192*sqrt(2)*sin(4*d*x + 4*c) - 384*sqrt(2)*sin(2*d*x + 2*c) + 192*I*
sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 1) + (-192*I*sqrt(2)*cos(4*d*x + 4*c) - 384*I*sq
rt(2)*cos(2*d*x + 2*c) + 192*sqrt(2)*sin(4*d*x + 4*c) + 384*sqrt(2)*sin(2*d
*x + 2*c) - 192*I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (96*sqrt(2)*cos(4*d*x +
 4*c) + 192*sqrt(2)*cos(2*d*x + 2*c) + 96*I*sqrt(2)*sin(4*d*x + 4*c) + 192*
I*sqrt(2)*sin(2*d*x + 2*c) + 96*sqrt(2))*log(2*sqrt(2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt
(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (96*sqrt(2)
*cos(4*d*x + 4*c) + 192*sqrt(2)*cos(2*d*x + 2*c) + 96*I*sqrt(2)*sin(4*d*x +
 4*c) + 192*I*sqrt(2)*sin(2*d*x + 2*c) + 96*sqrt(2))*log(-2*sqrt(2)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ (96*I*sqrt(2)*cos(4*d*x + 4*c) + 192*I*sqrt(2)*cos(2*d*x + 2*c) - 96*sqrt
(2)*sin(4*d*x + 4*c) - 192*sqrt(2)*sin(2*d*x + 2*c) + 96*I*sqrt(2))*log(2*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2) + (-96*I*sqrt(2)*cos(4*d*x + 4*c) - 192*I*sqrt(2)*co
s(2*d*x + 2*c) + 96*sqrt(2)*sin(4*d*x + 4*c) + 192*sqrt(2)*sin(2*d*x + 2*c)
 - 96*I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + (96*I*sqrt(2)*cos(4*d*x +
4*c) + 192*I*sqrt(2)*cos(2*d*x + 2*c) - 96*sqrt(2)*sin(4*d*x + 4*c) - 192*s
qrt(2)*sin(2*d*x + 2*c) + 96*I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + (-9
6*I*sqrt(2)*cos(4*d*x + 4*c) - 192*I*sqrt(2)*cos(2*d*x + 2*c) + 96*sqrt(2)*
sin(4*d*x + 4*c) + 192*sqrt(2)*sin(2*d*x + 2*c) - 96*I*sqrt(2))*log(2*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2) + 512*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) - 1536*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 512*I*sin(
5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1536*I*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*sqrt(e)/((-1024*I*a*e^4*cos(4*
d*x + 4*c) - 2048*I*a*e^4*cos(2*d*x + 2*c) + 1024*a*e^4*sin(4*d*x + 4*c) +
2048*a*e^4*sin(2*d*x + 2*c) - 1024*I*a*e^4)*d)
```

Fricas [A] time = 2.43101, size = 1669, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(-I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(1/2*I*d*x + 1/2*I*c) + (a*d
*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)*sqrt(9/
16*I/(a*d^2*e^7))*log(4/3*I*a*d*e^4*sqrt(9/16*I/(a*d^2*e^7)) + sqrt(2)*sqrt
(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(
1/2*I*d*x + 1/2*I*c)) - (a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x
 + 2*I*c) + a*d*e^4)*sqrt(9/16*I/(a*d^2*e^7))*log(-4/3*I*a*d*e^4*sqrt(9/16*
I/(a*d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)) + (a*d*e^4*e^(4*I*d*x +
4*I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)*sqrt(-9/16*I/(a*d^2*e^7))
*log(4/3*I*a*d*e^4*sqrt(-9/16*I/(a*d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(
2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*
I*c)) - (a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c) + a*d*
e^4)*sqrt(-9/16*I/(a*d^2*e^7))*log(-4/3*I*a*d*e^4*sqrt(-9/16*I/(a*d^2*e^7))
 + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)))/(a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*
e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\cos(dx+c))^{\frac{7}{2}}\sqrt{ia\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

3.688 $\int (e\cos(c+dx))^m (a+ia\tan(c+dx))^n\,dx$

Optimal. Leaf size=105

$$-\frac{i2^{n-\frac{m}{2}}(a+ia\tan(c+dx))^n(e\cos(c+dx))^m(1+i\tan(c+dx))^{\frac{1}{2}(m-2n)}\,{}_2F_1\left(-\frac{m}{2},\frac{1}{2}(m-2n+2);1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

```
[Out] ((-I)*2^(-m/2 + n)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, (2 + m - 2*n)
/2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((m - 2*n)/2)*(a
+ I*a*Tan[c + d*x])^n)/(d*m)
```

Rubi [A] time = 0.230718, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3515, 3505, 3523, 70, 69}

$$-\frac{i2^{n-\frac{m}{2}}(a+ia\tan(c+dx))^n(e\cos(c+dx))^m(1+i\tan(c+dx))^{\frac{1}{2}(m-2n)}\,{}_2F_1\left(-\frac{m}{2},\frac{1}{2}(m-2n+2);1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-m/2 + n)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, (2 + m - 2*n)
/2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((m - 2*n)/2)*(a
+ I*a*Tan[c + d*x])^n)/(d*m)
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int (e\sec(c+dx))^{-m}(a+ia\tan(c+dx)$$
$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia$$
$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\text{Subst}}{d}$$
$$= \frac{\left(2^{-1-\frac{m}{2}+n}a(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^n\left(\right.\right.}{}$$
$$= -\frac{i2^{-\frac{m}{2}+n}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{1}{2}(2+m-2n);1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+\right.}{dm}$$

Mathematica [A] time = 13.2095, size = 192, normalized size = 1.83

$$\frac{i2^{n-m}\left(1+e^{2i(c+dx)}\right)\left(e^{idx}\right)^n\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\cos^{-m}(c+dx)\sec^{-n}(c+dx)(\cos(dx)+i\sin(dx))^{-n}(a}{d(m-2n)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(-m + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(
1 + E^((2*I)*(c + d*x)))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^m*(e*C
os[c + d*x])^m*Hypergeometric2F1[1, (2 + m)/2, 1 - m/2 + n, -E^((2*I)*(c +
d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(m - 2*n)*Cos[c + d*x]^m*Sec[c + d*x]^n
*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F] time = 0.725, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^m\,(a+ia\tan(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^m\,(i\,a\tan(dx+c)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{1}{2}\left(ee^{(2i\,dx+2i\,c)}+e\right)e^{(-i\,dx-i\,c)}\right)^{m}\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*(2*a*e^(2*I*d
*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(e\cos(dx+c))^{m}(i\,a\tan(dx+c)+a)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

3.689 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx$

Optimal. Leaf size=86

$$-\frac{ia^2 2^{2-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(\frac{m-2}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

```
[Out] ((-I)*2^(2 - m/2)*a^2*(e*Cos[c + d*x])^m*Hypergeometric2F1[(-2 + m)/2, -m/2
, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

Rubi [A] time = 0.219014, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3515, 3505, 3523, 70, 69}

$$-\frac{ia^2 2^{2-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(\frac{m-2}{2},-\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(2 - m/2)*a^2*(e*Cos[c + d*x])^m*Hypergeometric2F1[(-2 + m)/2, -m/2
, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \left((e\cos(c+dx))^m(e\sec(c+dx))^m\right)\int (e\sec(c+dx))^{-m}(a+ia\tan(c+dx))^2\,d$$

$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia\tan$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\text{Subst}\left(\int\right.}{d}$$

$$= \frac{\left(2^{1-\frac{m}{2}}a^3(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{m/2}\right)\text{Subst}\left(\int\right.}{d}$$

$$= -\frac{i2^{2-\frac{m}{2}}a^2(e\cos(c+dx))^m\,{}_2F_1\left(\frac{1}{2}(-2+m),-\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(}{dm}$$

Mathematica [A] time = 1.58629, size = 125, normalized size = 1.45

$$-\frac{ia^2 2^{2-m}e^{i(c+dx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{m-1}(\tan(c+dx)-i)^2\,{}_2F_1\left(1,\frac{m+2}{2};3-\frac{m}{2};-e^{2i(c+dx)}\right)\cos^{2-m}(c+dx)(e\cos(c+dx))}{d(m-4)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(2 - m)*a^2*E^(I*(c + d*x))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*
x)))^(-1 + m)*Cos[c + d*x]^(2 - m)*(e*Cos[c + d*x])^m*Hypergeometric2F1[1,
(2 + m)/2, 3 - m/2, -E^((2*I)*(c + d*x))]*(-I + Tan[c + d*x])^2)/(d*(-4 + m
))
```

Maple [F] time = 0.419, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^m\,(a+ia\tan(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ia\tan(dx+c)+a)^2\,(e\cos(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*(e*cos(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{4\left(\frac{1}{2}\left(ee^{(2i\,dx+2i\,c)}+e\right)e^{(-i\,dx-i\,c)}\right)^{m}a^{2}e^{(4i\,dx+4i\,c)}}{e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(4*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*a^2*e^(4*I*
d*x + 4*I*c)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)^{2}(e\cos(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2*(e*cos(d*x + c))^m, x)
```

3.690 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx$

Optimal. Leaf size=82

$$-\frac{ia2^{1-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

```
[Out] ((-I)*2^(1 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, m/2, 1 - m/2
, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

Rubi [A] time = 0.174969, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3515, 3505, 3523, 70, 69}

$$-\frac{ia2^{1-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*2^(1 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, m/2, 1 - m/2
, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int (e\sec(c+dx))^{-m}(a+ia\tan(c+dx))$$

$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia\,\mathrm{t}$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\mathrm{Subst}\,(}{d}$$

$$= \frac{\left(2^{-m/2}a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{m/2}\right)\mathrm{Subst}}{d}$$

$$= -\frac{i2^{1-\frac{m}{2}}a(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\,\mathrm{ta}}{dm}$$

Mathematica [A] time = 7.57816, size = 131, normalized size = 1.6

$$-\frac{a2^{1-m}e^{i(c+2dx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m(\tan(c+dx)-i)(\cos(dx)-i\sin(dx))\,{}_2F_1\left(1,\frac{m+2}{2};2-\frac{m}{2};-e^{2i(c+dx)}\right)\cos^{1-m}(c}{d(m-2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -((2^(1 - m)*a*E^(I*(c + 2*d*x))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))
)^m*Cos[c + d*x]^(1 - m)*(e*Cos[c + d*x])^m*Hypergeometric2F1[1, (2 + m)/2,
 2 - m/2, -E^((2*I)*(c + d*x))]*(Cos[d*x] - I*Sin[d*x])*(-I + Tan[c + d*x])
)/(d*(-2 + m)))
```

Maple [F] time = 0.506, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^m\,(a+ia\tan(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (i\,a\tan(dx+c)+a)\,(e\cos(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{2\left(\frac{1}{2}\left(ee^{(2i\,dx+2i\,c)}+e\right)e^{(-i\,dx-i\,c)}\right)^{m}ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(2*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*a*e^(2*I*d*
x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int(e\cos(c+dx))^{m}\,dx+\int i\,(e\cos(c+dx))^{m}\tan(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c)),x)
```

```
[Out] a*(Integral((e*cos(c + d*x))**m, x) + Integral(I*(e*cos(c + d*x))**m*tan(c
+ d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(i\,a\tan(dx+c)+a)\,(e\cos(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

3.691 $\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=86

$$-\frac{i2^{-\frac{m}{2}-1}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+4}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{adm}$$

```
[Out] ((-I)*2^(-1 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, (4 + m)/2, 1
- m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a*d*m)
```

Rubi [A] time = 0.238043, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3515, 3505, 3523, 70, 69}

$$-\frac{i2^{-\frac{m}{2}-1}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+4}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{adm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*2^(-1 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, (4 + m)/2, 1
- m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a*d*m)
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \left((e\cos(c+dx))^m(e\sec(c+dx))^m\right)\int \frac{(e\sec(c+dx))^{-m}}{a+ia\tan(c+dx)}\,dx$$

$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia\tan(c+dx))^{-m/2}(a+$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\text{Subst}\left(\int (a-iax)^{-1-\frac{m}{2}}(a+\right.}{d}$$

$$= \frac{\left(2^{-2-\frac{m}{2}}(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{m/2}\right)\text{Subst}\left(\int \left(\frac{1}{2}+\frac{ix}{2}\right)^{-2-\frac{m}{2}}(a-\right.}{d}$$

$$= -\frac{i2^{-1-\frac{m}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{4+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{m/2}}{adm}$$

Mathematica [A] time = 59.1586, size = 147, normalized size = 1.71

$$\frac{2^{-m-1}e^{-i(c+2dx)}\left(1+e^{2i(c+dx)}\right)^2\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m(\cos(dx)+i\sin(dx))\,{}_2F_1\left(1,\frac{m+2}{2};-\frac{m}{2};-e^{2i(c+dx)}\right)\cos^{-m-1}(c+dx)(e}{ad(m+2)(\tan(c+dx)-i)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2^(-1 - m)*(1 + E^((2*I)*(c + d*x)))^2*((1 + E^((2*I)*(c + d*x)))/E^(I*(c
+ d*x)))^m*Cos[c + d*x]^(-1 - m)*(e*Cos[c + d*x])^m*Hypergeometric2F1[1, (2
 + m)/2, -m/2, -E^((2*I)*(c + d*x))]*(Cos[d*x] + I*Sin[d*x]))/(a*d*E^(I*(c
+ 2*d*x))*(2 + m)*(-I + Tan[c + d*x]))
```

Maple [F] time = 0.868, size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^m}{a+ia\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(\frac{1}{2}\left(ee^{(2i\,dx+2i\,c)}+e\right)e^{(-i\,dx-i\,c)}\right)^{m}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(-2i\,dx-2i\,c)}}{2\,a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(1/2*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*(e^(2*I*d
*x + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c)/a, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{m}}{i\,a\tan(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^m/(I*a*tan(d*x + c) + a), x)
```

3.692 $\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$-\frac{i2^{-\frac{m}{2}-2}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+6}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm}$$

```
[Out] ((-I)*2^(-2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, (6 + m)/2, 1
- m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a^2*d*m)
```

Rubi [A] time = 0.236781, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3515, 3505, 3523, 70, 69}

$$-\frac{i2^{-\frac{m}{2}-2}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+6}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```


```
[Out] ((-I)*2^(-2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, (6 + m)/2, 1
- m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a^2*d*m)
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int \frac{(e\sec(c+dx))^{-m}}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia\tan(c+dx))^{-m}$$
$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\text{Subst}\left(\int (a-iax)^{-1-}\right.}{d}$$
$$= \frac{\left(2^{-3-\frac{m}{2}}(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{m/2}\right)\text{Subst}\left(\int \left(\frac{1}{2}+\frac{ix}{2}\right)^{-3-\frac{m}{2}}\right.}{ad}$$
$$= -\frac{i2^{-2-\frac{m}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{6+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{m/}}{a^2dm}$$

Mathematica [A] time = 69.3477, size = 154, normalized size = 1.79

$$-\frac{i2^{-m-2}e^{-2i(c+2dx)}\left(1+e^{2i(c+dx)}\right)^3\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m(\cos(dx)+i\sin(dx))^2\,{}_2F_1\left(1,\frac{m+2}{2};-\frac{m}{2}-1;-e^{2i(c+dx)}\right)\cos^{-m-}}{a^2d(m+4)(\tan(c+dx)-i)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(-2 - m)*(1 + E^((2*I)*(c + d*x)))^3*((1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x)))^m*Cos[c + d*x]^(-2 - m)*(e*Cos[c + d*x])^m*Hypergeometric2F1[
1, (2 + m)/2, -1 - m/2, -E^((2*I)*(c + d*x))]*(Cos[d*x] + I*Sin[d*x])^2)/(a
^2*d*E^((2*I)*(c + 2*d*x))*(4 + m)*(-I + Tan[c + d*x])^2)
```

Maple [F] time = 1.24, size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^m}{(a+ia\tan(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(\frac{1}{2}\left(ee^{(2i\,dx+2i\,c)}+e\right)e^{(-i\,dx-i\,c)}\right)^{m}\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)e^{(-4i\,dx-4i\,c)}}{4\,a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*(e^(4*I*d
*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/a^2, x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^{m}}{(i\,a\tan(dx+c)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^m/(I*a*tan(d*x + c) + a)^2, x)
```

3.693 $\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{ia2^{\frac{1}{2}-\frac{m}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+1}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] ((-I)*2^(1/2 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, (1 + m)/2,
 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2))/(d*m*Sq
rt[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.286386, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3515, 3505, 3523, 70, 69}

$$-\frac{ia2^{\frac{1}{2}-\frac{m}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+1}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*2^(1/2 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, (1 + m)/2,
 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2))/(d*m*Sq
rt[a + I*a*Tan[c + d*x]])
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int (e\sec(c+dx))^{-m}\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia\tan$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\text{Subst}\left(\int(\right.}{d}$$

$$= \frac{\left(2^{-\frac{1}{2}-\frac{m}{2}}a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}+\frac{m}{2}}\right)\text{Subst}\left(\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{i2^{\frac{1}{2}-\frac{m}{2}}a(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{1+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 0.807458, size = 106, normalized size = 1.01

$$\frac{i2^{-m}\left(1+e^{2i(c+dx)}\right)\sqrt{a+ia\tan(c+dx)}\left(ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\,{}_2F_1\left(1,\frac{m+2}{2};\frac{3-m}{2};-e^{2i(c+dx)}\right)}{d(m-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*(1 + E^((2*I)*(c + d*x)))*((e*(1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x))
)^m*Hypergeometric2F1[1, (2 + m)/2, (3 - m)/2, -E^((2*I)*(c + d*x))]*Sqrt[a
 + I*a*Tan[c + d*x]])/(2^m*d*(-1 + m))
```

Maple [F] time = 0.388, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^m\sqrt{a+ia\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\tan\left(dx+c\right)+a}\left(e\cos\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{2}\left(\frac{1}{2}\left(ee^{(2i\,dx+2i\,c)}+e\right)e^{(-i\,dx-i\,c)}\right)^{m}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2)*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{i\,a\tan\left(dx+c\right)+a}\left(e\cos\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

3.694 $\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal. Leaf size=104

$$-\frac{i2^{-\frac{m}{2}-\frac{1}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+3}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] ((-I)*2^(-1/2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, (3 + m)/2,
1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2))/(d*m*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rubi [A] time = 0.290182, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3515, 3505, 3523, 70, 69}

$$-\frac{i2^{-\frac{m}{2}-\frac{1}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{m+3}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Cos[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*2^(-1/2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m/2, (3 + m)/2,
1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2))/(d*m*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3505

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3523

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a*c)/f, Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int \frac{(e\sec(c+dx))^{-m}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia\tan(c+dx))^{-m/}$$
$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\mathrm{Subst}\left(\int (a-iax)^{-1-\frac{m}{2}}\right.}{d}$$
$$= \frac{\left(2^{-\frac{3}{2}-\frac{m}{2}}a(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}+\frac{m}{2}}\right)\mathrm{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{3}{2}}\right.}{d\sqrt{a+ia\tan(c+dx)}}$$
$$= -\frac{i2^{-\frac{1}{2}-\frac{m}{2}}(e\cos(c+dx))^m\,{}_2F_1\left(-\frac{m}{2},\frac{3+m}{2};1-\frac{m}{2};\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1+}{2}}}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] time = 14.8944, size = 143, normalized size = 1.38

$$\frac{i4^{-m}\left(1+e^{2i(c+dx)}\right)\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\left(ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m{}_2F_1\left(1,\frac{m+2}{2};\frac{1-m}{2};-e^{2i(c+dx)}\right)\cos^{-m}(c+dx)}{d(m+1)\sqrt{a+ia\tan(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Cos[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*(1 + E^((2*I)*(c + d*x)))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^m*
((e*(1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^m*Hypergeometric2F1[1, (2 +
 m)/2, (1 - m)/2, -E^((2*I)*(c + d*x))])/(4^m*d*(1 + m)*Cos[c + d*x]^m*Sqrt
[a + I*a*Tan[c + d*x]])
```

Maple [F] time = 0.363, size = 0, normalized size = 0.

$$\int (e\cos(dx+c))^m\frac{1}{\sqrt{a+ia\tan(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^m}{\sqrt{ia\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2}\left(\frac{1}{2}\left(ee^{(2i\,dx+2i\,c)}+e\right)e^{(-i\,dx-i\,c)}\right)^m\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(-i\,dx-i\,c)}}{2a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(2)*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-I*d*x - I*c)
/a, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a(i\tan(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*cos(c + d*x))**m/sqrt(a*(I*tan(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\cos(dx+c))^m}{\sqrt{ia\tan(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

3.695 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx$

Optimal. Leaf size=183

$$-\frac{a\left(3b^2-a^2(1-m)\right)\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{3}{2};-\tan^2(e+fx)\right)}{f(1-m)}+\frac{b(d\cos(e+fx))^m}{\cdots}$$

```
[Out] -((a*(3*b^2 - a^2*(1 - m))*(d*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (2 + m
)/2, 3/2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x])/(f*(1 - m))
) + (b*(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^2)/(f*(2 - m)) + (b*(d*Cos[e
 + f*x])^m*(2*(b^2 - a^2*(3 - m))*(1 - m) + a*b*(4 - m)*m*Tan[e + f*x]))/(f
*m*(2 - 3*m + m^2))
```

Rubi [A] time = 0.285021, antiderivative size = 175, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3515, 3512, 743, 780, 245}

$$\frac{a\left(a^2-\frac{3b^2}{1-m}\right)\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{3}{2};-\tan^2(e+fx)\right)}{f}+\frac{b(d\cos(e+fx))^m\left(2(1-\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (a*(a^2 - (3*b^2)/(1 - m))*(d*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (2 + m
)/2, 3/2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x])/f + (b*(d*C
os[e + f*x])^m*(a + b*Tan[e + f*x])^2)/(f*(2 - m)) + (b*(d*Cos[e + f*x])^m*
(2*(b^2 - a^2*(3 - m))*(1 - m) + a*b*(4 - m)*m*Tan[e + f*x]))/(f*m*(2 - 3*m
 + m^2))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \left((d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))^3\,d$$

$$=\frac{\left((d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\right)\mathrm{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}dx,x,b\,\mathrm{ta}\right.}{bf}$$

$$=\frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))^2}{f(2-m)}+\frac{\left(b(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\right.}{}$$

$$=\frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))^2}{f(2-m)}+\frac{b(d\cos(e+fx))^m\left(2\left(b^2-a^2(3-m\right.\right.}{fm\left(2-\right.}$$

$$=-\frac{a\left(3b^2-a^2(1-m)\right)(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{3}{2};-\tan^2(e+fx)\right)\sec^2(e}{f(1-m)}$$

Mathematica [A] time = 59.0595, size = 215, normalized size = 1.17

$$\frac{\cos^3(e+fx)(a+b\tan(e+fx))^3(d\cos(e+fx))^m\left(b\left(\left(3a^2(m-2)+2b^2\right)\left(\sec^2(e+fx)^{m/2}-1\right)+ab(m-2)m\tan^3(e+fx\right.\right.}{f(m-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]^3*(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^3*(a^3*(-2 + m)*m*H
ypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)
*Tan[e + f*x] + b*((2*b^2 + 3*a^2*(-2 + m))*(-1 + (Sec[e + f*x]^2)^(m/2)) -
 b^2*m*Tan[e + f*x]^2 + a*b*(-2 + m)*m*Hypergeometric2F1[3/2, 1 + m/2, 5/2,
 -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]^3)))/(f*(-2 + m)*m*(a
*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [F] time = 0.644, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^m\left(a+b\tan\left(fx+e\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)+a\right)^{3}\left(d\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3*(d*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{3}\tan\left(fx+e\right)^{3}+3\,ab^{2}\tan\left(fx+e\right)^{2}+3\,a^{2}b\tan\left(fx+e\right)+a^{3}\right)\left(d\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*(d*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)+a\right)^{3}\left(d\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3*(d*cos(f*x + e))^m, x)
```

3.696 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx$

Optimal. Leaf size=155

$$\frac{\left(b^2-a^2(1-m)\right)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{f(1-m)(m+1)\sqrt{\sin^2(e+fx)}}-\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m}+$$

```
[Out] -((a*b*(2 - m)*(d*Cos[e + f*x])^m)/(f*(1 - m)*m)) + ((b^2 - a^2*(1 - m))*Co
s[e + f*x]*(d*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - m)*(1 + m)*Sqrt[Sin[e + f*x]^2]) + (b
*(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(1 - m))
```

Rubi [A] time = 0.238488, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3515, 3508, 3486, 3772, 2643}

$$\frac{\left(b^2-a^2(1-m)\right)\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{f(1-m)(m+1)\sqrt{\sin^2(e+fx)}}-\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m}+$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```


```
[Out] -((a*b*(2 - m)*(d*Cos[e + f*x])^m)/(f*(1 - m)*m)) + ((b^2 - a^2*(1 - m))*Co
s[e + f*x]*(d*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - m)*(1 + m)*Sqrt[Sin[e + f*x]^2]) + (b
*(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(1 - m))
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3508

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(
m + 1)), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \left((d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx)$$

$$= \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))}{f(1-m)} + \frac{\left((d\cos(e+fx))^m(d\sec(e+fx))\right.}{}$$

$$= -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))}{f(1-m)} + \frac{(}{}$$

$$= -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))}{f(1-m)} + \frac{(}{}$$

$$= -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{\left(b^2-a^2(1-m)\right)\cos(e+fx)(d\cos(e+fx}{f(1-m)(1+}$$

Mathematica [C] time = 3.73468, size = 330, normalized size = 2.13

$$\frac{\cos(e+fx)(a+b\tan(e+fx))^2(d\cos(e+fx))^m\left(\sqrt{\sin^2(e+fx)}\left(-\frac{a^2\cos(e+fx)\cot(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{m+1}-\frac{b^2\csc(e+}{}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*(d*Cos[e + f*x])^m*(-((2^(1 - m)*a*b*((1 + E^((2*I)*(e + f*x)
))/E^(I*(e + f*x)))^m*Cos[e + f*x]^(1 - m)*Hypergeometric2F1[1, m/2, 1 - m/
2, -E^((2*I)*(e + f*x))])/m) + (2^(1 - m)*a*b*E^((2*I)*(e + f*x))*((1 + E^(
(2*I)*(e + f*x)))/E^(I*(e + f*x)))^m*Cos[e + f*x]^(1 - m)*Hypergeometric2F1
[1, (2 + m)/2, 2 - m/2, -E^((2*I)*(e + f*x))])/(-2 + m) + (-((b^2*Csc[e + f
*x]*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Cos[e + f*x]^2])/(-1 + m
)) - (a^2*Cos[e + f*x]*Cot[e + f*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, Cos[e + f*x]^2])/(1 + m))*Sqrt[Sin[e + f*x]^2])*(a + b*Tan[e + f*x])^
2)/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^2)
```

Maple [F] time = 0.371, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^m\left(a+b\tan\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)+a\right)^2 \left(d\cos\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2*(d*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2\right)\left(d\cos\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*cos(f*x + e))^m
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)+a\right)^2 \left(d\cos\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2*(d*cos(f*x + e))^m, x)
```

3.697 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx$

Optimal. Leaf size=90

$$-\frac{a\sin(e+fx)(d\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{df(m+1)\sqrt{\sin^2(e+fx)}}-\frac{b(d\cos(e+fx))^m}{fm}$$

```
[Out] -((b*(d*Cos[e + f*x])^m)/(f*m)) - (a*(d*Cos[e + f*x])^(1 + m)*Hypergeometri
c2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(d*f*(1 + m)*
Sqrt[Sin[e + f*x]^2])
```

Rubi [A] time = 0.0999522, antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3515, 3486, 3772, 2643}

$$-\frac{a\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{f(m+1)\sqrt{\sin^2(e+fx)}}-\frac{b(d\cos(e+fx))^m}{fm}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

```
[Out] -((b*(d*Cos[e + f*x])^m)/(f*m)) - (a*Cos[e + f*x]*(d*Cos[e + f*x])^m*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1
+ m)*Sqrt[Sin[e + f*x]^2])
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3486

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[(b*(d*Sec[e + f*x])^m)/(f*m), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \left((d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))\,dx$$
$$= -\frac{b(d\cos(e+fx))^m}{fm} + \left(a(d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int (d\sec(e+fx))^{-}$$
$$= -\frac{b(d\cos(e+fx))^m}{fm} + \left(a\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\cos(e+fx))^m\right)\int \left(\frac{\cos(e+fx)}{d}\right.$$
$$= -\frac{b(d\cos(e+fx))^m}{fm} - \frac{a\cos(e+fx)(d\cos(e+fx))^m\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e\right.}{f(1+m)\sqrt{\sin^2(e+fx)}}$$

Mathematica [C] time = 1.02678, size = 203, normalized size = 2.26

$$\frac{(d\cos(e+fx))^m\left(-a(m-2)m\sin(2(e+fx))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)-2b\left(m^2-m-2\right)\sqrt{\sin^2(e+fx)}\,{}_2F_1\left(1,\frac{m}{2};\right.\right.}{2f(m-2)m(m+1)\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Cos[e + f*x])^m*(-2*b*(-2 - m + m^2)*Hypergeometric2F1[1, m/2, 1 - m/2,
 -E^((2*I)*(e + f*x))]*Sqrt[Sin[e + f*x]^2] + 2*b*m*(1 + m)*Hypergeometric2
F1[1, (2 + m)/2, 2 - m/2, -E^((2*I)*(e + f*x))]*Sqrt[Sin[e + f*x]^2]*(Cos[2
*(e + f*x)] + I*Sin[2*(e + f*x)]) - a*(-2 + m)*m*Hypergeometric2F1[1/2, (1
+ m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[2*(e + f*x)]))/(2*f*(-2 + m)*m*(1 +
m)*Sqrt[Sin[e + f*x]^2])
```

Maple [F] time = 0.558, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^m\left(a+b\tan\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\tan\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\tan\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(d\cos\left(e+fx\right)\right)^{m}\left(a+b\tan\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\tan\left(fx+e\right)+a\right)\left(d\cos\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

3.698 $\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$

Optimal. Leaf size=140

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m F_1\left(\frac{1}{2};1,\frac{m+2}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af}+\frac{b(d\cos(e+fx))^m\,{}_2F_1\left(1,-\frac{m}{2};\dots\right)}{fm\left(a^2+b^2\right)}$$

```
[Out] (b*(d*Cos[e + f*x])^m*Hypergeometric2F1[1, -m/2, 1 - m/2, (b^2*Sec[e + f*x]
^2)/(a^2 + b^2)])/((a^2 + b^2)*f*m) + (AppellF1[1/2, 1, (2 + m)/2, 3/2, (b^
2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^2)
^(m/2)*Tan[e + f*x])/(a*f)
```

Rubi [A] time = 0.210547, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3515, 3512, 757, 429, 444, 68}

$$\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m F_1\left(\frac{1}{2};1,\frac{m+2}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af}+\frac{b(d\cos(e+fx))^m\,{}_2F_1\left(1,-\frac{m}{2};\dots\right)}{fm\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] (b*(d*Cos[e + f*x])^m*Hypergeometric2F1[1, -m/2, 1 - m/2, (b^2*Sec[e + f*x]
^2)/(a^2 + b^2)])/((a^2 + b^2)*f*m) + (AppellF1[1/2, 1, (2 + m)/2, 3/2, (b^
2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^2)
^(m/2)*Tan[e + f*x])/(a*f)
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \left((d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int \frac{(d\sec(e+fx))^{-m}}{a+b\tan(e+fx)}\,dx$$

$$= \frac{\left((d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{a+x}\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{\left((d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\right)\text{Subst}\left(\int \left(\frac{a\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{a^2-x^2}+\frac{x\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{-a^2+x^2}\right)dx, x, b\tan(e+f\right.}{bf}$$

$$= \frac{\left((d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\right)\text{Subst}\left(\int \frac{x\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{-a^2+x^2}\,dx, x, b\tan(e+fx)\right)}{bf}+\frac{\left(a(d\cos(e\right.}{}$$

$$= \frac{F_1\left(\frac{1}{2};1,\frac{2+m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\tan(e+fx)}{af}$$

$$= \frac{b(d\cos(e+fx))^m\,{}_2F_1\left(1,-\frac{m}{2};1-\frac{m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{\left(a^2+b^2\right)fm}+\frac{F_1\left(\frac{1}{2};1,\frac{2+m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+\right.}{}$$

Mathematica [C] time = 14.1019, size = 1132, normalized size = 8.09

$$\frac{}{f(a+b\tan(e+fx))\left(-\frac{1}{2}bmF_1\left(m;\frac{m}{2},\frac{m}{2};m+1;\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)\sec^2(e+fx)^{-m/2}\left(\frac{b(\tan(e+fx)+i)}{a+b\tan(e+fx)}\right)^{m/2}\left(\frac{b\sec^2(e+}{a+b\tan(e}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Cos[e + f*x])^m*(b*(-1 + (Sec[e + f*x]^2)^(-m/2)) + a*m*Hypergeometric2
F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - (b*AppellF1[m, m/2, m
/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*
((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x])
)/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2)))/(f*(a + b*Tan[e + f
*x])*(a*m*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec[e + f*x
]^2 - (b*m*Tan[e + f*x])/(Sec[e + f*x]^2)^(m/2) + (b*m*AppellF1[m, m/2, m/2
, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Ta
n[e + f*x]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Ta
n[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2) - (b*((b*(
-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a
+ b*Tan[e + f*x]))^(m/2)*(-((a - I*b)*b*m^2*AppellF1[1 + m, 1 + m/2, m/2, 2
 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e
 + f*x]^2)/(2*(1 + m)*(a + b*Tan[e + f*x])^2) - ((a + I*b)*b*m^2*AppellF1[1
 + m, m/2, 1 + m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b
*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(1 + m)*(a + b*Tan[e + f*x])^2)))/(Sec[e
 + f*x]^2)^(m/2) - (b*m*AppellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e
 + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*T
an[e + f*x]))^(-1 + m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2
)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*
Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2*(Sec[e + f*x]^2)^(m/2)) - (b*m*Ap
pellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b
*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I
 + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 + m/2)*(-((b^2*Sec[e + f*x]^2*(
I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[
e + f*x])))/(2*(Sec[e + f*x]^2)^(m/2)) + a*m*Sec[e + f*x]^2*(-Hypergeometri
c2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^2)^(-1 - m/2))
))
```

Maple [F] time = 0.216, size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{m}}{a+b\tan\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^{m}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\cos\left(fx+e\right)\right)^{m}}{b\tan\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\cos\left(e+fx\right)\right)^{m}}{a+b\tan\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m/(a+b*tan(f*x+e)),x)
```

```
[Out] Integral((d*cos(e + f*x))**m/(a + b*tan(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(d\cos\left(fx+e\right)\right)^{m}}{b\tan\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

3.699 $\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$

Optimal. Leaf size=227

$$\frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m F_1\left(\frac{3}{2};2,\frac{m+2}{2};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f}+\frac{\tan(e+fx)\sec^2(e+fx)^{\prime}}{}$$

```
[Out] (2*a*b*(d*Cos[e + f*x])^m*Hypergeometric2F1[2, -m/2, 1 - m/2, (b^2*Sec[e +
f*x]^2)/(a^2 + b^2)])/((a^2 + b^2)^2*f*m) + (AppellF1[1/2, 2, (2 + m)/2, 3/
2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e + f
*x]^2)^(m/2)*Tan[e + f*x])/(a^2*f) + (b^2*AppellF1[3/2, 2, (2 + m)/2, 5/2,
(b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e + f*x]
^2)^(m/2)*Tan[e + f*x]^3)/(3*a^4*f)
```

Rubi [A] time = 0.276633, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3515, 3512, 757, 429, 444, 68, 510}

$$\frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m F_1\left(\frac{3}{2};2,\frac{m+2}{2};\frac{5}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f}+\frac{\tan(e+fx)\sec^2(e+fx)^{\prime}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```


```
[Out] (2*a*b*(d*Cos[e + f*x])^m*Hypergeometric2F1[2, -m/2, 1 - m/2, (b^2*Sec[e +
f*x]^2)/(a^2 + b^2)])/((a^2 + b^2)^2*f*m) + (AppellF1[1/2, 2, (2 + m)/2, 3/
2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e + f
*x]^2)^(m/2)*Tan[e + f*x])/(a^2*f) + (b^2*AppellF1[3/2, 2, (2 + m)/2, 5/2,
(b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e + f*x]
^2)^(m/2)*Tan[e + f*x]^3)/(3*a^4*f)
```


Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \left((d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int \frac{(d\sec(e+fx))^{-m}}{(a+b\tan(e+fx))^2}\,dx$$

$$= \frac{\left((d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(a+x)^2}\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{\left((d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\right)\text{Subst}\left(\int \left(\frac{a^2\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{\left(a^2-x^2\right)^2} - \frac{2ax\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{\left(a^2-x^2\right)^2} + \frac{x^2\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{\left(-a^2+x^2\right)^2}\right.\right.}{bf}$$

$$= \frac{\left((d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\right)\text{Subst}\left(\int \frac{x^2\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{\left(-a^2+x^2\right)^2}\,dx, x, b\tan(e+fx)\right)}{bf} - \frac{(2a(d\,c}{}$$

$$= \frac{F_1\left(\frac{1}{2};2,\frac{2+m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\tan(e+fx}{a^2f}$$

$$= \frac{2ab(d\cos(e+fx))^m\,{}_2F_1\left(2,-\frac{m}{2};1-\frac{m}{2};\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{\left(a^2+b^2\right)^2fm} + \frac{F_1\left(\frac{1}{2};2,\frac{2+m}{2};\frac{3}{2};\frac{b^2\tan^2(e+fx)}{a^2},-\tan\right.}{}$$

Mathematica [C] time = 3.12785, size = 361, normalized size = 1.59

$$-\frac{2(m+4)\sec^2(e+fx)(d\cos(e+fx))^m(}{bf(m+3)(a+b\tan(e+fx))^5\left((m+2)\cos(e+fx)\left((a+ib)F_1\left(m+4;\frac{m}{2}+1,\frac{m}{2}+2;m+5;\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-2*(4 + m)*AppellF1[3 + m, 1 + m/2, 1 + m/2, 4 + m, (a - I*b)/(a + b*Tan[e
 + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(d*Cos[e + f*x])^m*Sec[e + f*x]^2
*(a*Cos[e + f*x] + b*Sin[e + f*x])^5)/(b*f*(3 + m)*((2 + m)*((a + I*b)*Appe
llF1[4 + m, 1 + m/2, 2 + m/2, 5 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I
*b)/(a + b*Tan[e + f*x])] + (a - I*b)*AppellF1[4 + m, 2 + m/2, 1 + m/2, 5 +
 m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])])*Cos[e
+ f*x] + 2*(4 + m)*AppellF1[3 + m, 1 + m/2, 1 + m/2, 4 + m, (a - I*b)/(a +
b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(a*Cos[e + f*x] + b*Sin[e
+ f*x]))*(a + b*Tan[e + f*x])^5)
```

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^m}{\left(a+b\tan\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^m}{\left(b\tan\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d\cos\left(fx+e\right)\right)^m}{b^2\tan\left(fx+e\right)^2+2\,ab\tan\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^m/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(e+fx\right)\right)^m}{\left(a+b\tan\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral((d*cos(e + f*x))**m/(a + b*tan(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d\cos\left(fx+e\right)\right)^m}{\left(b\tan\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

3.700 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx$

Optimal. Leaf size=187

$$\frac{\cos^2(e+fx)(d\cos(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{\frac{m+2}{2}}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{\frac{m+2}{2}}(a+b\tan(e+fx))^{n+1}F_1\left(n+1;\frac{m+2}{2},\frac{m+2}{2};n+2\right.}{bf(n+1)}$$

```
[Out] (AppellF1[1 + n, (2 + m)/2, (2 + m)/2, 2 + n, (a + b*Tan[e + f*x])/(a - Sqr
t[-b^2]), (a + b*Tan[e + f*x])/(a + Sqrt[-b^2])]*Cos[e + f*x]^2*(d*Cos[e +
f*x])^m*(a + b*Tan[e + f*x])^(1 + n)*(1 - (a + b*Tan[e + f*x])/(a - Sqrt[-b
^2]))^((2 + m)/2)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^((2 + m)/2))/
(b*f*(1 + n))
```

Rubi [A] time = 0.202884, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3515, 3512, 760, 133}

$$\frac{\cos^2(e+fx)(d\cos(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{\frac{m+2}{2}}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{\frac{m+2}{2}}(a+b\tan(e+fx))^{n+1}F_1\left(n+1;\frac{m+2}{2},\frac{m+2}{2};n+2\right.}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, (2 + m)/2, (2 + m)/2, 2 + n, (a + b*Tan[e + f*x])/(a - Sqr
t[-b^2]), (a + b*Tan[e + f*x])/(a + Sqrt[-b^2])]*Cos[e + f*x]^2*(d*Cos[e +
f*x])^m*(a + b*Tan[e + f*x])^(1 + n)*(1 - (a + b*Tan[e + f*x])/(a - Sqrt[-b
^2]))^((2 + m)/2)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^((2 + m)/2))/
(b*f*(1 + n))
```

Rule 3515

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3512

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[(d^(2*IntPart[m/2])*(d*Sec[e + f*x])^(2*FracP
art[m/2]))/(b*f*(Sec[e + f*x]^2)^FracPart[m/2]), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 760

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[-(a*c), 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + (e*q)/c))^p*
(1 - (d + e*x)/(d - (e*q)/c))^p), Subst[Int[x^m*Simp[1 - x/(d + (e*q)/c), x
]^p*Simp[1 - x/(d - (e*q)/c), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \left((d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx$$

$$= \frac{\left((d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\right)\text{Subst}\left(\int (a+x)^n\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}dx, x,\right.}{bf}$$

$$= \frac{\left(\cos^2(e+fx)(d\cos(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{1+\frac{m}{2}}\left(1-\frac{a+b\tan(e+fx)}{a+\frac{b^2}{\sqrt{-b^2}}}\right)^{1+}\right.}{}$$

$$= \frac{F_1\left(1+n;\frac{2+m}{2},\frac{2+m}{2};2+n;\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)\cos^2(e+fx)(d\cos(}{bf(}$$

Mathematica [C] time = 22.4844, size = 698, normalized size = 3.73

$$\overline{f\left(2n(b-a\tan(e+fx))F_1\left(n+1;\frac{m}{2}+1,\frac{m}{2}+1;n+2;\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)+2(n-m)\tan(e+fx)(a+b\tan(e\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (2*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b),
 (a + b*Tan[e + f*x])/(a + I*b)]*(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^(1
 + n))/(f*(2*b*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b*Tan[e + f*x]
)/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2 + 2*n*AppellF1[
1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[
e + f*x])/(a + I*b)]*(b - a*Tan[e + f*x]) + (b*(2 + m)*(1 + n)*((a - I*b)*A
ppellF1[2 + n, 1 + m/2, 2 + m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a
+ b*Tan[e + f*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 2 + m/2, 1 + m/2,
3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)])*Sec
[e + f*x]^2*(a + b*Tan[e + f*x]))/((a - I*b)*(a + I*b)*(2 + n)) + 2*(-m + n
)*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b),
(a + b*Tan[e + f*x])/(a + I*b)]*Tan[e + f*x]*(a + b*Tan[e + f*x]) + (m*Appe
llF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b
*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(a + b*Tan[e + f*x]))/(-I + Tan[e
+ f*x]) + (m*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a + b*Tan[e + f*x])/
(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(a + b*Tan[e + f*
x]))/(I + Tan[e + f*x])))
```

Maple [F] time = 0.239, size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^m\left(a+b\tan\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{m}\left(b\tan\left(fx+e\right)+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d\cos\left(fx+e\right)\right)^{m}\left(b\tan\left(fx+e\right)+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(d\cos\left(fx+e\right)\right)^{m}\left(b\tan\left(fx+e\right)+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
            print("both result and optimal complex");
         fi;
         #both result and optimal complex
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      else #result contains complex but optimal is not
         if debug then
            print("result contains complex but optimal is not");
         fi;
         return "C";
      end if
   else # result do not contain complex
        # this assumes optimal do not as well
        if debug then
            print("result do not contain complex, this assumes optimal do
 not as well");
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
           if debug then
              print("leaf_count_result<=2*leaf_count_optimal");
           fi;
           return "A";
        else
           if debug then
              print("leaf_count_result>2*leaf_count_optimal");
           fi;
           return "B";
        end if
   end if
  else #ExpnType(result) > ExpnType(optimal)
     if debug then
        print("ExpnType(result) > ExpnType(optimal)");
     fi;
     return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

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#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
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                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

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def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
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        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
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